Ideology and Endogenous Constitutions

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Abstract

A legislature has to reach a collective decision in either of two states of nature. In the first state, legislators vote over an ideological issue. In this state, legislators may not vote for the pragmatic (optimal) policy and, instead, confirm voters’ ideological bias. In the second state, the legislature handles an issue that is not ideologically charged. We find the optimal majority rule under the veil of ignorance. The chosen majority rule affects (i) the nature and quality of policy reforms, (ii) the probability that changes are made (iii) the incentives of legislators with ideological constituencies to become agenda setter. We find that in some cases the optimal majority rule is hump-shaped with respect to ideological polarization. Finally, we test some of our theoretical predictions.

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Key Words: Voting Rule, Ideology, Consociational vs. Majoritarian Democracy.

1 Introduction

What type of constitution should a divided society adopt? In answering this question, many scholars have argued that in countries with deep ethnic, religious and other cultural cleavages, the adversarial (Westmister) democratic institutions are unsuitable. According to consociationalistic theory (see for instance Lijphart, 1977), the constitution of plural...
societies should instead favor power-sharing, grand coalitions of all groups, proportionality and mutual vetoes in decision-making.\footnote{Lijphart (1977, p. 28) argues that “in a political system with clearly separate and potentially hostile population segments, virtually all decisions are perceived as entailing high stakes, and strict majority rule places a strain on the unity and peace of the system.” In his view, "... the anxieties and hostilities attending the political process may be countered by removing its competitive features as much as possible” (Lijphart,1969, p. 216).} The consociational experiences in a few divided European countries (such as, Austria, Belgium, the Netherlands and Switzerland) are often cited as concrete examples of how democracy can be stable and (relatively) effective in a plural society.\footnote{In recent years, the recommendations of consociationalistic theory have been proposed (and sometimes adopted) to promote stability in deeply divided countries such as, Bosnia, South Africa, Afghanistan, Iraq, etc.} The optimality of consociationalism is, however, highly controversial. The most serious concern is that consociational arrangements in a deeply divided society may cause a permanent deadlock and lead to ineffective governance.\footnote{The view that more veto players should lead to fewer reforms is rooted in virtually all formal models of legislative bargaining. This occurs because adding veto players (weakly) reduces the winset of the status quo, that is the set of policies that can beat the status quo. See for instance Tsebelis (2002). For a critical assessment of consociationalism, see Horowitz (1984, ch. 14).}

This paper builds a stylized model to investigate whether and under which conditions consociational forms of government (in our model, democracies where the voting rule is unanimity) are less effective than majoritarian democracies in making decisions and passing reforms. A result of this paper is that in the presence of ideological polarization more constraints on the executive (i.e., higher majority requirements to pass a policy change) may actually lead to more and better reforms. Moreover, in some circumstances we find an \textit{inverted U-relationship} between the optimal majority requirement and the extent of ideological division within the country. In particular, an unanimous voting rule could be optimal for an intermediate level of ideological polarization, but not when ideological bias is either too small or too severe.

In this paper, policy decisions are made by playing an agenda-setting game.\footnote{An agenda-setting game is described as follows. An agenda setter (one could think of him as being the chief executive) proposes an alternative. The other legislators observe the proposal and decide whether to accept or reject it. In case the proposal does not pass, the status quo is maintained.} However, while the standard agenda-setting game assumes that legislators are policy-motivated, in this paper we suppose that in a particular state of the world (that we denote as ideological)
legislators’ utilities depend on a one-dimensional policy outcome but also on electoral concerns, which are a function of the legislator’s position in the voting game. In particular, we suppose that legislators, each one representing a different constituency, suffer a cost if they vote (or if they sponsor) a bill that is not in tune with their constituents. Indeed, many studies on American politics have shown that legislators whose roll call voting records do not align with their constituents receive a smaller vote share.\textsuperscript{5}

To simplify the analysis and make the model more transparent, we suppose that in a particular state of the world all legislators agree on what the "right" policy is. However, we also suppose that some constituents have different views about the policy that should be implemented. Consequently, legislators who represent those constituents would suffer an electoral cost if they vote in favor of the "right" policy. Why do legislators disagree with their voters? Our interpretation is that legislators in this particular state of the world are pragmatic (i.e., they treat this issue purely on its merit) and overcome pre-existent divisions, while ideological voters apply dogmatic solutions. In our model, ideological polarization is measured by the distance between the pragmatic policy and the policy preferred by the most ideological voters.\textsuperscript{6}

It is important to emphasize that we are not claiming that all the issues that legislatures handle fall into the category described above. In fact, in many other situations disagreement among voters does not arise from ideological bias. Think, for instance, to the obvious example of a legislature voting over a redistributive program. In this case, legislators representing voters with different incomes would genuinely disagree about the policy to implement. However, we believe that the issues that we categorize as ideological frequently occur: in many situations, pragmatic decisions are not chosen because of ideological bias.\textsuperscript{7} To the extent that the probability of the ideological state is positive, the optimal majority rule (which in our model is decided before knowing the type of decision that the legislature


\textsuperscript{6}The term ideological polarization (or bias) is used here with a negative connotation. It refers to the fact that different constituencies hold different beliefs and that these beliefs, which cannot be right at the same time, rely on distorted perceptions of reality. Recently, Benabou (2008) and Benabou and Tirole (2006) have developed models to explain why groups sustain and rigidly maintain a system of distorted beliefs.

\textsuperscript{7}See for instance Caplan (2007), who argues that biased beliefs by voters are one of the reasons why in practice sound policies are not implemented.
will be dealing with) will partly take into account how it affects voting behavior in the ideological state.

To understand why higher majority requirements may lead to more and better reforms when the issue is ideologically charged, consider the following example. Suppose that a legislature is voting whether to partially pull soldiers out of a foreign country. Suppose that all legislators agree that a gradual reduction of soldiers is the optimal (or pragmatic) decision. Notice, however, that legislators from the right-wing party may suffer an electoral cost if they vote in favor of a drawdown of troops because conservative voters in their constituency are ideologically in favor of keeping the status quo.\(^8\)

To continue our example, suppose that the current status quo is extremely inefficient. In this case, all legislators, including the ones with extreme constituencies, would benefit from a withdrawal. That is, passing the pragmatic proposal and suffering the electoral cost would be preferable to keeping the status quo. However, those legislators would benefit even more if the withdrawal is passed without their votes. This would allow them to obtain the policy outcome they prefer and, at the same time, to take a position that panders to their voters’ beliefs. Then, with some probability, legislators vote against the optimal proposal in the hope that it will be voted by the other legislators. As a result, proposals that benefit all legislators may not pass exactly when they are more needed (i.e., when the status quo is extremely inefficient). Notice that this problem would not occur under unanimity rule because in this case legislators cannot hope to see a proposal passed without having to take a position.\(^9\)

Another advantage of an unanimous voting rule, besides solving a coordination problem when the status quo is inefficient, is to lead to a better compromise by making proposals by ideological legislators less biased towards the extremes. This occurs because an ideological agenda setter has to moderate his proposal to make it acceptable to the legislator representing the opposed ideological constituency. This discussion explains why simple majority rule is less preferable than unanimity rule in a time of severe crisis.

However, there are also situations in which simple majority rule is preferable to unanimity rule. Suppose in fact that the status quo is only moderately inefficient. In this case, it would not be possible to pass a reform under unanimity rule because there is no policy change that

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\(^8\) By the same token, legislators from the left-wing party may suffer an electoral cost because their voters, who are in favor of a complete drawdown, may see a vote for a partial withdrawal as a betrayal.

\(^9\) A similar result is obtained by the public good literature, where it is known that unanimity rule may solve the free-riding problem.
would make the electoral cost worth incurring for all legislators. Instead, policy changes that improve upon the status quo would be feasible under simple majority rule. This is why when the status quo is moderately inefficient, voting under simple majority rule leads to a better equilibrium outcome.

Besides the location of the status quo, the identity of the agenda setter is also crucial in determining the equilibrium outcome of the voting game. In our model, we suppose that nature selects the agenda setter from the set of legislators that decide to run for this position. Since reforms proposed by an ideological legislator are likely to be more biased than reforms proposed by a non-ideological one, it is important to investigate whether the voting rule affects the incentives to run for the agenda setter’s position. In our model, we show that consociational arrangements, where all legislators must vote for a policy change and take a position accordingly, give ideological legislators larger incentives to run for office. The intuition being that since a position cost is suffered for sure under unanimity rule whenever a reform is made, an ideological legislator is keen on running for the agenda setter’s position in order to be able to choose the position cost the optimally trades off electoral costs and policy motivations. Conversely, under simple majority rule, an ideological legislator realizes that he may escape the position cost when the proposal is accepted by a coalition that excludes him. This possibility reduces his incentives to run for office. Then, it follows that reforms are more likely to be proposed by an ideological legislator when the majority requirement is more stringent. Of course, this is a potential drawback of consociational arrangements that constitutional designers should take into account when recommending a consensus democracy to a divided country. In Section 3.8, we argue that this concern is more serious when ideological bias is concentrated on one side of the political spectrum.

After determining the costs and benefits of each majority rule, this paper finds the optimal voting rule that constituents would choose under the veil of ignorance: that is, before knowing the location of the status quo and before knowing which type of issue (whether ideological or not) will be dealt with by the legislature. Moreover, we investigate whether the optimal voting rule depends on the extent of ideological polarization that is present in the country. We find that in some circumstances the optimal majority rule is hump-shaped with respect to ideological polarization. In particular, unanimity rule is not optimal when ideological bias is severe. This suggests that consociational arrangements may not be desirable in a society that is deeply divided along cleavages of religious, ideological, linguistic, cultural,
The remainder of the paper is organized as follows. In Section 2, we review the related literature. In Section 3 we introduce the model and discuss equilibrium behavior in the ideological state. In Section 4, we compare welfare under various voting rules depending on the extent of ideological polarization. In Section 5, we test some of the empirical predictions of our model. Section 6 concludes.

2 Review of the Literature

A very large literature investigates which voting rule a society should adopt to make collective decisions. More than two centuries ago, Rousseau (1762) argued that "the more grave and important the questions discussed, the nearer should the opinion that is to prevail approach unanimity." Along similar lines, Wicksell (1896) advocates unanimity rule to avoid the possibility that the government can reduce an individual below his status quo utility. Buchanan and Tullock (1962) argue that choosing the optimal majority rule involves a trade-off between the costs of expropriation, which decrease in the number of individuals whose agreement is required to make decisions, and some decision-time costs, which increase with the majority rule. Rae (1969) studies the choice of a voting rule by a group of individuals who are uncertain on whether or not they will gain or loose from a future collective decision and finds that simple majority rule is optimal since it maximizes total ex-ante utility.

In a public good provision model, Aghion and Bolton (2004) show that on the one hand a low majority rule provides higher ex-post flexibility (hence, more efficient public good provision) by lowering the cost of compensating vested interests, but on the other it provides little protection from expropriation. In their model, they show that unanimity is dominated by less stringent voting rules from the standpoint of ex-ante efficiency.

In a related paper, Aghion, Alesina, and Trebbi (2004) study the problem of choosing at the constitutional stage (when individuals are ex-ante identical) the optimal size of the super-majority that is needed to pass legislation. Of particular interest for this paper is their analysis of how the optimal majority rule depends on the degree of polarization of preferences. In their model, a polarized society is a society in which the distribution of the individuals' ex-post gains and losses from legislation has a thick lower tail. Proposition 3 in their paper shows that for a sufficiently large degree of risk aversion, more polarization...
increases the optimal share of votes needed by the executive to pass a policy change, since more checks and balances lower the risk of being, ex post, unsatisfied with the new legislation. On the other hand, they show that when the voting rules are written by those who will likely control political power after the Constitution is ratified, one should observe fewer constraints on the executive in more polarized countries.

