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Impact of Portfolio Behavior on the Macroeconomy

New Micro Finance literature has data on portfolio behavior

- Finds that many households don't invest as our model say.
- Micro behavior in models too sophisticated relative to data.
- Modeling miscue bad aesthetically, but may be interesting.
- Actual behavior prevents financial markets limiting risk.

Distill behavior down to 3 facts which can help us understand:

1. Household Consumption Behavior
2. The Distribution of Wealth
3. Asset Prices

A. Non-particiaption: Many don't use available assets

- Many hold little or no stocks - over 50% in US hold none - despite equity premium.
- Participation strongly increasing in wealth, but still limited - 10% of wealthiest households hold no equity.
- Many who hold equities only do so in a small way - under-participation.

(See Guiso/Sodini 2012)

B. Inertia: Many make only very infrequent adjustments

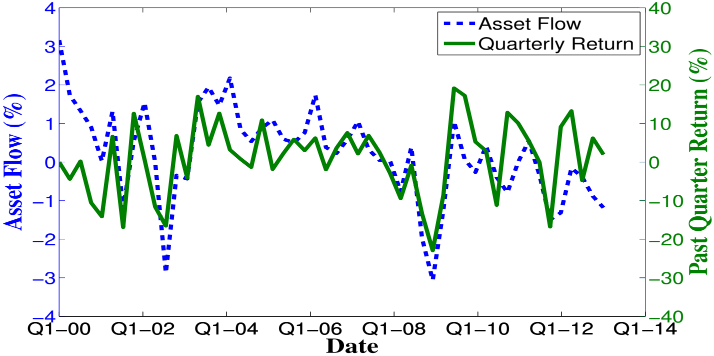
- In TIAA-CREF panel 44% made no change to flow/allocation over ten years (Ameriks/Zeldes 2004).
- Survey of US households owning equities in 2008, 57% conducted no trades (ICI survey).
- Italian survey of brokerage investors found 45% had one trade or less per year (Alvarez/Guiso/Lippi 2011).
- Inertia main driver of asset allocation (Brunnermeier/Nagel 2008)

C. Mistiming: Many adjust based on past returns

- equity mutual fund investments are pro-cyclical while returns are counter-cyclical, so miss-time the market.
- mistiming holds for individual funds (Morningstar).
- during Great Recession big outflow from equity to bond mutual funds right around trough.

Trading Behavior of Equity Mutual Funds

$Infl_t = A_t - A_{t-1}(1 + r_t)$: Returns & net inflows correlated (0.50).



mutual fund investors mistime the market losing **2%** per year. But these are reallocations so someone is gaining here too.

Observed Portfolio Behavior Very Different

Micro behavior very different from our models.

Households should buy equities because of equity premium.

A. But many don't, **Non-participation**.

Equity premium is very volatile and households should respond.

B. But many don't respond at all, **Inertia**.
C. Many respond the wrong way, **Mistiming**.

Evaluate whether this behavior is important by largely imposing it.

Consumption and Asset Markets ?

HH consumption is volatile and highly correlated with income

- Consumption behavior suggests asset markets are incomplete

Puzzle because asset market look pretty complete

- Very large number of different stocks and bonds
- Also more exotic securities and low cost entry

Can portfolio behavior explain this? If don't use assets properly exposed to a lot of risk.

Wealth Distribution ?

The distribution of wealth is substantially more skewed

than the distribution of income.

(See Budria/Diaz-Gimenez/Quadrini/Rios Rull 2002).

Can portfolio behavior help explain this? Big differences in returns could lead to big differences in wealth.

- "Sophisticated investors" invest more in equities and earn higher returns (Calvert/Campbell/Sodini 2007)
- Equity market participation increases with wealth. (Guiso/Sodini 2012)

Price of risk is high. Risk-free rate is low.

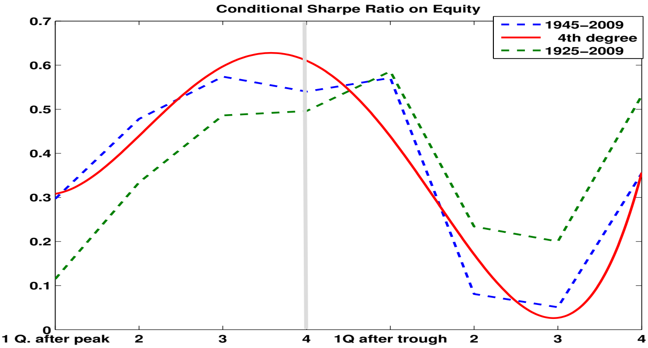
- Mehra/Prescott (1985): problem for representative CRRA consumer because aggregate consumption is too smooth

Pricing of risk is very counter-cyclical. Risk-free is very stable.

