The Macrodynamics of Sorting Between Workers and Firms

Jeremy Lise[1]   Jean-Marc Robin[2]

[1]UCL

[2]Sciences Po and UCL

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Questions

- What is the role of worker and job heterogeneity in explaining the macrodynamics of (un)employment?
- How does the business cycle affect sorting, i.e. the joint distribution of workers and tasks?

The method

- We develop a sequential auction model with heterogeneous workers and tasks, and aggregate productivity shocks.
- We study the quantitative implications of the model by fitting to US aggregate labor market data from 1951-2012.

Sequential auctions

(Postel-Vinay & Robin, Ecta 2002)

- Workers have limited bargaining power (say zero).
- But they can search on the job and trigger Bertrand competition between employers.
  - The amount of search frictions allows to move the cursor between the pure monopsony model and the competitive model.
- Whether employed or unemployed workers are always paid the best Remain option.
  - Technically, this considerably simplifies Bellman equations by comparison to the standard Nash bargaining model, which allows to incorporate lots of heterogeneity.
- After poaching workers' payoffs lie inside the bargaining set.
  - The sequential auction framework therefore offers an alternative to Nash bargaining.

Builds on

- Postel-Vinay & Robin (Ecta 2002): two-sided heterogeneity but no sorting
- Robin (Ecta 2011): only worker heterogeneity and aggregate shocks; plus a form of sorting between worker ability and the aggregate shock
- Lise, Meghir & Robin (RED, 2016): exogenous worker heterogeneity, idiosyncratic shocks to firm heterogeneity and sorting
- This paper has exogenous worker heterogeneity, endogenous firm heterogeneity, sorting, and aggregate shocks.
- The sequential auction framework gives the model a recursive structure.

Related Literature

- Models of aggregate shocks with (one sided) heterogeneity
  - Directed search and wage posting: Menzio & Shi (2010a,b, 2011), Kaas & Kircher (2011), Schaal (2016);
  - Random search and wage posting: Moscarini & Postel-Vinay (2011a,b), Coles & Mortensen (2011);
- Cyclical behavior of labor productivity and labor market variables
  - Shimer (2005), Hall (2005), Hagedorn & Manovskii (2008, 2010), Gertler & Trigari (2009), ...
- Sorting between workers and firms (or unemployed and vacancies)
  - Shimer & Smith (2001), Eekhout & Kircher (2011), Bagger & Lentz (2012), Barlevy (2002), Sahin, Song, Topa & Violante (2012), Hagedorn, Law & Manovskii (2012) ...
- There is still very little work with two-sided heterogeneity. Yet there is a lot of interest in understanding the evolution of match quality in recessions and booms.

1. THE MODEL

Time, agents and aggregate shocks

- Time is discrete and indexed by t.
- There is a continuum of workers indexed by type $x \in [0, 1]$, with distribution $\ell(x)$.
- There is a continuum of potential jobs indexed by $y \in [0, 1]$.
- The aggregate state of the economy is z_t.

Distributions of workers and jobs at end of $t-1$

- $h_t(x,y)$ is the distribution of worker-firm matches at the beginning of period t (prior to realization of z_t)
- $u_t(x)$ is the distribution of unemployed workers at the beginning of period t (prior to realization of z_t):

$$u_t(x) = \ell(x) - \int h_t(x,y)\,\mathrm{d}y$$

Timing

- At the beginning of period t, z_t is updated to z' from $z_{t-1} = z$ according to a Markov transition probability $\pi(z, z')$.
- Following the realization of z_t the timing is assumed to be:
  1. Separations occur.
  2. Workers search for a job and firms post vacancies.
  3. Meetings occur.

Following the realization of z_t job separations occur.

Job separations

- Let $P_t(x,y)$ denote the present value an (x,y) match given the aggregate state of the economy at t.
- Let $B_t(x)$ be the value of unemployment to a type-x worker.

