Skill-biased change in entrepreneurial technology*

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Abstract

In contrast to the very large literature on skill-biased technical change among workers, there is hardly any work on the importance of skills for the entrepreneurs who employ those workers, and in particular on their evolution over time. This paper proposes a simple theory of skill-biased change in entrepreneurial technology that fits with cross-country, historical and micro evidence. For this, it introduces two additional features into an otherwise standard occupational choice, heterogeneous firm model à la Lucas (1978): technological change does not benefit all potential entrepreneurs equally, and there is a positive relationship between an individual's potential payoffs in working and in entrepreneurship. If some firms consistently benefit more from technological progress than others, they stay closer to the frontier, and the others fall behind. Because wages rise for all workers, low-productivity entrepreneurs will then at some point exit and become workers. As a consequence, the entrepreneurship rate falls with income per capita, average firm size and firm size dispersion increase with income per capita, and “entrepreneurship out of necessity” falls with income per capita. The paper also documents, for two of the facts for the first time, that these are exactly the relationships prevailing in cross-country data. Quantitatively, the model fits the U.S. experience well. Using the parameters from a calibration to the U.S., the model also explains cross-country patterns reasonably well.

JEL codes: E24, J24, L11, L26, O30

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1 Introduction

In contrast to the very large literature on skill-biased technical change among workers, there is hardly any work on the importance of skills for the entrepreneurs who employ those workers, and in particular on their evolution over time. This paper proposes a simple theory of skill-biased change in entrepreneurial technology that fits with cross-country, historical and micro evidence.

Technological change is taken for granted as the main historical driver of growth in developed economies. While different types of technological change apart from the neutral variety have received a lot of attention in the recent literature (see e.g. Greenwood, Hercowitz and Krusell (1997) on investment-specific technological change, Krusell, Ohanian, Rios-Rull and Violante (2000) on capital-skill complementarity, or Katz and Murphy (1992) on skill-biased technical change and the demand for workers), there has been barely any work on how technological change affects entrepreneurs. Yet, entrepreneurs need to implement the technologies that they and their employees then operate, so the effect of technical change on entrepreneurs is of crucial importance for how technology subsequently affects labor demand, wages and employment. This paper aims to fill that gap by proposing and taking seriously a very simple theory of technology and entrepreneurship.

If changes in technology affect incentives to become an entrepreneur, the evolution of technology helps shape the firm size distribution. Section 2 presents evidence that this is indeed the case. It uses data from the Global Entrepreneurship Monitor, a survey conducted in around 50 countries that focusses on obtaining internationally comparable information on entrepreneurs. To the best of my knowledge, this is the first paper using information from that survey for macroeconomic analysis. The section establishes four facts. First, the entrepreneurship rate falls with per capita income across countries. Second, average firm size increases with per capita income. The first fact is very close to the negative relationship between the self-employment rate and per capita income across countries shown by Gollin (2007). The second one was shown by Lucas (1978) for U.S. history and here is extended to the cross-country dimension. The next two facts are new: Third, the standard deviation of firm size increases with per capita income both across countries and in U.S. history. Fourth, the fraction of entrepreneurs claiming to have chosen their occupation “out of necessity and not to pursue an opportunity” falls with per capita income across countries. Lucas (1978) and Gollin (2007) provide explanations for the first two facts, but their models do not fit the other two facts.

The data thus show a relationship between the level of development and features of the

firm size distribution. The paper shows that this can be explained in an otherwise standard occupational choice model à la Lucas (1978) with two additional features: technological change not benefitting all potential entrepreneurs equally, and a positive relationship between an individual's potential payoffs in working and in entrepreneurship.

Anyone who has programmed a VCR or tried to set up a home computing network will appreciate that while technological progress brings productivity advances, it often goes along with increased complexity of technology. This is even more so for firms, and not just for large or “high-tech” ones. Consider the corner shop owner contemplating the installation of bar code scanners. This allows automating inventory control, but requires managing the related computing infrastructure. Or consider the owner of a car repair shop who needs to master the increasing amount of computing power of customers' cars. This allows for faster diagnostic checks, but also requires mastering technology that is quite distinct from the core technologies used in that business.

As the menu of available technologies expands, raising aggregate productivity (assuming love of variety, as in Romer 1987), individual firms have to cope with increasing complexity of technology. To reflect this, the key assumption in the model, which otherwise is a standard occupational choice model à la Lucas (1978), is that, while advances in the technological frontier raise all firms' productivity, they do not affect all firms equally. Some firms absorb more of a given technology improvement than others, or are more able to use a new technological opportunity than others. As a result, some firms remain close to the frontier and use a production process involving many, highly specialized inputs, while others fall behind the frontier, use a simpler production process, and fall behind in terms of relative productivity.

The second crucial assumption is that agents differ in their labor market opportunities and that more productive workers can also manage more complex technologies if they become entrepreneurs. Occupational choice between employment and entrepreneurship closes the model. Because advances in the technological frontier do not benefit every potential entrepreneur equally, the position of the frontier then governs occupational choice. The more advanced the frontier, the greater the benefit from being able to stay close to it, as other firms fall behind. Because in equilibrium, advances in the frontier also raise wages, entrepreneurs' outside option improves, and marginal entrepreneurs exit. The result is a “history”, explored in Sections 4 and 5, in which high-productivity entrepreneurs are gradually drawn into the market as their productivity improves more than others'. Their entry raises labor demand and the wage, implying

that low-productivity entrepreneurs eventually find employment more attractive and exit.¹

The need for skills to deal with a broad array of technologies at the same time is in line with Lazear's (2004, 2005) finding that entrepreneurs tend to have more general skills than employees.² It also fits with evidence from the burgeoning recent literature on CEOs and CEO pay, which shows that the importance of general skills has risen of late (see e.g. Murphy and Zabojnik 2004, Rajan and Wulf 2006, Frydman 2007).³ These skills are usually measured as the variety of someone's experience of different industries, companies, functions within companies (e.g. production, marketing, finance), and thus technologies. The main reasons for this phenomenon suggested by that literature are a growing need to master more technologies at the same time (in particular, the rising importance of finance) and broader responsibilities that come from flatter hierarchies made possible by advancing information technology. If entrepreneurs want to benefit from the new possibilities put on the menu by technological advances, they need to keep up with technological developments. The degree to which they can do so determines how many benefits they reap from technological progress.⁴

While the effects of this development on organizational hierarchies and CEO pay have received a lot of attention recently,⁵ the general equilibrium implications have not been studied. Yet, they are substantial, as incentives for entrepreneurship determine not just individual occupational choice and entrepreneurs' incomes, but also the firm size distribution, aggregate labor demand, the level of aggregate technology that is actually in place, and output. Analyzing this is the main contribution of this paper. While the assumptions made here on the use of technology are admittedly much simpler than those in the micro literature, they make it possible to

¹As there is an across-the-board productivity increase in the model as the frontier advances, it also allows for certain tasks that used to be at the technological frontier to be achieved by entrepreneurs behind the frontier as technological advances. Think e.g. about multimedia; a professional can now do on a single computer what in earlier times would have required much more resources. Yet, the frontier moves on – the professional benefits, but entrepreneurs closer to the frontier now can use even more advanced technology.

