Entry, Trade Costs and International Business Cycles *

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Abstract

This paper studies the aggregate implications of adding entry and fixed costs of exporting to the standard international business cycle model. Building upon Ghironi and Melitz (2005), we present a two-country model in which the extensive margins of production and trade interact with the usual labor and investment choices. We find that entry and exporting decisions have quantitative implications for aggregate quantities and prices. In particular, we indentify a important interaction between entry and investment, as the two can be used for consumption smoothing. Entry dampens the volatility of investment and helps the model to have a better fit for the second moments of all aggregate variables. Regarding prices, the model delivers a negative correlation between the ratio of relative consumption and the real exchange rate, reverting the so-called Backus-Smith Puzzle, but only under certain assumptions of how price indices are constructed. Given the novel results of the model, we explore the role of each intensive margin and find a strong complementarity between the two. Absent any, the model loses some of its nice features. Finally, we discuss how results vary under different financial structures. Our findings indicate that the degree of completeness in international financial markets matter for the micro dynamics of the model, even with low persistent shocks.

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1 Introduction

Recent studies on international trade have identified an important role for modeling entry and exporting decisions by heterogeneous firms, improving upon standard models in explaining trade patterns and the effects of trade liberalizations. These improvements, however, have not spilled over so clearly into the business cycle literature as suggested by the seemingly conflicting results from the works of Ghironi and Melitz (2005), (henceforth GM) and Alessandria and Choi (2007), (henceforth AC). GM embed Melitz (2003) framework into a two country model with no capital stock, perfectly inelastic labor supply and incomplete financial markets. They find that entry to both domestic and foreign markets play a key role for international business cycles statistics and for the dynamics of the real exchange rate. AC, on the other hand, introduce firm-specific dynamics in productivity and sunk and fixed costs of exporting finding no significant departures from the benchmark Backus, Kehoe and Kydland (1992) (henceforth BKK) model. The goal of this paper is to present a fully-fledged two country model in which entry, exporting decisions, labor and capital are endogenous. By doing so, we address the question of what both entry and exporting margins can offer to the standard model once they interact with the usual labor and investment choices, an issue that both GM and AC models are silent about.

Our environment is similar to GM and builds upon their model by adding physical capital accumulation and endogenous labor supply. In our model, each firm differs in their specific level of productivity and is subject to a common aggregate productivity shock. There are fixed and variable (iceberg) costs of exporting, and sunk entry costs for entering the domestic market. A competitive producer combines domestic and foreign varieties to provide a final good, which can be consumed or invested. On the consumer side, there is a representative household that chooses hours worked, consumption, investment and a portfolio of assets that varies across the alternative financial structures that we consider: financial autarky, international bond trading and complete markets.

We find that adding the two extensive margins of production and exporting is important for the dynamics of aggregate variables, in particular for the volatilities of investment and net exports. Entry of new firms acts as a close substitute for investment in serving as a consumption smoothing device for the household, with the difference that entry requires domestic labor while investment demands domestic and, importantly, foreign varieties. Absent entry, the model delivers an excessive volatility of investment and trade balance, underpinned by a strong shift of production to the more efficient economy. Once entry is considered the volatility of investment falls, allowing the model to match the volatility of net exports without relying, as many papers on the literature, on ad-hoc adjustments costs or low values for the trade elasticity. Also regarding quantities, we find that the once investment and elastic labor supply are introduced, the correlation of relative output is much lower than the correlation of relative consumptions in the two countries, reverting the finding by GM of less output-consumption anomaly. Furthermore, we find that factors of productions are negatively correlated across countries in line with most international models. This negative commovement also extends to the creation of new firms.

Secondly, we find that our model, in line with GM, can revert the so called Backus-Smith puzzle, as it predicts a negative correlation between relative consumption of the two countries and the real exchange rate. Nevertheless, the result of this negative correlation should be regarded with caution as it relies on how price indices are measured. Risk sharing imposes a positive correlation between the ratio of consumption of the two countries and the real exchange rate constructed from the theoretical price indices that fully accounted for availability of new varieties. We depart from the the welfare-base measure of the real exchange rate by adjusting price indices so they are measured as weighthed averages of domestic and imported goods prices. However, even after controlling for "love for variety effects", this measure contemplates margins that are arguably not captured in the data. For instance, updates in the consumption expenditure surveys to account for changes in patterns of consumption presumably occur less often than at quarterly basis, which is the time frequency of the model. If we restrict the measure of the CPI in our model to account for changes in only a fixed number of existing varieties, we find that the RER dynamics are identical to those in BKK type of models, counterfactually predicting a high positive correlation between the ratio of consumptions and the RER.