- Assuming no fixed investment in job posts, matches are endogenously destroyed iff $P_t(x,y) < B_t(x)$.
- If $P_t(x,y) \geq B_t(x)$, exogenous job destruction occurs with probability δ.
- The layoff rate is thus

$$\underbrace{\mathbf{1}\{P_t(x,y) < B_t(x)\}}_{\text{endogenous}} + \underbrace{\delta \times \mathbf{1}\{P_t(x,y) \geq B_t(x)\}}_{\text{exogenous}}$$

Distributions at $t+$ after job separations

- The distribution of worker-firm matches that survive the destruction shocks is

$$h_{t+}(x,y) = (1-\delta)\,\mathbf{1}\{P_t(x,y) \geq B_t(x)\}\,h_t(x,y)$$

- The distribution of unemployed workers after any job separation is

$$\begin{aligned} u_{t+}(x) &= \ell(x) - \int h_{t+}(x,y)\,\mathrm{d}y \\ &= u_t(x) + \int \Big[\underbrace{\mathbf{1}\{P_t(x,y) < B_t(x)\} + \delta\,\mathbf{1}\{P_t(x,y) \geq B_t(x)\}}_{\text{job separations}}\Big] h_t(x,y)\,\mathrm{d}y \end{aligned}$$

Following the realization of z_t and job separations workers search for a job.

Aggregate search effort

- Workers search both when unemployed and employed.
- Together these workers produce aggregate search effort

$$L_t = \int u_{t+}(x)\,\mathrm{d}x + s \iint h_{t+}(x,y)\,\mathrm{d}x\,\mathrm{d}y$$

where s is the relative effectiveness of search effort by the employed.

Following the realization of z_t and job separations firms post vacancies.

Vacancy creation

- The cost of posting v vacancies is an increasing, convex function $c(v)$.
- Firms of type y choose to post $v_t(y)$ vacancies so as to equate the marginal cost of a recruiting to the marginal return

$$c'[v_t(y)] = q_t J_t(y)$$

  where $J_t(y)$ denotes the value of a vacancy and q_t the probability of a contact per vacancy (derived later).
- The aggregate number of vacancies solves

$$V_t \equiv \int v_t(y)\,\mathrm{d}y = \int (c')^{-1}\left(q_t J_t(y)\right)\,\mathrm{d}y$$

Then workers and vacancies meet.

Meeting rates

- The total measure of meetings between workers and firms at time t is given by
$$M_t = M(L_t, V_t)$$
- The probability an unemployed worker contacts a vacancy is $\lambda_t = M_t/L_t$.
- The probability an employed worker contacts a vacancy is $s\lambda_t$.
- The probability per unit of recruiting intensity $v_t(y)$, that a firm contacts a searching worker is $q_t = M_t/V_t$.

VALUES

The value of unemployment

- The planning horizon for workers and firms is infinite.
- The present value of unemployment is the expected discounted sum of future earnings conditional on being employed in period t and given z_t and distributions h_{t+}
- In period t, home production is $b(x, z_t)$.
- In period $t + 1$,
  - unemployed workers expect to receive offers with probability λ_t.
  - Firms make take it or leave it offers to unemployed workers.

The value of unemployment

- Hence, whether or not unemployed workers receive an offer, the continuation value is their reservation value $B_{t+1}(x)$.
- Workers (and firms) are risk neutral and discount the future at rate r.

$$
\begin{aligned}
B_t(x) &= b(x, z_t) \\
&+ \frac{1}{1+r}\mathbb{E}_t\left[(1-\lambda_{t+1})\, B_{t+1}(x) + \lambda_{t+1}\int B_{t+1}(x)\frac{v_{t+1}(y)}{V_{t+1}}\,\mathrm{d}y\right] \\
&= b(x, z_t) + \frac{1}{1+r}\mathbb{E}_t B_{t+1}(x)
\end{aligned}
$$

The value of unemployment

Therefore $B_t(x) = B(x, z_t)$ with

$$B(x,z) = b(x,z) + \frac{1}{1+r}\int B(x,z')\,\pi(z,z')\,\mathrm{d}z'$$

This is a simple linear equation.

