Modernization of Agriculture and Long-Term Growth[†]

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Work in Progress

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Abstract

Three events occurred concurrently around 1820 in England during the industrial revolution: (1) after persistent decline in the relative price of industrial to agricultural products, this price began to stabilize to a constant level; (2) agricultural mechanization was initiated; and (3) per capita GDP in England ended long-term stagnation, taking off to sustained growth. We explore the causal linkages of these observations with a two-sector growth model in which modernization of agriculture plays a central role. The model suggests a falling relative price with TFP growth in industry; and, as the price drops to a threshold level, farmers switch from a traditional technology to a modern technology that uses industry-supplied inputs. Once agricultural modernization begins, the economy escapes from the Malthusian trap. Quantitative analysis based on historical data from England between 1700 and 1909 accounts well for the observed changes in relative price, per capita income growth, agricultural mechanization and other aspects of English growth experience.

Keywords: long-term growth, relative price, agricultural modernization, structural transformation, Malthusian stagnation, transition dynamics, industrial revolution, England

"The man who farms as his forefathers did cannot produce much food no matter how rich the land or how hard he works. The farmer who has access to and knows how to use what science knows about soils, plants, animals, and machines can produce abundance of food though the land be poor. Nor need he work nearly so hard and long. He can produce so much that his brothers and some of his neighbors will move to town to earn their living." —T W Schultz (1964)

1 Introduction

The transition from stagnation to sustained improvement in living standard is a recent phenomenon. Angus Maddison's estimates of GDP per capita for the world indicate dramatic differences in growth experience between earlier historical times and the last two centuries (Maddison 2001). Prior to 1820, the world economy was in a Malthusian state with little growth: per capita product in that year was only 50% higher than the level estimated for ancient Rome. Since 1820, however, the world's per capita output has doubled three times. The switch from stagnation to growth has been a central concern to economists interested in development and growth because of the enormous welfare implications of sustained progress and the desire to understand the vast differences in living standards among nations.

The existing theoretical literature has offered several explanations for the transition from stagnation to growth. One set of studies emphasizes the role of endogenous fertility and investment in human capital (Becker et al. 1990, Tamura 1998, and Lucas 2002). An exogenous increase in the returns to human capital induces households to choose smaller family size and invest more in children, thus triggering the takeoff in per capita income growth. Another line of ideas places endogenous technological progress and population change at the center of their explanations (Kremer 1993, Goodfried and McDermott 1995, Galor and Weil 2000, and Jones 2001). For example, growing population can result in the acceleration of technological progress, which eventually leads to the transition from

a Malthusian to a modern steady state. In a recent study, Hansen and Prescott (2002) argue that what triggers modern growth is the adoption of a less land-intensive production technology that, although available throughout history, was not previously profitable to operate by individual firms. The economy is trapped in the Malthusian regime, when it only uses a land-intensive traditional technology that is subject to diminishing returns. These models have formed the basis for understanding the transition from constant to growing living standards, and are capable of generating broad patterns of the takeoff observed in the world history.

In this paper, we propose a new explanation regarding the timing and mechanisms for the transition from stagnation to growth that highlights the idea of T. W. Schultz (1964): modernization of agriculture is essential for long-term development and growth. Our model is motivated by three concurrent events that occurred around 1820 in England during the industrial revolution:

- The first is the well know fact about per capita income growth. As Figure 1 indicates, per capita GDP for the English economy ended its long-term stagnation and began to take off to a path of sustained growth.
- The second, and perhaps a less well-known fact, is that adoption and systematic diffusion of farm machinery also began around 1820 (Walton 1979; Overton 1996). Figure 4 indicates the percentage of farms in British regions that owned various agricultural machinery, which were provided by industry. The use of farm mechanical innovations was scant around 1810, but in the second half of the century, the adoption became widespread.¹

¹For centuries in the world development, the advancements in agricultural productivity were derived primarily from the experiences of farm people. But starting around 1820, application of scientific knowledge and inputs provided by industry have become the engine of rapid agricultural productivity growth (Huffamn and Evenson 1993; Johnson, 1997). This process of agricultural modernization, characterized by the use

- The third, and the least well-known fact, is the changes in relative price of industrial to agricultural products in England (see Figure 2). The relative price experienced persistent declines for more than a century, hitting a low point around 1820, and then stayed at that level in the following century.

We argue in this paper that the three concurrent events occurred in England during the industrial revolution – the change from decline to stabilization in the relative price, agricultural modernization, and the takeoff in per capita GDP from stagnation to growth – were not merely a historical coincident. Instead, the events were causally linked. We will show that the relative price of industrial to agricultural products declines with TFP growth in industry; and, when the price drops to a threshold level, farmers would find it profitable to switch from a traditional technology to a modern technology that uses industry-supplied inputs. Once agricultural modernization begins, the economy takes off from the Malthusian trap to a path of sustained income growth.

To explore the causal relationships underlying the three observations, we develop a model of structural transformation and long-term growth in which the adoption of modern agricultural technology triggers the takeoff from Malthusian stagnation.² A central element of the model is the choice of two technologies potentially available to farmers: a traditional technology that uses labor and land, where the latter is in fixed supply, thus implying diminishing returns to scale. The alternative is a modern technology that uses one additional intermediate input, which is produced outside of agriculture. This input represents farm machinery in our paper, but may also include other industry-supplied factors, such as chemical fertilizers and high-yield seeds. The cost of the input is determined endogenously, depending in part on the total factor productivity of the industrial sector, which grows exogenously. Farm-

of modern intermediate inputs, include the adoption of mechanical, chemical, biological and agronomic innovations.

²The two sectors are agriculture and nonagriculture. We also refer nonagriculture as the industrial sector.

ers start with the traditional technology. They begin to use the modern technology when the price of the intermediate input falls below a threshold level so that its adoption yields higher profits than using traditional technology. Once agricultural modernization begins, the economy starts the transition towards modern growth.

In the traditional economy, slow TFP growth in experience-based farming systems determines high agricultural labor share and low per capita income. Rich land endowment and temporary increase in agricultural productivity may lead to temporary structural transformation and per capita income increase. However, high income induces population growth, which in turn reduces per worker output in agriculture because of fixed supply of land. TFP growth in industry cannot generate sustained structural transformation, nor income growth, because labor is needed to assure sufficient food supply. Without modernizing agriculture, the economy cannot break out from the Malthusian trap.

In the model, agricultural modernization is inevitable given positive rates of total factor productivity growth in the industry. When the relative price of industrial product drops below a critical level, farmers will find it profitable to use the modern technology. The speed of adoption depends partly on the equilibrium adjustments across the two sectors. But once the adoption begins, structural transformation accelerates and the economy steps on a path of sustained growth. The critical link is that, when modernization begins, TFP growth in industry will join force with TFP growth in agriculture, contributing directly to agricultural labor productivity through the use of intermediate inputs, thus facilitating structural change. During the transition, TFP growth in agriculture and industry will both contribute to per capita income growth in the economy, taking off from the Malthusian trap. As growth persists, the share of labor in agriculture will approach zero in the limit, and the rate of per capita income growth will converge to the rate of growth in industry.

To examine the model's implications in light of empirical facts, we compiled from published sources aggregate historical data for the English economy for the period between 1700

and 1909, which covers the complete process of the industrial revolution. The data series consist of constructed indices of real GDP per capita in England, prices for agricultural output, prices for principal industrial products, agricultural mechanization, employment share in agriculture, real average wages of adult farm workers, and land rent. We also rely on studies of historical performance of the English economy to infer exogenous improvements in total factor productivity in agricultural and non-agricultural production. Then, we calibrate the model to the English economy and generate the time paths for four key aggregate variables – per capita income, relative price of industrial to agricultural product, fraction of labor in agriculture, and the extent of agricultural mechanization. When time permits, our analysis will also cover the time paths of real wages and rental price for land. Our preliminary quantitative analysis accounts well for the observed changes in English growth experience during the industrial revolution. The findings numerical simulations support a coherent and unified view on the importance of agricultural modernization in making the transition from stagnation to growth.

