On policy interactions among nations: when do cooperation and commitment matter?

Hubert Kempf∗ Leopold von Thadden†

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Abstract

This paper offers a comprehensive framework to study commitment and cooperation issues in games with multiple policymakers. To reconcile some puzzles in the recent literature on policy interactions in monetary unions, we prove that games characterized by different commitment and cooperation schemes can have the same equilibrium if certain spillover effects vanish at the common equilibrium of these games. We provide a detailed discussion of these spillovers, showing that, in general, commitment and cooperation are non-trivial issues. Yet, models of the linear-quadratic variety with multiple policymakers can generate a ‘symbiotic’ result where commitment and cooperation issues are irrelevant and where the unique equilibrium of any game is the bliss point. The proposed framework is sufficiently general to allow for a broad discussion of policymaking in monetary unions and policy interactions among nations.

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∗Direction de la recherche, Banque de France and Université Paris-1 Panthéon-Sorbonne. e-mail: hubert.kempf@banque-france.fr.
†European Central Bank, Kaiserstrasse 29, D-60311 Frankfurt/Main, Germany. e-mail: leopold.von_thadden@ecb.int. The views expressed in this paper are those of the authors and do not necessarily reflect the views of the Banque de France and the European Central Bank.
1 Introduction

The literature on policy interactions among nations proves to be quite puzzling. Paradoxes abound, and very different views on possible gains and costs from policy cooperation have been developed. Similarly, the necessity to impose constraints on some or all policymakers is controversially debated, as witnessed, for example, by the recent debate on the Stability and Growth Pact devised in the European Monetary Union. There are also many examples for institutional constraints which have first been seen with high expectations before being entirely dismissed or abandoned. The currency board solution is quite telling in this respect. Once (in the '90s) seen as a major way to anchor expectations and obtain credibility, it is now commonly viewed as having been badly conceived in the case of Argentina and seen as a factor of the Argentinian crisis of 2001.

These controversies have clear counterparts at the academic level. A good example of the unsettling state of the discussion on policy interactions is provided by recent and conflicting analyses on policymaking in monetary unions. For example, Cooper and Kempf (2004) show that the ability of a central bank in a monetary union to commit with respect to national fiscal authorities affects the outcome of the policy-mix, as this device helps to resist pressures to monetize national deficits. Leaning in the same direction, Chari and Kehoe (2004) consider a monetary union model which does not allow for any direct spillovers between countries and has nevertheless the feature that equilibrium outcomes depend sensitively on patterns of cooperation and commitment, primarily driven by private sector coordination failures within countries and their relationship to monetary policy which is commonly shared between countries. However, in striking contrast to these findings, Dixit and Lambertini (2003) consider a monetary union model which allows for direct spillovers between countries and has nevertheless the feature that policymakers attain the same equilibrium outcome, irrespective of whether polices are coordinated or not and irrespective of the order of moves of players.

Given these conflicting views, we are in need of a comprehensive analysis of policy interactions which would provide us with useful tools for the evaluation of various commitment and coalitions assumptions. In the present paper our goal is more modest: it is to provide some clues for understanding why some authors do or do not obtain an irrelevance result with respect to commitment and coalitions assumptions.

To this end, we set up a generic model for the analysis of policy interactions among independent but interdependent policymakers. We provide some propositions which develop special conditions under which commitment patterns and coalition structures do not matter. More specifically, we prove that games characterized by different commitment and cooperation schemes can have the same equilibrium if certain spillover effects vanish at the common equilibrium of these games. We provide a detailed discussion of these spillovers, showing that, in general, commitment and cooperation are non-trivial issues. Yet, assuming consensus on the target values of all players, models of the linear-quadratic variety with multiple policymakers can generate a ‘symbiotic’ result where commitment and cooperation issues are irrelevant and where the unique equilibrium of any game corresponds to the social optimum.

As we show, these propositions can be used to resolve some of the puzzling findings on
the (ir)relevance of cooperation and commitment in the literature on monetary unions. Moreover, going beyond monetary union issues, the proposed framework is sufficiently general to allow for a broad discussion of policy interactions among nations. The remainder of the paper is structured as follows. Section 2 develops a general framework to study commitment and cooperation issues in games with multiple policymakers. We then offer a number of general propositions on the (ir)relevance of commitment patterns and coalition structures. In Section 3, we apply these propositions to discuss some recent contributions on policy interactions in monetary unions. In Section 4 we look at some contributions in the literature on international policy coordination. Section 5 concludes. Proofs and some technical issues are delegated to the Appendix.

2 A unifying framework for policy analysis

2.1 Agents and policymakers

There are \( N \) nations with index \( i \). In each nation, there coexist individual agents and policymakers. A player in this economy is either an individual agent or a policymaker. We refer to a player, be it an individual agent or a policymaker, as \( \xi \), and the set of players as \( \Xi \). His action is denoted by \( x_\xi \).

The world economy consists of a set of individual agents \( M \), with \( M \) agents indexed by \( j \). The population of the \( i \)th nation is denoted by \( m_i \). Agents are stacked according to their nationality. The \( m_1 \) first agents belong to Nation 1, the next \( m_2 \) agents belong to Nation 2 such that \( M = \sum_{i=1}^{n} m_i \).

There are \( P \) policymakers with index \( p \). The set of policymakers is denoted by \( \mathcal{P} \). A policymaker can be “national” or “international”. In principle, a policymaker may control a set of instruments. Here, for simplicity, we assume that a policymaker controls a unique instrument. There are \( p_i \) national policymakers in the \( i \)th nation. Moreover, there are \( p_{int} \) international policymakers, taking into consideration the payoffs of agents belonging to different nations. A central bank in a monetary union formed by sovereign nations is such an international policymaker. In sum, we have \( P = p_{int} + \sum_{i=1}^{n} p_i \). Of course, \( \Xi = M \cup \mathcal{P} \). The number of players is equal to \( X = M + P \).

Given these notations, we can present the payoff functions of all these players which express the interdependence both within and between nations. When we refer to a player in general, we denote the payoff function of player \( \xi \) as follows:

\[
V_\xi = V_\xi(x)
\]

where \( x \) denotes the vector of actions \((x_\xi, x_{-\xi})\). When we distinguish between the two types of players, we introduce the following notations: \( a_{ij} \) refers to the action of private agent \( j \) in country \( i \), \( a_i \) to the vector of actions taken by private agents in country \( i \), \( \tau_{ij} \) refers to the action of policymaker \( j \) in country \( i \), \( \tau_i \) to the vector of actions taken by policymakers in country \( i \). The payoff function of agent \( j \) is denoted as follows:

\[
U_j = U_j(a_1, \ldots, a_i, \ldots, a_M, \tau_1, \ldots, \tau_p, \ldots, \tau_P), \quad \forall j \in \mathcal{M}.
\]
The payoff function of policymaker $p$ is denoted as follows:

$$V_p = V_p(a_1, \ldots, a_i, \ldots, a_M, \tau_1, \ldots, \tau_p, \ldots, \tau_P), \quad \forall p \in P.$$  

The payoff function of a national policymaker may coincide with the payoff function of the representative private sector member of its own country. This case is to be discussed below in some applications.