²For more and refined evidence on this from the U.S., Germany, Canada and Italy, see Baumol (2004), Wagner (2006), Åstebro and Thompson (2007) and Silva (2007).

³Of course, not all CEOs are entrepreneurs. Their job content however is rather similar, with the main difference being the importance of the willingness to take risk. As this will not play a role in this paper, they are an informative group of comparison.

⁴This is qualitatively different from the need for employees to keep up with technology: employees need to apply a given technology, while entrepreneurs need to choose and coordinate the technologies used in a firm's production process. So even if technological progress had de-skilling elements in the 19th century, as argued by James and Skinner (1985) and by Cain and Paterson (1986), replacing skilled workers with machinery still made increasing demands on entrepreneurs to understand and coordinate the new technologies that now were available in addition to the old ones. The setting here thus does not depend on complementarity between capital and workers' skills; all that is needed is that keeping up with advancing technology is costly for entrepreneurs.

⁵Important references include Garicano (2000), Gabaix and Landier (2008) and Terviö (2008). For a survey of the CEO literature see Bertrand (2009).

transparently obtain a full set of general equilibrium results and compare these to the evidence.

The calibration exercise in Section 6 shows that the model fits the U.S. experience, including the evolution of average firm size, well. Using parameter values from this calibration, it can also reproduce the cross-country patterns reported in Section 2 reasonably well.

Skill-biased change in entrepreneurial technology thus is a convenient way of taking results from the micro literature on entrepreneurs and skills to macroeconomics. In addition, the concrete model proposed here fits the U.S. experience well and helps to explain cross-country difference in the entrepreneurship rate, in average firm size, in the dispersion of firm size, and in the share of necessity entrepreneurs.

2 Cross-country evidence on entrepreneurship and the firm size distribution

Obtaining data on the firm size distribution across countries is notoriously hard because measurement is not harmonized across countries. The relatively new Global Entrepreneurship Monitor (GEM) dataset is an exception.⁶ To the best of my knowledge, this is the first paper using GEM data across countries for macroeconomic analysis. As this is a new dataset and probably is not well known to macroeconomists, I briefly present it in the next subsection.

The remainder of the section then shows four facts on occupational choice and the firm size distribution across countries obtained using the GEM data: entrepreneurship and the self-employment rate fall with per capita income, average firm size increases with per capita income, the standard deviation of firm size increases with per capita income, and the fraction of entrepreneurs claiming to have chosen their occupation “out of necessity and not to pursue an opportunity” falls with per capita income. The first two facts are known yet worth revisiting, while the last two are new.

2.1 The Global Entrepreneurship Monitor (GEM) survey

The GEM is an individual-level survey run by London Business school and Babson College now conducted in more than 50 countries. Country coverage has been expanding since its inception in 1999, with data for several years available for most countries. The micro data is in the public domain, downloadable at <http://www.gemconsortium.org/>. Most developed economies are

⁶Another exception are some OECD publications such as Bartelsman, Haltiwanger and Scarpetta (2004) that provide information on OECD countries and a limited number of other countries. Their numbers arise from an effort to harmonize official data, while the GEM approach is more bottom up.

represented, plus a substantial number of transition and developing economies, ensuring that the data covers a wide variety of income levels.⁷

The survey focusses on entrepreneurship. That is, while the survey overall is conducted by local research organizations or market research firms and is supposed to be representative of a country's population, it contains only limited demographic information (e.g. education) on non-entrepreneurs. It contains much richer information on entrepreneurs, including on firms in the start-up phase (a particular focus of the survey). For entrepreneurs, data on the number of employees is available, but not much financial information. Importantly, the survey is designed as to obtain harmonized data across countries. It is thus built to allow cross-country comparisons, the purpose for which it is used here.

To obtain data on entrepreneurship rates and necessity entrepreneurship, I use country averages of the country-level data covering the years 2002-2008 available on the GEM website for 66 countries. Micro data is available for 1999 to 2005 and covers fewer countries. I use it to obtain statistics on the firm size distribution, for which no country-level numbers are reported. As the initial years of the survey may be less reliable, I use the micro data for the period 2001-2005. For this period, data is available for 47 countries, though not for all years for all countries. Pooling the available years for each country, the number of observations per country is between 2,000 in some developing economies and almost 80,000 in the UK, with a cross-country average of 11,700. This is sufficient for computing the summary statistics of the firm size distribution that I use in the following. Unfortunately, in many countries, there are not enough observations for obtaining reliable estimates for detailed size classes, so I am limited to using these summary statistics. I consider someone an entrepreneur if they declare running a firm that they own and they have already paid wages (possibly to themselves, for the self-employed). I then obtain firm size data for these firms, truncating the distribution at 1000 employees to reduce measurement error.

The GEM dataset is very useful because of the harmonized data collection. Moreover, it allows establishing all facts of interest using one single dataset. However, it is of course important to know that results hold more generally, and are not due to specificities of the survey. Therefore, I compare the facts presented here to results from other sources. Moreover, Reynolds et al. (2005), Acs, Desai and Klapper (2008) and Ardagna and Lusardi (2008) show that GEM data tend to align well with data from other sources.

⁷Inclusion in the survey depends on an organization within a country expressing interesting and financing data collection.

2.2 The facts

Figure 1 plots statistics on entrepreneurship and the firm size distribution against 2005 real GDP per capita at purchasing power parity from the Penn World Tables (Summers and Heston 1991, Heston, Summers and Aten 2009). Each subfigure illustrates one of the following four facts:⁸

Fact 1 *The entrepreneurship rate falls with income per capita (see Figure 1(a)).*

Gollin (2007) shows using ILO data that this is also true for the self-employment rate. Moreover, it is comforting that the two data sources yield similar results. Although the negative relationship between the entrepreneurship rate and per capita income is very robust, it does not seem to be well known. A possible reason for that is that the population of entrepreneurs under consideration matters. The fact holds for broad measures of entrepreneurship that include small firms and, in particular, the self-employed. When considering only incorporated firms, the relationship is reversed. This can be seen for instance using data from the World Bank Group Entrepreneurship Survey, which covers only limited liability corporations. (See Figure 2, which plots “entrepreneurship density” from there against per capita income.) The positive slope obtained here is often attributed to differences in regulation.