Development economists have long emphasized the importance of agriculture in growth (e.g., Johnston and Mellor, 1961; and Schultz, 1964.). Jorgenson (1961) shows that a high enough agricultural productivity growth rate is a necessary condition for persistent growth of GDP per worker. More recently, Gollin, Parente and Rogerson (2002 and 2004) use a two-sector neoclassical growth model to show the importance of productivity changes in agriculture for understanding both the transition and the cross-country income differences today. Among endogenous growth models, Matsuyama (1992) showed a positive link between agricultural productivity and economic growth with learning-by-doing in the manufacturing sector for a closed economy. Our theory departs from the existing literature by emphasizing the feedback effect of nonagricultural growth on agriculture and by endogenizing the change in agricultural productivity growth and the timing of this change. We are also the first to calibrate a two-sector general equilibrium model to account for observed growth experience

of an individual country with respect to a set of key aggregate variables.

This is a highly preliminary and incomplete draft. Section 2 presents the model. In section 3, we analyze the equilibrium in a traditional economy without the adoption of modern agricultural technology. Section 4 studies the transition from traditional to modern economy. Section 5, provides descriptions of data. In section 6, we present a few stylized patterns of the English economy based on historical data for the period 1700-1909. We also present some preliminary results on numerical simulations. Section 6 concludes.

2 The Model

Consider an economy in discrete time. There is a fixed amount of land $\bar{Z}$ and N_t identical individuals in period t . Each individual owns $z_t = N_t^{-1}\bar{Z}$ amount of the land and one unit of time, which is supplied inelastically to work in the labor market.³ Let w_t be the wage rate and r_t be the rental rate of land. Then, an individual's income is

$$y_t = w_t + r_t z_t.$$

There are two consumption goods, agricultural and nonagricultural. Let the agricultural good be the numeraire and p_t be the price of the nonagricultural good. Each individual household consumes a constant $\bar{c}$ amount of agricultural good (c_{at}) and spends the remaining income on the consumption of nonagricultural good (c_{nt}). Therefore, we have

$$c_{at} = \bar{c}, \tag{1}$$

$$c_{nt} = p_t^{-1}(y_t - \bar{c}). \tag{2}$$

³For notation simplicity, we prohibit trading in land ownership. Since households are identical in this economy, the assumption is not substantial.

Each individual lives for one period and, at the end of period t , gives birth to g_t children. The land owned by the parent will be divided equally among the children. We assume that the population growth rate is a function of per capita income, $g_t = g(y_t)$. Thus,

$$N_{t+1} = g(y_t)N_t. \quad (3)$$

Note that we use the agricultural good as the numeraire, so the per capita income y_t is not the same as the usual measure of national income per capita, which is deflated by a GDP deflator. Rather, y_t is a measure of household's ability to purchase the agricultural good. This corresponds well to the living standard measures used in economic history literature, where they are often calculated as the ratio of nominal income to the price of some commonly consumed food products.

Production Technologies

The nonagricultural good is produced with a linear production technology:

$$Y_{nt} = A_{nt}L_{nt},$$

where A_{nt} is the TFP in the nonagricultural sector.

Two technologies are potentially available for farm production. The traditional technology only uses land and labor as inputs:

$$Y_{at}^T = Z_t^{1-\sigma} (A_{at}L_{at})^\sigma, \quad 0 < \sigma < 1.$$

Here, Z_t and L_{at} are land and labor inputs, respectively, A_{at} is a TFP parameter in traditional agriculture, σ is the labor share, and superscript T denotes traditional technology. The modern agricultural technology (with superscript M) uses an intermediate input X_t , as well as the traditional inputs—land and labor:

$$Y_{at}^M = [Z_t^{1-\sigma} (A_{at} L_{at})^\sigma]^{1-\alpha} X_t^\alpha, \quad 0 < \alpha < 1.$$

The intermediate input is produced outside of agriculture and has a factor share of α . We assume that the production of one unit of intermediate input requires π units of nonagricultural output. Hence, the price of intermediate input is πp_t .

Since the production technologies have constant returns to scale, we assume without losing generality that there is one representative firm in each of the two sectors. Both firms behave competitively, taking output and factor prices as given and choosing factor inputs to maximize profits.

The representative firm (or farm) in agriculture has the following profit maximization problem:

$$\max_{Z_t^T, Z_t^M, L_{at}^T, L_{at}^M, X_t} \left\{ \begin{aligned} & (Z_t^T)^{1-\sigma} (A_{at} L_{at}^T)^\sigma + [(Z_t^M)^{1-\sigma} (A_{at} L_{at}^M)^\sigma]^{1-\alpha} X_t^\alpha \\ & - \pi p_t X_t - r_t Z_t - w_t L_{at} \end{aligned} \right\}, \quad (4)$$

subject to the quantity constraints:

$$Z_t^T + Z_t^M = Z_t, \text{ and } L_{at}^T + L_{at}^M = L_{at}.$$

The profit maximization problem of the nonagricultural firm is

$$\max_{L_{nt}} \{p_t A_{nt} L_{nt} - w_t L_{nt}\} \quad (5)$$

Technology Adoption in Agriculture

If the representative farm adopts the modern technology and allocates $Z_t^M (> 0)$ amount of land and $L_{at}^M (> 0)$ amount of labor to the production using modern technology, then,

from (4), the optimal quantity of the intermediate input the farm should use is given by

$$X_t = \left(\frac{\alpha}{\pi p_t} \right)^{1/(1-\alpha)} (Z_t^M)^{1-\sigma} (A_{at} L_{at}^M)^\sigma$$

and the net output produced by the modern agricultural technology is

$$\hat{Y}_{at}^M = Y_{at}^M - \pi p_t X_t = (1 - \alpha) \left(\frac{\alpha}{\pi p_t} \right)^{\alpha/(1-\alpha)} (Z_t^M)^{1-\sigma} (A_{at} L_{at}^M)^\sigma.$$

In comparison, if the farm uses the same amounts of land and labor for production using the traditional technology, the output is $(Z_t^M)^{1-\sigma} (A_{at} L_{at}^M)^\sigma$. Clearly, the farm will adopt modern technology only if

$$(1 - \alpha) \left(\frac{\alpha}{\pi p_t} \right)^{\alpha/(1-\alpha)} \geq 1. \quad (6)$$

When the equality holds in (6), the farm is indifferent between the two technologies; either or both maybe used. So, the modern agricultural technology will not be adopted if the cost of using intermediate input (πp_t) is too high. To find out the conditions for adopting modern agricultural technology, we need to solve the equilibrium level of the price p_t . Before doing that, let us first define competitive equilibrium in our model economy.

Definition 1 *A competitive equilibrium consists of sequences of prices $\{w_t, r_t, p_t\}_{t \geq 0}$, firm allocations $\{L_{at}^T, L_{at}^M, L_{nt}, Z_t^T, Z_t^M, X_t\}_{t \geq 0}$, consumption allocations $\{c_{at}, c_{nt}\}_{t \geq 0}$, and population sizes, $\{N_t\}$, such that the followings are true:*

1. *Given the sequence of prices, the firm allocations solve the profit maximization problems in (4) and (5).*
2. *Consumption allocations are given by (1) and (2).*

3. All markets clear:

$$Y_{at} = N_t \bar{c}, \quad (7)$$

$$Y_{nt} = N_t c_{nt} + \pi X_t + G_t, \quad (8)$$

$$N_t = L_{at}^T + L_{at}^M + L_{nt}, \quad (9)$$

$$\bar{Z} = Z_t^T + \bar{Z}_t^M. \quad (10)$$

4. Population growth rate is given by equation (3).

In the appendix, we prove the following proposition for a competitive equilibrium:

Proposition 1 *Let $\Phi_l \equiv (1 - \alpha)^{\frac{\alpha-1}{\alpha}} \sigma \alpha^{-1} \bar{c}^{\frac{\sigma-1}{\sigma}}$ and $\Phi_h = (1 - \alpha)^{-\frac{1-\sigma}{\sigma}} \Phi_l$. In agricultural production, the farm only uses the traditional technology if*

$$\frac{A_{nt}}{\pi A_{at} (\bar{Z}/N_t)^{\frac{1-\sigma}{\sigma}}} \leq \Phi_l;$$

only uses the modern technology if

$$\frac{A_{nt}}{\pi A_{at} (\bar{Z}/N_t)^{\frac{1-\sigma}{\sigma}}} \geq \Phi_h;$$

and uses both technologies if

$$\Phi_l < \frac{A_{nt}}{\pi A_{at} (\bar{Z}/N_t)^{\frac{1-\sigma}{\sigma}}} < \Phi_h.$$

Proof: All proofs of propositions are in the appendix.

