HUMAN CAPITAL ACCUMULATION AND LABOUR MARKET EQUILIBRIUM

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Abstract

The objective of this paper is to analyse an equilibrium search model with on-the-job search and human capital accumulation. In our model wages are dispersed because firms pay workers of the same productivity different wages and workers of different productivities earn different wages. New entrants to the labour market increase their wages mainly through on-the-job search. As workers gain more experience and move up the offer distribution, job-to-job transitions become less frequent and human capital accumulation dominates wage growth. This interaction generates a wage distribution that exhibits a density with a unique mode and a long and decreasing right tail as observed in the data.

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1 Introduction

We investigate individual wage dynamics in the context of an equilibrium labour market model where workers accumulate human capital while working. It will be shown the analysis leads to new insights into two important areas in labour economics - the nature of equilibrium in search markets, and the study of on-the-job human capital accumulation. Indeed, the work reported on here can be perceived as a tentative first step at a more unified approach to these two areas.

Although there is no free lunch, it is accepted by many that individuals accumulate human capital freely by working. Typist become better typist while working as typists, economists become more productive by doing economics, etc. This seems both an important and intuitive idea. A related idea now common among labour economists is that human capital can be dichotomized into general human capital and firm specific human capital. A worker who enjoys an increase in general human capital becomes more productive at all jobs, whereas accumulating firm specific human capital implies a worker is only more productive at that firm. Workers who change job, or those who are laid off, loose there firm specific human capital but keep there general human capital. Putting the above two ideas together, plus assuming a worker's wage is an increasing function of both his/her general and specific human capital, leads to at least the rudiment of a theory of how the wages of workers evolve through their working lives. Indeed, there is now a significant and insightful literature that has exploited the above reasoning to empirically investigate how the wages of workers evolve through time (see, for example, Altonji and Shakotko, 1987, Topel, 1991, Altonji and Williams, 2005, and Dustmann and Meghir, 2005).

The objective of most contributions has been to determine and distinguish between the return to total experience in the labour market and the return to job tenure (seniority). This is not a simple task and the results obtained are still disputed. One reason for the confusion is that there is no consensus about the labour market structure within which this human capital accumulation occurs. Without such a specification it is an extremely difficult task to satisfactorily identify the two expected returns. The focus here is to embed the accumulation of human capital within the context of an explicit equilibrium labour market model. Within such a context it will be shown how the natural working of the labour market, and its interaction with the accumulation human capital, lead to many new insights into how the wages of workers change through time.

There is now a large literature investigating equilibrium in a labour market where firms post wages and workers (both unemployed and employed) search for new job opportunities. A large proportion of such studies have utilized a framework first investigated by Burdett and Mortensen (1998) (henceforth B/M). Four elements of this framework play a major role in what follows. First, each firm posts a wage it pays to any worker who is employed at the firm. The wage paid per unit of time completely describes the payoff from the job. Second, workers, both employed and unemployed, obtain new job offers from time to time (at some Poisson rate). Third, there is job destruction at some exogenous rate. If a job is destroyed the worker is laid off and becomes unemployed. Finally, firms do not respond to new offers received by its employees. Using these restrictions (and others) B/M were able to show there exists a unique equilibrium and this can be described by a non-degenerate distribution of wage offers. In a B/M equilibrium an employed worker faces a constant wage while employed at a firm. The wage of a worker is increased only when a better wage offer is received and

accepted. Of course the worker can be laid off. This model and the many extensions it has generated provides a simple intuitive framework to analyze several labour market issues. Most contributions have concentrated on analyzing the equilibrium characteristics, such as the distribution of wage offers, in a stationary state environment. Less attention has been paid to the labour market history of workers within such an environment.

Within the model used in the present study the wage faced by an employed worker increases as he/she works for a firm and accumulates human capital. An employed worker also enjoys an increase in wage when changing job. There are, of course, several other reasons why the wage faced by a worker may change through time. These will not be discussed in the body of this paper. Nevertheless, three important examples of such seem worth mentioning here. First, a worker's wage may increase without the worker changing job due to his/her current employer responding to an outside offer. Postel-Vinay and Robin (2002) have elegantly shown this can be an equilibrium phenomena within the context of a modified B/M model. Second, a worker's wage may change as the employer learns about the worker's abilities. Utilizing pioneering work by Jovanovic (1979), Moscarini (2006) has been able to show learning about an employee's ability can be part of a market equilibrium. Thirdly, firms may post an enforceable wage/tenure contract. Indeed, Burdett and Coles (2003) have demonstrated that if workers are risk averse within an otherwise standard B/M market model, the optimal contract posted by a firm is one where wages increase with tenure. Further, the contracts posted by firms differ such that in equilibrium employed workers change jobs as a better contract is received.

In what follows we specify a reasonably standard B/M model in which employed workers freely accumulate general human capital. It should be admitted at the start that two

restrictions we use in specifying the model are somewhat not particularly intuitive. First, we assume that a worker's utility flow when unemployment is proportional to that worker's productivity. Although most UI schemes imply a worker's UI income is independent of his/her productivity, it is not that difficult to rationalize this restriction. For example, assume a worker's utility flow when unemployed is the sum of the constant UI payment received, plus the housework production generated. The desired restriction now follows if we assume housework production is an increasing function of a worker's productivity. As we shall show, with this restriction the analysis is relatively straightforward - without it the analysis becomes complicated and messy. Second, we assume throughout that workers accumulate only general human capital. Firm specific human capital accumulation is ruled out by assumption. At present we are unable to provide a useful analysis of a labour market where both types of human capital accumulation occurs.

2 The Model

The model extends the basic Burdett and Mortensen (1998) framework by assuming workers' productivity increases over time while employed. On-the-job learning thus generates disperse worker productivities. The aim is to consider how such learning opportunities affect market outcomes in the B/M environment. Specifically how do experience effects change the equilibrium distribution of wages paid?

As in B/M , we only consider steady state and assume time is continuous with an infinite horizon. There is a continuum of firms and workers, each of measure one. All firms are equally productive and have a constant return to scale technology. Workers have identical

preferences but have heterogeneous productivities, denoted y . All labour market entrants have the same initial productivity, which is normalized to one. On-the-job learning implies a worker's productivity increases at rate $\rho > 0$. Thus after x years of work experience, a worker's productivity $y = e^{\rho x}$.

As in B/M there is no wage discrimination within a firm: a firm pays each worker the same rate per unit produced. Each firm is thus characterized by what we term a pay rate θ , where an employee with productivity y is paid flow wage $w = \theta y$.¹ The firm enjoys corresponding flow profit $(1 - \theta)y$ per employee. Let $F(\theta)$ denote the proportion of firms offering a pay rate no greater than θ . Let $\underline{\theta}, \bar{\theta}$ denote the infimum and supremum of the support of F .

There are job destruction shocks - each employed worker is displaced into unemployment according to a Poisson process with parameter $\delta > 0$. While unemployed a worker's productivity y remains constant. Also while unemployed a worker receives unemployment benefits from the government. Typically it is assumed that benefit payments are proportional to the unemployed worker's last wage.² Unfortunately this assumption makes this framework intractable. Following Postel-Vinay and Robin (2002), we instead simplify by assuming unemployment benefit received is by , where $0 < b < 1$, and so is proportional to the worker's productivity.

