Deconstructing Life-Cycle Expenditure with Home Production*

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Abstract

We incorporate home production in a dynamic stochastic general equilibrium model of household consumption and saving with illiquid housing and endogenous collateralized borrowing constraint. We show that such a model is capable of explaining life-cycle patterns of households’ time use and their consumption of different categories. Specifically, households’ market hours increase initially as households age, then decline sharply. By contrast, households’ home hours are much more stable and start to increase only later in life. Households’ consumption of market good, home good, and housing services all exhibit hump shape over the life cycle, with market good having the most pronounced hump, followed by home good, and then housing services. A plausibly parameterized version of our model predicts that the interaction of labor efficiency profile and the availability of pension fund explain households’ time use over the life cycle. The resulting income profile as well as endogenous borrowing constraints account for the initial humps in all three consumption goods. The consumption profiles in second half of the life cycle are mostly driven by the complementarity of home hours, home good, and housing in home production.

JEL Classification: D13, E21, J22

Key Words: Consumption, Home Production, Life Cycle

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1 Introduction

This paper incorporates explicitly home production in a dynamic stochastic general equilibrium model of the life-cycle consumption behavior of households. The model is further augmented with housing and endogenous borrowing constraints. We show that such a framework provides a unified theory for explaining both households’ time use and consumption of different categories including housing over the life cycle. The paper is motivated by two observations.

First, the explicit introduction of home production in otherwise standard dynamic general equilibrium models has proven to be useful in understanding a variety of macro-economic issues ranging from domestic as well as international business cycles, fiscal polices, and asset equilibrium puzzles. A key way in which models with home production differ from standard dynamic general equilibrium models (that do not include home production) is that home production allows households to substitute between different margins, both in labor supply and in total consumption. The existing literature on home production, however, has almost exclusively focused on labor supply despite that consumption constitutes almost two-thirds of total output. Our paper fills in this gap by bringing consumption as an additional discipline to the home production literature.

Second, while it is well established that total household consumption, durable as well as nondurable, exhibit a strong hump shape over the life cycle even after adjusting for economic growth and household size, much less attention has been paid to the substantial heterogeneity across lifecycle profiles of individual consumption expenditures. Using recent Consumption Expenditure Survey, we confirm the findings in Aguiar and Hurst (2008) that most of the decline in nondurable expenditure late in households’ life cycle is driven by categories related to households’ time spent at market work such as food away from home, transportation, and clothing/personal care. The other categories that are more closely related to households’ time spent at home production including food at home, household operation, as well as utilities, by contrast, do not decline nearly as much. Furthermore, adding to the “housing puzzle” on old age homeowners’ housing behavior, i.e., homeownership and house value do not decline much if at all toward the

\textsuperscript{2}See Rupert, Rogerson, and Wright (2000) and papers cited there. One exception is Heathcote (2002) who studies the effect of home production and retirement.
end of households’ life cycle, we show that even for renters, housing services hardly decline.\textsuperscript{4}

We argue that home production can explain this observed heterogeneity across the three broadly defined consumption categories in the second half of households’ life cycle. Our theory is, thus, much more general than the Beckerian theory proposed in Aguiar and Hurst (2008) which emphasizes intra-temporal substitution between time and each category of consumption expenditure. Additionally, our model generates data-testable predictions for the life-cycle dynamics of other observable characteristics such as labor supply and wealth accumulation beyond just consumption.

To summarize, our model has three key features. First, we explicitly model housing along both the extensive margin of owning versus renting and the intensive margin of house size. Additionally, housing adjustment is costly for homeowners but costless for renters. The latter assumption differentiates homeownership from renting. Second, households are subject to collateralized borrowing constraints. Put it simply, households can only borrow up to a fraction of their house value. This second feature together with the standard assumption of labor efficiency profile and uninsurable labor income risk in the consumption literature are largely responsible for the increasing consumption profiles in the early part of households’ life cycle as in Fernandez-Villaverde and Krueger (2002). Third, at each time period, households divide their time between market hour, home hour, and leisure. Home production takes home hour, housing, and home good as inputs to produce a composite good that is valued by households together with market good and leisure. As households age, their market labor efficiency declines and they assign more and more of their time to home production. As a result, consumption of market good declines drastically. Consumption of home good and housing services will also decline, however, given the complementarity of home good and housing services with home hours in home production, the decline is much more muted. For homeowners, adjustment cost and bequest motive further contribute to the slow decline in homeownership rate and house value.

The rest of the paper is organized as follows. In section 2, we present our empirical evidence. In section 3, we present the model. We discuss our calibration strategy in section 4 and results in section 5. Section 6 concludes.

\textsuperscript{4}This latter observation poses challenges to the traditional theory of bequest as well adjustment cost of older homeowners’ housing consumption behavior. For more discussion on the housing puzzle, see Venti and Wise (2002) and Davidoff (2006).
2 Empirical Results

In this section, we present our empirical findings on time use and consumption over the life cycle. We first study time use profiles using data from the American Time Use Survey (http://www.bls.gov/tus/), then we study consumption profiles of market good, home good, and housing services using data from the Consumption Expenditure Survey (http://www.bls.gov/cex/). We separate households into owners and renters and deal explicitly with problems of household size, cohort effect, and survey effect using the strategy first developed in Gourinchas and Parker (2002) and adopted in Aguiar and Hurst (2008) and many others.