This paper is also related to the literature on the political economy of reforms. Alesina and Drazen (1992) study a model in which two policy makers must agree on a reform plan. This assumption is meant to describe a consensus-democracy where a single policy maker cannot unilaterally pick the reform plan that is less costly to his constituents. In their model, although everyone would benefit from a reform, each policy maker has an incentive to "wait-and-see", in the hope that the other policy maker will concede before him and will accept to bear a larger share of the adjustment burden. As a result, in general reforms are inefficiently delayed. A similar coordination problem arises in our model. A clear implication of the war-of-attrition model is that in a democracy where the executive faces few checks and balances, reforms would occur earlier, since it would be costly for the opposition to veto the reform plan. Moreover, this model predicts that the delay decreases when the cost of staying with the status quo is high.

We now review the empirical literature that analyzes the political determinants of economic reforms. Alesina, Ardagna and Trebbi (2006) study episodes of budget and inflation stabilizations in a large sample of countries. According to their results, less constrained executives and presidential systems adjust more substantially. Moreover, they show that crises lead to more drastic adjustments with stronger governments. Hamman and Prati (2002) investigate which factors explain the success over time of an inflation stabilization. Their results show that inflation stabilizations are more likely to fail when the chief executive has more institutional constraints. More surprisingly, they also show that less cohesive governments are less likely to fail. In their views, the latter result can be attributed to the successful stabilizations implemented in some countries by a national unity government. Lora and Olivera (2004) focus on different array of reforms (namely, trade reforms, financial reforms, tax reforms, privatizations, and labor reforms) in Latin America from 1985 to 1995.

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10 In Rodrik and Fernandez (1991), where decisions are instead made by simple majority voting, delay is caused by the uncertainty regarding the distribution of gains and losses from the reform.

11 Indeed, as shown by Spolaore (2004), when the executive has no constraints, reforms occur too often (that is, even when they are not socially optimal) and the costs of the reform are unevenly distributed.
In their analysis, political variables do not seem to have a large explanatory power. However, when looking at labor reforms, they find that reforms are more likely to occur when the legislature has fewer parties and, somewhat counter-intuitively, when the percentage of legislative seats held by the head of government’s party is low. Persson (2005) looks at trade liberalizations and reforms of property right protection and finds that the passage from an authoritarian regime to a parliamentary and/or to a democracy with proportional representation raises the probability of opening the economy and passing structural reforms.

Tentatively, the aforementioned empirical evidence seems to suggest that for some reforms (a budget stabilization, for example), but not for others (a labor reform, for example), less constrained executives (that is, democracies with fewer veto players) are more effective. Indeed, it is reasonable to expect that the number of veto players will have different implications in the two cases. Consider for instance the decision to reduce the budget deficit. This is, to a large extent, a distributive problem, which consists in choosing the size of the adjustment and the distribution of the burden of the stabilization. In this case, a legislator with veto power can prevent, for example, a reduction of the transfers that his constituents receive; it does not mean that this legislator can stop the stabilization if the other legislators decide to sustain its entire cost. It is then intuitive that fewer veto players facilitate reforms because more individuals can be obliged to finance the stabilization. On the other hand, consider a reform of labor regulation. In this case, the legislature is picking a policy as opposed to a distribution of costs. If a change is decided on this subject matter, the utility of all individuals will be defined with respect to the new policy. Exercising veto power has more clear cut consequences: the veto player can prevent the change to the status quo by rejecting the proposal. However, as our model shows, in this case it is not obvious that having fewer veto powers will lead to more (and better) reforms.

3 The Model

We consider an economy that lasts two periods: \( t = 0,1 \). At time \( t = 1 \), a three-person legislature \( N = \{l, c, r\} \) has to reach a single policy decision. The indexes \( l, c, \) and \( r \) stand for the legislator who represents, respectively, the left, center, and right-leaning constituency.\(^{12}\)

\(^{12}\)For simplicity, we focus on a three-person committee. This is the smallest committee size for which we can obtain interesting results in terms of voting rules.
The decision that the legislature makes depends on the specific issue that arises at $t = 1$. The nature of the issue is determined by the realization of a random variable. With probability $\vartheta$ the state of nature is $I$ (where $I$ denotes the ideological state) and with complementary probability $1 - \vartheta$ the state of nature is $R$ (where $R$ denotes the residual state). We will describe the policy choice of state $I$ in great detail in Section 3.1. For the moment, it suffices to say that the decision in state $I$ will be a one-dimensional policy outcome. In this paper, we are agnostic about the particular decision that is made in state $R$. Since the set of issues that legislatures handle is obviously large, it would be unrealistic to try to cover it by means of only two states. Hence, the issue arising in state $R$ is meant to summarize all the issues that are not of the type described in Section 3.1.

At time $t = 0$, the constitution is chosen under the veil of ignorance (before knowing the state of nature). The constitution specifies the majority rule that is necessary to approve the policy decision at time $t = 1$. Since the legislature includes three members, constituents can choose among the following voting rules: unanimity rule (denoted by $U$), simple majority rule (denoted by $SM$) and autocracy (denoted by $A$). An important assumption is that the majority rule cannot be contingent on the realized state of nature or any other states (such as, the location of the status quo or the identity of the agenda setter in the legislature). The choice of the constitution will be described in Section 3.7.

### 3.1 Ideological State

In this section, we describe legislators’ preferences and the timing of events in the ideological state. We emphasize from the outset that we make two simple amendments to an otherwise standard agenda-setting game. First, we suppose that legislators have "position-taking" preferences as well as preferences over policy outcomes and, second, we endogenize the probabilities of being recognized to be the agenda setter.

The utility of each policy maker $i = l, c, r$ in state $I$ is given by

$$u_i(p_i, x, I) = -x^2 - \theta_i(p_i - i)^2.$$  \hspace{1cm} (1)

Utility (1) depends on the one-dimensional policy outcome $x$, where $x \in Q = [-\overline{q}, \overline{q}]$, with

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13The assumption that the constitution is incomplete (not state-contingent) is also made by Aghion and Bolton (2003) and Aghion, Alesina, and Trebbi (2004).
Notice that all legislators agree that the "right" policy outcome in state \( I \) is \( x = 0 \).\(^{14}\) Moreover, utility also depends on \( p_i \), the position taken by legislator \( i \) in the voting game, where \( p_i \) also belongs to \( Q \).\(^{15}\) Unlike \( x \), which is a collective decision, \( p_i \) is chosen individually by each committee member. In our model, committee members are either constrained or unconstrained in choosing \( p_i \). In particular, member \( i \) is constrained to choose \( p_i = x \) if he has voted in favor of \( x \) or he has actually proposed \( x \) (the latter possibility occurs only when \( i \) is recognized as agenda setter).\(^{16}\) If instead \( i \) rejects proposal \( x \), he is free to choose his position. The index \( i \) denotes the legislator but also the policy that minimizes his position cost. We suppose that \( c = 0 \) and that \( l \) and \( r \) are symmetrically on opposite sides of 0. That is, \( r \in (0, \bar{q}] \) and \( l \in [-\bar{q}, 0) \), with \( r = -l \).\(^{17}\)

As discussed in the Introduction, the position cost is meant to capture, in a reduced-form way, the electoral cost for a politician of not pandering to his voters’ beliefs. It is commonly believed that many position-taking activities politicians engage in cannot be explained by only appealing to legislators’ preferences for policy outcomes.\(^{18}\) Improving electoral chances appears to be a driving factor behind many position-taking activities. For instance, it would explain why legislators sponsor bills that have no chance of passing, why they deliver floor speeches to an almost empty chamber, and why they carefully ponder their votes even when the winning outcome is already clear. As Mayhew puts it: “politicians often get rewarded for taking positions rather than achieving effects” (Mayhew 1974, p. 51). The parameter \( \theta_i \) is then a measure of the relative intensity of electoral concerns (the second term) versus policy motivations (the first term). For simplicity, we assume that legislator \( c \) does not suffer a position cost and that the parameter \( \theta_i \) is the same for \( l \) and \( r \): that is, \( \theta_c = 0 \) and

\(^{14}\)This assumption simplifies the analysis. If we assume some disagreement on policy outcomes, the main intuition behind many of our results would continue to hold.

\(^{15}\)Models where legislators have position-taking preferences have been studied by the literature on lobbying. For example, Snyder (1991) studies a voting model where lobbyists condition their bribes on the legislators’ voting records. His goal is to investigate how lobbyists select the optimal bribe that allows them to obtain their preferred policy outcome. See also Dal Bo (2006) and Snyder and Ting (2005).

\(^{16}\)We make the assumption that votes are publicly known.

\(^{17}\)The main thrust of our results would not change if, instead, we assumed, for example, that the two legislators represent, respectively, moderate and extreme left-wing voters.

\(^{18}\)Mayhew (1974), Jacobson (1987), Kingdon (1989), and Arnold (1990) argue that legislators worry about the potential electoral implications of their votes and, consequently, behave strategically when choosing a position on a roll call vote.
\( \theta_l = \theta_r \). In what follows, we will then use \( \theta \) to denote \( \theta_l \) and \( \theta_r \). Throughout, we assume \( \theta \in (0, 1] \).\(^{19}\)

The value \( \iota \equiv (r - l)/2 \) measures the distance between the pragmatic policy and the most preferred policy by an ideological voter. Consequently, it summarizes the degree of ideological polarization in the society. Why is \( \iota > 0 \)? Where does the disagreement between legislators and constituencies come from? We suppose here that while legislators treat the issue in state \( I \) on its merit (they are pragmatic and agree on the practical solution to this issue), their constituencies are dogmatic. For example, one could think that legislator \( r \) represents voters with a pro-market bias while legislator \( l \) represents voters with an anti-market bias.\(^{20}\) Throughout this paper, we will often refer to \( l \) and \( r \) as the ideological legislators. Since the constituency of \( c \) is unbiased, we will instead refer to \( c \) as the non-ideological legislator.

The timing in state \( I \): First, nature selects the status quo \( q \), which is a random variable uniformly distributed on the policy space \( Q \). Afterwards, the legislative bargaining game begins. This game has two stages: the candidacy stage and the voting stage.

Candidacy Stage and Recognition Probabilities. The candidacy stage is necessary to determine the identity of the agenda setter in the legislature. In most models of legislative bargaining (all inspired by the pioneering work of Baron and Ferejohn, 1986), the agenda setter is chosen from the set of all legislators according to some exogenous probabilities.\(^{21}\) In this paper, we assume instead that the recognition probabilities are endogenous. More specifically, we suppose (i) that legislators simultaneously decide whether or not to run for the agenda setter’s position and (ii) that the agenda setter is randomly selected from the set of legislators that decided to run for office. We denote \( i \)'s decision whether or not to run for the agenda setter’s position by \( s_i \in \{0, 1\} \) where \( s_i = 1 \) (resp. \( s_i = 0 \)) indicates that \( i \) runs (does not run) for the agenda setter’s position.\(^{22}\) The benefit of becoming agenda setter is given by the possibility of setting the agenda. Choosing \( s_i = 1 \) entails no direct

\(^{19}\)If legislators care very little about policy outcomes (that is, \( \theta \) is much larger than one) legislators would not agree on anything since by rejecting any proposal they are free to choose the position that minimizes the position-taking term.


\(^{21}\)An exception is Yildirim (2006), where legislators exert effort to be the proposer.

\(^{22}\)The fact that the candidacy stage occurs after the status quo is known is not essential for our results.
cost, but it has an indirect cost: the agenda setter is bound to position himself according to his proposal. Since $c$ does not suffer a position cost, we will assume that $s_c = 1$ and focus on the candidacy strategies of the two ideological legislators.