Each employed and unemployed worker receives a job offer according to a Poisson process with parameter $\lambda > 0$. Search is random in that a job offer θ is considered as a random draw

¹ For simplicity, we avoid the complication of allowing a firm to offer different pay rates to workers of different experience. See Carrillo-Tudela (2008) for an equilibrium search model with contract posting in which firms offer different contracts to workers of different experience.

² For examples of such models see Albrecht and Axell (1984) and Albrecht and Vroman (2005).

from F . If a job offer is rejected, the worker remains in his/her current state and there is no recall. We make the standard tie-breaking assumptions: an unemployed worker accepts a job offer if indifferent to accepting it or remaining unemployed, while an employed worker quits only if the job offer is strictly preferred.

Workers die and so exit the labour market according to a Poisson process with parameter $\phi > 0$. ϕ also describes the inflow of new labour market entrants with initial productivity $y = 1$ (and so ensures a unit mass of workers in a steady state). As there is worker death, for simplicity the rate of time preference is set to zero (i.e. there is no discounting). We require $\phi > \rho$ to ensure discounted payoffs are finite.

All agents are risk neutral. Each firm chooses wage rate θ to maximize steady state flow profit, taking into account the search strategies of workers. Given the distribution of job offers F , each worker uses a search strategy to maximize expected discounted lifetime income. Section 2 describes the optimal search strategies of workers given F . Section 3 defines and characterizes a Market Equilibrium where F is consistent with profit maximizing wage posting strategies by firms. Section 4 discusses the results.

3 Optimal Search Strategies.

Let $W^U(y)$ denote the expected lifetime payoff of an unemployed worker with productivity y using an optimal search strategy. Let $W^E(y, \theta)$ denote the expected lifetime payoff of a worker with productivity y , currently employed at pay rate θ , when using an optimal search strategy. As it is immediate that $W^E(y, \theta)$ is strictly increasing in θ (it is always better to be employed at a firm with a larger pay rate) an employed worker will quit to an outside

offer θ' if and only if $\theta' > \theta$. Thus the flow Bellman equation for employed workers implies:

$$(\phi + \delta)W^E(y, \theta) = \theta y + \rho y \frac{\partial W^E}{\partial y} + \lambda \int_{\theta}^{\bar{\theta}} [W^E(y, \theta') - W^E(y, \theta)] dF(\theta') + \delta W^U(y). \quad (1)$$

The first term on the right hand side describes flow earnings, the second describes increased value through on-the-job learning, the third describes the capital gain by receiving a preferred outside offer $\theta' > \theta$, while the last corresponds to the welfare loss through being laid-off.

As there is no on-the-job learning while unemployed (and no depreciation), the flow Bellman equation describing $W^U(y)$ is instead

$$\phi W^U(y) = by + \lambda \int_{\underline{\theta}}^{\bar{\theta}} \max[W^E(y, \theta') - W^U(y), 0] dF(\theta'). \quad (2)$$

where a job offer is received at rate λ and conditional on the realized draw θ' , the worker either accepts it and enjoys welfare gain $W^E(y, \theta') - W^U(y)$, or remains unemployed with productivity y .

Note that a worker's income, whether unemployed or employed, is always proportional to y . As on-the-job learning is also proportional to y and workers are risk neutral, the above Bellman equations imply value functions of the form

$$W^U(y) = \alpha^U y$$

$$W^E(y, \theta) = \alpha^E(\theta) y$$

where we now determine α^U and $\alpha^E(\cdot)$.

The Bellman equation (2) implies the unemployed worker's optimal strategy is to accept any offer θ' satisfying $W^E(y, \theta') \geq W^U(y)$. As W^E is increasing in θ' , the worker will accept any offer $\theta' \geq \theta^R$ where the reservation pay rate θ^R is given by $\alpha^E(\theta^R) = \alpha^U$. Claim 1 now

characterizes α^U and $\alpha^E(\cdot)$. It is convenient first to define

$$q(\theta) = \phi + \delta + \lambda(1 - F(\theta)),$$

which is the rate any given employee exits employment from a firm with pay rate θ .

Claim 1: $W^E(y, \theta) = \alpha^E(\theta)y$ and $W^U(y) = \alpha^U y$ satisfy the above Bellman equations if and only if

(i) $\alpha^E(\cdot)$ is the solution to the ordinary differential equation

$$\frac{d\alpha^E}{d\theta} = \frac{1}{q(\theta) - \rho}, \quad (3)$$

with boundary condition $\alpha^E(\theta) = (\bar{\theta} + \delta\alpha^U)/(\delta + \phi - \rho)$ at $\theta = \bar{\theta}$, and

(ii) α^U satisfies :

$$\phi\alpha^U = b + \lambda \int_{\theta^R}^{\bar{\theta}} [\alpha^E(\theta') - \alpha^U] dF(\theta'). \quad (4)$$

with θ^R satisfying $\alpha^E(\theta^R) = \alpha^U$.

Proof:

Given the above functional forms for W^U and W^E , the Bellman equation (2) is equivalent to (4). The Bellman equation (1) is equivalent to

$$(\phi + \delta)\alpha^E(\theta) = \theta + \rho\alpha^E(\theta) + \lambda \int_{\theta}^{\bar{\theta}} [\alpha^E(\theta') - \alpha^E(\theta)] dF(\theta') + \delta\alpha^U \quad (5)$$

which is a functional equation for $\alpha^E(\cdot)$. Differentiating with respect to θ yields the stated differential equation for $\alpha^E(\cdot)$ and putting $\theta = \bar{\theta}$ yields its boundary value $\alpha^E(\bar{\theta})$. This completes the proof of Claim 1.

To solve the conditions of Claim 1, we reduce them to two equations for α^U and θ^R . First putting $\theta = \theta^R$ in (5) and using $\alpha^E(\theta^R) = \alpha^U$ implies

$$\phi\alpha^U = \theta^R + \rho\alpha^U + \lambda \int_{\theta^R}^{\bar{\theta}} [\alpha^E(\theta') - \alpha^U] dF(\theta').$$

Figure 1: The locuses LL and II describe (6) and (7), respectively. Existence of α^u, θ^R occurs at the intersection of these two curves. Locus II is graphed to produce a $\theta^R > 0$.

Comparing this equation with (4) establishes (6) described in Lemma 1 below. Second we can integrate (4) by parts. Using (3) and that $\alpha^E(\theta^R) = \alpha^U$ then yields (7) in Lemma 1. Thus (6) and (7) in Lemma 1 describe a pair of equations which determine (α^U, θ^R) . We now establish a solution exists and is unique.

Lemma 1: *For any F , (α^U, θ^R) satisfy the pair of equations*

$$\rho\alpha^U = b - \theta^R \quad (6)$$

$$\phi\alpha^U = b + \int_{\theta^R}^{\bar{\theta}} \frac{\lambda(1 - F(\theta))}{q(\theta) - \rho} d\theta. \quad (7)$$

Furthermore a solution exists, is unique and implies $\alpha^U > 0$ and $\theta^R < b$.

Proof: Consider Figure 1 which plots these two equations, where (α^U, θ^R) occurs at the intersection.