It is worth noting that many of our empirical results have already been documented in the literature as we discussed in the introduction. Our contribution here is two-fold. We are one of the first to bring in the American Time Use Survey (ATUS) to re-examine households time use (cite the NBER paper). We also bring in the most recent consumption data for our analysis. Our differentiation between homeowner and renter is novel and important in explaining older age households’ “housing puzzle.”

2.1 Life Cycle Profiles of Time Use

American Time Use Survey, carried out by the Bureau of Census under the contract with the Bureau of Labor Statistics, measure the amount of time people spend doing various activities, such as work, childcare, housework, watching television, volunteering, and socializing. The data is strictly cross sectional as respondents are interviewed only once. Households are top coded at age 80.\textsuperscript{5} The survey started in 2003 with the most recent one ending in 2007.

We include in our sample the 2005 to 2007 ATUS since ATUS started reporting household house tenure in 2005. We focus on households whose head is between the ages of 20 and 80 (inclusive) but exclude those whose head is either in school or in the military at the time of the survey. Our final sample consists of 33,639 households, about evenly split across the three survey years. Close to 75 percent of households in our sample own homes.

We use household as our unit of observation. Market hours are defined as time the household (husband as well as wife) spend working, job searching and commuting. Home hours consists of time spent doing house work, shopping, pet care, car care, personal care, child care, adult care, shop search, child care service, professional service, personal care service, and house work service. We treat the rest as leisure.

\textsuperscript{5}All households between age 80 and 84 are assigned age 80 and those that are 85 and above are assigned age 85.
For those households who were interviewed on Saturday or Sunday (holidays are viewed the same as Sunday in ATUS), we approximate their weekday hours by the average hours of those interviewed on weekdays, in the same year, of the same education, and gender.\textsuperscript{6} We focus on time use shares on week days as they best correspond to our model. We adjust all hours by family size, cohort, and survey year effects. In particular, following Aguira and Hurst (2008), we identify life cycle from cohort effects by using the multiple cross sections in our model and use cross sectional differences in family size and interview year, respectively, to identify family size and interview year effects. Specifically, we estimate the following equations,

\[ H^k_{it} = \beta_0 + \beta_{age} \text{AGE}_{it} + \beta_{cohort} \text{COHORT}_{it} + \beta_{interviewyear} \text{INTERVIEWYEAR}_{it} + \beta_{familysize} \text{FAMSIZE}_{it} + \varepsilon^k_{it}, \]

where $H^k_{it}$ represents time use in category $k$ (market work or home work), $\text{AGE}_{it}$ is a vector of 60 one-year age dummies, $\text{COHORT}_{it}$ is vector including 12 five-year age of birth-cohort dummies, $\text{INTERVIEW}_{it}$ is a vector of 2 one-year interview dummies, and $\text{FAMSIZE}_{it}$ is a vector of family structure dummies that include 10 family size dummies. The coefficients on the age dummies, $\beta_{age}$, capture the impact of life cycle conditional on cohort, family size, and interview effects.\textsuperscript{7}

Figure 1 panels a and b chart the share of market hour and home hour over the age of the household head. As can be seen, homeowners increase their supply of market hours between ages 20 and 30. The share is roughly stable at about 28 percent between ages 30 and 50 and then starts to decline sharply. By age 65, market hours accounts for less than 5 percent of households total hours. Home hours experience two waves of increases, between ages 20 and 30 and between ages 50 and 65. Renters’ market hours follow approximately those of homeowners, their home hours, by comparison, stay roughly constant at about 18 percent through out the life cycle. More importantly, renters tend to have lower home hours than homeowners especially after age 60.

\subsection{2.2 Life Cycle Profiles of Consumption Expenditure}

The Consumer Expenditure Survey (CEX), also carried out by the Bureau of Census under the contract with the Bureau of Labor Statistics, collects households demographic characteristics and consumption information. The data is a rotating panel with each

\textsuperscript{6}Deleting those interviewed on Saturday, Sunday or holidays will result in loss of close to half of the data. Our results, however, are robust to the treatment of these households.

\textsuperscript{7}In principle, one cannot include age, cohort, and year dummies simultaneously in the regression analysis as there exists a linear relationship between age, cohort and year. We can include year dummies in our analysis only because our cohort dummies are at five-year interval instead of one-year interval.
Figure 1: Time Use over the Lifecycle (Source: American Time Use Survey 2005-2007)

Figure 2: Consumption Expenditure over the Lifecycle (Source: Consumer Expenditure Survey 2003-2006)
household being interviewed from 2 to 5 quarters, and every quarter 25 percent of the sample is replaced by new households. The short-panel dimension of CEX makes the direct use of panel techniques nearly infeasible. We, thus, pool all the data together and treat it as one cross-section.

Consistent with our time use data, we include in our consumption sample the 2003 to 2006 CEX data. We delete the fifth interviews from the sample as they are identical to the first interview. For those who appeared in all of the four remaining interviews, we add their quarterly consumption together to arrive at an annual estimate. For the rest of the households, we first calculate a quarterly average consumption for each category and then multiply it by four to obtain an annual estimate.

We include in our market good food away from home, alcohol, apparel, transportation, personal care, tobacco, other lodging, telephone, and water and other public services. We include in our home good food at home, reading, entertainment, maintenance, repairs, insurance, and other expenses, household operation, and utilities. For homeowners’ housing services, we use the reported rental value of owned residence.

We delete from our sample households who reported zero or negative market good plus home good, renters who reported less than $300 annual rents, and homeowners who reported less than $1000 annual rent value. All consumption data are adjusted by the 2000 Chained Consumer Price Index. Our final sample consists of 48,158 households and about 65 percent are home owners.