### 2.2 Coalitions

Players may form coalitions. These coalitions may link players within a nation or cover players in different nations. A coalition is a subset of players who cooperate. Any coalition $C_\theta$ is defined by two characteristics: $i/$ it maximizes the joint welfare of its members; $ii/$ it decides over the actions to be taken by its members. Coalitions may be formed either by individual agents or by policymakers. For example, a trade-union is an agent-based coalition, and a fiscal cooperation scheme is a coalition of national treasuries. Mixed coalitions between individuals and policymakers are ruled out. The cardinal of a coalition $C_\theta$ is denoted by $\kappa^c_\theta$. For notational simplicity, we define coalitions in a broad sense so that they also include singletons (i.e. players acting in isolation) as special cases. An agent-based coalition $C_\theta$ is a subset of $\mathcal{M}$, leading to the restriction:

$$1 \leq \kappa^c_\theta \leq M.$$  

Similarly, in the case of a policymaker-based coalition, its cardinal is formed of at most $P$ members:

$$1 \leq \kappa^p_\theta \leq P.$$  

If $\kappa^p_\theta$ is equal to $M$ ($P$), then the grand agent-based (policymaker-based) international coalition forms; if the coalition covers all individual agents (policymakers) in a nation, then it is an agent-based (policymaker-based) national coalition for the $i$-th nation. If a coalition covers agents (policymakers) belonging to different nations, it is an agent-based (policymaker-based) international coalition. Notice that a membership to a coalition is different from the standard definition of membership. In particular, all agents belong to one coalition only (including singletons), and we rule out coalitions between private agents and policymakers. We denote by $\Theta$ the number of coalitions.

Then we define a partition of the economy as follows:

**Definition 1** A partition $C = \{C_1, \ldots, C_\theta, \ldots, C_\Theta\}$ is a set of coalitions such that:

$i/$

$$C_\theta \cap C_{\theta'} = \emptyset \quad \text{for } \theta \neq \theta'$$

$ii/$

$$\bigcup_{\theta=1}^\Theta C_\theta = \Xi.$$  

3
Some coalitions may be impossible (for legal, institutional, historical or custom reasons). In particular, it may be that some international coalitions are impossible. Any coalition $C_\theta$ fixes the strategies of its members simultaneously at a given stage $t$ and maximizes the joint welfare of its members:

$$W_\theta = \sum_{\xi \in C_\theta} \omega_\xi V_\xi(x)$$

In the case of agent-based or policymaker-based coalitions, we get:

Agent-based coalition : $W_\theta = \sum_{j \in C_\theta} \omega_j U_j(a_1, \ldots, a_j, \ldots, a_M, \tau_1, \ldots, \tau_p, \ldots, \tau_P)$

Policymaker-based coalition : $W_\theta = \sum_{p \in C_\theta} \omega_p V_p(a_1, \ldots, a_j, \ldots, a_M, \tau_1, \ldots, \tau_p, \ldots, \tau_P)$

where $\omega_j$ and $\omega_p$ denote the weights of agent $j$ and policymaker $p$ in their respective coalitions.

2.3 Commitment

We denote by $\Gamma$ a multi-stage multi-coalitions extensive form game. There are $T^\Gamma$ stages in this game, and we denote by $T^\Gamma$ the set of stages: $\{1, \ldots, T\}$. We assume that each player is allocated to a given stage. A player may form a coalition with players affected at the same stage and only plays once in the entire game, at a certain stage. This is the standard assumption made in macroeconomic games, excluding repeated games. Hence, to describe a game it is convenient to distinguish between i) the order of moves of players (for short: “commitment patterns”), determining at which stage any player acts, and ii) the “coalition structures” at each stage. In brief, a game is characterized by a commitment pattern $C$ and a coalition structure $C$. Notice that different commitment patterns imply different coalition structures (since singletons are considered as coalitions), but not vice versa. Formally we use the following:

**Definition 2** A commitment pattern $C$ is an application from $\Xi$ into the set of stages $T^\Gamma$.

We denote by $X_\xi$ the strategy space of player $\xi$, i.e. $x_\xi \in X_\xi$ for all $\xi \in \Xi$. Moreover, we denote by $W(\Gamma)$ the payoff vector consisting of the payoffs of all players involved in the game:

$$W(\Gamma) = (V_1, \ldots, V_\xi, \ldots, V_X).$$

In sum, a game $\Gamma$ is characterized by:

- a coalition structure $C$,
- a commitment pattern $C$,
- the information sets at each stage of the game, assuming there are $T$ stages in the extensive-form of the game (We denote by "$1$" the first period; $T$ denotes the last stage.)
- the strategy spaces $X_\xi$ for all players $\xi \in \Xi$.
- the payoff vector $W(\Gamma)$. 

4
2.4 Spillovers

This subsection offers a characterization of the marginal welfare effects of the actions of players. Given the existence of coalitions, it is important to distinguish between different types of spillovers. Generally speaking, the welfare effects of the action of any player can be decomposed into three distinct effects, namely the effects on his own welfare, the effects on the welfare of his coalition members (within-coalition spillover effects), and the effects on the welfare of players belonging to different coalitions (between-coalition spillover effects).

In the context of multi-stage games these effects do not only include contemporaneous effects on coalitions acting at the same stage, but also future effects on coalitions acting at subsequent stages. Spillover effects between agents will play a crucial role in the rest of our analysis. We shall use the following definition.

**Definition 3** For a given commitment pattern and coalition structure \((C, C')\) and a given vector of actions \(x\), consider a representative player \(\xi\) with action \(x_\xi\). Moreover, consider two further players \(\xi'\) and \(\xi''\), where \(\xi\) and \(\xi'\) belong to the same coalition \(C_θ\), while \(\xi''\) belongs to a coalition different from \(C_θ\). Then, to classify the impact of the action of agent \(\xi\) on other players, we refer to \(\frac{\partial V_{\xi'}(x)}{\partial x_\xi}\) and \(\frac{\partial V_{\xi''}(x)}{\partial x_\xi}\) as a within-coalition spillover effect and a between-coalition spillover effect, respectively.

Notice that between coalition spillover effects may link two agents acting at different stages in the game. In this case, we shall refer to “non-synchronous between-coalition spillover effects”. Since we assume that there is no coalition formed of policymakers and agents, spillovers between a policymaker and a private agent are necessary between-coalition spillovers. Moreover, in the special case in which a particular coalition is made up of a singleton player within-coalition effects, by construction, are equal to zero.

