A Online Appendix: Estimating $\sigma$ with Markups

This appendix is not intended for publication. We allow for goods market imperfections, derive the relationship between labor’s share and investment prices, and estimate the resulting model.

A.1 Extending the Model

We introduce exogenous, time-varying markups to the growth model by assuming that aggregate’s profits are given by:

$$\mu^i_t Y^i_t - w^i_t N^i_t - R^i_t K^i_t$$  \hspace{1cm} (27)

Where $\mu^i_t$ is the markup. These profits are then lump-sum rebated to households. The equilibrium is therefore as before, except that factor shares are given by:

$$\mu^i_t LS^i_t = (1 - \alpha_k)(Y^i_t)^{1-\sigma}/\sigma$$  \hspace{1cm} (28)

$$\mu^i_t KS^i_t = \alpha_k (Y^i_t/K^i_t)^{\frac{\sigma}{2}}$$  \hspace{1cm} (29)

$$PS^i_t = 1 - LS^i_t - KS^i_t = 1 - \frac{1}{\mu^i_t}$$  \hspace{1cm} (30)

Where the final equation defines profit’s share of income. Notice that we are still assuming that labor is inelastically supplied, although this has no effect on our analysis.

A.2 Empirical Model

We first write capital’s share to get a log-linear regression equation:

$$1 - \mu^i_t s^i_t = (\alpha^i_k)^{\sigma} \left( \frac{A^i_t}{\mu^i_t R^i_t} \right)^{\sigma - 1}$$  \hspace{1cm} (31)

Taking the log difference from $t$ to $t+1$, assuming constant $A^i$, substituting for $R$, and approximating $\log(1 - \mu^i_{t+1} s^i_{t+1})$ around $\mu^i_{t+1} = mu^i_t$ and $s^i_{t+1} = s^i_t$ yields the estimating equation:

$$\frac{\mu^i_{t+1} s^i_{t+1}}{1 - \mu^i_{t+1} s^i_{t+1}} \left( \beta^i_s + \beta^i_{\mu} \right) = \text{cons} + (\sigma - 1) \left( \beta^i_{\mu} + \beta^i_P + \beta^i_{\xi} \right) + \nu^i$$  \hspace{1cm} (32)
If we had time series for markups then we could estimate this model. We construct the average growth in markups in two stages. First we calculate the average level of markups and capital’s share and then we impute the trend in these two variables using observable investment rates.

In order to calculate the averages, we will use the fact that capital’s share can be written in terms of investment when the economy is in steady state as

\[
\frac{R^i K^i}{Y^i} = \frac{P^i X^i}{Y^i} \left( \frac{\frac{1}{\beta} + \delta - 1}{\delta} \right).
\]  

(33)

So, by assigning values to \( \beta \) and \( \delta \) and identifying the steady state as the average over of a variable, we know the capital’s share for each country from

\[
KS^i = \left( \frac{P^i X^i}{Y^i} \right) \left( \frac{\frac{1}{\beta} + \delta - 1}{\delta} \right).
\]  

(34)

Labor’s share is taken from the original data, so we can compute average markup from

\[
\mu^i = \left( KS^i + LS^i \right)^{-1}.
\]  

(35)

This is still not enough to estimate \( \sigma \); we need information on the average growth rate or trend in capital’s share. We therefore consider an imputation proposed by KN, in which the trend in the capital’s share is equated to the observed trend in the investment rate. That is, we will impute:

\[
\beta_{KS}^i = \beta_{IR}^i
\]  

(36)

Where \( IR^i_t = \frac{P^i X^i}{Y^i} \). This imputation may introduce substantial errors along a transition path, because the investment rate is not always monotonic, whereas capital’s share is. With this caveat in mind, we can now compute

\[
\mu_t^i = \frac{1}{LS_t^i + KS_t^i}.
\]  

(37)

We then take the log differences of each side:

\[
\Delta \log \mu_{t+1}^i = -\Delta \log \left( LS_{t+1}^i + KS_{t+1}^i \right)
\]  

(38)
We then approximate the right-hand side around $LS_{t+1}^i = LS_i^i$ and $KS_{t+1}^i = KS_i^i$:

$$\Delta \log \mu_{t+1} \approx -\left( \frac{LS_i^i}{LS_i^i + KS_i^i} \Delta \log LS_{t+1}^i + \frac{KS_i^i}{LS_i^i + KS_i^i} \Delta \log KS_{t+1}^i \right)$$  \hspace{1cm} (39)

And finally we replace levels with their averages and log differences with trends to get the imputed growth in the markup:

$$\beta_i^i \approx -\left[ \frac{LS_i^i}{LS_i^i + KS_i^i} \beta_{LS}^i + \frac{KS_i^i}{LS_i^i + KS_i^i} \beta_{KS}^i \right]$$  \hspace{1cm} (40)

This allows us to estimate $\sigma$ from Equation 32.

### A.3 Estimation Results

We present our estimation results in Table 17 for the four basic data sources and each exclusion restriction. We estimate only $\hat{\sigma}$ with robust regression because imputing markups introduces some extreme outliers. The estimates with markups are near one and most are below, with only one estimate significantly above one. The average across all estimates is $\hat{\sigma} = 0.983$, which implies that investment prices cannot account for the global decline in labor’s share.
Table 17: Model with Markups Estimates

<table>
<thead>
<tr>
<th></th>
<th>Hybrid, PWT</th>
<th>Exact</th>
</tr>
</thead>
<tbody>
<tr>
<td></td>
<td>$\hat{\sigma}$</td>
<td>90% Conf. Interval</td>
</tr>
<tr>
<td>$T_{min} = 10$</td>
<td>1.031</td>
<td>[0.976,1.086]</td>
</tr>
<tr>
<td>$T_{min} = 15$</td>
<td>0.918</td>
<td>[0.852,0.984]</td>
</tr>
</tbody>
</table>

<table>
<thead>
<tr>
<th></th>
<th>Corporate, PWT</th>
</tr>
</thead>
<tbody>
<tr>
<td>$T_{min} = 10$</td>
<td>0.972 [0.905,1.038]</td>
</tr>
<tr>
<td>$T_{min} = 15$</td>
<td>0.909 [0.836,0.982]</td>
</tr>
</tbody>
</table>

<table>
<thead>
<tr>
<th></th>
<th>Hybrid, WDI</th>
</tr>
</thead>
<tbody>
<tr>
<td>$T_{min} = 10$</td>
<td>1.056 [1.012,1.100]</td>
</tr>
<tr>
<td>$T_{min} = 15$</td>
<td>1.017 [0.935,1.099]</td>
</tr>
</tbody>
</table>

<table>
<thead>
<tr>
<th></th>
<th>Corporate, WDI</th>
</tr>
</thead>
<tbody>
<tr>
<td>$T_{min} = 10$</td>
<td>0.990 [0.921,1.058]</td>
</tr>
<tr>
<td>$T_{min} = 15$</td>
<td>0.969 [0.875,1.063]</td>
</tr>
</tbody>
</table>

<table>
<thead>
<tr>
<th></th>
<th>Average</th>
</tr>
</thead>
<tbody>
<tr>
<td></td>
<td>0.983</td>
</tr>
</tbody>
</table>