We use the same strategy outlined in the previous subsection to identify life cycle profiles of the three consumption categories with the exception that we take log of our consumption data. The results are presented in Figure 2 panels a and b in log deviations from age 20. As we can see, for homeowners, the market good, home good, and housing services all move up substantially from age 20 to age 40. The hump in housing services, however, is the least pronounced. The increase there is about 40 percent as opposed to over 60 percent in market and home goods. Starting in late 50s, consumption in all three categories begin to decline with market goods experience the most significant drop. The decline in home goods is much more muted, while the decline in housing services is only about one-third the size of the decline in market goods.

For renters, the humps in all three consumption categories are reached much faster, at mid 20s, as opposed to mid 40s for homeowners. The humps, however, are much less pronounced. Furthermore, the increase in home good, and housing services are almost the same as that of market good. In the second half of the life cycle, however, the profiles followed those of the homeowners. Market good consumption dropped substantially, home good consumption also dropped but not nearly as much. Housing services, by

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8We have also experimented with
contrast, come down only slightly.

3 The Model

We consider a model that is a modified extension of Fernandez-Villaverde and Krueger (2002), Gervais (2002), Heathcote (2002), and Yang (forthcoming), among many others. In particular, it is a discrete-time overlapping generation economy with an infinitely lived government. The government taxes labor income and provides pension to retirees. The model has several key features. First, consumers value leisure and a composite consumption good that consists of a market good and a home produced good. We model home production technology along the lines of Benhabib, Rogerson, and Wright (1991), Greenwood and Hercowitz (1991), and Rupert, Rogerson, and Wright (2000). Second, households face uninsurable idiosyncratic shocks to their labor efficiency units. Finally, we restrict intertemporal trade by endogenous borrowing constraints collateralized by housing.

3.1 Technology and Timing

There is only one type of market good produced according to the aggregate market production function

\[ F^m(K, N^m) = K^\alpha (N^m)^{1-\alpha}, \]

where \( K \) is the aggregate market capital stock and \( N^m \) is the aggregate market labor input. The final good can be consumed, invested in physical capital, or transformed into durable good or housing. Physical capital, durable good, and housing depreciate at rates \( \delta^k \), \( \delta^d \) and \( \delta^h \), respectively.

Home production takes durable good, housing, and labor as its inputs. In particular, the home work technology has the following CES functional form

\[ F^H(D, H, N_h) = \{\omega_2[\omega_1 D^{\frac{1}{\gamma_1}} + (1 - \omega_1)H^{\frac{1}{\gamma_1}}]^{\frac{1}{1-\frac{1}{\gamma_1}}} + (1 - \omega_2)(N_h)^{-\frac{1}{\gamma_2}}\}^{\frac{1}{1-\frac{1}{\gamma_2}}}, \]

where \( D \) denotes durable good, \( H \) denotes housing, and \( N_h \) denotes labor input to home production. Parameters \( \gamma_1 \) and \( \gamma_2 \) govern the substitutability between durable good, housing, and home time in home production.

At the beginning of each period, households observe their idiosyncratic labor shocks and may receive a bequest. Then, labor and capital are supplied to firms and market
production takes place. Home production also takes place using labor, durable good, and housing. After production, households receive factor payments and make their consumption and asset allocation decisions. At the end of the period, market capital, housing and durable good all depreciate, and uncertainty about early death is revealed. Accidental bequests from those early death are distributed to new agents in the next period.

3.2 The Rental Market

Following Gervais (2002), there is a two-period-lived financial institution that supplies rental housing. At the end of first period, it accepts deposits and buys residential capital. In the second period, it repays deposits with interest rate $r$. Residential capital is rented to agents at a price $\eta$ per unit. At the end of the second period, the financial institution sells the undepreciated residential stock to a new agency. The no-arbitrage condition implies that the rental rate on housing is given by

$$\eta = r + \delta^h.$$  

3.3 Demographics

During each model period, a continuum of consumers is born. They immediately begin working and consuming. Each consumer retires at $t = T_r$ and dies by the end of age $T$. Each consumer faces a positive probability of dying, given by $(1 - \lambda_t)$, where $0 \leq \lambda_t \leq 1$. The probability of dying is exogenous and independent of other household characteristics. Since the demographic patterns are stable, agents at age $t$ make up a constant fraction of the population at any point in time. Annuity markets are assumed to be absent and accidental bequests are assumed to be uniformly distributed.

3.4 Consumer’s Maximization Problem

3.4.1 Preferences

Individuals derive utility from consumption of a composite good $c$ that consists of a market produced nondurable good, $c_m$, and a home produced good, $c_h$, and leisure, $l$. Preferences are assumed to be time separable, with a constant discount factor $\beta$. The momentary utility function from consumption is of the constant relative-risk aversion class given by

$$U(c, l) = \frac{(c^{\sigma}l^{1-\sigma})^{1-\gamma} - 1}{1 - \gamma},$$
where

\[ c = [\omega_3 c_m \left( \frac{1}{\gamma^3} \right) + (1 - \omega_3) c_h \left( \frac{1}{\gamma^3} \right)]^{\frac{1}{\gamma}}, \]

\( \sigma \) (0 \leq \sigma \leq 1) represents the relative weight of the composite consumption good in utility, \( \gamma \) denotes the risk aversion parameter, \( \omega_3 \) denotes the relative weight of market good in the composite consumption good, \( \gamma_3 \) measures the degree of substitution between the market good and the home produced good.