2.5 Equivalence of Subgame perfect Nash equilibria

Any proper subgame played at \(t\), \(0 < t \leq T\) is denoted by \(G_t\). Denoting by \(H_t\) the history of actions decided before \(t\), \(G_t\) depends on \(H_t\). We denote by \(x | H_t\) the restriction on strategy profiles to be consistent with a particular history \(H_t\). Notice that each coalition, since playing only once, is assigned to a unique date \(t\). Hence we denote by \(C | H_t\) the subset of coalitions which have not played in \(H_t\) and will play at \(t \leq \lambda \leq T\). Let equilibrium profiles of \(x\) be denoted by \(z\). Then,

**Definition 4** A strategy profile/vector of actions \(z\) of the game \(\Gamma\) is a subgame perfect Nash equilibrium if, for every proper subgame \(G_t\), the restriction \(z | H_t\) constitutes a Nash equilibrium of \(G_t\).

Evidently, different games can be played in this multi-nation economy, varying in terms of coalition structures and commitment patterns. In the following we establish some conditions which can be used to compare two different games \(\Gamma\) and \(\Gamma'\). This comparison implies to keep track of both cooperation and commitment issues. A sufficient (but rather restrictive) condition for equivalence of two apparently different games is the following:
Proposition 1 Consider two games $\Gamma$ and $\Gamma'$, characterized by $(C, \xi)$ and $(C', \xi')$, respectively. Denote by $z$ and $z'$ their subgame perfect Nash equilibria. The two games are equivalent / have the same solution if, for the game $\Gamma'$ ($\Gamma$), there are no within-coalition spillover effects between any pair of players $(\xi, \xi')$ belonging to a coalition which does not belong simultaneously to $C$ and $C'$, and no between-coalition spillovers between agents in coalitions playing at different stages for the vector of actions $z$ ($z'$).

Proof: see appendix.

This proposition gives us conditions such that two different games can have the same subgame perfect Nash equilibrium despite differences in terms of commitment or cooperation. These conditions are related to the absence of certain spillover effects at the vectors of equilibrium actions $z$ ($z'$). Notice that Proposition 1 does not require the absence of all spillover effects at $z$ ($z'$). These non-vanishing spillover effects may be of two varieties: they may be within-coalition effects in coalitions which exist in both games or between-coalition spillovers existing between coalitions acting at the same stage. In other words, the equivalent equilibria are not necessarily the solution of the simultaneous Nash game, obtained when all players act as singletons. Hence the presence of coalitions is not sufficient to rule out the case that two games are identical, that is accept the same solution.

To shed further light on the nature of Proposition 1 it is constructive to look into two special cases. First, we denote by $\Gamma^{Nash}$ the game which is played by the $N$ individual agents and the $P$ policymakers simultaneously (i.e. no commitment as there is no sequential stages) and without any sort of coalition. (Here we abstract from the case where the policymakers themselves are coalitions). We denote by $z^{Nash}$ the equilibrium of this “no-commitment and no-cooperation” game. Then, Proposition 1 can be further extended as follows:

Corollary 1 Any two extensive-form games $\Gamma$ and $\Gamma'$, characterized by different commitment patterns and/or coalition structures, are solved by the cooperative equilibrium, identical to $z^{Nash}$, if there are no marginal spillovers between any pair of players at $z^{Nash}$.

Remark: While it is well-known that the Nash equilibrium coincides with the social planner's solution when there are no spillover effects, the Corollary follows directly from Proposition 1.

This Corollary states conditions under which cooperation and commitment never matter. These conditions appear to be quite stringent but they cannot be ruled out.

Second, games may differ in a subset of their extensive forms. In many applications differences in commitment patterns or coalition structures are restricted to subgames, while early stages are identical for the two games under considerations. It is straightforward to adapt the reasoning of Proposition 1 to such a special constellation. Consider two games $\Gamma$ and $\Gamma'$. Let the sequence of moves in $\Gamma$ ($\Gamma'$) be denoted by $T$ ($T'$). Suppose that the two games have in common the first stages up to $T - 1 \leq \min(T, T')$. $T$ is a single node and proper subgames, $G_T$ and $G'_T$ may be defined at $T$. By this, we mean that the coalitions, the commitment patterns and the information sets involved in $\Gamma$ and $\Gamma'$ are identical up
to $T - 1$ but differ in the subsequent stages. Hence all differences between $\Gamma$ and $\Gamma'$ are captured by:

$$G_T \neq G'_T.$$ 

Given our assumption that players can play only once, and that the two games are identical up to stage $T - 1$, the two games, $G_T \neq G'_T$ involve the same subset of players. Then we can offer the following:

**Proposition 2** Consider two games $\Gamma$ and $\Gamma'$ which are identical up to $T - 1$ but have different commitment patterns or coalitions structures in the subsequent stages. If both proper subgames $G_T$ and $G'_T$ satisfy the equivalence conditions of Proposition 1 then $\Gamma$ and $\Gamma'$ admit the same subgame perfect equilibrium $z$.

**Proof:** Starting from the proof of Proposition 1, Proposition 2 follows directly from backward induction.

This proposition tells us that for certain games not all within-coalition or between-coalition spillovers need to vanish in equilibrium if one wants to establish the equivalence of games in line with the logic of the previous subsection. It is only those which occur in subgames which make the two games different.

We then define $G^{Nash}_T$ as the “no-commitment and no-cooperation” Nash game involving the same players as $G_T$ and $G'_T$. Let $z^{Nash}_T$ denote the equilibrium of $G^{Nash}_T$. The previous corollary can be adapted such that one can use the equilibrium $z^{Nash}_T$ of the one-stage Nash game $G^{Nash}_T$ as a reference point for comparisons:

**Corollary 2** Consider a game $\Gamma$ which is solved by the subgame perfect Nash equilibrium $z$. If there are at $z^{Nash}_T$ no marginal spillovers between any players acting at stage $T$ and later, then $z$ is a subgame perfect Nash equilibrium for any possible extensive-form game characterized by arbitrary commitment patterns and coalition structures at stage $T$ and later.

This corollary differs from the previous one because it does not imply that the Nash equilibrium is the solution to any game, including the game corresponding to the grand coalition. Here it is not true that the Nash equilibrium is equal to the social optimum. This comes from the fact that there may well exist non-zero within-coalition and between-coalition occurring prior to stage $T$.

### 2.6 The linear-quadratic model for policy analysis

The results presented in the previous section will shed some light on the issue of policy interactions in a multi-player model where preferences are quadratic and constraints are linear. This species of macromodel has a long established tradition, dating back to H. Theil. It is still very much in use! Woodford (2003) argues that such a model is a good approximation for policy analysis purposes.