The value of a match

- The present value of a match (x, y) at t, $P_t(x, y)$, is the expected discounted sum of worker and employer future earnings.
- In period t, the output of a match (x, y) is $p(x, y, z_t)$.
- In period $t + 1$,
  - The employee meets a firm of type y' with probability $s\lambda_{t+1} v_{t+1}(y')/V_{t+1}$.
  - Firms engage in Bertrand competition.
    - The worker moves to firm y' if $P_{t+1}(x, y') > P_{t+1}(x, y)$ and s/he pockets $P_{t+1}(x, y)$.
    - The worker stays if $P_{t+1}(x, y') \leq P_{t+1}(x, y)$ and the match continues with value $P_{t+1}(x, y)$.

The value of a match

- Hence the continuation values is either unemployment $B_{t+1}(x)$ or the current match value $P_{t+1}(x,y)$ whether the worker moves or stays.

$$P_t(x,y) = p(x,y,z_t) + \frac{1}{1+r}\mathbb{E}_t\Big[\underbrace{(1-\delta)\mathbf{1}\left\{P_{t+1}(x,y) \geq B_{t+1}(x)\right\}}_{\text{no layoff}} P_{t+1}(x,y)$$

$$+\underbrace{\left[\mathbf{1}\{P_{t+1}(x,y) < B_{t+1}(x)\} + \delta\,\mathbf{1}\{P_{t+1}(x,y) \geq B_{t+1}(x)\}\right]}_{\text{layoff}} B_{t+1}(x)\Big].$$

- The continuation value does not depend on distribution $h_{t+1}(x,y)$.

The surplus of a match

- Define match surplus as $S_t(x, y) = P_t(x, y) - B_t(x, y)$.
- There is a solution $S_t(x, y) = S(x, y, z_t)$ such that

$$S(x, y, z) = s(x, y, z) + \frac{1-\delta}{1+r} \int S(x, y, z')^{+} \, \pi(z, z') \, \mathrm{d}z'$$

where $s(x, y, z) = p(x, y, z) - b(x, z)$ and we denote $x^{+} = \max\{x, 0\}$.

Expected firm profit on a new match

Given that a the firm meets a searching worker, the expected firm profit depends on whether the contacted worker is employed or unemployed:

$$
\begin{aligned}
J_t(y) &= \int \frac{u_{t+}(x)}{L_t}[P_t(x,y) - B_t(x)]^+ \, \mathrm{d}x \\
&\qquad + \iint \frac{sh_{t+}(x,y')}{L_t}[P_t(x,y) - P_t(x,y')]^+ \, \mathrm{d}x \, \mathrm{d}y' \\
&= \int \frac{u_{t+}(x)}{L_t} S_t(x,y)^+ \, \mathrm{d}x \\
&\qquad + \iint \frac{sh_{t+}(x,y')}{L_t}[S_t(x,y) - S_t(x,y')]^+ \, \mathrm{d}x \, \mathrm{d}y'
\end{aligned}
$$

Law of motion for updating worker distributions

- At the end of the period we have the distribution of jobs

$$h_{t+1}(x,y) = h_{t+}(x,y)\left[1 - \underbrace{\int s\lambda_t \frac{v_t(y')}{V_t}\mathbf{1}\{S_t(x,y') > S_t(x,y)\}\,\mathrm{d}y'}_{\text{exit because of poaching}}\right]$$
$$+ \underbrace{\int h_{t+}(x,y')s\lambda_t \frac{v_t(y)}{V_t}\mathbf{1}\{S_t(x,y) > S_t(x,y')\}\,\mathrm{d}y'}_{\text{entry by poaching}}$$
$$+ \underbrace{u_{t+}(x)\lambda_t \frac{v_t(y)}{V_t}\mathbf{1}\{S_t(x,y) \geq 0\}}_{\text{entry from unemployment}}$$

- And unemployment

$$u_{t+1}(x) = u_{t+}(x)\left[1 - \int \lambda_t \frac{v_t(y)}{V_t}\mathbf{1}\{S_t(x,y) \geq 0\}\,\mathrm{d}y\right]$$

Computation of the stochastic search equilibrium

1. Once and for all, solve for the fixed point in $S(x, y, z)$ independently of the actual realization of aggregate productivity shocks.
2. Then recursive: Given an initial distribution of workers across jobs, $h_0(x, y)$, and a realized sequence of aggregate productivity shocks $\{z_0, z_1, ..., z_T\}$ we can solve for the sequence of distributions of unemployed worker types, worker-firm matches, and vacancies $\{v_t(y), h_{t+1}(x, y)\}_{t=0}^{T}$.