- Equity premium as measured by excess returns, dividend yields, Sharpe ratios are all very counter-cyclical; Lettau/Ludvigson (2010)

Conditional Sharpe Ratio $= E\left\{R - R_f\right\} / \sigma\{R - R_f\}$

4-quarter holding period equity returns using NBER dating



Our Segmentation Mechanism

If many households are saving via low return/low risk portfolios

- low return means low wealth accumulation.
- ability to smooth low, so risk exposer high

If some households save via high return/high risk portfolios

- leads to higher and more cyclically volatile wealth.
- can smooth well, but aggregate risk exposure high

Small number of people exposed to a lot of aggregate risk clearing markets can lead to better asset prices.

Cyclical variation in their wealth can lead to cyclical risk pricing.

Prior Literature

Segmented markets has long history.

- E.g. Heaton and Lucas (1996) and Guvenen (2009) - 2 representative agents incomplete markets models.
- E.g. Gomes and Michaelides (2007) have related work that stresses differences in risk aversion and IES. Also Dumas/Lyasoff (2012)

One new thing is we are using trading behavior. So we can have

- Rich financial markets
- Different attitude towards aggregate risk
- Market clearing group smaller than all stockholders.

Hoping for more action than w. endogenous incompleteness: Kehoe/Levine (1993), and Alvarez/Jermann (2000).

Give Progress Report

This research joint work with Yili Chien and Hanno Lustig

A. Review of Economic Studies (2011)

- impose portfolio fact **A Non-participation**
- allows for different portfolio restrictions
- Find model's results closer to data but volatility failure

B. AER (2012)

- impose portfolio facts **A** and **B Inertia**
- Greatly increases risk pricing volatility

C. New paper

- imposes fact **A** and rationalize fact **C Mistiming**
- expands method

New Method

- develop **multiplier method** for segmented asset markets
- utilize **recursive multiplier** as a state variable
  - building on Basak/Cuoco (1998), Marcet/Marimon (1999)
- use **measurability restrictions** to get portfolio restrictions
  - building on Aiyagari/Marcet/Sargent/Seppala et al. (2002) and Lustig/Sleet/Yeltekin (2002)
- construct **analytic** consumption sharing rule and SDF
  - extends Chien/Lustig (2006) complete markets result
- leads to simple **quantitative** method
  - works like Krusell/Smith (1997)

Method Yields

Perturbed version of Breeden-Lucas stochastic discount factor

$$m_{t+1} \equiv \beta \left(\frac{C_{t+1}}{C_t} \right)^{-\alpha} \left(\frac{h_{t+1}}{h_t} \right)^{+\alpha}.$$

- standard part from a representative CRRA agent
- this is the new part

How are we going to get this?

- Use multiplier ζ as state variable
- Derive aggregation result - h moment of multiplier distribution
- Equilibrium is fixed point $F\left[h_{t+1}/h_t\right] = \left[h_{t+1}/h_t\right]$.
- Compute via simple iterative method.

Perturbed version of Breeden-Lucas stochastic discount factor

$$m_{t+1} \equiv \beta \left(\frac{C_{t+1}}{C_t} \right)^{-\alpha} \left(\frac{h_{t+1}}{h_t} \right)^{+\alpha}.$$

Need h_{t+1} / h_t to exhibit the right volatility.

Key ingredients:

- idiosyncratic and aggregate risk
- net wealth bounds
- portfolio restrictions

Road Map

1. Describe Physical Economy
2. Complete Markets Equilibrium; $h_t = h_{t+1}$ so boring.
3. Add frictions so h_{t+1}/h_t, prices and behavior interesting.
4. Allow us to add portfolio fact A Non-participation
5. Get results: some successes + but 1 failure
6. Go next to portfolio facts B Inertia and C Mistiming.