In order to better assess the role of the entry and exporting margins, we re-calibrate the model to different shares of labor accounted by new firms and different shares of exporting firms in steady state. The idea is that as we reduce the share of entrants and increase the percentage of exporters we go back to the standard BKK benchmark. We identify a *complementary role* between entry and the fixed exporting cost. If any is absent, the model delivers results that remain close to the standard model. In addition, conditional on the two margins being at play simultaneously, we find that the novel properties of the model hold only if labor demand by entrants is relatively high, (higher than what we estimate for new firms but closer to our numbers at the establishment level), but display much less sensitivity to changes in the fraction of exporting firms. In a complementary analysis, we control directly for the elasticities of entry and exporting with respect to the productivity shock. We find that for the benchmark results to prevail, the model requires a fair amount of elasticity of entry, which maps into a high volatility of firm entry, which is consistent with the data.

Finally, we discuss how our results change with the degree of international financial integration. Although the interaction between aggregate variables under alternative financial structures is well understood in the BKK model (see, for instance, Heathcote and Perri (2002)), it is not so the case for models with varying number of firms and endogenous export decisions. GM only consider the case of international bond trading, while AC restrict their attention to an economy with complete markets. Here, we alternatively assume financial autarky, international bond trading and complete markets. The most salient finding is that under complete markets not only there is a negative correlation between relative consumptions and the RER persists, provided that price indices are measured as average of domestic and imported varieties and not with the theoretical price indices, but also there is a negative correlation between the relative consumption ratio and the terms of trade, result that is in line with evidence documented in Corsetti, Dedola and Leduc (2008).

The rest of the paper is organized as follows. Section 2 briefly discusses other relevant papers in the literature, section 3 describes the benchmark model, and section 4 offers a discussion on the comparability of price indexes in the model and in the data. In section 5 we start with our quantitative study, and show the results under our benchmark calibration. Section 6 studies the role of entry and the fixed costs of exporting, while in section 7 we discuss the role of the elasticity of substitution across countries and the financial structure. In section 8 we conclude.

2 Related Literature

This paper belongs to the family of international business cycle models started by BKK, who were the first to document that the two country-model extension of the Kydland and Prescott model (1992) was at odds with the data in several dimensions. Since then, many alternative specifications have been proposed to improve the fit of the basic model, mainly on three fronts: aggregate volatilities, international correlations and predictions on international relative prices. Regarding the first one, the literatures has tackled the issue of the exacerbated volatility of investment and net exports of the benchmark model by adding market frictions, such as adjustment costs of investment, or by assuming a low elasticity of substitution between imported and domestic goods, which is not supported by the estimates of empirical studies in the trade literature[1]. Concerning the low correlation of cross-country output relative to consumption, works by Baxter and Cruccini (1995), Kollman (1996) and Heathcote and Perri (2002), have investigated the effect of frictions in international markets that prevent or completely preclude the possibility of international risk sharing. Close to this literature is the paper by Kehoe and Perri (2002), who introduce frictions in credit markets that inhibits capital flows as a result of imperfect enforceability of contracts. They find that adding endogenous incomplete markets to the standard model helps to dampen the volatility of next exports and to improve the predictions of the model on international correlations.

More recently, GM have suggested that modeling entry and exporting decisions as in the trade literature can also help to improve the benchmark model. Our work is directly related to theirs, as described earlier. Our extensions make their model fit into the international business cycle

[1]For instance, the trade elasticity parameter estimated by Eaton and Kortum (2002) has a lower bound of 3.6. Most international papers use 1.5 as BKK and recently, Corsetti, Dedola and Leduc (2008) manage to solve some of the problems of the benchmark model using a value of 0.9.

literature.

