Advance Information and Asset Prices*

Rui Albuquerque and Jianjun Miao

December 6, 2007

Abstract

This paper provides an explanation for momentum and reversal in stock returns within a rational expectations framework in which investors are heterogeneous in their information and investment opportunities. We assume that informed agents privately receive advance information about company earnings that materializes into the future. While this information is immediately incorporated into prices, stock prices underreact to it causing short-run momentum. Stock prices may appear to move in ways unrelated to current fundamentals. When the information materializes, the stock price reverts back to its long run mean mimicking an overreaction pattern.

Key words: advance information, momentum and reversal effects, underreaction, overreaction, rational expectations equilibrium

JEL Classification: G11, G12, G14.

*Contacts: Albuquerque, CEPR and Boston University School of Management, 595 Commonwealth Avenue, Boston, MA 02215. Email: ralbuque@bu.edu. Tel.: 617-353-4614. Miao, Department of Economics, Boston University, 270 Bay State Road, Boston, MA 02215 and Department of Finance, Hong Kong University of Science and Technology, Clear Water Bay, Hong Kong. Email: miaoj@bu.edu. Tel.: 617-353-6675. The usual disclaimer applies.

1. Introduction

Many empirical studies have documented evidence of the momentum and reversal effects in the aggregate and cross-sectional stock returns. The momentum effect refers to the phenomena that stock returns tend to exhibit unconditional positive serial correlation in the short to medium run. A related phenomenon is that conditional on observable public events, stocks tend to experience post-event drift in the same direction as the initial event impact. The reversal effect refers to the phenomena that stock returns are negatively correlated in the long run, and that stock returns are positively related to price-scaled variables such as the book-to-market ratio.¹

The momentum and reversal effects provide a serious challenge to the efficient markets hypothesis and standard risk-based models. Many researchers have recently shifted attention to behavioral models, where investors are boundedly rational and arbitrage is limited. A common behavioral interpretation is that investors underreact to current news and overreact to consistent patterns of news pointing in the same direction.

In this paper, we provide a rational, heterogeneous-agent model with asymmetric information that can deliver both the momentum and reversal effects. We start with a benchmark model which is a simplified version of Wang (1994). In this benchmark model, two types of agents, informed and uninformed, trade in the financial market. Informed agents can invest in both publicly traded assets (a stock and a risk-free bond) and in a private investment opportunity. Informed agents have private information about the persistent and transitory components of dividends or earnings as well as the return on the private investment. Uninformed agents can invest in the stock and bond only and their information consists of past earnings and stock price realizations. Based on this public information they try to infer informed agents' private information. As discussed in Wang (1994), informed agents trade for speculative and rebalancing reasons. Uninformed agents trade for noninformational reasons only because they cannot distinguish between informed agents' trading motives. They are willing to trade with informed agents because they may perceive informed agents trade for rebalancing reasons.

When informed agents trade for rebalancing reasons due to an increase in the return on the private investment, informed agents sell the stock and invest in the private opportunity. The current stock price falls to attract uninformed agents to buy, who in turn expect high stock returns in the next period. This trading decreases the current stock return and raises next period's stock return. Thus, stock returns generated by rebalancing trades tend to reverse themselves.

¹See Bernard (1992), Cutler, Poterba, and Summers (1991), DeBondt and Thaler (1985), Fama and French (1988), Jegadeesh and Titman (1993), Poterba and Summers (1988), Rouwenhorst (1998), among others.

When informed agents trade for speculative reasons, the stock price changes to reflect these agents' expectations of the stock's future payoffs. These expectations are fulfilled later on as private information becomes public. Thus, a price change generated by a speculative trade implies future returns of the same sign, and stock returns generated by speculative trades tend to continue themselves.

The serial correlation of excess stock returns depends on the relative importance of the speculative and rebalancing trades. We show that in our benchmark model, the effect of speculative trades is always dominated by the effect of rebalancing trades. Thus, our benchmark model cannot generate short-run momentum and long-run reversals simultaneously.

In order to explain these phenomena in a unified way, we extend our benchmark model by introducing a new mechanism that may generate momentum from both speculative and rebalancing trades. Our novel model ingredient is to assume that informed agents possess private advance information about a firm's future performance, such as earnings, in addition to their other private information. While the stock price incorporates this information, uninformed investors underreact to it, causing possible return continuation. At the same time, informed agents rebalance their portfolios by investing in their private investments. As before, this puts downward pressure in prices, but combined with the good advance information it creates room for the stock price to grow further in the future. Therefore, following a good advance information signal both speculative and rebalancing trades may help generate momentum. As the stock price incorporates advance information, stock price changes may be unrelated to current changes in fundamentals. After this advance information is materialized, the stock price gradually reverts to its long-run mean giving rise to what appears to be an overreaction effect.

We show that with a single piece of advance information, the short-run momentum and long-run reversal effects can occur when this information is about next period's earnings innovations. An undesirable prediction of this case is that momentum lasts for only a few periods, which seems inconsistent with empirical evidence. When the single piece of advance information is about many-period-ahead earnings innovations, the model may generate counterfactual cyclic return dynamics. We thus extend this model by assuming that informed agents receive increasingly precise signals about earnings innovations as they are closer to materialize. In this case, stale information is useful for forecasting and informed agents trade on this information.² As a result, the effect of speculative trades can last for a long period, causing long-lived momentum effects. After a sustained streak of good news, the stock price appears to have overshoot

²See Bernhardt and Miao (2005) for a strategic model of stock prices with stale information. See Tetlock (2007) for empirical evidence of the importance of stale information.

its fundamental value and ultimately must revert itself.

To the best of our knowledge, there is no rational model in the extent literature that can explain the momentum and reversal effects simultaneously in a unified way.³ Berk, Green and Naik (1999) show that a rich variety of return patterns, including momentum effects, result from the variation of risk exposures over the life-cycle of a firm’s endogenously chosen projects.⁴ Johnson (2002) provides a standard model of firm cash flows discounted by an ordinary pricing kernel, that can deliver the momentum effect. His key idea is that expected dividend growth rates vary over time and growth rate risk varies with the growth rates. Both models, however, cannot deliver the long-horizon reversal effect. Fama and French (1993, 1996) show that many of the long-horizon results—such as return reversals, the book-to-market effect, and the earnings to price ratio effect—can be largely subsumed within a three-factor model. They interpret their model as a variant of the APT or ICAPM. However, it is controversial to interpret the Fama and French factors as risk factors. In addition, Fama and French (1996) point out that the momentum result of Jegadeesh and Titman (1993) constitutes the “main embarrassment” for their three-factor model.

A common behavioral interpretation of the momentum and reversal effects is based on under- and over-reaction to news. Daniel et al. (1998) present a model based on investor overconfidence and self-attribution. In their model, investors are overconfident about the precision of their private signals. In addition, they update their confidence in a biased manner as they observe the outcomes of their actions. Thus, they overreact to private information and underreact to public information. The Barberis et al. (1998) model is based on different psychology phenomena, i.e., conservatism and the representativeness heuristic. Both of the preceding models study a representative agent framework. Hong and Stein (1999) analyze a model with two types of agents—news watchers and momentum traders. Each news watcher observes some private information, but fails to extract other newswatchers’ information from prices. Momentum traders can profit by trend chasing, but they use simple strategies, leading to overreaction at long horizons.

Daniel and Titman (2006) dispute both the behavioral and risk-based interpretations that the reversal and book-to-market effects are a result of high expected returns on stocks of distressed firms with poor past performance. They decompose individual firm returns into two components, one that is associated with past performance, based on a set of accounting performance measures, and one that is orthogonal to past performance. They show that future

³In addition to the papers in the main text, see Lewellen and Shanken (2002) and Makarov and Rytchkov (2006) for other related rational models.

⁴See Gomes, Kogan and Zhang (2003) for an extension in a general equilibrium framework.

returns are unrelated to the accounting measures of past performance, which they call tangible information, but are strongly negatively related to the component of news about future performance, which is unrelated to past performance. They refer to this last component as intangible information. We may interpret the advance information in our model as intangible information. Our advance information is also unrelated to past performance, but impacts prices. Using a rational expectations, risk-based model with asymmetric information, we show that the presence of this intangible information is important to generate the momentum and reversal effects. In the language of Daniel and Titman (2006), in our model, reversal effects are generated by the reversal of the intangible component of returns.

We organize the rest of the paper as follows. Section 2 presents the model. Section 3 studies a benchmark model without advance information. Section 4 analyzes the equilibrium when informed agents possess a single piece of advance information about future earnings. Section 5 extends this model to incorporate multiple-pieces of advance information. Section 6 concludes. Proofs are relegated to appendices.

2. The Model

Time is discrete and indexed by $t = 1, 2, \dots$. Consider an economy with a single good that can be either consumed or invested. There are two types of infinitely-lived agents in the economy, informed and uninformed. Informed and uninformed agents differ in their information structure and investment opportunities. The fraction of informed agents is $\lambda \in (0, 1)$ and the fraction of uninformed agents is $1 - \lambda$.

2.1. Preferences

All agents have expected exponential utility with an identical constant absolute risk aversion parameter γ . We assume agents are myopic and derive utility from next period's accumulated wealth. Thus, at time t preferences are represented by

$$E_t \left\{ -e^{-\gamma W_{t+1}} \right\}, \quad (1)$$

where E_t is the expectation operator conditional on an agent's information at time t and W_{t+1} is the wealth level at time $t + 1$. The assumption of myopic preferences rules out hedging demands and simplifies our analysis significantly. Introducing a hedging demand, however, does not change our key insights.⁵

⁵Other papers also adopt myopic preferences for tractability (see, e.g., Bacchetta and van Wincoop (2004, 2006), Campbell et al. (1993), and Llorente et al. (2002)).

2.2. Investment opportunities

Investors can trade publicly a riskless bond and a risky stock in the economy. The riskless bond is assumed to have an infinitely elastic supply at a positive constant interest rate r . Let $R = 1 + r$ denote the gross interest rate in the economy. The stock is in unit supply.

The stock has earnings D_t at time t . The earnings process is described by,

$$D_t = F_t + \varepsilon_t^D, \quad (2)$$

where F_t follows an AR(1) process,

$$F_t = a_F F_{t-1} + \varepsilon_t^F, \quad 0 < a_F < 1. \quad (3)$$

Earnings have both a persistent component F_t with persistence given by a_F and a temporary component ε_t^D . We assume that shocks to both components, ε_t^D and ε_t^F , are independently and identically distributed (i.i.d.) normal random variables with mean zeros and variance σ_D^2 and σ_F^2 , respectively. The firm is assumed to distribute 100 percent of its earnings as dividends. We therefore use the terms earnings and dividends interchangeably.

In addition to the publicly traded assets, there is a risky investment opportunity that is available only to the informed agents. This investment opportunity has constant returns to scale. Its return between period t and $t + 1$ is $R + q_{t+1}$, where the excess return q_{t+1} satisfies

$$q_{t+1} = Z_t + \varepsilon_{t+1}^q. \quad (4)$$

Here, Z_t is the expected excess return to the private investment opportunity and follows an AR(1) process:

$$Z_t = a_Z Z_{t-1} + \varepsilon_t^Z, \quad 0 < a_Z < 1, \quad (5)$$

Thus, the return to the private investment opportunity has both a persistent component given by Z_t and a transitory component given by ε_t^q . We assume that shocks to both components, ε_t^q and ε_t^Z , are i.i.d. normal random variables with mean zeros and variance σ_q^2 and σ_Z^2 . We also assume all shocks are uncorrelated except for ε_t^D and ε_t^q . Assume $\sigma_{Dq} = E[\varepsilon_t^D \varepsilon_t^q] > 0$ so that the stock and the private investment are substitutes.

2.3. Information structure

All agents observe the realizations of earnings and market prices of the stock. Informed agents have private information about the persistent component F_t of the stock and the return Z_t on the private investment, while uninformed agents do not. In addition to these pieces of private

information, informed agents receive news about future earnings announcements which we label advance information. The advance information is modeled as a noisy signal of future earnings innovations. In the analysis below, we will consider in turn two information structures:

1. *Single piece of advance information.* At time t , informed agents receive a single signal

$$S_t = \varepsilon_{t+k}^D + \varepsilon_t^S, \quad (6)$$

about time $t + k$ earnings innovations, where ε_t^S is an i.i.d. normal random variable with mean zero and variance σ_S^2 . Assume ε_t^S is independent of all other shocks. The information set of the informed agents is given by:

$$\mathcal{F}_t^i = \{D_s, F_s, P_s, Z_s, S_s : s \leq t\}, \quad (7)$$

and the information set of the uninformed is given by

$$\mathcal{F}_t^u = \{D_s, P_s : s \leq t\}. \quad (8)$$

2. *Multiple pieces of advance information.* At time t , informed agents receive a vector of signals $(S_t^k, \dots, S_t^1)$ about earnings innovations at $t + 1$ through $t + k$:

$$S_t^k = \varepsilon_{t+k}^D + \varepsilon_t^{S_k}, \dots, S_t^1 = \varepsilon_{t+1}^D + \varepsilon_t^{S_1}.$$

Assume that each $\varepsilon_t^{S_n}$ is an i.i.d. normal random variable with mean zero and variance $\sigma_{S_n}^2$. Also assume it is independent of any other shocks. The informed agents' information sets are given by

$$\mathcal{F}_t^i = \left\{ D_s, F_s, P_s, Z_s, (S_s^n)_{n=1, \dots, k} : s \leq t \right\}, \quad (9)$$

and uninformed agents' information sets are given by (8).

The modeling of advance information in (6) is stylized in many respects and can be generalized to deliver better quantitative predictions. However, we think that the qualitative mechanisms we describe survive these generalizations. First, advance information news at time t refers to a known date into the future $t + k$. This need not be the case and in fact the relevant future date associated with advance information is usually uncertain, perhaps one that can be influenced by the firm. Second, investors need not obtain advance information in every period. While this is a critical abstraction for stationarity of the model and tractability, it gains more realism when the period considered is 1 quarter or 1 year as opposed to 1 day. Finally, the informativeness of the advance information need not be constant and may vary over time or with the news themselves. We discuss the implications of this possibility in Section 5.

2.4. Equilibrium

A rational expectations equilibrium can be defined in the usual way. We will focus on a stationary equilibrium in which the stock price is a stationary function of state variables. The key step to define an equilibrium is to formulate the agents' portfolio choice problems. We start with an informed agent's decision problem. We use the superscript i to index a variable associated with an informed agent. Let P_t denote the time t stock price and $Q_t = P_t + D_t - RP_{t-1}$ the stock's excess return at time t . An informed agent solves the following problem

$$\max -E_t^i [\exp(-\gamma W_{t+1}^i)] \quad (10)$$

subject to the budget constraint:

$$\begin{aligned} W_{t+1}^i &= \theta_t^i (P_{t+1} + D_{t+1}) + \alpha_t^i (R + q_{t+1}) + (W_t^i - (\theta_t^i P_t + \alpha_t^i)) R \\ &= \theta_t^i Q_{t+1} + \alpha_t^i q_{t+1} + W_t^i R, \end{aligned} \quad (11)$$

where E_t^i denotes the conditional expectation operator given the information set $\mathcal{F}_t^i$, W_t^i denotes the wealth level, θ_t^i denotes the fraction of the stock held by i , and α_t^i denotes the invested amount in the private investment opportunity. Similarly, we write an uninformed agent's decision problem as follows:

$$\max -E_t^u [\exp(-\gamma W_{t+1}^u)] \quad (12)$$

subject to

$$W_{t+1}^u = RW_t^u + \theta_t^u Q_{t+1}, \quad (13)$$

where E_t^u denotes the conditional expectation operator given the information set $\mathcal{F}_t^u$. Note that an uninformed agent does not have any private investment opportunities.

The market clearing condition is given by

$$\lambda \theta_t^i + (1 - \lambda) \theta_t^u = 1. \quad (14)$$

3. A benchmark model without advance information

As a benchmark, we start with a model in which agents do not have advance information. This model is similar to Wang (1994) with two differences. First, we assume that agents are myopic and ignore hedging demands. Second, we assume that uninformed agents do not have any information about the persistent component of earnings beyond what they can infer from earnings and price realizations. Our simplified model allows us to make our analysis more transparent when deriving new results regarding momentum and reversal effects absent from Wang (1994).

3.1. Equilibrium stock price

We follow a similar solution methodology to that in Wang (1994). Our approach is more general and can be easily adapted to the models with advance information analyzed later. The key step is to determine the stock price function. To do so, we first define the state variable as $\mathbf{x}_t = (F_t, Z_t)^\top$ and the unforecastable shock vector as $\boldsymbol{\varepsilon}_t = (\varepsilon_t^D, \varepsilon_t^F, \varepsilon_t^Z, \varepsilon_t^q)^\top$. Note that $\boldsymbol{\varepsilon}_t \sim N(0, \Sigma)$, where Σ is the covariance matrix. We then conjecture that the price function takes the following form:

$$P_t = -p_0 + \mathbf{p}_i \mathbf{x}_t + \mathbf{p}_u \hat{\mathbf{x}}_t^u, \quad (15)$$

where $\hat{\mathbf{x}}_t^u = E_t^u[\mathbf{x}_t]$, p_0 is a constant, and $\mathbf{p}_i = [p_{i1}, p_{i2}]$ and $\mathbf{p}_u = [p_{u1}, p_{u2}]$ with $p_{u2} = 0$. In general, one may include $\hat{Z}_t^u$ in the price function in that $p_{u2} \neq 0$. However, from the current price P_t the uninformed agents can infer the following sum:

$$P_t + p_0 - p_{u1} \hat{F}_t^u = p_{i1} F_t + p_{i2} Z_t \equiv \Pi_t, \quad (16)$$

since $\hat{F}_t^u$ is observable by the uninformed agents. Thus, $p_{i1} F_t + p_{i2} Z_t$ represents the information content of the equilibrium price. This implies that $p_{i1} F_t + p_{i2} Z_t = p_{i1} \hat{F}_t^u + p_{i2} \hat{Z}_t^u$. We then obtain

$$\hat{Z}_t^u = Z_t + \frac{p_{i1}}{p_{i2}} (F_t - \hat{F}_t^u). \quad (17)$$

Using this equation, we can eliminate $\hat{Z}_t^u$ in (15), and thus set $p_{u2} = 0$.

Proposition 1 *Consider the benchmark model without advance information. If there is a solution to the system of equations given in Appendix A, then the economy has a stationary rational expectations equilibrium in which the equilibrium stock price is given by*

$$P_t = -p_0 + p_{i1} F_t + p_{i2} Z_t + p_{u1} \hat{F}_t^u, \quad (18)$$

where $p_0, p_{i1}, p_{u1} > 0, p_{i2} < 0$ and

$$p_{i1} + p_{u1} = a_F / (R - a_F). \quad (19)$$

This result is similar to Wang (1994) in spite of our simplifying assumptions.⁶ The sign properties of the coefficients are intuitive. The negative constant (or $p_0 > 0$) in the price function reflects the discount on the price to compensate for the risk in future earnings. The properties that $p_{i1}, p_{u1} > 0$ indicate that the price increases with the actual value of the persistent component in earnings and its estimate by uninformed investors. This persistent component

⁶Wang (1994) gives the same restrictions on the coefficients in the price function without a proof.

helps describe expected future earnings. The negative coefficient on Z_t ($p_{i2} < 0$) reveals that the stock and the private investment opportunity are substitutes. When the expected return on the private investment opportunity is high (i.e., Z_t is high), informed agents sell stock to invest in the private investment. This causes the stock price to drop.

Equations (18) and (19) reveal that the sum of the coefficients of the persistent component of earnings and its estimate by the uninformed agent is equal to the constant $a_F/(R - a_F)$. This constant is the coefficient on F_t if the stock price is equal to the expected present value of future earnings discounted by R .

Equation (18) demonstrates that the equilibrium price does not reveal the informed agents' private information. That is, uninformed agents cannot distinguish between persistent shocks to earnings and persistent shocks to expected returns to the private investment: Good news about future earnings (high F_t) or bad private investment opportunities (low Z_t) can both cause informed agents to buy the stock and its price to rise. Observing price and earnings is thus insufficient for uninformed agents to identify the two shocks. This implies that information asymmetry persists in the equilibrium.

The fact that the stock and the private investment opportunity are substitutes, i.e., $p_{i2} < 0$, also has implications for the forecast errors that uninformed agents make: Uninformed agents' forecast errors on the persistent components F_t and Z_t are positively correlated (see (17)). That is, if an uninformed agent underestimates the level of F_t it will also underestimate the level of Z_t .

3.2. Uninformed agents' forecast problem

Uninformed agents do not observe the persistent component F_t on earnings and Z_t on the private investment. They forecast these variables using available information as described by $\mathcal{F}_t^u$ in (8).

Proposition 2 *Consider the benchmark model without advance information. Given $\mathcal{F}_t^u = \{D_s, P_s : s \leq t\}$, the conditional expectations are given by the following steady-state Kalman filtering equation:*

$$\begin{bmatrix} \hat{F}_t^u \\ \hat{Z}_t^u \end{bmatrix} = \begin{bmatrix} a_F \hat{F}_{t-1}^u \\ a_Z \hat{Z}_{t-1}^u \end{bmatrix} + \mathbf{K} \begin{bmatrix} D_t - E_{t-1}^u[D_t] \\ \Pi_t - E_{t-1}^u[\Pi_t] \end{bmatrix}, \quad (20)$$

where $\mathbf{K}$ is a 2×2 matrix with elements $k_{11}, k_{12}, k_{21} > 0$ and $k_{22} < 0$.

The intuition behind the filtering equation (20) is as follows. The first term on the right-hand side gives the expectation based on information prior to period t . The second term gives

the update in expectations based on new information from unexpected fluctuations in earnings and the stock price. The sign restrictions on the elements of the Kalman gain matrix $\mathbf{K}$ reveal several properties.⁷ First, unexpected high earnings (i.e., $D_t - E_{t-1}^u[D_t] > 0$) can be attributed to a high transitory shock ε_t^D or an unexpected increase in the persistent component of earnings, ε_t^F . Because uninformed agents cannot tell these two shocks apart they increase their estimate of F_t , overreacting to transitory shocks ε_t^D . Hence, $k_{11} > 0$. Because an unexpected positive earnings surprise makes uninformed agents revise their forecast of F_t upwards, and because forecast errors of Z_t and F_t are correlated (see (17)), the uninformed agents also revise upwards their expectation of Z_t . Hence, $k_{21} > 0$. Second, an unexpected increase in $\Pi_t = p_{i1}F_t + p_{i2}Z_t$ may indicate an increase in F_t or a decrease in Z_t . Uninformed agents do not observe these two components separately, and thus raise their estimate of F_t and decrease their estimate of Z_t . This explains why $k_{12} > 0$ and $k_{22} < 0$.

3.3. Excess stock returns

Using Propositions 1-2, we can derive excess stock returns and agents' estimates of these returns. Formally, in Appendix A, we show that

$$Q_{t+1} = e_0 + e_{i2}Z_t + e_{i1}(F_t - \hat{F}_t^u) + \mathbf{b}_Q\varepsilon_{t+1}, \quad (21)$$

where $e_0 = rp_0$ is the mean excess returns, $\mathbf{b}_Q$ is a constant vector, and $e_{i2} > 0$ and $e_{i1} > 0$ are constants given in Appendix A. This equation reveals that changes in the expected return on the private investment Z_t or changes in the uninformed agents' estimation error $(F_t - \hat{F}_t^u)$ affect the excess stock return. In addition, exogenous shocks ε_{t+1} to dividends and returns on private investment also affect the excess stock return.