(6) is linear, has slope $-1/\rho$ and $\alpha^U = 0$ at $\theta^R = b$. Differentiating (7) with respect to θ^R implies the second locus is strictly decreasing with slope

$$\left[\frac{d\alpha^U}{d\theta^R} \right]_{eqn(7)} = -\frac{1}{\phi} \left(\frac{\lambda(1 - F(\theta^R))}{q(\theta^R) - \rho} \right),$$

and α^U is strictly positive and finite at $\theta^R = b$. Simple algebra using the parameter restriction $\phi > \rho$ establishes this latter locus is flatter (strictly) than the first locus for all θ , even when $\theta^R < \underline{\theta}$. Continuity now implies these two loci must have a unique intersection at some $\theta^R < b$. This completes the proof of Lemma 1.

In Burdett and Mortensen (1998) with no experience effects, the reservation pay rate $\theta^R = b$. Here with learning on-the-job, optimal job search instead implies $\theta^R < b$. An unemployed worker is willing to sacrifice current income to invest in increased productivity, and thus higher wages in the future.

Two comparative static results are useful for what follows. First suppose an increase in ρ ; there is greater learning on-the-job. It is straightforward to show this implies θ^R decreases; a greater on-the-job learning rate ρ implies workers will accept lower pay rates. This reflects the previous argument - an unemployed worker is willing to sacrifice current income to invest in increased productivity.

A reasonable conjecture (confirmed below) is that by increasing the value of employment, a greater ρ , in equilibrium, leads to greater pay rate competition across firms. Consider then an alternative distribution F_1 which first order stochastically dominates F ; i.e. $F_1 \leq F$ for all θ . Ceteris paribus this implies a shift up in the II locus in Figure 1 and an unambiguous decrease in θ^R . Thus as firms become more competitive on wages, unemployed workers are willing to accept a lower pay rate. This occurs as workers can continue to look for better

paid employment while employed and they accumulate more skills on-the-job which are more valuable when pay rates are more generous. Thus a curious property of this market with on-the-job learning is that as job offers become more competitive, the unemployed are willing to work for lower pay.

4 Market Equilibrium

As described in Figure 1, equations (6) and (7) map the distribution of offers F into a (unique) reservation pay rate θ^R for unemployed workers. In this section we consider profit maximizing behavior by firms and so determine equilibrium F .

Let U denote the steady state number of unemployed workers and so $1 - U$ workers are employed. Each employed worker is characterized by (y, θ) ; their current productivity and pay rate θ . Let $H(y, \theta)$ denote the distribution of productivities and wage rates across employed workers; i.e. $H(y, \theta)$ is the proportion of employed workers with productivity no greater than y earning pay rate no larger than θ . Let $N(y)$ denote the c.d.f. describing the distribution of productivities across unemployed workers.

As there is no discounting, the arguments in Burdett and Coles (2003) imply steady state flow profit equals the hiring rate of the firm, multiplied by the expected (discounted) profit of each hire. If the firm offers pay rate $\theta < \theta^R$, it attracts no employees and so makes zero profit. Consider instead offers with $\theta \geq \theta^R$. In this case steady state flow profit can be written as

$$\begin{aligned} \pi(\theta) = & \lambda U \int_{y=1}^{\infty} \left[\int_{x=0}^{\infty} e^{-q(\theta)x} (1 - \theta) y e^{\rho x} dx \right] dN(y) \\ & + \lambda(1 - U) \int_{\theta'=\underline{\theta}}^{\theta} \int_{y=1}^{\infty} \left[\int_{x=0}^{\infty} e^{-q(\theta)x} (1 - \theta) y e^{\rho x} dx \right] dH(y, \theta'). \end{aligned}$$

The first term is the steady state flow profit due to attracting unemployed workers with productivity $y \in [1, \infty)$; $\lambda U dN(y)$ is the hiring inflow of such workers and the inside bracketed integral is the expected discounted profit per hired unemployed worker with productivity y , taking into account that productivity on-the-job increases at rate ρ and the worker leaves the firm at rate $q(\theta)$. The second term is the flow profit due to attracting employed workers on wage rates $\theta' < \theta$ with productivity $y \in [1, \infty)$; $\lambda(1 - U)dH(y, \theta')$ describes the hiring inflow of each such worker and the inside bracketed integral is again the expected discounted profit per hire. Integrating over x yields:

$$\pi(\theta) = \frac{\lambda(1 - \theta)}{q(\theta) - \rho} \left[U \int_{y'=1}^{\infty} y' dN(y') + (1 - U) \int_{\theta'=\underline{\theta}}^{\theta} \int_{y'=1}^{\infty} y' dH(y', \theta') \right]. \quad (8)$$

As $\theta^R < b$ (Lemma 1), a firm can always guarantee strictly positive profit by offering pay rate $\theta = b < 1$. Thus there is no loss in generality in assuming firms make strictly positive profit in a Market Equilibrium. We now formally define such an equilibrium.

A Market Equilibrium is a quintuple $\{\theta^R, U, N(\cdot), H(\cdot), F(\cdot)\}$ such that

- (i) θ^R is the optimal reservation pay rate of any unemployed worker;
- (ii) $U, N(\cdot), H(\cdot)$ are consistent with steady state turnover given wage offers $F(\cdot)$ and optimal worker search strategies;
- (iii) the constant profit condition is satisfied; i.e.,

$$\pi(\theta) = \bar{\pi} > 0 \text{ for all } \theta \text{ where } dF(\theta) > 0;$$

$$\pi(\theta) \leq \bar{\pi} \quad \text{for all } \theta \text{ where } dF(\theta) = 0.$$

We begin with a standard result which much simplifies the analysis.

Lemma 2: *A Market Equilibrium implies (i) $F(\cdot)$ contains no mass points; (ii) $F(\cdot)$ has a connected support and (iii) $\underline{\theta} = \theta^R$.*

As the arguments are already well known (see B/M) we only sketch the proof. A contradiction argument establishes there cannot be a mass point in F . If there were, say at $\theta = \theta^m$, steady state would imply a mass of employees on wage θ^m . But offering wage rate $\theta = \theta^m + \varepsilon$ where $\varepsilon > 0$ but very small would yield a slightly lower profit per hire but a large increase in the hiring rate, and this deviation would strictly increase profit (and so contradict equilibrium). A contradiction argument also establishes the support of F must be connected. Otherwise if there were a hole, say for $\theta \in [\theta_L, \theta_H]$ then the offer $\theta = \theta_L$ would yield strictly greater profit (as both offers attract the same number of workers) which contradicts θ_H being an optimal offer. Finally a contradiction argument also establishes $\underline{\theta} = \theta^R$. $\underline{\theta} < \theta^R$ would imply a firm offering $\theta = \underline{\theta}$ obtains zero profit, which contradicts strictly positive profit. $\underline{\theta} > \theta^R$ would instead imply offering $\theta = \theta^R$ makes strictly greater profit than offering $\underline{\theta}$ which contradicts optimality of $\underline{\theta}$.

Next we determine steady state U, N , and H . The model is then closed as F must imply the constant profit condition as described in the definition of a Market Equilibrium.

4.1 Equilibrium Unemployment U

As Lemma 2(iii) states $\underline{\theta} = \theta^R$ in a Market Equilibrium, each unemployed worker accepts the first job offer received. Therefore the flow into unemployment is $\phi + \delta(1 - U)$, while the flow out is $(\lambda + \phi)U$. Hence, steady state U can be written as

$$U = \frac{\phi + \delta}{\phi + \delta + \lambda}.$$

4.2 Equilibrium $N(y)$

$UN(y)$ describes the number of unemployed workers with productivity no greater than y .