Our model is also related to the work by AC, as we share with them the basic BKK structure and the extensive margin of exports. Their model, though, is different from ours as they consider idiosyncratic shocks to the firms , let firms rather than households to accumulate physical capital and, more importantly, consider a fixed number of firms over the cycle, so firm creation is precluded . Our work also shares similar features to Corsetti, Martin and Pessenti (2009), in which an entry decision by heterogeneous producers is explicitly modeled. They focus, however, on current account adjustments rather than in business cycle properties and assume an exogenous set of non-tradable goods, lacking the extensive exporting margin we have.

As we mentioned in the introduction, our results support the modeling of the two extensive margins, as we found them highly complementarity in order to have significant departures from the canonical BKK. In addition, we contribute to the existing literature by identifying an important role for entry, which acting as a substitute for investment helps to dampen its volatility and the volatility of the trade balance. On this front, entry resembles adjustment costs or other market frictions that keeps investment volatility subdue, yet it doesn't dampen the volatility of output and employment as these frictions do. Relative to Kehoe and Perri (2002), our model can deliver the counter cyclicality of the trade balance that their model cannot predict and has a better fit for the second moments of output, consumption and employment given the strong constraint to international flows of investment that their model has as a result of the limited commitment assumption. Nonetheless, entry and exporting decisions don't bring any improvement on the comovements of international variables. In fact, the interaction between entry, investment and endogenous labor reverts the improvement in the output-consumption anomaly found by GM.

On the empirical side, the motivation of adding fixed exporting costs within a framework of changing number of firms or varieties can be found in the works by of Bernard et al. (2003) , who showed that only 21% of manufacturing plants export in the US. Tang (2006) and Hummels and Klenow (2005) document the importance of the extensive margin for trade flows in the US. Regarding entry of new establishments or product creation the works by Bernard, Redding and Schoot (2006), Broda and Weistein (2006), Axarloglou (2003) report the procyclicality of these variables and discuss the welfare implications of an expansion in set of available goods.

3 The Model

Our model economy is composed of a representative household, competitive producers of a final good and a continuum of heterogeneous producers of intermediate goods that operate under monopolistic competition. There are two countries, home and foreign, that can trade intermediate goods and we will alternatively assume international financial markets to be complete, to allow for international trade in a single uncontingent bond, or to preclude any trade in financial assets across countries. Unless otherwise necessary, we will restrict attention to domestic agents, omitting the presentation of foreign households and firm's problems for expositional purposes. When required to do so, we will distinguish foreign variables with a star.

3.1 Producers of Final Goods

In every period, there exist a set of domestically produced intermediate goods Ω_t, and a set of imported varieties $\Omega^*_{x,t}$ that are available for the production of the final good[2]. The final good can be used both for consumption or investment purposes, and cannot be internationally traded. Exporting is costly and entails the payment of an iceberg cost $\tau > 1$ for each unit of the intermediate good that is shipped overseas. In addition, trade is costly because firms must incur a per-period fixed export cost to gain access to the foreign good's markets. We shall describe the features of the exporting activity in more detail later in the paper, but for the purposes of this subsection it is crucial to understand that the set $\Omega^*_{x,t}$ changes over time as the economy is hit by aggregate productivity shocks or is subject to trade liberalizations. Furthermore, the set Ω_t will also be time varying since our model features entry and exit along the business cycle.

Let $z \in \Omega$ be a particular variety, the producers of final goods combine intermediate inputs according to:

$$Y_t = \left[\left[\int_{z \in \Omega_t} y_t(z)^{\frac{(\theta-1)}{\theta}} dz \right]^{\frac{\theta}{\theta-1}\frac{\rho-1}{\rho}} + \left[\int_{z \in \Omega^*_{x,t}} y^*_{x,t}(z)^{\frac{(\theta-1)}{\theta}} dz \right]^{\frac{\theta}{\theta-1}\frac{\rho-1}{\rho}} \right]^{\frac{\rho}{\rho-1}} \tag{1}$$

Our specification of the technology is flexible enough to capture potentially different elasticities of substitution within the set of domestic varieties, and between domestic and foreign ones. Here θ denotes the former, and ρ refers to the latter. The demand for the domestic variety z is denoted by $y_t(z)$, while imports of the home country are referred as $y^*_{x,t}(z)$. An identical technology is assumed for the foreign country.