The informed agents' estimate of excess stock returns depends on Z_t and $(F_t - \hat{F}_t^u)$:

$$E_t^i[Q_{t+1}] = e_0 + e_{i2}Z_t + e_{i1}(F_t - \hat{F}_t^u). \quad (22)$$

An increase in Z_t makes informed agents want to substitute stocks for private investments, and thus lowers the stock price P_t and raises the future excess stock return Q_{t+1} . If uninformed agents underestimate F_t in the sense that $F_t > \hat{F}_t^u$, they are likely to revise their estimates upward in the next period as more information on F_t is revealed. As a result, the stock price is expected to rise in period $t + 1$. Informed agents observe the size of uninformed agents' underestimation and thus expect the excess stock return to rise.

⁷Wang (1994) gives the same restrictions on $\mathbf{K}$ without a proof.

The uninformed agents' estimate of excess stock return depends on their estimates of the expected return on the private investment Z_t alone:

$$E_t^u [Q_{t+1}] = e_0 + e_{i2} \hat{Z}_t^u. \quad (23)$$

Changes in Z_t change the excess stock return for reasons discussed previously. However, uninformed agents do not observe Z_t and thus their estimates of Q_{t+1} depend on the estimated Z_t .

3.4. Optimal portfolios

We now solve the agents' optimal portfolio choice problems. It is straightforward to derive the informed agents' optimal portfolio:

$$\theta_t^i = \frac{E_t^i [Q_{t+1}]}{\gamma (\sigma_Q^i)^2 \left(1 - (\rho_{Qq}^i)^2\right)} - \frac{\rho_{Qq}^i E_t^i [q_{t+1}]}{\gamma \sigma_Q^i \sigma_q^i \left(1 - (\rho_{Qq}^i)^2\right)}, \quad (24)$$

where $\sigma_Q^i = \sqrt{\text{Var}_t^i(Q_{t+1})}$, $\sigma_q^i = \sqrt{\text{Var}_t^i(q_{t+1})}$, and

$$\rho_{Qq}^i = \text{Cov}_t^i(Q_{t+1}, q_{t+1}) / \sqrt{\text{Var}_t^i(Q_{t+1}) \text{Var}_t^i(q_{t+1})}.$$

Note that the preceding conditional variances and covariance are independent of time t due to the property of normal random variables. Similarly, the optimal portfolio for an uninformed agent is given by

$$\theta_t^u = \frac{1}{\gamma} \frac{E_t^u [Q_{t+1}]}{\text{Var}_t^u(Q_{t+1})}. \quad (25)$$

Equations (24)-(25) reveal that the optimal portfolios are mean-variance efficient reflecting the trade-off between expected return and risk. Since all agents are myopic and maximize utility from terminal wealth, there is no hedging demand. Using the conditional expectations of excess stock returns derived in the previous subsection, we can provide a sharper characterization of the optimal portfolios.

Proposition 3 *Consider the benchmark model without advance information. The equilibrium trading strategies satisfy*

$$\theta_t^i = f_0^i + f_Z^i Z_t + f_F^i (F_t - \hat{F}_t^u), \quad (26)$$

$$\theta_t^u = f_0^u + f_Z^u \hat{Z}_t^u, \quad (27)$$

where $f_0^i, f_0^u, f_F^i, f_Z^u > 0$ and $f_Z^i < 0$ are constants.

Equation (26) shows that informed agents trade in the stock when Z_t or $(F_t - \hat{F}_t^u)$ change. An increase in Z_t leads informed agents to sell the stock, giving rise to their rebalancing trading. The term $(F_t - \hat{F}_t^u)$ is the forecast error of the uninformed agents in estimating future earnings; it gives rise to informed investors' speculative trading in the stock market. When uninformed investors underestimate F_t in the sense that $F_t > \hat{F}_t^u$, the informed investors speculative trading induces them to buy stocks leading to price increases. Informed investors expect to sell in the future at even higher prices when uninformed investors revise their forecasts upwards.

Equation (27) reveals that the uninformed agents' optimal stockholding changes only when their expectation of the return Z_t on the informed agents' private investment changes. Using (17) to rewrite (27) we get

$$\theta_t^u = f_0^u + f_Z^u Z_t + f_Z^u \frac{p_{i1}}{p_{i2}} (F_t - \hat{F}_t^u).$$

This expression demonstrates that trading by uninformed agents is subject to adverse selection. When they trade because informed agents rebalance their portfolios in response to changes in the private investment return as given by Z_t , uninformed agents trade at favorable prices and earn abnormal returns. However, uninformed agents cannot tell these trades from the speculative trades of informed agents. In the later, they trade at unfavorable prices and earn a negative return: Uninformed agents are sellers at times when they underestimate persistent shocks to earnings, i.e. $f_Z^u (F_t - \hat{F}_t^u) p_{i1}/p_{i2} < 0$ when $F_t - \hat{F}_t^u > 0$.

3.5. Momentum and reversal effects

While Wang (1994) uses a similar model to analyze properties of trading volume, we instead focus on the serial correlation properties of excess stock returns. Empirical evidence documents short-run momentum and long-run reversal effects in returns. We now examine these effects in the benchmark model without advance information. We focus on excess stock returns, while some empirical studies use stock returns.

Proposition 4 *Consider the benchmark model without advance information. For any $n \geq 1$, we have*

$$E[Q_{t+n}|Q_t] = e_0 + e_{i2} a_Z^{n-1} \frac{Cov(Z_t, Q_t)}{Var(Q_t)} Q_t = e_0 + (1 - Ra_Z) a_Z^{n-1} f_Q Q_t, \quad (28)$$

where $e_0, e_{i2} > 0$ and $f_Q < 0$ are constants.

This proposition shows how Q_t can forecast n -period-ahead single-period returns. Define $Q_{t,t+n} = \sum_{j=1}^n Q_{t+j}$ as the cumulative return on a strategy initiated at time t to buy (sell) and

hold the stock for n consecutive periods after observing a high (low) return Q_t . We can then easily derive:

$$E[Q_{t,t+n}|Q_t] = ne_0 + (1 - Ra_Z) f_{Qn} Q_t,$$

where $f_{Qn} = (1 + a_Z + \dots + a_Z^{n-1}) f_Q$. Thus, the properties of momentum and reversal follow from the properties of $E[Q_{t+n}|Q_t]$ given in equation (28). This equation demonstrates that the sign of the correlation between Q_{t+n} and Q_t is the same as the sign of $(Ra_Z - 1)$ for all $n \geq 1$. This means that the model cannot predict both short-run momentum and long-run reversals in excess returns simultaneously. To see this, note that short-run momentum requires that $Cov(Q_{t+1}, Q_t) > 0$ or $1 - Ra_Z < 0$. However, this condition also implies that $Cov(Q_{t+n}, Q_t) > 0$ for all $n \geq 1$. As a result, we cannot obtain the long-run return reversal effect documented in empirical studies.

We now turn to the intuition behind proposition 4 for $n = 1$. As discussed in Section 3.3, stock returns are driven by rebalancing trades, speculative trades, and unpredictable shocks to earnings and private investment returns. Necessarily only the first two components are serially correlated. Consider the effects of rebalancing trades due to an increase in expected private investment returns, Z_t . In this case, informed agents sell the stock and invest in the private investment, causing the current stock price P_t and excess stock returns Q_t to fall, *ceteris paribus*. Uninformed agents buy the stock in expectation of high excess stock returns Q_{t+1} . On the other hand, a high Z_t tends to follow from a high Z_{t-1} , because they are persistent. A high Z_{t-1} causes previous period's stock price P_{t-1} to fall due to rebalancing reasons. This price drop raises current excess stock returns, Q_t , *ceteris paribus*. This effect dominates when the persistence of expected private investment returns is high enough in that $1 - Ra_Z < 0$, causing continuation in excess stock returns. Otherwise, excess stock returns tend to reverse themselves.

Consider next the effects of speculative trades. Suppose earnings are unexpectedly high because of a shock to persistent earnings. Current stock prices increase. Uninformed agents raise their expectation of future earnings, but underreact to the shock. This underreaction gives valuable private information to informed agents who know that the price has not increased to reflect the full value of future expected earnings. Over time as uninformed agents correct their expectations, the price increases further. Thus, returns generated by speculative trades tend to continue themselves.

Proposition 4 demonstrates that the effect of rebalancing trades always dominates the effect of speculative trades. Thus, excess stock returns are negatively serially correlated if and only

if $1 - Ra_Z > 0$.⁸

We next consider the case with $n > 1$. Because changes in the return on the private investment are persistent, the effect of rebalancing trades is also persistent, causing excess stock returns in period $t + n$ to be correlated with excess stock returns in period t . This correlation follows the same sign as the correlation between successive period returns. Its impact decays at the rate a_Z as n increases.

3.6. Earnings announcement drift

The empirical literature on earnings announcements documents a persistent price drift after earnings surprises. In our model with heterogeneous investor expectations, we assume that earnings surprises are computed with respect to all available public information. Thus, we define earnings surprises as $D_t - E_{t-1}^u(D_t)$. We have the following result:

Proposition 5 *Consider the benchmark model without advance information. For any $n \geq 1$, we have*

$$E[Q_{t+n}|D_t - E_{t-1}^u(D_t)] = e_0 + e_{i2}a_Z^{n-1}E[Z_t|D_t - E_{t-1}^u(D_t)] = e_0 + a_Z^{n-1}d_1[D_t - E_{t-1}^u(D_t)], \quad (29)$$

where e_0 and $e_{i2}, d_1 > 0$ are constants.

To understand this proposition, we start with $n = 1$. An earnings surprise can forecast tomorrow's excess stock return Q_{t+1} to the extent that it can forecast the uninformed agents' conditional expectation of the private investment return Z_t as revealed by equation (23). The intuition is similar to that in the previous subsection. Now, as discussed in Section 3.1, a positive earnings surprise calls for an upward revision of the estimate of F_t and likewise for the estimate of Z_t because forecast errors in these estimates are positively correlated by equation (17). This raises the uninformed agents' estimate of Z_t and their forecast of the excess return Q_{t+1} . The discussion of the case with $n \geq 2$ follows a similar argument to that given in the previous subsection and is omitted.

Intuitively, when uninformed agents observe a positive earnings surprise they learn, among other things, that they have been underestimating the persistent component of earnings. This leads to an immediate price increase. However, because forecast errors are positively correlated, they also learn that they have been underestimating the expected private return Z_t . They then expect more portfolio rebalancing by informed agents as they move into the private

⁸A similar condition is suggested in other papers with different models (see Wang (1993) and Makarov and Rytchkov (2006)). We note that the same result applies when $\sigma_{Dq} < 0$. Its proof is available upon request.

investment opportunity. Portfolio rebalancing trades partly offset the stock price increase from re-estimating F_{t-1} and raise expected stock returns.

Empirical studies of price drift after earnings announcements consider the return to a buy (sell) and hold strategy initiated at time t and hold for n periods after a positive (negative) earnings surprise. In the benchmark model, the return to this strategy is characterized by the following equation:

$$E [Q_{t,t+n}|D_t - E_{t-1}^u(D_t)] = ne_0 + d_{1n} [D_t - E_{t-1}^u(D_t)] ,$$

where $d_{1n} = (1 + a_Z + \dots + a_Z^{n-1}) d_1$. This equation reveals that positive surprises lead to positive returns because $d_{1n} > 0$. In addition, the returns to the buy and hold strategy increase with the investment horizon since $\partial (d_{1n}) / \partial n > 0$, but at a decreasing rate since $\partial^2 (d_{1n}) / \partial n^2 < 0$.

4. Equilibrium with a single piece of advance information

In the next two sections, we analyze models with advance information. We start with the case where informed agents receive a single piece of advance information each period. In particular, we assume that the informed and uninformed agents' information sets are given by (7) and (8), respectively.⁹

In the presence of advance information, solving an equilibrium is nontrivial. The key is to construct suitable state variables. We define the state vector as

$$\mathbf{x}_t = (F_t, Z_t, \varepsilon_{t+k}^D, \dots, \varepsilon_t^D, \boldsymbol{\varepsilon}_{t+k}^q, \dots, \boldsymbol{\varepsilon}_{t+1}^q)^\top . \quad (30)$$

The state vector includes all future realizations of the transitory shocks to earnings and returns on private investment because informed agents can use their private information to forecast them. Note that we also include ε_t^D as part of the state vector for the technical reason to make the forecasting problem easy to solve. We show below that it is not priced in equilibrium and that it does not appear in the asset demand functions, because ε_t^D has already been paid out in the form of earnings at time t and has no value for forecasting future earnings.

4.1. Stock price

We conjecture that the equilibrium stock price function takes the following form

$$P_t = -p_0 + \mathbf{p}_i \hat{\mathbf{x}}_t^i + \mathbf{p}_u \hat{\mathbf{x}}_t^u, \quad (31)$$

⁹Bacchetta and van Wincoop (2004, 2006) construct models of investor heterogeneity with advance information to study the role of higher order expectations. However, they do not address the questions we study here.

where $\hat{\mathbf{x}}_t^i = E_t^i[\mathbf{x}_t]$, $\hat{\mathbf{x}}_t^u = E_t^u[\mathbf{x}_t]$, p_0 is a constant, and $\mathbf{p}_i$ and $\mathbf{p}_u$ are row vectors to be determined in equilibrium. In addition, we set $p_{u2} = 0$ as in the benchmark model in Section 3. The reason is that the equality $\mathbf{p}_i \hat{\mathbf{x}}_t^i = \mathbf{p}_i \hat{\mathbf{x}}_t^u$ holds in equilibrium because $\mathbf{p}_i \hat{\mathbf{x}}_t^i$ is in the information set of uninformed investors. Therefore,

$$\hat{Z}_t^u = \frac{1}{p_{i2}} \mathbf{p}_i \hat{\mathbf{x}}_t^i - \frac{\mathbf{p}_i \mathbf{I}_{-2}}{p_{i2}} \hat{\mathbf{x}}_t^u, \quad (32)$$

where $\mathbf{I}_{-2}$ denotes the matrix that is the same as the identity matrix except that the (2,2) element equals zero. Thus, we can eliminate $\hat{Z}_t^u$ in the price function. Also, like equation (17), equation (32) indicates that uninformed agents' forecast errors are perfectly linearly correlated.

Proposition 6 *Consider the model with a single piece of advance information. If there is a solution to the system of equations given in Appendix B, then the economy has a stationary rational expectations equilibrium in which the equilibrium stock price is given by*

$$\begin{aligned} P_t = & -p_0 + p_{i1}F_t + p_{i2}Z_t + p_{u1}\hat{F}_t^u \\ & + \sum_{j=1}^k \{p_{i,k+3-j}E_t^i[\varepsilon_{t+j}^D] + p_{u,k+3-j}E_t^u[\varepsilon_{t+j}^D]\} \\ & + \sum_{j=1}^k \left\{ p_{i,2k+4-j}E_t^i[\varepsilon_{t+j}^q] + p_{u,2k+4-j}E_t^u[\varepsilon_{t+j}^q] \right\}, \end{aligned} \quad (33)$$

where $p_0 > 0$, and

$$p_{i1} + p_{u1} = a_F / (R - a_F), \quad (34)$$

$$p_{i2} = \frac{-e_{i2}}{R - a_Z}, \quad (35)$$

$$p_{ij} + p_{uj} = \frac{1}{R^{3+k-j}}, \text{ for } 3 \leq j \leq k+2, \quad (36)$$

$$p_{ij} + p_{uj} = -\frac{e_{i2}}{R^{2k+3-(j-1)}}, \text{ for } j \geq k+4, \quad (37)$$

where e_{i2} is some constant. The constant $e_{i2} > 0$ iff $Cov_t^i(Q_{t+1}, q_{t+1}) > 0$.

The interpretation of the first line of equation (33) is similar to that of (18) in the benchmark model. Unlike the benchmark model, we are unable to prove $p_{i2} < 0$ because we are unable to show $Cov_t^i(Q_{t+1}, q_{t+1}) > 0$ analytically. This positive covariance is intuitive. It reflects the fact that the stock and the private investment are substitutes because earnings are positively correlated with the unexpected return on the private investment $\sigma_{Dq} > 0$. We will verify this result numerically below.

The second and third lines of equation (33) reflect the effects of advance information. In the presence of advance information about earnings innovations in the future, informed agents forecast these innovations and their forecasts will be incorporated into the stock price. Since uninformed agents can observe the stock price, they will also forecast the earnings innovations based on the stock price information and their forecasts will be incorporated into the stock price as well. This explains the second line of (33). The terms on the price function that appear on the third line of (33) arise because earnings innovations are correlated with innovations of the return on private investment: As informed agents learn about ε_{t+k}^D they also improve their forecast on ε_{t+k}^q and can better forecast private investment returns. This information is important for the stock price as it anticipates the future rebalancing trades of informed agents.

Equation (36) and the second line of (33) reveal that the combined impact on the stock price of the estimates of the future earnings innovations is positive, and this impact dies out at the rate $1/R$. That is, the advance information news has a larger impact when the news is closer to be materialized. This effect arises because the shocks ε_{t+k}^D are only due in k periods and cash flows k -periods ahead are discounted by R^{-k} .

By contrast, equation (37) and the third line of (33) reveal that the combined impact on the stock price of the estimates of the innovations of the future returns on the private investment is negative, and this impact dies out at the rate $1/R$. As with earnings innovations, the estimates of the next period private investment return innovations have the largest impact. The negative impact arises because the private investment and the stock are substitutes. When the estimate of the future private investment return is high, the informed agents sell the stock to invest in the private investment, thus lowering the stock price.

Unlike in Wang (1994), in our model with advance information both the informed and uninformed agents must solve forecasting problems.¹⁰ We next turn to these problems and solve the informed agent's forecasting problem first.

4.2. Informed agents' forecast

Informed agents use their private signals on k -period-ahead earnings innovations to learn about the growth potential in both the stock and private investment. Their information processing problem is greatly simplified for two reasons. First, their information set includes knowledge of all past values of the persistent components of earnings and private investment returns. Second, their information set includes that of uninformed agents, which means that informed agents do

¹⁰ Albuquerque et al. (2007) generalize Wang (1994) to an international economy and also require both informed and uninformed agents to solve forecasting problems.

not learn from the price level.

Proposition 7 *Consider the model with a single piece of advance information. Given the information set $(\mathcal{I})$, the informed agents' conditional expectations are given by:*

$$\begin{aligned} E_t^i [\varepsilon_{t+k-j}^D] &= \frac{\sigma_D^2}{\sigma_S^2 + \sigma_D^2} S_{t-j}, & 0 \leq j \leq k-1 \\ E_t^i [\varepsilon_{t+k-j}^q] &= \frac{\sigma_{Dq}}{\sigma_S^2 + \sigma_D^2} S_{t-j}, & 0 \leq j \leq k-1. \end{aligned}$$

The signals S_j , $j \leq t$, do not help forecast any other variable.

The information content of a signal S_t about earnings ε_{t+k}^D is given by $\sigma_D^2 / (\sigma_S^2 + \sigma_D^2)$. If the signal is infinitely precise and $\sigma_S^2 = 0$ then $\sigma_D^2 / (\sigma_S^2 + \sigma_D^2) = 1$ and S_t reveals the true innovation value. If the signal is worthless and $\sigma_S^2 = \infty$ then $\sigma_D^2 / (\sigma_S^2 + \sigma_D^2) = 0$ and the informed agents do not use this information to update their expectation of future earnings. The signal S_t is also informative about shocks to the private investment return because $E[\varepsilon_t^D \varepsilon_t^q] = \sigma_{Dq} > 0$. The information content of the signal is given by $\sigma_{Dq} / (\sigma_S^2 + \sigma_D^2)$. Because signals are i.i.d., a signal observed today S_t about future earnings ε_{t+k}^D contains no new information about other innovations ε_{t+k-j}^D for $0 \leq j \leq k-1$. Likewise, S_t contains no new information about innovations ε_{t+k-j}^q for $0 \leq j \leq k-1$. Finally, because ε_{t+k}^D is uncorrelated with every other shock, the signal S_t cannot be used to forecast any other future variable.

It is useful to write the forecasting problem of informed agents as a filtering problem in terms of the state-space system representation. We write

$$\mathbf{x}_t = \mathbf{A}_x \mathbf{x}_{t-1} + \mathbf{B}_x \boldsymbol{\varepsilon}_t, \quad (38)$$

where the matrices $\mathbf{A}_x$ and $\mathbf{B}_x$ are defined in Appendix B. We also construct the unforecastable shock vector (based on period $t-1$ information) as

$$\boldsymbol{\varepsilon}_t = (\varepsilon_{t+k}^D, \varepsilon_t^F, \varepsilon_t^Z, \varepsilon_{t+k}^q, \varepsilon_t^S)^\top.$$

This vector is normally distributed with mean zero and covariance matrix $\Sigma = E[\boldsymbol{\varepsilon}_t \boldsymbol{\varepsilon}_t^\top]$, where the only nonzero covariance is $\sigma_{Dq} = E[\varepsilon_{t+k}^D \varepsilon_{t+k}^q] > 0$.

The informed agents' observable signals are summarized in the vector $y_t^i = (D_t, F_t, Z_t, S_t)^\top$. This vector satisfies:

$$\mathbf{y}_t^i = \mathbf{A}_{yi} \mathbf{x}_t + \mathbf{B}_{yi} \boldsymbol{\varepsilon}_t.$$

where $\mathbf{A}_{yi}$ and $\mathbf{B}_{yi}$ are given in Appendix B. Then we have the steady-state Kalman filter:

$$\hat{\mathbf{x}}_t^i = \mathbf{A}_x \hat{\mathbf{x}}_{t-1}^i + \mathbf{K}_i \hat{\boldsymbol{\varepsilon}}_t^i, \quad (39)$$

where the innovation

$$\hat{\varepsilon}_t^i \equiv \mathbf{y}_t^i - E_{t-1}^i [\mathbf{y}_t^i], \quad (40)$$

is normally distributed with mean zero and covariance matrix $\Sigma_i = E [\hat{\varepsilon}_t^i (\hat{\varepsilon}_t^i)^\top]$. The expressions of the Kalman gain matrix $\mathbf{K}_i$ and the matrix Σ_i are given in Appendix B. We note that the first two components of $\hat{\mathbf{x}}_t^i$ are given by F_t and Z_t since they are observable. Also, since D_t and F_t are observable, $\hat{\mathbf{x}}_t^i$ contains ε_t^D .