Let us write such a model as follows, using our previous setting. Consider an economy with $\Xi$ players indexed by $\xi$. The state of the economy is described by a linear model, that
is there exists a \((\Xi \times 1)\)-vector \(y\), which depends linearly on the \((\Xi \times 1)\)-vector of actions \(x\):
\[
y = \mathbf{y} + Bx.
\] (1)
with \(\mathbf{y}\) being a vector of constants and the matrix \(B\) being invertible. The \(\xi\)-th element of \(y\), \(y_\xi\), characterizes the situation of agent \(\xi\). Similarly, \(y^*\) is a \((\Xi \times 1)\)-vector of target values, with \(\xi\)-th element \(y^*_\xi\). It is assumed that the target values are shared by all agents.
The payoff function corresponding to player \(\xi\) is a quadratic function of squared gaps to the targets. It is given by:
\[
V_\xi = \frac{1}{2} \left[ \omega_1^\xi (y_1^* - y_1)^2 + \cdots + \omega_\xi^\xi (y^*_\xi - y_\xi)^2 + \cdots + \omega_\Xi^\Xi (y^*_\Xi - y_\Xi)^2 \right]
\] (2)
with \(\omega_\xi^\xi > 0\) and \(\omega_\xi^\xi \geq 0\). Notice that individual payoffs depend on other players’ actions through the model itself (the \(B\) matrix) and the specification of the payoff functions. These payoff functions may differ as the vector of weights \(\left(\omega_j^\xi\right)\) may be specific to agent \(\xi\).

**Proposition 3** For an economy described by (1) and (2), the equilibrium \(z^{Nash} = B^{-1} \left[ y^* - \mathbf{y} \right]\) of the “no-commitment and no-cooperation” Nash game \(\Gamma^{Nash}\) is a subgame perfect Nash equilibrium for any possible extensive-form game characterized by different commitment patterns and coalition structures. Since the target values \(y^*_\xi\) are shared by all players, this solution is identical the social planner’s solution.

**Proof:** see appendix.

Proposition 3 states that in the case of a linear-quadratic model of the type described above neither commitment nor cooperation matter. The explanation directly derives from the Corollary to Proposition 1: in the linear-quadratic model, all marginal spillovers are null at the Nash equilibrium. Hence, \(z^{Nash}\) is a subgame perfect Nash equilibrium for any possible extensive-form game characterized by different commitment patterns and coalition structures.
This proposition is reminiscent of the Tinbergen rule. Actually it may be seen as a generalized Tinbergen rule in a game-theoretic environment. It is central to stress that this result relies, not only on the quadratic nature of the problem, but also on the fact that each player disposes of an instrument and therefore that the number of instruments matches the number of arguments in any payoff function. Moreover, there is no disagreement about the target values of all players.

Proposition 3 implies that if there is no target conflict between players, then the solution to any game is the one that a social planner would have chosen. This “symbiotic” result, to use the term coined by Dixit and Lambertini, comes from the fact that in this case, the Nash solution is equivalent to the social planner’s solution, given that there are enough instruments to reach the various objectives.

This proposition can be readily extended to the introduction of an aggregate variable characterizing the economy as a whole, linearly entering each individual payoff function in the form of a squared gap to a common target value, as well as the introduction of an additional player, controlling an additional instrument.
3 Applications: Monetary Unions

The previous propositions can be used to shed light on a number of puzzling and seemingly contradictory results that have recently been established on the (ir-) relevance of cooperation and commitment between policymakers. These issues have been particularly controversially discussed in recent contributions which address the desirable design of policymaking in monetary unions. Therefore, we devote this section entirely to monetary union issues.

When do cooperation and commitment matter in a monetary union? In general, the possible existence of spillovers within countries (related to private actors), of spillovers between countries (related to fiscal and private actors) and of a common monetary policy (affecting players in all countries) creates a number of channels which make this question non-trivial, i.e. it is clear that, in general, commitment and cooperation (i.e. coalition structures) do matter, within countries and between countries.

Against this general insight two recently established findings seem particularly puzzling. On the one hand, Dixit and Lambertini (2003) consider a model which allows for spillovers between players acting in different countries and which nevertheless has the feature that fiscal and monetary policymakers attain the same equilibrium outcome, irrespective of the commitment patterns of the policymakers and irrespective of whether policies are coordinated between countries or not. By contrast, Chari and Kehoe (2004) consider a model which does not allow for any spillovers between players acting in different countries and which nevertheless has the feature that equilibrium outcomes depend sensitively on commitment patterns and on whether policies are coordinated between countries or not.

However, within the general framework established above, it is straightforward to resolve this puzzle. To this end, let us consider a monetary union with $N$ member countries, indexed by $i = 1, 2, ..., N$. For each country there exists a single fiscal policymaker. Moreover, there exists a single monetary policymaker (central bank) operating for the monetary union as a whole. Let $\pi$ denote the action of the central bank. Moreover, let $a_{ij}$ denote the action of private agent $j$ in country $i$. Then, the vector $x$ of actions of all players is given by $x = (a, \tau, \pi)$, with $\tau = (\tau_i, \tau_{-i})$ and $a = (a_i, a_{-i})$, where $a_i$ can be further decomposed into $a_i = (a_{ij}, a_{i,-j})$. We consider the following set of generic payoff functions:

- Payoff function of a (representative) private agent $j$ in country $i$:
  $$U_{ij} = U_{ij}(a, \tau, \pi).$$

- Payoff function of fiscal policymaker in country $i$:
  $$V_i = V_i(a, \tau, \pi)$$

- Payoff function of cooperating fiscal policymakers:
  $$V^{FC} = \sum_{i=1}^{n} \omega_i^F V_i = \sum_{i=1}^{n} \omega_i^F V_i(a, \tau, \pi)$$
where $\omega_i^F$ denotes the fiscal weight attached to country $i$ in the collective fiscal payoff function.

- Payoff function of the central bank:

$$V^M = \sum_{i=1}^{n} \omega_i^M V_i = \sum_{i=1}^{n} \omega_i^M V_i(a, \tau, \pi).$$

where $\omega_i^M$ denotes the country-specific monetary weight maintained by the central bank.\(^1\)

### 3.1 The Dixit and Lambertini (2003) model

Using this notation, the model of Dixit and Lambertini can be represented within our broad and deterministic set-up as follows. Two assumptions are particularly important. First, the payoff function $V_i$ allows, in general, for fiscal spillover effects between countries. Second, there exists a uniform private sector throughout the monetary union such that there are no spillovers linked to private actions, be they within countries or between countries. Specifically, private sector behaviour reduces to $a_{ij} = a$ for all $i, j$, leading to

$$U_{ij} = U_i = U = U(a, \pi)$$

$$V_i = V_i(a, \tau_i, \tau_{-i}, \pi).$$

Should policymakers care about commitment patterns? And should fiscal policymakers in the member countries form a coalition or not? The framework of Dixit and Lambertini has the striking feature that it gives rise to a general irrelevance proposition which can be summarized as follows:

**Result Dixit/Lambertini (2003):** Assume there exist fiscal spillover effects between countries. Yet, fiscal cooperation in a monetary union is irrelevant under arbitrary commitment patterns of policymakers.