Let $\rho_i(s_i, s_{-i})$ denote the recognition probability that ideological legislator $i$ is selected if his effort decision is $s_i$ when the effort decision of the other ideological legislator is $s_{-i}$. Then, $\rho_i(1, 1)$ denotes the probability that legislator $i$ is selected when all legislators run for office (recall that $s_c = 1$). We make the following simplifying assumptions on the recognition probabilities:

$$
\rho_i(0, s_{-i}) = 0 \quad \forall s_{-i} \quad \text{and} \quad \rho_i(1, 0) = \rho_i(1, 1) < \frac{1}{2} \quad \text{for } i = l, r. \quad \text{(RP1&2)}
$$

The first assumption implies that legislator $i$ by choosing $s_i = 0$ can make sure that he is not recognized. According to the second assumption, the decision of legislator $-i$ does not affect the probability that legislator $i$ is recognized. This implies, since recognition probabilities must add to one, that when legislator $i$ does not run for office the probability that the other ideological legislator is recognized does not change, while the probability that the non-ideological legislator is recognized increases accordingly. This assumption is made to simplify the strategic interaction in the candidacy stage since it implies that the candidacy decision of one ideological legislator does not influence the marginal gain of running for office of the other ideological legislator (see Lemma 2 in Section 3.5).

**Voting stage.** After the identity of the agenda setter is known, policy $x$ is chosen through a simple agenda-setting game à la Romer and Rosenthal (1978). As is standard in that model, the agenda setter makes a take-it-or-leave-it proposal to the legislature.\footnote{This assumption, which is often made in the legislative bargaining literature, is a way of capturing the} The other
legislators vote simultaneously and they can either approve or reject the proposal. If the proposal is rejected, the default policy \( q \) is implemented. Notice that a proposal passes if it satisfies the majority requirement specified in the constitution. Under autocracy (or full delegation), proposals by the agenda setter do not need to be approved in order to be implemented; under simple majority rule the agenda setter needs one acceptance vote to pass his proposal, while under unanimity rule two acceptance votes are necessary. Finally, on the basis of the voting records, which we suppose to be public, positions are determined.

We now describe the solution concept in the voting game. Knowing the status quo, each ideological legislator decides whether or not to run for the agenda setter’s position. Let the (pure) candidacy strategy be denoted by \( s_i : Q \to \{0, 1\} \), where \( s_i = 1 \) indicates that individual \( i \) is willing to become agenda setter. Conditional on \( i \) being the recognized agenda setter, \( i \)'s (pure) proposal strategy is a map from the status quo policy into a proposal: \( x_i : Q \to Q \). Legislator \( i \)'s (mixed) voting strategy is a map \( v_i : Q \times Q \to \Delta(\{\text{accept}, \text{reject}\}) \), where \( \Delta(\{\text{accept}, \text{reject}\}) \) denote the set of probability distributions over the two pure strategies accept and reject. In other words, given the realized status quo and given the proposal, \( i \) casts his vote. Finally, on the basis of the voting records, the position \( p_i \) is determined. A strategy for \( i \) is then given by \( \chi_i \equiv \{s_i, x_i, p_i\} \). A strategy profile \( \chi \equiv \{\chi_1, \chi_c, \chi_r\} \) is a subgame perfect equilibrium if it is a Nash equilibrium in any subgame.

Before solving the agenda-setting game for all possible majority rules, we define the acceptance correspondence \( A_i : Q \to Q \). This correspondence indicates the set of acceptable policies to legislator \( i \) for any status quo:

\[
A_i(q) = \{x \in Q : -x^2 - \theta_i(x - i)^2 \geq -q^2\}.
\]

We say that legislator \( i \) finds policy \( x \) acceptable if the utility of implementing \( x \) and taking position \( p_i = x \) is greater or equal than the utility of maintaining the status quo and minimizing the position cost (that is, choosing \( p_i = i \)). Notice that a legislator never votes in favor of a policy that is not acceptable. The fact that a policy is acceptable, however, is not sufficient, as we will see, to conclude that it will be accepted.

Recalling that \( \theta_c = 0 \), the acceptance correspondence for \( c \) is \( A_c(q) = \{x \in Q : x \leq |q|\} \). That is, \( c \) finds acceptable all proposals that are closer to the pragmatic policy. For member

\footnote{idea that in practice there exist institutional features that allow some legislators to control the agenda. For a discussion, see Baron (1998), Diermeier and Feddersen (1998), and Huber (1996).}
\(i\), with \(i = l, r\), we have

\[
A_i(q) = \left[ \frac{\theta_i - \sqrt{(\theta_i)^2 - (1 + \theta)(\theta_i^2 - q^2)}}{1 + \theta}, \frac{\theta_i + \sqrt{(\theta_i)^2 - (1 + \theta)(\theta_i^2 - q^2)}}{1 + \theta} \right].
\]

Notice that the more inefficient the status quo (that is, the further the status quo is from zero), the larger the acceptance set. Figure 2 depicts the acceptance correspondences of legislators \(r\) and \(l\). For a given ideology level \(\iota\), we now partition the policy space \(Q\) in the following sets, \(Q_1, Q_2, Q_3\) and \(Q'_3\).

\[
Q_1 = \left( -\frac{\sqrt{\theta}}{\sqrt{1 + \theta}} \iota, + \frac{\sqrt{\theta}}{\sqrt{1 + \theta}} \iota \right)
\]

\[
Q_2 = \left( -\sqrt{\theta} \iota, -\frac{\sqrt{\theta}}{\sqrt{1 + \theta}} \iota \right) \cup \left[ \frac{\sqrt{\theta}}{\sqrt{1 + \theta}} \iota, \sqrt{\theta} \iota \right)
\]

\[
Q'_3 = \left( \frac{\sqrt{\theta}(1 + 4\theta)}{\sqrt{1 + \theta}} \iota, -\sqrt{\theta} \iota \right) \cup \left[ \sqrt{\theta} \iota, \frac{\sqrt{\theta}(1 + 4\theta)}{\sqrt{1 + \theta}} \iota \right)
\]

\[
Q''_3 = \left[ -\bar{q}, -\frac{\sqrt{\theta}(1 + 4\theta)}{\sqrt{1 + \theta}} \iota \right] \cup \left[ \frac{\sqrt{\theta}(1 + 4\theta)}{\sqrt{1 + \theta}} \iota, \bar{q} \right)
\]

The interval \(Q_1\) includes status quo policies near to the pragmatic policy, \(x = 0\). From Figure 2, notice that if the status quo belongs to \(Q_1\), the acceptance correspondences of \(l\) and \(r\) are empty. This implies that under unanimity or simple majority rule reforms are not possible since a decisive coalition would have to include at least one ideological legislator. The intuition is that the status quo is sufficiently efficient that no ideological policy maker is willing to accept a reform which would force him to take a position away from the ideal point of his constituency.

If the status quo belongs to \(Q_2\), the acceptance correspondence of \(l\) and \(r\) are not empty. However, note that there is no alternative in \(Q\) that would be acceptable to both ideological

\[\text{\textsuperscript{24}To make Figure 2 more legible, the boundaries of the acceptance correspondences are drawn linear in } q.\]
legislators (i.e., the two correspondences do not intersect in this range of status quo policies). It follows that under unanimity rule reforms are not possible. Conversely, under simple majority rule a coalition including $c$ and either $l$ or $r$ would find a policy change acceptable.

If the status quo belongs to $Q_3$ where $Q_3 ≡ Q'_3 \cup Q''_3$, policy changes are possible for all majority rules. In particular, when $q = ±\sqrt{\theta}t$, we have $A_l(q) \cap A_r(q) = \emptyset$. As $q$ worsens, more policies (besides the pragmatic one) become acceptable to both ideological legislators. Notice that when $q \in Q''_3$, the status quo is so inefficient that, for example, the left-wing legislator would be willing to accept $\theta r/(1 + \theta)$, the policy reform preferred by the right-wing legislator.

So far we have characterized the set of acceptable policies and shown that under some voting rules and for some status quo policies reforms are not possible. In the next sections, we characterize equilibrium behavior in the legislative bargaining game for each majority rule. We start from the voting subgame and find proposal strategies and voting decisions. Then, in Section 3.5, we move backwards to the candidacy stage and study the decision to run for the agenda setter’s position.

### 3.2 The Voting Stage: Unanimity

Under unanimity rule, in order to pass a reform all legislators must approve the policy change and, consequently, all legislators must position themselves according to the new policy. We
now find the proposal strategy that each legislator would choose if he is recognized as agenda setter.

To begin with, suppose that $c$ is the agenda setter. Two cases must be considered. First, notice that when $q \in Q_3$ policy $x = 0$ is acceptable to both $l$ and $r$. Then, $c$ optimally proposes 0. This proposal is accepted by each ideological legislator because a single rejection would keep the status quo. Second, suppose that $q \in Q_1 \cup Q_2$. As argued in the previous section, reforms are not possible in this case. Any proposal by $c$ would be rejected. Consequently, ideological legislators are free to choose $p_l = l$ and $p_r = r$.

Suppose instead that $l$ is the agenda setter. (The proposal strategy when $r$ is recognized is symmetric). Various cases must be considered. First, suppose $q \in Q_1 \cup Q_2$. Knowing that his position will be determined by his proposal, and knowing that reforms are not possible in this case, the agenda setter is better off proposing $x_l = l$, which is rejected by the legislature but minimizes his position cost. When $q \in Q_3$ the agenda setter is willing to propose a policy reform. Legislator $l$ chooses $x_l$ to maximize (1) subject to the constraints that $x_l = p_l$ and that the proposal must be accepted by the other legislators. Looking at Figure 2, one can see that for sufficiently inefficient status quo (when $q \in Q_0'$) the acceptance constraint is not binding and $l$ is able to propose $\theta l/(1 + \theta)$, that is, the reform that optimally balances electoral concerns and policy motivations. Finally, if $q \in Q_3'$ the acceptance constraint is binding: in order to pass a reform, $l$ must propose a policy that is less ideologically biased and, therefore, acceptable to $r$. In particular, when $q \in Q_3'$ he will propose the left endpoint of the interval $A_r(q)$. The next proposition summarizes the previous discussion.

**Proposition 1** Under unanimity rule, the policy outcome in the voting subgame is $x = q$ when $q \in Q_1 \cup Q_2$ regardless of the identity of agenda setter. When $q \in Q_3$ and $c$ is the agenda setter, $x = 0$. When $q \in Q_3'$ and $r$ is selected, we have

\[
x = \frac{\theta r - \sqrt{(\theta r)^2 - (1 + \theta)(\theta^2 r - q^2)}}{1 + \theta}.
\]

When instead $q \in Q_3'$ and $l$ is selected,

\[
x = \frac{\theta l + \sqrt{(\theta l)^2 - (1 + \theta)(\theta^2 l - q^2)}}{1 + \theta}.
\]

Finally, when $q \in Q_0'$ the policy outcomes are

\[
x = \frac{\theta l}{1 + \theta}.
\]
and

\[ x = \frac{\theta r}{1 + \theta} \]

when the agenda setter is, respectively, \( l \) and \( r \).

Notice that when \( c \) is the agenda setter, decisions are either the optimal one or the status quo. As ideology bias decreases, we observe more and better reforms. In fact, the set of status quo policies for which reforms are not possible becomes smaller and, also, reforms by ideological legislators become closer to the optimal one.

### 3.3 The Voting Stage: Simple Majority

Under simple majority rule reforms must not be unanimous. This implies that it is possible for an ideological legislator to see a reform implemented without suffering a position cost.

