Commitment, advertising and efficiency of
two-sided investment in competitive search
equilibrium.

PRELIMINARY VERSION

Adrian Masters

Department of Economics, University at Albany, SUNY

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Abstract

This paper examines the role of commitment and advertising in the labor market for the determination of the levels of wages, human capital and physical capital. In a competitive search framework it is shown that when the characteristics of jobs or workers become common knowledge (so that the other side of the market can use those characteristics as a basis for search) the efficient outcome pertains. Which side of the market advertises a particular characteristic (or a requirement for that characteristic) does not matter for the outcomes. When there is no wage commitment but investments are common knowledge the Hosios condition is shown to bring about efficiency on every margin.

1 Introduction

This paper examines the role of commitment and advertising in shaping labor market outcomes. Various institutional arrangements are considered which differ in terms of what either side of the market can commit to and the extent to which those commitments become common knowledge. Characteristics of a vacancy or job seeker that are common knowledge are used by individuals on the other side of the market as a basis for directing their search. The general framework encompasses many of the standard models of search and matching. The aim is to assess the extent to which efficiency results obtained in those models continue to apply when workers make educational investments and firms make physical capital investments in their jobs.

Initial interest in competitive (e.g. Moen [1997]) and directed (e.g. Acemoglu and Shimer [1999a]) search models arose because of the efficiency properties of the equilibria. This interest has persisted because of (at least a perceived) sense that they provide extra realism over the more established, “random” search approach (see Shimer [2007]). Certainly, workers apply for jobs on the basis of what they know about them. But (as pointed out by Rogerson et al [2005]), for characteristics of a vacancy to affect workers’ search decisions, the prospective employer has to have both some degree of commitment to those characteristics and, some way to inform job seekers of them. Here I will make a distinction between characteristics to which individuals have simply committed versus those to which they have committed and advertised. (For brevity of exposition “advertisement” or “common knowledge” of the value of an individual’s choice with respect to some characteristic will always subsume commitment.)

The general environment is one of a fixed number of workers (the death and birth rates are the same) and a set of firms which can create as many atomistic vacancies as they like. Workers acquire irreversible human capital and firms invest in job specific physical capital prior to market entry. Invest-

ments cannot be augmented after match formation and so they involve full commitment by workers and firms. Output occurs in any match according to a neoclassical production function which requires both physical and human capital as inputs. When some aspect of a job or worker to which they have committed is also advertised it becomes common knowledge. Potential partners can then direct their search on the basis of these advertised characteristics. So when anyone offers a particular value of a characteristic they create a submarket at that value for that characteristic. In all submarkets, unemployed workers and vacancies find each other according to a constant returns to scale matching function.

Rogerson et al [2005] point out that in the standard competitive search environment it does not matter whether it is the firms who post wages and the workers search or the other way around. Accordingly, I show more generally that for any given characteristic of a job or worker that is common knowledge, it does not matter for the equilibrium allocation how it became common knowledge. For example, if workers advertise their human capital investments and firms direct their search accordingly, the implied efficiency condition for human capital investment is identical to that when firms commit to, and advertise, a human capital requirement for their job. Consequently, it is sufficient that some characteristic becomes common knowledge to induce competitive search with respect to that characteristic.

The generalization of the Moen [1997] (and Acemoglu and Shimer [1999a]) efficiency result is that whenever characteristics of jobs and workers to which they commit become common knowledge, competitive search provides both sides of the market with the “right” incentives. Thus, when posted wages and, human and physical capital investments are common knowledge, all the equilibrium conditions coincide with a Social Planner’s optimality conditions. To see why this happens, consider how the market would react if the prevailing choice with respect to some characteristic were sub-optimal. A new entrant could simply choose a value closer to the efficient one. Total utility in

the generated submarket will be higher than that in the sub-optimal market, the deviant would therefore increase his matching rate while making sure that he does at least as well out of any match as before.

Whenever the committed to value of a characteristic is private information, there is a moral hazard problem. How this affects the outcome depends on the extent to which private marginal costs and benefits of any choice coincide with social costs and benefits. The question becomes: can the efficient allocation be supported as an equilibrium under each or any of the possible institutional arrangements?

If either side of the market can commit to a wage that is not advertised the marginal cost of changing the posted value is zero - search frictions ensure that the other side of the market will always accept a wage that is slightly worse for them than the efficient one. The marginal benefits of altering the wage are bounded away from zero. The economy unravels as in Diamond [1971] and the only equilibrium is the “trivial” outcome in which workers simply enjoy leisure and firms do not create vacancies.

What happens when investment is private information depends on how wages are determined. The two remaining possibilities are that wages are common knowledge or that there is no commitment to them at all. In the former case they are determined by competitive search so that, given investment levels, the wage will generate efficient vacancy creation. In the latter situation, wages are determined by generalized Nash bargaining.

Wages being determined by competitive search while investments are private information highlights an asymmetry in the model. Firms become residual claimants on match output and directly incur all the costs and benefits from their own physical capital investments. It does not matter whether physical capital is advertised or not, firms always receive their marginal product and, given the wage and prevailing level of human capital, they make efficient investments. Consequently, wage postings and human capital choices being common knowledge is sufficient for efficiency. On the other

hand, human capital choices have to be common knowledge for workers to make the right decisions. Search frictions ensure that when human capital is private information, and workers take the wage as given, the marginal benefit is zero while the marginal cost is strictly positive. Only the trivial outcome can be supported as an equilibrium.

When wages are determined by bargaining and investments are private information the model becomes essentially that of Acemoglu [1996] and Masters [1998] within the Pissarides [2000] framework. Consistent with those papers there is a hold-up problem that means both sides of the market underinvest. Using the same thought experiment as above recall that at the efficient allocation, both workers and firms equate social marginal costs to marginal benefits. Ex post bargaining implies that the marginal private benefit is reduced by some factor - their bargaining power. Lowering investment levels raises marginal benefit and, for workers, reduces marginal cost. It is well known that the Hosios [1990] condition, which equates the bargaining power of the firm to the elasticity of the matching function with respect to the vacancies, is sufficient to generate efficient vacancy creation in the Pissarides [2000] model. Indeed, taking investments as given, this also holds true in the current framework. There is, however, no bargaining power that is also able to overcome the hold-up problem.

When wages are determined by bargaining and investments are common knowledge, under the Hosios condition there is no hold-up. Despite the fact that once consummated the match output is divided by an *ex post* rule, the fact that either side can create a market at any level of investment means they can use their decisions to attract match partners. Away from the Hosios rule, matching externalities still distort outcomes but operate on intensive as well as extensive margins. The side with excess bargaining power will match too slowly and overinvest. The other side will match too quickly and underinvest.