Let P_t be the price of the home final good, taken as given by the competitive producer of such good, and let $p(z)$ and $p^*_x(z)$ denote the price of a domestic and a foreign variety z respectively. Then, the problem of the producer of final goods is given by:

$$\max_{y_t(z), y_{x,t}(z)} P_t Y_t - \int_{z \in \Omega_t} p_t(z) y_t(z) dz - \int_{z \in \Omega^*_{x,t}} p^*_{x,t}(z) y_{x,t}(z) dz$$

subject to the technology in (1). The solution to this problem gives the following demand and

[2] The superscript indicates the origin of the good, and the subscript x is to denote that the good is exported by the source. So, for instance, Ω_t is the set of domestically produced intermediate goods available in the home country, while $\Omega^*_{x,t}$ are those goods that are produced abroad and available for export to the home economy

aggregate price functions:

$$y_t(z) = \left(\frac{p_t(z)}{P_t}\right)^{-\theta} \left(\frac{P_{D,t}}{P_t}\right)^{\theta-\rho} Y_t \tag{2}$$

$$y^*_{x,t}(z) = \left(\frac{p^*_{x,t}(z)}{P_t}\right)^{-\theta} \left(\frac{P^*_{X,t}}{P_t}\right)^{\theta-\rho} Y_t \tag{3}$$

$$P_t = \left[P_{D,t}^{1-\rho} + P_{X,t}^{1-\rho}\right]^{\frac{1}{1-\rho}} \tag{4}$$

where we have defined $P_{D,t} \equiv \left[\int_{z\in\Omega_t} N_{D,t}p(z)^{1-\theta}dz\right]^{\frac{1}{1-\theta}}$ and $P^*_{X,t} \equiv \left[\int_{z\in\Omega^*_{x,t}} N^*_{X,t}p^*_x(z)^{1-\theta}dz\right]^{\frac{1}{1-\theta}}$. The aggregate price level in the home country is thus a CES aggregator of two other CES aggregates of domestic and imported varieties. The demand functions depend negatively on the variety's price relative to the aggregate price level, $\frac{p_t(z)}{P_t}$, with price elasticity given by θ. Moreover, so long $\theta \neq \rho$, the demand for domestic varieties will also be a function of the aggregate price of domestic varieties relative to the aggregate price index in the economy, $\frac{P_{D,t}}{P_t}$. An equivalent relationship holds for home country's imports. Finally, since the final good will be either consumed or invested, in equilibrium[3] we will have $Y_t = C_t + I_t$.

3.2 Producers of Intermediate Goods

We start the analysis of the intermediate goods sector describing the optimization problem of an already existing mass of producers, then switching to the characterization of entry and exit into this sector.

3.2.1 Pricing and the Decision to Export

Firms are differentiated by their productivity z, so there is no loss of generality in indexing varieties with productivities, as we have done above[4]. At any point in time there exists a mass $N_{D,t}$ of firms that are actively producing and selling in domestic markets. These firms act under monopolistic competition, so they take the demand functions for their variety as given to maximize their profits. We assume that production is done with a Cobb Douglas production function with identical factor shares across varieties:

[3]We summarize the full equilibrium of the economy later in the paper.

[4]Although we refer to the production units in our model as firms, we acknowledge that these do not correspond to what firms are in reality, where a firm may embody a number of different establishments with different productivities. Implicitely, thus, we are imposing a firm in the model to actually be equivalent to an establishment. Furthermore, it is also arguable that a variety is actually a product, so the model can be re-interpreted as a description of product innovation along the cycle

$$Y_t(z) = Z_t z \left[k_t(z)\right]^{\alpha} \left[l_t(z)\right]^{1-\alpha}$$

where $k_t(z)$ and $l_t(z)$ represent the demand for capital stock and labor units from a producer with productivity z. The notation extends naturally to the foreign country with the inclusion of the star superscript. Heterogeneity is reflected in differences in the TFP component of each firm z, which we assumed fixed[5]. All firms are subject to a common aggregate productivity factor Z_t, which evolves stochastically in our model .

Conditional on entry, firms can decide to engage in exporting activity. The decision is non trivial because we assume there exists a per period fixed export cost f_x, measured in effective units of labor[6], that makes only a subset of firms to find exporting profitable.