Fact 2 *Average firm size in terms of employment increases with income per capita (see Figure 1(b)).*

This fact is of course just the inverse of fact 1, as high entrepreneurship rates must necessarily imply smaller average employment.⁹ Indeed, already Lucas (1978) reported that over U.S. history (1900-70), average firm size increased with per capita income. Figure 3(a) reports measures of average firm size close to those used by Lucas (the two series labelled “Historical statistics”) and more recent data. All of these series, with the exception of the period 1900-1930, show an increasing trend, which of course occurs simultaneously with increasing per capita income. Firm size thus increases with per capita income both within (at least in the U.S.) and across countries.

⁸All regression lines plotted in the figure are significant at least at the 5% level.

⁹Strangely enough, this simple relationship often seems to escape policy discussions bent on promoting entrepreneurship.

(a) The entrepreneurship rate

(b) Average employment

(c) The standard deviation of employment

(d) The share of necessity entrepreneurs

Figure 1: Entrepreneurship, the firm size distribution and per capita income.

Fact 3 *The dispersion of firm size in terms of employment increases with income per capita (see Figure 1(c)).*

This probably is the fact for which the GEM data contribute most, as it seems impossible to obtain from other sources in a consistent way for more than a small number of countries. The figure shows a clear positive relationship between the standard deviation of firm size and per capita income. The only previous mention of such a relationship I could find is Bartelsman et al. (2004), who show that firm size dispersion is substantially higher in industrialized countries compared to emerging markets, using OECD data for a much smaller set of countries. Interestingly, the relationship also holds in recent U.S. history. SBA data include the number of firms in different size classes since the 1980s and thereby allow computing an approximate

Figure 2: Limited liability companies and per capita income

measure of the standard deviation of firm size. This is plotted in Figure 3(b) and also exhibits a clear upward trend over time. Unfortunately, I am not currently aware of data that would allow extending this series further into the past.

Finally, it is interesting to consider information the GEM data provides on entrepreneurs' motivations. The survey identifies "opportunity" and "necessity" entrepreneurs. This classification is based on the answer to the question: "Are you involved in this start-up/firm to take advantage of a business opportunity or because you have no better choices for work?" While strictly speaking this question is ill-defined – after all, choosing entrepreneurship implies that it must have been the best choice – it arguably still conveys information on how strongly the respondent identifies with the term "opportunity". Indeed, upon closer inspection, the answer to the question turns out to be significantly related to a firm's current size and growth expectations: necessity entrepreneurs have less education, run smaller firms and expect much less growth (see also Ardagna and Lusardi 2008, Poschke 2009). These patterns are consistent across countries. It can thus be taken to be informative about a firm's current state, and also reveals that founders can, to some degree, anticipate how successful their venture will be. This leads us to the final fact:

Fact 4 *The share of "necessity entrepreneurs" falls with income per capita (see Figure 1(d)).*¹⁰

¹⁰Interestingly, this is the most statistically significant of the facts. Indeed, once per capita income is controlled

Figure 3: Average firm size (employment) and the standard deviation of firm size over U.S. history

Sources: Historical statistics: U.S. Department of Commerce (1975); Census: various Census reports; SBA: Small Business Administration Office of Advocacy data, available at <http://www.sba.gov/advo/research/data.html>.

While there are necessity entrepreneurs in all economies, including in rich ones, their proportion among entrepreneurs is much higher the poorer the country. In countries with low per capita income there are thus more entrepreneurs, but a larger fraction of them chooses entrepreneurship “out of necessity”. The share of “opportunity” entrepreneurs is thus smaller.

Lucas (1978) in his seminal occupational choice framework explains Fact 2 by allowing for complementarity in production between the capital and labor inputs. More productive economies accumulate more capital, which with the complementarity raises wages more than profits, reducing the share of entrepreneurs and thus raising the average size of firms. Gollin (2007) explicitly introduces self-employment as an option and then uses a similar framework to fit self-employment rates across countries (Fact 1).

In each of these cases, the agents who choose entrepreneurship are the fraction of the population that is best at it. Increases in productivity raise the threshold and reduce that fraction. While this implies that in richer countries, there are fewer and larger firms, this mechanism does not explain Facts 3 and 4. In contrast, a more homogeneous population of entrepreneurs may well reduce the standard deviation of firm size, and the fact that it is always the more able in-

for, other factors such as measures of regulation do not have a significant impact on the share of necessity entrepreneurs. While admittedly power of a simple cross-sectional regression is low, this underlines the importance of per capita income for explaining variations in necessity entrepreneurship.

dividuals who choose entrepreneurship do not allow dealing satisfactorily with entrepreneurship out of necessity. The model developed in the next Section addresses these points and thus is able to explain all four facts.

3 A simple model

The economy consists of a unit continuum of agents and an endogenous measure of firms. Agents differ in their endowment of effective units of labor $a \in [0, 1]$ that they can rent to firms. Refer to this endowment as “ability”. Differences in ability can be thought of as skill differences. They are observable, and the distribution of ability in the population can be described by a *pdf* $\phi(a)$.

Agents value consumption c of a homogeneous good, which is also used as the numéraire. They choose between work and entrepreneurship to maximize consumption.¹¹ The outcome of this choice endogenously determines the measures of workers and of firms in the economy.

Consumption maximization implies that individuals who choose to be workers supply their entire labor endowment. Denoting the wage rate per effective unit of labor by w , a worker’s labor income then is wa .

Skills and technology. Entrepreneurs run firms and earn their firm’s profits. Let firm i ’s “technology” be M_i , and assume that it is defined such that firms with a higher level of technology, relative to their competitors, are more profitable. Also assume, realistically, that more able individuals can use better technologies, and $M_i = M(a)$, with $M'(a) > 0$.¹² For concreteness, suppose that $M = \bar{M}^a$, where $\bar{M} \geq 1$ is a parameter capturing the frontier level of technology and a is the entrepreneur’s ability. Then, the most able entrepreneurs ($a = 1$) operate at the frontier, the least able ones ($a = 0$) at the lowest level, and intermediate ones at some distance to the frontier. With the expressions chosen here, the position of a firm relative to the frontier, $m(a, \bar{M}) = M(a)/\bar{M} = \bar{M}^{a-1}$, is bounded between 0 and 1. Crucially, for low levels of the frontier, all firms are close to it (and if $\bar{M} = 1$, all firms are at the frontier). The higher the frontier, the more dispersed the levels of technology of potential firms. (Those of actually active firms will depend on occupational choice.)

This specification thus captures the effect of increasing technological complexity on individual firms: as the frontier advances, some firms can stay close to the frontier and use these better

¹¹Concave utility would not affect qualitative results.

¹²Jovanovic (1994) shows that with different assumptions, different occupational choice outcomes are possible. Indeed, even with this natural assumption the occupational choice outcomes and dynamics will be quite rich.

technologies. Other firms can only use some of them, so while they do use technologies, their distance to the frontier increases: they benefit in absolute terms, but lose in relative ones. Since more skilled entrepreneurs are the ones who benefit most from technological improvements, this is “skill-biased change in entrepreneurial technology”.