Proposition 1 shows three important factors that have direct effects on the adoption of modern technology in agriculture. First, since the modern technology requires the use of

intermediate input, the cost of producing this input has a negative effect on its adoption. Second, high productivity in traditional agriculture, as measured by the TFP parameter A_{at} and land-to-population ratio $\bar{Z}/N_t$, also has a negative effect on using modern technology. Finally, high productivity in the nonagricultural sector (A_{nt}) reduces the price of nonagricultural good and therefore the cost of intermediate input. Consequently, it has a positive effect on the adoption decision.

3 Traditional Economy

We call the economy a traditional economy when farmers only use traditional agricultural technology. From Proposition 1, if the initial land to population ratio $\bar{Z}/N_0$ is sufficiently high and/or the initial relative TFP of nonagricultural sector, A_{n0}/A_{a0} , is sufficiently low, the economy starts out as a traditional one. The following proposition states the determination of key variables in a traditional economy.

Proposition 2 *Let $\tilde{A}_{at} = A_{at} (\bar{Z}/N_t)^{\frac{1-\sigma}{\sigma}}$, then, in a traditional economy, we have*

$$p_t = \sigma \bar{c}^{\frac{\sigma-1}{\sigma}} \frac{\tilde{A}_{at}}{A_{nt}}, \quad (11)$$

$$w_t = \sigma \bar{c}^{\frac{\sigma-1}{\sigma}} \tilde{A}_{at}, \quad (12)$$

$$r_t = (1 - \sigma) \bar{c} \frac{N_t}{\bar{Z}}, \quad (13)$$

$$y_t = \left[1 - \sigma + \sigma \bar{c}^{-\frac{1}{\sigma}} \tilde{A}_{at} \right] \bar{c}, \quad (14)$$

$$\frac{L_{at}}{N_t} = \bar{c}^{\frac{1}{\sigma}} \tilde{A}_{at}^{-1}. \quad (15)$$

Proposition 2 shows that in a traditional economy per capita income and labor allocation are determined by the variable $\tilde{A}_{at}$, which is a measure of productivity in traditional

agriculture. This variable itself is determined by the TFP in traditional agriculture A_{at} and the land to population ratio $\bar{Z}/N_t$. Higher agricultural TFP and/or higher per capita land endowment lead to higher labor productivity in agriculture, higher per capita income, and lower agriculture's share of employment.

From equation (15) and (14), it is clear that a traditional economy can go through a sustained structural change (i.e., declining share of employment in agriculture) and per capita income growth only if there is sustained growth in $\tilde{A}_{at}$. By its definition, we have

$$\begin{aligned}\frac{\tilde{A}_{at+1}}{\tilde{A}_{at}} &= \frac{A_{at+1}}{A_{at}} \left(\frac{N_{t+1}}{N_t} \right)^{-\frac{1-\sigma}{\sigma}} = \frac{A_{at+1}}{A_{at}} [g(y_t)]^{-\frac{1-\sigma}{\sigma}} \\ &= \frac{A_{at+1}}{A_{at}} \left[g \left((1 - \sigma + \sigma \bar{c}^{-\frac{1}{\sigma}} \tilde{A}_{at}) \bar{c} \right) \right]^{-\frac{1-\sigma}{\sigma}}.\end{aligned}$$

If A_{at} grows at a constant rate $\gamma_a \geq 1$, then, the equation above becomes

$$\tilde{A}_{at+1} = \gamma_a \left[g \left((1 - \sigma + \sigma \bar{c}^{-\frac{1}{\sigma}} \tilde{A}_{at}) \bar{c} \right) \right]^{-\frac{1-\sigma}{\sigma}} \tilde{A}_{at}. \quad (16)$$

We make the following assumption about function $g(\cdot)$.

Assumption 1 (i) $g(\bar{c}) < 1$; (ii) there is a $\hat{y} > \bar{c}$ such that $g(\hat{y}) > \gamma_a^{\frac{\sigma}{1-\sigma}}$; and (iii) $g(\cdot)$ is continuous and strictly increasing over the interval $[0, \hat{y})$, decreasing over the interval $[\hat{y}, \infty)$, and $\lim_{y \rightarrow \infty} g(y) = 1$.

Under this assumption, starting at a low level of income, population growth rate increases with income and reaches its peak at a certain income level, after which the population growth rate decreases with income and eventually converges to one. This hump-shaped function for population growth rate is consistent with typical patterns of demographic transition.

Proposition 3 Under Assumption 1, there exists a unique steady state solution to the difference equation (16) such that the corresponding income per capita $y^* \in (\bar{c}, \hat{y})$.

Thus, if the economy never adopts the modern technology in agriculture, there will be no sustained growth and the economy always settles down at a Malthusian steady state with constant income per capita y^* . From equation (14), we know that in the steady state, the value of $\tilde{A}_{at}$, $\tilde{A}_a^*$, is determined by the equation

$$y^* = \left[1 - \sigma + \sigma \bar{c}^{-\frac{1}{\sigma}} \tilde{A}_a^* \right] \bar{c}.$$

Since $y^* > \bar{c}$, $\tilde{A}_a^* > 0$. Therefore, in the steady state, the population size is given by the equation:

$$\tilde{A}_a^* = A_{at} \left(\bar{Z} / N_t \right)^{\frac{1-\sigma}{\sigma}}$$

or

$$N_t = \left(\frac{A_{at}}{\tilde{A}_a^*} \right)^{\frac{\sigma}{1-\sigma}} \bar{Z}.$$

So, in a traditional economy, the effects of agricultural TFP and land endowment on agricultural labor productivity are completely offset by the long-run adjustment of population size. As a result, the labor productivity in agriculture is independent of these two factors in the Malthusian steady state.

4 Transition to Modern Economy

The previous section shows that there would be no sustained income growth unless modern technology is adopted in agriculture. Suppose the economy starts with the steady state equilibrium at $t = 0$. Will there ever be a time at which the modern technology will be adopted in agriculture?

Proposition 1 suggests that the modern agricultural technology will be adopted if

$$\frac{A_{nt}}{\pi \tilde{A}_{at}} > \Phi_l. \quad (17)$$

In the steady state, $\tilde{A}_{at} = \tilde{A}_a^*$, which is a constant. So, as long as A_{nt} grows without bound, there will be a point of time at which the inequality (17) holds. We can also see this from the behavior of relative price of nonagricultural good. From (11), we know that $p_t = \sigma \bar{c}^{\frac{\sigma-1}{\sigma}} \tilde{A}_{at}/A_{nt}$. In the Malthusian steady state, $\tilde{A}_{at} = \tilde{A}_a^*$ so we have

$$p_t = \sigma \bar{c}^{\frac{\sigma-1}{\sigma}} \tilde{A}_a^*/A_{nt}, \quad (18)$$

which declines monotonically as long as the TFP in nonagriculture grows. So, at some point of time the price of nonagricultural good will be so low that farmers will find it cheap enough to use as input for agricultural production. At that point, the economy starts a transition from the traditional agricultural technology to modern agricultural technology. During the transition, both technologies will be used. We call such an economy a mixed economy.