Given our general discussion in Section 2, this result is puzzling for two reasons. First, as concerns the interaction between fiscal policymakers, Dixit and Lambertini allow, in principle, for the existence of fiscal *within-coalition spillover effects* in $V_i$, making cooperation issues (i.e. the possible formation of coalitions) non-trivial. Second, as concerns the interaction between monetary policy and fiscal policies, Dixit and Lambertini allow, in principle, for *between-coalition spillover effects*, making also commitment patterns non-trivial. Notwithstanding these two features, the driving force behind this strong result is easily identified if one recognizes that the analysis is conducted within a linear-quadratic framework in line with the general discussion in Section 2.3. Specifically, Dixit and Lambertini reserve the scalar $a$, summarizing union-wide private sector actions, for private

\(^1\)Hence, by assuming $V_i^F = V_i^M = V_i$, we restrict in this section possible differences between monetary and fiscal policy objectives to the weighting factors $\omega_i^F$ and $\omega_i^M$. Implications of the assumption $V_i^F \neq V_i^M$ are discussed, in particular, in Dixit and Lambertini (2003b).
sector inflation expectations, i.e. \( a \equiv \pi^e \), and all equilibria satisfy the assumption of rational expectations such that \( \pi^e \equiv \pi \). This feature can be recovered from writing \( U \) as

\[
U = U(a, \pi) = \frac{1}{2} (\pi - \pi^e)^2,
\]

i.e. \( \pi^e = \pi \) results from a minimization of the squared inflation forecast error. Moreover, the policy objective \( V_i \) represents a weighted sum of squared deviations of (country-specific) output \((y_i)\) and (union-wide) inflation values from target values, denoted by \( y^*_i \) and \( \pi^* = 0 \), respectively, such that

\[
V_i = \frac{1}{2} \left[ \omega_i (y^*_i - y_i)^2 + \pi^2 \right],
\]

while the output levels depend linearly on the vector of actions \( x = (\pi^e, \tau_i, \tau_{-i}, \pi) \):

\[
y_i = \overline{y}_i + \sum_{k=1}^{n} b_{ik} \tau_k + b_i (\pi^e - \pi^e).
\]

By construction of \( U, V_i, V^{FC} \) and \( V^M \), there is consensus on the target values between all players under all conceivable cooperation and commitment schemes. Hence, as shown in the Appendix, it is straightforward to establish that this economy satisfies all the requirements of Proposition 3, i.e. all marginal spillovers, including those related to the private sector, vanish at the equilibrium of the ‘no-commitment and no-cooperation’ Nash game \( \Gamma^{\text{Nash}} \). Because of this feature, the irrelevance proposition extends, in fact, to all possible commitment patterns not only with respect to all policymakers, but also the private sector. Moreover, the outcome of any such game is the social optimum, as all players always attain their target values.\(^2\)

The key result of Dixit and Lambertini is refreshing and provocative at the same time since it challenges the conventional wisdom that the existence of spillovers should create meaningful commitment and cooperation problems. Certainly, the model is special in many ways. For example, private sector actions are restricted to the assumption of rational inflation expectations at the union-wide level. Similarly, there is no role for country-specific inflation effects on national output levels, i.e. possible tensions between such effects and policy reactions of the central bank to union-wide inflation are ruled out. However, any refinements of the model along these lines, as long as the crucial characteristics of the linear-quadratic set-up are maintained, would not challenge the irrelevance proposition. Hence, the evident limitations of this proposition are linked to more fundamental concerns.\(^3\) First, it is clear that linear quadratic frameworks, while being convenient and

\(^{2}\)The corresponding summary in Dixit and Lambertini (2003) is as follows: “If the monetary and fiscal authorities in a monetary union have identical output and inflation goals, those goals can be achieved without the need for fiscal coordination, without the need for monetary commitment, irrespective of which authority moves first and despite any disagreement about the relative weights of the two objectives.”

\(^{3}\)There exist hybrid monetary unions models, like Calmfors (2001), which respect for some, but not all reduced form equations, the linear-quadratic structure. Yet, to use them as counterexamples to the reasoning of Dixit and Lambertini is not entirely satisfactory.
widely used approximations, are, by construction, not generic. Second, the irrelevance
proposition of Dixit and Lambertini requires that the number of independent instruments
available to policymakers matches the number of policy objectives in $V_i$. The consequences
of these two features are addressed in turn in the next two Sections.

3.2 The Chari and Kehoe (2004) model

Results very different from Dixut and Lambertini are obtained by Chari and Kehoe. In
a sense, this is not surprising since they study a general class of economies in which the
very special features of linear-quadratic economies do not apply. Yet, the striking feature
of their analysis is that they manage to establish non-trivial results on cooperation and
commitment in a framework which deliberately rules out any spillovers between players
acting in different countries.

To reconstruct this reasoning in terms of our broad framework, the two crucial assumptions
invoked by Chari and Kehoe can be summarized as follows. First, the payoff functions
$U_{ij}$ and $V_i$ rule out, in general, any spillover effects between players acting in different
countries, be they private or fiscal. Second, the payoff function $U_{ij}$ features private sector
spillovers between agents within any country $i = 1, 2, \ldots, N_i$, leading to

\[
U_{ij} = U_{ij}(a_{ij}, a_{i,-j}, \tau_i, \pi) \\
V_i = \sum_{j \in M_i} U_{ij} = \sum_{j \in M_i} U_{ij}(a_{ij}, a_{i,-j}, \tau_i, \pi).
\]

The main proposition of Chari and Kehoe, adopted to our framework, can be summarized
as follows:

**Result Chari and Kehoe (2004)** Assume there are no spillovers between any players
acting in different countries. Then, fiscal cooperation is still relevant under certain com-
mitment patterns. Specifically, i) if monetary policy can commit (moves first) and fiscal
policy moves last, the outcomes of fiscal cooperation and fiscal non-cooperation are identi-
cal. However, ii) if monetary policy cannot commit (moves last) and fiscal policy moves
first, these two outcomes differ because of a time consistency problem of monetary policy,
related to the interaction with the private sector.