Physical Economy with Macro and Micro Risk

- Aggregate output $Y_t = \exp(z_t) Y_{t-1}$ comes in two forms
  - tree 1: *tradeable output* $(1-\gamma) Y_t$ depends on z^t
  - tree 2: *non-tradeable output* $\gamma Y_t \eta_t$ depends on η_t too.
- Idiosyncratic shocks
  - η are i.i.d. across households and $E\{\eta_t | z^t\} = 1$
- Aggregate history is z^t and individual history is (z^t, η^t)
  - $\pi(z^t, \eta^t)$ is probability of observing z^t and η^t
- Continuum of ex ante identical households with preferences

$$E_0 \left\{ \sum_{t \geq 1}^{\infty} \beta^t \frac{c_t^{1-\alpha}}{1-\alpha} \right\}$$

With standard Arrow-Debreu economy, individual i chooses consumption sequence $c_t(z^t, \eta^t)$ to

$$\max_{\{c_t(z^t,\eta^t)\}} E_0 \left\{ \sum_t \beta^t c_t(z^t, \eta^t)^{1-\alpha} / (1-\alpha) \right\}$$

$$\text{s.t. } E_0 \sum_t \left\{ \gamma Y(z^t)\eta_t - c_t(z^t, \eta^t) \right\} P(z^t) + \omega_0 \geq 0.$$

- ω_0 is the price of a claim to tradeable output $(1-\gamma) Y_t(z^t)$.
- $\gamma Y(z^t)\eta_t$ is risky nontraded ("labor") income
- $P(z^t)$ is the state price, and $P(z^t)\pi(z^t, \eta^t)$ is the present-value price

Consumption F.O.C.

First-order conditions for consumption take the form:

$$\beta^t c_t(z^t, \eta^t)^{-\alpha} = \zeta_i P(z^t).$$

where ζ_i is the multiplier on his present value budget constraint.

Denote his consumption function by $c_t(\zeta_i, P(z^t))$ where

$$c_t(\zeta_i, P(z^t)) = \left(\zeta_i P(z^t)/\beta^t\right)^{-1/\alpha}$$

ζ_i is constant over time here and makes a very good state variable.

We don't really need $P(z^t)$, since

$$C\left(z^t\right) = \sum_i c_t(\zeta_i, P(z^t))\mu_i,$$

$$\frac{c_t(\zeta_i, P(z^t))}{\sum_i c_t(\zeta_i, P(z^t))\mu_i} = \frac{\left(\zeta_i P(z^t)/\beta^t\right)^{-1/\alpha}}{\sum_i \left(\zeta_i P(z^t)/\beta^t\right)^{-1/\alpha} \mu_i}$$

Which simplifies to

$$c(\zeta, z^t) = \left(\frac{\zeta_i^{-1/\alpha}}{\sum_i \zeta_i^{-1/\alpha}\mu_i}\right) C\left(z^t\right).$$

- $h \equiv \sum_i \zeta_i^{-1/\alpha}\mu_i$ is the key moment where α is risk aversion.
- $C\left(z^t\right) = Y\left(z^t\right)$

Don't need prices to determine discount rates $P(z^{t+1})/P(z^t)$ since

$$\beta^t c(\zeta_i, z^t)^{-\alpha} = \zeta_i P(z^t)$$

and tomorrow's f.o.c. implies that

$$\frac{P(z^{t+1})}{P(z^t)} = \frac{\beta^{t+1} c(\zeta_i, z^{t+1})^{-\alpha}/\zeta_i}{\beta^t c(\zeta_i, z^t)^{-\alpha}/\zeta_i}.$$

If we replace $c_t(\zeta, z^t)$ using our consumption functions, we get

$$m_{t+1} = \frac{P(z^{t+1})}{P(z^t)} = \beta \left(\frac{C_{t+1}}{C_t}\right)^{-\alpha} \left(\frac{h}{h}\right)^{\alpha},$$

but this last term will cancel out with complete markets.

Now for Something More Interesting

With Complete Markets results aren't exciting because can share all risks efficiently and therefore h constant.

To make more interesting:

1. Define net savings function and use it to
2. Add net financial wealth bounds
3. Construct stock and bond from $(1-\gamma)Y$ w/ fixed leverage.
4. Use net savings again to add limited asset use

Use net savings function to impose these restrictions on allocations and stay within the Arrow-Debreu framework.

Remember that "Arrow" = "Arrow-Debreu",

so households position in Arrow bonds at (z^t, η^t), $a(z^t, \eta^t)$,

must be consistent with their consumption plan, or

$$E_t \left[\sum_{\tau \geq t} P(z^\tau) \left(\gamma Y(z^\tau) \eta_\tau - c(\zeta, z^\tau) \right) \right] \pi(z^t, \eta^t)$$

$$+ a(z^t, \eta^t) P(z^t) \pi(z^t, \eta^t) \geq 0.$$

Hence, any floor on how low $a(z^t, \eta^t)$ can be is also a ceiling on $[\cdot]$.