Proposition 4 *In a mixed economy,*

$$p_t = p_{mixed} \equiv \frac{\alpha}{\pi} (1 - \alpha)^{\frac{1-\alpha}{\alpha}}, \quad (19)$$

$$w_t = p_{mixed} A_{nt}, \quad (20)$$

$$r_t = (1 - \sigma) \left(\frac{\sigma}{p_{mixed}} \frac{\tilde{A}_{at}}{A_{nt}} \right)^{\frac{\sigma}{1-\sigma}} \frac{N_t}{\bar{Z}}, \quad (21)$$

$$y_t = p_{mixed} A_{nt} + (1 - \sigma) \left(\frac{\sigma}{p_{mixed}} \frac{\tilde{A}_{at}}{A_{nt}} \right)^{\frac{\sigma}{1-\sigma}}, \quad (22)$$

$$\frac{L_{at}}{N_t} = \left(\frac{\sigma}{p_{mixed}} \frac{\tilde{A}_{at}}{A_{nt}} \right)^{\frac{1}{1-\sigma}} \tilde{A}_{at}^{-1}, \quad (23)$$

$$\frac{Z_t^M}{\bar{Z}} = \frac{1 - \alpha}{\alpha} \left[\left(\frac{A_{nt}}{\pi \tilde{A}_{at}} \Phi_l^{-1} \right)^{\frac{\sigma}{1-\sigma}} - 1 \right]. \quad (24)$$

The time paths of key variables in the economy with mixed technologies differ from the variables' time paths in the traditional economy. In particular, note the structural breaks for the following variables:

- Agriculture's employment share, L_{at}/N_t : In the traditional economy, the share is a decreasing function of A_{at} and $\bar{Z}/N_t$ (see equ. 15). During the transition with mixed technologies, the share of employment in agriculture is also a decreasing function of A_{nt} —TFP growth in industry would reduce the cost of modern input X_t , inducing farmers to use more X_t and less labor, which leads to faster structural change.
- Relative price of nonagricultural good: In the traditional steady state, $\tilde{A}_{at}$ is a constant and p_t declines as A_{nt} increases (see equ. 11). During the transition, p_t remain at a constant level.
- Per capita income growth y_t : In the traditional economy, changes in y_t is limited due to the slow growth of A_{at} (see equ. 14). But during the transition, A_{nt} contributes directly

to the growth of y_t ; the economy breaks away from the Malthusian trap. Clearly, a structural break occurs to the growth rate of y_t when the agricultural modernization begins.

- Use of modern input in agriculture: In the traditional economy, there is no use of modern technology. During the transition, if the TFP in nonagriculture, A_{nt} grows faster enough, $A_{nt}/(\pi\tilde{A}_{at})$ increases over time, and the percentage of land (and labor) that is used for modern agricultural production increases from zero to one as $A_{nt}/(\pi\tilde{A}_{at})$ moves from Φ_l to Φ_h (see equ. 24).

4.1 Modern Growth

As long as A_{nt} grows faster enough, the relative productivity $A_{nt}/(\pi\tilde{A}_{at})$ will eventually exceed the threshold Φ_h and the economy will use modern technology only. In this case, we have

Proposition 5 *Let $\tilde{A}_{at}^M = \left(\tilde{A}_{at}\right)^{\frac{\sigma(1-\alpha)}{\alpha+\sigma(1-\alpha)}} A_{nt}^{\frac{\alpha}{\alpha+\sigma(1-\alpha)}}$. Then, in a modern economy, we have*

$$p_t = \sigma(1-\alpha) \left(\frac{\alpha}{\sigma(1-\alpha)\pi} \right)^{\frac{\alpha}{\alpha+\sigma(1-\alpha)}} \bar{c}^{-\frac{(1-\sigma)(1-\alpha)}{\alpha+\sigma(1-\alpha)}} \frac{\tilde{A}_{at}^M}{A_{nt}}, \quad (25)$$

$$w_t = \sigma(1-\alpha) \left(\frac{\alpha}{\sigma(1-\alpha)\pi} \right)^{\frac{\alpha}{\alpha+\sigma(1-\alpha)}} \bar{c}^{-\frac{(1-\sigma)(1-\alpha)}{\alpha+\sigma(1-\alpha)}} \tilde{A}_{at}^M, \quad (26)$$

$$r_t = (1-\alpha)(1-\sigma)\bar{c}^{\frac{N_t}{Z}}, \quad (27)$$

$$y_t = (1-\alpha) \left[1 - \sigma + \sigma \bar{c}^{-\frac{1}{\alpha+\sigma(1-\alpha)}} \left(\frac{\alpha}{\sigma(1-\alpha)\pi} \right)^{\frac{\alpha}{\alpha+\sigma(1-\alpha)}} \tilde{A}_{at}^M \right] \bar{c}, \quad (28)$$

$$\frac{L_{at}}{N_t} = \bar{c}^{\frac{1}{\alpha+\sigma(1-\alpha)}} \left(\frac{\sigma(1-\alpha)\pi}{\alpha} \right)^{\frac{\alpha}{\alpha+\sigma(1-\alpha)}} \tilde{A}_{at}^{M-1}. \quad (29)$$

Note that in the modern economy, income per worker is a linear function of $\tilde{A}_{at}^M = \left(\tilde{A}_{at}^T\right)^{\frac{\sigma(1-\alpha)}{\alpha+\sigma(1-\alpha)}} A_{nt}^{\frac{\alpha}{\alpha+\sigma(1-\alpha)}}$, which is a geometric average of the TFPs in both sectors. That is,

TFP growth in both sectors will contribute to the growth of per capita income. The growth rate of $\tilde{A}_{at}^M$ is given by

$$\begin{aligned}\frac{\tilde{A}_{at+1}^M}{\tilde{A}_{at}^M} &= \left(\frac{\tilde{A}_{at+1}^T}{\tilde{A}_{at}^T} \right)^{\frac{\sigma(1-\alpha)}{\alpha+\sigma(1-\alpha)}} \left(\frac{A_{nt+1}}{A_{nt}} \right)^{\frac{\alpha}{\alpha+\sigma(1-\alpha)}} \\ &= \left(\frac{A_{at+1}}{A_{at}} \right)^{\frac{\sigma(1-\alpha)}{\alpha+\sigma(1-\alpha)}} \left(\frac{A_{nt+1}}{A_{nt}} \right)^{\frac{\alpha}{\alpha+\sigma(1-\alpha)}} [g(y_t)]^{-\frac{(1-\sigma)(1-\alpha)}{\alpha+\sigma(1-\alpha)}}.\end{aligned}$$

Suppose that both A_{at} and A_{nt} grows at constant rates, γ_a and γ_n respectively. Then, we have

$$\frac{\tilde{A}_{at+1}^M}{\tilde{A}_{at}^M} = (\gamma_a)^{\frac{\sigma(1-\alpha)}{\alpha+\sigma(1-\alpha)}} (\gamma_n)^{\frac{\alpha}{\alpha+\sigma(1-\alpha)}} [g(y_t)]^{-\frac{(1-\sigma)(1-\alpha)}{\alpha+\sigma(1-\alpha)}}.$$

So, as long as

$$(\gamma_a)^{\sigma(1-\alpha)} (\gamma_n)^\alpha > [g(\hat{y})]^{(1-\sigma)(1-\alpha)},$$

the $\tilde{A}_{at}^M$ will grow without bound and so will the per capita income.

Summarizing all the results above, we have

Proposition 6 *Under Assumption 1 and the assumption that $(\gamma_a)^{\sigma(1-\alpha)} (\gamma_n)^\alpha > [g(\hat{y})]^{(1-\sigma)(1-\alpha)}$, the economy that starsts in a Malthusian steady state will at some point moves into a mixed economy and eventually to a modern economy with sustained growth of income per capita. During this process, the relative price of nonagricultural goods decline in the traditional economy, stays constant in the mixed economy, and then declines further in the modern economy. The employment share of agriculture will start to decline in the mixed economy and converges to zero in the modern economy.*

5 Data

We have compiled from published sources aggregate historical data for the English economy between 1700 and 1909;⁴ this time period covers the complete process of the industrial revolution. We construct and analyze data series on a decennial basis, with an emphasis on long-term trend. A decade consists of ten years starting with a rounded year of ten, i.e., 1700-1709.⁵ The data series consist of constructed indices of real GDP per capita in England, prices for agricultural output, prices for principle industrial products, agricultural mechanization, fertilizer utilization, labor share in agriculture, real average day wages of adult farm workers, and land rent. We also rely on studies of historical performance of the English economy to infer exogenous improvements in total factor productivity in agricultural and non-agricultural production. Our data are drawn heavily from the works of Gregory Clark for earlier periods of the English economy and the works of B. R. Mitchell on British historical statistics for later periods.