In this economy, a population ability distribution will induce occupational choice between working and entrepreneurship, and correspondingly an ability distribution of workers and a productivity distribution of firms.

Firm profits. Firms employ labor in different, differentiated activities to produce the homogeneous consumption good. A firm’s level of technology M gives the number of differentiated activities in a firm and thus corresponds to the complexity of its production process, or the extent of division of labor in the firm. The assumption that more able individuals can run firms with better technology thus concretely means that they can manage more complex production processes, while others are limited to simple ones.¹³

A firm’s production technology is summarized by the production function

$$y_i = X_i^\gamma, \quad X_i = \left(\int_0^{M_i} n_{ij}^{\frac{\sigma-1}{\sigma}} dj \right)^{\frac{\sigma}{\sigma-1}}, \quad \gamma \in (0, 1), \sigma > 1, \quad (1)$$

where y_i is output of firm i , X_i is an aggregate of the differentiated labor inputs n_{ij} it uses, and M_i indicates the degree of complexity of its technology. The production function exhibits decreasing returns to scale. This can be interpreted to reflect any entrepreneur’s limited span of control, as in Lucas (1978). It also ensures that firm size is determinate, implying a firm size distribution given any distribution of M over firms. The elasticity of substitution among inputs is given by σ . Given that M differs across firms and that thus not all firms use all types of differentiated inputs, it is natural to assume that they are substitutes ($\sigma > 1$). Heterogeneity in M plays a role as long as they are imperfect substitutes, as shown below.¹⁴ Importantly,

¹³This appears to be a very natural way of introducing heterogeneity. Galí (1995) uses a similar setup, but allows a representative firms to optimally choose its degree of differentiation. If differentiation is costless, the greatest degree is optimal. This may not be true if it entails costly complexity. Heterogeneity in the cost of complexity would induce different choices of input differentiation. Assuming that the cost of managing complexity decreases in a and all entrepreneurs choose M would induce a qualitatively similar relation between M and a as the more direct assumption made in the text.

¹⁴The formulation in equation (1) is isomorphic to one where final goods firms use (a heterogeneous number of) differentiated intermediate products, intermediates are produced using a production function that is linear in labor, and there is perfect competition in each intermediate goods sector. Monopolistic competition in intermediate goods can also be accommodated easily and would just require a remapping of parameters. In the quantitative exercise in Section 6, a more general specification is chosen in which intermediates are produced using capital and labor with constant returns to scale.

the production function exhibits love of variety, and firms with larger M are more productive. (Therefore, in the following I will sometimes refer to M as “productivity”.) An increase in the frontier technology $\bar{M}$ then increases all firms’ productivity, but benefits those close to the frontier the most.

The firm’s profit maximization problem can be solved using a typical two-stage approach: choose inputs n_{ij} to minimize the cost of attaining a given level of the input aggregate X , and then choose X to maximize profit. The solution to the latter will depend on a firm’s productivity M .

Denoting desired output by $\bar{y}_M$ and defining $\bar{X}_M = \bar{y}_M^{1/\gamma}$, the solution to the cost minimization problem yields the firm’s labor demand function for each activity j as

$$n_j(M) = \left(\frac{w}{\lambda_{\bar{X}_M}} \right)^{-\sigma} \bar{X}_M \quad \forall j, \quad (2)$$

where λ is the marginal cost of another unit of X . With constant returns to scale for transforming the differentiated labor inputs into X , λ is independent of X and equals $M^{\frac{1}{1-\sigma}} w$, and the demand for each n_j becomes

$$n_j(M) = M^{\frac{-\sigma}{\sigma-1}} \bar{X}_M \quad \forall j. \quad (3)$$

Because of greater specialization in firms using more complex technologies, their marginal cost of X , λ , is lower. As a consequence, they demand more of each input. Because a larger M allows a firm to produce more output of a given quantity of inputs, I will in the following refer to M as the firm’s productivity. While M does not equal TFP, it maps one-to-one with TFP.

Choice of $\bar{X}_M$ to maximize profits yields optimal output and profits as

$$y(M) = \left(\frac{w}{\gamma} \right)^{\frac{-\gamma}{1-\gamma}} M^{\frac{1}{\sigma-1} \frac{\gamma}{1-\gamma}}, \quad \pi(M) = (1 - \gamma)y(M). \quad (4)$$

Both output and profits increase in M and are convex in M if $\gamma > \frac{\sigma-1}{\sigma}$.¹⁵

Occupational choice. Occupational choice endogenously determines the distributions of workers’ ability and of firms’ technologies. Since both the firm’s and the worker’s problem are static, individuals choose to become a worker if $wa > \pi(M(a))$. Given the wage rate, the known value of an agent’s ability thus is sufficient for the choice.¹⁶

¹⁵A low γ implies more quickly decreasing returns to scale. As a result, optimal size responds less to productivity, and benefits from being more productive are not as large. High σ implies that inputs are more substitutable, so the benefit of being able to use more of them declines.

¹⁶We abstract from entry costs, sunk investment, search or other issues that would make the problem dynamic without necessarily substantially affecting results. For a related setting with search, see Poschke (2009).

Because profits are continuous, increasing and convex in a , while wages are linear in a , it is clear that there is a threshold a_H above which it is optimal to become an entrepreneur. If $a_H < 1$ (the upper bound on a), high-productivity firms are active in the economy. At the same time, from (4), $\pi(M(0)) > 0 = w \cdot 0$, so that agents with ability between 0 and a threshold a_L become entrepreneurs. In analogy with the evidence reported in Section 2, refer to them as “entrepreneurs out of necessity”. Individuals with $a \in (a_L, a_H)$ choose to become workers.

The existence of necessity entrepreneurs is due to the specific way in which technology and its relationship with ability is modelled here and need not arise with other ways of modelling heterogeneity in productivity and its relation to ability. Yet, while the specification chosen here delivers their existence somewhat directly, as if via a shortcut, their occupational choice arises naturally in more general settings with heterogeneity in productivity and pre-entry uncertainty about a project’s merits, as shown in Poschke (2009). More precisely, even if expected profits of the lowest-ability potential entrepreneur are zero or negative, this is not what matters because of the ability to reject bad projects. Once only sufficiently good projects are accepted, low-ability agents will choose entrepreneurship if projects that are preferred to employment exist and they are sufficiently likely to find them. That paper also provides empirical evidence on the phenomenon of low-ability entrepreneurship and its relationship with potential wages that fits with the setting adopted here.