Similarly any restriction portfolio restriction on how savings can go from $(z^{t-1}, \eta^{t-1}) \to (z^t, \eta^t)$ will also limit $[\cdot]$.

So define the present-value of net savings from state (z^t, η^t) as

$$S(\zeta, z^t, \eta_t) = E_t \left[\sum_{\tau \geq t} P(z^\tau) \left(\gamma Y(z^\tau) \eta_\tau - c(\zeta, z^\tau) \right) \right] \pi(z^t, \eta^t).$$

Now we can focus on allocations since

$$S(\zeta, z^t, \eta_t) + a(z^t, \eta^t) \pi(z^t, \eta^t) P(z^t) = 0$$

where $a(z^t, \eta^t)$ is the beginning of period net financial wealth.

With net wealth bounds, we cannot have ζ constant since

$$a_t(z^t,\eta^t)\pi(z^t,\eta^t)P(z^t) \geq D\left(z^t\right),$$

implies that

$$S(\zeta,z^t,\eta_t) \leq D\left(z^t\right).$$

So, we need to allow ζ to vary to satisfy these constraints

$$S(\zeta_t\left(z^t,\eta^t\right),z^t,\eta_t) \leq D\left(z^t\right)$$

and

$$\zeta_t = \zeta_{t-1} - \varphi_t,$$

where φ_t is the multiplier on the bound.

(Note can still short assets even if $D\left(z^t\right) = 0$.)

4. Heterogeneous Trading Technologies

Traded Assets include Arrow bonds, stocks and risk-free bonds.

Have 2 classes and 3 types of Traders:

- active traders who manage their portfolio
  1. **aggregate-complete market traders** (z):
     - trade claims only on z_{t+1} realizations
- passive traders who have fixed portfolios
  2. **diversified traders** (div):
     - hold the market in stocks and bonds
  3. **non-participants** (np):
     - only a risk-free bond with return $R_t^f(z^{t-1})$

Types ranked here from best to worst. Non-participants hits fact A.

For passive traders with fixed portfolio shares, need

$$saving(z^{t-1}, \eta^{t-1}) R^p(z^t) = a(z^t, \eta^t),$$

where $R^p(z^t)$ is the return on their portfolio between z^{t-1} and z_t.

This implies that a simple restriction on $a_t(z^t, \eta^t)$. Rewrite as

$$\begin{aligned} \frac{S(\zeta(z^t, \eta^t), z^t, \eta_t)}{R^p(z^t)} &= \frac{S(\zeta_t(\tilde{z}^t, \tilde{\eta}_t), \tilde{z}^t, \tilde{\eta}^t)}{R^p(\tilde{z}^t)} \\ \text{if } z^{t-1} &= \tilde{z}^{t-1} \text{ and } \eta^{t-1} = \tilde{\eta}^{t-1} \end{aligned}$$

So, need to allow ζ to vary to satisfy these constraints too and

$$\zeta_t = \zeta_{t-1} + \upsilon_t - \varphi_t,$$

where υ_t is portfolio multiplier and φ_t is bound multiplier. lag

Recursive multiplier adjusts according to

$$\zeta_t = \zeta_{t-1} + v_t - \varphi_t,$$

and consumption for type i is given by

$$c_i(z^t, \eta^t) = \frac{\zeta_i(z^t, \eta^t)^{-1/\alpha}}{h(z^t)} C(z^t),$$

where h is the cross-sectional moment

$$h(z^t) = \sum_i \left(\sum_{\eta^t} \zeta_i(z^t, \eta^t)^{-1/\alpha} \pi(\eta^t) \right) \mu_i.$$

Our SDF is

$$m_{t+1} \equiv \beta \left(\frac{C_{t+1}}{C_t} \right)^{-\alpha} \left(\frac{h_{t+1}}{h_t} \right)^{+\alpha}.$$

Around the Next Corner

Baseline Model

- Impose portfolio fact **A** with non-participants (bonds only)
- Calibrate and compute outcomes
- Compare to the data and determine successes/failures

Baseline Calibration

- period is a year and (discount rate) $\beta = .95$
- Preferences: CRRA with $\alpha = 5$
- Endowments: $z_t \in \{z_h, z_l\}$ and $\eta_t \in \{\eta_h, \eta_l\}$
  - aggregate consumption growth: iid version of Merha-Prescott
  - Idiosyncratic risk calibrated to Storesletten/Telmer/Yaron (2004), but no concentration of idio. risk in recessions
  - calibrated to focus on internal propagation
- choose γ to match collateralizable wealth-to-income ratio
- Types: 10% active, 40% passive diversified, 50% bond-only

Baseline Results - Risk-free Rate

	Data	Base Model	RA Model
R_f	**1.05**	1.93	13.0
$\sigma(R_f)$	**1.56**	0.06	0

Baseline Model (Base) doing very well on the risk free rate, especially compared to standard representative agent model (RA).