For $y \geq 1$, steady state turnover requires

$$\phi + \delta[1 - U]H(y, \bar{\theta}) = [\phi + \lambda]UN(y),$$

where the left hand side describes the inflow (new labour market entrants with productivity $y = 1$ and employed workers with productivity less than y who lose their jobs) and the right hand side describes the outflow. Solving for $N(y)$ (and using the above solution for U) yields

$$N(y) = \frac{\phi(\phi + \delta + \lambda) + \lambda\delta H(y, \bar{\theta})}{(\phi + \lambda)(\phi + \delta)} \quad \text{for all } y \geq 1. \quad (9)$$

4.3 Equilibrium $H(y, \theta)$

$(1 - U)H(y, \theta)$ describes the number of employed workers with productivity no greater than y and pay rate no more than θ . For $\theta \geq \underline{\theta}$ and $y \geq 1$, the total outflow of workers from this pool, over any instant of time $dt > 0$, is

$$(1 - U)H(y, \theta)[q(\theta)]dt + (1 - U) \left[H(y, \theta) - H\left(\frac{y}{1 + \rho dt}, \theta\right) \right] + O(dt^2).$$

The first term specifies the number of workers who either leave employment, or quit to a job with $\theta' > \theta$. The second describes those who exit this pool as their productivity increases above y through on-the-job learning. The last term corrects for the fact that some do both but this term has the property $O(dt^2)/dt \rightarrow 0$ as $dt \rightarrow 0$. The inflow into this pool over small time period dt , is $UN(y)\lambda F(\theta)dt$ which is simply those unemployed workers with productivity no greater than y who find a job no better than θ . Setting inflow equal to outflow, rearranging appropriately and then letting $dt \rightarrow 0$ yields the following partial

differential equation for H

$$q(\theta)H(y, \theta) + \rho y \frac{\partial H(y, \theta)}{\partial y} = (\phi + \delta)F(\theta)N(y).$$

which is more usefully rearranged as

$$\frac{\partial H}{\partial y} + \frac{q(\theta)}{\rho y} H = \frac{(\phi + \delta)F(\theta)N(y)}{\rho y}, \quad (10)$$

for $\theta \in [\underline{\theta}, \bar{\theta}]$. Note this differential equation is linear in H . Integration using the integrating factor $y^{\frac{q(\theta)}{\rho}}$ implies

$$\left[H y^{\frac{q(\theta)}{\rho}} \right]_{y'=1}^y = \int_{y'=1}^y y'^{\frac{q(\theta)}{\rho}} \frac{(\phi + \delta)F(\theta)N(y')}{\rho y'} dy'.$$

As $H(1, \theta) = 0$, we obtain

$$H(y, \theta) = \frac{(\phi + \delta)F(\theta)}{\rho} y^{-\frac{q(\theta)}{\rho}} \int_1^y y'^{\frac{q(\theta)}{\rho}-1} N(y') dy'. \quad (11)$$

The next Lemma now solves for steady state $N(\cdot)$ and $H(\cdot, \cdot)$.

Lemma 3: *A Market Equilibrium implies distribution functions*

$$N(y) = 1 - \frac{\lambda \delta}{(\phi + \lambda)(\phi + \delta)} y^{-\left(\frac{\phi(\phi + \delta + \lambda)}{\rho(\phi + \lambda)}\right)} \text{ for all } y \geq 1, \quad (12)$$

$$H(y, \theta) = \frac{(\phi + \delta)F(\theta)}{q(\theta)} [1 - y^{-\frac{q(\theta)}{\rho}}] - \frac{\delta F(\theta)}{q(\theta) - \phi F(\theta)} [y^{-\left(\frac{\phi(\phi + \delta + \lambda)}{\rho(\phi + \lambda)}\right)} - y^{-\frac{q(\theta)}{\rho}}] \text{ for all } \theta \in [\underline{\theta}, \bar{\theta}] \text{ and } y \geq 1. \quad (13)$$

Proof:

Set $\theta = \bar{\theta}$. (10) implies $H(y, \bar{\theta})$ satisfies the differential equation

$$\frac{\partial H}{\partial y} + \frac{\phi + \delta}{\rho y} H = \frac{(\phi + \delta)N(y)}{\rho y} \text{ for all } y \geq 1.$$

Now use (9) to substitute out $N(\cdot)$. Simplifying yields the differential equation

$$\frac{\partial H}{\partial y} = \frac{\phi(\phi + \delta + \lambda)}{\rho(\phi + \lambda)} \frac{1 - H}{y} \text{ for all } y \geq 1,$$

where $H = H(y, \bar{\theta})$. As this differential equation is separable in H and y , integration is straightforward and the boundary condition $H(1, \bar{\theta}) = 0$ implies

$$H(y, \bar{\theta}) = 1 - y^{-\left(\frac{\phi(\phi + \delta + \lambda)}{\rho(\phi + \lambda)}\right)}. \quad (14)$$

Using this in (9) and simplifying yields the desired result (12). Using (12) to substitute out $N(\cdot)$ in (11) and some algebra then establishes (13). This completes the proof of Lemma 3.

Note that $H(y, \bar{\theta})$ describes the distribution of productivities across employed workers, while $N(y)$ describes unemployed worker productivities. $N(1) > 0$ implies there is a mass of unemployed workers with productivity $y = 1$. But for $y > 1$, both (14) and (12) imply these productivity densities decline geometrically at the same rate, which depends on the turnover parameters λ, δ, ϕ and the on-the-job learning parameter ρ .

4.4 Equilibrium $F(\theta)$

We now turn to the constant profit condition, $\pi(\theta) = \bar{\pi}$ for all $\theta \in [\underline{\theta}, \bar{\theta}]$, where (8) implies:

$$\pi(\theta) = \frac{\lambda(1 - \theta)}{q(\theta) - \rho} \left[U \int_{y'=1}^{\infty} y' dN(y') + (1 - U) \int_{y'=1}^{\infty} \int_{\theta'=\underline{\theta}}^{\theta} y' \frac{\partial^2 H(y', \theta')}{\partial y' \partial \theta'} d\theta' dy' \right]. \quad (15)$$

Using the solutions for N, H identified in Lemma 3, Claim 2 now computes these integrals.

Claim 2: *In a steady state,*

$$\begin{aligned} \int_{y'=1}^{\infty} y' dN(y') &= \frac{\phi(\phi + \delta + \lambda)}{(\phi + \delta)} \left[\frac{(\phi + \delta - \rho)}{\phi(\phi + \delta + \lambda) - \rho(\phi + \lambda)} \right], \\ \int_{y'=1}^{\infty} \int_{\theta'=\underline{\theta}}^{\theta} y' \frac{\partial^2 H(y', \theta')}{\partial y' \partial \theta'} d\theta' dy' &= \frac{\phi F(\theta)}{q(\theta) - \rho} \left[\frac{(\phi + \delta + \lambda)(\phi + \delta - \rho)}{\phi(\phi + \delta + \lambda) - \rho(\phi + \lambda)} \right]. \end{aligned}$$

Proof:

See Appendix.