2. ESTIMATION

A parametric specification

- Meeting function

$$M_t = M(L_t, V_t) = \min\{\alpha\sqrt{L_t V_t}, L_t, V_t\}, \quad \alpha > 0$$

- Vacancy costs

$$c(v) = \tfrac{c_0}{1+c_1} v^{1+c_1}, \quad c_0 > 0, \quad c_1 > 0$$

- Value added

$$p(x, y, z) = z\left(p_1 + p_2 x + p_3 y + p_4 x^2 + p_5 y^2 + p_6 xy\right)$$

- Home production

$$b(x) = 0.7 \times p(x, y^*(x, 1), 1) \qquad y^*(x, 1) = \arg\max_y S(x, y, 1)$$

- Worker type distribution

$$x \sim \text{Beta}(\beta_1, \beta_2)$$

- Aggregate shocks

$$\ln z_t = \rho \ln z_{t-1} + \sigma\sqrt{1-\rho^2}\varepsilon_t, \quad \varepsilon_t \sim \mathcal{N}(0, 1)$$

Estimation

- We HP filter the log transformed data (1951-2012).
- We calculate means, volatilities (standard deviations) and correlations.
- We estimate the model parameters by method of simulated moments.
- The model is solved at a weekly frequency and the simulated data is then aggregated (exactly as the BLS data) to form quarterly moments.

Identification

- α, s, and δ (mobility) are identified from transition rates between unemployment and employment, between jobs, and from employment to unemployment.
- σ and ρ (process for z) are identified from aggregate output (GDP).
- c (vacancy cost) is identified from vacancies.
- β (worker heterogeneity) is identified from unemployment duration patterns (number of workers unemployed 5, 15 and 27 or more weeks).
- p (match value added) is identified from the cross-sectional dispersion in value added per job across firms (from Bloom et al., 2014).

MODEL FIT

Moments

Right amplification of aggregate shocks.

Fitted Moments	Data	Model	Fitted Moments	Data	Model
sd[GDP]	0.033	0.034	sd[UE]	0.127	0.127
	(0.003)			(0.011)	
sd[U]	0.191	0.203	sd[EU]	0.100	0.095
	(0.018)			(0.011)	
sd[U^{5p}]	0.281	0.315	sd[EE]	0.095	0.112
	(0.027)			(0.005)	
sd[U^{15p}]	0.395	0.413	sd[V/U]	0.381	0.306
	(0.038)			(0.029)	
sd[U^{27p}]	0.478	0.439	sd[V]	0.206	0.105
	(0.045)			(0.015)	
			sd[sd labor prod]	0.039	0.038
				(0.005)	

Note: Newey-West standard errors in brackets.