Prices are expressed in units of the final good at destination. Then, $\rho_{x,t}(z) = \frac{p_{x,t}(z)}{P_t^*}$ is the price set by a domestic producer for its sales in the foreign market, while the real value of domestic varieties sold in the home country is $\rho(z) = \frac{p(z)}{P}$. Firms can aggregate domestic and foreign profits flows converting the latter into units of the home final good through the real exchange rate $Q_t = \frac{\varepsilon P_t^*}{P_t}$.

Given these definitions, an intermediate producer maximizes (5) subject to (6), (7) and (8).

$$\max \rho_t(z) y_t(z) + x(t) Q_t \rho_{x,t}(z) y_{x,t}^*(z) - w_t l_t(z) - r_t^k k_t(z) - x(t) \frac{w_t}{Z_t} f_x \tag{5}$$

$$y_t(z) + x(t) \tau y_{x,t}^*(z) = Z_t z \left[k_t(z)\right]^{\alpha} \left[l_t(z)\right]^{1-\alpha} \tag{6}$$

$$y_t(z) = \left(\frac{p_t(z)}{P_t}\right)^{-\theta} \left(\frac{P_{D,t}}{P_t}\right)^{\theta-\rho} Y_t \tag{7}$$

$$y_{x,t}^*(z) = \left(\frac{p_{x,t}^*(z)}{P_t}\right)^{-\theta} \left(\frac{P_{X,t}^*}{P_t}\right)^{\theta-\rho} Y_t \tag{8}$$

The term $x(t)$ is an indicator function that captures the export status of the firm, being equal to 1 when it exports and 0 otherwise. Letting $MC_{p,t}(z)$[7] denote the marginal cost of production, the optimal pricing rule is defined by:

[5]We will be more specific about the evolution of the idiosyncratic productivity term when we describe the problem of entry and exit.

[6]Our proof of existence of a unique symmetric steady state relies on the assumption that fixed exporting costs and entry costs are measured in the same units. This proof is in appendix A.1.

[7]With Cobb Douglas production function and labor share 1-α, the marginal cost is of the form $\frac{r^{\alpha} w^{1-\alpha}}{Zz} \left(\frac{1}{\alpha}\right)^{\alpha} \left(\frac{1}{1-\alpha}\right)^{1-\alpha}$, where r is the rental rate of capital stock in units of the home final good, and w is the real wage.

$$\rho_t(z) = \frac{\theta}{\theta - 1} MC_{p,t}(z) \tag{9}$$

$$\rho_{x,t}(z) = Q_t^{-1} \frac{\theta}{\theta - 1} \tau MC_{p,t}(z) = Q_t^{-1} \tau \rho_t(z) \tag{10}$$

which is the familiar result for problems with monopolistic competition and aggregate CES technology. Notice that marginal costs are a negative function of the firm-specific productivity term z, hence, more productive firms set lower prices. The choice of factors of production is determined by:

$$r_t^k = MC_{p,t}(z) \alpha \frac{Y_t(z)}{k_t(z)} \tag{11}$$

$$w_t = MC_{p,t}(z)(1 - \alpha) \frac{Y_t(z)}{l_t(z)} \tag{12}$$

Under monopolistic competition, prices no longer equals marginal costs, implying that factor demands will be different from those in the efficient allocation[8]. Explicit formulas for the demand for factors of production are derived in appendix A.2.

Finally, the substitution of the optimal choices back into the profit function gives that profits from domestic sales, $d_{D,t}(z)$, are:

$$d_{D,t} = \frac{1}{\theta} \left[\rho_t(z)\right]^{1-\theta} \left(\frac{P_{D,t}}{P_t}\right)^{\theta-\rho} Y_t$$

and profits from export sales[9] ,$d_x(z)$:

$$d_{x,t} = \begin{cases} \frac{Q_t}{\theta} \left[\rho_{x,t}(z)\right]^{1-\theta} \left(\frac{P_{X,t}}{P_t^*}\right)^{\theta-\rho} Y_t^* - \frac{w_t}{Z_t} f_x & \text{if firm } z \text{ exports} \\ 0 & \text{otherwise} \end{cases}$$