Claim 2 and (15) yields the constant profit condition:

$$\frac{\lambda(1-\theta)}{q(\theta)-\rho} \left[\frac{\phi(\phi+\delta-\rho)}{\phi(\phi+\delta+\lambda)-\rho(\phi+\lambda)} + \frac{\lambda F(\theta)}{q(\theta)-\rho} \left(\frac{\phi(\phi+\delta-\rho)}{\phi(\phi+\delta+\lambda)-\rho(\phi+\lambda)} \right) \right] = \bar{\pi},$$

for all $\theta \in [\underline{\theta}, \bar{\theta}]$, which thus determines F . Long but straightforward algebra finds this condition simplifies to

$$\lambda(1-\theta) \left[\frac{\phi(\phi+\delta-\rho)(\phi+\delta-\rho+\lambda)}{\phi(\phi+\delta+\lambda)-\rho(\phi+\lambda)} \right] = \bar{\pi}(q(\theta)-\rho)^2. \quad (16)$$

Noting that $q = \phi + \delta + \lambda(1-F)$ and that F must be increasing over θ , this quadratic equation for F now yields Lemma 4.

Lemma 4: *A Market Equilibrium implies*

$$F(\theta) = \frac{\phi + \delta + \lambda - \rho}{\lambda} - \left[\frac{\phi(\phi + \delta - \rho)[\phi + \delta - \rho + \lambda]}{\lambda \bar{\pi}(\phi(\phi + \delta + \lambda) - \rho(\phi + \lambda))} \right]^{1/2} (1 - \theta)^{1/2}. \quad (17)$$

Somewhat surprisingly, the functional form describing equilibrium F is the same as in B/M . Given this simple solution for F , we now obtain a closed form solution for a Market Equilibrium.

4.5 A Market Equilibrium.

Suppose a Market Equilibrium implies firms make steady state profit $\bar{\pi} > 0$ and $\underline{\theta}$ is the infimum of F . As $F = 0$ at $\theta = \underline{\theta}$ then (17) implies $\bar{\pi}, \underline{\theta}$ are related as

$$\frac{\phi + \delta + \lambda - \rho}{\lambda} = \left[(1 - \underline{\theta}) \frac{\phi(\phi + \delta - \rho)[\phi + \delta - \rho + \lambda]}{\lambda \bar{\pi}(\phi(\phi + \delta + \lambda) - \rho(\phi + \lambda))} \right]^{1/2}.$$

It is convenient to use this condition to substitute out $\bar{\pi}$ in (17). Thus the equilibrium offer distribution can be written as

$$F = \widehat{F}(\theta \mid \underline{\theta}) = \left(\frac{\phi + \delta - \rho + \lambda}{\lambda} \right) \left[1 - \left(\frac{1 - \theta}{1 - \underline{\theta}} \right)^{1/2} \right]. \quad (18)$$

The goal now is to solve for equilibrium $\underline{\theta}$ and so determine F . This requires solving the following problem. Consider an arbitrary value for $\underline{\theta} \leq b$. If $\underline{\theta}$ is an equilibrium value, then (18) implies $F = \widehat{F}(\theta \mid \underline{\theta})$ is the equilibrium distribution of offers. Given this explicit solution for F , we then solve the conditions of Lemma 1 for the workers' optimal θ^R . Let $\theta^R = \theta^R(\underline{\theta})$ denote that solution. As a Market Equilibrium requires $\theta^R(\underline{\theta}) = \underline{\theta}$ (by Lemma 2(iii)) this condition ties down $\underline{\theta}$.

Theorem 1. *For any $\rho < \phi$ a Market Equilibrium exists, is unique and implies reservation pay rate*

$$\theta^R = \frac{b(\phi - \rho)(\phi + \delta - \rho + \lambda)^2 - \rho\lambda^2}{\phi(\phi + \delta - \rho + \lambda)^2 - \rho\lambda^2}. \quad (19)$$

Equilibrium F is given by (18) with $\underline{\theta} = \theta^R$, and the steady state distribution functions N, H are as described in Lemma 3.

Proof: In the Appendix we show that solving $\theta^R(\underline{\theta}) = \underline{\theta}$ yields the unique solution (19) for $\underline{\theta}$. Given the equilibrium distribution F as described in Theorem 1 then, by construction, the unemployed worker's optimal reservation pay rate $\theta^R = \underline{\theta}$. Further U, N, H are consistent with steady state turnover and F ensures the constant profit condition holds for all $\theta \in [\underline{\theta}, \bar{\theta}]$. All that remains is to show there is no other offer which is profit increasing. Offering $\theta < \underline{\theta}$ yields zero profit as all workers reject such offers (as $\theta^R = \underline{\theta}$). Conversely offering $\theta > \bar{\theta}$ yields strictly less profit than offering $\theta = \bar{\theta}$ as it attracts no additional workers. Thus the above identifies the Market Equilibrium. This completes the proof of Theorem 1.

Note for given $\underline{\theta}$, the offer distribution $F = \widehat{F}(\theta \mid \underline{\theta})$ described in equation (18) is first order stochastically increasing in ρ . Thus, ceteris paribus, greater on-the-job learning rates tends to generate more competitive pay rate offers. But some algebra using (19) implies θ^R decreases with ρ ; unemployed workers become less selective in a Market Equilibrium. This latter outcome reflects the comparative static arguments given at the end of Section 2. A curious feature of equilibrium, then, is that higher ρ implies firms tend to be more competitive on pay rates while unemployed workers become less selective which leads to a fall in $\underline{\theta}$. This interaction reflects the dual role played by a firm's pay rate offer: the firm wishes to set a low pay rate to extract the search rents of new recruits but wishes to offer higher pay rates to prevent subsequent quits. In a simpler framework with no on-the-job learning, Stevens (2004), Burdett and Coles (2003) show that firms can increase profit by offering contracts where wages increase with tenure. For tractability, however, here we have ruled out such contracts.

5 Discussion

A surprising feature of the analysis is that the distribution of pay rate offers F has the same functional form as that identified in B/M . As in B/M , it follows that the density of pay rates received by employed workers is increasing in θ . In B/M where wage $w = \theta$, this implies the density of wages paid to workers has an increasing right tail which is inconsistent with the data. But here instead wages are $w = \theta y$. The distribution of wages paid thus depends both on wage increases due to quit turnover, and wage increases due to on-the-job learning and increased productivity. Here we investigate the equilibrium interaction of these two effects. For ease of exposition from now on assume b sufficiently large that $\underline{\theta} > 0$.

Recall that $H(y, \theta)$ describes the distribution of (y, θ) across employed workers. Lemma 3 implies

$$H(y, \theta) = \frac{(\phi + \delta)F}{q} [1 - y^{-\frac{q}{\rho}}] - \frac{\delta F}{q - \phi F} [y^{-\frac{\phi(\phi + \delta + \lambda)}{\rho(\phi + \lambda)}} - y^{-\frac{q}{\rho}}] \quad (20)$$

where equilibrium $F = F(\theta)$ is given by Theorem 1.

