Online Appendix C: Measurement error correction using repeated observations

This appendix explains how we implement the measurement error correction technique proposed by Hausman et al. (1991) and generalized by Schennach (2004) in our production function estimation. They propose a way to identify the moments of the “true” variable without measurement error using repeated observations of the variable of interest—capital stock in our case. We develop a way of implementation that is computationally easy.

Following the previous studies, we make statistical assumptions on measurement errors. Specifically, two different noisy measures of capital stock (denoted by $K_{Ait}$ and $K_{Bit}$) are assumed to satisfy

$$
\varepsilon_{Ait} \bot \ln K^*_i, \quad \forall \tau = t \text{ and } t - 1,
$$

$$
E[\ln X_{it}|\ln K^*_i, \varepsilon_{Ait}] = E[\ln X_{it}|\ln K^*_i], \quad \forall t,
$$

$$
E[\varepsilon_{Bit}|\ln K^*_i, \varepsilon_{Ait}] = 0, \quad \forall \tau = t \text{ and } t - 1,
$$

where $\varepsilon_{Ait}$ and $\varepsilon_{Bit}$ denote the measurement error in $\ln K^*_i$ and $\ln K^*_i$, respectively; $K^*_i$ denotes the “true” capital stock; and $X_{it}$ represents any other variables and their lags used in the estimation (gross output, labor input, etc). For regularity, all moments and cross-moments used in the estimation are assumed to be finite. Our first measure of capital stock ($K_{Ait}$) is obtained by applying the perpetual inventory method to the change in total real assets. Our second measure of capital stock ($K_{Bit}$) is the industry-level real capital stock multiplied by plant $i$’s share in one of the following variables: material inputs, electricity input, fuel input, and the sum of electricity and fuel inputs.

We construct a new variable $\hat{K}^*_i$ from two measures of capital stock (i.e., $K_{Ait}$ and $K_{Bit}$) that can be used in estimation in place of a noisy measure of capital stock. In principle, one can derive the true moments of $K^*_i$ using $K_{Ait}$ and $K_{Bit}$ and replace all biased sample moments by the sample analogue of the true moments in the estimation formulas. However, such an approach would add computational burden and be prone to human errors. Instead, in our approach, once we obtain $\hat{K}^*_i$ that has the same sample moments as true capital stock $K^*_i$, we can simply replace a noisy measure of capital stock by $\hat{K}^*_i$ in estimation, while using the same formulas and functions in statistical packages (such as those for generalized method of moments, GMM).

Now, let us explain how to construct $\ln \hat{K}^*_i$ and $\ln \hat{K}^*_{i,t-1}$ that have the same sample moments and the sample cross-moments with other variables as those of true capital stock $\ln K^*_i$ and $\ln K^*_{i,t-1}$. Note that $\ln \hat{K}^*_{i,t-1}$ does not need to be actually the lag of $\ln \hat{K}^*_i$, as far as each of these variables has the same sample moments that $\ln K^*_i$ and $\ln K^*_{i,t-1}$ must have, respectively. Although it is possible to construct these variables keeping the lag structure between the two, it is unnecessary and computationally more challenging.

We begin with the construction of $\ln \hat{K}^*_i$. We first construct $\ln \tilde{K}^*_i$. We then use the estimated $\tilde{K}^*_i$ to construct $\hat{K}^*_i$.
that is a mean-bias corrected version of $\ln K_{A,i,t-1}$, as follows:

$$
\ln K_{A,i,t-1}^* := \ln K_{A,i,t-1} - E[\ln K_{A,i,t-1}] + E[\ln K_{B,i,t-1}].
$$

We can then express the following moments of $\ln K_{i,t}^*$ as follows:

$$
\begin{align*}
E[(\ln K_{i,t-1}^*)^2] &= E[\ln K_{A,i,t-1}^* \ln K_{B,i,t-1}], \quad (E.13) \\
E[X_{it}\ln K_{i,t-1}^*] &= E[X_{it}\ln K_{A,i,t-1}^*], \quad (E.14) \\
E[\ln K_{i,t-1}^* \ln K_{i,t}^*] &= E[\ln K_{A,i,i,t,t-1}^* \ln K_{B,i,t-1}]. \quad (E.15)
\end{align*}
$$

Relationships (E.14) and (E.15) imply that $\ln K_{i,t-1}^*$ must retain the same sample cross-moments that $\ln K_{A,i,t-1}^*$ has with any $\ln X_{it}$ and $\ln K_{B,i,t-1}$. At the same time, $\ln K_{i,t-1}^*$ must have the second moment implied by (E.13).