For $a_H < 1$, the resulting occupational choice pattern then is as depicted in Figure 4, which plots the value of entrepreneurship (solid line) and of employment (line with crosses) against a . Low- and high- a agents become entrepreneurs, with intermediate- a individuals choosing to become workers. This pattern persists when also considering additional heterogeneity that is orthogonal to that in a , e.g. differences in taste for entrepreneurship or in attitudes towards risk. This two-sided occupational choice pattern fits with evidence on the propensity to be an entrepreneur across the education and wage distribution reported in Poschke (2009). It differs from the pattern usually obtained in this type of model, e.g. the individuals with the highest entrepreneurial ability (Lucas 1978) or the lowest risk aversion (Kihlstrom and Laffont 1979) choosing entrepreneurship. The self-employed in Gollin (2007) also have relatively high entrepreneurial ability and potential wages.

Equilibrium. An equilibrium of this economy consists in a wage rate w and an allocation of agents to activities such that, taking w as given, agents choose optimally between work and entrepreneurship, firms demand labor optimally, and the labor market clears.

Figure 4: The values of employment ($W(a)$) and entrepreneurship ($V(a)$)

Denoting the density of firms over a by $\nu(a)$, their total measure by B and total effective labor supply by $N \equiv \int_{a_L}^{a_H} a\phi(a)da$, the equilibrium wage rate then is obtained from labor market clearing as

$$w = \gamma \left[\frac{B}{N} \int \nu(a) M(a)^{\frac{1}{\sigma-1} \frac{\gamma}{1-\gamma}} da \right]^{1-\gamma}. \quad (5)$$

The model is easy to extend to capital as an input, to the production of intermediate goods outside the firm, with perfect or monopolistic competition, and to other dimensions of heterogeneity, e.g. in tastes or in risk aversion. The quantitative exercise in Section 6 will employ such a more general model.

4 The technological frontier and entrepreneurship

Changes in the technological frontier affect incentives to become a worker or an entrepreneur both through their effect on potential profits and on wages. A convenient exercise for analyzing these changes is to trace out a “history” of different, increasing levels of the technological frontier $\bar{M}$ and to explore the effects of advances in that frontier on occupational choice and characteristics of the firm size distribution. As the economy under consideration is static, and its equilibrium depends on the state of the technological frontier, this implies analyzing the effect of changes (advances) in that frontier on the equilibrium.

As technology advances, some firms stay close to the advancing frontier, while others fall behind. As a result, profits as a function of ability change, the populations of firms and workers change, and the equilibrium wage rate changes. Using $M(a) = \bar{M}^a$, recall that profits and the wage are given by

$$\pi(a, \bar{M}) = (1 - \gamma) \left(\frac{w}{\gamma} \right)^{\frac{-\gamma}{1-\gamma}} \bar{M}^{a \frac{1}{\sigma-1} \frac{\gamma}{1-\gamma}} \quad (6)$$

$$w(\bar{M}) = \gamma \left[\frac{B}{N} \int \nu(a) \bar{M}^{a \frac{1}{\sigma-1} \frac{\gamma}{1-\gamma}} da \right]^{1-\gamma}. \quad (7)$$

To see the effect of advances in the technological frontier, consider their elasticities with respect to $\bar{M}$.

$$\varepsilon(\pi(\cdot), \bar{M}) = a \frac{1}{\sigma-1} \frac{\gamma}{1-\gamma} - \frac{\gamma}{1-\gamma} \varepsilon(w, \bar{M}) \quad (8)$$

$$\varepsilon(w(\cdot), \bar{M}) = \frac{\gamma}{\sigma-1} \int \nu(a) a \bar{M}^{a \frac{1}{\sigma-1} \frac{\gamma}{1-\gamma}} da \left[\int \nu(a) \bar{M}^{a \frac{1}{\sigma-1} \frac{\gamma}{1-\gamma}} da \right]^{-1} \quad (9)$$

Equation (8) shows that an advance in the frontier has two effects on profits: it improves every firm's technology (the first term), but it also raises the wage rate (the second term), which is a drag on profits. As the effect of higher wages is independent of a , it is clear that only firms with high enough a actually benefit from aggregate technology improvements. Low- a firms lose more to the wage increase than they gain from the productivity improvement. This implies that the composition of the firm size distribution changes as technology advances. Wages, in contrast, unambiguously increase with advances in the technological frontier.

Note that if all agents had the same ability a , $\varepsilon(w, \bar{M})$ would reduce to $\frac{a\gamma}{\sigma-1}$, which would then equal $\varepsilon(\pi(a), \bar{M})$. As a consequence, wages and profits would increase in sync with technological advances, and occupational choice would remain unaffected, i.e. the thresholds a_L and a_H constant. Only with heterogeneity in a do some agents benefit more than others from advances in the frontier, and occupational choices change.

For an individual with ability a , a marginal advance in the frontier makes becoming an entrepreneur more attractive if

$$\Delta \varepsilon(a, \bar{M}) \equiv \varepsilon(\pi(\cdot), \bar{M}) - \varepsilon(w(\cdot), \bar{M}) = a\eta - \eta \int \nu(a) a \bar{M}^{a\eta} da \left[\int \nu(a) \bar{M}^{a\eta} da \right]^{-1} > 0, \quad (10)$$

where $\eta \equiv \frac{1}{\sigma-1} \frac{\gamma}{1-\gamma}$.¹⁷ Advances in the frontier thus affect the occupational choices of agents of different ability differently. For the most productive entrepreneurs, $\Delta \varepsilon(\cdot)$ will always be

¹⁷Strictly speaking, because the expression is in terms of a difference of elasticities, > 0 is required for marginal

positive. This is because for $a \in (0, 1]$ and for any $\nu(a)$, both integrals in (10) are strictly positive. Moreover, $a\bar{M}^{a\eta} < \bar{M}^{a\eta}$ for $a \in [0, 1)$, implying that the ratio of integrals is between 0 and 1. Similarly, $\Delta\varepsilon$ is strictly negative for the worst entrepreneurs. This implies that as the technological frontier advances, the best entrepreneurs gain, and the worst ones lose.

Intuitively, whether a firm gains or loses depends on its productivity relative to a complicated moment of the productivity distribution. This is because advances in the frontier increase labor demand and wages, and thereby all firms' costs. They also improve firms' productivity – but only firms that can make use of most of the advance in the frontier benefit sufficiently from this. Low- a firms that benefit only slightly from advances in the frontier are exposed to the wage increase, while their own productivity improves only mildly.

What is more, as the frontier continues to advance, the winners become more concentrated. This is because

$$\frac{\partial \Delta\varepsilon}{\partial \bar{M}} = -\frac{\eta^2 \int \nu(a) \bar{M}^{a\eta} da \int \nu(a) a^2 \bar{M}^{a\eta} da - [\int \nu(a) a \bar{M}^{a\eta} da]^2}{\bar{M} [\int \nu(a) \bar{M}^{a\eta} da]^2} < 0. \quad (11)$$

This implies that even firms that at low levels of $\bar{M}$ benefit from increases in the frontier see these benefits reduced and eventually turn negative as the frontier advances further. Only for firms with $a = 1$ is it certain that $\Delta\varepsilon$ cannot turn negative. For firms with $a = 0$, in contrast, it is always negative. For high enough a , $\Delta\varepsilon$ is positive for low $\bar{M}$, eventually turns negative and ultimately pushes $\pi(a, \bar{M})$ below wa . The next section explores the evolution of occupational choice as captured by a_L and a_H and its implications for the firm size distribution and entrepreneurship.