4.3. Uninformed agents' forecast

We next consider an uninformed agent's forecasting problem. Because informed agents know more than uninformed agents, the most that uninformed agents can hope to learn is what informed agents know. Therefore, it is sufficient for uninformed agents to track the dynamics of the state vector (39). The uninformed agents' observation is summarized by the vector $\mathbf{y}_t^u = (D_t, \mathbf{p}_i \hat{\mathbf{x}}_t^i)^\top$. We write the observation system as

$$\mathbf{y}_t^u = \mathbf{A}_{yu} \hat{\mathbf{x}}_t^i,$$

where we can write $\mathbf{A}_{yu} = [\mathbf{c}_1^\top + \mathbf{c}_{k+3}^\top, \mathbf{p}_i^\top]^\top$. By the Kalman filtering theory, we have:

Proposition 8 *Consider the model with a single piece of advance information. Uninformed agents' conditional forecast of the state vector is given by the steady-state Kalman filters:*

$$\hat{\mathbf{x}}_t^u = \mathbf{A}_x \hat{\mathbf{x}}_{t-1}^u + \mathbf{K}_u \hat{\varepsilon}_t^u, \quad (41)$$

and

$$\hat{\varepsilon}_t^u \equiv \mathbf{y}_t^u - E_{t-1}^u [\mathbf{y}_t^u], \quad (42)$$

where $\hat{\varepsilon}_t^u$ is normally distributed with mean zero and covariance matrix

$$\Sigma_u = \mathbf{A}_{yu} \mathbf{A}_x \Omega_u \mathbf{A}_x^\top \mathbf{A}_{yu}^\top + \mathbf{A}_{yu} \mathbf{K}_i \Sigma_i \mathbf{K}_i^\top \mathbf{A}_{yu}^\top.$$

Moreover, the covariance matrix $\Omega_u = E_t^u [(\hat{\mathbf{x}}_t^i - \hat{\mathbf{x}}_t^u) (\hat{\mathbf{x}}_t^i - \hat{\mathbf{x}}_t^u)^\top]$ and the Kalman gain matrix $\mathbf{K}_u$ satisfy:

$$\Omega_u = (\mathbf{A}_x \Omega_u \mathbf{A}_x^\top + \Sigma_{xx}) - \mathbf{K}_u \mathbf{A}_{yu} (\mathbf{A}_x \Omega_u \mathbf{A}_x^\top + \Sigma_{xx}), \quad (43)$$

where

$$\mathbf{K}_u = (\mathbf{A}_x \Omega_u \mathbf{A}_x^\top + \Sigma_{xx}) \mathbf{A}_{yu}^\top [\mathbf{A}_{yu} (\mathbf{A}_x \Omega_u \mathbf{A}_x^\top + \Sigma_{xx}) \mathbf{A}_{yu}^\top]^{-1}, \quad (44)$$

and $\Sigma_{xx} = \mathbf{K}_i \Sigma_i \mathbf{K}_i^\top$.

We are now ready to solve for the estimates of excess stock returns and optimal portfolios of informed and uninformed agents.

4.4. Excess stock returns

In Appendix B, we use Propositions 6-8 to derive the excess stock return

$$Q_{t+1} = e_0 + e_{i2} (Z_t + E_t^i [\varepsilon_{t+1}^q]) + e_{i1} (F_t - \hat{F}_t^u) + e_D (\varepsilon_{t+1}^D - E_t^i [\varepsilon_{t+1}^D]) + \mathbf{b}_Q^1 \varepsilon_{t+1} \quad (45)$$

$$+ \sum_{j=1}^k \left\{ e_{i,k+3-j} (E_t^i [\varepsilon_{t+j}^D] - E_t^u [\varepsilon_{t+j}^D]) - e_{u,2k+4-j} (E_t^i [\varepsilon_{t+j}^q] - E_t^u [\varepsilon_{t+j}^q]) \right\},$$

where $\mathbf{b}_Q^1$ is a constant vector, $e_0, e_{i1}, \dots, e_{i,k+3}, e_{u,k+4}, \dots, e_{u,2k+3}$, and e_D are constants given in Appendix B. The first line of equation (45) is similar to equation (21) for the benchmark model with two differences. First, informed agents received a signal about the transitory component of earnings, ε_{t+1}^D , in period $t-k+1$. This information generates speculative trading when the signal is obtained and informed agents revise their estimates. Thus, informed agents' estimation error $\varepsilon_{t+1}^D - E_t^i [\varepsilon_{t+1}^D]$ affect excess stock returns. Second, informed agents' forecast of the innovations ε_{t+1}^q to the return on the private investment in period $t+1$ also affects the excess stock return. The intuition is that the informed agents' signal about ε_{t+1}^D in period $t-k+1$ is useful for forecasting ε_{t+1}^q because ε_{t+1}^D and ε_{t+1}^q are correlated (see Proposition 7). The informed agents estimates of ε_{t+1}^q generate rebalancing trades and thus affect excess stock returns.

The second line of equation (45) reveals that the uninformed agents' error in estimating future earning innovations ε_{t+1}^D and private return innovations ε_{t+1}^q relative to the informed agents' estimates influences excess stock returns. The reason is that the informed agents can profit from this estimation error by speculative trading.

Using equation (45), we can easily derive the informed and uninformed agents' estimates of excess returns:

$$E_t^i [Q_{t+1}] = e_0 + e_{i2} (Z_t + E_t^i [\varepsilon_{t+1}^q]) + e_{i1} (F_t - \hat{F}_t^u) \quad (46)$$

$$+ \sum_{j=1}^k \left\{ e_{i,k+3-j} (E_t^i [\varepsilon_{t+j}^D] - E_t^u [\varepsilon_{t+j}^D]) - e_{u,2k+4-j} (E_t^i [\varepsilon_{t+j}^q] - E_t^u [\varepsilon_{t+j}^q]) \right\},$$

and

$$E_t^u [Q_{t+1}] = e_0 + e_{i2} (\hat{Z}_t^u + E_t^u [\varepsilon_{t+1}^q]). \quad (47)$$

The error terms $e_D (\varepsilon_{t+1}^D - E_t^i [\varepsilon_{t+1}^D])$ and $\mathbf{b}_Q^1 \varepsilon_{t+1}$ are unforecastable by both informed and uninformed agents and thus do not change the various agents' estimates of excess stock returns.

4.5. Optimal portfolios

As in Section 3.2, optimal portfolios are given by equations (24) and (25). We use the previously derived conditional expectations of excess stock returns to derive the following:

Proposition 9 *Consider the model with a single piece of advance information. The equilibrium trading strategies satisfy*

$$\begin{aligned} \theta_t^i = & f_0^i + f_Z^i Z_t + f_{q0}^i E_t^i [\varepsilon_{t+1}^q] + f_F^i (F_t - \hat{F}_t^u) \\ & + \sum_{j=1}^k \left\{ f_{Dj}^i (E_t^i [\varepsilon_{t+j}^D] - E_t^u [\varepsilon_{t+j}^D]) + f_{qj}^i (E_t^i [\varepsilon_{t+j}^q] - E_t^u [\varepsilon_{t+j}^q]) \right\}, \end{aligned} \quad (48)$$

and

$$\theta_t^u = f_0^u + f_Z^u (\hat{Z}_t^u + E_t^u [\varepsilon_{t+1}^q]), \quad (49)$$

where $f_0^i, f_0^u > 0$, $f_Z^i, f_F^i, f_Z^u, f_{Dj}^i$, and f_{qj}^i are constants. Also, $f_Z^i, f_{q0}^i < 0$, and $f_Z^u > 0$ iff $Cov_t^i(Q_{t+1}, q_{t+1}) > 0$.

This proposition shows that the informed agents trade for both speculative and rebalancing reasons as in the benchmark model. The return on the private investment determines the rebalancing trades. Unlike the benchmark model, both the expected and shock components, Z_t and ε_{t+1}^q , contribute to the rebalancing trading. The effect of the forecast of ε_{t+1}^q is present because at $t-k+1$ informed agents received advance information about the earnings innovations ε_{t+1}^D , which are positively correlated with ε_{t+1}^q . The forecast of ε_{t+1}^q is then part of expected private investment returns and, when the assets are substitutes and $Cov_t^i(Q_{t+1}, q_{t+1}) > 0$, leads to rebalancing trades.

Turn to the speculative trading. The terms $E_t^i [\varepsilon_{t+j}^D] - E_t^u [\varepsilon_{t+j}^D]$ and $E_t^i [\varepsilon_{t+j}^q] - E_t^u [\varepsilon_{t+j}^q]$ give the errors of the uninformed agents in estimating future earnings and future returns on the private investment, relative to the informed agents' estimation. These errors cause the stock price to fluctuate. With private information about the advance information, the informed agents can observe these price fluctuations and take speculative positions against expected future corrections of the uninformed agents' expectations.

The uninformed agents trade only for noninformational reasons. They are willing to trade with the informed agents because part of the informed agents' trading is for rebalancing reasons. When the future return $q_{t+1} = Z_t + \varepsilon_{t+1}^q$ on the private investment increases, the informed agents rebalance their portfolios by selling the stock if the two assets are substitutes. The uninformed agents are willing to take the other side of this rebalancing trade because they buy at a low price (i.e. $p_{i2} < 0$). However, there is an adverse selection problem when uninformed agents trade with informed agents; uninformed agents do not know for sure whether informed agents are rebalancing their portfolios or trading for speculative reasons. This is because the forecast error uninformed agents make on Z_t , i.e. $Z_t - \hat{Z}_t^u$, is correlated with the forecast error in all other state variables which are responsible for the speculative positions of informed agents.

4.6. Momentum and reversal effects

Because of advance information the properties of momentum and reversal in stock returns can differ substantially from the benchmark model. As in Section 3.3, in order to study the properties of $E[Q_{t+n}|Q_t]$ commonly used to measure momentum and reveal effects, we only need to focus on $E[Q_{t+n}|Q_t]$.

Proposition 10 *Consider the model with a single piece of advance information. For any $n \geq 1$, we have*

$$E[Q_{t+n}|Q_t] = e_0 + e_{i2} \frac{a_Z^{n-1} \text{Cov}(Z_t, Q_t) + \mathbf{1}_{\{n \leq k\}} \text{Cov}(E_t^i[\varepsilon_{t+n}^q], Q_t)}{\text{Var}(Q_t)} Q_t, \quad (50)$$

where $e_0 > 0$ and $e_{i2} > 0$ if and only if $\text{Cov}_t^i(Q_{t+1}, q_{t+1}) > 0$. The indicator function $\mathbf{1}_{\{n \leq k\}}$ equals 1 if $n \leq k$ and 0 otherwise.

This proposition decomposes the correlation between Q_{t+n} and Q_t into two parts. The first part is determined largely by the covariance between the stock return and the expected return on the private investment, $\text{Cov}(Z_t, Q_t)$. Its sign depends on the persistence a_Z of the expected return on the private investment, much like in the benchmark model. In particular, if the persistence is sufficiently high, then the covariance is positive. Otherwise, it is negative.

The second part reflects the effect of advance information. This part is represented by the term $\text{Cov}(E_t^i[\varepsilon_{t+n}^q], Q_t)$ that plays a role if and only if $n \leq k$. After period $t+k$, all advance information up to date t loses its value and the correlation between Q_{t+n} ($n > k$) and Q_t is determined as in the benchmark model without advance information. From the analysis in Section 3, we deduce that we must have two conditions for our model to generate short-run momentum and long-run reversals simultaneously. First, the persistence a_Z must be sufficiently small. If it is too large, we cannot generate long-run reversals. Second, given a small value of a_Z , we must have $\text{Cov}(E_t^i[\varepsilon_{t+n}^q], Q_t) > 0$. Otherwise, we cannot generate short-run momentum.

We now analyze the mechanism to deliver $\text{Cov}(E_t^i[\varepsilon_{t+n}^q], Q_t) > 0$ for any $k \geq 1$ and $n \leq k$. Consider the effects of a good signal S_{t+n-k} received in period $t+n-k$ about future earnings, ε_{t+n}^D , for $n \leq k$. First, this good signal leads to an increase in the stock price P_t and in the excess stock return Q_t . Because uninformed agents underreact to it, it also leads to speculative trading. This results in a positive covariance $\text{Cov}(E_t^i[\varepsilon_{t+n}^q], Q_t) > 0$, because $E_t^i[\varepsilon_{t+n}^q]$ is positively correlated with $E_t^i[\varepsilon_{t+n}^D]$ as shown in Proposition 7. Second, the increase in $E_t^i[\varepsilon_{t+n}^q]$ leads to a drop in the stock price P_t and hence in Q_t , because informed agents rebalance their portfolios and sell the stock. If the rebalancing effect is not too strong to cause the price to fall following the good advance information signal, we obtain $\text{Cov}(E_t^i[\varepsilon_{t+n}^q], Q_t) > 0$.

To further describe the mechanism that generates momentum, suppose a_Z is small. Recall that when a_Z is small, our benchmark model without advance information predicts a negative correlation between Q_t and Q_{t+n} . Consider first the case of one-period-ahead advance information, $k = 1$. Suppose informed agents receive a good signal about future earnings ε_{t+1}^D . The good news leads to an increase in price P_t and in the excess stock return Q_t by the amount $E_t^i[\varepsilon_{t+1}^D]$ (see (33)). In addition, uninformed agents underreact to the private signal because there are several reasons why the stock price could have gone up. Therefore, $E_t^i[\varepsilon_{t+1}^D] - E_t^u[\varepsilon_{t+1}^D] > 0$, giving rise to an increase in informed agents' speculative demand for the stock. On the other hand, a good signal about ε_{t+1}^D helps informed agents forecast a high return in their private investments as well, $E_t^i[\varepsilon_{t+1}^q] > 0$, leading to rebalancing trades. When the two assets are substitutes these rebalancing trades put downward pressure on the stock price partly off-setting the effect of higher expected future earnings. If the rebalancing motive is strong enough, but not so strong to cause the stock price to fall, informed agents sell the stock even though they expect its price to increase, acting as contrarian investors. Uninformed agents follow trend-chasing strategies, buying as the price increases: They are willing to buy stocks as they expect a high return Q_{t+1} next period. Consequently, we may obtain one-period momentum in that $Cov(Q_t, Q_{t+1}) > 0$. Because private advance information is about the transitory component of the next period's earnings, the effect of date t advance information disappears after period $t+1$ and, all else equal, prices are expected to revert to their long run mean starting from $t+1$. After $t+1$, informed agents' trading mimics a trend-chasing strategy while uninformed agents act as contrarians.

We next consider the case of advance information about ε_{t+k}^D with $k > 1$. We argue that serial correlation in one-period returns may display a cyclical pattern. The intuition follows from Proposition 6, which shows that the impact of advance information on the stock price is larger when the advance information is closer to be materialized due to discounting. In addition, Proposition 6 shows that informed agents underreact to the advance information about k -period-ahead earnings. Thus, before date $t+k$, the previously discussed speculative trading effect is not sufficiently strong. In the meantime, in response to the date t advance information about ε_{t+k}^D , informed agents change their rebalancing trades, represented by the third term on the right-hand side of (48), only at date $t+k-1$. Consequently, the second term in (50), which represents the effect of advance information before date $t+k$, may be dominated by the first term $Cov(Z_t, Q_t)$, which arises when there is no advance information. This causes negative correlation between Q_{t+n} and Q_t , i.e., $Cov(Q_{t+n}, Q_t) < 0$ for $n < k$. When the good advance information gets closer to be materialized, the price increases more and the effects of

speculative and rebalancing trades get stronger. In particular, in period $t + k$, the two effects combine to generate $Cov(Q_{t+k}, Q_t) > 0$, as analyzed in the case with $k = 1$. After period $t + k$, the date t advance information has no effect on future returns, and thus stock returns revert to the long-run mean in that $Cov(Q_{t+n}, Q_t) < 0$ for $n > k$.

We now conduct some numerical experiments to illustrate our previous intuition. Table 1 shows the slope coefficients on the forecast of single-period returns Q_{t+n} conditional on Q_t as well as the slope coefficient of the forecast of cumulative returns $Q_{t,t+n}$ conditional on Q_t . We find that when $k = 1$, our model generates two-period momentum followed by a stock return reversal. The two-period cumulative return is positive because the first period positive return compensates for the second period negative return. We also find that when $k = 4$, cumulative returns for all horizons up to 10 are negative and there is no momentum effect.

[Insert Table 1 Here.]

4.7. Earnings announcement drift

Advance private information about future transitory earnings also alters the nature of earnings price drift. As in Section 3.4, we only need to focus on $E[Q_{t+n}|D_t - E_{t-1}^u(D_t)]$. We have the following result:

Proposition 11 *Consider the model with a single piece of advance information. For any $n \geq 1$, we have*

$$\begin{aligned} E[Q_{t+n}|D_t - E_{t-1}^u(D_t)] &= e_0 + e_{i2}a_Z^{n-1}E[Z_t|D_t - E_{t-1}^u(D_t)] \\ &\quad + e_{i2}\frac{\mathbf{1}_{\{n \leq k-1\}}Cov(E_t^i[\varepsilon_{t+n}^q], D_t - E_{t-1}^u(D_t))}{Var(D_t - E_{t-1}^u(D_t))}[D_t - E_{t-1}^u(D_t)], \end{aligned} \quad (51)$$

where e_0 is a constant and $e_{i2} > 0$ if and only if $Cov_t^i(Q_{t+1}, q_{t+1}) > 0$. The indicator function $\mathbf{1}_{\{n \leq k-1\}}$ equals 1 if $n \leq k - 1$ and 0 otherwise.

The intuition behind the first line of equation (51) is similar to that behind equation (29) in the benchmark model. That is, a positive earnings surprise calls for an upward revision of the uninformed agents' estimates of F_t , and hence Z_t since their estimation errors are perfectly correlated. Consequently, uninformed agents raise their expectations of excess stock returns, causing positive correlation between Q_{t+n} and $D_t - E_{t-1}^u(D_t)$ when Z_t is persistent.

The term in the second line of equation (51) reflects the effect of advance information. Clearly, this term is zero for $n \geq k$ because the forecast error $D_t - E_{t-1}^u(D_t)$ does not convey information about ε_{t+n}^D , and hence ε_{t+n}^q , for $n \geq k$. This term is nonzero for $n < k$ and is

generally negative for the following reason. Consider $k > 1$, and to fix ideas let $k = 2$ and $n = 1$. When the private investment and the stock are substitutes, it is intuitive that the coefficients of both Z_t and $E_t^i[\varepsilon_{t+n}^q]$ in the price function (33) are negative. This implies that the errors in the uninformed agents' estimates of Z_t and $E_t^i[\varepsilon_{t+n}^q]$ are negatively correlated. As argued above, a positive earnings surprise calls for an upward revision in the uninformed agents' estimates of Z_t . Thus, we deduce that it calls for a downward revision in the uninformed agents' estimates of $E_t^i[\varepsilon_{t+n}^q]$, causing a negative correlation between $E_t^i[\varepsilon_{t+n}^q]$ and $D_t - E_{t-1}^u(D_t)$. We conclude that the model with advance information may dampen the effect of earnings announcement drift.

Despite this dampening effect, Table 2 illustrates numerically that our model with advance information can still generate the earnings announcement drift phenomenon. The table displays the slope coefficients of the forecast of single-period returns Q_{t+n} conditional on the earnings announcement $D_t - E_{t-1}^u[D_t]$ as well as the slope coefficients of the forecast of cumulative returns $Q_{t,t+n}$ conditional on $D_t - E_{t-1}^u[D_t]$ for various values of n and k .

[Insert Table 2 Here.]

5. Equilibrium with multiple pieces of advance information

The analysis thus far has shown that the benchmark model without advance information in Section 3 cannot generate momentum and reversal effects simultaneously. In addition, we have shown in Section 4 that the model with one-period-ahead advance information can generate short-run momentum followed by reversals in stock returns. However, momentum lasts only for a very few periods. We also show that when informed agents receive a single piece of long-horizon advance information, the model generates a counterfactual cyclic behavior of serial correlation in one-period excess stock returns.

In order to generate long-lived momentum followed by long-run reversals, we extend the model in Section 4 to incorporate multiple pieces of advance information. Specifically, we assume that in each period t informed agents receive signals about earnings in each period from $t + 1$ to $t + k$. At time t they have received k correlated signals about period $t + 1$ earnings because they also receive signals about these in the past $k - 1$ periods. Thus, the informed and uninformed agents' information sets are given by (9) and (8), respectively. This assumption is quite natural as new information, say about end-of-quarter earnings, is likely to arrive at intermediate periods as the quarter nears its end. In addition, past stale information is still useful for forecasting and thus affects stock prices.

The intuition behind this modeling device is that the successive advance information news about the same future earnings can generate long-lived large speculative trading effects and momentum. An important modeling issue is how to specify the quality of signals. Because up to period t informed agents will have received $k - 1$ signals on ε_{t+1}^D already, the stock price increasingly reveals ε_{t+1}^D to the uninformed agents, reducing the motive for speculative trading by informed agents. It is therefore possible that, with too much information in previous periods, only the rebalancing trade motive is at work, generating negative serial correlation in returns. In order to obtain long-lived momentum when $k > 1$ it is thus needed that the advance information increases in quality as we approach the earnings realization, i.e., $\sigma_{S_k}^2 > \dots > \sigma_{S_1}^2$. In this case, return reversals occur after at least k periods as the advance information effect dissipates and the stock price overshoots its long-run mean. This appears to be the overreaction pattern in the behavioral story.

In appendix C, we show that after casting the problem into vector and matrix forms, we can use the previous solution method to show that the equilibrium in this section displays the same form as in Section 4. The only difference is that the informed agents' forecasting problem is different because they now have multiple pieces of advance information. We omit the detailed derivation here and turn to a numerical analysis.

For ease of exposition, we focus on the case with $k = 2$. We first discuss two limiting results. First, when the signal about two-period-ahead earnings is completely uninformative (i.e., $\sigma_{S_2} = \infty$), the model becomes that in Section 4 with $k = 1$. Consequently, our previous results in Section 4 apply here. Second, when the signal about the two-period-ahead earnings innovation is extremely precise (i.e., $\sigma_{S_2} \rightarrow 0$), we find numerically that asymmetric information increases so greatly that there is no trading in equilibrium. As a result, stock returns are serially uncorrelated. The intuition is as follows. The stock price incorporates the information about the persistent and transitory components of earnings as well as the expected return on the private investment. Uninformed agents use the earnings realizations and the stock price to infer the value of these variables, but may attribute changes in earnings innovations to changes in the persistent components of earnings or to changes in the private investment return. When informed agents receive very precise information about earnings innovations, they trade on this information more aggressively and uninformed agents believe that informed agents' trading is generated by a speculative motive and not by a rebalancing motive. Because uninformed agents know that they will lose if they trade with informed agents when their trades are solely motivated by speculative reasons, they refrain from trading.

We now turn to intermediate values of σ_{S_2} . Table 3 displays the slope coefficients of the

forecast of single-period returns Q_{t+n} conditional on Q_t as well as the slope coefficients of the forecast of cumulative returns $Q_{t,t+n}$ conditional on Q_t for various values of σ_{S_2} . This table reveals that our model with advance information about two successive periods earnings innovations can generate momentum and reversal effects simultaneously. In addition, the duration of momentum depends on the precision of the advance information signals. In particular, when the signal about two-period-ahead earnings innovations becomes more precise relative to the signal about one-period-ahead earnings innovations, the momentum effect lasts longer. On the other hand, when this signal is sufficiently imprecise, the momentum effect disappears.

[Insert Table 3 Here.]

Table 4 displays the slope coefficients of the forecast of single-period returns Q_{t+n} conditional on the earnings announcement $D_t - E_{t-1}^u[D_t]$ as well as the slope coefficients of the forecast of cumulative returns $Q_{t,t+n}$ conditional on $D_t - E_{t-1}^u[D_t]$ for various values of σ_{S_2} . This table shows that the earnings announcement drift effect is stronger as the signal about two-period-ahead earnings innovations becomes less precise relative to the signal about one-period-ahead earnings innovations.

[Insert Table 4 Here.]