We find the proposal strategy of each legislator. We start by supposing that \( c \) is the agenda setter. Before finding his proposal strategy, we study the voting rules of legislators \( l \) and \( r \). For any proposal \( x_c \in Q \), the payoffs to \( l \) (the row player) and \( r \) (the column player) are given by the following matrix

<table>
<thead>
<tr>
<th></th>
<th>Accept</th>
<th>Reject</th>
</tr>
</thead>
<tbody>
<tr>
<td>Accept</td>
<td>(-\theta(x_c - l)^2 - x_c^2, -\theta(x_c - r)^2 - x_c^2)</td>
<td>(-\theta(x_c - l)^2 - x_c^2, -x_c^2)</td>
</tr>
<tr>
<td>Reject</td>
<td>(-x_c^2, -\theta(x_c - r)^2 - x^2)</td>
<td>(-q^2, -q^2)</td>
</tr>
</tbody>
</table>

Three cases must be considered. First, when \( q \in Q_1 \) there is no proposal that is acceptable to either one of the two legislators. In this case, reforms are not possible since rejecting is a dominant strategy for both ideological legislators. Second, suppose that \( q \in Q_2 \). In this case, there are reforms that are acceptable to one ideological legislator, but not to both. Suppose for instance that \( c \) proposes a (relatively) left-wing proposal which is acceptable to \( l \) but not to \( r \). In this case, \( l \) accepts this proposal because he realizes that his vote is pivotal since rejecting is a dominant strategy for \( r \). Therefore, the only equilibrium is \((\text{Accept}, \text{Reject})\). Conversely, when \( x_c \) is a (relatively) right-wing proposal which is not acceptable to \( l \), the only equilibrium is \((\text{Reject}, \text{Accept})\). We now determine the optimal \( x_c \) when \( q \in Q_2 \). Legislator \( c \) chooses the best proposal among the ones that are acceptable to at least one of the two ideological legislators. Since \( r = |l| \), it easy to see that legislator \( c \) is
indifferent between proposing

\[ x'(q) = \frac{\theta l + \sqrt{(\theta l)^2 - (1 + \theta)(\theta l^2 - q^2)}}{1 + \theta} \]  

(3)

and

\[ x''(q) = \frac{\theta r - \sqrt{(\theta r)^2 - (1 + \theta)(\theta r^2 - q^2)}}{1 + \theta} \].  

(4)

where (3) is the right endpoint of the interval that contains \( l \)'s acceptable policies while (4) is the left endpoint of the interval containing \( r \)'s acceptable policies. Since \( x'(q) \) (resp. \( x''(q) \)) is not acceptable to \( r \) (resp. \( l \)), \( l \) accepts \( x'(q) \) while \( r \) accepts \( x''(q) \).

Third, suppose that \( q \in Q_3 \). In this case, the status quo is so inefficient that there exist reforms that are acceptable to all policy makers. In other words, when \( q \in Q_3 \) we have that the intersection between \( A_l(q) \) and \( A_r(q) \) is not empty and coincides with the interval \([x'(q), x''(q)]\). In particular, policy \( x = 0 \) is also acceptable. However, notice that proposing policy zero (or any other policy in the interval \([x'(q), x''(q)]\)) may lead to a coordination problem. To see this, notice that \(-\theta(x_c - \nu)^2 - x_c^2 > -q^2\) for \( i = l, r \) when \( x_c \) belongs to both \( A_l(q) \) and \( A_r(q) \). This implies that as a group, both ideological legislators are better off if they both accept rather if they both reject. However, we also have \(-x_c^2 > -\theta(x_c - \nu)^2 - x_c^2\). This implies that if one player is going to accept the proposal, the other is better off rejecting. By doing so, the legislator that rejects the proposal is able to escape the position cost \(-\theta(x_c - \nu)^2\) and, at the same time, see the reform implemented. When the proposal belongs to \([x'(q), x''(q)]\) and \( q \in Q_3 \) the above game has an equilibrium in mixed-strategies in which \( l \) and \( r \) randomize between accepting and rejecting proposal \( x_c \). \footnote{Focusing on the equilibrium in mixed-strategies is a tractable way of introducing uncertainty in the model, so that the probability of being pivotal is never one. An alternative possibility would be to consider a game in which players have a slight amount of incomplete information about the others’ payoffs. The two frameworks are however closely related (see Fudenberg and Tirole, 1991, Ch. 6.7)} Let \( \gamma_r \) (resp. \( \gamma_l \)) be the probability that \( r \) (resp. \( l \)) accepts. Then, in a mixed-strategy equilibrium \( l \) must be indifferent between the two pure strategies:

\[-\theta(x_c - l)^2 - x_c^2 = \gamma_r (-x_c^2) + (1 - \gamma_r) (-q^2) \].  

(5)

The left-hand side of the equality is the utility of accepting proposal \( x_c \). If instead \( l \) rejects \( x_c \), a reform occurs only with probability \( \gamma_r \) and the status quo is maintained with complementary probability \( 1 - \gamma_r \). We solve for \( 1 - \gamma_r \) and obtain
\[(1 - \gamma_r) = \frac{\theta(x_c - l)^2}{q^2 - x_c^2}. \tag{6}\]

The probability of \(r\) rejecting the proposal is increasing in the extent of ideological polarization and decreasing in the distance of the status quo from \(x_c\). In a similar manner, we can solve for \((1 - \gamma_l)\). Therefore, in a mixed-strategy equilibrium, with probability \((1 - \gamma_r)(1 - \gamma_l)\) a Pareto-improving policy change is not approved. It should be noticed that the game described above has also two (degenerate) pure-strategy equilibria, \((Accept, Reject)\) and \((Reject, Accept)\), in which one legislator accepts for sure. The two equilibria correspond to different beliefs about the player that concedes. We do not focus here on those equilibria since our goal is to demonstrate that in a model with position taking-preferences there are instances in which a policy that benefits all players is not accepted.\(^{26}\)

The discussion above points out that when the proposal is acceptable to both ideological legislators, with some probability a policy change does not occur. Given that \(c\) anticipates this outcome, we now determine the optimal proposal by \(c\) when \(q \in Q_3\). To do this, two steps are necessary. First, we find \(c\)’s best proposal among the ones in \([x'(q), x''(q)]\). In fact, since the rejection probabilities are a function of \(x_c\), it might be the case that a proposal slightly to the left or to the right of zero, but still acceptable to both ideological legislators, increases \(c\)’s expected payoff compared to proposing \(x_c = 0\). The following lemma states that the pragmatic policy is \(c\)’s best proposal among the proposals that lead to a coordination problem.

**Lemma 1.** Let \(q\) belong to \(Q_3\). Suppose that \(c\) expects \(l\) and \(r\) to randomize between acceptance and rejection when they receive a proposal \(x_c \in (x'(q), x''(q))\). Then, policy 0 is \(c\)’s preferred proposal among the ones in the interval \((x'(q), x''(q))\).

The proof of Lemma 1 is presented in the Appendix. Second, we find the best proposal among the ones that do not lead to a coordination problem. Notice that by offering a policy outside the interval \([x'(q), x''(q)]\) the voting game has only one pure-strategy equilibrium in which only one ideological legislator accepts with probability one since his vote is pivotal.

\(^{26}\)A similar assumption is usually made in the war of attrition literature (e.g., Alesina and Drazen, 1991) who does not focus on asymmetric equilibria in which reforms are implemented with no delay because one of the two players concedes immediately.
In other words, \( \mathcal{C} \) may propose a bad reform in order to be sure that his proposal will be accepted! However, notice that the maximization problem of \( \mathcal{C} \) over the set \( Q \setminus [x'(q), x''(q)] \) would not be defined since the admissible set is open. To avoid this open set existence problem, we suppose that when the proposal is at the endpoints of the interval \([x'(q), x''(q)]\) no coordination problem arises.\(^{27}\) Therefore, it is straightforward to see that \( \mathcal{C} \) will be indifferent between \( x'(q) \) and \( x''(q) \) which are the best proposals among the ones that assure an outright approval.

We now write down \( \mathcal{C} \)'s indirect utility as a function of \( q \) when \( q \in Q_3 \):

\[
\max \left\{ -\frac{\theta^2 \nu^4}{q^2}, -\left( \frac{\theta l + \sqrt{(\theta l)^2 - (1 + \theta)(\theta l^2 - q^2)}}{1 + \theta} \right)^2, -q^2 \right\}
\]

(7)

The first argument in the max operator is \( \mathcal{C} \)'s expected utility of offering \( x_c = 0 \). To compute it, notice that when \( \mathcal{C} \) proposes the pragmatic policy, with probability \( (q^2 - \theta \nu^2)/q^2 \) the proposal passes, and with complementary probability the status quo is maintained. It is important to note that this term is increasing as the absolute value of \( q \) becomes larger. This occurs because a bad status quo reduces the probability of rejection. The second argument is the utility of proposing either \( x'(q) \) or \( x''(q) \).\(^{28}\) Either one of these two policies would be accepted by one of the two ideological legislators with a probability equal to one. Notice that the second argument is equal to zero when \( q = \pm \sqrt{\theta} \nu \) and is decreasing as the absolute value of \( q \) increases. Finally, the third argument is the utility of keeping the status quo.

First, note that when \( q \in Q_3 \) legislator \( \mathcal{C} \) never proposes the status quo.\(^{29}\) This implies that one can disregard the third argument. Then, it follows that the proposal strategy by \( \mathcal{C} \) will be a cut-off rule, where the cut-off \( \nu(\nu) \in \left( \sqrt{\theta} \nu, \sqrt{q} \right] \) is implicitly defined by the following equality

\[
\frac{\theta \nu^2}{q} = \frac{\theta l + \sqrt{(\theta l)^2 - (1 + \theta)(\theta l^2 - q^2)}}{1 + \theta}.
\]

(8)

If \( \nu(\nu) > \sqrt{q} \), we set \( \nu(\nu) = \sqrt{q} \). According to this rule, legislator \( \mathcal{C} \) proposes policy \( x = 0 \) only

\(^{27}\)That is, when the proposal is either \( x'(q) \) or \( x''(q) \), we focus on the pure-strategy equilibrium in which, respectively, legislator \( l \) and legislator \( r \) accept with probability one.

\(^{28}\)Since \( \mathcal{C} \) is indifferent between these two proposals, without any loss of generality in writing (7) we supposed that \( \mathcal{C} \) proposes \( x'(q) \).

\(^{29}\)For example, it is easy to show that proposing \( x = 0 \) would be preferable to keeping the status quo for all \( q \in Q_3 \).
when \( q \in [-\overline{q}, -f(\overline{q})] \cup (f(\overline{q}), \overline{q}) \). When instead \( q \in \left[-f(\overline{q}), -\sqrt{\overline{q}}\right] \cup \left[\sqrt{\overline{q}}, f(\overline{q})\right] \), \( c \) proposes either \( x'(q) \) or \( x''(q) \).

After finding the proposal strategy of the non-ideological legislator, we now determine the proposal strategies of the two ideological players. Two cases must be considered. First, when \( q \in Q_1 \), policy changes are not possible since an ideological agenda setter is not willing to take a position different from the one that minimizes his position cost. In this case, the proposal will be the policy that minimizes his position cost. This proposal is obviously rejected by the other ideological legislator but also by \( c \) since, when \( q \in Q_1 \), \( c \) prefers keeping \( q \). \(^{30}\) Second, when \( q \in Q_2 \cup Q_3 \), the agenda setter is willing to suffer a position cost in order to pass a reform. The agenda setter chooses the proposal to maximize (1) subject to the constraint that \( x_i = p_i \) and subject to the constraint that the proposal must be accepted by at least one other legislator. It easy to see that the acceptance constraint is not binding under simple majority rule. In fact, proposal \( \overline{x} = \theta / (1 + \theta) \), with \( i = l, r \), is accepted by \( c \) when \( q \in Q_2 \cup Q_3 \).

Proposition 2 summarizes the previous discussion (see also Figure 3).