5 A “history” of entrepreneurship and the firm size distribution

Historically, every successful development experience has been characterized by improvements in total factor productivity. This section explores the predictions of the model for occupational choice and the firm size distribution along a “history” of an advancing technological frontier. As the model is static, every $\bar{M}$ induces an equilibrium occupational choice, summarized by the thresholds a_L and a_H , and a firm productivity distribution implied by these choices. Let $\bar{\mathbf{M}} = \{\bar{M}_0, \bar{M}_1, \dots, \bar{M}_T\}$, $\bar{M}_0 = 1$, be a strictly increasing sequence of real numbers and refer to

agents ($a = a_L$ or a_H). For entrepreneurs who are inframarginal at the old level of the frontier, $\pi(a, \bar{M}) > w(\bar{M})a$, and even equal (positive) elasticities of π and w with respect to $\bar{M}$ would make entrepreneurship more attractive in absolute terms. So for them, increases in the frontier make entrepreneurship more attractive if $\Delta\varepsilon(a, \bar{M}) > b(a, \bar{M})$, where b is a continuous function of a and $\bar{M}$ with $b(a_L, \bar{M}) = b(a_H, \bar{M}) = 0$, $b(a, \bar{M}) < 0$ for $a < a_L$ and for $a > a_H$, and $b(a, \bar{M}) > 0$ for $a_L < a < a_H$.

it as the history of $\bar{M}$. Analyzing the equilibrium of the model economy for each element of $\bar{\mathbf{M}}$ then yields a “history” of occupational choice and the firm size distribution.

The sequence $\bar{\mathbf{M}}$ can also be interpreted as a list of different countries’ technological states at a point in time. It then induces a cross-section of occupational choices and firm size distributions. This interpretation is pursued in the following section. To evaluate the quantitative fit, the model is slightly extended and calibrated in that section. This is not necessary for the qualitative history explored in the present section.

Figure 5 shows the values of working (dashed) and of entrepreneurship (solid) as functions of ability for increasing levels of $\bar{M}$. The value of working is linear in a , while that of entrepreneurship is convex. Together with $\pi(M(0)) > 0 = w \cdot 0$, this implies that there is a lower threshold a_L such that individuals with $a < a_L$ become entrepreneurs. The shapes of the functions also imply the existence of an upper threshold a_H such that individuals with $a > a_H$ become entrepreneurs.

Figures 5(a) to 5(c) show the values of working and of entrepreneurship for *two* values of $\bar{M}$ per figure, a low one (x_1 , blue) and a higher one (x_2 , red), for $x = a, b, c$. At very low levels of $\bar{M}$ ($\bar{M} = \bar{M}_{a_1}$), it may occur that a_H exists but is larger than one, so no high-productivity firms are active. All firms are run by the lowest-ability agents. Whether such a situation ever occurred is somewhat in doubt, maybe $\bar{M}$ was larger than this even at very primitive levels of technology.

As $\bar{M}$ increases to $\bar{M}_{a_2}$, entrepreneurship becomes more attractive for high-ability agents, and the threshold a_H drops below 1, to a_{H2} in Figure 5(a). High-productivity firms enter the market. Depending on the distribution of a , this may initially be very few firms. The lower threshold a_L does not necessarily drop immediately as a_H falls. In fact it may rise (not depicted). The reason is that in an economy that consists almost exclusively of low-productivity firms, firms at a_L are actually very productive compared to other firms. So they may initially still benefit from advances in the frontier.

As the frontier advances further, however, a_H decreases further (from 1 to 2 on the x -axis in Figure 5(b)). These new, relatively productive entrants drive up aggregate labor demand and put upward pressure on wages (see Figure 5(b), the slope of the wage line increases). As a consequence, entrepreneurship becomes less attractive to low-ability agents, and a_L falls. Once most low-productivity firms are gone (say at $\bar{M} = \bar{M}_{c_1}$, with a_L at the leftmost 1 on the x -axis in Figure 5(c)), firms with $a = a_H$, while run by relatively high-ability individuals, actually have low productivity compared to other firms in the economy. Thus, when the frontier advances to

(a) $\bar{M}_{a_1}, \bar{M}_{a_2}$

(b) $\bar{M}_{b_1}, \bar{M}_{b_2}$

(c) $\bar{M}_{c_1}, \bar{M}_{c_2}$

(d) $\bar{M}_{d_1}, \bar{M}_{d_2}$

Figure 5: Occupational choice for increasing levels of $\bar{M}$ ($\bar{M}_{a_i} < \bar{M}_{b_j} < \bar{M}_{c_k} < \bar{M}_d \forall i, j, k$, $\bar{M}_{x_1} < \bar{M}_{x_2} \forall x = a, b, c$)

$\bar{M}_{c_2}$, profits increase for firms with $a = a_H$, but less so than wages, and the upper threshold a_H shifts up again, from the rightmost 1 to 2 in Figure 5(c).

As $\bar{M}$ increases further, a_L will fall further, but approach zero only asymptotically. The upper threshold a_H also continues to rise, albeit at a slow pace. ($\partial \Delta \varepsilon(a, \bar{M}) / \partial \bar{M}$, while always negative for $a < 1$, falls in absolute value as $\bar{M}$ increases.) As a result, for very high levels of the frontier, almost all firms have high productivity.

Figure 6(a) depicts the “time series” of a_L (dashed line) and a_H (solid line), just with $\bar{M}$ instead of actual time on the x -axis. At very low levels of technology, $a_H > 1$ and only low-productivity firms are active. As the frontier advances, potential profits of high-productivity

firms increase relative to wages, and eventually a_H falls below 1. As technology improves further, potential profits of high-productivity firms, who are best placed to benefit from the advance in the frontier, increase further, and a_H drops further. Initially, firms at a_L do not suffer from this: their productivity also increases, just less than that of high-productivity firms. Indeed, initially, firms with $a = a_L$ are among the most productive active firms. As a_H drops, however, this changes, and their relative productivity declines. As a consequence, at some point the wage increases due to improved productivity make employment more attractive for marginal low-productivity firms, and a_L starts falling. As wages continue to rise faster than low-productivity firms' profits, a_L drops further. At some point, this happens even for firms with $a = a_H$, and a_H starts to rise, though very slowly.