6. Conclusion

In this paper, we present a rational expectations, heterogeneous-agent model with asymmetric information that can deliver the momentum and reversal effects in a unified way. Our key insight is to assume that informed agents possess advance information about future earnings. The stock price underreacts to this information and uninformed agents can profit by following trend-chasing strategies. Advance information also makes prices move in ways that are unrelated to current fundamentals. These price movements can predict future returns. After a sustained streak of good news the advance information materializes and the price starts reverting back to its long-run mean giving the appearance of having overshoot its fundamental value. Thus, our model provides a rational account of the underreaction and overreaction phenomena.

We can extend our model in a number of directions. First, for tractability, we have assumed myopic preferences and ignored hedging demands. While it is not difficult to introduce hedging demand, we believe that this extension does not change the main predictions and insights of our model. Second, our model focuses on the implications of advance information for momentum and reversal effects. It would be interesting to study the implications for trading volume,

as in Wang (1994) and Llorente et al. (2002). Third, we follow Wang (1994) and assume a hierarchical information structure. It would be interesting to consider the case where information is symmetrically dispersed (see, e.g., Bacchetta and van Wincoop (2004, 2006)). In this case, higher order expectations play an important role. Finally, there are alternative ways to model advance information. For example, our model specifies advance information about the transitory component of dividends. Allowing advance information to be informative about the persistent component of earnings will not change our key insights. In addition, we assume that advance information is private to informed agents only. It would be interesting to consider the case where advance information is public to all agents in the economy.

Appendices

A Proofs for the benchmark model

We first prove Proposition 2 since we need to solve the agents' forecasting problems before we derive an equilibrium.

Proof of Proposition 2: An uninformed agent's forecasting problem is the classical Kalman filtering problem. To solve this problem, we use the state-space system representation. Let

$$\mathbf{x}_t = \mathbf{A}_x \mathbf{x}_{t-1} + \mathbf{B}_x \boldsymbol{\varepsilon}_t. \quad (\text{A.1})$$

where

$$\mathbf{A}_x = \begin{bmatrix} a_F & 0 \\ 0 & a_Z \end{bmatrix}, \quad \mathbf{B}_x = \begin{bmatrix} 0 & 1 & 0 & 0 \\ 0 & 0 & 1 & 0 \end{bmatrix}. \quad (\text{A.2})$$

The uninformed agent has signals $\mathbf{y}_t = (D_t, \Pi_t)^\top$, which satisfies

$$\mathbf{y}_t = \mathbf{A}_y \mathbf{x}_t + \mathbf{B}_y \boldsymbol{\varepsilon}_t, \quad (\text{A.3})$$

where

$$\mathbf{A}_y = \begin{bmatrix} 1 & 0 \\ p_{i1} & p_{i2} \end{bmatrix}, \quad \mathbf{B}_y = \begin{bmatrix} 1 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 \end{bmatrix}. \quad (\text{A.4})$$

Define

$$\boldsymbol{\Sigma}_{xx} = \mathbf{B}_x \boldsymbol{\Sigma} \mathbf{B}_x^\top = \begin{bmatrix} \sigma_F^2 & 0 \\ 0 & \sigma_Z^2 \end{bmatrix}, \quad \boldsymbol{\Sigma}_{yy} = \mathbf{B}_y \boldsymbol{\Sigma} \mathbf{B}_y^\top = \begin{bmatrix} \sigma_D^2 & 0 \\ 0 & 0 \end{bmatrix} \quad (\text{A.5})$$

and $\boldsymbol{\Omega}_t = E_t^u [(\mathbf{x}_t - \hat{\mathbf{x}}_t^u)(\mathbf{x}_t - \hat{\mathbf{x}}_t^u)^\top]$. Then by Kalman filtering

$$\hat{\mathbf{x}}_{t+1}^u = \mathbf{A}_x \hat{\mathbf{x}}_t^u + \mathbf{K}_t [\mathbf{y}_{t+1} - E_t^u [\mathbf{y}_{t+1}]] \quad (\text{A.6})$$

$$\boldsymbol{\Omega}_t = (\mathbf{A}_x \boldsymbol{\Omega}_{t-1} \mathbf{A}_x^\top + \boldsymbol{\Sigma}_{xx}) - \mathbf{K}_t \mathbf{A}_y (\mathbf{A}_x \boldsymbol{\Omega}_{t-1} \mathbf{A}_x^\top + \boldsymbol{\Sigma}_{xx}) \quad (\text{A.7})$$

where

$$\mathbf{K}_t = (\mathbf{A}_x \boldsymbol{\Omega}_{t-1} \mathbf{A}_x^\top + \boldsymbol{\Sigma}_{xx}) \mathbf{A}_y^\top [\mathbf{A}_y (\mathbf{A}_x \boldsymbol{\Omega}_{t-1} \mathbf{A}_x^\top + \boldsymbol{\Sigma}_{xx}) \mathbf{A}_y^\top + \boldsymbol{\Sigma}_{yy}]^{-1} \quad (\text{A.8})$$

As in Wang (1994), we focus on the steady-state Kalman filtering. Let $\boldsymbol{\Omega}$ be the solution to the Riccati equation

$$\boldsymbol{\Omega} = (\mathbf{A}_x \boldsymbol{\Omega} \mathbf{A}_x^\top + \boldsymbol{\Sigma}_{xx}) - \mathbf{K} \mathbf{A}_y (\mathbf{A}_x \boldsymbol{\Omega} \mathbf{A}_x^\top + \boldsymbol{\Sigma}_{xx}), \quad (\text{A.9})$$

where

$$\mathbf{K} = (\mathbf{A}_x \boldsymbol{\Omega} \mathbf{A}_x^\top + \boldsymbol{\Sigma}_{xx}) \mathbf{A}_y^\top [\mathbf{A}_y (\mathbf{A}_x \boldsymbol{\Omega} \mathbf{A}_x^\top + \boldsymbol{\Sigma}_{xx}) \mathbf{A}_y^\top + \boldsymbol{\Sigma}_{yy}]^{-1}. \quad (\text{A.10})$$

We then obtain the following the steady-state filters:

$$\hat{\mathbf{x}}_t^u = \mathbf{A}_x \hat{\mathbf{x}}_{t-1}^u + \mathbf{K} \hat{\varepsilon}_t^u, \quad (\text{A.11})$$

and

$$\mathbf{y}_t = \mathbf{A}_y \mathbf{A}_x \hat{\mathbf{x}}_{t-1}^u + \hat{\varepsilon}_t^u. \quad (\text{A.12})$$

where $\hat{\varepsilon}_t^u = \mathbf{y}_t - E_{t-1}^u[\mathbf{y}_t]$ is the innovation, which is normally distributed with mean of zero and covariance matrix

$$\begin{aligned} E[\hat{\varepsilon}_t^u \hat{\varepsilon}_t^{u\top}] &= E[(\mathbf{y}_t - \mathbf{A}_y \mathbf{A}_x \hat{\mathbf{x}}_{t-1}^u)(\mathbf{y}_t - \mathbf{A}_y \mathbf{A}_x \hat{\mathbf{x}}_{t-1}^u)^\top] \\ &= E[(\mathbf{A}_y \mathbf{x}_t + \mathbf{B}_y \varepsilon_t - \mathbf{A}_y \mathbf{A}_x \hat{\mathbf{x}}_{t-1}^u)(\mathbf{A}_y \mathbf{x}_t + \mathbf{B}_y \varepsilon_t - \mathbf{A}_y \mathbf{A}_x \hat{\mathbf{x}}_{t-1}^u)^\top] \\ &= E[(\mathbf{A}_y \mathbf{A}_x (\mathbf{x}_{t-1} - \hat{\mathbf{x}}_{t-1}^u) + (\mathbf{A}_y \mathbf{B}_x + \mathbf{B}_y) \varepsilon_t) \\ &\quad ((\mathbf{x}_{t-1} - \hat{\mathbf{x}}_{t-1}^u)^\top \mathbf{A}_x^\top \mathbf{A}_y^\top + \varepsilon_t^\top (\mathbf{A}_y \mathbf{B}_x + \mathbf{B}_y)^\top)] \\ &= \mathbf{A}_y \mathbf{A}_x \mathbf{\Omega} \mathbf{A}_x^\top \mathbf{A}_y^\top + (\mathbf{A}_y \mathbf{B}_x + \mathbf{B}_y) \mathbf{\Sigma} (\mathbf{A}_y \mathbf{B}_x + \mathbf{B}_y)^\top, \end{aligned}$$

because by construction

$$E[(\mathbf{x}_{t-1} - \hat{\mathbf{x}}_{t-1}^u) \varepsilon_t^\top] = E[E_{t-1}((\mathbf{x}_{t-1} - \hat{\mathbf{x}}_{t-1}^u) \varepsilon_t^\top)] = 0.$$

Thus, we have

$$\text{Var}(\hat{\varepsilon}_t^u) = \mathbf{A}_y \mathbf{A}_x \mathbf{\Omega} \mathbf{A}_x^\top \mathbf{A}_y^\top + (\mathbf{A}_y \mathbf{B}_x + \mathbf{B}_y) \mathbf{\Sigma} (\mathbf{A}_y \mathbf{B}_x + \mathbf{B}_y)^\top. \quad (\text{A.13})$$

Post-multiplying both sides of (A.9) by $\mathbf{A}_y^\top$ and subtracting $\mathbf{K} \mathbf{\Sigma}_{yy}$ from both sides yields

$$\mathbf{\Omega} \mathbf{A}_y^\top - \mathbf{K} \mathbf{\Sigma}_{yy} = (\mathbf{A}_x \mathbf{\Omega} \mathbf{A}_x^\top + \mathbf{\Sigma}_{xx}) \mathbf{A}_y^\top - \mathbf{K} [\mathbf{A}_y (\mathbf{A}_x \mathbf{\Omega} \mathbf{A}_x^\top + \mathbf{\Sigma}_{xx}) \mathbf{A}_y^\top + \mathbf{\Sigma}_{yy}] = 0.$$

This equality can be written as

$$\begin{bmatrix} \omega_{11} & \omega_{12} \\ \omega_{21} & \omega_{22} \end{bmatrix} \begin{bmatrix} 1 & p_{i1} \\ 0 & p_{i2} \end{bmatrix} = \begin{bmatrix} k_{11} & k_{12} \\ k_{21} & k_{22} \end{bmatrix} \begin{bmatrix} \sigma_D^2 & 0 \\ 0 & 0 \end{bmatrix},$$

where $\mathbf{\Omega} = (\omega_{ij})$ with $\omega_{12} = \omega_{21}$. Therefore, we get the following 4 equations

$$\begin{bmatrix} \omega_{11} & \omega_{11} p_{i1} + \omega_{12} p_{i2} \\ \omega_{21} & \omega_{21} p_{i1} + \omega_{22} p_{i2} \end{bmatrix} = \sigma_D^2 \begin{bmatrix} k_{11} & 0 \\ k_{21} & 0 \end{bmatrix}.$$

Solving yields

$$k_{11} = \omega_{11}/\sigma_D^2 > 0, \quad k_{21} = \omega_{21}/\sigma_D^2 > 0, \quad (\text{A.14})$$

$$\omega_{12} = -p_{i1}/p_{i2} \omega_{11} > 0, \quad \omega_{21} = -p_{i2}/p_{i1} \omega_{22} > 0, \quad (\text{A.15})$$

where we have used the sign restrictions in Proposition 1.

Now post-multiply both sides of (A.9) by $(\mathbf{A}_x \mathbf{\Omega} \mathbf{A}_x^\top + \mathbf{\Sigma}_{xx})^{-1}$ to get

$$\mathbf{\Omega} (\mathbf{A}_x \mathbf{\Omega} \mathbf{A}_x^\top + \mathbf{\Sigma}_{xx})^{-1} = \mathbf{I} - \mathbf{K} \mathbf{A}_y$$

or

$$\begin{aligned} & \omega_{11} \begin{bmatrix} 1 & -\frac{p_{i1}}{p_{i2}} \\ -\frac{p_{i1}}{p_{i2}} & \left(\frac{p_{i1}}{p_{i2}}\right)^2 \end{bmatrix} \begin{bmatrix} a_F^2 \omega_{11} + \sigma_F^2 & -a_Z a_F \frac{p_{i1}}{p_{i2}} \omega_{11} \\ -a_F a_Z \frac{p_{i1}}{p_{i2}} \omega_{11} & a_Z^2 \left(\frac{p_{i1}}{p_{i2}}\right)^2 \omega_{11} + \sigma_Z^2 \end{bmatrix}^{-1} \\ &= \begin{bmatrix} 1 - k_{11} - k_{12} p_{i1} & -k_{12} p_{i2} \\ -k_{21} - k_{22} p_{i1} & 1 - k_{22} p_{i2} \end{bmatrix} \end{aligned}$$

Simplify this equation to obtain:

$$\begin{aligned} & \frac{\omega_{11}}{\Delta} \begin{bmatrix} a_Z (a_Z - a_F) \left(\frac{p_{i1}}{p_{i2}}\right)^2 \omega_{11} + \sigma_Z^2 & \frac{p_{i1}}{p_{i2}} [a_F (a_Z - a_F) \omega_{11} - \sigma_F^2] \\ -\frac{p_{i1}}{p_{i2}} [a_Z (a_Z - a_F) \left(\frac{p_{i1}}{p_{i2}}\right)^2 \omega_{11} + \sigma_Z^2] & -\left(\frac{p_{i1}}{p_{i2}}\right)^2 [a_F (a_Z - a_F) \omega_{11} - \sigma_F^2] \end{bmatrix} \\ &= \begin{bmatrix} 1 - k_{11} - k_{12} p_{i1} & -k_{12} p_{i2} \\ -k_{21} - k_{22} p_{i1} & 1 - k_{22} p_{i2} \end{bmatrix}, \end{aligned} \quad (\text{A.16})$$

with

$$\begin{aligned} \Delta &= (a_F^2 \omega_{11} + \sigma_F^2) \left(a_Z^2 \left(\frac{p_{i1}}{p_{i2}}\right)^2 \omega_{11} + \sigma_Z^2 \right) - \left(a_F a_Z \frac{p_{i1}}{p_{i2}} \omega_{11} \right)^2 \\ &= \sigma_F^2 a_Z^2 \left(\frac{p_{i1}}{p_{i2}}\right)^2 \omega_{11} + a_F^2 \sigma_Z^2 \omega_{11} + \sigma_F^2 \sigma_Z^2 > 0 \end{aligned} \quad (\text{A.17})$$

Now using the top right hand corner equation:

$$\frac{\omega_{11}}{\Delta} \frac{p_{i1}}{p_{i2}^2} [\sigma_F^2 - (1 - a_F^2) \omega_{11} + (1 - a_F a_Z) \omega_{11}] = k_{12}, \quad (\text{A.18})$$

with $\sigma_F^2 - (1 - a_F^2) \omega_{11} > 0$ and $a_F, a_Z \in (0, 1)$, we obtain $k_{12} > 0$. Similarly, we have

$$-\frac{\omega_{11}}{\Delta} \frac{p_{i1}}{p_{i2}} \left[a_Z (a_Z - a_F) \left(\frac{p_{i1}}{p_{i2}}\right)^2 \omega_{11} + \sigma_Z^2 \right] = -k_{21} - k_{22} p_{i1}. \quad (\text{A.19})$$

Since

$$a_Z (a_Z - a_F) \left(\frac{p_{i1}}{p_{i2}}\right)^2 \omega_{11} + \sigma_Z^2 = \sigma_Z^2 - (1 - a_Z^2) \omega_{22} + (1 - a_F a_Z) \omega_{22} > 0,$$

we obtain

$$-p_{i1} k_{22} = k_{21} - \frac{\omega_{11}}{\Delta} \frac{p_{i1}}{p_{i2}} \left[a_Z (a_Z - a_F) \left(\frac{p_{i1}}{p_{i2}}\right)^2 \omega_{11} + \sigma_Z^2 \right] > 0 \quad (\text{A.20})$$

implying $k_{22} < 0$. Q.E.D.

Proof of Proposition 1: We will use the market-clearing condition (14) to verify that the equilibrium price function takes the form in (15) and that the coefficients satisfy the sign restrictions. We start with the agents' optimal portfolios. We first derive the conditional expectations, variances, and covariance of the excess returns. We use the conjectured pricing function (15) and Kalman filters (A.11)-(A.12) to rewrite the excess return as

$$\begin{aligned}
Q_{t+1} &= P_{t+1} + D_{t+1} - RP_t \\
&= -p_0 + \mathbf{p}_i \mathbf{x}_{t+1} + p_{u1} \mathbf{c}_1 \hat{\mathbf{x}}_{t+1}^u + F_{t+1} + \varepsilon_{t+1}^D - R(-p_0 + \mathbf{p}_i \mathbf{x}_t + p_{u1} \mathbf{c}_1 \hat{\mathbf{x}}_t^u) \\
&= rp_0 + ((\mathbf{p}_i + \mathbf{c}_1) \mathbf{A}_x - R\mathbf{p}_i) \mathbf{x}_t + p_{u1} \mathbf{c}_1 (\mathbf{A}_x - R\mathbf{I}) \hat{\mathbf{x}}_t^u \\
&\quad + ((\mathbf{p}_i + \mathbf{c}_1) \mathbf{B}_x + \mathbf{c}_1) \varepsilon_{t+1} + p_{u1} \mathbf{c}_1 \mathbf{K} (\mathbf{y}_{t+1} - E_t^u [\mathbf{y}_{t+1}]) \\
&= e_0 + \mathbf{e}_i \mathbf{x}_t + \mathbf{e}_u \hat{\mathbf{x}}_t^u + \mathbf{b}_Q \varepsilon_{t+1}.
\end{aligned} \tag{A.21}$$

Here the coefficients are defined as

$$e_0 = rp_0, \tag{A.22}$$

$$\mathbf{e}_i = (\mathbf{p}_i + \mathbf{c}_1) \mathbf{A}_x - R\mathbf{p}_i + p_{u1} \mathbf{c}_1 \mathbf{K} \mathbf{A}_y \mathbf{A}_x \begin{bmatrix} 1 \\ -p_{i1}/p_{i2} \end{bmatrix} \mathbf{c}_1, \tag{A.23}$$

$$\mathbf{e}_u = p_{u1} \mathbf{c}_1 \left[\mathbf{A}_x - R\mathbf{I} - \mathbf{K} \mathbf{A}_y \mathbf{A}_x \begin{bmatrix} 1 \\ -p_{i1}/p_{i2} \end{bmatrix} \mathbf{c}_1 \right], \tag{A.24}$$

$$\mathbf{b}_Q = (\mathbf{p}_i + \mathbf{c}_1) \mathbf{B}_x + \mathbf{c}_1 + p_{u1} \mathbf{c}_1 \mathbf{K} (\mathbf{A}_y \mathbf{B}_x + \mathbf{B}_y), \tag{A.25}$$

where $\mathbf{c}_j$ is the standard row vector with the j^{th} element being 1 and the rest being zero. Note that in deriving equation (A.21), we have used the following:

$$\begin{aligned}
\hat{\varepsilon}_{t+1}^u &= \mathbf{y}_{t+1} - E_t^u [\mathbf{y}_{t+1}] \\
&= \mathbf{A}_y \mathbf{x}_{t+1} + \mathbf{B}_y \varepsilon_{t+1} - E_t^u [\mathbf{A}_y \mathbf{x}_{t+1} + \mathbf{B}_y \varepsilon_{t+1}] \\
&= \mathbf{A}_y \mathbf{A}_x (\mathbf{x}_t - \hat{\mathbf{x}}_t^u) + (\mathbf{A}_y \mathbf{B}_x + \mathbf{B}_y) \varepsilon_{t+1} \\
&= \mathbf{A}_y \mathbf{A}_x \begin{bmatrix} 1 \\ -p_{i1}/p_{i2} \end{bmatrix} (F_t - \hat{F}_t^u) + (\mathbf{A}_y \mathbf{B}_x + \mathbf{B}_y) \varepsilon_{t+1} \\
&= \mathbf{A}_y \mathbf{A}_x \begin{bmatrix} 1 \\ -p_{i1}/p_{i2} \end{bmatrix} \mathbf{c}_1 \mathbf{x}_t - \mathbf{A}_y \mathbf{A}_x \begin{bmatrix} 1 \\ -p_{i1}/p_{i2} \end{bmatrix} \mathbf{c}_1 \hat{\mathbf{x}}_t^u + (\mathbf{A}_y \mathbf{B}_x + \mathbf{B}_y) \varepsilon_{t+1}
\end{aligned} \tag{A.26}$$

where the next-to-last step uses (17) again. Note that from (A.24) it must true that $e_{u2} = 0$.

We next use (A.21) to derive the conditional variances and covariance of excess returns:

$$(\sigma_Q^i)^2 = Var_t^i(Q_{t+1}) = \mathbf{b}_Q \Sigma \mathbf{b}_Q^T, \tag{A.27}$$

$$(\sigma_q^i)^2 = Var_t^i(q_{t+1}) = \sigma_q^2, \tag{A.28}$$

$$Cov_t^i(Q_{t+1}, q_{t+1}) = \mathbf{b}_Q E_t^i [\boldsymbol{\varepsilon}_{t+1} \boldsymbol{\varepsilon}_{t+1}^\top] \mathbf{c}_4^\top = \mathbf{b}_Q \Sigma \mathbf{c}_4^\top = (1 + p_{u1} k_{11}) \sigma_{Dq} \quad (\text{A.29})$$

which is positive by (A.14) and the guess that $p_{u1} > 0$. In addition,

$$\begin{aligned} (\sigma_Q^u)^2 &= Var_t^u(Q_{t+1}) = E_t^u [(\mathbf{e}_i(\mathbf{x}_t - \hat{\mathbf{x}}_t^u) + \mathbf{b}_Q \boldsymbol{\varepsilon}_{t+1})(\mathbf{e}_i(\mathbf{x}_t - \hat{\mathbf{x}}_t^u) + \mathbf{b}_Q \boldsymbol{\varepsilon}_{t+1})^\top] \quad (\text{A.30}) \\ &= E_t^u [\mathbf{e}_i(\mathbf{x}_t - \hat{\mathbf{x}}_t^u)(\mathbf{x}_t - \hat{\mathbf{x}}_t^u)^\top \mathbf{e}_i^\top + \mathbf{e}_i(\mathbf{x}_t - \hat{\mathbf{x}}_t^u) \boldsymbol{\varepsilon}_{t+1}^\top \mathbf{b}_Q^\top \\ &\quad + \mathbf{b}_Q \boldsymbol{\varepsilon}_{t+1}(\mathbf{x}_t - \hat{\mathbf{x}}_t^u)^\top \mathbf{e}_i^\top + \mathbf{b}_Q \boldsymbol{\varepsilon}_{t+1} \boldsymbol{\varepsilon}_{t+1}^\top \mathbf{b}_Q^\top] \\ &= \mathbf{e}_i \Omega \mathbf{e}_i^\top + \mathbf{b}_Q \Sigma \mathbf{b}_Q^\top, \end{aligned}$$

where we have used $E[(\mathbf{x}_t - \hat{\mathbf{x}}_t^u) \boldsymbol{\varepsilon}_{t+1}^\top] = 0$ to derive that last equality.