Moments

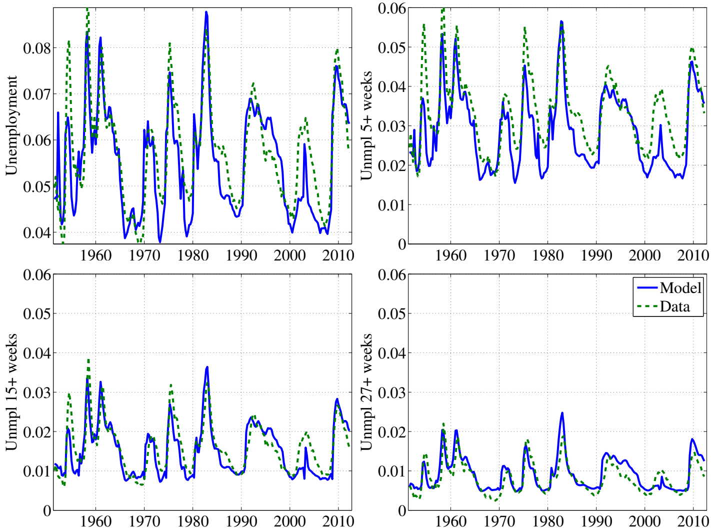
Right signs for correlations

Fitted Moments	Data	Model	Fitted Moments	Data	Model
autocorr$[GDP]$	0.932	0.991	corr$[UE, GDP]$	0.878	0.978
	(0.132)			(0.122)	
corr$[U, GDP]$	–0.860	–0.983	corr$[EU, GDP]$	–0.716	–0.910
	(0.124)			(0.133)	
corr$[V, GDP]$	0.721	0.996	corr$[UE, EE]$	0.695	0.977
	(0.149)			(0.108)	
corr$[V, U]$	–0.846	–0.975	corr[sd labor prod, GDP]	–0.366	–0.365
	(0.119)			(0.260)	

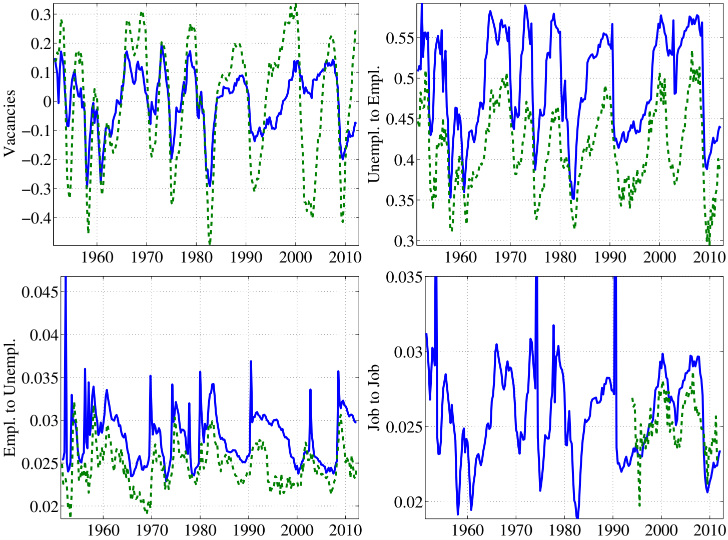
Note: Newey-West standard errors in brackets.

Unemployment prediction given filtered z_t

- We first filter out z_t so as to exactly fit GDP (depends on h_{t+}).
- Then we predict the other variables (h_{t+1} in particular).