Advancing technology does not lift all boats here. By assumption, the most able agents benefit most from advances in the technological frontier, as they can deal more easily with the increased complexity, and use a larger fraction of the new technologies. Low-ability entrepreneurs benefit less. In fact, increasing wages due to higher productivity at top firms (wage earners always gain from technological improvements) mean that the least productive firms' profits fall as technology improves. As a consequence, marginal low-productivity entrepreneurs convert to become wage earners, and eventually also do better, though not necessarily immediately (see e.g. agents with a just above a_{L2} in Figure 5(b)). The lowest-ability agents ($a = 0$) always lose. This negative effect of technology improvements on low-productivity firms that operates through wage increases appears very reasonable.¹⁸

Figure 6(b) depicts the consequences of this development: the entrepreneurship rate (solid) falls as technology improves. While high-productivity firms replace the exiting low-productivity ones, they operate at a larger scale, so their number is smaller. Similarly, the share of necessity entrepreneurs ($a < a_L$, dashed line, right axis) and employment in their firms (dash-dot line, right axis) fall and ultimately go to zero.

This development in the model parallels the evidence from Section 2, which reported entrepreneurship rates and rates of necessity entrepreneurship that fall in income across countries and in U.S. history. The same is true for average firm size (see Figure 6(c)). If some agents are better placed than others to benefit from technological advances, they benefit more and may drive the others out of the market. As a consequence, marginal small firms exit, average firm size grows, and fewer, more productive firms remain. Only a few necessity entrepreneurs remain

¹⁸This is also the effect generating an entrepreneurship rate that falls with aggregate productivity in Lucas (1978) and in Gollin (2007), however without necessity entrepreneurship and with declining size dispersion.

(a) The thresholds a_L and a_H

(b) The entrepreneurship rate

(c) Average firm size and firm size dispersion

(d) Wage inequality

Figure 6: Model “time series”

active.

Of course, there are still small firms in rich countries like the U.S.. Possible reasons for this are that the process analyzed here is not as advanced yet (maybe Figure 5(b) is closest to the relevant recent empirical experience), that other factors such as tastes also matter, implying that some small firms remain active *despite* their low productivity (the quantitative exercise in the next Section allows for this), or that in some industries or markets returns to scale decrease very quickly, implying a less convex (in a and $\bar{M}$) profit function and longer activity of low-productivity firms.

The statement that an intermediate stage of the process such as that in Figure 5(b) may

be the empirically relevant also fits with the model history of firm size dispersion reported in Figure 6(c) (dashed line). Firm size dispersion starts to increase substantially only once a_H falls below 1 and high-productivity firms enter. In the limit, it falls again, as low-productivity firms account for barely any employment and a_H rises again. The increasing part is the one that fits with steep parts of the average firm size and entrepreneurship rate series, and that fits with the evidence on firm size dispersion.

Finally, the model also generates evolving wage inequality, reported in Figure 6(d). The mechanism generating changes in wage inequality is new, to my knowledge. As low-productivity entrepreneurs convert to employment they become low-wage employees. Therefore, wage inequality increases as a_L falls (or as a_H increases). It is not clear how important this mechanism is empirically. Of course, this is not the only mechanism driving inequality in the model. Income inequality among entrepreneurs, or among entrepreneurs and wage earners, increases even more, as the most productive entrepreneurs benefit from productivity advances while the least productive ones actually lose. More results on this type of inequality are reported in the next section.¹⁹

Summarizing the model “time series”, the model thus is consistent with the facts reported in Section 2 that the entrepreneurship rate and the share of necessity entrepreneurs fall with per capita income and that average firm size increases with per capita income. For “interior”, i.e. not too extreme, thresholds a_L and a_H the model also generates firm size dispersion that increases with per capita income.

6 Quantitative exercise: occupational choice and entrepreneurship across countries

How well do the historical evolution of one country and cross-country patterns accord? This is a test of how relevant the mechanisms in the model are relative to other factors affecting entrepreneurship and the firm size distribution.

To explore this, I calibrate the model to the U.S. experience and then evaluate how well it fits across a broad set of countries; in particular, how well it mimics the empirical relationships shown in Section 2.

¹⁹Note that the important impact of $\bar{M}$ on endogenous variables implies that it is an important parameter. In this model, the level of productivity can not simply be normalized.

6.1 Generalized model

For the quantitative exercise, it is useful to generalize the very stylized model from Section 3 slightly. I introduce three modifications: production of intermediates with capital and labor, heterogeneity in taste for entrepreneurship, and a more general specification of $M(a)$.

Capital. In the simple model in Section 3, the differentiated activities used for producing final output use labor only. The aggregate input X has constant returns to scale in all labor inputs. Replace this by

$$X = \left(\int_0^{M_i} (n_j^\alpha k_j^{1-\alpha})^{\frac{\sigma-1}{\sigma}} dj \right)^{\frac{\sigma}{\sigma-1}}, \quad (12)$$

i.e., production of intermediates with capital and labor.²⁰ This allows setting α and γ to match income shares in the data. Firms' optimization is as in Section 3, taking the wage rate w and the rental rate of capital r as given. Households, who own the capital stock and rent it to firms, now face a capital accumulation decision. Their Euler equation, evaluated at the steady state of the economy they live in (thus, given its $\bar{M}$), prescribes equating the rental rate of capital net of depreciation to the rate of time preference. Assuming a common rate of time preference ρ and a common depreciation rate δ , this implies $r = \rho + \delta$. The firm's optimality condition for capital then pins down the aggregate capital stock.

Taste heterogeneity. In the model of Section 3, only agents with $a < a_L$ or $a > a_H$ become entrepreneurs. Given the one-to-one mapping between a and M , this implies a bimodal firm size distribution with only low- and high-productivity firms, but no firms with intermediate productivity. This is clearly counterfactual. Incorporating heterogeneity in tastes for entrepreneurship into the model allows to "fill in" the hole in the middle of the firm size distribution, while also adding realism. Indeed, most empirical studies of entrepreneurship point to some role for heterogeneity in tastes or risk aversion for entrepreneurship (see e.g. Hamilton 2000).

Thus, suppose that agents differ in their taste for entrepreneurship τ . This taste is defined such that individuals choose entrepreneurship if $\tau\pi(a) > w \cdot a$. $\tau > 1$ then implies "enjoyment" of entrepreneurship. If agents enjoy entrepreneurship enough, they will choose it even if $\pi(a) < w \cdot a$. Whether on average agents enjoy entrepreneurship is an empirical question; therefore the

²⁰For comparability with Lucas (1978) and Gollin (2007), a CES production function for intermediates could also be used. This would require calibrating an additional parameter.

distribution of τ has to be calibrated, and the mean could be different from 1. A mean below 1 would indicate that on average, individuals do not enjoy entrepreneurship.