The compilation of data requires an extensive review of statistical sources as well as historical studies of the British economy. Reliability and completeness are two important criteria for data selection. We draw data from authoritative historical studies to form the benchmark case, and then analyze the reliability of the benchmark in light of alternative data. While data collection is still incomplete, in what follows, we explain data sources for a subset of the variables that are essential for understanding this preliminary draft.

⁴The quantitative analysis will focus on aggregate economic performance of England, rather than the United Kingdom. This choice reflects the fact that key economic variables in early historical periods are available for England, but not for Wales, Scotland, and Northern Ireland.

⁵By the same principle, we choose 1909 as the ending year despite the fact that many indices of historical variables has an ending year 1912. Similar to early periods, the ten years of data starting with 1900 and ending with 1909 form a decade. We do not use three years of data between 1910 and 1912 to represent economic performance for the next decade, as it may introduce sharp temporal fluctuations due to limited sample points.

5.1 GDP Per Capita

Clark (2001) calculates a long series of aggregate nominal GDP for England and Wales based on the income approach. Because this series ends in the decade 1860-9, supplementary information is needed to extend the coverage to 1900-9. Mitchell (1988) reports yearly aggregate nominal GDP for the United Kingdom covering the period 1855-1980. We take arithmetic average of Mitchell's yearly GDP numbers within individual decades to form a decennial GDP series for the period 1860-1909. To derive nominal GDP per capita for the period 1700-1869, we divide the aggregate GDP series for England and Wales by the population of England and Wales. For the period 1870-1909, we divide the aggregate GDP series for the United Kingdom by the population of England, Wales, Scotland and Northern Ireland. With the assumption that GDP per capita was the same across regions in the United Kingdom, as economic historians often assume in the construction of income data (e.g., Clark, 2001), we take this constructed index as nominal GDP per capita for England for the period 1700-1909.

In order to build a historical index of real GDP per capita, we again use information from the above two sources. Clark (2001) provides a GDP deflator based on price series of 12 broad commodity groups; however, this series also ends in the decade 1860-9. To continue building a GDP deflator for the remaining decades, we use the Rousseaux's overall price index, which is an unweighted average of the indices of total agricultural products and of principle industrial products (Mitchell, 1988). Connecting the two deflator series and applying it to the nominal GDP per capita generate an index of real GDP per capita for the period 1700-1909.

5.2 Relative Prices

Prices of agricultural products: Based on published sources, Clark (2004) uses a consistent method to construct an annual price series for English agricultural output in the years 1209-1912. The series consists of information from 26 commodities: wheat, barley, oats, rye, peas, beans, potatoes, hops, straw, mustard seed, saffron, hay, beef, mutton, pork, bacon, tallow, eggs, milk, cheese, butter, wool, firewood, timber, cider, and honey. We take the arithmetic average of farm price indices within decades to form our decennial agricultural price series for the period 1700-1909.

Prices of nonagricultural products: There is no single data source that provides aggregate price series on nonagricultural production for the English economy in that historical period. However, the work of Mitchell and Deane (1962) on British historical statistics contains sufficient information that enables the construction of a long price series for principle industrial products. For the periods between 1700 and 1800, we use the Schumpeter-Gilboy price indices for producer's goods, which consist of 12 industrial products – bricks, coal, lead, pantiles, hemp, leather backs, train oil, tallow, lime, glue, and copper. This series ends in 1801.

To continue with the price series for the period between 1800 and 1913, we adopt the Rousseaux price index for principle industrial products, which has significant overlapping of product coverage with the Schumpeter-Gilboy price index (Mitchell and Deane, 1962). From 1800 to 1850, the Rousseaux price index covers coal, pig iron, mercury, tin, lead, copper, hemp, cotton, wool, flax, tar, tobacco, hides, skins, tallow, hair, silk, and building wood. For the years between 1850 and 1909, the index covers coal, pig iron, tin, lead, copper, hemp, cotton, wool, linseed oil, palm oil, flax, tar, jute, tobacco, hides, skins, foreign tallow, native tallow, silk, and building wood. We connect the two price indices and use it as constructed price series for the nonagricultural sector.

5.3 Modern Agricultural Inputs

Walton (1979) and Overton (1996) document the diffusion of farm mechanization in England in the early 1800s. In the next section, we provide some details on data sources and information on the adoption of agricultural machineries. We plan to construct a quantitative index on the diffusion of mechanization in England for the interested period based on information on adopting eight types of machines, as documented in Figure 4.

5.4 TFP Growth

There is no single study that provides historical estimates for total factor productivity growth in agriculture and non-agricultural production for the English economy. However, two separate studies provide systematic estimates on A_{at} and A_{nt} . Drawing data from multiple sources on land rental values, wages, returns on capital and output prices, Clark (2002) presents estimates of total factor productivity by decades for English agriculture all the way from 1500 to 1912. His estimates for the period 1700 to 1909 are taken to compute decennial growth rate of TFP in agriculture. We regress the logarithm of the decennial TFP index on a time variable, and use the estimated time coefficient as decennial exogenous improvements in productivity in traditional agriculture ($\Delta \ln A_a$), which is consistent with a constant TFP growth, as specified in the model.

The pioneering work of Deane and Cole (1969) presents estimates of aggregate economic performance of the British economy for the period 1688-1959. However, as most economic historians agree, output growth during the industrial revolution was much slower than the view associated with the original estimates of Deane and Cole. To obtain an estimate for A_{nt} , we rely on the revised estimates of Crafts and Harley (1992) as primary data source. Their results are widely accepted among economic historians, and used in recent quantitative studies of the British aggregate performance (e.g., Stokey, 2001).

We use estimates of output growth per worker of British industrial production to approximate exogenous improvements in TFP for the nonagricultural sector, an approach that is consistent with our model specification of linear production technology. More specifically, we first use the indices of British industrial production for the period 1700-1857, as covered in Crafts and Harley (1992), to compute the rate of industrial output growth ($\Delta Y_n/Y_n$). Taking their estimates of labor force growth ($\Delta L_n/L_n$), we obtain the parameter of TFP growth in the nonagricultural sector, i.e., $\Delta A_n/A_n = \Delta Y_n/Y_n - \Delta L_n/L_n$.

6 Quantitative Analysis

In this section, we describe the growth experience in England for the period between 1700 and 1909, calibrate our benchmark economy to account for observed changes in key variables for the English economy, and use the calibrated model to investigate the quantitative effects of initial conditions and policies on the timing of transition.

6.1 The English Economy from 1700 to 1909

Table 1 presents historical statistics of England for the period between 1700 and 1909. In the first century, real per capita GDP and wages fluctuated around constant levels, exhibiting typical features of a Malthusian regime. Gradual declines in the share of labor in agriculture is consistent with slow increases in agricultural productivity during that period. Starting in the early 1800s, however, per capita GDP growth, changes in real wages and structural transformation began to accelerate. Between 1820 and 1909, per capita GDP increased 1.59 times, wages were nearly doubled, and agricultural labor share dropped from 33% to 10%. By 1909, England was far ahead of other countries in the extent of structural transformation. These coordinated changes in key economic variables mark the emergency from the Malthusian regime.

The escape from Malthusian stagnation occurred concurrently with the diffusion of farm mechanization in England in the early 1800s. Despite the sparsity of historical data, John Walton used creatively farm sales advertisements to quantify the adoption of farm machines for selective regions in England and Wales for the years from 1753 to 1880 (Walton 1979; Overton 1996, 123). Figure 4 reports the percentage of dispersal sales of farm stocks that contain the sale of specific types of farm machinery.⁶ The use of drills, haymaking machines, and chaff and threshing machines started around 1810. Horse hoes and turnip cutters began to appear around 1820. The adoption of these machines, except for threshing machines, continued an uptrend until 1880.

As equations (18) implies, the relative price of industrial to agricultural good should fall continuously in the Malthusian steady state as a result of growth of the nonagricultural TFP. Then, during the transition to modern growth, the relative price should stay constant, as equation (19) demonstrates. In Figure 2, the observed English experience shows exactly this pattern. The relative price index declined persistently from above 2 to 1 in 1820, and then fluctuated around that level until 1900-9.