It follows that more productive firms set lower prices and earn higher profits. This is a feature of CES demand functions, which exhibit elasticities with respect to price that are greater than one in absolute value ($\theta > 1$). This property is vital for the characterization of the subset of firms that will export. More specifically, since profits are increasing in productivity and the fixed export cost imposes a non-convexity that leads to a decreasing average *total cost* curve, it is the most productive firms the ones that are more likely to export. In particular, since the relationship between profits and productivity is monotonic, we can characterize the export choice with a cut-off rule of the form:

8 This is because factor demands are not chosen so as to equalize factor prices with the value of marginal products, as in the efficient allocation

9 We can split domestic from export profits because firms produce under constant returns to scale.

$$z_{x,t} = \inf \{ z : d_{x,t}(z) > 0 \} \tag{13}$$

where all firms whose productivity is greater than or equal to the cutoff productivity $z_{x,t}$ decide to export. Provided the lowest productivity firm is sufficiently low to ensure that $z_{x,t}$ is in the interior of the support of the distribution, there will always exist a subset of firms that decide not to export, endogenously determining a non-traded sector in the economy. Importantly, the size of this sector will differ across countries and change over time, in response to aggregate technology shocks. This adjustment in the extensive margin of trade is an new feature allowed by our economy with heterogeneous firms and is one of the main channels through which the micro-level heterogeneity feeds back into aggregate dynamics.

3.2.2 Entry, Exit and Ownership of Firms

Entry is modelled following Melitz (2003). There is an infinite pool of forward looking potential entrants that consider paying a sunk entry cost f_e of effective units of labor to get a productivity draw z from a common known distribution $G(z)$, identical in both countries, with support $[z_{\min,}\infty)$. We assume no fixed costs of operation in the domestic markets, hence all firms in the support have positive profits. The idiosyncratic productivity is constant over the lifetime of the firm, which can be interrupted at any time with exogenous probability δ. These assumptions combined imply that there is a fixed *ex-post* distribution of productivities that is identical to the *ex-ante* distribution $G(z)$, keeping the aggregation of the model tractable over the business cycle[10].

Since productivities are drawn after payment of the entry cost, prospective entrants consider the average value of a firm in making their entry decisions. This is determined by the expected present value of average total profits and must equal the entry cost in equilibrium:

$$f_e \frac{w_t}{Z_t} = \tilde{v}_t$$

Here $\tilde{v}_t = E_t \sum_{\tau=t+1}^{\infty} \left[\beta\left(1-\delta\right)\right]^{\tau-t} \frac{U_c(c_\tau,l_\tau)}{U_c(c_t,l_t)} \tilde{d}_\tau$ is the present value of the expected stream of average profits $\tilde{d}_t$, discounted at the stochastic discount factor augmented to account for the exogenous probability of exit δ.

In order to give persistence to the evolution of the mass of domestic producers, we assume a one period lag for the new entrants before they become operative. In this way, the mass of domestic producers is a state variable in the model, that evolves according to:

$$N_{D,t} = \left(1-\delta\right)\left(N_{D,t-1} + N_{e,t-1}\right) \tag{14}$$

[10]So long our focus is on the aggregate implications of entry and exit decisions, and fixed exporting costs; we don't view these assumptions as excesively restrictive. For certain, it wouldn't be reasonable ones if we were interested in matching firm-level moments of the data along the business cycle (such as firm growth and size).

We conclude this subsection establishing the property rights on the existing firms in the economy. GM conveniently assume, and we here imitate, that firms are owned by a mutual fund on which households can trade shares in the stock markets. The mutual fund pays dividends equal to the average total profits of active firms, $\tilde{d}_t$; and shares can be traded at a price $\tilde{v}_t$. The definition of the average value of the firm as the present value of the expected stream of average profits is a relationship that follows, as we shall see below, from the first order condition of the representative household's maximization problem.

3.2.3 Aggregation

In previous sections we argued that firms' idiosyncratic productivities will be distributed according to $G(z)$, and we showed that there exists a cut-off productivity separating the set of exporting and non-exporting firms. The mass of exporting firms is then:

$$N_{x,t} = [1 - G(z_{x,t})]\, N_{D,t}$$

which pins down the share exporting firms directly from the assumed distribution of productivities and the cut-off.