Vacancies and mobility prediction given filtered z_t



PARAMETER ESTIMATES

Estimated parameters

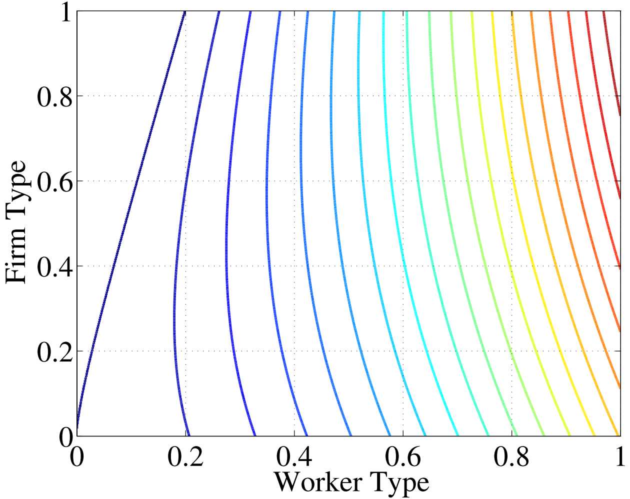
Parameters precisely estimated

Matching	α	0.497	Worker heterogeneity	β_1	2.148
$M = \min\{\alpha\sqrt{LV}, L, V\}$		(0.083)	Beta(β_1, β_2)		(0.192)
Search intensity	s	0.027		β_2	12.001
		(0.007)			(1.951)
Vacancy posting costs	c_0	0.028	Value added	p_1	0.003
$c[v(y)] = \frac{c_0}{1+c_1} v(y)^{1+c_1}$		(0.014)	$p(x,y,z) =$		(0.006)
	$1+c_1$	1.084	$z(p_1 + p_2 x + p_3 y$	p_2	2.053
		(0.040)	$+p_4 x^2 + p_5 y^2 + p_6 xy)$		(0.684)
Exogenous separation	δ	0.013		p_3	-0.140
		(0.001)			(0.504)
Productivity shocks	σ	0.071		p_4	8.035
Gaussian copula (σ,ρ)		(0.009)			(5.422)
	ρ	0.999		p_5	–1.907
		(0.001)			(0.355)
				p_6	6.596
					(0.835)

Note: r is fixed at 0.05 annually.

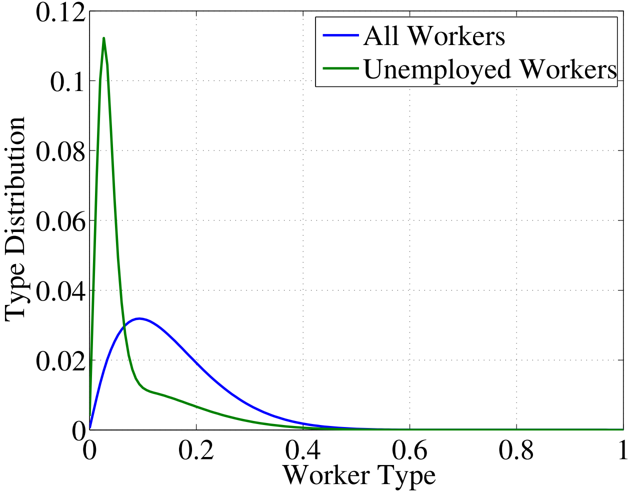
Production function

Varies more across workers than firms



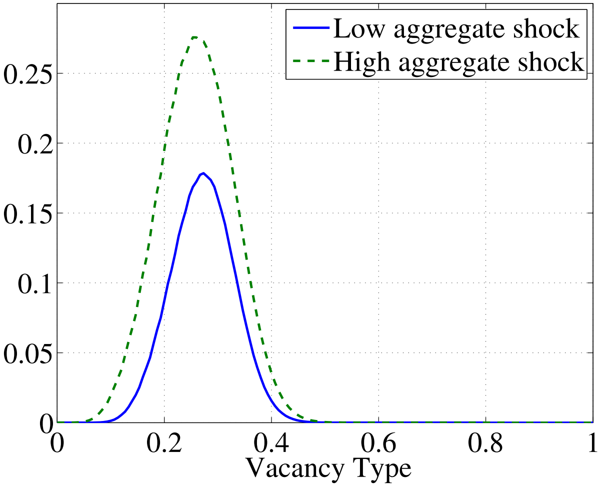
Worker ability distributions

Unemployed are mostly low ability workers.



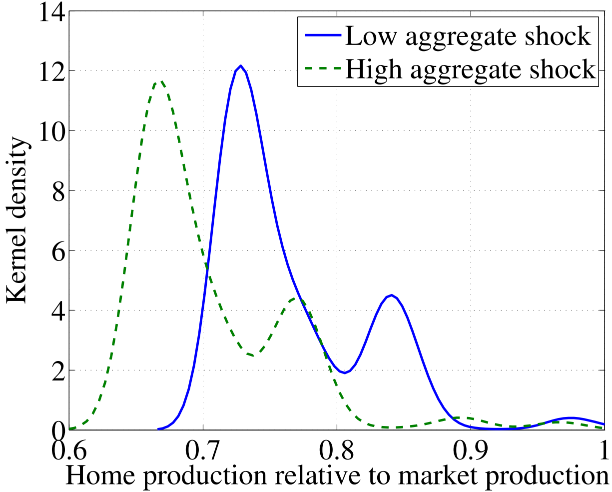
Equilibrium vacancy creation $v(y)$

More vacancies are created in booms. No lateral shift.