Given that in the model, income from both employment and entrepreneurship is certain, there is no role for risk aversion. It would be natural to allow for fluctuations in both, or at least in entrepreneurs' incomes, as this is well-documented to be more volatile than employment income. If this is the case, heterogeneity in risk aversion would also make some individuals more willing than others to engage in entrepreneurship for any given comparison of expected profits and wages. Supposing that wages are certain and that $\pi(a)$ are expected profits, with actual profits fluctuating around it, a sufficiently risk-averse agent may not choose entrepreneurship even if $\pi(a) > w \cdot a$. τ below 1 may thus also be interpreted as risk aversion. For now, stochastic entrepreneurship income is left for future work.

The technological frontier and complexity. How much additional complexity do advances in the technological frontier comport? The simple specification of $M(a)$ chosen in Section 3 restricted this relationship. But it is of course an empirical issue. Therefore, in this section, let a firm's position relative to the frontier be given by

$$m(a) = \frac{M(a)}{\bar{M}} = \bar{M}^{\frac{a-1}{\lambda}}, \quad (13)$$

implying

$$M(a) = m(a)\bar{M} = \bar{M}^{\frac{a-1}{\lambda}+1}. \quad (14)$$

The lower λ , the faster low-ability entrepreneurs fall behind the technological frontier as it advances. Note that this relationship contains two parameters: λ and $\bar{M}$, which is an important parameter in its own right. However, they enter equation (14) sufficiently differently that both can be calibrated, using information from the U.S. time series.

6.2 Calibration

The model is calibrated to U.S. data. Some parameters can be set using standard numbers from the literature, while the remaining ones are calibrated to match a set of moments describing the U.S. economy.

The share parameters γ and α are set to generate a profit share of income of 10% and a labor share of two thirds. This implies a γ of 0.9 and an α of 0.74. The elasticity of substitution among intermediate inputs is set to 4, which is about the 75th percentile of the distribution of

σ across 4-digit industries estimated by Broda and Weinstein (2006). Setting the rate of time preference to 4% and the depreciation rate to 10% per annum implies a rental rate of capital of 14%.

For the remaining parameters, first suppose that the ability and taste distributions are lognormal. A lognormal ability distribution implies that the wage distribution would be lognormal if everyone was an employee. With taste heterogeneity, entrepreneurs will come from across the ability distribution, and the wage distribution will be close to lognormal. For tastes, a lognormal distribution also seems natural, as they affect payoffs multiplicatively. Letting $\ln a \sim N(\mu_a, \sigma_a)$ and $\ln \tau \sim N(\mu_\tau, \sigma_\tau)$, the remaining moments to be calibrated are $\mu_a, \sigma_a, \mu_\tau, \sigma_\tau, \lambda$ and $\bar{M}$.

Note that the $\bar{M}$ resulting from the calibration will describe the U.S. level of technology. To evaluate the cross-country patterns, it will be necessary to set other countries' $\bar{M}$ relative to the U.S. level such that the output ratios match the data.

Data and model moments are shown in Table 1. U.S. data is for the year 2000, or close years where that year is not available. The first two targets are computed from GEM data for the U.S.. The entrepreneurship rate from other sources, e.g. Hipple (2004), is similar. Wage inequality is measured as the ratio between the 90th and the 10th percentile of the wage distribution and is taken from Autor, Katz and Kearney (2008, Figure 2.A). Firm size dispersion is also from the GEM data.²¹ The fraction of firms with profits smaller than the average wage is from Hamilton (2000, Figure 1 and Table 3). The elasticity of average firm size with respect to output per capita for the U.S. is computed using data from 1900 to 1970.²²

Table 1: Calibration: Data and model moments

	model	data
share necessity entrepreneurs	0.13	0.13
entrepreneurship rate	0.098	0.104
90/10 wage ratio	4.85	4.81
firm size iqr/ $\bar{n}$	0.40	0.40
fraction firms with $\pi < \bar{w}$	0.67	0.67
$\varepsilon(\bar{n}, Y)$ (US)	0.51	0.50

²¹The interquartile range is a measure that is robust to outliers and therefore particularly useful in the present context, as the model is not built to generate a fat upper tail of the firm size distribution.

²²Both series are from the very useful publication U.S. Department of Commerce (1975). GNP per capita is Series F4. Average firm size is computed using Series V20, D127, D137 and D139. Unfortunately, the series run only until 1970, and subsequent series on the number of firms count firms differently. Still, it is the longest consistently measured series available.

Values of the calibrated parameters are reported in Table 2. On average, individuals do not like entrepreneurship (the implied average τ in the population is around 0.5), and thus require a premium before they take it up. There is substantial variation, however.

Table 2: Calibrated parameter values

from external sources:					
γ	α	σ	ρ	δ	
0.9	0.74	4	0.04	0.1	
from fitting U.S. target moments:					
μ_a	σ_a	μ_τ	σ_τ	c_m	$\bar{M}$
-1.9341	0.6283	-2.9626	2.1243	2.3744	44.9997

Before moving to the cross-country results, it is interesting to consider an additional result for the U.S.. Overall income inequality in the model increases more than wage inequality, as entrepreneurs' incomes lie at the extremes of the income distribution. Figure 7 reports the income shares of the top 10% and 1% in the U.S. income distribution for the data (from Piketty and Saez 2006, for 1950-2002) and for the model, plotted against U.S. GDP per capita relative to its level in 2002. It is not surprising that inequality in the model does not reach its level in the data, as the model has no mechanism generating a fat right tail of the income distribution. What is remarkable, however, is that, apart from the jump in U.S. income inequality that is known to have occurred in the 1980s (at about 75% of 2002 GDP per capita), the trend in the model captures a large portion of the trend in the data.

6.3 Results

The model fits the U.S. experience quite well. To evaluate the fit with other countries, each country is assigned the $\bar{M}$ that replicates the output per capita ratio to the U.S. observed in the data. This $\bar{M}$ is then taken to be the country's state of technology. Figure 8 plots the entrepreneurship rate, average firm size, firm size dispersion and share of necessity entrepreneurs generated by the model for these levels of $\bar{M}$ against the data. The straight line in each graph is the OLS fit discussed in Section 2. The slightly curved lines are the outcomes generated by

Figure 7: Income concentration at the top, U.S., 1950-2002, model and data

Source: Piketty and Saez (2006).

the model.²³

Given that it was calibrated to the U.S., the model fits the cross-country experience rather well. Of course, just as in the “time series” in Section 5, it predicts that the entrepreneurship rate and necessity entrepreneurship fall with per capita income, while average firm size and the dispersion of firm size increase with it. The model generates a non-linear relationship that in some cases seems to fit reasonably well.

7 Conclusion

²³The model predictions should always fit the U.S., but they do not always cross the U.S. data label. This is because the software places the label next to, not on top of, the data point, which itself is invisible. The OLS line, in contrast, of course need not fit the U.S..

(a) The entrepreneurship rate

(b) Fraction necessity entrepreneurs

(c) Average firm size

(d) Firm size dispersion

Figure 8: Entrepreneurship and the firm size distribution versus output per capita (relative to the U.S. level): model (green curved line) and data (red line: OLS fit)

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