### 3.4.2 Labor Productivity

In the market economy, all consumers of the same birth cohort face the same exogenous age-\-efficiency profile, \( e_t \). Each worker \( i \) also faces stochastic productivity shocks \( \varepsilon^i_t \), which follows a Markov process

\[ \ln \varepsilon^i_t = \rho \ln \varepsilon^i_{t-1} + v^i_t, \quad v^i_t \sim N(0, \sigma^2 \varepsilon^i). \]

This Markov process, \( Q_\varepsilon^i \), is the same for all households and there is no uncertainty over the aggregate labor endowment. The total productivity of a worker at age \( t \) is given by the product of the worker's age-\( t \) productivity shock and age-\( t \) deterministic efficiency index: \( e_t \varepsilon^i_t \).

After age \( T_r \), households retire and receive Social Security income. We assume the Social Security benefits level is the same for agents after age \( T_r \).\[^9\] To capture the disincentive from actuarially unfair delayed retirement credit on labor supply, we assume that labor income earned by those who are above age 65 is subject to an additional tax.

### 3.4.3 Transaction Costs

Due to the heterogeneity and the spatial fixity of housing, both potential buyers and sellers in the housing market are forced to spend considerable amount of time and resources to acquire information about the value of a specific housing unit. As a consequence, there are both implicit and explicit search costs associated with moving (Chinloy (1980)). These include the opportunity costs of time associated with market search, brokerage and agent fees, recording fees, legal fees, and origination fees. Moreover, households have to physically move to a new house, which entails moving costs and psychological costs of changing neighborhoods (Smith, Rosen and Fallis (1988)).

\[^9\] A more realistic assumption is that the Social Security benefit is a concave function of the accumulated lifetime contributions. Under this assumption, however, the accumulated contribution becomes a state variable, which increases the computation time dramatically.
We consider non-convex transaction costs in the housing stock, similar to those in Grossman and Laroque (1990). The specification of the transaction costs is

\[
\phi(h, h') = \begin{cases} 
0 & \text{if } h' \in [(1 - \mu_1)h, (1 + \mu_2)h] \\
\rho_1 h + \rho_2 h' & \text{otherwise.}
\end{cases}
\]

This formulation of transaction costs allow households to change their level of housing consumption without moving by undertaking housing renovation up to a fraction \(\mu_2\) of the value of the house, or by allowing depreciation up to a fraction \(\mu_1\) of the value of the house. If the house depreciates by more than a fraction \(\mu_1\) of the value, or appreciates by more than a fraction \(\mu_2\) of the value, We assume that the house has been sold. In those cases, the household must pay the transaction costs as a fraction \(\rho_1\) of its selling value. Buying a property incurs a fraction \(\rho_2\) of its buying value. Finally, an owner occupied property has to exceed a minimum size \(h\).

### 3.4.4 Borrowing Constraints

We assume that only collateralized credit is available and that the borrowing rate, mortgage rate, and deposit interest rate are all equal. This implies that mortgages and deposits are perfect substitutes. We use \(a_t\) to denote the net asset position. To buy a house, a household must satisfy a minimum down payment requirement equal to a fraction \(\theta\) of the value of the house. Housing also serves as collateral for loans (through home equity loans or refinancing) up to a fraction \((1 - \theta)\). Therefore, at any given period the household’s financial assets must satisfy\(^{10}\)

\[
a' \geq - (1 - \theta)h'.
\]

In addition, to rule out negative bequests, net worth is bounded below by 0 according to

\[
(1 + r)a' + (1 - \delta^h)h' + (1 - \delta^d)d' \geq 0.
\]

Finally, there is a minimum house size \(h\) (\(h > 0\)) that can be purchased, i.e.,

\[
h' \geq h.
\]

\(^{10}\)For a household without a house, the borrowing constraint reduces to the standard form \(a' \geq 0\).
3.4.5 Renting Shock

In a model where households differ only by age, income, and wealth, rich households tend to be homeowners and poor tend to be renters. In the US, a fraction of rich households are renters. As reported by the Bureau of Labor Statistics in 2000, 12% of households whose income is in the top quintile are renters, and 25% of those whose income is in the fourth quintile are renters. The existence of high-income renters may be due to heterogeneity in house prices, job mobility, preferences, or family composition. For example, a high-income household that lives in a city, where house prices are much higher than the national average, might not be able to afford to buy a house. Additionally, an individual who dislikes the responsibility of owning a home or is likely to move to another city for work might choose to be a renter.

To capture factors other than age, income, and wealth that affect household’s renting/owning decision, we assume households face renting shocks. A household who receives a renting shock is not allowed to own and can only rent. Let \( q_t \) denote the probability of receiving a renting shock at age \( t \). The shock is exogenous and independent of other household characteristics.