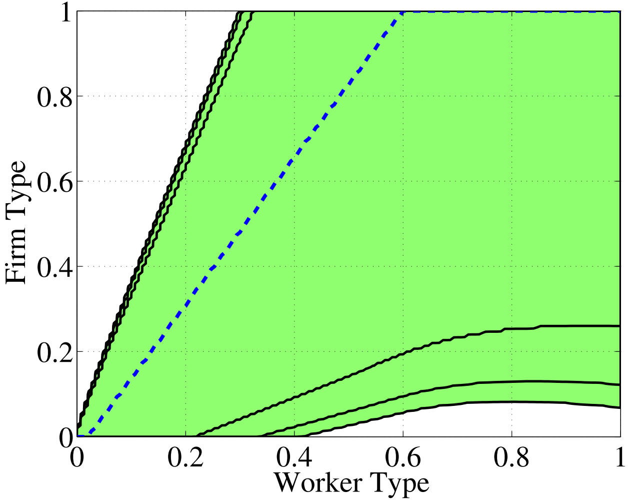
Relative home-to-market productivity $b(x)/p(x,y,z)$

This is not a small surplus economy ($b/p \ll 1$)



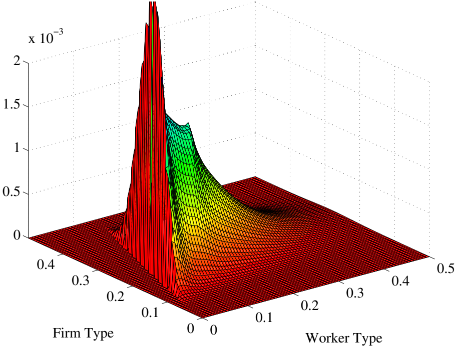
Feasible matches

In booms, there is more mismatch. In recessions, shrinks toward optimal matches.

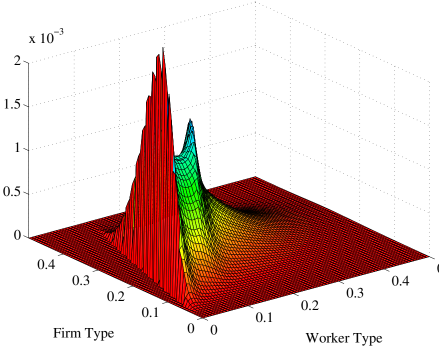


Distribution of matches

Once employed they move more quickly to better matches in booms than in recessions.



z at the 1st decile



z at 9th decile

CONCLUSION

Summary

- We develop a sequential auction model with heterogeneous workers and tasks, and aggregate productivity shocks.
- The model fits the US time-series data 1951-2012 and exactly propagates the technology shock to unemployment rates.
- In booms, workers initially accept worse matches on average than in recessions. Once employed they move more quickly to better matches in booms than in recessions.

What about wages?

- There is a simple way of maintaining the recursive structure of the model and of tracking wage distributions at the same time.
- Simply assume that wage contracts are state-contingent and employers commit to a fixed surplus sharing until the next poaching event:

$$W_t(\sigma, x, y) = B_t(x) + \sigma S_t(x, y)$$

Wages

- Solving for wages, we obtain

$$w_t(\sigma, x, y) = \sigma p(x, y, z_t) + (1-\sigma) b(x, z_t) - \Delta$$

- Δ is a discount for future renegotiation opportunities:

$$\Delta = \frac{1-\delta}{1+r}\mathbb{E}_t\left[\mathbf{1}\left\{S_{t+1}(x,y) \geq 0\right\} s\lambda_{t+1} \int I_{t+1}(\sigma, x, y, y')\frac{v_{t+1}(y')}{V_{t+1}}\,\mathrm{d}y'\right]$$

where

$$I_{t+1}(\sigma, x, y, y') = \begin{cases} (1-\sigma)S_{t+1}(x,y) & \text{if j2j mobility} \\ S_{t+1}(x,y') - \sigma S_{t+1}(x,y) & \text{if counteroffer} \\ 0 & \text{if status quo} \end{cases}$$

THANK YOU!