Baseline Results - Equity Premium

	Data	Base Model	RA Model
$E\left[R_{lc} - R_f\right]$	7.53	5.78	3.08
$\frac{E[R_{lc}-R_f]}{\sigma(R_{lc}-R_f)}$	**0.44**	0.38	0.19
$\frac{\sigma(m)}{E(m)}$	-	0.41	0.19

Doing much better on the leveraged claim too, but results sensitive to nature of claim. So focus on market price of risk (MPR).

If correlation m+dividends $= 1$, then MPR $=$ Sharpe Ratio.

Consumption is volatile and correlated with income.

Extent depends on asset trading technology:

- Consumption of traders with *worst asset trading technology* is subject to more risk but little aggregate risk.
- Consumption of traders with *better asset trading technologies* is subject to less risk, but more aggregate risk.

Consistent with Malloy/Moskowitz/Vissing-Jorgensen (2007) findings on consumption risk:

- stockholders = low risk but high aggregate risk
- nonstockholders = high risk but low aggregate risk.

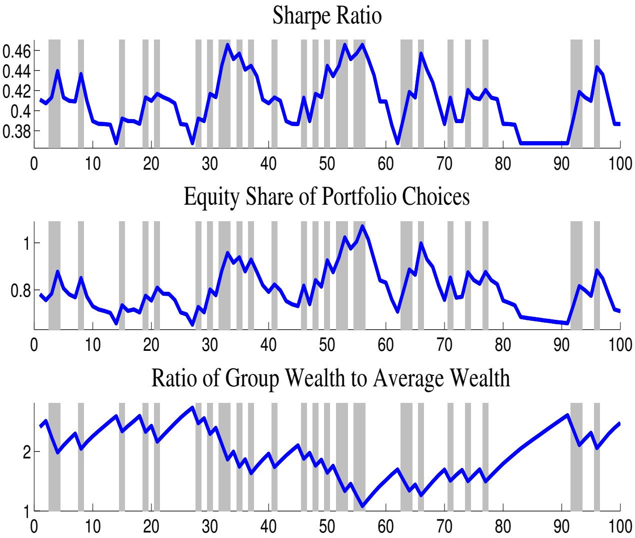
Baseline Results - Portfolio+Wealth

Active Trader's Equity Exposure and Relative Wealth by Group

	mean	standard deviation
Active Traders Avg.		
Equity Share ω_z	0.80	0.11
Group Wealth Ratio		
Active W_z / W	2.15	0.57
Nonpart. W_{np} / W	0.84	0.10

- Active trader's high equity investment leads to high return, high wealth and high return+wealth volatility.
- Non-participants have reverse - low return, low wealth and low return+wealth volatility.

Figure: Baseline Case



Baseline Results - Volatility

	Data	Base Model	RA Model
$\text{Std}\left[\frac{E[R_{lc}-R_f]}{\sigma(R_{lc}-R_f)}\right]$	**50**	-	-
$\text{Std}\left[\frac{\sigma(m)}{E(m)}\right]$	-	2.78	0.0

The MPR is counter-cyclical in the data.

Data estimate of conditional volatility Lettau/Ludvigson (2010).

(Annual version of Campbell/Cochrane gets 21%.)

Our volatility is way too low. Focus on increasing this.

Diversified traders in benchmark rebalance portfolio every period.

- every period they trade to restore position
- means they buy in bad times and sell in good.
- reduces impact of active traders' wealth variation on prices

Paper 2: Adds Inertia

- Targets portfolio fact **B** - very little trading or adjusting.
- Changed diversified traders to intermittent rebalancers

Intermittent rebalancers

- spend out of bond fund
- let equity grow with its return (reinvesting dividend)
- rebalancing every 3 periods, restoring debt/equity to target.

How this changes their portfolio behavior:

- if equity returns high, value of their equities grows rapidly
- as a result equity share of their portfolio fluctuates.