We now use equation (A.21) to derive the informed and uninformed agents' conditional expectations of excess returns:

$$E_t^i[Q_{t+1}] = e_0 + \mathbf{e}_i \mathbf{x}_t + \mathbf{e}_u \hat{\mathbf{x}}_t^u, \quad (\text{A.31})$$

$$E_t^u[Q_{t+1}] = e_0 + (\mathbf{e}_i + \mathbf{e}_u) \hat{\mathbf{x}}_t^u, \quad (\text{A.32})$$

and

$$E_t^i[q_{t+1}] = Z_t = \mathbf{c}_2 \mathbf{x}_t. \quad (\text{A.33})$$

We rewrite (17) as

$$\hat{Z}_t^u = \frac{1}{p_{i2}} \mathbf{p}_i \mathbf{x}_t - \frac{p_{i1}}{p_{i2}} \hat{F}_t^u.$$

Substituting this equation into (A.31)-(A.32) yields

$$E_t^i[Q_{t+1}] = e_0 + \mathbf{e}_i \mathbf{x}_t + e_{u1} \hat{F}_t^u \quad (\text{A.34})$$

$$\begin{aligned} E_t^u[Q_{t+1}] &= e_0 + e_{i1} \hat{F}_t^u + e_{i2} \hat{Z}_t^u + e_{u1} \hat{F}_t^u \quad (\text{A.35}) \\ &= e_0 + e_{i1} \hat{F}_t^u + e_{i2} \left[\frac{1}{p_{i2}} \mathbf{p}_i \mathbf{x}_t - \frac{p_{i1}}{p_{i2}} \hat{F}_t^u \right] + e_{u1} \hat{F}_t^u \\ &= e_0 + \frac{e_{i2}}{p_{i2}} \mathbf{p}_i \mathbf{x}_t + \left(e_{i1} - e_{i2} \frac{p_{i1}}{p_{i2}} + e_{u1} \right) \hat{F}_t^u. \end{aligned}$$

Substituting the preceding equations into (24) and (25), we obtain

$$\theta_t^i = \frac{e_0 + \mathbf{e}_i \mathbf{x}_t + e_{u1} \hat{F}_t^u}{\gamma (\sigma_Q^i)^2 \left(1 - (\rho_{Qq}^i)^2 \right)} - \frac{\rho_{Qq}^i \mathbf{c}_2 \mathbf{x}_t}{\gamma \sigma_Q^i \sigma_q^i \left(1 - (\rho_{Qq}^i)^2 \right)}, \quad (\text{A.36})$$

$$\theta_t^u = \frac{e_0 + \frac{e_{i2}}{p_{i2}} \mathbf{p}_i \mathbf{x}_t + \left(e_{i1} - e_{i2} \frac{p_{i1}}{p_{i2}} + e_{u1} \right) \hat{F}_t^u}{\gamma (\sigma_Q^u)^2}. \quad (\text{A.37})$$

We now use the market-clearing condition (14) to determine the coefficients p_0 , $\mathbf{p}_i$ and $\mathbf{p}_u$. Using this condition and equations (A.36)-(A.37), we have

$$1 = \frac{\lambda}{\gamma} \left[\frac{e_0 + \mathbf{e}_i \mathbf{x}_t + e_{u1} \hat{F}_t^u}{\left(\sigma_Q^i\right)^2 \left(1 - \left(\rho_{Qq}^i\right)^2\right)} - \frac{\rho_{Qq}^i \mathbf{c}_2 \mathbf{x}_t}{\sigma_Q^i \sigma_q^i \left(1 - \left(\rho_{Qq}^i\right)^2\right)} \right] + (1 - \lambda) \frac{e_0 + \frac{e_{i2}}{p_{i2}} \mathbf{p}_i \mathbf{x}_t + \left(e_{i1} - e_{i2} \frac{p_{i1}}{p_{i2}} + e_{u1}\right) \hat{F}_t^u}{\gamma \left(\sigma_Q^u\right)^2}.$$

Matching coefficients yields the following four equilibrium restrictions:

$$e_0 = \gamma \frac{\left(\sigma_Q^i\right)^2 \left(1 - \left(\rho_{Qq}^i\right)^2\right) \left(\sigma_Q^u\right)^2}{\lambda \left(\sigma_Q^u\right)^2 + (1 - \lambda) \left(\sigma_Q^i\right)^2 \left(1 - \left(\rho_{Qq}^i\right)^2\right)}, \quad (\text{A.38})$$

$$0 = \lambda \frac{e_{i1}}{\left(\sigma_Q^i\right)^2 \left(1 - \left(\rho_{Qq}^i\right)^2\right)} + (1 - \lambda) \frac{e_{i2} \frac{p_{i1}}{p_{i2}}}{\left(\sigma_Q^u\right)^2}, \quad (\text{A.39})$$

$$0 = \lambda \left[\frac{e_{i2}}{\left(\sigma_Q^i\right)^2 \left(1 - \left(\rho_{Qq}^i\right)^2\right)} - \frac{\rho_{Qq}^i}{\sigma_Q^i \sigma_q^i \left(1 - \left(\rho_{Qq}^i\right)^2\right)} \right] + (1 - \lambda) \frac{e_{i2}}{\left(\sigma_Q^u\right)^2}, \quad (\text{A.40})$$

and

$$0 = \lambda \frac{e_{u1}}{\left(\sigma_Q^i\right)^2 \left(1 - \left(\rho_{Qq}^i\right)^2\right)} + (1 - \lambda) \frac{e_{i1} - e_{i2} \frac{p_{i1}}{p_{i2}} + e_{u1}}{\left(\sigma_Q^u\right)^2}. \quad (\text{A.41})$$

Equation (A.40) gives the solution for e_{i2} :

$$e_{i2} = \frac{\lambda \left(\sigma_Q^u\right)^2}{\lambda \left(\sigma_Q^u\right)^2 + (1 - \lambda) \left(\sigma_Q^i\right)^2 \left(1 - \left(\rho_{Qq}^i\right)^2\right)} \frac{\text{Cov}_t^i(Q_{t+1}, q_{t+1})}{\left(\sigma_q^i\right)^2}. \quad (\text{A.42})$$

Equation (A.39) gives the solution for e_{i1} :

$$e_{i1} = -\frac{p_{i1}}{p_{i2}} \frac{(1 - \lambda) \left(\sigma_Q^i\right)^2 \left(1 - \left(\rho_{Qq}^i\right)^2\right)}{\lambda \left(\sigma_Q^u\right)^2 + (1 - \lambda) \left(\sigma_Q^i\right)^2 \left(1 - \left(\rho_{Qq}^i\right)^2\right)} \frac{\text{Cov}_t^i(Q_{t+1}, q_{t+1})}{\left(\sigma_q^i\right)^2}. \quad (\text{A.43})$$

Equation (A.41) gives the solution for e_{u1} :

$$e_{u1} = \frac{p_{i1}}{p_{i2}} \frac{(1-\lambda) \left(\sigma_Q^i\right)^2 \left(1 - \left(\rho_{Qq}^i\right)^2\right)}{\lambda \left(\sigma_Q^u\right)^2 + (1-\lambda) \left(\sigma_Q^i\right)^2 \left(1 - \left(\rho_{Qq}^i\right)^2\right)} \frac{Cov_t^i(Q_{t+1}, q_{t+1})}{\left(\sigma_q^i\right)^2}. \quad (\text{A.44})$$

Having solved for e_{i1} , e_{i2} and e_{u1} as a function of the price coefficients we can use (A.22)-(A.24) to determine these coefficients. First, equation (A.22) implies

$$p_0 = r^{-1} e_0 > 0,$$

verifying our conjectured sign in Proposition 1. The rest gives us a system of 3 equations in 3 unknowns p_{i1} , p_{i2} , and p_{u1} . As noted before, we have $e_{u2} = 0$ since we have substituted our $\hat{Z}_t^u$ in (A.21). We can verify that equation (A.24) implies that this is always true. To explicitly derive those 3 equations, we first use (A.24) to derive

$$e_{u1} = p_{u1} [a_F (1 - k_{11} - p_{i1} k_{12}) - R + a_Z p_{i1} k_{12}]. \quad (\text{A.45})$$

We then add equations (A.23) and (A.24) to derive

$$\mathbf{e}_i + \mathbf{e}_u = (\mathbf{p}_i + \mathbf{c}_1) \mathbf{A}_x - R \mathbf{p}_i + p_{u1} \mathbf{c}_1 (\mathbf{A}_x - R \mathbf{I}).$$

That is,

$$e_{i1} + e_{u1} = -(p_{i1} + p_{u1}) (R - a_F) + a_F, \quad (\text{A.46})$$

$$e_{i2} = -p_{i2} (R - a_Z). \quad (\text{A.47})$$

Note that the system of three equations (A.45)-(A.47) along with (A.42)-(A.44) do not admit an analytical solution for p_{i1} , p_{i2} and p_{u1} . An equilibrium exists if this system has a solution. A simple iterative numerical procedure can be used to solve this system. We should emphasize that when solving this system we must substitute the equations for conditional variances and covariance (A.27)-(A.37) and the equations for the Kalman gain matrix (A.9)-(A.10).

Even though we cannot solve the equilibrium explicitly, we can derive restrictions on the coefficients in the price function. First, adding equations (A.43) and (A.44) yields $e_{i1} + e_{u1} = 0$. Equation (A.46) then implies equation (19) in Proposition 1. We then take the ratio of (A.44) and (A.42), and substitute for e_{i2} using (A.47) to derive

$$p_{i1} = -(R - a_Z) e_{u1} \frac{(1-\lambda) \left(\sigma_Q^i\right)^2 \left(1 - \left(\rho_{Qq}^i\right)^2\right)}{\lambda \left(\sigma_Q^u\right)^2}. \quad (\text{A.48})$$

We conclude that $p_{i1} > 0$. Suppose to the contrary $p_{i1} < 0$. Then equation (A.48) implies that $e_{u1} > 0$. We next use (A.45) to show that $p_{u1} < 0$. To this end, we use the two equations implied in the top row of (A.16) to substitute for $1 - k_{11} - p_{i1}k_{12}$ and $p_{i1}k_{12}$, respectively. We then obtain

$$\begin{aligned}
& a_F(1 - k_{11} - p_{i1}k_{12}) - R + a_Z p_{i1}k_{12} \\
= & \frac{\omega_{11}}{\Delta} a_F \left[a_Z(a_Z - a_F) \left(\frac{p_{i1}}{p_{i2}} \right)^2 \omega_{11} + \sigma_Z^2 \right] - R \\
& - \frac{\omega_{11}}{\Delta} a_Z \left(\frac{p_{i1}}{p_{i2}} \right)^2 [a_F(a_Z - a_F) \omega_{11} - \sigma_F^2] \\
= & \frac{\omega_{11}}{\Delta} \left(a_F \sigma_Z^2 + a_Z \left(\frac{p_{i1}}{p_{i2}} \right)^2 \sigma_F^2 \right) - R. \\
= & \frac{a_Z \left(\frac{p_{i1}}{p_{i2}} \right)^2 \sigma_F^2 \omega_{11} + a_F \sigma_Z^2 \omega_{11}}{a_Z^2 \left(\frac{p_{i1}}{p_{i2}} \right)^2 \sigma_F^2 \omega_{11} + a_F^2 \sigma_Z^2 \omega_{11} + \sigma_F^2 \sigma_Z^2} - R,
\end{aligned}$$

We now show this expression is negative and thus deduce $p_{u1} < 0$. It suffices to show that

$$\frac{\omega_{11}}{\Delta} \left(a_F \sigma_Z^2 + a_Z \left(\frac{p_{i1}}{p_{i2}} \right)^2 \sigma_F^2 \right) = \frac{a_Z \left(\frac{p_{i1}}{p_{i2}} \right)^2 \sigma_F^2 \omega_{11} + a_F \sigma_Z^2 \omega_{11}}{a_Z^2 \left(\frac{p_{i1}}{p_{i2}} \right)^2 \sigma_F^2 \omega_{11} + a_F^2 \sigma_Z^2 \omega_{11} + \sigma_F^2 \sigma_Z^2} < 1,$$

where we have substituted equation (A.17). This inequality is equivalent to

$$a_Z(1 - a_Z) \left(\frac{p_{i1}}{p_{i2}} \right)^2 \sigma_F^2 \omega_{11} + (1 - a_F) a_F \sigma_Z^2 \omega_{11} < \sigma_F^2 \sigma_Z^2.$$

This inequality is true since we can show that

$$\begin{aligned}
& a_Z(1 - a_Z) \left(\frac{p_{i1}}{p_{i2}} \right)^2 \sigma_F^2 \omega_{11} + (1 - a_F) a_F \sigma_Z^2 \omega_{11} \\
= & a_Z(1 - a_Z) \sigma_F^2 \omega_{22} + (1 - a_F) a_F \sigma_Z^2 \omega_{11} \\
< & a_Z(1 - a_Z) \frac{\sigma_F^2 \sigma_Z^2}{(1 - a_Z^2)} + (1 - a_F) a_F \frac{\sigma_F^2 \sigma_Z^2}{(1 - a_F^2)} \\
= & \frac{a_Z}{1 + a_Z} \sigma_F^2 \sigma_Z^2 + \frac{a_F}{1 + a_Z} \sigma_F^2 \sigma_Z^2 \\
< & \sigma_F^2 \sigma_Z^2,
\end{aligned}$$

where we have used (A.15) and the definition of Ω to derive

$$\begin{aligned}
\left(\frac{p_{i1}}{p_{i2}} \right)^2 \omega_{11} &= \omega_{22} = \text{Var} \left(Z_t - \hat{Z}_t \right) = \frac{\sigma_Z^2}{1 - a_Z^2} - \text{Var} \left(\hat{Z}_t \right) \\
\omega_{11} &= \text{Var} \left(F_t - \hat{F}_t \right) = \frac{\sigma_F^2}{1 - a_F^2} - \text{Var} \left(\hat{F}_t \right),
\end{aligned}$$

and we note

$$a_Z, a_F \in (0, 1) \text{ and } \frac{a_Z}{1 + a_Z} + \frac{a_F}{1 + a_Z} < 1.$$

Therefore, equation (A.45) and $e_{u1} > 0$ imply that $p_{u1} < 0$, which contradicts with (19), because equation (19) implies that $p_{i1} + p_{u1}$ must be positive.

Thus, we must have $p_{i1} > 0$. We then use (A.48) to deduce that $e_{u1} < 0$. Since we have shown that the expression in the square bracket in equation (A.45) is negative, we conclude that $p_{u1} > 0$. It follows from (A.29) that $Cov_t^i(Q_{t+1}, q_{t+1}) > 0$. Use this result and equation (A.42) to obtain $e_{i2} > 0$. By this result, equation (A.47), and $a_Z < R$, we obtain $p_{i2} < 0$. Q.E.D.

Proof of Proposition 3: We only need to consider conditional expectations of future returns in (24) and (25) since their coefficients are constant. Equation (A.33) gives $E_t^i[q_{t+1}] = Z_t$. Since $e_{i1} + e_{u1} = 0$, we can rewrite equations (A.34) and (A.35) as

$$E_t^i[Q_{t+1}] = e_0 + e_{i2}Z_t + e_{i1}(F_t - \hat{F}_t^u), \quad (\text{A.49})$$

$$E_t^u[Q_{t+1}] = e_0 + e_{i2}\hat{Z}_t^u. \quad (\text{A.50})$$

Plugging the preceding conditional expectations in equations (24) and (25) yields the desired result. Q.E.D.

Proof of Proposition 4: We use equation (A.50) to compute:

$$E[Q_{t+1}|Q_t] = E[E_t^u[Q_{t+1}]|Q_t] = e_0 + e_{i2}E[E_t^u[Z_t]|Q_t] \quad (\text{A.51})$$

$$= e_0 + e_{i2}E[Z_t|Q_t] = e_0 + e_{i2}\frac{Cov(Z_t, Q_t)}{Var(Q_t)}Q_t, \quad (\text{A.52})$$

where e_0 and $e_{i2} > 0$ are constants as shown in the proof of Proposition 1.

Now we use (15) to compute the unconditional covariance $Cov(Z_t, Q_t)$:

$$\begin{aligned} Cov(Z_t, Q_t) &= E[Z_t(p_{i1}F_t + p_{u1}\hat{F}_t^u + p_{i2}Z_t + F_t + \varepsilon_t^D \\ &\quad - R(p_{i1}F_{t-1} + p_{u1}\hat{F}_{t-1}^u + p_{i2}Z_{t-1}))] \\ &= E[p_{i2}Z_t^2 - p_{i2}RZ_tZ_{t-1} + p_{u1}Z_t\hat{F}_t^u - p_{u1}a_ZRZ_{t-1}\hat{F}_{t-1}^u] \\ &= p_{i2}(1 - Ra_Z)Var(Z_t) + p_{u1}(1 - Ra_Z)E(Z_t\hat{F}_t^u), \end{aligned}$$

where we use the fact that $E[Z_tF_t] = E[Z_tF_{t-1}] = E[Z_t\varepsilon_t^D] = 0$.

Multiplying by Z_t in both sides of (17) and taking expectations, we obtain:

$$E[Z_t\hat{Z}_t^u] = Var(Z_t) - \frac{p_{i1}}{p_{i2}}E[Z_t\hat{F}_t^u].$$

We then derive:

$$\frac{p_{i1}}{p_{i2}} E \left[Z_t \hat{F}_t^u \right] = \text{Var} (Z_t) - \text{Var} \left(\hat{Z}_t^u \right)$$

using the fact that

$$E \left(Z_t \hat{Z}_t^u \right) = E \left[E_t^u \left(Z_t \hat{Z}_t^u \right) \right] = E \left[\hat{Z}_t^u \hat{Z}_t^u \right] = \text{Var} \left(\hat{Z}_t^u \right).$$

Finally we obtain:

$$\text{Cov} (Z_t, Q_t) = (1 - Ra_Z) p_{i2} \left\{ \text{Var} (Z_t) + \frac{p_{u1}}{p_{i1}} \left[\text{Var} (Z_t) - \text{Var} \left(\hat{Z}_t^u \right) \right] \right\}$$

Define

$$f_Q = p_{i2} e_{i2} \frac{\text{Var} (Z_t) + \frac{p_{u1}}{p_{i1}} \left[\text{Var} (Z_t) - \text{Var} \left(\hat{Z}_t^u \right) \right]}{\text{Var} (Q_t)}.$$

To show this expression is negative, we only need to prove $\text{Var} (Z_t) > \text{Var} \left(\hat{Z}_t^u \right)$ because Proposition 1 shows that $p_{i2} < 0$, and $p_{i1}, p_{u1} > 0$. To this end, we use the equation:

$$Z_t = \hat{Z}_t^u + \frac{p_{i1}}{p_{i2}} \left(\hat{F}_t^u - F_t \right).$$

Squaring both sides of this equation and taking expectations, we obtain:

$$\begin{aligned} \text{Var} (Z_t) &= \text{Var} \left(\hat{Z}_t^u \right) + \left(\frac{p_{i1}}{p_{i2}} \right)^2 \text{Var} \left(\hat{F}_t^u - F_t \right) + 2 \frac{p_{i1}}{p_{i2}} E \left[\hat{Z}_t^u \left(\hat{F}_t^u - F_t \right) \right] \\ &= \text{Var} \left(\hat{Z}_t^u \right) + \left(\frac{p_{i1}}{p_{i2}} \right)^2 \text{Var} \left(\hat{F}_t^u - F_t \right) + 2 \frac{p_{i1}}{p_{i2}} E \left[E_t^u \left[\hat{Z}_t^u \left(\hat{F}_t^u - F_t \right) \right] \right] \\ &= \text{Var} \left(\hat{Z}_t^u \right) + \left(\frac{p_{i1}}{p_{i2}} \right)^2 \text{Var} \left(\hat{F}_t^u - F_t \right) + 2 \frac{p_{i1}}{p_{i2}} E \left[\hat{Z}_t^u E_t^u \left[\hat{F}_t^u - F_t \right] \right] \\ &= \text{Var} \left(\hat{Z}_t^u \right) + \left(\frac{p_{i1}}{p_{i2}} \right)^2 \text{Var} \left(\hat{F}_t^u - F_t \right) \\ &> \text{Var} \left(\hat{Z}_t^u \right), \end{aligned}$$

as desired. Finally, for $n > 1$:

$$\begin{aligned} E [Q_{t+n} | Q_t] &= E [E_t^u [Q_{t+n}] | Q_t] = e_0 + e_{i2} E [E_t^u [Z_{t+n-1}] | Q_t] \\ &= e_0 + a_Z^{n-1} e_{i2} E [Z_t | Q_t] = e_0 + a_Z^{n-1} e_{i2} \frac{\text{Cov} (Z_t, Q_t)}{\text{Var} (Q_t)} Q_t. \end{aligned}$$

We can then use the previous analysis to obtain the desired result. Q.E.D.