To construct such $\ln K_{i,t-1}^*$, we use a property of ordinary least squares (OLS) and a geometric interpretation of statistical variables. We first run a regression of $\ln K_{A,i,t-1}^*$ on the log of all the other variables used in the estimation (gross output, labor input, etc) as well as $\ln K_{B,i,t-1}$ and save the fitted values as $\ln \hat{K}_{A,i,t-1}^*$. Note that sample cross-moments of $\ln \hat{K}_{A,i,t-1}^*$ and the log of other variables are the same as those of $\ln K_{A,i,i,t-1}^*$, from a property of OLS, that is:

$$
\hat{E}[\ln \hat{K}_{A,i,i,t-1}^* \ln X_{it}] = \hat{E}[(\ln K_{A,i,i,t-1}^* + e_{A,i,i,t-1}^*)\ln X_{it}] = \hat{E}[\ln \hat{K}_{A,i,i,t-1}^* \ln X_{it}],
$$

where $\hat{E}[\cdot]$ is the sample analog of the expectation operator and $e_{A,i,i,t-1}^*$ is the OLS residual. By linearity of summation, any linear combinations of $\ln K_{A,i,i,t-1}^*$ and $\ln \hat{K}_{A,i,i,t-1}^*$ also have the same sample cross-moments with the log of other variables. We choose one linear combination that has the sample second moment implied by (E.13), using the (multi-dimensional) Pythagorean theorem with a geometric interpretation. Specifically, we construct $\ln \hat{K}_{i,t-1}^*$ as follows:

$$
\begin{align*}
\ln \hat{K}_{i,t-1}^* &= \ln \hat{K}_{A,i,i,t-1}^* + \theta_{tag} \left( \ln K_{A,i,i,t-1}^* - \ln \hat{K}_{A,i,i,t-1}^* \right), \\
\theta_{tag} &= \sqrt{\frac{\hat{E}[(\ln K_{i,t-1}^*)^2] - \hat{E}[(\ln \hat{K}_{A,i,i,t-1}^*)^2]}{\hat{E}[(\ln K_{A,i,i,t-1}^*)^2] - \hat{E}[(\ln \hat{K}_{A,i,i,t-1}^*)^2]}}.
\end{align*}
$$

By construction, $\ln \hat{K}_{i,t-1}^*$ has the desired first and second sample moments and cross-moments with the log of other variables.

Next, we construct $\ln \hat{K}_{i,t}^*$ in a similar way but with additional steps. Relationship (E.14) implies that $\ln \hat{K}_{i,t}^*$ must retain the same sample cross-moments that $\ln K_{A,i,t}^*$ has with any $\ln X_{it}$. At the same time, $\ln \hat{K}_{i,t}^*$ must have the cross-moment with $\hat{K}_{i,t-1}^*$ implied by (E.15), in addition to the second moment implied by (E.13). This requires additional steps, compared to the case of $\ln \hat{K}_{i,t-1}^*$. 

50
Specifically, we construct $\hat{K}_{it}$ as follows:

$$\ln \hat{K}_{it} := \ln \hat{K}_{Ait}^{u} + \theta_{level} \left( \ln K_{Ait}^{u} - \ln \hat{K}_{Ait}^{u} \right),$$

$$\hat{K}_{Ait}^{u} := \ln \hat{K}_{Ait}^{u} + \left( \frac{\beta_b - \beta_a}{\beta_a - \beta_c} \right) \left( \ln \hat{K}_{Ait}^{u} - \ln \hat{K}_{Ait}^{u} \right),$$

$$\theta_{level} := \frac{\hat{E} \left[ (\ln K_{Ait}^{u})^2 \right] - \hat{E} \left[ (\ln \hat{K}_{Ait}^{u})^2 \right]}{\sqrt{\ln \hat{E} \left[ (\ln K_{Ait}^{u})^2 \right] - \hat{E} \left[ (\ln \hat{K}_{Ait}^{u})^2 \right]}},$$

where $\ln \hat{K}_{Ait}^{u}$ is the fitted value from the regression of $\ln K_{Ait}^{u}$ on all other variables in log form as well as $\ln K_{Ait}^{u}$; $\ln \hat{K}_{Ait}^{u}$ is the fitted value from a regression of $\ln K_{Ait}^{u}$ on all other variables in log form (without $\ln K_{Ait}^{u}$); and lastly, $\beta_a$, $\beta_b$, and $\beta_c$ are the coefficients on $\ln \hat{K}_{it}$ in the regression of $\ln K_{Ait}^{u}$, $\ln K_{Bit}$, and $\ln K_{Ait}$ on $\ln \hat{K}_{Ait}^{u}$ (without the constant term), respectively.

Note that $\hat{K}_{it}$ or $\hat{K}_{i,t-1}$ does not always exist, because $\theta_{level}$ or $\theta_{lag}$ may take an imaginary number. The non-existence of $\hat{K}_{it}$ or $\hat{K}_{i,t-1}$ means that the assumptions that we pose are not feasible and lead to a contradiction. For example, the covariance of $\ln K_{Ait}$ and $\ln K_{Bit}$—which must be the variance of $\ln K_{it}$—can be negative in the data. In this case, this technique of measurement error correction cannot be implemented in any way. In our approach, we notice this problem by observing that $\hat{K}_{it}$ does not exist. In other words, the existence of variables $\hat{K}_{it}$ and $\hat{K}_{i,t-1}$ ensures that our assumptions as a whole are not infeasible.

Using these two constructed variables, we apply the Wooldridge-Levinsohn-Petrin estimator to estimate production functions. We simply replace the log of the original capital stock variable and its lag by $\ln \hat{K}_{it}$ and $\ln \hat{K}_{i,t-1}$, respectively. We do not include the second- and the third-order polynomials terms involving capital stock, because $\ln \hat{K}_{it}$ and $\ln \hat{K}_{i,t-1}$ only replicate up to the second moments of the true capital stock. Similarly, we use the one-step GMM, instead of the two-step GMM, because the optimal weighting matrix in the two-step GMM uses up to the fourth moments. For the standard errors, we use panel bootstrap with 1,000 replications.