Still passive traders since not managing their portfolio

Paper 2: Adds Inertia

Enhanced Segmented Markets Mechanism

- Intermittant rebalances run up their equity/debt ratio in good times and down in bad.
- create less aggregate risk in good times and more in bad.
- Force the amount of aggregate risk being absorbed by active traders to be more counter-cyclical.
- Found increase in volatility of risk pricing to 25% (with true MP calibration)

Huge improvement, but still a big gap with the data.

New Paper Adds Miss-Timing

Paper 3 targets portfolio fact **C**

- many who do adjust their portfolio mistime the market.
- tricky: since adjusting portfolio natural to think of as active
- resolution: rationalize their trading with different beliefs

However we first need to extend our method.

Previously, all households had same CRRA preferences, discount rates and beliefs.

Now agent of type i has preferences

$$\sum_{t\geq 1,(z^t,\eta^t)}^{\infty} (\beta_i)^t u^i(c_t)\tilde{\pi}^i(z^t,\eta^t),$$

- $u^i(c_t)$ is strictly concave
- own discount rate β_i
- $\tilde{\pi}^i(z^t,\eta^t)$ probability agent i assigns to (z^t,η^t).

How did we do this? Magic - see new paper! supplement

Quantitative Experiments

Compare baseline economy to one where 1/2 active traders have

1. **More volatile beliefs**
2. Less Patient
3. Less Risk Averse

Other types: 40% passive diversified, 50% bond-only

All types survive in long run because

borrowing constraint + ido risk = precautionary savings

and low risk-free rate pushes downward on wealth.

Volatile beliefs: trader form their belief $\tilde{\pi}(z^t, \eta^t)$

- with probability κ on the ergodic transition $\pi(z_{t+1}|z_t)$ and
- with probability $1-\kappa$ by the observed transition frequencies during the past 4 periods.

Consistent with forecasting in a nonstationary world

- Structural break tests without structure have no power.
- Bayesian who thinks that the transition matrix might have changed a fixed number of periods ago.
- Similar strategies are followed by many forecasting models which truncate the data or overweight recent observations.

Literature Review under Construction

A few cites (with more to add) are:

1. Delong, Shleifer, Summers and Waldman (1990, 1991) consider the stability impact of positive feedback traders.
2. Sandroni (2000) and Blume and Easley (2006) examined market selection for rational expectations.
3. Able (2002) considers the impact of pessimism on the risk-free rate.
4. Bhamra and Uppal (2010) consider a two-agent continuous time model with differences in risk aversion and beliefs.
5. Cogley and Sargent (2012) consider diverse beliefs with Bayesian learning.
6. Cvitanic, Jouini, Malamud and Napp (2011) have heterogeneous agents with single endowment good.

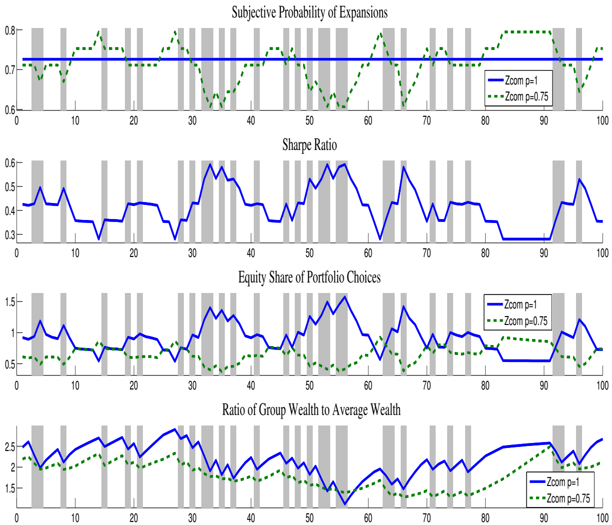
Baseline Model vs. Volatile Beliefs

(weight κ on ergodic, **St**=standard Vol=volatile-belief active trader)

	Base Model	$\kappa = .75$	
			for asset prices
$\frac{\sigma(m)}{E(m)}$	0.41	0.42	average MPR about the same
$Std\left\{\frac{\sigma(m)}{E(m)}\right\}$	2.78	8.52	but 3 times more volatile
			average equity shares
St: $E(\omega_z)$	0.80	**0.90**	goes up
Vol: $E(\omega_{\tilde{z}})$	-	0.66	because volatile lower
			equity share variability
St: $Std(\omega_z)$	0.11	**0.26**	more variable
Vol: $Std(\omega_{\tilde{z}})$	-	0.14	because volatile is less
			corr of Sharpe Ratio and eq. sh.
St: $Corr(\omega_z, SR)$	0.93	**0.98**	time market correctly
Vol: $Corr(\omega_{\tilde{z}}, SR)$	-	-0.97	volatile mistime market lose 2%

Lowering κ gets more volatility. Goes up with true MP calibration.