Although the principle motivation for this work is theoretical, the results

are not without empirical implications. I am, however, not aware of any consistently collected data on the content of job postings. Looking in the help wanted columns of newspapers and on line at *Monster.com* reveals significant variation. As advertising is costly, it is safe to assume that content is designed to attract the best set of applicants for the job. Specifying an occupation is important in this regard but is ignored here on the presumption that markets are fully segmented by occupation. The quality of the job (which corresponds here to the physical capital investment by the firm), the compensation package (the wage) and the educational requirements (human capital) are further possibilities. As was pointed out above, if firms advertise jobs they need only specify wages and a human capital requirement. If they cannot commit to the wage then an indication of job quality will help.

An additional, practical implication of this analysis points to a benefit of the internet as a means of matching workers to firms. Historically, we observe firms indicating educational requirements for positions. But the effectiveness of their advertisements in directing workers search depends on the extent to which the firms can commit to those requirements.¹ What job matching web sites can provide is the possibility that workers post their human capital investments and firms direct their search accordingly. With full commitment, both institutional arrangement produce identical outcomes. However, as workers are committed to their own educational achievements, in reality firms searching across workers should achieve a preferable outcome.

There are essentially two different frameworks that have been collectively referred to as either competitive or directed search. What I will refer to as competitive search, following Moen [1997], operates in continuous time. Deviations open up whole new submarkets in which firms and workers meet sequentially according to a matching function. Directed search, typified by Acemoglu and Shimer [1999a], is either a one-shot or a discrete time economy.

¹Menzio [2007] shows that aspects of a job to which there is no contractual commitment can still help direct search by generating self-fulfilling beliefs about job quality.

Individual deviations affect the expected number of applicants in any one period so that search (at least for those able to advertise) is non-sequential. Rogerson et al [2005] point out that for wage formation, it does not matter which side of the market gets to advertise. The results below generalize that result to multiple characteristics of jobs and workers. More important though is that with respect to advertised wages, the results from both frameworks have been essentially the same - hence the interchangeability of the names. Indeed, this similarity in the results passes through to the current environment in which jobs and workers have multiple characteristics as long as the characteristics that are pay-off relevant to both sides are made common knowledge. A distinction emerges, however, when neither side is able to advertise such a characteristic. In the competitive search framework employed here, the sequential nature of meeting leads to the unraveling of any non-trivial equilibrium as in Diamond [1971]. This does not necessarily happen with directed search. Facing the prospect of competing for positions with a number of other applicants means that individuals tend to deviate from symmetric equilibria because of a discontinuity in their expected pay-off function as in Burdett and Judd [1983]. By using the competitive search framework, I abstract from these potentially interesting competition effects in favour of simplicity.²

There are three other papers that include investment along with competitive or directed search that I am aware of. Acemoglu and Shimer [1999a,b] allow for physical capital investment by firms in a model where firms post and advertise wages. There is no hold-up problem because the wages are taken as given by the firms. In Kim [2007] the human capital investments are common knowledge and form the basis for search by firms even though wages are determined by Nash bargaining. The current framework incorporates both

²Directed search also has some remaining controversy as to how to deal with multiple applications - the canonical model restricts applicants to one per period. See Albrecht et al [2006] and Gallenianos and Kircher [2007] for progress on that front.

of these models and therefore all of their results.

The remainder of the paper is organized as follows. Section 2 describes the general environment. Section 3 derives the Planner's optimality conditions. Analysis of the decentralized models are provided in Section 4. It begins with a description of the general equilibrium concept followed by an exposition of the generalized equivalence result. I then look at specific institutional arrangements and compare the equilibrium conditions to the Planners optimality conditions for each. Section 5 concludes.

2 Environment

A continuum of workers exist in continuous time. Longevity is distributed exponentially with parameter δ . Every one who dies is replaced by a new entrant so that δ represents both the birth and death rate for workers. This means that the total population is fixed - normalized to 1. All workers are risk neutral. Other than that induced by death (and job destruction) there is no discounting.

New entrant workers can instantaneously acquire human capital h at a cost of $c(h)$. The cost function $c(\cdot)$ is strictly increasing and strictly convex with $c(0) = c'(0) = 0$ and $\lim_{h \rightarrow \infty} c'(h) = \infty$. Once educated they look for work.

There are a large number of firms which can each create any number of atomistic vacancies. A vacancy is characterized by its set-up cost k which can also be interpreted as the quantity of specific capital invested in the job. From the moment of creation, jobs are subject to destruction shocks at a Poisson arrival rate λ . A match between a worker of type h and job of type k produces $f(k, h)$ units of the consumption good. Here, $f(k, h)$ is a standard neoclassical production function, homogeneous of degree 1, and increasing in both arguments. It is strictly concave and satisfies the Inada conditions.³

³Specifically, $\lim_{x_i \rightarrow 0} \frac{\partial f(x_1, x_2)}{\partial x_i} = \infty$, for $i = 1, 2$ and $x_j > 0$, $j \neq i$, and

Employment lasts until either the worker dies or the job is destroyed; there is no on-the-job search.

Matching can occur in any of a large number of potential submarkets. The number of meetings M^j per unit time in submarket j is given by $M^j = M(u_j, v_j)$ where u_j is the mass of workers in the market and v_j is the mass of vacancies. The meeting function, M , is strictly increasing in both arguments, twice differentiable, strictly concave and homogeneous of degree 1. This means that workers in submarket j meet firms at the Poisson arrival rate $m(\theta_j) = M^j/u_j$ where $\theta_j \equiv v_j/u_j$ and $m(\theta) \equiv M(1, \theta)$. Vacancies created for that submarket can expect to encounter unemployed workers at the rate $m(\theta_j)/\theta_j$. I further impose that $\lim_{\theta \rightarrow \infty} m'(\theta) = 0$, $\lim_{\theta \rightarrow 0} m'(\theta) = \infty$ and that $\lim_{\theta \rightarrow 0} \theta m'(\theta)/m(\theta) < 1$.⁴ Moving between active submarkets is costless for both vacancies and workers seeking employment. Individuals can share their search time how ever they like between markets. Whenever multiple active submarkets are identical with each other in terms of the market tightness θ and the demographic distributions of workers and vacancies I will call that one submarket. This simply avoids the proliferation of allocations that are pay-off equivalent. It means that participants can direct their search according to generally observed pay-off relevant characteristics of the

$\lim_{x_i \rightarrow \infty} \frac{\partial f(x_1, x_2)}{\partial x_i} = 0$, for $i = 1, 2$ and $x_j > 0$, $j \neq i$.