Notice that part i) of this result follows directly from Proposition 2. If fiscal policy moves
last and if, by assumption, there are no fiscal within-coalition spillovers it is evident
that fiscal cooperation becomes irrelevant. Correspondingly, part ii) of the result reflects,
broadly speaking, that this same reasoning does not go through if fiscal policy moves first
and if there exist non-synchronous between-coalition spillovers related to monetary policy
(which moves last), in line with the logic of Proposition 1.\textsuperscript{4} The main contribution of Chari
and Kehoe is to discuss thoroughly the role of private sector behaviour in this context.
Specifically, it is well-known that monetary policy, if it lacks commitment with respect

\textsuperscript{4}More specifically, part i) of the result is obtained under the sequence of moves: 1) monetary policy, 2)
private sector, 3) fiscal policy. By contrast, part ii) of the result is obtained under the sequence of moves
1) fiscal policy, 2) private sector, 3) monetary policy.
to the other actors, may be a source of non-synchronous spillovers in a monetary union, reflecting speculations of a last-round bailout motive of monetary policy. However, as discussed in detail by Chari and Kehoe, for this argument to prevail under their rather stringent assumptions it is crucial that non-cooperative private sector behaviour reinforces these spillovers such that monetary policy cannot undo them at the margin by means of a simple envelope theorem argument. Economically speaking, if private sector agents expect a monetary reaction to earlier fiscal decisions, but the private sector itself suffers within each country from a coordination problem, then this creates a fiscal cooperation problem in the first place which cannot be undone by monetary policy at a later stage. Hence, for a fiscal cooperation problem to exist under the assumptions maintained by Chari and Kehoe it is not enough that fiscal policy moves prior to monetary policy. Instead, this constellation needs to be enriched by a (plausible) lack of private sector coordination which breaks the logic of the envelope theorem.

Notice, however that in the general class of economies studied by Chari and Kehoe, fiscal cooperation is always relevant if one allows within $V_i$ for fiscal spillover effects between countries, irrespective of whether the lack of commitment of monetary policy may induce an additional fiscal cooperation problem.

### 3.3 The role of the consolidated budget constraint

Apart from the special characteristics of a linear quadratic set-up, there is a second and independent feature of the Dixit and Lambertini model which drives their irrelevance proposition. Specifically, there exist, using our notation, $N+1$ policymakers with the same number of policy objectives, as captured by the intention to close $N$ (country-specific) output gaps $(y^*_i - y_i)$ and the single (and union-wide) inflation gap $\pi^* - \pi$. Moreover, policymakers have access to $N+1$ instruments, as given by the actions $(\tau, \pi)$, which can be independently chosen.

This latter assumption is not entirely convincing if one sees the reduced form equations as an approximation to a model in which all policy instruments are assumed to have budgetary implications. Under this assumption, one rather needs to respect that all instruments are jointly tied together by some version of a consolidated budget constraint of the public sector, as stressed, in particular, by Cooper and Kempf (2004). For the sake of illustration, let us assume that the linearized budget constraint can be represented as

$$\sum_{i=1}^{n} \alpha_i \tau_i + \alpha_\pi \pi = 0.$$  \hspace{1cm} (3)

Hence, for (3) to be satisfied in equilibrium, not all $N+1$ instruments can be chosen independently. Instead, any feasible commitment pattern needs to respect that there

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5 Here: reference to the literature which has stressed this point before: i.e. role of fiscal externalities in a monetary union which may trigger a monetary bail-out etc.

6 In the Appendix we summarize this reasoning in a more detailed way by adapting the main insights from Chari and Kehoe to our framework.

7 Otherwise the model of Cooper and Kempf (2004) is very different from Dixit and Lambertini (2003). In particular, it is not of the linear-quadratic variety.
exists at least one player who adjusts his instrument passively to satisfy (3). For a given set of coefficients $\alpha$, this implies that, in general, it will not be possible to simultaneously achieve all $N + 1$ policy objectives.\(^8\) Alternatively, this reasoning relates to the insight that for non-trivial policy trade-offs to exist the number of independent instruments needs to be smaller than the number of objectives. This can be most clearly seen if one shuts down all fiscal effects within the model of Section 3.1, by letting $b_{ik} = 0 \ \forall i, k$. Then, the analysis collapses to the standard monetary model of Barro and Gordon (1983) where the relevance of monetary commitment is well-known, reflecting the trade-off faced by monetary policy to meet output and inflation objectives with a single instrument.

4 Applications: International policy cooperation

5 Conclusion

In this paper, we set up a general framework to address the importance of commitment patterns and cooperation schemes in policy games between various policymakers. We prove that the nature of spillover effects between agents is of key relevance to answer this issue. To this end, we offer a simple classification of spillover effects between agents which distinguishes between within-coalition and between-coalition spillover effects. Based on this classification, we provide two general propositions which prove that under some conditions, linked to these spillover effects, commitment and cooperation schemes do not matter. In particular, linear-quadratic models can well lead to the conclusion that commitment and cooperation issues are entirely irrelevant. Yet, the conditions which are responsible for this puzzling result are shown to be rather restrictive and, more importantly, these conditions have no longer any bite in a generic, non-linear environment. We then apply these theoretical results to review a number of recent, seemingly contradictory contributions on policy interactions in monetary unions and on issues related to the international policy cooperation.

\(^8\)From a broader perspective, the neglect of the budget constraint could be rationalized if one thinks about policy actions without (direct) budgetary incidence, like reform measures which affect the competitiveness of industries etc. Moreover, in a narrow fiscal context, one could assume that national treasuries have access to (lump-sum) balancing items which are not related to the spillovers between countries.
6 Appendix

6.1 Proof of Proposition 1

We consider two-stage games, $\Gamma$ and $\Gamma'$ allowing for coalitions among subsets of agents. Some notation is therefore necessary. We concentrate on two-stage games. It is easy to generalize the proof to more stages.

Consider $\Gamma$. We partition the set of players into two subsets $\Xi_1$ and $\Xi_2$. $\Xi_1 (\Xi_2)$ is formed of players making their decision at stage 1 (2). At each stage, coalitions may be active. There are $K$ ($L$) coalitions at stage 1 (2), denoted by $C_k$ ($C_l$). We denote by $C_1$ ($C_2$) the set of coalitions formed in stage 1 (2).

For a given structure of coalitions, the game is solved by subgame perfection. So any subgame-perfect Nash equilibrium $z$ of the two-stage game satisfies the following conditions:

- At stage 2, for any $C_l \in C_2$, 
  \[ \omega_\xi \frac{dV_\xi(z)}{dx_\xi} + \sum_{\xi' \in C_l, \xi' \neq \xi} \omega_{\xi'} \frac{dV_{\xi'}(z)}{dx_{\xi'}} = 0, \quad \forall \xi \in C_l, \]  
  \[ (4) \]

  where the first term captures the effect of the action of player $\xi$ on its welfare while the second term describes the within-coalition spillover effects on the coalition members in the coalition $C_l$. Since stage 2 is the last stage of the game, there are by construction non-synchronous effects on subsequent coalitions. Given that it is the joint welfare of the coalition $C_l$ which is maximized, weights are taken into account.