3.4.6 The Household’s Recursive Problem

In a stationary equilibrium, the interest rate is constant at \( r \) and the wage rate \( w \) is also constant. The household’s state variables are given by \((m, t, a, h, d, \varepsilon)\), which denote the agent’s renting shock, age, financial assets, housing stock carried from the previous period, durable good carried over from the previous period, and labor productivity, respectively. If \( h = 0 \), then this household was renting in the previous period. If \( m = 1 \), then this household is not allowed to own a house. Denote \( V(0, t, a, h, d, \varepsilon) \) as the value function of an agent in state \((0, t, a, h, d, \varepsilon)\). If \( m = 0 \), the consumer chooses whether to rent or to be a homeowner by comparing the value of being a homeowner, \( V^o(t, a, h, d, \varepsilon) \), against the value of renting a house, \( V^r(t, a, h, d, \varepsilon) \), so that, \( V(0, t, a, h, d, \varepsilon) = \max\{V^o, V^r\} \). If \( m = 1 \), the agent can only rent, and thus \( V(1, t, a, h, d, \varepsilon) = V^r(t, a, h, d, \varepsilon) \). If the agent chooses to rent, then

\[
V^r(t, a, h, d, \varepsilon) = \max_{\{c_m, d', a', n_m, n_h\}} \left\{ U(c, 1 - n_m - n_h) + \beta \lambda \left[(1 - q_t)E(V(0, t + 1, a', 0, d', \varepsilon') + q_t E(V(1, t + 1, a', 0, d', \varepsilon'))]\right\}
\]
subject to

\[ c_m + a' + d' + \eta s + \varphi(h, 0) = (1 - \tau)(1 - \tau_R 1(t \geq T^r))[e_t \varepsilon w_n] + pen 1(t \geq T^r) \]
\[ + b_1(t = 0) + (1 + r) a + (1 - \delta^h) \eta h + (1 - \delta^d) d \]

\[ c_m \geq 0, \ s \geq 0, \ a' \geq 0, \ d' \geq 0, \ 0 \leq n_m, n_h \leq 1, \]

where \( \tau \) denotes social security income tax, \( \tau_R \) denotes additional income tax for working retirees, \( w \) denotes wage, \( pen \) is pension after retirement. In any subperiod, an agent’s resources depend on asset holdings, \( a \), labor endowment, \( e_t \varepsilon \), or pension \( pen \), housing stock, \( h \), durable stock \( d \), and received bequests, \( b_t \). \( 1(.) \) is an indicator function which takes a value of 1 if the statement inside the parenthesis is correct and zero otherwise. Note that, agents receive pension only after retirement, and bequest only when they were first born. The composite consumption good \( c \) is defined as in equation (6), the home produced good is defined as

\[ c_h = F^h(d', s, n_h). \]

If the agent chooses to own, then

\[ V^o(t, a, h, d, \varepsilon) = \max_{\{c_m, d', h', a', n_m, n_h\}} \left\{ U(c, 1 - n_m - n_h) + \beta \lambda_t [(1 - q_t) E(V(0, t + 1, a', h', d', \varepsilon')) + q_t E(V(1, t + 1, a', h', d', \varepsilon'))] \right\} \]

subject to

\[ c_m + a' + d' + \varphi(h, h') = (1 - \tau)(1 - \tau_R 1(t \geq T^r))[e_t \varepsilon w_n] + pen 1(t \geq T^r) \]
\[ + b_1(t = 0) + (1 + r) a + (1 - \delta^h) \eta h + (1 - \delta^d) d \]

\[ c_m \geq 0, \ h \geq h, \ d' \geq 0, \ 0 \leq n_m, n_h \leq 1, \]

in addition to borrowing constraints (9) and (10). While \( c_m \) is similarly defined as in equation (6), the home produced good is now defined as

\[ c_h = F^h(d', h', n_h). \]

### 3.5 Definition of Stationary Equilibrium

A formal definition of a stationary equilibrium is provided in Appendix A. The model is solved numerically. Appendix B describes the computation algorithm in greater detail.
4 Calibration

We chose some of our parameters according to estimates in the literature. Others are chosen so that the data generated by the model’s equilibrium match a given set of aggregate targets. None of the parameters, however, are calibrated against the life-cycle profiles documented in the empirical section.

One period in the model is equal to 5 years. At age 25, each person enters into the model. The retirement age $T_r$ is set at age 65, and the maximum life expectancy $T$ is set at 90. Figure 3, panel b, shows the $\lambda_5$s, the vector of 5-year conditional survival probabilities. We use the mortality probabilities for people born in 1960, weighted by gender.

The term $\alpha$ is the share of income that goes to the nonresidential stock of capital and is set at 0.263. This capital share is lower than that in other calibrations which abstract from housing. We set $\delta^k$ to be 5.9% and $\delta^h$ to be 1.4%. The rate $r$ is the interest rate on capital net of depreciation and is 8.2%. Yang (forthcoming) explains the rationale behind these choices in greater detail. The term $\delta^d$ is set to be 20%.

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11Since one period in this model corresponds to 5 years in real life, we adjust parameters in the model accordingly. We report parameters at annual frequency, unless stated otherwise.

12We use weighted mortality probability because the model is a gender neutral model.
The deterministic age-profile of the unconditional mean of labor productivity, $e_t$, taken from French (2005), is shown in Figure 3, panel a. Labor-efficiency profile is hump-shaped, with a peak at age 50. The persistence $\rho_y$ and variance $\sigma^2_y$ of the stochastic productivity process are estimated from Panel Study of Income Dynamics data, aggregated into 5 years in order to be consistent with the model period (Altonji and Villanueva (2002)). The persistence is low and variance is high because this refers to income in a 5-year period. The discretized income process produces a cross-sectional income distribution that is similar to the data reported in De Nardi (2004). For simplicity, we assume the labor efficiency profile for home production to be constant.

The retirement benefit is calculated to mimic the Old Age and Survivor Insurance component of Social Security system. We set the Social Security tax, $\tau = 10.6\%$, and additional income tax for those who work after normal retirement age, $\tau_R = 15\%$.