**Proposition 2.** Under simple majority rule, the policy outcome in the voting subgame is \( x = q \) when \( q \in Q_1 \) regardless of the identity of the agenda setter. When \( q \in Q_2 \cup Q_3 \) the policy outcomes are

\[
x = \frac{\theta l}{1 + \theta}
\]

and

\[
x = \frac{\theta r}{1 + \theta}
\]

when the agenda setter is, respectively, \( l \) and \( r \). When \( q \in \left[-f(\overline{q}), -\sqrt{\overline{q}}\right] \cup \left[\sqrt{\overline{q}}, f(\overline{q})\right] \) and \( c \) is the agenda setter, the policy outcome is either

\[
x = \frac{\theta l + \sqrt{(\theta l)^2 - (1 + \theta)(\theta^2 l - q^2)}}{1 + \theta}
\]

\(^{30}\) This is consistent with the evidence that legislators often sponsor bills that are sure to die in committee. Indeed, a large fraction of all bills that are referred to committees do not reach the floor. In the 102nd US Congress, for example, 10,238 bills and joint resolutions were introduced in the House and Senate. Only 1,201 were not were stopped at the committee stage and just 667 bills passed both chambers. See Oleszek (1996).
or

\[ x = \frac{\theta r - \sqrt{(\theta r)^2 - (1 + \theta)(\theta^2 r - q^2)}}{1 + \theta}. \]

Finally, when \( q \in \left[ -\bar{q}, -f(i) \right) \cup \left( f(i), \bar{q} \right] \) and \( c \) is the agenda setter, policy \( x = 0 \) is implemented with probability \( (1 - \gamma)^2 \) while \( x = q \) is implemented with probability \( 1 - (1 - \gamma)^2 \), where \( \gamma = (q^2 - \theta t^2)/q^2 \).

In the remainder of this section, we compare the equilibrium outcome under simple majority to the one obtained under unanimity rule.

First, notice that when the status quo is close to the efficient policy (i.e., \( q \in Q_1 \)) unanimity and simple majority are equivalent since under both constitutions the status quo is not changed. When instead the status quo is relatively inefficient (i.e., \( q \notin Q_1 \)) the two majority rules have different welfare implications depending on the location of the status quo.

**Remark 1.** *When the status quo policy is extremely inefficient, legislators make better reforms under unanimity than under simple majority rule.*

The reason is twofold. First, suppose that the agenda setter is not ideological. When \( q \) is extremely inefficient (i.e., \( q \in Q_3 \)), we have learned from Propositions 1 and 2 that the pragmatic policy is always implemented under unanimity but not necessarily under simple majority, due to a possible coordination problem. Moreover, we also saw that to avoid this problem, inefficient reforms may be proposed under simple majority rule even though the optimal policy would be acceptable to all legislators. Second, suppose that the agenda setter is ideological. In this case, unanimity is preferable since it puts more constraints on the proposer. This results into a better compromise and a less biased reform than the one that is reached under simple majority. These results would then provide a rationale for why national unity governments are usually formed in time of crisis.\(^{31}\)

**Remark 2.** *When the status quo policy is moderately inefficient, legislators make better reforms under simple majority than under unanimity rule.*

\(^{31}\)See for instance Laver and Schofield (1990) who have studied European coalition politics and shown that surplus majority governments are relatively frequent in such circumstances.
When the status quo is moderately efficient (i.e., \( q \in Q_2 \)), simple majority is preferable to unanimity rule simply because under the latter voting rule there are no reforms that are acceptable to all legislators. On the contrary, under simple majority rule there exist reforms that are acceptable to a coalition that includes \( c \) and one ideological legislator. Moreover, when the status quo is moderately inefficient reforms under simple majority rule pass with probability one.

### 3.4 The Voting Stage: Autocracy

Solving the voting game under autocracy is straightforward. Recall that an autocrat cannot be blocked by the legislature. To begin with, suppose that \( c \) is the autocrat. Then, policy \( x = 0 \) is proposed and implemented; the two ideological legislators obtain their preferred policy outcome and, at the same time, they are able to minimize their position cost by choosing \( p_i = i \). Suppose instead that the autocrat is an ideological legislator. The proposer chooses the policy outcome in order to maximize (1) subject to the constraint that \( x_i = p_i \). Since the agenda setter does not face an acceptance constraint, his proposal is \( \theta i / (1 + \theta) \).

**Proposition 3.** Under autocracy, for all \( q \in Q \) legislator \( c \) chooses \( x = 0 \), while legislators \( l \) and \( r \) choose, respectively,

\[
x = \frac{\theta l}{1 + \theta}
\]

and

\[
x = \frac{\theta r}{1 + \theta'}
\]
It should be noticed that unlike under simple majority and unanimity rule, under autocracy we have that (i) policy changes always (at least generically) occur and that (ii) reforms may actually move to a worse alternative than the status quo. The absence of status quo bias even when \( q \in Q_1 \) can be explained as follows. Recall that when \( q \in Q_1 \) an ideological legislator is not willing to suffer a position cost in order to pass a policy change. Under either simple majority or unanimity rule, an ideological agenda setter is able to escape the position cost by proposing \( x_1 = x \), a policy that the legislature would not accept. By doing so, he is able to minimize his position cost without worsening the policy outcome. Under autocracy this is not possible anymore since any proposal is automatically implemented. In other words, the downside of being an autocrat is that one cannot put the blame on the legislature for not accepting a proposal that one secretly dislikes. Interestingly, an ideological legislator would prefer being an agenda setter under simple majority rule than being an autocrat. In fact, his utility would be the same under both voting rules when \( q \in Q_2 \cup Q_3 \) since proposals coincide, but it would be lower under autocracy when \( q \in Q_1 \).

### 3.5 Candidacy Strategies and Agenda Setter Selection

So far, we have shown that the choice of the voting rule influences equilibrium proposals by affecting the acceptance constraint that the agenda setter faces. There is however a second channel through which voting rules affect equilibrium outcomes in our model. As it will be discussed in this section, voting rules provide differential incentives to run for the agenda setter’s position.

In this section, we proceed backwards and study the candidacy decisions of \( l \) and \( r \). (Recall that \( c \) is assumed to choose \( s_c = 1 \)) This will allow us to find the subgame perfect equilibrium of the bargaining game. We start by considering the candidacy strategies under autocracy and unanimity rule, which are simple to derive.

It is easy to see that under autocracy the two ideological legislators have no incentives to run for the agenda setter’s position. In fact, by not running they make sure that the pragmatic solution is implemented (this occurs because \( c \) becomes the agenda setter with probability one) and, at the same time, they are able to minimize their position cost.

Under unanimity rule, we obtain instead that whenever a reform is feasible (that is, when \( q \in Q_3 \)) both ideological legislators choose to run for office.\(^{32}\) In fact, \( l \) and \( r \) know that in

\(^{32}\text{We suppose that } l \text{ and } r \text{ never run for the agenda setter’s position if in the voting stage reforms are not} \)
the voting stage reforms will be approved and voted by all legislators and, consequently, the position cost will be suffered for sure. Therefore, running for office is a dominant strategy since it allows them to choose the position cost that solves the optimal trade-off between electoral concerns and policy motivations instead of having to accept $x = 0$ which is proposed by $c$.

We now determine the candidacy decisions of legislators $l$ and $r$ when voting is made under simple majority rule. Unlike under unanimity rule, ideological legislators have some chance of escaping the position cost if they do not run for office. For instance, they never incur the position cost if the other ideological legislator is selected since in that case proposals would be accepted by $c$ only. This implies that the incentive of legislator $i$ to choose $s_i = 1$ will, in general, depend on the candidacy strategy of the other ideological legislator. However, the assumptions that we made about the recognition probability function simplify the analysis and allow us to obtain a simple condition that determines $i$’s decision to run for office. Let $u_i(j)$ denote the expected utility of individual $i$ when $j$ is recognized as agenda setter.

**Lemma 2.** Under simple majority rule, legislator $i$, with $i = l, r$, runs for office if and only if $u_i(i) \geq u_i(c)$.

The proof of Lemma 2 is presented in the Appendix. This lemma states that an ideological legislator runs for office when the utility that derives from being the agenda setter is greater or equal than the utility that he obtains when $c$ is recognized.

Notice that the utility that an ideological legislator obtains when $c$ is elected depends on the proposal strategy that $c$ is expected to play in the voting subgame. Too see this, suppose for instance that $q \in Q_2$. If $c$ is elected, from Proposition 2 we know that he will be indifferent between making a left or a right-wing proposal. Suppose that all ideological legislators expect $c$ to make a left-wing proposal with probability one. In this case, we obtain that $l$ (resp. $r$) runs (resp. does not run) for office after realizing that he will (resp. will not) incur a position cost for sure if $c$ is elected.$^{33}$ The reason being that by running for office, $l$ hopes to have at least the chance of reducing his position cost by picking the possible. To see that this assumption is without any loss of generality, notice that when $q \notin Q_3$ the utility of an ideological legislator is the same whether or not he is recognized as agenda setter. In both cases, in fact, no policy change is made in the voting game, and $l$ and $r$ are able to choose $p_l = l$ and $p_r = r$.

$^{33}$ By the same token, we obtain that $r$ runs for office if he expects $c$ to propose a right-wing policy.
proposal that trades off electoral and policy concerns.\footnote{What if \( c \) can play mixed strategies? Suppose that mixtures over proposals occur before votes are cast. It can be shown that for some status quo policies and for some appropriate randomizing probabilities, both ideological legislators do not run for office. In other words, by selecting an "ambiguous" platform, \( c \) may give sufficient hope to both legislators of not being included in the winning coalition so as to induce them not to run for office. We do not emphasize much on this result, since it strongly relies on the assumption that \( |l| = r \). If this were not the case, \( c \) would not be indifferent between the two proposals and, consequently, \( c \) would not randomize in a subgame perfect equilibrium.}

When instead the absolute value of the status quo is greater than \( f(\nu) \), \( c \) is expected to propose the pragmatic policy. In this case, as the next lemma shows, both ideological legislators run for office under simple majority rule.

**Lemma 3.** Under simple majority rule, for all \( q \in Q_2 \) and for some status quo policies in \( Q_3 \) at least one ideological legislator does not run for office. Conversely, when \( q \) is such that \( \bar{q} > |q| > f(\nu) \), all ideological legislators run for office.

The proof of Lemma 4 is presented in the Appendix. We can now state the following proposition.

**Proposition 4.** Under autocracy, only \( c \) runs for office and the policy outcome is \( x = 0 \) for all \( q \in Q \). Under unanimity rule, all legislators run for office and, consequently, reforms are proposed by \( c \) only with probability \( 1 - \rho_l(1, 1) - \rho_r(1, 1) \). Under simple majority rule, for some \( q \in Q \) the recognition probability of \( c \) is strictly greater than \( 1 - \rho_l(1, 1) - \rho_r(1, 1) \).

Another way of stating Proposition 4 is the following.

**REMARK 3.** The higher the majority requirement, the lower the probability that reforms are proposed by a non-ideological agenda setter.

This finding points out a potential drawback of consociational arrangements. Namely, the possibility that under unanimous voting rules ideological legislators are more willing to exert effort to compete for the agenda setter's position, thereby biasing the final outcome. The intuition behind this result is that the higher the majority requirement, the lower the probability that an ideological legislator is able to avoid the position cost by choosing \( s_i = 0 \).
This decreases the expected payoff of not being agenda setter and, consequently, provides more incentives to run for office.\footnote{\textsuperscript{35}An opposite result is found, in a different context, by Yildirim (2007). By looking at a dynamic bargaining model à la Baron and Ferejohn (1986) in which agents must agree on the division of a fixed amount of surplus, he finds that as the voting rule becomes more stringent (implying that the prize from being the agenda setter gets smaller) the incentives to exert effort to become the proposer are reduced.}

### 3.6 Welfare Comparison in State $I$

We compare expected welfare as of time $t = 0$ (i.e., before knowing the state of nature and before knowing the realized status quo) under the three majority rules and see how the comparison changes as ideological bias increases. To begin with, we suppose that $\theta = 1$ and, consequently, at time $t = 1$ the ideological state occurs for sure. Then, the only uncertainty at time $t = 0$ is given by the location of the status quo. Recall that $q$ is distributed uniformly on $Q$. The welfare criterion that we adopt is the following. We find which voting rule is capable (in expected terms) to pass reforms that bring the policy outcome closer to $x = 0$.\footnote{\textsuperscript{36}We refer the reader to Section 3.7 for a discussion of this assumption.}

It is straightforward that in state $I$, autocracy is first-best since in equilibrium $c$ is elected with probability one and, consequently, the pragmatic solution is implemented regardless of the realized status quo and regardless of the extent of ideological polarization. We emphasize two caveats to this result. First, in this paper we assumed that all legislators agree that $x = 0$ is the pragmatic policy. If this were not the case, most of our results would still hold but it would not be true anymore that autocracy is first-best. Second, we are not claiming that autocracy is first-best in all circumstances. In many other policy decisions (which are made in the residual state $R$) it is likely that autocracy would lead to a poor outcome (e.g., expropriation).