Proof of Proposition 5: We use a similar argument as in the Proof of Proposition 4 to derive:

$$\begin{aligned}
E[Q_{t+1}|D_t - E_{t-1}^u(D_t)] &= E[E_t^u[Q_{t+1}]|D_t - E_{t-1}^u(D_t)] \\
&= e_0 + e_{i2}E[Z_t|\mathbf{c}_1\hat{\mathbf{e}}_t^u] \\
&= e_0 + e_{i2}\mathbf{c}_2E[\mathbf{x}_t|\mathbf{c}_1\hat{\mathbf{e}}_t^u] \\
&= e_0 + e_{i2}\frac{\mathbf{c}_2Cov(\mathbf{x}_t, \hat{\mathbf{e}}_t^{u\top})\mathbf{c}_1^\top}{\mathbf{c}_1Var(\hat{\mathbf{e}}_t^u)\mathbf{c}_1^\top}[D_t - E_{t-1}^u(D_t)] \\
&= e_0 + d_1[D_t - E_{t-1}^u(D_t)],
\end{aligned}$$

where d_1 is some constant. We next derive:

$$\begin{aligned}
Cov(\mathbf{x}_t, \hat{\mathbf{e}}_t^{u\top}) &= E[E_t^u[\mathbf{x}_t\hat{\mathbf{e}}_t^{u\top}]] = E[E_t^u[\mathbf{x}_t]\hat{\mathbf{e}}_t^{u\top}] = E[\hat{\mathbf{x}}_t\hat{\mathbf{e}}_t^{u\top}] \\
&= E[(\mathbf{A}_x\hat{\mathbf{x}}_{t-1} + \mathbf{K}\hat{\mathbf{e}}_t^u)\hat{\mathbf{e}}_t^{u\top}] = \mathbf{K}Var(\hat{\mathbf{e}}_t^u),
\end{aligned}$$

and

$$\begin{aligned}
Var(\hat{\mathbf{e}}_t) &= \mathbf{A}_y\mathbf{A}_x\mathbf{\Omega}\mathbf{A}_x^\top\mathbf{A}_y^\top + (\mathbf{A}_y\mathbf{B}_x + \mathbf{B}_y)\mathbf{\Sigma}(\mathbf{A}_y\mathbf{B}_x + \mathbf{B}_y)^\top \\
&= \begin{bmatrix} \omega_{11}a_F^2 + \sigma_D^2 + \sigma_F^2 & \sigma_F^2p_{i1} + a_F(a_F - a_Z)p_{i1}\omega_{11} \\ \sigma_F^2p_{i1} + a_F(a_F - a_Z)p_{i1}\omega_{11} & \sigma_F^2p_{i1}^2 + \sigma_Z^2p_{i2}^2 + p_{i1}^2\omega_{11}(a_F - a_Z)^2 \end{bmatrix}.
\end{aligned}$$

Therefore, the sign of d_1 is the sign of $Cov(Z_t, D_t - E_{t-1}^u(D_t))$:

$$k_{21}(\omega_{11}a_F^2 + \sigma_D^2 + \sigma_F^2) + k_{22}p_{i1}(\sigma_F^2 + a_F(a_F - a_Z)\omega_{11}).$$

We use (A.20) to derive that this expression is equal to

$$\begin{aligned}
&k_{21}(\omega_{11}a_F^2 + \sigma_D^2 + \sigma_F^2) + k_{22}p_{i1}(\sigma_F^2 + a_F(a_F - a_Z)\omega_{11}) \\
&= k_{21}\left(\omega_{11}a_F^2 + \sigma_D^2 + \sigma_F^2 + \frac{k_{22}p_{i1}}{k_{21}}(\sigma_F^2 + a_F(a_F - a_Z)\omega_{11})\right) \\
&= k_{21}\left(\omega_{11}a_F^2 + \sigma_D^2 + \sigma_F^2 + \frac{-k_{21} + \frac{\omega_{11}}{\Delta}\frac{p_{i1}}{p_{i2}}(\sigma_Z^2 + a_Z(a_Z - a_F)\omega_{22})}{k_{21}}(\sigma_F^2 + a_F(a_F - a_Z)\omega_{11})\right) \\
&= k_{21}\left(\sigma_D^2 + a_Fa_Z\omega_{11} - \sigma_D^2\frac{(\sigma_Z^2 + a_Z^2\omega_{22} - a_Za_F\omega_{22})(\sigma_F^2 + a_F^2\omega_{11} - a_Fa_Z\omega_{11})}{\sigma_F^2a_Z^2\omega_{22} + a_F^2\sigma_Z^2\omega_{11} + \sigma_F^2\sigma_Z^2}\right) \\
&= k_{21}a_Fa_Z\left(\omega_{11} + \sigma_D^2\frac{(a_F - a_Z)^2\omega_{11}\omega_{22} + \omega_{22}\sigma_F^2 + \omega_{11}\sigma_Z^2}{\sigma_F^2a_Z^2\omega_{22} + a_F^2\sigma_Z^2\omega_{11} + \sigma_F^2\sigma_Z^2}\right)
\end{aligned}$$

where the sign of d_1 is given by the sign of $k_{21}a_Za_F$, which is positive. Similarly, we can show

that for all $n \geq 2$,

$$\begin{aligned}
E[Q_{t+n}|D_t - E_{t-1}^u(D_t)] &= E[E_t^u[Q_{t+n}]|D_t - E_{t-1}^u(D_t)] \\
&= e_0 + e_{i2}E[E_t^u[Z_{t+n-1}]|\mathbf{c}_1\hat{\boldsymbol{\varepsilon}}_t^u] \\
&= e_0 + e_{i2}a_Z^{n-1}E[Z_t|\mathbf{c}_1\hat{\boldsymbol{\varepsilon}}_t^u] \\
&= e_0 + a_Z^{n-1}d_1[D_t - E_{t-1}^u(D_t)],
\end{aligned}$$

as desired. Q.E.D.

B Proofs for the model with a single piece of advance information

As in the benchmark model, we first prove Propositions 7-8 and then prove Proposition 6.

Proof of Proposition 7: We use the following state-space system representation

$$\mathbf{x}_t = \mathbf{A}_x \mathbf{x}_{t-1} + \mathbf{B}_x \boldsymbol{\varepsilon}_t,$$

where we write $\mathbf{A}_x$ and $\mathbf{B}_x$ as:

$$\mathbf{A}_x = \begin{bmatrix} a_F & & & & \\ & a_Z & & & \\ & & 0 & & \\ & & I_k & & \\ & & & 0 & \\ & & & & I_{k-1} & 0 \end{bmatrix}_{2k+3}, \quad \mathbf{B}_x = \begin{bmatrix} 0 & 1 & 0 & 0 & 0 \\ 0 & 0 & 1 & 0 & 0 \\ 1 & 0 & 0 & 0 & 0 \\ & & \dots & 0 & \\ 0 & 0 & 0 & 1 & 0 \\ 0 & & \dots & & 0 \end{bmatrix}_{[2k+3] \times 5}$$

Note that $\mathbf{A}_x$ has two columns with zeros only, column $k+3$ associated with ε_t^D and column $2k+3$ associated with $2k+3$. The informed agents' observable signals are summarized in the vector $y_t^i = (D_t, F_t, Z_t, S_t)^\top$. This vector satisfies:

$$\mathbf{y}_t^i = \mathbf{A}_{yi} \mathbf{x}_t + \mathbf{B}_{yi} \boldsymbol{\varepsilon}_t,$$

where we write $\mathbf{A}_{yi}$ as:

$$\mathbf{A}_{yi} = \begin{bmatrix} 1 & 0 & \dots & 1_{\text{at}[k+3]} & 0 & \dots & 0 \\ 1 & 0 & & & & & \\ 0 & 1 & 0 & & & & \\ 0 & 0 & 1 & 0 & \dots & & 0 \end{bmatrix}_{4 \times [2k+3]}, \quad \mathbf{B}_{yi} = \begin{bmatrix} \mathbf{0} \\ \mathbf{0} \\ \mathbf{0} \\ \mathbf{c}_5 \end{bmatrix}_{4 \times 5}.$$

Note that the first two components of $\hat{\mathbf{x}}_t^i$ are given by F_t and Z_t since they are observable. Also, since D_t and F_t are observable, $\hat{\mathbf{x}}_t^i$ contains ε_t^D .

We can now derive the steady-state Kalman filters as in Section 3.1:

$$\hat{\mathbf{x}}_t^i = \mathbf{A}_x \hat{\mathbf{x}}_{t-1}^i + \mathbf{K}_i \hat{\mathbf{e}}_t^i \quad (\text{B.1})$$

and

$$\mathbf{y}_t^i = \mathbf{A}_{yi} \mathbf{A}_x \hat{\mathbf{x}}_{t-1}^i + \hat{\mathbf{e}}_t^i, \quad (\text{B.2})$$

where the innovation $\hat{\mathbf{e}}_t^i = \mathbf{y}_t^i - E_{t-1}^i [\mathbf{y}_t^i]$ is normally distributed with mean zero and variance

$$\boldsymbol{\Sigma}_i = E [\hat{\mathbf{e}}_t^i (\hat{\mathbf{e}}_t^i)^\top] = \mathbf{A}_{yi} \mathbf{A}_x \boldsymbol{\Omega}_i \mathbf{A}_x^\top \mathbf{A}_{yi}^\top + (\mathbf{A}_{yi} \mathbf{B}_x + \mathbf{B}_{yi}) \boldsymbol{\Sigma} (\mathbf{A}_{yi} \mathbf{B}_x + \mathbf{B}_{yi})^\top. \quad (\text{B.3})$$

The covariance matrix $\boldsymbol{\Omega}_i = E_t^i [(\mathbf{x}_t - \hat{\mathbf{x}}_t^i)(\mathbf{x}_t - \hat{\mathbf{x}}_t^i)^\top]$ and the Kalman gain matrix $\mathbf{K}_i$ satisfy:

$$\boldsymbol{\Omega}_i = (\mathbf{A}_x \boldsymbol{\Omega}_i \mathbf{A}_x^\top + \boldsymbol{\Sigma}_{xx}) - \mathbf{K}_i \mathbf{A}_{yi} (\mathbf{A}_x \boldsymbol{\Omega}_i \mathbf{A}_x^\top + \boldsymbol{\Sigma}_{xx}), \quad (\text{B.4})$$

and

$$\mathbf{K}_i = (\mathbf{A}_x \boldsymbol{\Omega}_i \mathbf{A}_x^\top + \boldsymbol{\Sigma}_{xx}) \mathbf{A}_{yi}^\top [\mathbf{A}_{yi} (\mathbf{A}_x \boldsymbol{\Omega}_i \mathbf{A}_x^\top + \boldsymbol{\Sigma}_{xx}) \mathbf{A}_{yi}^\top + \boldsymbol{\Sigma}_{yy}]^{-1}, \quad (\text{B.5})$$

where we define

$$\boldsymbol{\Sigma}_{xx} = \mathbf{B}_x \boldsymbol{\Sigma} \mathbf{B}_x^\top, \quad \boldsymbol{\Sigma}_{yy} = \mathbf{B}_{yi} \boldsymbol{\Sigma} \mathbf{B}_{yi}^\top. \quad (\text{B.6})$$

As in Proposition 2, we have $\boldsymbol{\Omega}_i \mathbf{A}_{yi}^\top = \mathbf{K}_i \boldsymbol{\Sigma}_{yy}$ and $\boldsymbol{\Omega}_i (\mathbf{A}_x \boldsymbol{\Omega}_i \mathbf{A}_x^\top + \boldsymbol{\Sigma}_{xx})^{-1} = \mathbf{I} - \mathbf{K}_i \mathbf{A}_{yi}$. We will use these two equations to simplify the informed agents' forecast problem. For expositional convenience, we consider the case with $k = 1$. The solution for general $k > 1$ is similar. Let $\boldsymbol{\Omega}_i = (\omega_{ij})_{5 \times 5}$. Using the first equation yields

$$\begin{bmatrix} \omega_{11} + \omega_{14} & \omega_{11} & \omega_{12} & \omega_{13} \\ \omega_{21} + \omega_{24} & \omega_{21} & \omega_{22} & \omega_{23} \\ \omega_{31} + \omega_{34} & \omega_{31} & \omega_{32} & \omega_{33} \\ \omega_{41} + \omega_{44} & \omega_{41} & \omega_{42} & \omega_{43} \\ \omega_{51} + \omega_{54} & \omega_{51} & \omega_{52} & \omega_{53} \end{bmatrix} = \sigma_S^2 \begin{bmatrix} 0 & 0 & 0 & k_{14} \\ 0 & 0 & 0 & k_{24} \\ 0 & 0 & 0 & k_{34} \\ 0 & 0 & 0 & k_{44} \\ 0 & 0 & 0 & k_{54} \end{bmatrix}.$$

We thus obtain:

$$\begin{aligned} \omega_{1j} &= \omega_{j1} = 0, \\ \omega_{2j} &= \omega_{j2} = 0, \\ \omega_{4j} &= \omega_{j4} = 0, \\ \sigma_S^2 k_{j4} &= \omega_{j3}. \end{aligned}$$

That is,

$$\boldsymbol{\Omega}_i = \begin{bmatrix} 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & \sigma_S^2 k_{34} & 0 & \sigma_S^2 k_{54} \\ 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & \sigma_S^2 k_{54} & 0 & \omega_{55} \end{bmatrix}.$$

Using the second equation, we can derive

$$\begin{aligned}
& \begin{bmatrix} 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & k_{54}\sigma_S^2 \frac{\sigma_{qD}}{\sigma_{qD}^2 - \sigma_q^2 \sigma_D^2} - k_{34}\sigma_S^2 \frac{\sigma_q^2}{\sigma_{qD}^2 - \sigma_q^2 \sigma_D^2} & 0 & k_{34}\sigma_S^2 \frac{\sigma_{qD}}{\sigma_{qD}^2 - \sigma_q^2 \sigma_D^2} - k_{54}\sigma_S^2 \frac{\sigma_D^2}{\sigma_{qD}^2 - \sigma_q^2 \sigma_D^2} \\ 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & \omega_{55} \frac{\sigma_{qD}}{\sigma_{qD}^2 - \sigma_q^2 \sigma_D^2} - k_{54}\sigma_S^2 \frac{\sigma_q^2}{\sigma_{qD}^2 - \sigma_q^2 \sigma_D^2} & 0 & -\omega_{55} \frac{\sigma_D^2}{\sigma_{qD}^2 - \sigma_q^2 \sigma_D^2} + k_{54}\sigma_S^2 \frac{\sigma_{qD}}{\sigma_{qD}^2 - \sigma_q^2 \sigma_D^2} \end{bmatrix} \\
& = \begin{bmatrix} 1 - (k_{11} + k_{12}) & -k_{13} & -k_{14} & -k_{11} & 0 \\ -(k_{21} + k_{22}) & 1 - k_{23} & -k_{24} & -k_{21} & 0 \\ -(k_{31} + k_{32}) & -k_{33} & 1 - k_{34} & -k_{31} & 0 \\ -(k_{41} + k_{42}) & -k_{43} & -k_{44} & 1 - k_{41} & 0 \\ -(k_{51} + k_{52}) & -k_{53} & -k_{54} & -k_{51} & 1 \end{bmatrix}.
\end{aligned}$$

Equating terms gives

$$\begin{aligned}
k_{34} &= \frac{\sigma_D^2}{\sigma_S^2 + \sigma_D^2}, k_{54} = \frac{\sigma_{qD}}{\sigma_S^2 + \sigma_D^2}, \\
\omega_{55} &= \sigma_q^2 \left(1 - \frac{\sigma_D^2}{\sigma_S^2 + \sigma_D^2} \rho_{qD}^2 \right).
\end{aligned}$$

And generalizing to $k \geq 1$ yields:

$$\mathbf{K}_i = \begin{bmatrix} 0 & 1 & 0 & 0 \\ 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & \frac{\sigma_D^2}{\sigma_S^2 + \sigma_D^2} \\ \mathbf{0}_{k-1} & \mathbf{0}_{k-1} & \mathbf{0}_{k-1} & \mathbf{0}_{k-1} \\ 1 & -1 & 0 & 0 \\ 0 & 0 & 0 & \frac{\sigma_{qD}}{\sigma_S^2 + \sigma_D^2} \\ \mathbf{0}_{k-1} & \mathbf{0}_{k-1} & \mathbf{0}_{k-1} & \mathbf{0}_{k-1} \end{bmatrix}.$$

Substituting this equation into equation (B.1) yields the desired result. Q.E.D.

Proof of Proposition 8: It follows from the standard filtering theory (see, e.g., Wang (1994)). So we omit the detailed proof. Q.E.D.

Proof of Proposition 6: We first use the conjectured price function (31) to derive

$$\begin{aligned}
Q_{t+1} &= P_{t+1} + D_{t+1} - RP_t \\
&= -p_0 + \mathbf{p}_i \hat{\mathbf{x}}_{t+1}^i + \mathbf{p}_u \mathbf{I}_{-2} \hat{\mathbf{x}}_{t+1}^u + F_{t+1} + \varepsilon_{t+1}^D - R(-p_0 + \mathbf{p}_i \hat{\mathbf{x}}_t^i + \mathbf{p}_u \mathbf{I}_{-2} \hat{\mathbf{x}}_t^u) \\
&= e_0 + \mathbf{e}_i \hat{\mathbf{x}}_t^i + \mathbf{e}_u \hat{\mathbf{x}}_t^u + \mathbf{b}_Q \hat{\varepsilon}_{t+1}^i,
\end{aligned} \tag{B.7}$$

where

$$e_0 = rp_0, \quad (\text{B.8})$$

$$\mathbf{e}_i = (\mathbf{p}_i + \mathbf{c}_1 + \mathbf{c}_{k+3}) \mathbf{A}_x - R\mathbf{p}_i + \mathbf{p}_u \mathbf{I}_{-2} \mathbf{K}_u \mathbf{A}_{yu} \mathbf{A}_x \left(\mathbf{I}_{-2} - \mathbf{c}_2^\top \frac{1}{p_{i2}} \mathbf{p}_i \mathbf{I}_{-2} \right), \quad (\text{B.9})$$

$$\mathbf{e}_u = \mathbf{p}_u \mathbf{I}_{-2} \mathbf{A}_x - R\mathbf{p}_u \mathbf{I}_{-2} - \mathbf{p}_u \mathbf{I}_{-2} \mathbf{K}_u \mathbf{A}_{yu} \mathbf{A}_x \left(\mathbf{I}_{-2} - \mathbf{c}_2^\top \frac{1}{p_{i2}} \mathbf{p}_i \mathbf{I}_{-2} \right), \quad (\text{B.10})$$

$$\mathbf{b}_Q = (\mathbf{p}_i + \mathbf{c}_1 + \mathbf{c}_{k+3}) \mathbf{K}_i + \mathbf{p}_u \mathbf{I}_{-2} \mathbf{K}_u \mathbf{A}_{yu} \mathbf{K}_i. \quad (\text{B.11})$$

and where we have used

$$\begin{aligned} \hat{\varepsilon}_{t+1}^u &= \mathbf{y}_{t+1}^u - \mathbf{A}_{yu} \mathbf{A}_x \hat{\mathbf{x}}_t^u \\ &= \mathbf{A}_{yu} \hat{\mathbf{x}}_{t+1}^i - \mathbf{A}_{yu} \mathbf{A}_x \hat{\mathbf{x}}_t^u \\ &= \mathbf{A}_{yu} \mathbf{A}_x (\hat{\mathbf{x}}_t^i - \hat{\mathbf{x}}_t^u) + \mathbf{A}_{yu} \mathbf{K}_i \hat{\varepsilon}_{t+1}^i \\ &= \mathbf{A}_{yu} \mathbf{A}_x \left(\mathbf{I}_{-2} - \mathbf{c}_2^\top \frac{1}{p_{i2}} \mathbf{p}_i \mathbf{I}_{-2} \right) (\hat{\mathbf{x}}_t^i - \hat{\mathbf{x}}_t^u) + \mathbf{A}_{yu} \mathbf{K}_i \hat{\varepsilon}_{t+1}^i, \end{aligned}$$

noting that ε_{t+1}^D –and not its expectation– is in $\hat{\mathbf{x}}_{t+1}^i$ at time $t+1$. The last equality follows from (32):

$$\hat{Z}_t^i - \hat{Z}_t^u = \frac{1}{p_{i2}} \mathbf{p}_i \mathbf{I}_{-2} (\hat{\mathbf{x}}_t^u - \hat{\mathbf{x}}_t^i),$$

and the fact that

$$\begin{aligned} \hat{\mathbf{x}}_t^i - \hat{\mathbf{x}}_t^u &= \mathbf{I}_{-2} (\hat{\mathbf{x}}_t^i - \hat{\mathbf{x}}_t^u) + \mathbf{c}_2^\top (\hat{Z}_t^i - \hat{Z}_t^u) \\ &= \mathbf{I}_{-2} (\hat{\mathbf{x}}_t^i - \hat{\mathbf{x}}_t^u) - \mathbf{c}_2^\top \frac{1}{p_{i2}} \mathbf{p}_i \mathbf{I}_{-2} (\hat{\mathbf{x}}_t^i - \hat{\mathbf{x}}_t^u) \\ &= \left(\mathbf{I}_{-2} - \mathbf{c}_2^\top \frac{1}{p_{i2}} \mathbf{p}_i \mathbf{I}_{-2} \right) (\hat{\mathbf{x}}_t^i - \hat{\mathbf{x}}_t^u). \end{aligned}$$

We next derive the conditional expectations:

$$E_t^i [Q_{t+1}] = e_0 + \mathbf{e}_i \hat{\mathbf{x}}_t^i + \mathbf{e}_u \hat{\mathbf{x}}_t^u, \quad (\text{B.12})$$

and

$$E_t^u [Q_{t+1}] = e_0 + (\mathbf{e}_i + \mathbf{e}_u) \hat{\mathbf{x}}_t^u, \quad (\text{B.13})$$

We use (32) to substitute out $\hat{Z}_t^u$ in $E_t^u [Q_{t+1}]$ (because likely $e_{i2} \neq 0$) and derive:

$$\begin{aligned}
E_t^u [Q_{t+1}] &= e_0 + (\mathbf{e}_i + \mathbf{e}_u) \hat{\mathbf{x}}_t^u \\
&= e_0 + \mathbf{e}_i \left[\mathbf{I}_{-2} \hat{\mathbf{x}}_t^u + \mathbf{c}_2^\top \hat{Z}_t^u \right] + \mathbf{e}_u \hat{\mathbf{x}}_t^u \\
&= e_0 + \mathbf{e}_i \left[\mathbf{I}_{-2} \hat{\mathbf{x}}_t^u + \mathbf{c}_2^\top \hat{Z}_t^i - \frac{1}{p_{i2}} \mathbf{c}_2^\top \mathbf{p}_i \mathbf{I}_{-2} (\hat{\mathbf{x}}_t^u - \hat{\mathbf{x}}_t^i) \right] + \mathbf{e}_u \hat{\mathbf{x}}_t^u \\
&= e_0 + \mathbf{e}_i \left(\mathbf{c}_2^\top \mathbf{c}_2 + \frac{1}{p_{i2}} \mathbf{c}_2^\top \mathbf{p}_i \mathbf{I}_{-2} \right) \hat{\mathbf{x}}_t^i + \left[\mathbf{e}_i \left(\mathbf{I}_{-2} - \frac{1}{p_{i2}} \mathbf{c}_2^\top \mathbf{p}_i \mathbf{I}_{-2} \right) + \mathbf{e}_u \right] \hat{\mathbf{x}}_t^u \\
&= e_0 + \tilde{\mathbf{e}}_i \hat{\mathbf{x}}_t^i + \tilde{\mathbf{e}}_u \hat{\mathbf{x}}_t^u.
\end{aligned}$$

It is easy to derive

$$E_t^i [q_{t+1}] = E_t^i [Z_t + \varepsilon_{t+1}^q] = (\mathbf{c}_2 + \mathbf{c}_{2k+3}) \hat{\mathbf{x}}_t^i.$$

We can also derive the conditional variances:

$$Var_t^i (Q_{t+1}) = \mathbf{b}_Q \Sigma_i \mathbf{b}_Q^\top,$$

$$Var_t^i (q_{t+1}) = \sigma_q^2,$$

$$\begin{aligned}
Cov_t^i (Q_{t+1}, q_{t+1}) &= \mathbf{b}_Q E_t^i [\hat{\varepsilon}_{t+1}^i \varepsilon_{t+1}^q] \\
&= \mathbf{b}_Q E_t^i [\hat{\varepsilon}_{t+1}^i \mathbf{x}_t^\top] \mathbf{c}_{2k+3}^\top \\
&= \mathbf{b}_Q \mathbf{A}_{yi} \mathbf{A}_x \Omega_i \mathbf{c}_{2k+3}^\top,
\end{aligned}$$

where we have used the following result to derive the last equation,

$$\begin{aligned}
E_t^i [\hat{\varepsilon}_{t+1}^i \mathbf{x}_t^\top] &= E_t^i [\mathbf{A}_{yi} \mathbf{A}_x (\mathbf{x}_t - \hat{\mathbf{x}}_t^i) \mathbf{x}_t^\top + (\mathbf{A}_{yi} \mathbf{B}_x + \mathbf{B}_{yi}) \varepsilon_{t+1} \mathbf{x}_t^\top] \\
&= E_t^i [\mathbf{A}_{yi} \mathbf{A}_x (\mathbf{x}_t - \hat{\mathbf{x}}_t^i) \mathbf{x}_t^\top] \\
&= E_t^i [\mathbf{A}_{yi} \mathbf{A}_x (\mathbf{x}_t - \hat{\mathbf{x}}_t^i) (\mathbf{x}_t - \hat{\mathbf{x}}_t^i)^\top] \\
&= \mathbf{A}_{yi} \mathbf{A}_x \Omega_i.
\end{aligned}$$