- At stage 1, for any $C_k \in C_1$, 
  \[ \omega_\xi \left[ \frac{\partial V_\xi(z)}{\partial x_\xi} + \sum_{C_l \in C_2} \sum_{\xi' \in C_l} \frac{\partial V_\xi(z)}{\partial x_{\xi'}} \frac{\partial x_{\xi'}}{\partial x_\xi} \right] 
  + \sum_{\xi' \in C_k, \xi' \neq \xi} \omega_{\xi'} \left[ \frac{\partial V_{\xi'}(z)}{\partial x_{\xi'}} + \sum_{C_l \in C_2} \sum_{\xi'' \in C_l} \frac{\partial V_{\xi''}(z)}{\partial x_{\xi''}} \frac{\partial x_{\xi''}}{\partial x_{\xi'}} \right] 
  = 0, \quad \forall \xi \in C_k, \forall C_k \in C_1. \]  
  \[ (5) \]

  Four effects need to be distinguished. The first term captures the direct effect of the action of player $\xi$ on its own welfare, while the second term describes the indirect effect on its own welfare through actions taken by players in coalitions formed in the second period. This second term is non-zero if there are non-synchronous between-coalition spillovers effects between $\xi$ and at least one player $\xi''$, i.e. if $\frac{\partial V_\xi(z)}{\partial x_{\xi'}}$ and $\frac{\partial x_{\xi''}}{\partial x_{\xi'}}$ are non-zero. The third term describes the within-coalition spillover effects of the action $x_{\xi'}$ on the coalition members in the coalition $C_k$. Finally, the fourth term captures the indirect welfare effect on the coalition members in the coalition $C_k$. 

15
through actions taken by players in coalitions formed in the second period. This fourth term is non-zero if there are non-synchronous between-coalition spillovers effects between at least one other member of $C_k$ and at least one player $\xi''$ playing at stage 2.

Correspondingly, one can derive the set of conditions applying to $\Gamma'$. To ensure that (4) and (5) admit the same solutions for two games $\Gamma$ and $\Gamma'$, the (sufficient) conditions summarized in Proposition 1 are derived from the following two-step procedure. Firstly, to undo the effects of different commitment patterns in $\Gamma$ and $\Gamma'$, all non-synchronous between-coalition spillover effects are required to be zero at the vector of actions $z (z')$. This requirement ensures that the second and fourth term discussed above vanish in equilibrium. Secondly, a set of condition is needed which addresses the effects of different coalitions structures in $\Gamma$ and $\Gamma'$. Certainly, a sufficient condition would be to require that the within-coalition spillover effects for all coalitions formed in $\Gamma$ and $\Gamma'$ need to vanish for the vector of actions $z (z')$, implying that (4) and (5) reduce for both games to $\frac{\partial V(z)}{\partial x} = 0, \forall \xi \in \Xi$. Yet, having controlled for possible differences in commitment patterns already in step 1, it is clear that (4) and (5) admit for $\Gamma$ and $\Gamma'$ the same solutions also under the weaker condition that the within-coalition spillover effects need to vanish only for those coalitions which are formed in $\Gamma'$, but not in $\Gamma$, and vice versa, for the vector of actions $z (z')$.

Remark at Corollary 1: If at $z^{\text{Nash}}$ all marginal spillovers between any pair of players vanish it is clear from (4) and (5) that $z^{\text{Nash}}$ is a subgame-perfect Nash equilibrium for any possible extensive-form game characterized by arbitrary commitment patterns and coalition structures.

This reasoning can be generalized to a game with more than 2 stages, as any $h - \text{stage}$ extensive form game can be restated as a sequence of 2-stage extensive games. This completes the proof.

6.2 Proof of Proposition 3

Using (1) in (2), we can express $V_\xi$ as:

$$V_\xi = \frac{1}{2} \left[ \omega^{y_1}_\xi (y^*_1 - \bar{y}_1 - \sum_{j=1}^{\Xi} b_{1j} x_j)^2 + ... + \omega^{y_\Xi}_\xi (y^*_\Xi - \bar{y}_\Xi - \sum_{j=1}^{\Xi} b_{\Xi j} x_j)^2 + ... + \omega^{y_\Xi}_\xi (y^*_\Xi - \bar{y}_\Xi - \sum_{j=1}^{\Xi} b_{\Xi j} x_j)^2 \right]$$

In general, any Nash equilibrium $z^{\text{Nash}}$ of the simultaneous “no-commitment and no-cooperation” Nash game $\Gamma^{\text{Nash}}$ being played by the $\Xi$ players satisfies the following set of conditions:

$$\frac{\partial V_\xi (z^{\text{Nash}})}{\partial x_\xi} = 0, \forall \xi \in \Xi.$$
In this linear-quadratic model, this set of equations can be expressed as follows:

\[
\frac{\partial V_\xi(z^{\text{Nash}})}{\partial x_\xi} = \sum_{k=1}^{\Xi} \omega_k^\xi b_k \left[ y_k^* - \bar{y}_k - \sum_{j=1}^{\Xi} b_{kj} x_j \right] = 0, \forall \xi \in \Xi
\]  

(6)

The marginal spillover effect of \( \xi' \) on agent \( \xi \)'s payoff is given by the following expression:

\[
\frac{\partial V_\xi(z^{\text{Nash}})}{\partial x_{\xi'}} = \sum_{k=1}^{\Xi} \omega_k^\xi b_k \left[ y_k^* - \bar{y}_k - \sum_{j=1}^{\Xi} b_{kj} x_j \right]
\]  

(7)

Let \( z^{\text{Nash}} = B^{-1}[y^* - \bar{y}] \). This vector evidently satisfies (6), as required for a Nash-equilibrium. Moreover, at \( z^{\text{Nash}} \) for any pair \( (\xi, \xi') \) equation (7) will then also be zero. Hence, the linear-quadratic case satisfies Corollary 1 to Proposition 2. QED.

6.3 The model of Dixit and Lambertini: a special case of Proposition 3

The model of Dixit and Lambertini described in Section 3.1 can be rewritten as follows such that it satisfies (1) and (2). First, define the inflation forecast error such that \( \pi^{fe} \equiv \pi - \pi^e \).