In contrast to the equilibrium welfare under autocracy, welfare under either simple majority or unanimity rule depends on the extent of ideological polarization. This occurs because $\nu$ affects (i) the set of initial status quo policies where reforms are possible, (ii) it determines how biased policy proposals are and (iii) it changes the incentives to run for the agenda setter’s position.

In order to understand how simple majority rule compares to unanimity rule, we investigate the validity of Remark 1 in light of what we have learned in the previous section.
Recall that Remark 1 states that each legislator (whether ideological or not) makes better decisions under unanimity rule than under simple majority when $q \in Q_3$. However, since we expect ideological legislators to make worse decisions than the non-ideological one, it matters how the majority rule, by changing the incentives to run for office, affects the equilibrium probability that each legislator is recognized. Since we have shown (see Proposition 4) that ideological legislators are less willing to run for the agenda setter’s position under simple majority than under unanimity rule, a natural question arises. Does Remark 1 still hold when we take into account the greater incentives to run for the agenda setter’s position under unanimity rule? As the next lemma shows, the answer is affirmative.

**Lemma 4** When $q \in Q_3$, expected welfare is higher (i.e., decisions are more pragmatic) under unanimity rule than under simple majority rule. The converse is true when $q \in Q_2$.

The intuition for the first part of Lemma 4 is that the stronger incentives for ideological legislators to run for office under unanimity rule are compensated by the tighter constraints that unanimity rule puts on the proposals by ideological legislators. This is why Remark 1 remains valid when we endogenize the candidacy decisions. The second part of Lemma 4 is straightforward since no reforms are made under unanimity rule when $q \in Q_2$ while policy changes always take place under simple majority rule.

Deriving the analytical expression for expected welfare under simple majority and unanimity rule turns out to be analytically involved. However, the next proposition provides sufficient conditions that allow us to conclude that unanimity rule is preferred to simple majority rule when $\iota$ is sufficiently small, but not when $\iota$ is large. The underlying intuition is the following. First, recall (from Lemma 4) that unanimity is strictly preferable to simple majority when $q \in Q_3$, while the converse is true when $q \in Q_2$. Further, after looking at Figure 2, notice that the set $Q_3$ becomes smaller as $\iota$ reaches its upper bound $\bar{q}$ (provided that $\theta$ is sufficiently close to one). In other words, when ideological bias is too severe, it is difficult to make all legislators agree on a reform except for very few (and inefficient) status quo policies. This is why for large $\iota$ simple majority rule is preferable to unanimity rule since it is very unlikely that the realized status quo will end being in $Q_3$. On the other hand, when $\iota$ goes to zero, the set $Q_3$ expands relatively to $Q_2$ and covers almost the entire

---

37 We write them down in the proof of Proposition 5 in the Appendix.
policy space. Since in this case the status quo will likely lie in the set \( Q_3 \), coordination problems are more likely to arise and unanimity rule becomes preferable to simple majority rule for low \( \iota \).

Let \( Eu(M, I) \) denote expected utility when the state is \( I \) under the voting rule \( M \), where \( M \in \{ A, SM, U \} \).

**Proposition 5.** Autocracy is first-best for all \( \iota \in [0, \bar{\iota}] \). Suppose that the probability that \( c \) is elected when \( s_l = s_r = 1 \) is (moderately) large. Then, there exists a \( \iota \in (0, \bar{\iota}) \) such for all \( \iota \in (0, \iota) \), we have

\[
\]

Suppose that \( \theta = 1 \). Then there exists a \( \bar{\iota} \in (0, 1) \) such for all \( \iota \in (\bar{\iota}, \bar{\iota}] \), we have

\[
\]

The proof of Proposition 5 is presented in the Appendix. In Figure 4 we draw expected welfare under simple majority rule and unanimity rule as a function of \( \iota \). We assumed \( \theta = 0.5 \), \( \rho_l(1,1) = \rho_r(1,1) = 0.25 \) and \( \bar{\iota} = 2.5 \). As predicted by Proposition 5, unanimity rule dominates simple majority rule for small values of \( \iota \) but not for values close to \( \bar{\iota} \).
4 Residual State and Constitutional Choice

Residual State. When \( \vartheta < 1 \), the constitution will also take into account the optimality of the voting rule in the residual state \( R \), occurring with probability \( 1 - \vartheta \). In this paper, we will be agnostic about the type of decision that is made in \( R \). As discussed before, this state summarizes all the contingencies and decisions that cannot be categorized as in Section 3.1. For example, the legislature could handle a distributive decision as in Alesina, Aghion and Trebbi (2004). In this case, the optimal voting rule in state \( R \) would have to solve the optimal trade-off between the need of protection from expropriation and the ex-post flexibility of implementing ex-ante efficient reforms. Alternatively, we could consider a spatial model of policy choice and suppose that legislators’ utilities are \( u_i(x) = -(x - \bar{x}_i)^2 \) with \( i = l, r, c \), where \( x \in X \subset R_+ \) could represent the amount of public good or the capital/income tax and \( \bar{x}_i \) the preferred policy by legislator \( i \).\(^{38}\)

We suppose that individuals are ex-ante identical. Then, as of time \( t = 0 \), equilibrium welfare in state \( R \) is \( Eu(SM, R) \), which is the same for all legislators. Equilibrium welfare in state \( R \) and the ranking among the three voting rules will clearly depend on the specifics of the setup that one assumes. For example, it could depend on the degree of risk aversion, on the difficulty of expropriation, on the existence of compensatory schemes, on the wealth distribution, etc. What is important for our analysis is that parameter \( \vartheta \) does not play a role in state \( R \).

Assumption Equilibrium behavior in \( R \) is independent on \( \vartheta \)

The assumption above allows us to "decouple" the two states. This implies that the ranking among voting rules in state \( R \) does not depend on \( \vartheta \). Given that we have three voting rules, we can order them in \( 2^3 \) possible ways. However, we assume that autocracy is the least preferable voting rule in state \( R \).\(^{39}\) This leaves us with two cases:

\[
Eu(SM, R) > Eu(U, R) > Eu(A, R). \tag{R1}
\]

\(^{38}\)In a preliminary draft of this paper, we solved for the optimal voting rule in this setup. We assumed, as in state \( I \), that decisions are made by playing an agenda-setting game. Results are the following. Regardless of the voting rule, all legislators run for office. Moreover, autocracy is never optimal since it leads to excessive uncertainty, which is undesirable ex ante because legislators are risk averse. The optimal voting rule turns out to be simple majority rule when the recognition probability of the center legislator is sufficiently large and unanimity rule otherwise. Detailed results are available upon request.

\(^{39}\)In a distributive problem, it would lead to expropriation. In spatial setting, it would lead to political
Figure 5: Optimal Constitution under R1

and

\[ Eu(U, R) > Eu(SM, R) > Eu(A, R). \]  \hspace{1cm} (R2)

In the first case, simple majority rule is preferred to unanimity rule, while the converse is true in the second case.

**Constitutional Stage.** The optimal (uncontingent) constitution \( M^* \) at time \( t = 0 \) solves the following problem

\[
\max_{M \in \{U, SM\}} \vartheta Eu(M, I) + (1 - \vartheta) Eu(M, R).
\]

That is, \( M^* \) should make policy zero more likely to occur in state \( I \) and maximize welfare in state \( R \). The choice of \( M^* \) obviously depends on the value of \( \vartheta \), but also on the cardinal utility differences among voting rules in each state.

The goal of this section is to show how the optimal majority rule \( M^* \) changes as ideological bias increases.

The (R1) case is indeed the most interesting. In this case, one can show that for sufficiently low \( \iota \) simple majority is optimal. On the other hand, as \( \iota \) increases, provided that \( Eu(SM, R) \) is not much greater than \( Eu(U, R) \) we have that unanimity rule becomes uncertainty, which is welfare reducing. In Alesina, Aghion and Trebbi (2004) the optimal voting rule is never lower than 50% of the votes.
optimal. On the other hand, when $\rho$ is very large and $\vartheta$ sufficiently important, autocracy may become optimal. Figure 5 draws the weighted sum of welfare in the two states as a function of $\rho$ for some given parameters. By taking the upper envelope of the three lines, where each line corresponds to welfare under a different voting rule, we obtain that for low $\rho$ simple majority rule is optimal. Unanimity becomes optimal as $\rho$ increases. For larger values of $\rho$ simple majority is again optimal and, for very large $\rho$, autocracy becomes the preferred majority rule at the constitutional stage. This implies that under the assumption (R1) we may obtain inverted U-relationship between the extent of ideological bias and the majority requirement. When instead assumption (R2) holds, the optimal majority rule for $\rho$ close to zero would be unanimity rule and the optimal majority rule would be decreasing in $\rho$.

5 Moderate vs. Extreme Left

In this section, we suppose that there are no right-wing voters but only moderate and extreme left-wing voters. Then, $N = \{l, \bar{l}, c\}$ with $l < \bar{l} < c = 0$. Legislator $l$ represents the extreme-left voters, while legislator $\bar{l}$ represents the more moderate (but still ideologically biased) ones. Suppose that the policy space is now $Q = [-\bar{q}, 0]$.

Instead of providing a full equilibrium characterization, we give a flavor of the main results. In Figure 5 we draw the acceptance sets of $l$ and $\bar{l}$. 
We define the following intervals:

\[ Q_{L1} = \left( \frac{\sqrt{\theta}}{1 + \theta}, 0 \right), \quad Q_{L2} = \left( \frac{\sqrt{\theta}}{1 + \theta}, \frac{\sqrt{\theta}}{1 + \theta} \right), \quad Q_{L3} = \left( \sqrt{\theta l}, \sqrt{\theta l} \right), \quad Q_{L4} = \left[ -q, \sqrt{\theta l} \right] \]

When \( q \in Q_{L1} \) reforms are clearly not possible, except under autocracy. When \( q \in Q_{L2} \) reforms by all legislators are suboptimal since \( x = 0 \) is not acceptable. When \( q \in Q_{L3} \) reforms proposed by \( c \) are optimal. Notice that in this case, the moderate left-wing legislator would accept the pragmatic policy with probability one since \( x = 0 \) is not acceptable to \( l \). Finally, when \( q \in Q_{L4} \), the optimal policy, which is now acceptable to both ideological legislators, may not be approved under simple majority rule because of a possible coordination problem.

There are various similarities with the symmetric setup. First, we obtain that unanimity rule is able to overcome the inefficiencies associated with a coordination problem when the status quo is very inefficient and \( c \) is the agenda setter. Second, the set of status quo policies for which policy changes are possible is smaller under unanimity rule than under simple majority rule. Third, under unanimity rule all legislators run for office whenever a reform is expected in the voting game. Finally, for status quo policies closer to zero, reforms are possible only under autocracy.