Accidental bequests are first distributed to new agents at age 25 to reproduce the distribution of capital endowments for households at age 25 according to SCF 1998 data. The remaining bequests are then distributed evenly to all agents alive, which endogenously determines $b_t$.

The down payment rate $\theta$ is set to be 0.1, which is commonly used in the housing literature. The probability of receiving a renting shock, is shown Figure 3, panel b. Gruber and Martin (2003) estimate the reallocation cost of tax and agency costs from CEX and find the median household spends 7 percent of a house’s value to sell it and 2.5 percent to purchase. In our simulation, we choose transaction costs from sales to be $\rho_1 = 7\%$ and from purchases to be $\rho_2 = 2.5\%$. Davidoff (2006) shows that homeowners over age 75, compared with younger owners of similar homes, spend about 0.8 percent of home value less per year on routine maintenance. We choose a big range and set $\mu_1 = \mu_2 = 5\delta_h$. That is to say, households can change their level of housing consumption by allowing depreciation or renovation up to 7% of the value of the house as an alternative to moving.

We take the risk aversion parameter, $\eta$, to be 1.5, from Attanasio, Banks, Meghir and Weber (1999), and Gourinchas and Parker (2002), who estimate it from consumption data. This value is in the commonly used range (1-5) in the literature.

The remaining parameters are chosen so that the model-generated data match a given set of aggregate targets. We choose $\beta$, $\omega_1$, $\omega_2$, $\omega_3$, $\varsigma_1$, $\varsigma_2$, $\varsigma_3$, $\sigma$, $h$, and pension

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14The purpose of this procedure is to generate some heterogeneity at the beginning of the life cycle. Otherwise, income will be the only factor to determine the renting/owning decision for agents at age 20. Assuming that all households start with zero capital does not change the results noticeably.
15Since the model does not allow negative wealth, negative wealth holdings in the data are treated as zero. Most households start with wealth endowments close to zero.
pen to match the capital-output ratio of 1.86, housing-output ratio of 1.19, durable-
housing ratio of 0.18, home hours-housing ratio of 1.67e-6, market goods-housing ratio
of 0.24, leisure-housing ratio of 7.4e-6, homeownership rate of 67.6%, and to balance the
government budget.\footnote{The first two are calculated using NIPA and FAT and the last is calculated from CEX 1984-2000 using samples of households whose heads were between age 25-90 in each year.}
<table>
<thead>
<tr>
<th>Parameters</th>
<th>Calibrations</th>
</tr>
</thead>
<tbody>
<tr>
<td><strong>Demographics</strong></td>
<td></td>
</tr>
<tr>
<td>$T$ maximum life expectancy</td>
<td>90</td>
</tr>
<tr>
<td>$T_r$ retirement age</td>
<td>65</td>
</tr>
<tr>
<td>$\lambda_t$ survival probability</td>
<td>see text</td>
</tr>
<tr>
<td><strong>Technology</strong></td>
<td></td>
</tr>
<tr>
<td>$\alpha$ capital share in National Income</td>
<td>0.263</td>
</tr>
<tr>
<td>$\delta^k$ annual depreciation rate of capital</td>
<td>5.9%</td>
</tr>
<tr>
<td>$\delta^h$ annual depreciation rate of housing</td>
<td>1.4%</td>
</tr>
<tr>
<td>$\delta^d$ annual depreciation rate of durables</td>
<td>20%</td>
</tr>
<tr>
<td><strong>Endowment</strong></td>
<td></td>
</tr>
<tr>
<td>$\epsilon_t$ age-efficiency profile</td>
<td>see text</td>
</tr>
<tr>
<td>$\rho_\gamma$ AR(1) coefficient of 5-year income process</td>
<td>0.85</td>
</tr>
<tr>
<td>$\sigma^2_\gamma$ innovation of 5-year income process</td>
<td>0.30</td>
</tr>
<tr>
<td><strong>Government policy</strong></td>
<td></td>
</tr>
<tr>
<td>$\tau$ Social Security tax</td>
<td>10.6%</td>
</tr>
<tr>
<td>$\tau_R$ Income tax on working retiree</td>
<td>15%</td>
</tr>
<tr>
<td>$pen$ Social Security benefit</td>
<td>0.44</td>
</tr>
<tr>
<td><strong>Housing market</strong></td>
<td></td>
</tr>
<tr>
<td>$h$ minimum housing size</td>
<td>2.9</td>
</tr>
<tr>
<td>$\theta$ down payment rate</td>
<td>0.1</td>
</tr>
<tr>
<td>$q_t$ renting shock probability</td>
<td>see text</td>
</tr>
<tr>
<td>$\rho_1$ transaction costs of selling a house</td>
<td>7%</td>
</tr>
<tr>
<td>$\rho_2$ transaction costs of buying a house</td>
<td>2.5%</td>
</tr>
<tr>
<td>$\mu_1$ maximum depreciation</td>
<td>7%</td>
</tr>
<tr>
<td>$\mu_2$ maximum renovation</td>
<td>7%</td>
</tr>
<tr>
<td><strong>Preference</strong></td>
<td></td>
</tr>
<tr>
<td>$\gamma$ risk aversion coefficient</td>
<td>1.5</td>
</tr>
<tr>
<td>$\varsigma_1$ substitutability between durable and housing</td>
<td>1.1</td>
</tr>
<tr>
<td>$\omega_1$ weight on durable</td>
<td>0.362</td>
</tr>
<tr>
<td>$\varsigma_2$ substitutability between durable and housing composite and time</td>
<td>0.6</td>
</tr>
<tr>
<td>$\omega_2$ weight on durable and housing composition</td>
<td>0.9997</td>
</tr>
<tr>
<td>$\varsigma_3$ substitutability between market and home goods</td>
<td>1.1</td>
</tr>
<tr>
<td>$\omega_3$ weight on market goods</td>
<td>0.248</td>
</tr>
<tr>
<td>$\sigma$ weight on consumption</td>
<td>0.412</td>
</tr>
<tr>
<td>$\beta$ discount factor</td>
<td>0.923</td>
</tr>
</tbody>
</table>