⁴These conditions are imposed so to ensure existence of non-trivial solutions to the efficiency conditions derived below. More specifically I will use that as θ approaches 0,

$$m'(\theta) \gg X \gg m(\theta) \gg \theta$$

and as θ approaches ∞ ,

$$\theta \gg m(\theta) \gg X \gg m'(\theta)$$

where X is any strictly positive constant and the symbol $\gg$ is defined such that

$$\lim_{n \rightarrow N} \phi(n) \gg \lim_{n \rightarrow N} \psi(n) \implies \lim_{n \rightarrow N} \frac{\psi(n)}{\phi(n)} = 0$$

The last restriction on $m(\cdot)$ requires that the contribution of unemployment to matching does not disappear as market tightness goes to zero.

individuals with whom they would like to match. Once meetings occur, all pay-off relevant characteristics of the other party are revealed so that there is no private information at the level of the individual encounter.

How much workers educate, how much capital is invested in any job, which realized matches are formed and how the output of any match is distributed between employer and worker depends on the specific institutional arrangements and informational assumptions made. Various arrangements are discussed below but first we need to consider what is the efficient allocation.

3 Efficiency

The fictitious central Planner weights every generation of individuals' utility equally into her measure of welfare. Consequently, she does not discount the future and simply chooses the best steady state for the economy. Because everyone is risk neutral welfare, W is simply equal to aggregate flow benefits less flow costs,

$$W = (1 - u)f(k, h) + ub - \delta c(h) - sk \quad (1)$$

where u is the aggregate unemployment rate and s is the flow number of vacancies created per unit time. In steady state the inflow to unemployment is $\delta + \lambda(1 - u)$. The outflow is $(m(\theta) + \delta)u$. Thus

$$u = \frac{\delta + \lambda}{m(\theta) + \delta + \lambda}$$

A steady state also implies that the rate of vacancy creation has to equal the rate at which jobs are destroyed. The total number of jobs in the economy is $v + (1 - u)$ where v is the mass of vacancies. So $s = \lambda(v + 1 - u)$. Substitution into equation (1) for s and u yields

$$W = W(k, h, \theta; b) \equiv \frac{m(\theta) [f(k, h) - \delta c(h) - \lambda k] + (\delta + \lambda) [b - \delta c(h) - \lambda \theta k]}{m(\theta) + \delta + \lambda} \quad (2)$$

The Planner's problem is to maximize W with respect to k , h and θ over the positive orthant. Depending on the size of b , $\{k, h, \theta\} = \{0, 0, 0\}$ is a potential solution. In which case $W = b$. Identification of non-trivial solutions requires first order conditions which yield,

$$m(\theta^*) [f_1(k, h) - \lambda] - \lambda(\delta + \lambda)\theta^* = 0 \quad (3)$$

$$m(\theta^*) [f_2(k^*, h^*) - \delta c'(h^*)] - \delta(\delta + \lambda)c'(h^*) = 0 \quad (4)$$

$$m'(\theta^*)[f(k^*, h^*) - b] - \lambda[\delta + \lambda + m(\theta^*) + (1 - \theta^*)m'(\theta^*)]k^* = 0 \quad (5)$$

Without further restrictions on the functional forms of f , m and c , $W(k, h, \theta; b)$ is not necessarily concave so existence of a solution to the first order conditions does not necessarily imply existence of a solution to the Planners problem. The next series of Lemmas is used to establish existence of solutions to both.

Lemma 1 *Any system of the form*

$$f_1(k, h) - A = 0 \quad (6)$$

$$f_2(k, h) - Bc'(h) = 0 \quad (7)$$

where A and B are strictly positive constants has a unique non-trivial solution, $\{\bar{k}, \bar{h}\}$.

Proof. As f is constant returns to scale, (6) implies $\bar{k}/\bar{h}$ is a constant. Thus $f_2(\bar{k}, \bar{h})$ is also a constant and since $c'(\cdot)$ is strictly increasing with range $\mathbb{R}_+$, (7) implies a unique $\bar{h} > 0$ must exist. Then $\bar{k}/\bar{h}$ being a strictly positive constant also implies that $\bar{k} > 0$. ■

Now consider equations (3) and (4). For a given value of θ , Lemma 1 implies that there is a unique non-trivial solution, $\{k^*(\theta), h^*(\theta)\}$.

Lemma 2 $\lim_{\theta \rightarrow 0} f(k^*(\theta), h^*(\theta))/k^*(\theta) > \lambda$.

Proof. Consider what happens as $\theta \rightarrow 0$, $\theta/m(\theta) \rightarrow 0$, so from (3), $f_1(k^*(\theta), h^*(\theta)) \rightarrow \lambda$ and $f_2(k^*(\theta), h^*(\theta)) \rightarrow \gamma$ where γ is a strictly positive constant. From (4), as $\delta + \lambda \gg m(\theta)$, $c'(h) \rightarrow 0$ which means that $\lim_{\theta \rightarrow 0} h^*(\theta) = \lim_{\theta \rightarrow 0} k^*(\theta) = 0$. As both f and k approach 0 we need to use L'Hospital's rule to evaluate their ratio. Thus,

$$\lim_{\theta \rightarrow 0} \frac{f(k^*(\theta), h^*(\theta))}{k^*(\theta)} = \frac{f_1 \frac{dk^*}{d\theta} + f_2 \frac{dh^*}{d\theta}}{\frac{dk^*}{d\theta}} = \lambda + \gamma \frac{dh^*}{dk^*} > \lambda$$

The last inequality follows from the fact that $\frac{dh^*}{dk^*} = \frac{h}{k}$. ■

Lemma 3 $\lim_{\theta \rightarrow \infty} k^*(\theta) = \lim_{\theta \rightarrow \infty} h^*(\theta) = 0$

Proof. From (3)

$$\begin{aligned} \lim_{\theta \rightarrow \infty} \frac{\theta}{m(\theta)} &= \infty \implies \lim_{\theta \rightarrow \infty} f_1(k^*(\theta), h^*(\theta)) = \infty \\ \implies \lim_{\theta \rightarrow \infty} \frac{k^*(\theta)}{h^*(\theta)} &= 0 \implies f_2(k^*(\theta), h^*(\theta)) = 0 \end{aligned}$$