There are however a few important differences compared to the previous setup. First, notice that under simple majority rule proposals by \( c \) will be mainly addressed to the moderate legislator \( \overline{l} \).\(^{40}\) This increases (resp. decreases) the incentives of \( \overline{l} \) (resp. \( l \)) to run for office. Then, in equilibrium under simple majority rule we obtain that proposals are mostly made by either \( c \) or \( \overline{l} \), with \( l \) occupying a passive role in the legislature.\(^{41}\) Second, and most importantly, it is not true anymore that unanimity rule forces ideological legislators to moderate their proposals. In the symmetric setup, unanimity rule leads to more centered proposals compared to simple majority rule because an ideological legislator has to make his proposal acceptable to the legislator representing the opposed ideological constituency. When ideological voters are on the same side with respect to the pragmatic policy, this moderating force is not at work anymore. In particular, it might now be the case that reforms by an ideological legislator are worse under unanimity than under simple majority rule. This

\(^{40}\)The only exception is when \( q \in Q_{L4} \). In this case, \( c \) may propose policy zero, which is acceptable to all legislators. Then, the winning coalition may include the extreme legislator.

\(^{41}\)Again, the only exception is when \( q \in Q_{L4} \). In this case, \( c \) may propose policy zero. Similarly to what we have shown in Lemma 3, all ideological legislators would then run for office.
happens because under unanimity rule a moderate left-wing legislator has to make the reform acceptable to all legislators, including $l$. Under simple majority rule, instead, the legislator representing more moderate voters could make proposals that are acceptable to $c$ only.

These differences have important implications concerning the optimality of consociational arrangements. In the symmetric setup, we showed that when $q$ belongs to $Q_3$ unanimity rule is preferable to simple majority rule. The reason was twofold: because unanimity rule allows to solve a possible coordination problem when $c$ is the proposer and because proposals by ideological legislators are more moderate under unanimity rule. In an asymmetric setup, the first reason is still present, but not necessarily the second one. Recalling that unanimity rule gives $l$ greater incentives to run for office than simple majority rule, this suggests that consociational arrangements might be less desirable for countries where ideological bias is highly skewed towards one side of the political spectrum.

Before concluding the section, we do some comparative statics with respect to the location of $l$. It is straightforward to see that as $l$ gets smaller (i.e., extremism increases), unanimity rule becomes undesirable since reforms that are acceptable to all legislators are possible only in very limited cases. When instead $l$ approaches $\bar{l}$, on the one hand the set of status quo policies in which reforms are possible under simple majority but not under unanimity rule becomes smaller. On the other hand, more policies are acceptable to both legislators and, consequently, coordination problems are more likely to arise. Therefore, one should expect, all other things being equal, unanimity rule to be more desirable relative to simple majority rule when $l$ is closer to $\bar{l}$.

6 A Look at the Data

In this section, we look at the cross sample of countries and see how, in practice, the degree of consociationalism in the decision making process of a country relates to its degree of ideological polarization.

This section is not meant to be a formal test of our theory. Of course, there are measurement problems for both variables. As an indicator of ideological polarization, we use various indices of ethno-linguistic and religious fractionalization. These measures may, of course, capture differences among sectorial groups that are not ideological. For instance, ethnic fractionalization is correlated with income and regional differences. Moreover, it is
also debatable whether or not these measures of fragmentation are a good proxy of polarization, that is of the distance of the two most extreme positions. The fractionalization index measures the probability that two randomly drawn individuals from the overall population belong to different groups.

To measure the degree of consociationalism, we use the XCONST variable from the Polity IV dataset. This variable captures the extent on constraints on the decision making power of chief executive. Usually, limits on the executive are imposed by the legislative, but also by other "accountability groups", such as the judiciary and the military. [...]

7 Conclusion

In this paper, we investigated the claim that in a plural society (that is, a society divided along cleavages of religious, linguistic, cultural, racial or ethnic nature) a consociational democracy is more suitable than a majoritarian democracy. We found that in some circumstances the optimal majority rule is hump-shaped with respect to ideological polarization. This suggests that consociational arrangements may not be desirable in a society that is deeply divided. But on the other hand, they may be suitable for societies that are moderately divided.

Before concluding, we relate our results to the ones obtained by the literature on the political economy of reforms (e.g. Alesina and Drazen, 1991, and Drazen and Grilli, 1993). The conventional wisdom in that literature is that there are two favorable conditions for reforms: having a bad status quo (e.g., being in an economic crisis) and having an executive with few constraints. Similarly to that literature, this model shows that reforms always occur if the executive has no constraints (i.e., under autocracy) and that reforms are not likely when the status quo is relatively efficient. However, in our model we also obtain that reforms under simple majority rule may not occur when the status quo is very inefficient, while they do occur with probability one when the status quo is moderately inefficient. Moreover, in the context of our model endowing all legislators with veto power may lead, for the reasons outlined in the paper, to more and better reforms.

In this paper we focused on two merits of unanimous voting rules: their capacity to solve coordination problems that prevent legislators from passing Pareto improving reforms and the fact unanimity rule helps moderate proposals by ideological legislators. As such, we
abstracted from other merits of consociationalism: namely, the possibility that unanimity may facilitate the trading of favors, improve trust, lead to a better quality of government (as measured, for instance, by large voter turnout and higher indexes of satisfaction with democracy), and also the possibility that consociationalism may indeed decrease the extent of ideological polarization. All these extensions are certainly worthy of future research.
APPENDIX

Proof of Lemma 1: Suppose \( q \in Q_3 \). When \( x_c \in [x'(q), x''(q)] \) and a coordination problem is expected to arise, \( c \)'s expected payoff is

\[
-\frac{q^2 \theta^2 (x_c - r)^2 (x_c - l)^2}{(q^2 - x_c^2)^2} - x_c^2 (1 - \frac{\theta^2 (x_c - r)^2 (x_c - l)^2}{(q^2 - x_c^2)^2}),
\]

which can be written as

\[
-\frac{\theta^2 (x_c^2 - r^2)^2}{(q^2 - x_c^2)^2} - x_c^2.
\]

We find its derivative with respect to \( x \):

\[
-\frac{4 \theta^2 x (x_c^2 - r^2)(q^2 - x_c^2) - 2 x \theta^2 (x_c^2) - 2x(q^2 - x_c^2)^2}{(q^2 - x_c^2)^2}.
\]

The derivative’s sign is given by the numerator, which can be written as

\[
-2x \left( 2 \theta^2 AB + \theta^2 A^2 + B^2 \right),
\]

where \( A \equiv (q^2 - x_c^2) \) and \( B \equiv (x_c^2 - r^2). \) Since proposals are acceptable to \( c \), we have \( A > 0 \). Notice that the term in parenthesis is strictly positive when \( B > 0 \), but it is also positive when \( B < 0 \). In fact, since \( \theta \in (0, 1] \) we have that

\[
2 \theta^2 AB + \theta^2 A^2 + B^2 > (\theta A + B)^2 > 0.
\]

Then, when \( x \) is positive (resp. negative), the derivative is negative (resp. positive). This implies that \( c \)'s expected utility from proposing in \( c \) is single-peaked and \( x_c^2 = 0 \) is \( c \)'s best proposal in \([x'(q), x''(q)]\), thus establishing the desired result.

Proof of Lemma 2: Recall that \( s_c = 1 \). Two cases must then be considered. First, if ideological legislator \( i \) expects \( -i \) to choose \( s_{-i} = 1 \), \( i \) runs for office if and only if

\[
\rho_i(1, 1) u_i(i) + (1 - \rho_i(1, 1) - \rho_{-i}(1, 1)) u_i(c) + \rho_{-i}(1, 1) u_i(-i) \geq (1 - \rho_{-i}(1, 0)) u_i(c) + \rho_{-i}(1, 0) u_i(-i).
\]

The left-hand (resp. right-hand) side is the expected utility when all legislators (resp. only \( -i \) and \( c \)) run for office.

Second, suppose instead that \( i \) does not expect \( -i \) to choose \( s_{-i} = 1 \). In this case \( i \) runs for office if and only if

\[
\rho_i(1, 1) u_i(i) + (1 - \rho_i(1, 0)) u_i(c) \geq u_i(c).
\]
Since we assumed that \( \rho_i(1,0) = \rho_i(1,1) \) for \( i = l, r \), the two inequalities above simplify to \( u_i(i) \geq u_i(c) \) .

**Proof of Lemma 3:** Suppose that \( q \in \left[ -f(i), \frac{\sqrt{\theta}}{1+\theta}l \right] \cup \left[ \frac{\sqrt{\theta}}{1+\theta}l, f(i) \right] \). Notice that this set includes all status quo policies from set \( Q_2 \) and some status quo policies from \( Q_3 \) since \( f(i) > \sqrt{\theta}l \). In this region of status quo policies, Proposition 2 tells us that \( c \) proposes either a left-wing or a right-wing policy. From Lemma 2, \( l \) runs for office if and only if

\[
-\left( \frac{\theta l}{1+\theta} \right)^2 - \theta\left( \frac{\theta l}{1+\theta} - l \right)^2 \geq -\eta q^2 - (1-\eta)\left( \frac{\theta r - \sqrt{(\theta r)^2 - (1+\theta)(\theta r^2 - q^2)}}{1+\theta} \right)^2. \tag{A1}
\]

To explain the right-hand side, notice that with probability \( \eta \), where \( \eta \in \{0,1\} \), legislator \( c \) makes an offer that is acceptable only to \( l \). Then, \( l \)'s acceptance constraint is binding. With complementary probability \( 1-\eta \), \( c \) proposes a policy that is acceptable to \( r \) only and \( l \) suffers no position cost. The equivalent condition for legislator \( r \) is

\[
-\left( \frac{\theta r}{1+\theta} \right)^2 - \theta\left( \frac{\theta r}{1+\theta} - r \right)^2 \geq -(1-\eta)q^2 - \eta\left( \frac{\theta l + \sqrt{(\theta l)^2 - (1+\theta)(\theta l^2 - q^2)}}{1+\theta} \right)^2. \tag{A2}
\]

Since \( \eta \in \{0,1\} \), two cases must be considered. Suppose \( \eta = 1 \). We first show that \( l \) always run for office. In fact, condition (A1) becomes

\[
-\left( \frac{\theta l}{1+\theta} \right)^2 - \theta\left( \frac{\theta l}{1+\theta} - l \right)^2 \geq -q^2, \tag{A3}
\]

which is always satisfied when \( q \in \left[ -f(i), -\frac{\sqrt{\theta}}{1+\theta}l \right] \cup \left[ \frac{\sqrt{\theta}}{1+\theta}l, f(i) \right] \). To prove the first part of Lemma 3, we show that \( r \) never run for office when \( q \in Q_2 \) and \( \eta = 1 \). When \( q = \pm \frac{\sqrt{\theta}}{1+\theta}l \) condition (A2) becomes

\[
-\left( \frac{\theta r}{1+\theta} \right)^2 - \theta\left( \frac{\theta r}{1+\theta} - r \right)^2 \geq -\left( \frac{\theta l}{1+\theta} \right)^2,
\]

which is clearly not satisfied. It easy to see that when \( \eta = 1 \) condition (A2) is not satisfied for all \( q \in \left[ -\sqrt{\theta}l, -\frac{\sqrt{\theta}}{1+\theta}l \right] \cup \left[ \frac{\sqrt{\theta}}{1+\theta}l, \sqrt{\theta}l \right] \). In particular, when \( q = \pm \sqrt{\theta}l \) condition (A2) becomes

\[
-\left( \frac{\theta r}{1+\theta} \right)^2 - \theta\left( \frac{\theta r}{1+\theta} - r \right)^2 \geq 0,
\]

which is again not satisfied. Therefore, it follows that for status quo policies sufficiently close to \( q = \sqrt{\theta}l \) (including then also policies in \( Q_3 \)) we have that \( r \) does not run for office.
Following a similar argument, one can show that for all $q$ in $Q_2$ and for some policies in $Q_3$, $l$ does not run for office when $\eta = 0$.

We now prove the all legislators run for office when $\bar{q} \gg |q| > f(\iota)$. Whenever $c$ is selected, $l$ and $r$ expect $c$ to propose 0 and, consequently, they expect a coordination problem to arise. The condition that determines $i$’s decision to run for office is:

$$-\left(\frac{\theta_i}{1 + \theta}\right)^2 - \theta\left(\frac{\theta_i}{1 + \theta} - i\right)^2 \geq -\theta i^2.$$ 

The right-hand side is the expected payoff of the mixed-strategy equilibrium for an ideological legislator. Notice that this inequality is always satisfied, implying that both ideological legislators run for office.