Table 1: Parameters used in the benchmark model
5 Numerical Results

This section studies the implications of the model economy for homeownership rates by age, and for the life-cycle profiles of consumption, hours and wealth for both homeowners and renters.

5.1 Homeownership

Figure 4 shows the fraction of homeowners at each age. In the model, most young agents rent while accumulating financial assets. As time goes by, more households have accumulated sufficient funds for down payments to become homeowners. Homeownership rates continue to be very high late in life. In this model, renting has several advantages over owning. First, since there is no minimum size in rental units, relatively poor households can rent relatively small units rather than buy a large one. Second, renters can adjust housing without paying transaction costs for trading houses. On the other hand, owning might dominate renting. Owned housing can be used as collateral and relaxes borrowing constraints. Also, in the absence of indivisibility and transaction costs of owner-occupied units, the cost of buying is $1 - \frac{1-\delta_h}{1+r} = \frac{r+\delta_h}{1+r}$, which is lower than the cost of renting, $\eta = r + \delta_h$, when the interest rate is positive. For young agents, who face future income shocks and on average receive lower income than middle-aged agents, renting is more attractive than owning. Once agents have accumulated a down payment and most uncertainty in income has been revealed, they choose to own.

5.2 Life-cycle Profiles of Hours

Figure 5 shows the life-cycle profiles of average fraction of time spent in working and home production in the model. Young agents, all starting with little wealth, work a lot to buy goods and to accumulate precautionary assets. As agents age, they spend more time at home and decrease market hours. There are dramatic drops of market hours and increase of home hours at age 65, the age at which Social Security benefits become available, both for renters and for homeowners. Social Security reduces market hours through two channels. First, Social Security annuity, as fixed income, reduces the marginal utility of consumption and increases the marginal utility of leisure, thus providing an incentive to work less. Second, the additional tax on labor income for those after age 65 reduces the marginal benefit of working in the market. Renters older than age 65 spend less time working than homeowners in the same age group. This is because renters hold relatively less financial assets than homeowners. The increase of resources after age 65 from Social Security benefits induces more deduction in market hours for
Homeowners spend more time in home production than renters in the same age group. This is because, under our parameterization, time and house are complements. Homeowners on average have more housing capital than renters, and thus spend more time at home.

5.3 Life-cycle Profiles of Consumption

Figure 6 shows, in percentage deviation from the corresponding value at age 25, the life-cycle profiles of average demand for market consumption, housing and durable goods for home-owners. Average market goods consumption for homeowners is hump-shaped and peaks at age 55. Market goods consumption at age 55 is about 45% more than that at age 25. After the peak, market goods consumption decreases dramatically with age. Market goods consumption at age 80 is about 10% less than that at age 25. Facing an increasing future income profile, young agents would like to borrow to finance their current consumption but they are borrowing-constrained. This explains why early in life consumption increases as income does. As households age, they start to decrease their market consumption due to the fact that the mortality rates are increasing along the life cycle. Since there are no perfect annuity markets to insure against mortality
risk, old agents discount their future consumption at a higher rate. This implies that the consumption profile for market goods is declining later in life.

The demand for housing in the model reproduces the empirically observed profiles, increasing early in life and downsizing slowly later in life. Households begin their economic lives without any housing stock. During the early part of their lives, because of the existence of borrowing constraints and the role of housing as collateral, they forego non-housing consumption and build housing stock quickly. Agents build up their highest housing stock at age 60. The elderly decrease their housing stock quite slowly, due to the existence of transaction costs. Old households are less likely to move than young households, because they can only live in the new house for a relatively short period of time.

Figure 7 shows the life-cycle profiles of average market consumption and demand for housing and durable goods for renters. Utility optimization implies that the ratio of housing to durable goods is constant for renters, thus the profile for durables coincides with the one for housing. Average market goods consumption for renters does not vary much before age 65 but decreases very dramatically after retirement.
Figure 6: Life-cycle profiles of consumption for homeowners

Figure 7: Life-cycle profiles of consumption for renters
5.4 Life-cycle Profiles of Wealth Composition

Figure 8 displays the evolution of the wealth portfolio over the life cycle for home owners. Young agents, who start with little wealth and expect to have much higher earnings in the future, do not hold much wealth. Early in life, households borrow as much as possible to buy houses, and thus save in the form of housing. As time progresses, agents have accumulated stocks of houses and start to increase their holding of financial assets. The profile of financial assets and housing assets intersect in the early 40s. Financial wealth holding peaks at age 55. Afterwards, households start to use their assets to finance consumption. At very old ages, homeowners borrow against their homes and take on debt.