Consider a worker with productivity $y > 1$. First note that all such workers have the same unemployment rate. For those who are currently employed, the conditional distribution of earned pay rates is $H(\theta | y) = \int_{\theta'=\underline{\theta}}^{\bar{\theta}} \frac{\partial^2 H(y, \theta')}{\partial y \partial \theta'} d\theta' / \int_{\theta'=\underline{\theta}}^{\bar{\theta}} \frac{\partial^2 H(y, \theta')}{\partial y \partial \theta'} d\theta'$ which by (20) reduces to:

$$H(\theta | y) = F \left[\frac{\delta + (\phi + \lambda)(1 - F)y^{-\frac{\lambda(q - \phi F)}{\rho(\phi + \lambda)}}}{q - \phi F} \right].$$

As new entrants have productivity $y = 1$, and as their first job is a random draw from F , note that $H(\theta | 1) = F$. Over time, employed workers search on-the-job for better paid work and quit whenever such a job opportunity arises. $H(\theta | y)$ is thus strictly decreasing in y ; i.e. the conditional distribution of earned pay rates is first order stochastically increasing in y . Employed workers with greater experience thus typically earn higher pay rates.

Note that as $y \rightarrow \infty$, this distribution function converges to $\delta F / (q - \phi F)$ which is finite. But as (14) implies the density of y across employed workers declines geometrically at rate $\left(\frac{\phi(\phi + \delta + \lambda)}{\rho(\phi + \lambda)} + 1 \right)$, it follows that at high enough productivities, the density of wages paid, where $w = \theta y$, must also decline geometrically at this rate. We shall return to this insight below.

Consider now a firm paying θ . Random matching implies the distribution of worker productivities at that firm is $H(y | \theta)$ as that describes the productivities y of all workers employed at all type θ firms. Now $H(y | \theta) = \int_{y'=1}^y \frac{\partial^2 H(y', \theta)}{\partial y' \partial \theta} dy' / \int_{y'=1}^{\infty} \frac{\partial^2 H(y', \theta)}{\partial y' \partial \theta} dy'$ and the

joint cdf (20) implies this reduces to:

$$H(y | \theta) = \left(1 - y^{-\frac{q}{\rho}}\right) - \frac{\delta q^2}{(\phi + \delta) [q - \phi F]^2} \left[y^{-\left(\frac{\phi(\phi + \delta + \lambda)}{\rho(\phi + \lambda)}\right)} - y^{-\frac{q}{\rho}} \right] - \frac{\lambda F \phi (1 - F) q}{\rho(\phi + \delta) [q - \phi F]} y^{-\frac{q}{\rho}} \log y.$$

This expression is most easily interpreted by first considering $\theta = \underline{\theta}$ and $\delta = 0$. Such a firm only attracts unemployed workers and the second two terms in the above expression are then zero. The first term thus describes the distribution of worker productivities by hiring new labour market entrants, taking into account that employee productivity grows at rate ρ and workers exit this firm's employment at rate q . The second term is the feedback due to job displacement, where $\delta > 0$ implies displaced workers with positive work experience re-enter the pool of unemployment and each firm hires from this unemployment pool. For $\theta > \underline{\theta}$, the third term takes into account that a firm offering θ attracts workers from firms offering $\theta' < \theta$ and those workers also hold previous experience.

A sufficient condition for first order stochastic dominance is $\delta > \phi$; i.e. $\delta > \phi$ guarantees $\partial[H(y | \theta)]/\partial\theta < 0$. Thus firms offering higher pay rates not only attract more workers (as in B/M), but the composition of its workforce is skewed towards more experienced and thus more productive workers. Note the restriction $\delta > \phi$ implies each worker expects to be laid off at least once over a working lifetime which would appear to be the most reasonable case.³

We now turn attention to the equilibrium wage distribution, which we denote $G(w)$. Let

³ To obtain this result, differentiation yields

$$\begin{aligned} \frac{\partial H(y | \theta)}{\partial \theta} = & -\frac{2\delta F' \phi (\phi + \lambda + \delta) q}{(\phi + \delta) [q - \phi F]^3} \left[y^{-\left(\frac{\phi(\phi + \delta + \lambda)}{\rho(\phi + \lambda)}\right)} - y^{-\frac{q}{\rho}} \right] - \left[\frac{\phi q \lambda^2 F' F [1 - F]}{\rho^2 (\phi + \delta) [q - \phi F]} \right] y^{-\frac{q}{\rho}} [\log y]^2 \\ & - y^{-\frac{q}{\rho}} \frac{\lambda F'}{\rho} \log y \left[1 + \frac{\phi^2 (\phi + \lambda + \delta) F [1 - F]}{(\phi + \delta) [q - \phi F]^2} + \frac{\phi q [1 - 2F]}{(\phi + \delta) [q - \phi F]} \right] \end{aligned}$$

The first two terms (with $y \geq 1$) are guaranteed to be negative. The third term is guaranteed to be negative if $F < \delta/\phi$. Thus $\delta > \phi$ is sufficient for first order stochastic dominance.

$h(y, \theta)$ denote the joint density of (y, θ) across employed workers. Thus given H defined by (20) above

$$\begin{aligned} h &= \partial^2 H / \partial y \partial \theta \text{ for } y \geq 1 \text{ and } \theta \in [\underline{\theta}, \bar{\theta}] \\ &= 0 \text{ otherwise.} \end{aligned}$$

As wages $w = \theta y$, then by definition:

$$G(w) = \int_{\theta'=\underline{\theta}}^{\bar{\theta}} \left[\int_{y'=1}^{w/\theta'} h(y, \theta') dy' \right] d\theta'.$$

Differentiating with respect to w implies the wage density is:

$$G'(w) = \int_{\theta'=\underline{\theta}}^{\bar{\theta}} h\left(\frac{w}{\theta'}, \theta'\right) \frac{1}{\theta'} d\theta'. \quad (21)$$

Although it is straightforward to compute an analytic solution for G , that solution is long and unwieldy. The solution reflects the interaction between (i) sorting dynamics as workers quit for better paid employment, (ii) worker productivity grows through on the job learning, and (iii) the distribution of pay rates F is endogenous. Nevertheless two useful insights are immediate.

First consider new entrants who have the lowest productivity level $y = 1$. As their first job has pay rate θ which is a random draw from F , then each labour market entrant enjoys starting wage $w_0 = \theta$ with distribution $F(\theta)$. Learning by doing implies wages then strictly increase with experience. But at the infimum of the wage distribution, $\underline{w} = \underline{\theta}$, the density of wages paid must be zero. Formally for $w \in [\underline{\theta}, \bar{\theta}]$, the wage density is⁴

$$G'(w) = \int_{\theta'=\underline{\theta}}^w h\left(\frac{w}{\theta'}, \theta'\right) \frac{1}{\theta'} d\theta'.$$

⁴ Note $\theta' > w$ implies $y = w/\theta' < 1$ and $h(\cdot) = 0$ for such y .

and so $G'(w) \rightarrow 0$ as $w \rightarrow \underline{w} = \underline{\theta} > 0$. Thus at low wages, the wage density must initially be increasing with w .

Second consider workers with wages $w > \bar{\theta}$. Using (20) it is straightforward to show $h(\frac{w}{\bar{\theta}}, \theta')$ is $O(w^{-(\frac{\phi(\phi+\delta+\lambda)}{\rho(\phi+\lambda)})-1})$ as $w \rightarrow \infty$. Thus equation (21) implies

$$G'(w) = O(w^{-(\frac{\phi(\phi+\delta+\lambda)}{\rho(\phi+\lambda)})-1}) \text{ as } w \rightarrow \infty.$$

Reflecting that as experience grows large, the distribution of employee pay rates $H(\theta \mid y)$ converges to its ergodic distribution $H(\theta \mid \infty)$, the density of wages paid ultimately declines geometrically at the same rate as the underlying distribution of worker productivities $H(y, \bar{\theta})$. Note then that the density of wages paid is increasing for sufficiently low w but, unlike B/M , the wage density must be decreasing for sufficiently high w .