We can also derive the uninformed agents' conditional variance:

$$\begin{aligned}
Var_t^u (Q_{t+1}) &= E_t^u [(\mathbf{e}_i (\hat{\mathbf{x}}_t^i - \hat{\mathbf{x}}_t^u) + \mathbf{b}_Q \hat{\varepsilon}_{t+1}^i) (\mathbf{e}_i (\hat{\mathbf{x}}_t^i - \hat{\mathbf{x}}_t^u) + \mathbf{b}_Q \hat{\varepsilon}_{t+1}^i)^\top] \\
&= \mathbf{e}_i \Omega_u \mathbf{e}_i^\top + \mathbf{b}_Q \Sigma_i \mathbf{b}_Q^\top.
\end{aligned}$$

where we have used the following result:

$$E_t^u [(\hat{\mathbf{x}}_t^i - \hat{\mathbf{x}}_t^u) \hat{\varepsilon}_{t+1}^i] = E_t^u [E_t^i [(\hat{\mathbf{x}}_t^i - \hat{\mathbf{x}}_t^u) \hat{\varepsilon}_{t+1}^i]] = E_t^u [(\hat{\mathbf{x}}_t^i - \hat{\mathbf{x}}_t^u) E_t^i [\hat{\varepsilon}_{t+1}^i]] = 0.$$

Now we use (24) and (25) to show that optimal stock holdings are linear functions of $\hat{\mathbf{x}}_t^i$ and $\hat{\mathbf{x}}_t^u$ by substituting the preceding conditional expectations:

$$\theta_t^i = \frac{e_0 + \mathbf{e}_i \hat{\mathbf{x}}_t^i + \mathbf{e}_u \hat{\mathbf{x}}_t^u}{\gamma \left(\sigma_Q^i \right)^2 \left(1 - \left(\rho_{Qq}^i \right)^2 \right)} - \frac{\rho_{Qq}^i (\mathbf{c}_2 + \mathbf{c}_{2k+3}) \hat{\mathbf{x}}_t^i}{\gamma \sigma_Q^i \sigma_q^i \left(1 - \left(\rho_{Qq}^i \right)^2 \right)}, \quad (\text{B.14})$$

and

$$\theta_t^u = \frac{e_0 + \tilde{\mathbf{e}}_i \hat{\mathbf{x}}_t^i + \tilde{\mathbf{e}}_u \hat{\mathbf{x}}_t^u}{\gamma \left(\sigma_Q^u \right)^2}. \quad (\text{B.15})$$

We use the market clearing condition (14) to determine the coefficients in the price function. Substituting (B.14) and (B.15) into (14) yields:

$$1 = \frac{\lambda}{\gamma} \left[\frac{e_0 + \mathbf{e}_i \hat{\mathbf{x}}_t^i + \mathbf{e}_u \hat{\mathbf{x}}_t^u}{\left(\sigma_Q^i \right)^2 \left(1 - \rho_{Qq}^2 \right)} - \frac{\rho_{Qq} (\mathbf{c}_2 + \mathbf{c}_{2k+3}) \hat{\mathbf{x}}_t^i}{\sigma_Q^i \sigma_q^i \left(1 - \rho_{Qq}^2 \right)} \right] + (1 - \lambda) \frac{e_0 + \tilde{\mathbf{e}}_i \hat{\mathbf{x}}_t^i + \tilde{\mathbf{e}}_u \hat{\mathbf{x}}_t^u}{\gamma \left(\sigma_Q^u \right)^2}$$

Matching coefficients on constant, $\hat{\mathbf{x}}_t^i$ and $\hat{\mathbf{x}}_t^u$, we obtain

$$e_0 = \gamma \frac{\left(\sigma_Q^i \right)^2 \left(\sigma_Q^u \right)^2 \left(1 - \rho_{Qq}^2 \right)}{\lambda \left(\sigma_Q^u \right)^2 + (1 - \lambda) \left(\sigma_Q^i \right)^2 \left(1 - \rho_{Qq}^2 \right)} > 0, \quad (\text{B.16})$$

$$\frac{\lambda}{\gamma} \left[\frac{\mathbf{e}_i}{\left(\sigma_Q^i \right)^2 \left(1 - \left(\rho_{Qq}^i \right)^2 \right)} - \frac{\rho_{Qq}^i (\mathbf{c}_2 + \mathbf{c}_{2k+3})}{\sigma_Q^i \sigma_q^i \left(1 - \left(\rho_{Qq}^i \right)^2 \right)} \right] + \frac{1 - \lambda}{\gamma} \frac{\mathbf{e}_i \left(\mathbf{c}_2^\top \mathbf{c}_2 + \frac{1}{p_{i2}} \mathbf{c}_2^\top \mathbf{p}_i \mathbf{I}_{-2} \right)}{\left(\sigma_Q^u \right)^2} = 0, \quad (\text{B.17})$$

$$0 = \frac{\lambda}{\gamma} \frac{\mathbf{e}_u}{\left(\sigma_Q^i \right)^2 \left(1 - \rho_{Qq}^2 \right)} + (1 - \lambda) \frac{\mathbf{e}_i \left(\mathbf{I}_{-2} - \frac{1}{p_{i2}} \mathbf{c}_2^\top \mathbf{p}_i \mathbf{I}_{-2} \right) + \mathbf{e}_u}{\gamma \left(\sigma_Q^u \right)^2}. \quad (\text{B.18})$$

Equations (B.8) and (B.16) imply that $p_0 > 0$. Using

$$\mathbf{e}_i \left(\mathbf{c}_2^\top \mathbf{c}_2 + \frac{1}{p_{i2}} \mathbf{c}_2^\top \mathbf{p}_i \mathbf{I}_{-2} \right) = e_{i2} \left[\frac{p_{i1}}{p_{i2}} \quad 1 \quad \frac{p_{i3}}{p_{i2}} \quad \dots \quad \frac{p_{i,2k+3}}{p_{i2}} \right]$$

together with (B.17), we have

$$\begin{aligned}
& \frac{\lambda}{\gamma} \frac{e_{i1}}{(\sigma_Q^i)^2 \left(1 - (\rho_{Qq}^i)^2\right)} + \frac{1-\lambda}{\gamma} \frac{e_{i2} \frac{p_{i1}}{p_{i2}}}{(\sigma_Q^u)^2} = 0, \\
& \frac{\lambda}{\gamma} \left[\frac{e_{i2}}{(\sigma_Q^i)^2 \left(1 - (\rho_{Qq}^i)^2\right)} - \frac{\rho_{Qq}^i}{\sigma_Q^i \sigma_q^i \left(1 - (\rho_{Qq}^i)^2\right)} \right] + \frac{1-\lambda}{\gamma} \frac{e_{i2}}{(\sigma_Q^u)^2} = 0, \\
& \frac{\lambda}{\gamma} \frac{e_{i3}}{(\sigma_Q^i)^2 \left(1 - (\rho_{Qq}^i)^2\right)} + \frac{1-\lambda}{\gamma} \frac{e_{i2} \frac{p_{i3}}{p_{i2}}}{(\sigma_Q^u)^2} = 0, \\
& \dots \\
& \frac{\lambda}{\gamma} \left[\frac{e_{i,2k+3}}{(\sigma_Q^i)^2 \left(1 - (\rho_{Qq}^i)^2\right)} - \frac{\rho_{Qq}^i}{\sigma_Q^i \sigma_q^i \left(1 - (\rho_{Qq}^i)^2\right)} \right] + \frac{1-\lambda}{\gamma} \frac{e_{i2} \frac{p_{i,2k+3}}{p_{i2}}}{(\sigma_Q^u)^2} = 0.
\end{aligned}$$

Solving these equation gives

$$\begin{aligned}
e_{u1} &= \frac{p_{i1}}{p_{i2}} \frac{(1-\lambda) (\sigma_Q^i)^2 (1 - \rho_{Qq}^2)}{\lambda (\sigma_Q^u)^2 + (1-\lambda) (\sigma_Q^i)^2 (1 - \rho_{Qq}^2)} \frac{Cov_t^i(Q_{t+1}, q_{t+1})}{(\sigma_q^i)^2} \\
e_{i2} &= \frac{\lambda (\sigma_Q^u)^2}{\lambda (\sigma_Q^u)^2 + (1-\lambda) (\sigma_Q^i)^2 (1 - (\rho_{Qq}^i)^2)} \frac{Cov_t^i(Q_{t+1}, q_{t+1})}{(\sigma_q^i)^2}, \\
e_{i1} &= -\frac{p_{i1}}{p_{i2}} \frac{(1-\lambda) (\sigma_Q^i)^2 (1 - (\rho_{Qq}^i)^2)}{\lambda (\sigma_Q^u)^2 + (1-\lambda) (\sigma_Q^i)^2 (1 - (\rho_{Qq}^i)^2)} \frac{Cov_t^i(Q_{t+1}, q_{t+1})}{(\sigma_q^i)^2}, \\
e_{i3} &= \frac{p_{i3}}{p_{i1}} e_{i1}, \\
& \dots \\
e_{i,2k+2} &= \frac{p_{i,2k+2}}{p_{i1}} e_{i1}, \\
e_{i,2k+3} &= \left[1 - \frac{p_{i,2k+3}}{p_{i2}} \frac{(1-\lambda) (\sigma_Q^i)^2 (1 - (\rho_{Qq}^i)^2)}{\lambda (\sigma_Q^u)^2 + (1-\lambda) (\sigma_Q^i)^2 (1 - (\rho_{Qq}^i)^2)} \right] \frac{Cov_t^i(Q_{t+1}, q_{t+1})}{(\sigma_q^i)^2}.
\end{aligned} \tag{B.19}$$

Turning now to (B.18), we use

$$\mathbf{e}_i \left(\mathbf{I}_{-2} - \frac{1}{p_{i2}} \mathbf{c}_2^T \mathbf{p}_i \mathbf{I}_{-2} \right) = \begin{bmatrix} e_{i1} - e_{i2} \frac{p_{i1}}{p_{i2}} & 0 & e_{i3} - e_{i2} \frac{p_{i3}}{p_{i2}} & \dots & e_{i,2k+3} - e_{i2} \frac{p_{i,2k+3}}{p_{i2}} \end{bmatrix}$$

to derive:

$$\begin{aligned}
0 &= \frac{\lambda}{\gamma \left(\sigma_Q^i\right)^2 \left(1 - \rho_{Qq}^2\right)} + (1 - \lambda) \frac{e_{i1} - e_{i2} \frac{p_{i1}}{p_{i2}} + e_{u1}}{\gamma \left(\sigma_Q^u\right)^2}, \\
0 &= \frac{\lambda}{\gamma \left(\sigma_Q^i\right)^2 \left(1 - \rho_{Qq}^2\right)} + (1 - \lambda) \frac{e_{u2}}{\gamma \left(\sigma_Q^u\right)^2}, \\
0 &= \frac{\lambda}{\gamma \left(\sigma_Q^i\right)^2 \left(1 - \rho_{Qq}^2\right)} + (1 - \lambda) \frac{e_{i3} - e_{i2} \frac{p_{i3}}{p_{i2}} + e_{u3}}{\gamma \left(\sigma_Q^u\right)^2}, \\
&\dots \\
0 &= \frac{\lambda}{\gamma \left(\sigma_Q^i\right)^2 \left(1 - \rho_{Qq}^2\right)} + (1 - \lambda) \frac{e_{i,2k+3} - e_{i2} \frac{p_{i,2k+3}}{p_{i2}} + e_{u,2k+3}}{\gamma \left(\sigma_Q^u\right)^2}.
\end{aligned}$$

Solving these equations yields:

$$\begin{aligned}
e_{u1} &= \frac{p_{i1}}{p_{i2}} \frac{(1 - \lambda) \left(\sigma_Q^i\right)^2 \left(1 - \rho_{Qq}^2\right)}{\lambda \left(\sigma_Q^u\right)^2 + (1 - \lambda) \left(\sigma_Q^i\right)^2 \left(1 - \rho_{Qq}^2\right)} \frac{Cov_t^i(Q_{t+1}, q_{t+1})}{\left(\sigma_q^i\right)^2}, \\
e_{u2} &= 0, \\
e_{u3} &= \frac{p_{i3}}{p_{i2}} \frac{(1 - \lambda) \left(\sigma_Q^i\right)^2 \left(1 - \rho_{Qq}^2\right)}{\lambda \left(\sigma_Q^u\right)^2 + (1 - \lambda) \left(\sigma_Q^i\right)^2 \left(1 - \rho_{Qq}^2\right)} \frac{Cov_t^i(Q_{t+1}, q_{t+1})}{\left(\sigma_q^i\right)^2}, \\
&\dots \\
e_{u,2k+3} &= \frac{p_{i,2k+3} - p_{i2}}{p_{i2}} \frac{(1 - \lambda) \left(\sigma_Q^i\right)^2 \left(1 - \rho_{Qq}^2\right)}{\lambda \left(\sigma_Q^u\right)^2 + (1 - \lambda) \left(\sigma_Q^i\right)^2 \left(1 - \rho_{Qq}^2\right)} \frac{Cov_t^i(Q_{t+1}, q_{t+1})}{\left(\sigma_q^i\right)^2}
\end{aligned} \tag{B.20}$$

To solve an equilibrium, we need to determine $1 + (2k + 3) + (2k + 2)$ price coefficients p_0 , $\mathbf{p}_i$, and $\mathbf{p}_u$ (note that $p_{u2} = 0$). Equations (B.8) and (B.16) determine p_0 . Equating equations (B.9) with equations in (B.19), we obtain $(2k + 3)$ equations. Equating equations (B.10) with equations in (B.20), we obtain $(2k + 3)$ equations. Note that the second equation $e_{u2} = 0$ is an redundant identity. Therefore, we essentially have $(2k + 2)$ equations. In summary, we obtain $(2k + 3) + (2k + 2)$ equations to solve for $(2k + 3) + (2k + 2)$ unknowns of $\mathbf{p}_i$, and $\mathbf{p}_u$. When solving these equations, we need to substitute in the preceding variances and covariances. If there is a solution, then we obtain a stationary equilibrium.

Now we derive restrictions on the coefficients in the price function. Adding equations in (B.19) and (B.20) yields:

$$e_{i,j} + e_{u,j} = 0, \text{ for all } j \neq 2, 2k + 3, \tag{B.21}$$

$$e_{i2} + e_{u2} = e_{i2}, \tag{B.22}$$

and

$$e_{i,2k+3} + e_{u,2k+3} = e_{i2}. \quad (\text{B.23})$$

Adding equations (B.9)-(B.10), we get

$$\mathbf{e}_i + \mathbf{e}_u = (\mathbf{p}_i + \mathbf{c}_1 + \mathbf{c}_{k+3}) \mathbf{A}_x - R\mathbf{p}_i + \mathbf{p}_u \mathbf{I}_{-2} \mathbf{A}_x - R\mathbf{p}_u \mathbf{I}_{-2}. \quad (\text{B.24})$$

Simplifying yields equations (34)-(37) and

$$p_{i,k+3} + p_{u,k+3} = 0. \quad (\text{B.25})$$

We next show that $p_{u,k+3} = p_{i,k+3} = 0$. Note that from equation (B.10) we get

$$e_{u,k+3} = -Rp_{u,k+3} + \mathbf{p}_u \mathbf{K}_{u,2} a_Z p_{i,k+3},$$

where $\mathbf{K}_{u,2}$ is the second column of matrix $\mathbf{K}_u$. Substituting for the equation for $e_{u,k+3}$ in (B.19) and $p_{i,k+3} + p_{u,k+3} = 0$ yields:

$$\frac{p_{i,k+3}}{p_{i2}} \frac{(1-\lambda) \left(\sigma_Q^i\right)^2 \left(1-\rho_{Qq}^2\right)}{\lambda \left(\sigma_Q^u\right)^2 + (1-\lambda) \left(\sigma_Q^i\right)^2 \left(1-\rho_{Qq}^2\right)} \frac{\text{Cov}_t^i(Q_{t+1}, q_{t+1})}{\left(\sigma_q^i\right)^2} = (R + \mathbf{p}_u \mathbf{K}_{u,2} a_Z) p_{i,k+3}.$$

Suppose $p_{i,k+3} \neq 0$. Then we write the above expression as

$$\frac{1}{p_{i2}} \frac{(1-\lambda) \left(\sigma_Q^i\right)^2 \left(1-\rho_{Qq}^2\right)}{\lambda \left(\sigma_Q^u\right)^2 + (1-\lambda) \left(\sigma_Q^i\right)^2 \left(1-\rho_{Qq}^2\right)} \frac{\text{Cov}_t^i(Q_{t+1}, q_{t+1})}{\left(\sigma_q^i\right)^2} = R + \mathbf{p}_u \mathbf{K}_{u,2} a_Z. \quad (\text{B.26})$$

Now use the last equation (for $2k+3$) in (B.10) to derive

$$e_{u,2k+3} = -Rp_{u,2k+3} + \mathbf{p}_u \mathbf{K}_{u,2} a_Z p_{i,2k+3}.$$

We substitute the equation for $e_{u,2k+3}$ in (B.20) and $p_{i,2k+3} + p_{u,2k+3} = -\frac{e_{i2}}{R}$ to derive

$$\frac{p_{i,2k+3} - p_{i2}}{p_{i2}} \frac{(1-\lambda) \left(\sigma_Q^i\right)^2 \left(1-\rho_{Qq}^2\right)}{\lambda \left(\sigma_Q^u\right)^2 + (1-\lambda) \left(\sigma_Q^i\right)^2 \left(1-\rho_{Qq}^2\right)} \frac{\text{Cov}_t^i(Q_{t+1}, q_{t+1})}{\left(\sigma_q^i\right)^2} - e_{i2} = p_{i,2k+3} (R + \mathbf{p}_u \mathbf{K}_{u,2} a_Z).$$

Substituting the equation for e_{i2} in (B.19) yields:

$$\left(\frac{p_{i,2k+3}}{p_{i2}} \frac{(1-\lambda) \left(\sigma_Q^i\right)^2 \left(1-\rho_{Qq}^2\right)}{\lambda \left(\sigma_Q^u\right)^2 + (1-\lambda) \left(\sigma_Q^i\right)^2 \left(1-\rho_{Qq}^2\right)} - 1 \right) \frac{\text{Cov}_t^i(Q_{t+1}, q_{t+1})}{\left(\sigma_q^i\right)^2} = p_{i,2k+3} (R + \mathbf{p}_u \mathbf{K}_{u,2} a_Z).$$

Replacing the value of $R + \mathbf{p}_u \mathbf{K}_{u,2} a_Z$ from (B.26), we get an impossibility. Therefore $p_{i,k+3} = 0 = p_{u,k+3}$. Q.E.D.

Proof of Proposition 9: As in the proof of Proposition 3, we focus on conditional expectations of future returns in (24) and (25). Using equations (B.12) and (B.21), we can derive the expression in (48). Using equations (B.13) and (B.21)-(B.23), we can derive

$$E_t^u [Q_{t+1}] = e_0 + e_{i2} E_t^u [Z_t + \varepsilon_{t+1}^q], \quad (\text{B.27})$$

and (49). Q.E.D.

Proof of Proposition 10: We proceed as in Proposition 4. Using (B.27) and the law of iterated expectations, we compute

$$\begin{aligned} E[Q_{t+1}|Q_t] &= E[E_t^u [Q_{t+1}] | Q_t] = E[E_t^i [Q_{t+1}] | Q_t] \\ &= e_0 + e_{i2} E[E_t^i (Z_t + \varepsilon_{t+1}^q) | Q_t] \\ &= e_0 + e_{i2} \frac{\text{Cov}(Z_t, Q_t) + \text{Cov}(E_t^i [\varepsilon_{t+1}^q], Q_t)}{\text{Var}(Q_t)} Q_t. \end{aligned}$$

We also have for general $n \geq 1$:

$$\begin{aligned} E[Q_{t+n}|Q_t] &= e_0 + e_{i2} E[Z_{t+n-1} + \varepsilon_{t+n}^q | Q_t] \\ &= e_0 + e_{i2} \frac{(\mathbf{c}_2 + \mathbf{c}_{2k+3}) \mathbf{A}_x^{n-1} \text{Cov}(\hat{\mathbf{x}}_t^i, Q_t)}{\text{Var}(Q_t)} Q_t \\ &= e_0 + e_{i2} \frac{a_Z^{n-1} \text{Cov}(Z_t, Q_t) + \mathbf{1}_{\{n \leq k\}} \text{Cov}(E_t^i [\varepsilon_{t+n}^q], Q_t)}{\text{Var}(Q_t)} Q_t \end{aligned}$$

noting that $(\mathbf{c}_2 + \mathbf{c}_{2k+3}) \mathbf{A}_x^{n-1} = a_Z^{n-1} \mathbf{c}_2 + \mathbf{1}_{\{n \leq k\}} \mathbf{c}_{2k+3-(n-1)}$, where $\mathbf{1}_{\{n \leq k\}}$ is an indicator function equal to 1 if $n \leq k$ and 0 otherwise.

The covariances $\text{Cov}(Z_t, Q_t)$ and $\text{Cov}(E_t^i [\varepsilon_{t+1}^q], Q_t)$ can be obtained from

$$\begin{aligned} \text{Cov}(\hat{\mathbf{x}}_t^i, Q_t) &= E \left(E_{t-1}^i \left[\hat{\mathbf{x}}_t^i \left(\hat{\mathbf{x}}_{t-1}^{i\top} \mathbf{e}_i^\top + \hat{\mathbf{x}}_{t-1}^{u\top} \mathbf{e}_u^\top + \hat{\varepsilon}_t^{i\top} \mathbf{b}_Q^\top \right) \right] \right) \\ &= E \left(E_{t-1}^i \left[\mathbf{A}_x \hat{\mathbf{x}}_{t-1}^i \hat{\mathbf{x}}_{t-1}^{i\top} \mathbf{e}_i^\top + \mathbf{A}_x \hat{\mathbf{x}}_{t-1}^i \hat{\mathbf{x}}_{t-1}^{u\top} \mathbf{e}_u^\top + \mathbf{A}_x \hat{\mathbf{x}}_{t-1}^i \hat{\varepsilon}_t^{i\top} \mathbf{b}_Q^\top \right] \right) \\ &\quad + E \left(E_{t-1}^i \left[\mathbf{K}_i \hat{\varepsilon}_t^i \hat{\mathbf{x}}_{t-1}^{i\top} \mathbf{e}_i^\top + \mathbf{K}_i \hat{\varepsilon}_t^i \hat{\mathbf{x}}_{t-1}^{u\top} \mathbf{e}_u^\top + \mathbf{K}_i \hat{\varepsilon}_t^i \hat{\varepsilon}_t^{i\top} \mathbf{b}_Q^\top \right] \right) \\ &= \mathbf{A}_x E \left[\hat{\mathbf{x}}_{t-1}^i \hat{\mathbf{x}}_{t-1}^{i\top} \right] \mathbf{e}_i^\top + \mathbf{A}_x E \left[\hat{\mathbf{x}}_{t-1}^i \hat{\mathbf{x}}_{t-1}^{u\top} \right] \mathbf{e}_u^\top + \mathbf{K}_i \Sigma_i \mathbf{b}_Q^\top, \end{aligned}$$

where

$$E \left[\hat{\mathbf{x}}_t^i \hat{\mathbf{x}}_t^{i\top} \right] = \mathbf{A}_x E \left[\hat{\mathbf{x}}_{t-1}^i \hat{\mathbf{x}}_{t-1}^{i\top} \right] \mathbf{A}_x^\top + \mathbf{K}_i \Sigma_i \mathbf{K}_i^\top = \sum_{n=0}^{\infty} \mathbf{A}_x^n [\mathbf{K}_i \Sigma_i \mathbf{K}_i^\top] \mathbf{A}_x^{n\top},$$

and

$$\begin{aligned} E \left[\hat{\mathbf{x}}_t^i \hat{\mathbf{x}}_t^{u\top} \right] &= E \left[E_t^u (\hat{\mathbf{x}}_t^i) \hat{\mathbf{x}}_t^{u\top} \right] = E \left[\hat{\mathbf{x}}_t^u \hat{\mathbf{x}}_t^{u\top} \right] \\ &= \mathbf{A}_x E \left[\hat{\mathbf{x}}_{t-1}^u \hat{\mathbf{x}}_{t-1}^{u\top} \right] \mathbf{A}_x^\top + \mathbf{K}_u \Sigma_u \mathbf{K}_u^\top = \sum_{n=0}^{\infty} \mathbf{A}_x^n [\mathbf{K}_u \Sigma_u \mathbf{K}_u^\top] \mathbf{A}_x^{n\top}. \end{aligned}$$

Finally,

$$\begin{aligned} Var(Q_t) &= E \left[\left(\mathbf{e}_i \hat{\mathbf{x}}_{t-1}^i + \mathbf{e}_u \hat{\mathbf{x}}_{t-1}^u + \mathbf{b}_Q \hat{\boldsymbol{\varepsilon}}_t^i \right) \left(\hat{\mathbf{x}}_{t-1}^{i\top} \mathbf{e}_i^\top + \hat{\mathbf{x}}_{t-1}^{u\top} \mathbf{e}_u^\top + \hat{\boldsymbol{\varepsilon}}_t^{i\top} \mathbf{b}_Q^\top \right) \right] \\ &= \mathbf{e}_i E \left[\hat{\mathbf{x}}_{t-1}^i \hat{\mathbf{x}}_{t-1}^{i\top} \right] \mathbf{e}_i^\top + (2\mathbf{e}_i + \mathbf{e}_u) E \left[\hat{\mathbf{x}}_{t-1}^u \hat{\mathbf{x}}_{t-1}^{u\top} \right] \mathbf{e}_u^\top + \mathbf{b}_Q \boldsymbol{\Sigma}_i \mathbf{b}_Q^\top. \end{aligned}$$

Q.E.D.