Figure: Variation in Beliefs



Simple Regime Switching Model

Rationalizing volatile beliefs: maybe their right.

Assume z_t follows a regime-switching process

- Probability of high growth high in good regime, low in bad.
- Given regime: i.i.d. draws for high/low growth rate shock.
- Regimes persistent and only realized z^t are observed.
- The transition rule for h' / h is largely unchanged.
- With enough regime persistence volatile belief do better.

Question: Can observed portfolio behavior help explain things?

Answer: Yes

- Increased and differential risk exposure explains a lot of consumption behavior
- Differential returns explains a lot of wealth skewness and correlation of wealth and equity participation
- Segmented markets and concentration of risk explains equity premium and low risk-free rate
- Time variation in wealth and risk exposure of active traders can explains a lot of risk pricing cyclicality.

Next we need to better explain this micro portfolio behavior.

Bonus Report on other 2 Experiments

1. Myopic active traders have lower wealth target, otherwise similar
   - Their portfolio behavior very similar just lower precautionary motive leads to lower wealth.
   - Absorb similar amounts of aggregate risk so prices not change.
2. Less risk averse active traders changes many things,
   - less risk averse active traders more willing to absorb risk
   - price of risk down, volatility up.

Method's Key Drawbacks

We only can have simple discrete shock process

- discrete shocks $z \in \{z^h, z^l\}$ and $\eta \in \{\eta^h, \eta^l\}$ which follow simple Markov process
- We use a finite history as the state.
- The number of states is $\#Z^{k+1} \times 2$ to capture $\{z_{t-5}, ..., z_t, z_{t+1}, \eta_t, \eta_{t+1}\}$ for our transitions.

Have not incorporated capital

- Leads to continuous state variables and transition rule for capital
- Can examine implications of risk pricing for capital accumulation and various feedbacks.

Lagrangian

Take a household i with debt bounds and subject to fix portfolio restriction, σ^i, as an example:

$$
\begin{aligned}
L \quad = \quad & \max_{\{c^i,\sigma\}} \min_{\{\chi,\nu,\varphi\}} \sum_{t=1}^{\infty} \beta^t \sum_{(z^t,\eta^t)} (c^i(z^t,\eta^t)^{1-\alpha}/(1-\alpha)\pi(z^t,\eta^t) \\
& +\zeta^i \left\{ \sum_{t\geq 1} \sum_{(z^t,\eta^t)} P(z^t,\eta^t) \left[\gamma Y(z^t)\eta_t - c^i(z^t,\eta^t) \right] + \varpi(z^0) \right\} \\
& + \sum_{t\geq 1} \sum_{(z^t,\eta^t)} \nu^i(z^t,\eta^t) \left\{ \begin{array}{c} S^i(z^t,h^t) \\ -P(z^t,\eta^t)\sigma(z^{t-1},\eta^{t-1})R^p(z^t) \end{array} \right\} \\
& + \sum_{t\geq 1} \sum_{(z^t,\eta^t)} \varphi^i(z^t,\eta^t) \left\{ D_t^i(z^t)P(z^t,h^t) - S^i(z^t,h^t) \right\}.
\end{aligned}
$$

Return main1

Define the recursive multiplier

$$\zeta^i(z^t, h^t) = \zeta^i + \sum_{(z^\tau, h^\tau) \preceq (z^t, h^t)} \left[\nu^i(z^\tau, h^\tau) - \varphi^i(z^\tau, h^\tau) \right].$$

ζ evolves:

$$\zeta^i(z^t, h^t) = \zeta^i(z^{t-1}, \eta^{t-1}) + \nu^i \left(z^t, \eta^t \right) - \varphi^i(z^t, \eta^t).$$

Rewrite this first-order condition

$$\beta^t u'(c(z^t, \eta^t)) = \zeta^i(z^t, h^t) P(z^t).$$

Return main1

We construct a reference trader for each type:

- CRRA flow utility $\bar{u}(c)$,
- a discount rate β,
- common beliefs π, and
- a social planning weight $1/\bar{\zeta}^i(z^t, \eta^t)$.