Then, introduce a new vector \( \tilde{y} = (\pi^{fe}, y, \pi) \), with \( \tilde{y} \) relating to the states of the three groups of agents: single private sector actor, country-specific fiscal policymakers, single monetary policymaker. Since

\[
\begin{align*}
\pi^{fe} &= \pi - \pi^e \\
y_i &= \bar{y} + \sum_{k=1}^{n} b_{ik} \tau_k + b_i (\pi - \pi^e) \\
\pi &= \pi,
\end{align*}
\]

\( \tilde{y} \) can be linearly linked to the instruments \( x = (\pi^e, \tau, \pi) \) in line with (1), i.e.

\[
\tilde{y} = \bar{y} + \hat{B} x,
\]

with \( \bar{y} = (0, \bar{y}, 0) \). Moreover, the target value of the inflation forecast error satisfies \( \pi^{fe*} = 0 \). Hence, the payoff function of all three types of (non-cooperative) players

\[
\begin{align*}
U &= \frac{1}{2} (\pi^{fe*} - \pi^{fe})^2 = \frac{1}{2} (\pi - \pi^e)^2 \\
V_i &= \frac{1}{2} \left[ \omega_i (y_i^* - y_i)^2 + (\pi^* - \pi)^2 \right] = \frac{1}{2} \left[ \omega_i (y_i^* - y_i)^2 + \pi^2 \right] \\
V^M &= \sum_{i=1}^{n} \omega_i^M V_i = \frac{1}{2} \left[ \sum_{i=1}^{n} \omega_i^M \omega_i (y_i^* - y_i)^2 + \pi^2 \right]
\end{align*}
\]

are in line with (2).
6.4 The model of Chari and Kehoe: main results

Consider the following four games discussed by Chari and Kehoe which we adapt to our notation. In all games the non-cooperative private sector players always move after the fiscal players.

1. Game CK 1: There is no cooperation between fiscal policymakers and no commitment of monetary policy: i/ the fiscal authorities set \( \tau_i \) non cooperatively, ii/ the private agents set \( a_{ij} \) non cooperatively, iii/ the central bank sets \( \pi \).

2. Game CK 2: There is cooperation between fiscal policymakers and no commitment of monetary policy: i/ the fiscal authorities jointly set the vector of fiscal instruments \( \tau \) with the aim to maximize \( V^{FC} \), ii/ the private agents set \( a_{ij} \) non cooperatively, iii/ the central bank sets \( \pi \).

3. Game CK 3: There is no cooperation between fiscal policymakers, and there is commitment of monetary policy: i/ the central bank sets \( \pi \), ii/ the fiscal authorities set \( \tau_i \) non cooperatively, iii/ the private agents set \( a_{ij} \) non cooperatively.

4. Game CK 4: There is cooperation between fiscal policymakers, and there is commitment of monetary policy: i/ the central bank sets \( \pi \), ii/ the fiscal authorities jointly set the vector of fiscal instruments \( \tau \) with the aim to maximize \( V^{FC} \), iii/ the private agents set \( a_{ij} \) non cooperatively.

The logic behind i) and ii) can be inferred from the following sketch, i.e. we do not reproduce the detailed proof from Chari and Kehoe. Consider a perfectly symmetric set-up and let \( z_g = (a, \tau, \pi) \), \( g = 1, 2, 3, 4 \) denote the solution vectors to the four games.

**Result ii)** obtains from comparing CK 1 and CK 2:

**Consider CK 1:** By backward induction, stage 3 gives rise to the first-order condition \( \frac{\partial U_{ij}(a_1)}{\partial \pi} = 0 \), leading to a solution for \( \pi \) such that \( \pi = \pi(a, \tau) \). Stage 2 gives rise to a first-order condition \( \frac{\partial U_{ij}(a_1)}{\partial a_{ij}} \frac{\partial a_{ij}}{\partial \tau_i} = 0 \) (where we use the envelope theorem which ensures \( \frac{\partial U_{ij}(a_1)}{\partial a_{ij}} \frac{\partial a_{ij}}{\partial \pi} = 0 \)), leading to a solution for \( a_{ij} \) such that \( a_{ij} = \pi(\tau) \). Stage 3 gives rise to a first-order condition
\[
\sum_{k \in M_i, k \neq j} \frac{\partial U_{ij}(z_1)}{\partial a_{ik}} \frac{\partial a_{ik}}{\partial \tau_i} + \frac{\partial U_{ij}(z_1)}{\partial \tau_i} = 0, \tag{8}
\]

where we use the envelope theorem which ensures \( \frac{\partial U_{ij}(a_1)}{\partial a_{ij}} \frac{\partial a_{ij}}{\partial \pi} = \frac{\partial U_{ij}(a_1)}{\partial a_{ij}} \frac{\partial a_{ij}}{\partial \tau_i} = 0 \). Two elements are crucial for the following analysis: i) since the common monetary policy moves last, this makes private sector actions in stage 2 dependent on the entire vector of fiscal actions \( \tau \). ii) The non-cooperative behaviour of private sector players within countries creates spillovers which become relevant at stage 1 for the fiscal players, i.e. there exist within-coalition spillover effects between fiscal players which are entirely related to the commitment of fiscal policy with respect to all other players. Under the assumption of non-cooperative fiscal policy in CK 1, these effects are not internalized.
Consider CK 2: By backward induction, stage 3 and 2 are identical to CK 1. Stage 3, again using the envelope theorem, gives rise to a first-order condition

\[
\sum_{k \in M_i, k \neq j} \frac{\partial U_{ij}(z_2)}{\partial a_{ik}} \sum_{l=1}^{n} \frac{\partial a_{ik}}{\partial \tau_l} + \frac{\partial U_{ij}(z_2)}{\partial \tau_i} = 0, \tag{9}
\]

i.e. within-coalition spillover effects between fiscal players are internalized, making the solutions to CK 1 and CK 2 generically different.

Result i) obtains from comparing CK 3 and CK 4.

Consider CK 3. By backward induction, stage 3 gives rise to the first-order condition \(\frac{\partial U_{ij}(z_3)}{\partial a_{ij}} = 0\), leading to a solution for \(a_{ij}\) such that \(a_{ij} = (a, \tau_i)\). Stage 2 gives rise to a first-order condition

\[
\sum_{k \in M_i, k \neq j} \frac{\partial U_{ij}(z_3)}{\partial a_{ik}} \frac{\partial a_{ik}}{\partial \tau_i} + \frac{\partial U_{ij}(z_3)}{\partial \tau_i} = 0, \tag{10}
\]

(where we use the envelope theorem which ensures \(\frac{\partial U_{ij}(z_3)}{\partial a_{ij}} \frac{\partial a_{ij}}{\partial \tau_i} = 0\), leading to a solution for \(\tau_i\) such that \(\tau_i = \tau_i(\pi)\). Stage 3 gives rise to a first-order condition \(\frac{\partial U_{ij}(z_3)}{\partial \pi} = 0\), where we use the envelope theorem which ensures \(\frac{\partial U_{ij}(z_3)}{\partial a_{ij}} \frac{\partial a_{ij}}{\partial \pi} = \frac{\partial U_{ij}(z_1)}{\partial \tau_i} \frac{\partial \tau_i}{\partial \pi} = 0\).

Consider CK 4: The solution of CK 4 satisfies the same first-order conditions as CK 3, since there are no within-coalition spillover effects between fiscal players at stage 2.
References