In (4) this means

$$\lim_{\theta \rightarrow \infty} f_2(k^*(\theta), h^*(\theta)) = 0 \implies \lim_{\theta \rightarrow \infty} c'(h^*(\theta)) = 0 \implies \lim_{\theta \rightarrow \infty} h^*(\theta) = 0$$

The last implication follows from assumptions on the form of $c(\cdot)$. ■

Now, from equation (5) define

$$\Gamma(\theta) \equiv m'(\theta)[f(k^*(\theta), h^*(\theta)) - b] - \lambda[\delta + \lambda + m(\theta) + (1 - \theta)m'(\theta)]k^*(\theta)$$

so that $\Gamma(\theta^*) = 0$ represents a solution to the system (3), (4), and (5). For values of θ very close to 0, $\Gamma(\theta) \approx m'(\theta)[f(k^*(\theta), h^*(\theta)) - b - \lambda k^*(\theta)]$ and, from Lemma 2,

$$\frac{d}{d\theta}[f(k^*(\theta), h^*(\theta)) - \lambda k^*(\theta)] > 0$$

So that, for b sufficiently small, there exists some $\tilde{\theta} > 0$ such that $\Gamma(\tilde{\theta}) > 0$.

On the other hand, as θ approaches ∞ , Lemma 3 implies that $k^*(\theta)$ and $h^*(\theta)$ approach 0. So that $\Gamma(\infty) < 0$. It follows from continuity that for small enough value of b , there exists at least one non-trivial solution to the system (3), (4), and (5).

Continuity of f , m and c and, their first and second derivatives does imply that any solution to the Planner's problem in the interior of the positive orthant must also solve the first order conditions. From the preceding analysis we know that if $b = 0$, $\Gamma(0) > 0$ and $\Gamma(\infty) \leq 0$ so that a solution to the planners problem must exist in that case - call it $\{k_0^*, h_0^*, \theta_0^*\}$. Let $W_0 \equiv W(k_0^*, h_0^*, \theta_0^*; 0)$ then a non-trivial solution to the Planner's problem exists for any $b < W_0$. This is because the Planner could always pick $\{k, h, \theta\} = \{k_0^*, h_0^*, \theta_0^*\}$ and, from equation (2),

$$W(k_0^*, h_0^*, \theta_0^*; b) > W_0 > b = W(0, 0, 0; b).$$

Meanwhile, large enough values of θ (taking account of the effect on k^* and h^*) clearly imply that $W < b$.

The up-shot is that the Planner's problem has a solution which is generically unique. For low enough values of b the solution is in the interior of the positive orthant otherwise $\{k^*, h^*, \theta^*\} = \{0, 0, 0\}$ and $W(k^*, h^*, \theta^*; b) = b$. In the sequel I will assume that b is always sufficiently small that the efficient allocation involves strictly positive investments on both sides of the market. It should be noted, though, that $b > 0$ plays no real part in the results. It is included because it is commonly incorporated in matching models and to show robustness of the results.

4 Decentralized models

Decentralizing the model means individuals get to make to make choices in their own private interest taking as given what they know about the choices others will make. What they know and the options they each have depends

on the specific institutional arrangement they face. An institutional arrangement specifies:

- the decisions to which each individual can commit
- whether or not decisions made with commitment are advertised (i.e. become common knowledge)

I rule out the possibility that individuals can make one choice and declare that they made a different one.

Firms decide on how many vacancies to create, the associated capital stock, k , the wage, w , to pay to anyone hired, and the required human capital h for the job. Workers decide on their own level of education, h , the required wage, w , and the required level of physical capital associated with an acceptable job, k . Choices to which individuals cannot commit are *vacuous*; the realized values are determined at the point of match formation. In principle. Firms and workers own capital investments are fully committed to (and cannot be further augmented once matching has occurred). Even still, the general framework allows for a very large number of possible institutional arrangements.

In continuous time there is no real sense of "timing within a period". However, when one side of the market gets to advertise their choice of a particular variable, new entrants will instantaneously react to that advertisement. Thus advertising a choice is analogous to making a the first move in a discrete time setting. For this reason I rule out institutional arrangements that permit both sides of the market to advertise decisions made with respect to the same variable and I rule out arrangements in which both sides commit to a variable neither can advertise. I also allow at most one side of the market to make wage commitments.

4.1 Equilibrium

The solution concept will be symmetric competitive search equilibrium as introduced by Moen (1997). This means that individual's optimal strategies maximize their expected utility while taking institutions and the strategies of everyone else as given. So, for instance, if a firm deviates from equilibrium behavior in a way he advertises (i.e. it becomes common knowledge), the deviation sets up a market to which workers will be immediately attracted. Workers will enter the deviant's market until they are indifferent between entering that market and remaining in the market specified by the equilibrium. If a firm deviates from equilibrium behavior in a way that he cannot advertise, no new market is made - the firm's ability to attract workers is unaffected by the deviation.

In any symmetric steady state equilibrium, $\{k^*, h^*, w^*, \theta^*\}$, we have

$$\lambda V_v = \frac{m(\theta^*)}{\theta^*} [V_j - V_v] \quad (8)$$

$$\lambda V_j = f(k^*, h^*) - w^* - \delta [V_j - V_v] \quad (9)$$

where V_v is the value to holding the vacancy open and V_j is the value to the filled job. If V_c is the value to creating a vacancy in that market then $V_c = -k + V_v$.

Similarly, for workers,

$$\delta V_u = b + m(\theta^*) [V_e - V_u] \quad (10)$$

$$\delta V_e = w^* + \lambda (V_u - V_e) \quad (11)$$

where V_u is the value to unemployment and V_e is the value to employment. Let V_b represent the equilibrium value to being born into this economy. Then $V_b = V_u - c(h)$.