The static allocation problem is given by

$$\sum_i \left\{ \beta^t \sum_{(z^t, h^t)} \frac{1}{\bar{\zeta}^i(z^t, \eta^t)} \bar{u}(\bar{c}(z^t, \eta^t)) \pi(z^t, \eta^t) - P(z^t) \bar{c}(z^t, \eta^t) \right\} \mu_i.$$

We can construct a mapping from our standard trader's multiplier to the reference trader

so that their consumptions are the same. $\bar{\zeta}^i(z^t, \eta^t)$:

$$\left(\frac{\bar{\zeta}^i(z^t, \eta^t)P(z^t)}{\beta^t}\right)^{-1/\bar{\alpha}} = u'^{-1}\left(\frac{\zeta^i(z^t, \eta^t)\pi(z^t, \eta^t)P(z^t)}{\beta_i^t \theta_t \tilde{\pi}^i(z^t, h^t)}\right).$$

With these multipliers for the reference traders:

- If the state-contingent consumption market clears in the economy with reference traders, it does in the original one too.
- We need the original only for their multiplier updating rule.

Our aggregation results on the consumption share and stochastic discount rate holds for the reference trader.

So

$$\frac{c^i(z^t,\eta^t)}{C(z^t)} = \frac{\bar{\zeta}^i(z^t,\eta^t)^{-1/\bar{\alpha}}}{h(z^t)}$$

and

$$\frac{P(z^{t+1})}{P(z^t)} = \beta \left(\frac{h(z^{t+1})}{h(z^t)}\right)^{\bar{\alpha}} \left(\frac{C(z^{t+1})}{C(z^t)}\right)^{-\bar{\alpha}}$$

where

$$h(z^t) = \sum_i \left\{ \sum_{z^t,\eta^t} \bar{\zeta}^i(z^t,\eta^t)^{-1/\bar{\alpha}} \pi(z^t,\eta^t) \right\} \mu_i$$

1. Fix set of **truncated histories** of length k: $\mathbf{z} \in Z^k$
2. In stage i, guess an aggregate weight forecasting function $H(\mathbf{z}, \mathbf{z}') = \{h^i(\mathbf{z}')/h^i(\mathbf{z})\}$ with truncated history $\mathbf{z} \to \mathbf{z}'$
3. This implies relative prices $Q(\mathbf{z}, \mathbf{z}') = \{\frac{P'}{P}(\mathbf{z}, \mathbf{z}')\}$
4. Solve system of equations for updating functions for $\zeta^i(z^t, \eta^t)$ for each type.
   i. If using reference traders map $\zeta^i(z^t, \eta^t) \to \bar{\zeta}^i(z^t, \eta^t)$.
5. Updating functions define new $H(\mathbf{z}, \mathbf{z}')$, computed by simulating long panels and finding conditional averages.
   i. Average is w.r.t. reference traders' multipliers if used.
6. iterate until convergence of $\{h^{i+1}(z^{k'})/h^{i+1}(z^k)\}$

Return main2

Results with Variation of 1/2 Active Traders

Variations Relative to Baseline: $\kappa = 1, \beta = .95, \alpha = 5$,

St=standard, Alt=alternative

	Baseline	$\kappa = .75$	$\beta = .925$	$\alpha = 2$
$\frac{\sigma(m)}{E(m)}$	**0.41**	0.42	0.43	0.27
$Std\left\{\frac{\sigma(m)}{E(m)}\right\}$	2.78	8.52	2.94	3.74
$E(R_f)$	**1.93**	2.03	1.97	2.62
$Std(R_f)$	**0.06**	0.40	0.08	0.20
St: $E(\omega_z)$	**0.80**	**0.90**	**0.82**	0.51
Alt: $E(\omega_{\tilde{z}})$	-	0.66	0.87	1.90
St: $Std(\omega_z)$	0.11	**0.26**	0.91	0.15
Alt: $Std(\omega_{\tilde{z}})$	-	0.14	0.94	0.13
St: $Corr(\omega_z, SR)$	**0.93**	**0.98**	0.91	0.99
Alt: $Corr(\omega_{\tilde{z}}, SR)$	-	-0.97	0.94	0.90
St: $E(W_z/W)$	2.15	**2.39**	**2.38**	1.32
Alt: $E(W_{\tilde{z}}/W)$	-	1.88	1.66	1.37
St: $Std(W_z/W)$	0.57	0.54	0.67	0.12
Alt: $Std(W_{\tilde{z}}/W)$	-	0.48	0.40	0.90