The last column of table 1 shows that real wages stayed around a constant level between 1700 and 1820, and then rose continuously. These observed patterns are also consistent with the implications of the model for real wages in the Malthusian regime, if the TFP growth in agriculture is low, and during the transition, if the TFP growth in industry is significant.

⁶The original data consist of 3,115 advertisements that appeared in the in the *Reading Mercury* and *Jackson's Oxford Journal* in Oxfordshire in England. A smaller sample of advertisements for the period from 1840 to 1867, marked by dotted lines in figure 4, comes from the two Shrewsbury weekly newspapers, Eddowes's Shropshire Journal and the Shrewsbury Chronicle, covering Shropshire and the adjacent parts of Herefordshire, Staffordshire, Cheshire and Wales.

6.2 Calibration of the Model

Each period in the model consists of 10 years, and the initial period of the model corresponds to 1700. Now we discuss how we choose the values for the parameters in the model.

Technology parameters. We set the labor share in traditional agriculture σ to 0.6, consistent with Hansen and Prescott (2002) and Ngai (2000). Following Restuccia, Yang and Zhu (2004), we set the share of intermediate input in modern agriculture α to 0.4. The value of land endowment $\bar{Z}$ and the cost of intermediate inputs π are both normalized to one.

Productivity growth rates. Clark (2002) provides estimates on the total factor productivity in English agriculture for the period between 1500 and 1910 based on estimated factor prices, their input shares and output prices. His estimates show that the TFP in agriculture grew at a slow rate prior to 1860 and then grew more rapidly between 1860 to 1910. Based on this fact, we assume that A_{at} grows at a constant rate γ_{a1} for the period between 1700 to 1860 and then grows at another constant rate γ_{a2} for the period after 1860. Since in our model the TFP in agriculture is A_{at}^σ , so the TFP growth rate in the two periods are γ_{a1}^σ and γ_{a2}^σ , respectively. For each of the two periods, $i = 1, 2$, we regress $\ln \text{TFP}$ on calendar year to get a slope coefficient ξ_i , which is the annual exponential growth rate. We then calculate γ_{ai} according to the formula $\gamma_{ai} = \exp(10\xi_i/\sigma)$ to get the decennial growth rate for A_{at} . [Dennis, please add discussion on the estimates of labor productivity growth in nonagriculture, or A_{nt}]

Initial values. We normalize the initial value of income y_0 to one, and we set N_0 to the population level in England and Wales in 1700. From equation (14) and (15), we can see that in the traditional economy

$$y_t = [1 - \sigma + \sigma \left(\frac{L_{at}}{N_t} \right)] \bar{c}.$$

>From Clark (2002??) we know that in 1700 in England (and Wales?) the fraction of labor in agriculture is 0.55. So the above equation can be used to pin down the value of $\bar{c}$ such that the model implied initial level of income y_0 is one. Given N_0 and the calibrated value of $\bar{c}$ we can then use equation (15) to pin down the value of A_{a0} such that the model implied value of L_{a0}/N_0 is 0.55. Because agricultural mechanization in England emerged in the beginning of the nineteenth century, more precisely around 1820 (Walton 1979; Overton, 1996), we choose the initial value of A_{n0} so that in our model the use of modern agricultural technology would be adopted in agriculture in that year.

Population growth profile.

6.3 Simulation of the Model

We simulate the English economy for the years from 1700 to 1910, focusing on four key variables—relative price of industrial product to agricultural product, the fraction of land used intermediate input in agricultural production, per capita income, and fraction of labor in agriculture.

As figures 5 to 8 show, the time sequences of the four variables predicted by the model perform well quantitatively in accounting for observed changes in the English economy for the period. Admittedly, the predications are not perfect. A discrepancy between simulated and actual changes in agricultural labor share is noticeable in figure 7: the observed decline is gradual and smooth, while the predicted time path has a kink in 1820, although the long-term trends are vary similar. Another discrepancy is that the predicted relative price of industrial versus agricultural good exceeds the actual price in the early 1700s, but again the long-term patterns are very similar.

The findings from simulation support a coherent and unified view on the importance of agricultural modernization in making the transition from stagnation to growth. For a

long time, a faster rate of growth in the industrial TFP than agricultural TFP may lead to declines in the price ratio (fig. 6). But, before the relative price hitting a low threshold level, farmers would not find it profitable to use farm machinery, thus agricultural production cannot break away from diminishing returns due to fixed supply of land. This defines the traditional regime in which real wages and per capita output remain constant. Structural transformation is slow, constrained by limited growth in agricultural productivity.

However, when the relative price of industrial product hits the low critical level, profit-maximizing farmers will begin to adopt modern agricultural inputs produced in the industrial sector. Agricultural modernization will initiate a virtuous cycle. Structural transformation will accelerate, as farmer have incentives to substitute modern agricultural inputs for labor (fig. 7). Per capita income growth will also break out to a higher rate of growth (fig. 5), as TFP growth in industry and agriculture will both contribute to aggregate growth [equ. (13) and (21)]. During the transition, the relative price will stay constant (fig.6), but the real income will rise, as it is driven by the fast TFP growth in industry.

Our simulation analysis is still incomplete. We plan to study the time paths of land prices and the use of modern agricultural inputs, after we compile the historical information on these variables.

7 Conclusion

(to be completed)

8 Appendix: Proof of the Propositions

Proof of Proposition 1.

Let $\tilde{A}_{at} = A_{at}(\bar{Z}/N_t)^{\frac{1-\sigma}{\sigma}}$. We first solve for the equilibrium prices in the three possible

cases.

(1) Traditional technology only: $Y_{at}^T = \bar{Z}^{1-\sigma} (A_{at} L_{at})^\sigma$. From the market clearing condition (7) we have $N_t \bar{c} = \bar{Z}^{1-\sigma} (A_{at} L_{at})^\sigma$, which implies that

$$\begin{aligned} \frac{L_{at}}{N_t} &= \bar{c}^{\frac{1}{\sigma}} \tilde{A}_{at}^{-1}, \\ w_t &= \sigma \frac{Y_{at}}{L_{at}} = \sigma \frac{Y_{at}}{N_t} \left(\frac{L_{at}}{N_t} \right)^{-1} = \sigma \bar{c}^{\frac{\sigma-1}{\sigma}} \tilde{A}_{at}, \\ p_t^T &= \frac{w_t^T}{A_{nt}} = \sigma \bar{c}^{\frac{\sigma-1}{\sigma}} \frac{\tilde{A}_{at}}{A_{nt}}. \end{aligned}$$

(2) Modern technology only: $Y_{at} = \left(\frac{\alpha}{\pi p_t} \right)^{\alpha/(1-\alpha)} \bar{Z}^{1-\sigma} (A_{at} L_{at})^\sigma$. Again, from the market clearing condition (7) we have $N_t \bar{c} = \left(\frac{\alpha}{\pi p_t} \right)^{\alpha/(1-\alpha)} \bar{Z}^{1-\sigma} (A_{at} L_{at})^\sigma$, which implies that

$$\frac{L_{at}}{N_t} = \bar{c}^{\frac{1}{\sigma}} \tilde{A}_{at}^{-1} \left(\frac{\alpha}{\pi p_t} \right)^{-\frac{\alpha}{\sigma(1-\alpha)}}.$$

But

$$w_t = \sigma(1-\alpha) \frac{Y_{at}}{L_{at}} = \sigma(1-\alpha) \frac{Y_{at}}{N_t} \left(\frac{L_{at}}{N_t} \right)^{-1} = \sigma(1-\alpha) \bar{c}^{\frac{\sigma-1}{\sigma}} \left(\frac{\alpha}{\pi p_t} \right)^{\frac{\alpha}{\sigma(1-\alpha)}} \tilde{A}_{at}.$$