**Proof of Lemma 4:** We first show that when $q \in Q_3$ welfare under unanimity rule dominates welfare under simple majority rule. From Lemma 3, when $\bar{q} \gg |q| > f(\iota)$ all legislators run for office under both voting rules. From Proposition 2 we know that in this range of status quo policies, proposals under unanimity rule are more pragmatic than proposals under simple majority rule. Then, it follows that unanimity rule dominates simple majority rule in this region of initial conditions. When $q \in \left[-f(\iota), -\sqrt{\theta} \right] \cup \left[\sqrt{\theta}, f(\iota)\right]$ the probability that proposals are made by $c$ is higher under simple majority rule. However, from Propositions 1 and 2 we know that under simple majority rule an ideological agenda setter makes decisions that are as pragmatic as the ones that are made by a non-ideological agenda setter under unanimity rule. Therefore, also in this range of status quo policies we obtain that unanimity rule is preferable to simple majority rule.

To complete the proof, we show that when $q \in Q_2$ welfare under simple majority rule dominates welfare under unanimity rule. This is straightforward since under unanimity rule (but not under simple majority rule) policy improving reforms are not passed.

**Proof of Proposition 5:** We first write the expression of $Eu(M, I)$ for all $M \in \{A, SM, U\}$. Ex ante expected utility under autocracy is

$$Eu(A, I) = 0.$$ 

Ex ante expected utility under unanimity rule is

$$Eu(U, I) = -\int_{Q_1 \cup Q_2} q^2 dF q - 2\rho \int_{Q_3} (x'(q))^2 dF q - 2\rho \int_{Q_3} \left(\frac{\theta l}{1 + \theta}\right)^2 dF q.$$
To explain this expression, notice that when \( q \in Q_1 \cup Q_2 \) reforms are not possible under unanimity rule. This explains the first term. From Proposition 4 we know that whenever reforms are possible, both ideological legislators run for office so that with probability equal to \( 2\rho \equiv \rho_l(1, 1) + \rho_r(1, 1) \) the proposal is ideologically biased. With remaining probability, the agenda setter is \( c \) and the chosen proposal is \( x = 0 \). The proposals of \( l \) and \( r \) depend on whether or not their acceptance constraints are binding. Without any loss of generality, since the proposals by \( l \) and \( r \) are equally distant from zero, in the expression above we supposed that when an ideological legislator is selected, \( l \)'s proposal is implemented. Proposals by \( l \) are equal to \( x_0(q) \) (which is defined in equation (3) in the paper) and \(-\theta l/(1 + \theta)\) when, respectively, \( q \in Q'_3 \) and \( q \in Q''_3 \).

We now write \( Eu(SM, \mathcal{I}) \). Recall from Lemma 3 that at least one ideological legislator runs for office when \( q \in Q_2 \cup Q_3 \). Moreover, when \( \bar{q} > |q| > f(i) \) all legislators run for office. We partition the set \([-f(i), \frac{\sqrt{\bar{q}}}{\sqrt{1+\theta}}], \left[\frac{\sqrt{\bar{q}}}{\sqrt{1+\theta}}, f(i)\right]\) in the following two sets: \( S_1, S_2 \). When \( q \in S_1 \), only one ideological legislator runs for office and is recognized with probability \( \rho \). When \( q \in S_2 \) both legislators run for office. From Proposition 2, whenever ideological legislator \( i \) is recognized, he proposes \( \theta i/(1 + \theta) \). Whenever \( c \) is recognized, he proposes either \( x'(q) \) (which is defined in equation (3) in the paper) or \( x = 0 \).

\[
Eu(SM, \mathcal{I}) = -\int_{Q_1} q^2dF - \rho \int_{S_1} \left(\frac{\theta_l}{1 + \theta}\right)^2 dF - (1 - \rho) \int_{S_1} (x'(q))^2 dF - 2\rho \int_{S_2} \left(\frac{\theta_l}{1 + \theta}\right)^2 dF - \rho \int_{S_2} (x'(q))^2 dF - 2\rho \int_{f(i)} \frac{\theta_l^2 i^4}{q^2} dF.
\]

From Proposition 1, 2, and 3, it follows that when \( i = 0 \), \( Eu(A, \mathcal{I}) = Eu(AM, \mathcal{I}) = Eu(U, \mathcal{I}) = 0 \). When \( i > 0 \), it is also easy to show that \( Eu(A, \mathcal{I}) > Eu(U, \mathcal{I}) \) and \( Eu(A, \mathcal{I}) > Eu(SM, \mathcal{I}) \). Recall in fact that autocracy always implements the first best while under the other constitutions the policy outcome will be biased with some positive probability (for example, when \( q \in Q_1 \)).

CLAIM 1: When \( \theta = 1 \) there exists a \( \bar{i} \in (0, \bar{q}) \) such that for all \( i \in (\bar{i}, \bar{q}] \), we have \( Eu(SM, \mathcal{I}) > Eu(U, \mathcal{I}) \).

Proof of Claim 1: First, notice that when \( \theta = 1 \) and \( i = \bar{q} \), we have that \( Q_3 = \{ \pm \bar{q} \} \).
The probability that a reform is passed under unanimity rule is then zero. On the other hand, \( Q_2 = \left( -\frac{1}{\sqrt{2}}, -\frac{1}{\sqrt{2}}, 1 \right) \cup \left[ \frac{1}{\sqrt{2}}, 1 \right] \). Then, reforms are passed with positive probability under simple majority rule. It follows that \( E(SM, I) > Eu(U, I) \) when \( \theta = 1 \) and \( \iota = \overline{\iota} \). Notice that \( Eu(U, I) \) is decreasing in \( \iota \). Notice instead that \( Eu(SM, I) \) may depend in a non trivial (possibly discontinuous) way on \( \iota \) since the candidacy strategies change with \( \iota \). Let \( Eu^j(SM, I) \) denote the expected utility under simple majority when \( j \) ideological legislators run for office, with \( j = 1, 2 \). Then, \( Eu(SM, I) \geq \min \{ Eu^1(SM, I), Eu^2(SM, I) \} \). From Proposition 2 we have that \( Eu^j(SM, I) \) is continuous and decreasing. Then, there exists a cutoff \( \tau \in (0, \overline{\iota}) \) such that for all \( \iota \in (\overline{\iota}, \overline{\iota}] \), we have that \( \min \{ Eu^1(SM, I), Eu^2(SM, I) \} > Eu(U, I) \) and this concludes the proof of Claim 1.

CLAIM 2: We show that the cutoff \( f(\iota) \leq 2\sqrt{\theta} \).

Proof of Claim 2: To show this we must show that at \( q = 2\sqrt{\theta} \) we have that

\[
-\frac{\theta^2 \iota^4}{q^2} > -\left( -\frac{\theta \iota + \sqrt{(\theta \iota)^2 - (1 + \theta)(\theta \iota^2 - q^2)}}{1 + \theta} \right)^2.
\]

Substituting for \( q \), we obtain

\[
-\frac{\theta}{4} > -\left( -\frac{\theta + \sqrt{3\theta + 4\theta^2}}{1 + \theta} \right)^2.
\]

After some algebra, one can easily verify that the equality is satisfied for all \( \theta \in (0, 1] \).

CLAIM 3: Suppose that the probability that \( c \) is selected when \( s_l = s_r = 1 \) is sufficiently large. There exists a \( \underline{\iota} \in (0, 1) \) such for all \( \iota \in (0, \underline{\iota}) \), we have

\[
Eu(U, I) > Eu(SM, I).
\]

Proof of Claim 3: We now find a lower bound for the utility difference \( Eu(SM, I) - Eu(U, I) \). One can show that

\[
Eu(U, I) - Eu(SM, I) > -\frac{2}{\overline{\iota} - q} \int_{\sqrt{\overline{\iota}}}^{\overline{\iota}} q^2 dq + \frac{2(1 - 2\rho)}{q - \overline{\iota}} \int_{f(i)}^\pi \theta^2 \iota^4 dq
\]

The right-hand side of the above expression is the difference between welfare under unanimity and simple majority rule in the following hypothetical scenario. We suppose that under
simple majority rule the legislature chooses the pragmatic policy when \( q \in Q_2 \) while under unanimity rule it selects (as in the true equilibrium) the status quo. Second, when \( \sqrt{\theta t} \leq |q| \leq f(\iota) \) we suppose that equilibrium behavior under simple majority and unanimity rule coincide. In the true equilibrium, under unanimity rule but not under simple majority rule the proposal is the optimal policy when \( c \) is the agenda setter. Moreover, under unanimity rule proposals are less biased than under simple majority rule when the agenda setter is ideological. Third, when \( f(\iota) < |q| \leq \bar{q} \) we suppose that the proposal by an ideological legislator is the same under both voting rules. In the true equilibrium, ideological legislators make better decision under unanimity rule. Moreover, when \( f(\iota) < |q| \leq \bar{q} \) and \( c \) is the agenda setter, we have that a coordination problem arises under simple majority rule while under unanimity rule the optimal policy is implemented. It is clear that the assumed scenario is very favorable (resp. unfavorable) for simple majority (resp. unanimity) rule. In other words, the right-hand side is a lower bound for \( E(U, \mathcal{I}) - Eu(SM, \mathcal{I}) \). Solving the integrals, we obtain:

\[
U(U, \mathcal{I}) - U(SM, \mathcal{I}) > -\frac{2}{q - q} \left[ \frac{1}{3} q^3 \right] \sqrt{\eta} \theta + \frac{2(1 - 2\rho)}{q - q} \left[ -\frac{\theta^2 \iota^4}{q} \right] f(\iota) \\
> -\frac{1}{q - q} \frac{2}{3} \iota^3 \left( \sqrt{\theta} - \frac{\sqrt{\theta}}{1 + \theta^3} \right) + \left( -\frac{\theta^2 \iota^4}{q} + \frac{\theta^2 \iota^4}{f(\iota)} \right)
\]

Using Claim 2, and defining \( \rho_c \equiv 1 - 2\rho \), we obtain

\[
U(U, \mathcal{I}) - U(SM, \mathcal{I}) > -\frac{\iota^3}{q - q} \left( \frac{2}{3} \sqrt{\theta} - \frac{2}{3} \frac{\sqrt{\theta}}{1 + \theta^3} \right) + \frac{2\rho_c}{q - q} \left( -\frac{\theta^2 \iota^4}{q} + \frac{\theta^2 \iota^4}{2\sqrt{\theta t}} \right) \\
= \frac{\iota^3}{q - q} \left( -\frac{2}{3} \sqrt{\theta} + \frac{2}{3} \frac{\sqrt{\theta}}{1 + \theta^3} + \rho_c \sqrt{\theta} - \frac{2\theta^2 \iota \rho_c}{q} \right)
\]

We define the constant

\[
C \equiv -\frac{2}{3} \sqrt{\theta} + \frac{2}{3} \frac{\sqrt{\theta}}{1 + \theta^3} + \rho_c \sqrt{\theta}.
\]

Notice that \( C \) does not depend on \( \iota \). For \( \rho_c \) sufficiently large \( C > 0 \) for all \( \theta \in (0, 1] \). Writing the above inequality as

\[
U(U, \mathcal{I}) - U(SM, \mathcal{I}) > -\frac{1}{q - q} \iota^3 \left( C - \frac{2\theta^2 \iota \rho_c}{q} \right)
\]
one can see that sufficiently small \( \iota \) the term in parenthesis is positive. This proves that unanimity rule dominates simple majority rule for sufficiently small \( \iota \).
References


[16] Bulow, Jeremy and Klemperer A generalized war of attrition AER


[34] Lijphart Arend 2007 Thinking about Democracy, Power Sharing and Majority Rule in Theory and Practice, Routledge