4.2 Equilibrium equivalence and the set of institutional arrangements.

In general, the equilibrium including the wage, $\{k^*, h^*, w^*, \theta^*\}$ can be characterized as the solution to the pair of problems

$$\begin{aligned} \{k^*, h^*, w^*, \theta^*\} &= \arg \max_{k_f, h_f, w_f, \theta_f} V_c(\tilde{k}_f, \tilde{h}_f, \tilde{w}_f, \tilde{\theta}_f; k^*, h^*, w^*, \theta^*) \\ &\text{subject to} \end{aligned} \quad (12)$$

$$\begin{aligned} V_b(\hat{k}_f, \hat{h}_f, \hat{w}_f, \hat{\theta}_f; k^*, h^*, w^*, \theta^*) &= V_b(k^*, h^*, w^*, \theta^*; k^*, h^*, w^*, \theta^*) \\ \{k^*, h^*, w^*, \theta^*\} &= \arg \max_{k_w, h_w, w_w, \theta_w} V_b(\tilde{k}_w, \tilde{h}_w, \tilde{w}_w, \tilde{\theta}_w; k^*, h^*, w^*, \theta^*) \\ &\text{subject to} \end{aligned} \quad (13)$$

$$V_c(\hat{k}_w, \hat{h}_w, \hat{w}_w, \hat{\theta}_w; k^*, h^*, w^*, \theta^*) = 0$$

Here $V_c(\tilde{k}, \tilde{h}, \tilde{w}, \tilde{\theta}; k^*, h^*, w^*, \theta^*)$ is the value to creating a vacancy in a market with $\{k, h, w, \theta\} = \{\tilde{k}, \tilde{h}, \tilde{w}, \tilde{\theta}\}$ given everyone else conforms to the equilibrium, $\{k^*, h^*, w^*, \theta^*\}$. And, $V_b(\hat{k}, \hat{h}, \hat{w}, \hat{\theta}; k^*, h^*, w^*, \theta^*)$ is the value to being born for a worker with $\{k, h, w, \theta\} = \{\hat{k}, \hat{h}, \hat{w}, \hat{\theta}\}$ given everyone else conforms to the equilibrium. What the variables which carry the tildes and the hats are equal to depends on the specific arrangement. So if firms are, for instance, able to advertise a human capital requirement, $\hat{h}_f = \tilde{h}_f = h_f$. If they can commit to a human capital requirement but cannot advertise their choice, then $\hat{h}_f = h^*$ but $\tilde{h}_f = h_f$. If they cannot commit to a human capital requirement, $\hat{h}_f = \tilde{h}_f = h^*$. Of course, no one can commit to a value for θ , the market tightness. For each problem, this is pinned down by the constraint. The value of θ^* comes from the competitive entry condition, $V_c = 0$. Capital investments are committed to prior to market entry but it is possible that neither side commits to a wage level. In that case, the wage is determined by Nash bargaining and is therefore itself a function of the other equilibrium values and the specific choices made by anyone deviating from equilibrium behavior.

If the Lagrange multiplier on the constraint in problem (12) is μ_f and Lagrange multiplier on the constraint in problem (13) is μ_w , a direct consequence of the first order conditions with respect to θ for each problem is that in any non-trivial equilibrium $\mu_f = 1/\mu_w$. What this tells us is that problems (12) and (13) represent a system of equivalent problems in that they have identical efficiency conditions.⁵ Specifically, the allocation does not depend on which side gets to advertise a particular variable. For instance, whenever firms advertise a human capital requirement, the outcome will be the same as if workers get to advertise their human capital investments. It is important to note that because the constraints do not apply to choices made that do not become common knowledge, this “market equivalence” only applies to characteristics which are advertised.

4.3 Institutional arrangements with wage commitment

4.3.1 General

Equations (8), (9) can be used to solve for V_v . Substituting this into the definition of V_c , we can define a function

$$V_c(k, h, w, \theta) \equiv \frac{m(\theta) [f(k, h) - w]}{\lambda[(\delta + \lambda)\theta + m(\theta)]} - k \quad (14)$$

Similarly, from equations (10), (11) and the definition of V_b , we can define a function

$$V_b(k, h, w, \theta) \equiv \frac{m(\theta)w + (\delta + \lambda)b}{\delta(\delta + \lambda + m(\theta))} - c(h) \quad (15)$$

Claim 4 *When either workers or firms post wages which are not advertised, there are no non-trivial equilibria.*

⁵Rogerson et al [2005] point out that this applies to wages in the standard directed and competitive search environments.

Proof. Fix a candidate non-trivial equilibrium, $\{k^*, h^*, w^*, \theta^*\}$ and consider what happens if firms can commit to a wage that workers do not observe. As a non-trivial equilibrium implies $w^* > b$, a direct implication of equations (10) and (11) is that $V_u < w^*/\delta$. If the firm were to lower its wage to δV_u , it would increase its profit while leaving its matching rate unaffected. Similarly as $V_j > V_v$ any worker who could secretly commit to a wage would set that wage higher than w^* . ■

This is a simple extension of the Diamond [1971] paradox .

4.3.2 Transparency

Under full transparency all the features of jobs and workers that are salient to match formation are common knowledge. The specific arrangement analyzed here is that firms advertise their physical capital input, the wage and an educational requirement. Workers simply pick their level of human capital and direct their search accordingly. Market equivalence implies that the same allocation would arise if any or all of the following were true:

1. instead of firms advertising an educational requirement, workers advertise their level of educational attainment.
2. instead of firms, workers commit to a wage and advertise it
3. instead of firms advertising physical capital, workers advertise a physical capital requirement.

The relevant problem is

$$\begin{aligned}
\{k^*, h^*, w^*, \theta^*\} &= \arg \max_{k, h, w, \theta} V_c(k, h, w, \theta) \\
\text{subject to} &: V_b(k^*, h^*, w^*, \theta^*) = V_b(k, h, w, \theta) \\
\text{and } V_c(k^*, h^*, w^*, \theta^*) &= 0
\end{aligned} \tag{16}$$

The variables with the tildes and carats depend on the particular institutional arrangement as explained earlier.

An important simplification that appears in problem (16) is the lack of direct dependence of the value to vacancy creation to a deviant firm on what everyone else is doing. This is because the influence of other firms' and workers' behavior on the deviant's choice operates solely through the constraint.

The appropriate Lagrangian for this arrangement is

$$\mathcal{L} = \frac{m(\theta) [f(k, h) - w]}{\lambda[(\delta + \lambda)\theta + m(\theta)]} - k - \mu_f [(V_b^* + c(h)) \delta (\delta + \lambda + m(\theta)) - (\delta + \lambda)b - m(\theta)w]$$

where $V_b^* \equiv V_b(k^*, h^*, w^*, \theta^*)$. The implied first order conditions are:

$$\begin{aligned} k &: \frac{m(\theta) [f_1(k, h) - w]}{\lambda[(\delta + \lambda)\theta + m(\theta)]} - 1 = 0 \\ h &: \frac{m(\theta) [f_2(k, h) - w]}{\lambda[(\delta + \lambda)\theta + m(\theta)]} - \mu_f \delta (\delta + \lambda + m(\theta)) c'(h) = 0 \\ w &: \frac{-m(\theta)}{\lambda[(\delta + \lambda)\theta + m(\theta)]} + \mu_f m(\theta) = 0 \\ \theta &: \frac{[f(k, h) - w] (\delta + \lambda) (\theta m'(\theta) - m(\theta))}{\lambda[(\delta + \lambda)\theta + m(\theta)]^2} - \mu_f [(V_b^* + c(h)) \delta - w] m'(\theta) = 0 \end{aligned}$$