Some further insight is obtained by considering the limiting market outcomes. Suppose search frictions increase and consider the limit $\lambda \rightarrow 0$. In this limit (19) implies $\underline{\theta} \rightarrow b(\phi - \rho)/\phi$ and (18) implies $\bar{\theta} \rightarrow \underline{\theta}$. As the possibility of an employee receiving an outside offer goes to zero, Market Equilibrium converges to the so-called Diamond paradox (Diamond, 1971): all firms offer the same pay rate and each worker is indifferent between being employed and unemployed. But as all firms pay the same rate, all wage dispersion is entirely due to worker productivity dispersion. Thus we have

$$G(w) \rightarrow 1 - \left(\frac{w}{\bar{\theta}^R}\right)^{-\frac{(\phi+\delta)}{\rho}} \text{ as } \lambda \rightarrow 0.$$

Conversely as $\lambda \rightarrow \infty$ the distribution of pay rates converges to a mass point at $\bar{\theta} = 1$; the competitive equilibrium. Thus we also have

$$G(w) \rightarrow 1 - w^{-\frac{\phi}{\rho}} \text{ as } \lambda \rightarrow \infty.$$

In both limiting cases, the distribution of pay rates becomes degenerate and the wage distribution is isomorphic to the underlying distribution of worker productivities. Further these wage distributions imply densities which are monotonically decreasing.

Conversely suppose $\lambda > 0$ but $\rho \rightarrow 0$. In that case it is straightforward to show the wage distribution converges to the one implied in Burdett and Mortensen (1998) - the wage density thus increases monotonically over $w \in [\underline{\theta}, \bar{\theta}]$. Note, however, that $\rho > 0$ implies the support of wages paid is unbounded, though as $\rho \rightarrow 0$, the density of wages paid becomes arbitrarily small for wages $w > \bar{\theta}$.

For intermediate values of λ, ρ , the equilibrium distribution of wages paid is the convolution of these two effects. On the job search yields a density of wages paid which is increasing at low wages, while a geometrically declining productivity distribution at long experiences implies the wage density must ultimately decline at this same rate. Although it is tempting to conjecture the wage density G' is single peaked we have been unable to establish this. Nevertheless this is always the case in the following simulations.

5.1 Simulations

We now perform some numerical simulations to show further properties of the model and the wage distribution. Using a year as a time unit, we set $\phi = 0.025$ such that workers have 40 years of expected working lifetime, $\rho = 0.01$ implying a productivity growth of 1% per annum, $\delta = 0.1$ such that the job turnover rate is of 10% per annum (US), $\lambda = 2$ such that we obtain an unemployment rate of 5.8%, consistent with OECD countries and $b = 0.8$ to keep $\theta^R > 0$. This yields a $\theta^R = 0.1904$, $\bar{\theta} = 0.9976$ and an average wage of 1.43.⁵

⁵ Performing Monte Carlo simulations on workers' employment histories we also obtain that the average labour market experience is 38 years, the average tenure is 3 years and a worker holds on average 14 jobs,

Figure 2: Wage density with human capital accumulation and on-the-job search.

Figure 2 shows the wage density in this case. It exhibits an increasing left tail and a decreasing and long right tail, with an interior mode in the neighbourhood of $\bar{\theta}$. The left tail resembles the B/M wage density, while the right tail follows from the distribution of workers productivities. As a comparison, Figure 2 also shows a fitted Pareto density that roughly matches the density, g , at the mode and average wage. Note that the right tail properties of g are similar to that of the Pareto density.

It is well known that introducing firm heterogeneity to the B/M model also yields a wage density function that exhibits an interior mode and a decreasing right tail. However, in this extension of the model wage competition between firms is not strong enough to produce a wage density with a sufficiently long right tail. To approximate the empirical wage density

from which 9 are due to job to job transitions and the rest are jobs taken after an unemployment spell. Note that the number of job held seems too high, while the average tenure too low. However, this is due to the fact that we are constraining unemployed and employed workers to have the same offer arrival rate.

one then needs to assume a distribution of firm productivities with an unrealistically long right tail or more wage competition.⁶ Our analysis shows that the interaction between workers' on-the-job search and human capital accumulation can also help to generate a wage density with a longer right tail.

The simulations are also helpful to perform (local) comparative statics of the wage density with respect to the parameters of the model. In particular, the previous section argues that reducing search frictions implies the model converges to the competitive equilibrium, while by increasing search frictions we recover the Diamond outcome. The simulations show that as we increase λ , $\bar{\theta}$ increases and converges to 1 and θ^R decreases and converges to 0.1333. Figure 3 shows the wage density for an arbitrary large value of λ . It implies that the density for $w < 1$ goes to zero and instead a mass point at $w = 1$ appears. Moreover, in this case the right tail of the wage density exactly coincides with the declining density of the productivity distribution. Figure 4, on the other hand, shows the wage density when λ is arbitrarily close to zero. In this case, both θ^R and $\bar{\theta}$ converge to 0.544. As in the previous case the left tail of the wage density converges to a mass point and the right tail follows the productivity distribution. The main difference, however, is that with less frictions the g exhibits less skewness and kurtosis measures and a fatter right tail. Comparing the wage densities in Figures 2, 3 and 4 then reveals how search frictions distort the equilibrium distribution of wages paid away from $H(y, \bar{\theta})$, generating a “hump” shaped wage density as a result of workers' on-the-job search.

We now turn to comparative statics with respect to ρ . As established earlier, a higher

⁶ Mortensen (2003) introduces more competition between firms by allowing them to choose their recruiting effort. Postel-Vinay and Robin (2002) also introduce more competition by allowing firms to perfectly discriminate workers by their reservation wages and counter offer their employees' outside offers.

Figure 3: A comparison between the wage and productivity densities for $\lambda = 100,000$.

Figure 4: A comparison of the wage and productivity densities for $\lambda = 0.001$

rate of human capital accumulation reduces unemployed workers' reservation wage.⁷ Interestingly the simulations show that ρ is also inversely related to $\bar{\theta}$. Although the distribution of offer pay rates is first order stochastically increasing in ρ , firms are able to extract a higher share of the match rents. Figure 5 shows a comparison between wage densities with different ρ . The main difference is that a lower ρ decreases the variance of the wage distribution as there is a smaller proportion of workers with higher productivities and hence earning higher wages. Recall that as $\rho \rightarrow 0$, the wage density converges to that of B/M . The upper bound of its support becomes bounded and it exhibits an increasing and convex right tail. In this limit, any wage increase is due to job-to-job transitions. Figure 5 shows that increasing the rate at which workers accumulate human capital distorts the B/M distribution, generating wage densities that are right skewed with longer left tails (as θ^R decreases) and “fatter” and long right tails.