Proof of Proposition 11: Letting $\mathbf{c}_1 \hat{\boldsymbol{\varepsilon}}_t^u = D_t - E_{t-1}^u(D_t)$, earnings drift is given by:

$$\begin{aligned} E[Q_{t+1} | \mathbf{c}_1 \hat{\boldsymbol{\varepsilon}}_t^u] &= E[E_t^u[Q_{t+1}] | \mathbf{c}_1 \hat{\boldsymbol{\varepsilon}}_t^u] = e_0 + e_{i2} E[E_t^u[Z_t + \varepsilon_{t+1}^q] | \mathbf{c}_1 \hat{\boldsymbol{\varepsilon}}_t^u] \\ &= e_0 + e_{i2} E[E_t^i[Z_t + \varepsilon_{t+1}^q] | \mathbf{c}_1 \hat{\boldsymbol{\varepsilon}}_t^u] \\ &= e_0 + e_{i2} (\mathbf{c}_2 + \mathbf{c}_{2k+3}) E[\hat{\mathbf{x}}_t^i | \mathbf{c}_1 \hat{\boldsymbol{\varepsilon}}_t^u] \\ &= e_0 + e_{i2} \frac{(\mathbf{c}_2 + \mathbf{c}_{2k+3}) Cov(\hat{\mathbf{x}}_t^i, \hat{\boldsymbol{\varepsilon}}_t^{u\top}) \mathbf{c}_1^\top}{\mathbf{c}_1 Var(\hat{\boldsymbol{\varepsilon}}_t^u) \mathbf{c}_1^\top} [\mathbf{c}_1 \hat{\boldsymbol{\varepsilon}}_t^u] \\ &= e_0 + e_{i2} \frac{Cov(Z_t, D_t - E_{t-1}^u(D_t)) + Cov(\varepsilon_{t+1}^q, D_t - E_{t-1}^u(D_t))}{Var(D_t - E_{t-1}^u(D_t))} [\mathbf{c}_1 \hat{\boldsymbol{\varepsilon}}_t^u]. \end{aligned}$$

Also, for general $n \geq 1$:

$$\begin{aligned} E[Q_{t+n} | \mathbf{c}_1 \hat{\boldsymbol{\varepsilon}}_t^u] &= E[E_t^u[Q_{t+n}] | \mathbf{c}_1 \hat{\boldsymbol{\varepsilon}}_t^u] = e_0 + e_{i2} E[E_t^u[Z_{t+n-1} + \varepsilon_{t+n}^q] | \mathbf{c}_1 \hat{\boldsymbol{\varepsilon}}_t^u] \\ &= e_0 + e_{i2} E[E_t^i[Z_{t+n-1} + \varepsilon_{t+n}^q] | \mathbf{c}_1 \hat{\boldsymbol{\varepsilon}}_t^u] \\ &= e_0 + e_{i2} (\mathbf{c}_2 + \mathbf{c}_{2k+3}) \mathbf{A}_x^{n-1} E[\hat{\mathbf{x}}_t^i | \mathbf{c}_1 \hat{\boldsymbol{\varepsilon}}_t^u] \\ &= e_0 + e_{i2} \frac{a_Z^{n-1} Cov(Z_t, \mathbf{c}_1 \hat{\boldsymbol{\varepsilon}}_t^u) + \mathbf{1}_{\{n \leq k-1\}} Cov(E_t^i[\varepsilon_{t+n}^q], \mathbf{c}_1 \hat{\boldsymbol{\varepsilon}}_t^u)}{Var(D_t - E_{t-1}^u(D_t))} [\mathbf{c}_1 \hat{\boldsymbol{\varepsilon}}_t^u], \end{aligned}$$

where again we have used $(\mathbf{c}_2 + \mathbf{c}_{2k+3}) \mathbf{A}_x^{n-1} = a_Z^{n-1} \mathbf{c}_2 + \mathbf{1}_{\{n \leq k\}} \mathbf{c}_{2k+3-(n-1)}$. We also used the fact that $Cov(E_t^i[\varepsilon_{t+k}^q], \mathbf{c}_1 \hat{\boldsymbol{\varepsilon}}_t^u) = 0$. This is because the time t earnings forecast at $t-1$, and hence the forecast error $D_t - E_{t-1}^u(D_t)$, do not convey any information about ε_{t+k}^q (or in fact about ε_{t+k}^D) as news about ε_{t+k}^D only arrive at time t .

To find the $Cov(Z_t, D_t - E_{t-1}^u(D_t))$ we compute

$$Cov(\hat{\mathbf{x}}_t^i, \hat{\boldsymbol{\varepsilon}}_t^{u\top}) = E[E_t^u[\hat{\mathbf{x}}_t^i \hat{\boldsymbol{\varepsilon}}_t^{u\top}]] = E[E_t^u[\hat{\mathbf{x}}_t^i] \hat{\boldsymbol{\varepsilon}}_t^{u\top}] = E[\hat{\mathbf{x}}_t^u \hat{\boldsymbol{\varepsilon}}_t^{u\top}] = \mathbf{K}_u \boldsymbol{\Sigma}_u.$$

Q.E.D.

C Solution for the model with multiple pieces of advance information

To solve the model with multiple pieces of advance information, we define the state vector in equation (30), as in Section 4. Unlike Section 4, we define the unforecastable error term based

on the period $t - 1$ information as

$$\boldsymbol{\varepsilon}_t = \left(\varepsilon_{t+k}^D, \varepsilon_t^F, \varepsilon_t^Z, \varepsilon_{t+k}^q, \varepsilon_t^{S_k}, \dots, \varepsilon_t^{S_1} \right)^\top.$$

It is normally distributed with mean zero and covariance matrix $\Sigma = E[\boldsymbol{\varepsilon}_t \boldsymbol{\varepsilon}_t^\top]$. The only positive covariance that shows up is between $E(\varepsilon_{t+k}^D \varepsilon_{t+k}^q) = \sigma_{Dq} > 0$. We can apply the same method in Section 4 to solve the model. The only modification lies in the inference problem of informed agents. This problem results in conditional expectations given by

$$\begin{aligned} E_t^i [\varepsilon_{t+k}^D] &= \rho_k S_t^k, \\ E_t^i [\varepsilon_{t+j}^D] &= E_{t-1}^i [\varepsilon_{t+j}^D] + \rho_j S_t^j, \quad 1 \leq j \leq k-1, \end{aligned}$$

and

$$\begin{aligned} E_t^i [\varepsilon_{t+k}^q] &= \frac{\sigma_{Dq}}{\sigma_D^2} \rho_k S_t^k, \\ E_t^i [\varepsilon_{t+j}^q] &= E_{t-1}^i [\varepsilon_{t+j}^q] + \frac{\sigma_{Dq}}{\sigma_D^2} \rho_j S_t^j, \quad 1 \leq j \leq k-1, \end{aligned}$$

where

$$\rho_k = \frac{\sigma_D^2}{\sigma_D^2 + \sigma_{S_k}^2}, \quad \rho_{k-1} = \frac{(1 - \rho_k) \sigma_D^2}{(1 - \rho_k) \sigma_D^2 + \sigma_{S_{k-1}}^2}, \dots, \rho_1 = \frac{(1 - \rho_k) \dots (1 - \rho_2) \sigma_D^2}{(1 - \rho_k) \dots (1 - \rho_2) \sigma_D^2 + \sigma_{S_1}^2}.$$

Because the signals $S_{t+j-k}^k, \dots, S_{t+j-1}^1$ arrive sequentially and are correlated, each of them contributes to lowering the conditional variance of earnings innovations ε_{t+j}^D for $0 < j < k$, but incrementally, the precision of each new signal changes with $\sigma_{S_j}^2$. Using these expressions we can construct a new Kalman gain matrix $\mathbf{K}_i$. Additional details are available upon request. Q.E.D.

References

- Albuquerque, Rui, Greg Bauer and Martin Schneider, 2007, International Equity Flows and Returns: A Quantitative Equilibrium Approach, *Review of Economic Studies* 74, 1-30.
- Bacchetta, Philippe, and Eric van Wincoop, 2004, Can Information Heterogeneity Explain the Exchange Rate Determination Puzzle?, *American Economic Review*, forthcoming.
- Bacchetta, Philippe, and Eric van Wincoop, 2006, Higher Order Expectations in Asset Pricing, University of Lausanne working paper.
- Barberis, Nicholas, Andrei Shleifer, Robert Vishny, 1998, A Model of Investor Sentiment, *Journal of Financial Economics* 49, 307-343.
- Berk, Jonathan, Richard C. Green, and Vasant Naik, 1999, Optimal Investment, Growth Options, and Security Returns, *Journal of Finance* 54, 1553-1607.
- Bernard, Victor L., 1992, Stock Price Reactions to Earnings Announcements, in Richard Thaler ed.: *Advances in Behavioral Finance* (Russell Sage Foundation, New York).
- Bernhardt, Dan and Jianjun Miao, 2004, Informed Trading when Information Becomes Stale, *Journal of Finance* 59, 339-390.
- Campbell, John, Sanford Grossman, and Jiang Wang, 1993, Trading Volume and Serial Correlation in Stock Returns, *Quarterly Journal of Economics* 108, 905-939.
- Cutler, David M., James M. Poterba, and Lawrence H. Summers, 1991, Speculative Dynamics, *Review of Economic Studies* 58, 529-546.
- Daniel, Kent, David Hirshleifer, and Avanidhar Subrahmanyam, 1998, Investor Psychology and Security Market Under- and Over-Reactions, *Journal of Finance* 53, 1839-1885.
- Daniel, Kent and Sheridan Titman, 2006, Market Reactions to Tangible and Intangible Information, *Journal of Finance* 61, 1605-1643.
- De Bondt, Werner F.M., and Richard H. Thaler, 1985, Does the Stock Market Overreact? *Journal of Finance* 40, 793-808.
- Fama, Eugene F., and Kenneth R. French, 1988, Permanent and Temporary Components of Stock Prices, *Journal of Political Economy* 96, 246-273.
- Fama, Eugene F., and Kenneth R. French, 1996, Multifactor Explanations of Asset Pricing Anomalies, *Journal of Finance* 51, 55-83.
- Fama, Eugene F., and Kenneth R. French, 1993, Common Risk Factors in the Returns on Stocks and Bonds, *Journal of Financial Economics* 33, 3-56.
- Gomes, João, Leonid Kogan, and Lu Zhang, 2003, Equilibrium Cross Section of Returns, *Journal of Political Economy* 111, 693-732.

- Harrison Hong, Terence Lim, and Jeremy C. Stein, 2000, Bad News Travels Slowly: Size, Analyst Coverage, and the Profitability of Momentum Strategies, *Journal of Finance* 55, 265-295.
- Hong, Harrison and Jeremy C. Stein, 1999, A Unified Theory of Underreaction, Momentum Trading, and Overreaction in Asset Markets, *Journal of Finance* 54, 2143-2184.
- Jegadeesh, Narasimhan, and Sheridan Titman, 1993, Returns to Buying Winners and Selling Losers: Implications for Stock Market Efficiency, *Journal of Finance* 48, 65-91.
- Johnson, Timothy C., 2002, Rational Momentum Effects, *Journal of Finance* 57, 585-608.
- Lewellen, Jonathan, and Jay Shanken, 2002, Learning, Asset-Pricing Tests, and Market Efficiency, *Journal of Finance* 57, 1113-1145.
- LIorente, Guillermo, Roni, Michaely, Guideon Saar, Jiang Wang, 2002, Dynamic Volume Return Relation of Individual Stocks, *Review of Financial Studies* 15, 1005-1047.
- Makarov, Igor, and Oleg Rytchkov, 2006, Forecasting the Forecasts of Others: Implications for Asset Pricing? working paper.
- Poterba, James M. and Summers, Lawrence H., 1988, Mean Reversion in Stock Prices: Evidence and Implications, *Journal of Financial Economics* 22, 27-59.
- Rouwenhorst, Geert K., 1998, International Momentum Strategies, *Journal of Finance* 53, 267-284.
- Tetlock, Paul, 2007, All the News That's Fit to Reprint: Do Investors React to Stale Information, working paper, Yale University.
- Wang, Jiang, 1993, A Model of Asset Prices under Asymmetric Information, *Review of Economic Studies* 60, 249-282.
- Wang, Jiang, 1994, A Model of Competitive Stock Trading Volume, *Journal of Political Economy* 102, 127-168.

Table 1.

MOMENTUM AND REVERSAL EFFECTS IN THE MODEL WITH A SINGLE PIECE OF ADVANCE INFORMATION

The columns labeled “One period” display the slope coefficients of the forecast of single-period returns from $t+n-1$ to $t+n$, Q_{t+n} , conditional on the current excess returns Q_t . The columns labeled “Cumulative” display the slope coefficients of the forecast of cumulative excess returns from period t to $t+n$, $Q_{t,t+n}$, conditional on the current excess returns Q_t . n is the holding period and k is the number of advance periods in the advance information. We set $\sigma_D = 0.5$, $\sigma_F = 0.01$, $\sigma_Z = 2$, $\sigma_q = 3$, $\sigma_S = 0.2$, $\sigma_{Dq} = 1.35$, $a_F = a_Z = 0.9$, $\lambda = 0.9$, $\gamma = 5$, and $r = 0.0025$.

$n \setminus k$	1		2		3		4	
	One period	Cumu- lative	One period	Cumu- lative	One period	Cumu- lative	One period	Cumu- lative
1	0.0337	0.0337	-0.0197	-0.0197	-0.0198	-0.0198	-0.0200	-0.0200
2	-0.0178	0.0160	0.0353	0.0156	-0.0178	-0.0377	-0.0180	-0.0380
3	-0.0160	-0.0000	-0.0161	-0.0005	0.0366	-0.0010	-0.0162	-0.0541
4	-0.0144	-0.0144	-0.0145	-0.0150	-0.0146	-0.0157	0.0378	-0.0163
5	-0.0130	-0.0274	-0.0131	-0.0281	-0.0132	-0.0288	-0.0133	-0.0295
6	-0.0117	-0.0391	-0.0118	-0.0399	-0.0118	-0.0407	-0.0119	-0.0415
7	-0.0105	-0.0495	-0.0106	-0.0504	-0.0107	-0.0513	-0.0107	-0.0522
8	-0.0094	-0.0590	-0.0095	-0.0600	-0.0096	-0.0609	-0.0097	-0.0618
9	-0.0085	-0.0675	-0.0086	-0.0685	-0.0086	-0.0696	-0.0087	-0.0705
10	-0.0077	-0.0752	-0.0077	-0.0762	-0.0078	-0.0773	-0.0078	-0.0784

Table 2.

EARNINGS ANNOUNCEMENT DRIFT IN THE MODEL WITH A SINGLE PIECE OF ADVANCE INFORMATION

The columns labeled “One period” display the slope coefficients of the forecast of single-period returns from $t + n - 1$ to $t + n$, Q_{t+n} , conditional on the earnings announcement $D_t - E_{t-1}^u[D_t]$. The columns labeled “Cumulative” display the slope coefficients of the forecast of cumulative excess returns from period t to $t + n$, $Q_{t,t+n}$, conditional on the earnings announcement $D_t - E_{t-1}^u[D_t]$. n is the holding period and k is the number of advance periods in the advance information. We set $\sigma_D = 0.5$, $\sigma_F = 0.01$, $\sigma_Z = 2$, $\sigma_q = 3$, $\sigma_S = 0.2$, $\sigma_{Dq} = 1.35$, $a_F = a_Z = 0.9$, $\lambda = 0.9$, $\gamma = 5$, and $r = 0.0025$.

$n \setminus k$	1		2		3		4	
	One period	Cumu- lative	One period	Cumu- lative	One period	Cumu- lative	One period	Cumu- lative
1	0.0593	0.0593	0.0505	0.0505	0.0469	0.0469	0.0435	0.0435
2	0.0533	0.1126	0.0501	0.1006	0.0421	0.0891	0.0390	0.0826
3	0.0480	0.1606	0.0451	0.1457	0.0423	0.1314	0.0350	0.1176
4	0.0432	0.2038	0.0406	0.1863	0.0380	0.1694	0.0356	0.1532
5	0.0389	0.2427	0.0365	0.2228	0.0342	0.2036	0.0321	0.1853
6	0.0350	0.2776	0.0329	0.2557	0.0308	0.2345	0.0289	0.2141
7	0.0315	0.3091	0.0296	0.2852	0.0277	0.2622	0.0260	0.2401
8	0.0283	0.3375	0.0266	0.3119	0.0250	0.2872	0.0234	0.2635
9	0.0255	0.3630	0.0240	0.3358	0.0225	0.3096	0.0210	0.2845
10	0.0230	0.3859	0.0216	0.3574	0.0202	0.3298	0.0189	0.3034

Table 3.

MOMENTUM AND REVERSAL IN THE MODEL WITH MULTIPLE PIECES OF ADVANCE INFORMATION

The columns labeled “One period” display the slope coefficients of the forecast of single-period returns from $t+n-1$ to $t+n$, Q_{t+n} , conditional on the current excess returns Q_t . The columns labeled “Cumulative” display the slope coefficients of the forecast of cumulative excess returns from period t to $t+n$, $Q_{t,t+n}$, conditional on the current excess returns Q_t . n is the holding period and σ_{S_2} is the variance of advance information about two-period ahead earnings. We set $k = 2$, $\sigma_D = 0.5$, $\sigma_F = 0.01$, $\sigma_Z = 2$, $\sigma_q = 3$, $\sigma_{S_1} = 1$, $\sigma_{Dq} = 1.35$, $a_F = a_Z = 0.9$, $\lambda = 0.9$, $\gamma = 5$, and $r = 0.0025$.

$n \setminus \sigma_{S_2}$	0.1		0.15		0.2		0.30	
	One Period	Cumu- lative	One Period	Cumu- lative	One Period	Cumu- lative	One Period	Cumu- lative
1	0.0180	0.0180	0.0267	0.0267	0.0222	0.0222	-0.0046	-0.0046
2	0.0033	0.0213	0.0018	0.0285	-0.0048	0.0173	-0.0204	-0.0250
3	-0.0018	0.0195	-0.0071	0.0213	-0.0153	0.0020	-0.0281	-0.0530
4	-0.0016	0.0179	-0.0064	0.0149	-0.0138	-0.0118	-0.0252	-0.0783
5	-0.0014	0.0164	-0.0058	0.0091	-0.0124	-0.0242	-0.0227	-0.1010
6	-0.0013	0.0151	-0.0052	0.0039	-0.0112	-0.0354	-0.0204	-0.1214
7	-0.0012	0.0140	-0.0047	-0.0008	-0.0101	-0.0454	-0.0184	-0.1398
8	-0.0011	0.0129	-0.0042	-0.0050	-0.0091	-0.0545	-0.0166	-0.1564
9	-0.0009	0.0120	-0.0038	-0.0088	-0.0081	-0.0626	-0.0149	-0.1713
10	-0.0009	0.0111	-0.0034	-0.0122	-0.0073	-0.0700	-0.0134	-0.1847

Table 4.

EARNINGS ANNOUNCEMENT DRIFT IN THE MODEL WITH MULTIPLE PIECES OF ADVANCE INFORMATION

The columns labeled “One period” display the slope coefficients of the forecast of single-period returns from $t+n-1$ to $t+n$, Q_{t+n} , conditional on the earnings announcement $D_t - E_{t-1}^u[D_t]$. The columns labeled “Cumulative” display the slope coefficients of the forecast of cumulative excess returns from period t to $t+n$, $Q_{t,t+n}$, conditional on the earnings announcement $D_t - E_{t-1}^u[D_t]$. n is the holding period and σ_{S_2} is the variance of advance information about two-period ahead earnings. We set $k = 2$, $\sigma_D = 0.5$, $\sigma_F = 0.01$, $\sigma_Z = 2$, $\sigma_q = 3$, $\sigma_{S_1} = 1$, $\sigma_{Dq} = 1.35$, $a_F = a_Z = 0.9$, $\lambda = 0.9$, $\gamma = 5$, and $r = 0.0025$.

$n \setminus \sigma_{S_2}$	0.1		0.15		0.2		0.30	
	One period	Cumu- lative	One period	Cumu- lative	One period	Cumu- lative	One period	Cumu- lative
1	0.0100	0.0100	0.0289	0.0289	0.0407	0.0407	0.0443	0.0443
2	0.0209	0.0309	0.0396	0.0685	0.0478	0.0885	0.0455	0.0898
3	0.0188	0.0497	0.0357	0.1042	0.0430	0.1315	0.0409	0.1307
4	0.0169	0.0666	0.0321	0.1363	0.0387	0.1701	0.0368	0.1676
5	0.0152	0.0819	0.0289	0.1652	0.0348	0.2050	0.0332	0.2007
6	0.0137	0.0956	0.0260	0.1912	0.0313	0.2363	0.0298	0.2306
7	0.0123	0.1079	0.0234	0.2146	0.0282	0.2645	0.0269	0.2574
8	0.0111	0.1191	0.0211	0.2356	0.0254	0.2899	0.0242	0.2816
9	0.0100	0.1291	0.0190	0.2546	0.0228	0.3128	0.0218	0.3034
10	0.0090	0.1381	0.0171	0.2717	0.0206	0.3333	0.0196	0.3229