To proceed, use the wage equation to solve for μ_f and substitute it into the remaining equations. Then substitute for V_b^* from equation (15). The implied necessary conditions for an equilibrium, $\{k^*, h^*, w^*, \theta^*\}$ are

$$m(\theta^*) f_1(k^*, h^*) - \lambda[(\delta + \lambda)\theta^* + m(\theta^*)] = 0 \quad (17)$$

$$m(\theta^*) f_2(k^*, h^*) - \delta[\delta + \lambda + m(\theta^*)] c'(h^*) = 0 \quad (18)$$

$$m(\theta^*) m'(\theta^*) (w^* - b) - \lambda[m(\theta^*) - \theta^* m'(\theta^*)][\delta + \lambda + m(\theta^*)] k^* = 0 \quad (19)$$

$$m(\theta^*) [f(k^*, h^*) - w^*] - \lambda[(\delta + \lambda)\theta^* + m(\theta^*)] k^* = 0 \quad (20)$$

Equation (20) comes from setting $V_c(k^*, h^*, w^*, \theta^*) = 0$.

It is immediate from (17) and (20) that firms receive the marginal product of their investments. Linear homogeneity of f , implies the same is true for workers. The system can be reduced to 3 equations in $\{k^*, h^*, \theta^*\}$ by eliminating w^* from (19) and (20) to get

$$m'(\theta^*)[f(k^*, h^*) - b] - \lambda[\delta + \lambda + m(\theta^*) + (1 - \theta^*)m'(\theta^*)]k^* = 0 \quad (21)$$

The market equilibrium under complete transparency and the Planner's model share the same characterization. If the right commitments can be made and those commitments are common knowledge, directed search can overcome the hold up problems associated with two-sided investments as well as eliminate the matching externalities.

It should be pointed out here that this particular framework could easily be extended to an environment with worker heterogeneity in their ability to acquire human capital. That is if worker i has cost function $c_i(h)$. Firms could either specify a wage and minimum human capital pair or an optimal wage function (of human capital) either way the matched pairs would look like those that emerge here with $c(\cdot)$ replaced by $c_i(\cdot)$ for each i .

4.3.3 Hidden Investments (under wage commitment)

First, it should be noted that whenever there is commitment with full transparency with respect to the wage, whether the physical capital investment of the firms is common knowledge or not is moot. This is because V_b does not depend on k at all. Condition (17) comes directly from maximizing V_c in (14) with respect to k - advertising physical capital investment in such situations is superfluous. Ultimately, this is a consequence of an asymmetry in the model. Taking the wage as given, workers do not care about match output but firms do.

Claim 5 *When human capital investment is hidden there is no non-trivial equilibrium*

Proof. Fix a non-trivial equilibrium, $\{k^*, h^*, w^*, \theta^*\}$, and consider the acceptance strategy of firms. They will be willing to hire any worker with $h \geq h_r$ where h_r solves.

$$\lambda V_v = f(k^*, h_r) - w^*.$$

This is because h_r is the level of human capital that provides firms their continuation value. From (9) and (8) it follows that for any non-trivial equilibrium, $h_r < h^*$. But, knowing this workers will pick $h = h_r$. (Education is costly and having more does not benefit workers in either matching or wages.) ■

4.4 Institutional arrangements without wage commitment

4.4.1 General

Following Pissarides [2000] wages are determined by generalized Nash bargaining after firms and workers have met. For the purpose of negotiations the continuation values of both parties, V_u and V_v are taken as given although each can depend on the specific choices made by the individuals prior to their meeting. So

$$V_e - V_u = \beta (V_j - V_v + V_e - V_u) \quad (22)$$

where β is the bargaining power of workers. As long as the match surplus, the right hand side of (22), is positive, there will be match formation and the values of V_e and V_j can be obtained from equations (11) and (9) respectively. Thus,

$$w = \delta V_u + \beta [f(k, h) - \lambda V_v - \delta V_u] \quad (23)$$

Eliminating V_j from equations (8) and (9), substituting for w and solving for V_v yields

$$V_v^B = \frac{(1 - \beta)m(\theta)[f(k, h) - \delta V_u]}{\lambda[(\delta + \lambda)\theta + (1 - \beta)m(\theta)]} \quad (24)$$

where the superscript B indicates that wage formation is by bargaining. Similarly eliminating V_e from equations (11) and (10), substituting for w and solving for V_u yields

$$V_u^B = \frac{\beta m(\theta)[f(k, h) - \lambda V_v]}{\delta[\delta + \lambda + \beta m(\theta)]} \quad (25)$$

Variables V_v^B and V_u^B represent respectively the value to holding a vacancy, and the value to unemployment when wages are determined by bargaining and the continuation values, V_u and V_v , of the participants on the other side of the market are taken as given.

In any equilibrium with bargaining, $\{k^*, h^*, \theta^*\}$, $V_v^B = V_v$ and $V_u^B = V_u$ then

$$V_v^B(k^*, h^*, \theta^*) \equiv \frac{(1 - \beta)m(\theta^*)[f(k^*, h^*) - b]}{\lambda[(\delta + \lambda)\theta^* + (1 - \beta + \theta^*\beta)m(\theta^*)]} \quad (26)$$

$$V_u^B(k^*, h^*, \theta^*) \equiv \frac{\theta^*\beta m(\theta^*)[f(k^*, h^*) - b] + [(\delta + \lambda)\theta^* + (1 - \beta)m(\theta^*)]b}{\delta[(\delta + \lambda)\theta^* + (1 - \beta + \theta^*\beta)m(\theta^*)]} \quad (27)$$

and free entry implies $V_c^B \equiv V_v^B - k^* = 0$. So,

$$(1 - \beta)m(\theta^*)[f(k^*, h^*) - b] - \lambda[m(\theta^*)(1 - \beta + \theta^*\beta) + (\delta + \lambda)\theta^*]k^* = 0 \quad (28)$$

We are now in a position to analyze specific institutional arrangements with bargaining

4.4.2 Complete ignorance

Neither the physical capital incorporated in a job nor the human capital investment of workers is common knowledge. Furthermore, neither side can commit to capital requirements. Thus firms seek to maximize V_c^B by picking k . As they cannot influence the arrival rate of workers by their choice, they take V_u as given. The relevant equation for V_v^B is therefore (24). The implied first order condition yields

$$(1 - \beta)m(\theta^*)[f_1(k^*, h^*) - \lambda] - \lambda(\delta + \lambda)\theta^* = 0 \quad (29)$$

Similarly for workers, they can choose h to maximize $V_b^B \equiv V_u^B - c(h)$. As their choice cannot influence the behavior of firms, V_u^B is obtained from (25) and the implied first order condition yields

$$\beta m(\theta^*) [f_2(k^*, h^*) - \delta c'(h^*)] - \delta(\delta + \lambda)c'(h^*) = 0 \quad (30)$$

Equations (28), (29) and (30) represent the equilibrium conditions for this model.