Figure 9 displays the evolution of the wealth portfolio over the life cycle for renters. Renters hold less financial assets and durables than homeowners. Compared with the data, the financial assets profiles for both owners and renters have humps that are more pronounced. Since we abstract from bequest motives, health expenditure uncertainty or other shocks, old agents in our model do not have bequest or precautionary saving motives and run down their assets much more quickly than in the data.\footnote{The risk of incurring substantial medical expenses such as out-of-pocket medical expenses and uninsurable nursing home expenses might generate precautionary savings and affect the wealth profile}
6 Conclusions

We extend a standard life-cycle model of consumption augmented with housing to incorporate home production. The model, thus, explicitly distinguishes between market and non-market related labor supply and consumption variables. We show that such a model can account for the observed life-cycle patterns in households’ time use as well as consumption of different categories. In particular, labor efficiency profile together with the availability of households’ retirement fund imply that households have incentives to drastically reduce their labor supply at around age 65. As they reduce their market hours, households allocate more of their time to home production and leisure. Since home production requires complementary inputs of home hour, housing, and home good. This implies that homeowners after retirement enjoy relatively more home hours than renters. On the consumption front, households initially increase their consumption of market good, home good, and housing as they accumulate more assets to relax their borrowing constraint. Toward retirement age, as households reduce their market hours, the cost for home production is lower. Consequently, consumption of market good declines since households substitute home produced domestic good for market good. 

(De Nardi, French and Jones (2005)). The effect of medical costs on the life-cycle consumption and saving in an environment with housing is left for future research.
Home good and housing also decline slightly as households approach the end of their life cycle but the decline is partially offset by the requirement of home production. Finally, we show that our results are robust to alternative model specifications.

7 Appendix

Appendix A. Definition of the Stationary Equilibrium

We focus on the stationary equilibrium of the economy where factor prices and agent distribution over state space are constant over time. Each agent’s state is denoted by $x$. Let $S$ denote the aggregate housing stock available for renting, $H$ denote the aggregate owner-occupied housing stock, $D$ the aggregate durable stock, $C_m$ the aggregate consumption of market good, $I_h$ the aggregate investment on housing, $I_d$ the aggregate investment on durable good, $I_k$ the aggregate investment on physical capital, and $T^c$ the total transaction costs for trading housing, $N_m$ aggregate market hours supplied, $N_h$ aggregate home hours supplied. An equilibrium is described as follows.

**Definition 1.** A stationary equilibrium is given by government policies tax rate $\tau$ and pension $pen$; an interest rate $r$ and a wage rate $w$; value functions $V(x)$; allocations $c_m(x), a'(x), h'(x), d'(x), s(x), n_m(x), n_h(x)$; bequest $b(t)$ for a person at age $t$; and a constant distribution of people over the state variables $x, v(x)$, such that the following conditions hold:

(i) Given the government policies, the interest rate, the wage, and the expected bequest, the value functions and allocations solve the above described maximization problem for a household with state variables $x$.

(ii) $v(.)$ is the invariant distribution of households over the state variables for this economy.
(iii) All markets clear.

\[ H = \int h'v(dx), \]
\[ S = \int_{h'=0}^{h'} s h(dx), \]
\[ D = \int d'v(dx), \]
\[ K = \int a'v(dx) - S, \]
\[ C_m = \int c_m v(dx), \]
\[ T^c = \int \varphi(h, h')v(dx), \]
\[ N_m = \int \varepsilon_t n_m v(dx), \]
\[ I_h = \int [h' - (1 - \delta^h)h]v(dx), \]
\[ I_d = \int [d' - (1 - \delta^d)d]v(dx), \]
\[ I_k = K' - (1 - \delta^d)K \]
\[ F^m(K, N_m) = C_m + I_k + I_h + I_d + T^c. \]

(iv) The price of each factor is equal to its marginal product.

\[ r = F^m_1(K, N^m) - \delta^k, \]
\[ w = F^m_2(K, N^m). \]

(v) Government budget is balanced at each period

\[ \tau \int \varepsilon_t w_n v(dx) + \tau R \int_{t \geq T^r} \varepsilon_t w_m v(dx) = \int_{t \geq T^r} p e v(dx). \]

(vi) The expected bequest is consistent with the actual bequest left

\[ \int_{t=0} b v(dx) + \int_{t=0} a(1 + r) v(dx) = \int (1 - \lambda_t)((1 + r)a' + (1 - \delta^h)h' + (1 - \delta^d)d')v(dx). \]
Appendix B: Computation of the Model

Because of the non-convex transaction costs on housing, we cannot use either Euler equation approximation or policy function iteration. Hence we solve the model using approximation of value functions.

To compute the steady state of our model, we first discretize the income process into 7 points following Tauchen and Hussey (1991). The state space for owner-occupied housing and asset holdings are discretized into unevenly spaced grids. The upper bounds on the grids are chosen to be large enough so that they do not constitute a constraint on the optimization problem.

We solve for the steady state equilibrium as follows:
1. Make an initial guess of interest rate $r$, the wage rate $w$ and pension.
2. Guess the size of accidental bequests.
3. Set value function after the last period to be 0 and solve the value function for the last period of life for each of the points of the grid. This yields policy functions and value function at the last period.
4. By backward induction, repeat step 3 until the first period in life.
5. Compute the associated stationary distribution of households by forward induction using the policy functions starting from the known distribution over types of age.
6. Check whether the associated accidental bequests are consistent the initial guess. If so, continue to step 7. If not, go back to step 2 and update accidental bequests.
7. Check whether market clearing conditions hold, and whether government budget is balanced. If so, an equilibrium is found. If not, go to step 1 and update initial guessed.
References