>From the labor market condition $w_t = p_t A_{nt}$, we have

$$\sigma(1-\alpha) \bar{c}^{\frac{\sigma-1}{\sigma}} \left(\frac{\alpha}{\pi p_t} \right)^{\frac{\alpha}{\sigma(1-\alpha)}} \tilde{A}_{at} = p_t A_{nt}$$

which yields the following

$$\begin{aligned} p_t^M &= \left(\frac{\sigma(1-\alpha) \tilde{A}_{at}}{A_{nt}} \right)^{\frac{\sigma(1-\alpha)}{\alpha+\sigma(1-\alpha)}} \left(\frac{\alpha}{\pi} \right)^{\frac{\alpha}{\alpha+\sigma(1-\alpha)}} \bar{c}^{-\frac{(1-\sigma)(1-\alpha)}{\alpha+\sigma(1-\alpha)}} \\ &= \sigma(1-\alpha) \left(\frac{\alpha}{\sigma(1-\alpha)\pi} \right)^{\frac{\alpha}{\alpha+\sigma(1-\alpha)}} \bar{c}^{-\frac{(1-\sigma)(1-\alpha)}{\alpha+\sigma(1-\alpha)}} \frac{\tilde{A}_{at}^M}{A_{nt}}, \end{aligned}$$

where

$$\tilde{A}_{at}^M = \left(\tilde{A}_{at}^T \right)^{\frac{\sigma(1-\alpha)}{\alpha+\sigma(1-\alpha)}} A_{nt}^{\frac{\alpha}{\alpha+\sigma(1-\alpha)}},$$

and

$$\begin{aligned} \frac{L_{at}}{L_t} &= \bar{c}^{\frac{1}{\alpha+\sigma(1-\alpha)}} \left(\frac{\sigma(1-\alpha)\pi}{\alpha} \right)^{\frac{\alpha}{\alpha+\sigma(1-\alpha)}} \tilde{A}_{at}^{M-1}, \\ w_t &= p_t A_{nt} = \sigma(1-\alpha) \left(\frac{\alpha}{\sigma(1-\alpha)\pi} \right)^{\frac{\alpha}{\alpha+\sigma(1-\alpha)}} \bar{c}^{-\frac{(1-\sigma)(1-\alpha)}{\alpha+\sigma(1-\alpha)}} \tilde{A}_{at}^M. \end{aligned}$$

(3) Both technologies are used: In this case, we have

$$\begin{aligned} w_t &= \sigma(1-\alpha) \left(\frac{\alpha}{\pi p_t} \right)^{\frac{\alpha}{1-\alpha}} A_{at}^\sigma \left(\frac{Z_t^M}{L_{at}^M} \right)^{1-\sigma} = \sigma A_{at}^\sigma \left(\frac{Z_t^T}{L_{at}^T} \right)^{1-\sigma} \\ 1 &= (1-\alpha) \left(\frac{\alpha}{\pi p_t} \right)^{\frac{\alpha}{1-\alpha}}. \end{aligned}$$

Thus, we have

$$\begin{aligned} \frac{Z_t^M}{L_{at}^M} &= \frac{Z_t^T}{L_{at}^T} = \frac{\bar{Z}}{L_{at}}, \\ w_t &= \sigma A_{at}^\sigma \left(\frac{\bar{Z}}{L_{at}} \right)^{1-\sigma} = \sigma \tilde{A}_{at}^\sigma \left(\frac{L_{at}}{N_t} \right)^{\sigma-1}, \\ p_t^{mixed} &= \frac{\alpha}{\pi} (1-\alpha)^{\frac{1-\alpha}{\alpha}}. \end{aligned}$$

>From the labor market condition, $w_t = p_t A_{nt}$ we have

$$\begin{aligned} \frac{L_{at}}{N_t} &= (1-\alpha)^{\frac{\alpha-1}{\alpha(1-\sigma)}} \left(\frac{\sigma\pi}{\alpha A_{nt}} \right)^{\frac{1}{1-\sigma}} \tilde{A}_{at}^{\frac{\sigma}{1-\sigma}}, \\ w_t &= \frac{\alpha}{\pi} (1-\alpha)^{\frac{1-\alpha}{\alpha}} A_{nt}. \end{aligned}$$

For given $\tilde{A}_{at}$ and A_{nt} , the economy uses traditional technology if and only if

$$(1 - \alpha) \left(\frac{\alpha}{\pi p_t^T} \right)^{\frac{\alpha}{1-\alpha}} \leq 1$$

and

$$(1 - \alpha) \left(\frac{\alpha}{\pi p_t^M} \right)^{\frac{\alpha}{1-\alpha}} < 1.$$

This requires that

$$\begin{aligned} p_t^T &= \sigma \bar{c}^{\frac{\sigma-1}{\sigma}} \frac{\tilde{A}_{at}}{A_{nt}} \geq \frac{\alpha}{\pi} (1 - \alpha)^{\frac{1-\alpha}{\alpha}} \\ p_t^M &= \left(\frac{\sigma(1-\alpha)\tilde{A}_{at}}{A_{nt}} \right)^{\frac{\sigma(1-\alpha)}{\alpha+\sigma(1-\alpha)}} \left(\frac{\alpha}{\pi} \right)^{\frac{\alpha}{\alpha+\sigma(1-\alpha)}} \bar{c}^{-\frac{(1-\sigma)(1-\alpha)}{\alpha+\sigma(1-\alpha)}} > \frac{\alpha}{\pi} (1 - \alpha)^{\frac{1-\alpha}{\alpha}}. \end{aligned}$$

The later inequality is equivalent to

$$(p_t^T)^{\frac{\sigma(1-\alpha)}{\alpha+\sigma(1-\alpha)}} > \left(\frac{\alpha}{\pi} \right)^{\frac{\sigma(1-\alpha)}{\alpha+\sigma(1-\alpha)}} (1 - \alpha)^{\frac{1-\alpha}{\alpha} \frac{\alpha(1-\sigma)+\sigma(1-\alpha)}{\alpha+\sigma(1-\alpha)}}$$

or

$$p_t^T > \frac{\alpha}{\pi} (1 - \alpha)^{\frac{\alpha(1-\sigma)+\sigma(1-\alpha)}{\alpha\sigma}} = \frac{\alpha}{\pi} (1 - \alpha)^{\frac{1-\alpha}{\alpha}} (1 - \alpha)^{\frac{1-\sigma}{\sigma}}.$$

Apparently, as long as $p_t^T \geq \frac{\alpha}{\pi} (1 - \alpha)^{\frac{1-\alpha}{\alpha}}$, then the above inequality is automatically satisfied.

Thus, the necessary and sufficient condition for the economy to use traditional technology is

$$\sigma \bar{c}^{\frac{\sigma-1}{\sigma}} \frac{\tilde{A}_{at}}{A_{nt}} \geq \frac{\alpha}{\pi} (1 - \alpha)^{\frac{1-\alpha}{\alpha}},$$

or

$$\frac{A_{nt}}{\pi A_{at} \left(\bar{Z}/N_t \right)^{\frac{1-\sigma}{\sigma}}} \leq \Phi_l$$

For given $\tilde{A}_{at}$ and A_{nt} , the economy uses modern technology if and only if

$$(1 - \alpha) \left(\frac{\alpha}{\pi p_t^T} \right)^{\frac{\alpha}{1-\alpha}} > 1$$

and

$$(1 - \alpha) \left(\frac{\alpha}{\pi p_t^M} \right)^{\frac{\alpha}{1-\alpha}} \geq 1.$$

Again, the two conditions can be written as

$$\begin{aligned} p_t^T &= \sigma \bar{c}^{\frac{\sigma-1}{\sigma}} \frac{\tilde{A}_{at}}{A_{nt}} < \frac{\alpha}{\pi} (1 - \alpha)^{\frac{1-\alpha}{\alpha}}, \\ p_t^T &= \sigma \bar{c}^{\frac{\sigma-1}{\sigma}} \frac{\tilde{A}_{at}}{A_{nt}} \leq \frac{\alpha}{\pi} (1 - \alpha)^{\frac{1-\alpha}{\alpha}} (1 - \alpha)^{\frac{1-\sigma}{\sigma}} \end{aligned}$$

Both will be satisfied if the later is satisfied. So, the necessary and sufficient condition for the economy to use modern technology only is

$$\sigma \bar{c}^{\frac{\sigma-1}{\sigma}} \frac{\tilde{A}_{at}}{A_{nt}} \leq \frac{\alpha}{\pi} (1 - \alpha)^{\frac{1-\alpha}{\alpha}} (1 - \alpha)^{\frac{1-\sigma}{\sigma}},$$

or

$$\frac{A_{nt}}{\pi A_{at} (\bar{Z}/N_t)^{\frac{1-\sigma}{\sigma}}} \geq \Phi_h.$$

When

$$\Phi_l < \frac{A_{nt}}{\pi A_{at} (\bar{Z}/N_t)^{\frac{1-\sigma}{\sigma}}} < \Phi_h,$$

neither traditional technology only nor modern technology only can be an equilibrium. The only possible equilibrium is when both technologies are used.