The simulations also show that increasing the displacement rate increases $\underline{\theta}$ and decreases $\bar{\theta}$. Note that under the assumption of the model a displaced worker faces the same offer distribution as any new entrant to the labour market. Hence, when displaced, a worker loses all the gains he/she made through job changes during the previous employment spell. In compensation for this future loss, unemployed workers now require a higher pay rate to accept a job. Increasing the displacement rate also implies that the wage density concentrates most of its mass at the left tail of the distribution, exhibiting a high kurtosis measure and a “thinner” right tail. Figure 6 shows a comparison between wage densities for different δ . In this case we have calibrated λ to keep the productivity distribution $H(y, \bar{\theta})$ unchanged. The objective is to compare two economies with the same productivity distribution but

⁷ For sufficient high values of ρ we find that unemployed workers' reservation pay rates are negative.

Figure 5: A comparison between wage densities for different rates of human capital accumulation.

with different rates of labour turnover. Note that since the productivity distribution is kept constant, the right tails of both wage densities coincide. The difference then arises from the proportion of workers at the left tail of the distribution. Lower turnover implies a “thinner” left tail and a smaller interior mode.

Finally, increasing the rate at which workers leave the market, ϕ , increases both $\underline{\theta}$ and $\bar{\theta}$, but reduces the range of F . Unemployed workers’ reservation pay rate increases for similar reasons as with δ . However, since a ϕ -shock pushes the worker out of the labour market, the increase in $\underline{\theta}$ is higher under increases in ϕ than in δ . As with the displacement rate, increasing ϕ increases the skewness of the wage density distribution. In this case, a higher chance that employed workers leave the market in the future implies the economy has less workers with higher experience earning high wages.

Figure 6: A comparison of two economies with the same productivity distribution but different rates of labour turnover.

6 Conclusions

Discuss the above insights in relation to the empirical literature which tries to decompose wages into experience and tenure effects. For example, the model above implies wage

$$\log w = \rho \log x + \log \theta,$$

and θ can be interpreted as the firm fixed effect. In contrast, the model of Burdett and Coles (2003) where wages increase with tenure τ might suggest an alternative functional form

$$\log w = \beta_{\theta} \log \tau + \log \theta$$

where the magnitude of any wage tenure effects, β_{θ} , are related to the firm's starting wage $w = \theta$. A large literature has tried to estimate β_{θ}, ρ . To understand why this might be problematic, suppose first there is no job displacement $\delta = 0$. In the model above there are no tenure effects, while in Burdett and Coles (2003) there are no experience effects. Yet in both cases wages increase while employed at any given firm (where some firms pay higher wages than other firms) and increase across any job switch. Identifying econometrically between tenure and experience effects without controlling for firm fixed effects would thus seem problematic .

This suggests two possible identifying routes. One would be to compare workers within a firm, with the same tenure and compare wages by experience. But this would only identify firm specific returns to general human capital. Job displacement, however, provides a more useful identification route. All unemployed workers clearly have zero tenure. Furthermore in the context of the above model, a displaced worker who is laid off with productivity $y = e^{\rho x}$

has expected re-employment wage

$$\log w^U(x) = \rho \log x + \log \int_{\underline{\theta}}^{\bar{\theta}} \theta dF(\theta).$$

Thus re-employment wages of displaced workers identifies experience effects. Though not articulated as an equilibrium search model, this identification procedure was followed by Dustmann and Meghir (2006). Once the return to experience is identified in this way, it is then possible to identify the return to tenure.

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APPENDIX

Proof of Claim 2:

(i) Lemma 3 implies

$$\int_{y'=1}^{\infty} y' dN(y') = N(1) + \int_{y'=1}^{\infty} \left(\frac{\phi(\phi + \delta + \lambda)}{\rho(\phi + \lambda)} \right) \frac{\lambda\delta}{(\phi + \lambda)(\phi + \delta)} (y')^{-\left(\frac{\phi(\phi + \delta + \lambda)}{\rho(\phi + \lambda)}\right)} dy'$$

and integrating yields the solution stated in the Claim.

(ii) Integrating over θ' implies

$$\int_{y'=1}^{\infty} \int_{\theta'=\underline{\theta}}^{\theta} y' \frac{\partial^2 H(y', \theta')}{\partial y' \partial \theta'} d\theta' dy' = \int_{y'=1}^{\infty} y' \left[\frac{\partial H(y', \theta')}{\partial y'} \right]_{\underline{\theta}}^{\theta} dy'$$

As $F(\underline{\theta}) = 0$, (13) then implies $H(y', \underline{\theta}) = 0$, and (10) then implies $\frac{\partial H(y', \underline{\theta})}{\partial y'} = 0$, this

reduces to

$$\int_{y'=1}^{\infty} \int_{\theta'=\underline{\theta}}^{\theta} y' \frac{\partial^2 H(y', \theta')}{\partial y' \partial \theta'} d\theta' dy' = \int_{y'=1}^{\infty} y' \frac{\partial H(y', \theta)}{\partial y'} dy'.$$

Now (13) implies

$$\frac{\partial H}{\partial y} = \frac{(\phi + \delta)F(\theta)}{q(\theta)} \frac{q(\theta)}{\rho} y^{-\frac{q(\theta)}{\rho}-1} + \frac{\delta F(\theta)}{q(\theta) - \phi F(\theta)} \left[\left(\frac{\phi(\phi + \delta + \lambda)}{\rho(\phi + \lambda)} \right) y^{-\left(\frac{\phi(\phi + \delta + \lambda)}{\rho(\phi + \lambda)}\right)-1} - \frac{q(\theta)}{\rho} y^{-\frac{q(\theta)}{\rho}-1} \right].$$

Using this in the previous expression and integrating, noting that $\phi > \rho$, yields

$$\begin{aligned} \int_{y'=1}^{\infty} \int_{\theta'=\underline{\theta}}^{\theta} y' \frac{\partial^2 H(y', \theta')}{\partial y' \partial \theta'} d\theta' dy' &= \frac{\phi F(\theta)}{q(\theta) - \rho} \left[1 - \frac{\delta F(\theta)}{q(\theta) - \phi F(\theta)} \right] \\ &+ \frac{\delta F(\theta)}{q(\theta) - \phi F(\theta)} \left[\frac{\phi(\phi + \delta + \lambda)}{\phi(\phi + \delta + \lambda) - \rho(\phi + \lambda)} \right]. \end{aligned}$$

Collecting terms and simplifying then yields the second expression in the Claim. This completes the proof of Claim 2.

Proof of Theorem 1 - Derivation of θ^R .

(6) and (7) in Lemma 1 imply θ^R is implicitly determined by

$$\theta^R = \frac{b(\phi - \rho)}{\phi} - \frac{\rho}{\phi} \int_{\theta^R}^{\bar{\theta}} \frac{\lambda(1 - F(\theta))}{q(\theta) - \rho} d\theta,$$

for any given F . Substituting $F = \widehat{F}(\theta \mid \underline{\theta})$ as described by (18) we obtain that

$$\theta^R(\underline{\theta}) = \frac{b(\phi - \rho)}{\phi} - \frac{\rho}{\phi} \int_{\theta^R(\underline{\theta})}^{\bar{\theta}} \left[1 - \frac{(\phi + \delta - \rho)}{(\phi + \delta - \rho + \lambda) \left(\frac{1-\theta}{1-\underline{\theta}} \right)^{1/2}} \right] d\theta.$$

Solving for $\theta^R(\underline{\theta}) = \underline{\theta}$ then gives the expression in (19).