Claim 6 *For b small enough there exists a non-trivial equilibrium under complete ignorance*

Proof. From Lemma 1, for given θ , (29) and (30) have unique non-trivial solution, $\{k_c(\theta), h_c(\theta)\}$. Then following identical logic to that in Lemma 2, as θ approaches 0, $f(k_c(\theta), h_c(\theta))/k_c(\theta) > \lambda$. Consequently, for small enough b there must be a value of θ such that LHS (28) is positive.

Following identical logic to that in Lemma 3 we have

$$\lim_{\theta \rightarrow \infty} \{k_c(\theta), h_c(\theta)\} = \{0, 0\}$$

Which means that for large enough values of θ , $f(k_c(\theta), h_c(\theta)) < b$ for any b so that LHS (28) becomes negative. ■

In general, the equilibrium conditions do not coincide with those of the first order conditions of Planner's problem, (3), (4) and (5). We can, however, ask if the Hosios [1990] result applies here. His result was that in the Pissarides [2000] framework if the bargaining power of the worker just so happened to equal the elasticity of the matching function with respect to unemployment, that the equilibrium conditions for the model of the decentralized economy were the same as the Planner's optimality conditions. In the current context that would mean setting

$$\beta = \beta_H \equiv \frac{m(\theta) - \theta m'(\theta)}{m(\theta)} \quad (31)$$

After making the substitution it is immediate that equations (28) and (5) are identical. It should also be clear that there is no substitution that will yield equations (29) and (30) from equations (3) and (4). Indeed, there is always under-investment in this model caused by the hold-up problem.⁶ The workers' inability to pre-contract with firms along with bargaining means both sides only get some fraction of their marginal contribution to match output.

Consistent with Hosios, then, given k and h , the “right” bargaining power delivers efficient vacancy creation but it does not lead to efficient investment. This should be viewed in comparison to Pissarides [2000] who finds that the Hosios condition is sufficient to bring about efficient search and advertising intensity decisions. The difference is that the investments here pertain to the productivity of the eventual match whereas the intensity choices he analyses only affect the matching rate.⁷

4.4.3 Transparency

Here, firms post their physical capital stock and a human capital requirement. Again, equilibrium equivalence means that either or both of the following would yield the same equilibrium conditions

1. rather than firms advertising an educational requirement, workers advertise their level of human capital investment.
2. rather than firms advertising their physical capital investment, workers advertise a physical capital requirement.

Because workers can respond to changes made by individual firms in their level of k or their required h , market tightness will adjust and the

⁶This model is essentially that of Acemoglu [1996] and Masters [1998] brought into the Pissarides [2000] framework.

⁷Moen [1997] shows that directed search induces efficient search intensity decisions.

continuation values for workers and firms will be the same for everyone in any submarket. Hence the appropriate equations for V_v^B and V_u^B are (26) and (27). Moreover the value of θ in any market to which a firm deviates will be such that workers are indifferent between entering that market and staying in the candidate equilibrium market. The problem to solve is:

$$\begin{aligned} \{k^*, h^*, \theta^*\} &= \arg \max_{k, h, \theta} V_v^B(k, h, \theta) - k \\ \text{subject to } V_u^B(k, h, \theta) &= V_u^B(k^*, h^*, \theta^*) \\ \text{where } V_v^B(k^*, h^*, \theta^*) &= k^* \end{aligned}$$

The last constraint is simply equation (28). Using μ for the multiplier on the constraint leads to the Lagrangian

$$\begin{aligned} \mathcal{L} &= \frac{(1 - \beta)m(\theta)[f(k, h) - b]}{\lambda[(\delta + \lambda)\theta + (1 - \beta + \theta\beta)m(\theta)]} \\ &\quad - \mu \left\{ \begin{aligned} &[V_u^B(k^*, h^*, \theta^*) + c(h)] \delta [(\delta + \lambda)\theta + (1 - \beta + \theta\beta)m(\theta)] \\ &- (\theta\beta m(\theta)[f(k, h) - b] + [(\delta + \lambda)\theta + (1 - \beta)m(\theta)]b \end{aligned} \right\} \end{aligned}$$

The first order conditions are

$$\begin{aligned} k &: \frac{(1 - \beta)m(\theta)f_1(k, h)}{\lambda[(\delta + \lambda)\theta + (1 - \beta + \theta\beta)m(\theta)]} - 1 - \mu\theta\beta m(\theta)f_1(k, h) = 0 \\ h &: \frac{(1 - \beta)m(\theta)f_2(k, h)}{\lambda[(\delta + \lambda)\theta + (1 - \beta + \theta\beta)m(\theta)]} - \mu \left\{ \begin{aligned} &c'(h)\delta [(\delta + \lambda)\theta + (1 - \beta + \theta\beta)m(\theta)] \\ &- \mu\theta\beta m(\theta)f_2(k, h) \end{aligned} \right\} = 0 \\ \theta &: \frac{(1 - \beta)[f(k, h) - b][m'(\theta)(\delta + \lambda)\theta + m(\theta)(\delta + \lambda + \beta m(\theta))]}{\lambda[(\delta + \lambda)\theta + (1 - \beta + \theta\beta)m(\theta)]^2} \\ &\quad - \mu \left\{ \begin{aligned} &[V_u^B(k^*, h^*, \theta^*) + c(h)] [\delta + \lambda + m'(\theta)(1 - \beta + \theta\beta) + \beta m(\theta)] \\ &- [\delta + \lambda + m'(\theta)(1 - \beta)]b - \beta(m(\theta) + \theta m'(\theta))f(k, h) \end{aligned} \right\} = 0 \end{aligned}$$

Now, substitute for $V_u^B(k^*, h^*, \theta^*)$ from (27) into the θ equation, solve for μ and substitute into the k and h equations respectively to obtain

$$(1 - \beta)m^2(\theta) [f_1(k, h) - \lambda] - \lambda(\delta + \lambda)\theta^2 m'(\theta) = 0 \quad (32)$$

$$\beta m^2(\theta) [f_2(k, h) - \delta c'(h)] - \delta(\delta + \lambda) [m(\theta) - \theta m'(\theta)] c'(h) = 0 \quad (33)$$

Equations (32), (33) and (28) are the equilibrium conditions for this model.