Proof of Proposition 2.

The derivations of expressions for price p_t , wage w_t and employment share L_{at}/N_t are

already given in the proof of Proposition 1. For the rental price of land, using the property of Cobb-Douglas production function, we have

$$r_t = (1 - \sigma)Y_{at}/\bar{Z} = (1 - \sigma)N_t\bar{c}/\bar{Z}.$$

Thus,

$$y_t = w_t + r_t\bar{Z}/N_t = \left[1 - \sigma + \sigma\bar{c}^{-\frac{1}{\sigma}}\tilde{A}_{at}\right]\bar{c}.$$

Proof of Proposition 3.

In the steady state, equation (16) becomes

$$1 = \gamma_a [g(y^*)]^{-\frac{1-\sigma}{\sigma}}$$

or

$$g(y^*) = \gamma_a^{\frac{\sigma}{1-\sigma}}.$$

Under Assumption 1, g is continuous and strictly increasing over the interval $[\bar{c}, \hat{y}]$ and $g(\bar{c}) < 1 \leq \gamma_a^{\frac{\sigma}{1-\sigma}} < g(\hat{y})$. Therefore, there exists a unique $y^* \in (\bar{c}, \hat{y})$ such that $g(y^*) = \gamma_a^{\frac{\sigma}{1-\sigma}}$. The corresponding $\tilde{A}_a^*$ is given by

$$y^* = \left[1 - \sigma + \sigma\bar{c}^{-\frac{1}{\sigma}}\tilde{A}_a^*\right]\bar{c}$$

or

$$\tilde{A}_a^* = \left[\frac{y^*}{\bar{c}} - 1 + \sigma\right]\sigma^{-1}\bar{c}^{\frac{1}{\sigma}} > 0.$$

Proof of Proposition 4.

For the rental price of land, we have

$$r_t = (1 - \sigma)Y_{at}^T/Z_t^T = (1 - \sigma) \left(A_{at} \frac{L_{at}^T}{Z_t^T} \right)^\sigma = (1 - \sigma) \left(A_{at} \frac{L_{at}}{\bar{Z}} \right)^\sigma.$$

Substituting the expression for L_{at}/N_t derived in the proof of Proposition 1 into the equation above yields equation (21). (22) can be derived directly from the expression for w_t derived in the proof of Proposition 1 and the expression for rental price that is just derived. To derive the expression for the percentage of land used for modern agricultural production, note that the goods market clearing conditions require that

$$\begin{aligned} N_t \bar{c} &= (Z_t^T)^{1-\sigma} (A_{at} L_{at}^T)^\sigma + [(Z_t^M)^{1-\sigma} (A_{at} L_{at}^M)^\sigma]^{1-\alpha} X_t^\alpha \\ &= (Z_t^T)^{1-\sigma} (A_{at} L_{at}^T)^\sigma + \left(\frac{\alpha}{\pi p_t} \right)^{\alpha/(1-\alpha)} (Z_t^M)^{1-\sigma} (A_{at} L_{at}^M)^\sigma. \end{aligned}$$

In the mixed economy, we have

$$(1 - \alpha) \left(\frac{\alpha}{\pi p_t} \right)^{\alpha/(1-\alpha)} = 1$$

or

$$\left(\frac{\alpha}{\pi p_t} \right)^{\alpha/(1-\alpha)} = (1 - \alpha)^{-1}.$$

So,

$$\begin{aligned} N_t \bar{c} &= (Z_t^T)^{1-\sigma} (A_{at} L_{at}^T)^\sigma + (1 - \alpha)^{-1} (Z_t^M)^{1-\sigma} (A_{at} L_{at}^M)^\sigma \\ &= Z_t^T \left(A_{at} \frac{L_{at}^T}{Z_t^T} \right)^\sigma + (1 - \alpha)^{-1} Z_t^M \left(A_{at} \frac{L_{at}^M}{Z_t^M} \right)^\sigma. \end{aligned}$$

>From the proof of Proposition 1, we have

$$\frac{L_{at}^T}{Z_t^T} = \frac{L_{at}^M}{Z_t^M} = \frac{L_{at}}{\bar{Z}}.$$

So, we have

$$\begin{aligned} N_t \bar{c} &= [Z_t^T + (1 - \alpha)^{-1} Z_t^M] \left(A_{at} \frac{L_{at}}{\bar{Z}} \right)^\sigma \\ &= [\bar{Z} + \alpha(1 - \alpha)^{-1} Z_t^M] \left(A_{at} \frac{N_t}{\bar{Z}} \right)^\sigma \left(\frac{L_{at}}{N_t} \right)^\sigma. \end{aligned}$$

Substituting the expression for L_{at}/N_t derived in the proof of Proposition 1 into the equation above and solving for $Z_t^M/\bar{Z}$ yields the equation (24).

Proof of Proposition 5.

Similar to that of Proposition 2.

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Table 1: Historical Statistics of England: 1700-1910

Year	Real per capita GDP ¹	Relative price ¹ (Pn/Pa)	Labor share in agriculture ²	Use of farm machinery No. ≥ 1 ³	Use of farm machinery No. ≥ 2 ³	Real wages ⁴
1700-9	0.80	2.14	0.55	0.00	0.00	1.01
1710-9	0.79	1.89	0.54	0.00	0.00	0.95
1720-9	0.83	1.83	0.53	0.00	0.00	0.97
1730-9	0.93	1.91	0.52	0.00	0.00	1.13
1740-9	0.85	1.95	0.52	0.00	0.00	1.10
1750-9	0.86	1.77	0.53	0.00	0.00	1.01
1760-9	0.84	1.83	0.49	0.00	0.00	0.98
1770-9	0.85	1.59	0.47	0.00	0.00	0.92
1780-9	0.82	1.71	0.44	0.00	0.00	0.97
1790-9	0.82	1.54	0.40	0.00	0.00	0.89
1800-9	0.84	1.29	0.37	0.07	0.00	0.81
1810-9	0.91	1.13	0.35	0.13	0.01	0.87
1820-9	1.00	1.00	0.33	0.43	0.07	1.00
1830-9	1.02	0.97	0.30	0.51	0.10	1.05
1840-9	1.06	0.96	0.26	0.72	0.24	1.11
1850-9	1.07	1.08	0.24	0.82	0.40	1.18
1860-9	1.08	1.03	0.21	0.89	0.57	1.18
1870-9	1.43	0.94	0.17	0.94	0.72	1.44
1880-9	1.76	0.90	0.15	0.98	0.86	1.69
1890-9	2.32	0.91	0.12	na	na	2.02
1900-9	2.37	1.04	0.10	na	na	2.08

Source: 1. See section 5.3 for variable descriptions; 2. Share of males in agriculture, Clark (2002, 12); 3. Mitchell (1962); these figures indicate the probability that a farm uses at least one or two machines for the specific decades; 4. Clark (2002, p.4).

Figure 1. Real Per Capita GDP: England, 1700-1909

Figure 2. Relative Price of Industrial to Agricultural Products:
England, 1700-1909

Figure 4. The diffusion of implements and machines in Oxfordshire and the Welsh borderland, 1750–1880. Source: Walton (1979), 25.

Figure 5: Real Per Capita Income, England: 1700-1909

Figure 6: Price Ratio of Industrial to Agricultural Product:
England, 1700-1909

Figure 7: Employment Share in Agriculture: England, 1700-1909

Figure 8: Agricultural Mechanization, England, 1700-1909