Claim 7 *For b small enough there exists a non-trivial equilibrium under transparency (without wage commitment)*

Proof. For given θ , Lemma 1 implies that there is a unique non-trivial solution to (32) and (33), $\{k_t(\theta), h_t(\theta)\}$. Then, as θ approaches 0 since $m(\theta) > \theta m'(\theta)$ and $m(\theta) \gg \theta$ we have $f_1(k_t(\theta), h_t(\theta)) \rightarrow \lambda$. Similar logic to that in Lemma 2, now implies that $f(k_t(\theta), h_t(\theta))/k_t(\theta) > \lambda$. Consequently, for small enough b there must be a value of θ such that LHS (28) is positive.

Now consider what happens to $\{k_t(\theta), h_t(\theta)\}$ when θ gets very large. In (32)

$$\begin{aligned} \lim_{\theta \rightarrow \infty} \frac{\theta^2 m'(\theta)}{m^2(\theta)} &= \infty \implies \lim_{\theta \rightarrow \infty} f_1(k_t(\theta), h_t(\theta)) = \infty \\ \implies \lim_{\theta \rightarrow \infty} \frac{k_t(\theta)}{h_t(\theta)} &= 0 \implies \lim_{\theta \rightarrow \infty} f_2(k_t(\theta), h_t(\theta)) = 0 \end{aligned}$$

But from (33)

$$\lim_{\theta \rightarrow \infty} f_2(k_t(\theta), h_t(\theta)) = 0 \implies \lim_{\theta \rightarrow \infty} c'(h_t(\theta)) = 0 \implies \lim_{\theta \rightarrow \infty} h_t(\theta) = 0 \implies \lim_{\theta \rightarrow \infty} k_t(\theta) = 0$$

Which means that for large enough values of θ , $f(k_t(\theta), h_t(\theta)) < b$ for any b so that LHS (28) becomes negative. ■

Again, we can ask if the Hosios condition plays any role here. In fact, it is simple to show that when we set $\beta = \beta_H$ from equation (31), equations (28), (32) and (33) become identical to equations (3), (4), and (5). That is, if the bargaining power of the workers is just right we get efficiency on every margin.

When (31) does not apply there may be under or over investment relative to the Planner's optimum. To see why this happens we want to obtain an equation involving the wage. Substituting from equation (27) into (23) and

setting $V_v = k^*$ yields

$$w = \frac{\beta[m(\theta^*)(1 - \beta) + \theta^*(m(\theta^*) + \delta + \lambda)]f(k^*, h^*) + (1 - \beta)[m(\theta^*)(1 - \beta) + \theta^*(\delta + \lambda)]b}{[(\delta + \lambda)\theta^* + (1 - \beta + \theta^*\beta)m(\theta^*)]} - \beta\lambda k^*$$

Using this to eliminate b from the free-entry condition (28) yields,

$$m(\theta^*)[f(k^*, h^*) - w^*] - \lambda[(\delta + \lambda)\theta^* + m(\theta^*)]k^* = 0$$

which is identical to equation (20). So, given equilibrium values $\{k^*, h^*, \theta^*\}$ workers and firms always get the share of output associated with the efficient allocation even when the equilibrium itself is not efficient. Essentially, the side of the market that has bargaining power higher than that implied by the Hosios condition will overinvest (i.e. set marginal productivity below their actual return). The matching externalities operate on the intensive as well as the extensive margin. For example, suppose firms advertise their investment and a human capital requirement, and $\beta < \beta_H$. (Firms have excess bargaining power.) Then, at the efficient allocation, the average social value to creating a vacancy exceeds the marginal social value but firms create vacancies on the basis of the former. If h and k were held constant there would simply be more vacancies created. However, in the current environment firms can offer higher levels of physical capital and require lower levels of human capital to attract more workers. In equilibrium, of course, the exercise is futile.

4.4.4 Hidden investment

When physical capital investment is the private information of the firms (and workers cannot advertise a required level of physical capital in jobs), it is straightforward to show that the relevant equilibrium conditions are (28), (29) and (33). Again there will be a non-trivial equilibrium as long as b is not too large. Similarly, when the education level of workers is their private information (and firms cannot advertise a required level of human capital of applicants), the relevant equilibrium conditions are (28), (32) and (30).

There will be a non-trivial equilibrium as long as b is not too large. In these models, under the Hosios condition all but one of the equilibrium conditions coincides with the Planner's optimality conditions.

5 Conclusion

This paper provides an assessment of the efficiency properties of various models of competitive search in the labor market. When characteristics of the job and worker that are pay-off relevant to both of them are common knowledge we find that the efficiency result of Moen [1997] passes through to the more general environment. When neither side can commit to a wage so that the terms of trade are determined by Nash bargaining the Hosios condition ensures efficient vacancy formation but its implications for investment depend on the extent to which firms and workers can advertise their investments. When investments are private information to the investor (only revealed in one-on-one meetings) the classical hold-up problem leads to under investment even under the Hosios condition. However, if investments become common knowledge, then competitive search disciplines investors to make the right choices.

A general implication of this analysis is that while competitive or directed search lead to desirable outcomes they require strong informational assumptions on the environment. Specifically, for the equilibrium condition associated with some job or worker characteristic to coincide with that of the optimally condition of Social Planner requires both commitment and common knowledge. When it is reasonable to make these assumptions depends on the specific context. Shimer [2007] suggests that directed forms of search models are superior to random search models because the former recognizes that job seekers are sentient. What emerges from this paper is that actors in random search environments are not so much stupid as ignorant and unless we have good reason to assume away that ignorance, random search should

be the model of how workers seek employment.

The results have policy implications. Specifically, transparency would help here - a job posting agency that required pay-off relevant aspect of vacancies be made public and accurate could achieve efficient matching and investment.

As for possible extensions, the analysis presented here has been simplified by the sequential nature of search in this continuous time competitive search framework. The directed search framework typified by Acemoglu and Shimer [1999a] captures the idea that firms can simultaneously compare workers and workers may negotiate with multiple employers at the same time. When for instance the human capital investment of the workers is private information, this would affect incentives to invest because of a competition effect that did not apply here. Another simplification made here is that advertising was not a choice. It might be the case that while a firm is committed to some minimum human capital level h , it may advertise as if the minimum requirement is $\hat{h} > h$.

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