In addition, it can be shown that all micro-level variables (individual prices, profits, etc) depend on the idiosyncratic productivity in the form[11] $z^{\theta-1}$. For instance, to shed light on this result, consider the profits from domestic sales having replaced prices from the optimal rule:

$$d_{D,t}(z) = \frac{1}{\theta}\left[\frac{r_t^{\alpha} w_t^{1-\alpha}}{Z_t}\left(\frac{1}{\alpha}\right)^{\alpha}\left(\frac{1}{1-\alpha}\right)^{1-\alpha}\right]^{1-\theta}\left(\frac{P_{D,t}}{P_t}\right)^{\theta-\rho} z^{\theta-1} Y_t$$

It follows that *average* domestic profits are given by:

$$\tilde{d}_{D,t} = \frac{1}{\theta}\left[\frac{r_t^{\alpha} w_t^{1-\alpha}}{Z_t}\left(\frac{1}{\alpha}\right)^{\alpha}\left(\frac{1}{1-\alpha}\right)^{1-\alpha}\right]^{1-\theta}\left(\frac{P_{D,t}}{P_t}\right)^{\theta-\rho} Y_t \int_{z_{\min}} z^{\theta-1} dG(z) \qquad (15)$$

Following Melitz (2003), we can construct a pair of statistics $\tilde{z}_D$ and $\tilde{z}_{x,t}$ (similarly for the foreign country) that summarize the entire distribution of productivities of all firms and the exporting firms in the economy:

[11] See Melitz (2003) for further proofs

$$\tilde{z}_D \equiv \left[\int_{z_{\min}}^{\infty} z^{\theta-1} dG(z)\right]^{\frac{1}{\theta-1}}$$

$$\tilde{z}_{x,t} \equiv \left[\frac{1}{1-G(z_{x,t})}\int_{z_{x,t}} z^{\theta-1} dG(z)\right]^{\frac{1}{\theta-1}}$$

Notice that the average productivity of domestic firms, $\tilde{z}_D$, is independent of time as a consequence of the assumption of no fixed costs of operation, which implies that all firms in the support produce at least domestically. The average productivity of exporters, on the other hand, changes over time in response to aggregate productivity shocks.

The particular form on which micro-level variables depend on productivity implies that we can refer to average values of the variable with its value at the average firm. Then, continuing with the example of domestic profits, we can write average domestic profits $\tilde{d}_{D,t}$ as the profits of the average firm $d_{D,t}(\tilde{z}_D)$. The results extends to average exporting profits $\tilde{d}_{x,t} \equiv d_{x,t}(\tilde{z}_{x,t})$, average relative prices $\tilde{\rho}_t \equiv \rho_t(\tilde{z}_D)$ and $\tilde{\rho}_{x,t} \equiv \rho_{x,t}(\tilde{z}_{x,t})$; and average total profits $\tilde{d}_t = \tilde{d}_{D,t} + [1-G(z_{x,t})]\,\tilde{d}_{x,t}$.

Finally, having characterized the microeconomics underpinning of the model more sharply, we can improve on the notation we used in the description of the final goods sector to characterize the consumption baskets, Ω_t and $\Omega^*_{x,t}$. We now know that the former is composed of all the firms in the economy and has mass equal to $N_{D,t}$, while the latter comprises the subset of firms with $z \in [z^*_{x,t}, \infty)$ and has mass equal to $N^*_{x,t}$. The aggregate price indexes, using the aggregation properties of the model, can therefore be written as:

$$P_D \equiv N_{D,t}^{\frac{1}{1-\theta}} \tilde{p}_t \tag{16}$$

$$P^*_x \equiv N_{x,t}^{\frac{1}{1-\theta}} \tilde{p}_{x,t} \tag{17}$$

In summary, there are two key features of the model that keep the aggregation simple and tractable at business cycle frequency. These are the assumptions of an exogenous exit rate and the absence of idiosyncratic productivity shocks, which tie the productivity distribution of firms to the ex ante distribution and keeps it isolated from the influence of the aggregate shocks. In addition, the shape of the CES demand function allows for the representation of aggregate variables in terms of sufficient statistics of the distribution.

3.3 Household's Problem

Each country is populated by a unit mass of households that choose consumption, investment in physical capital, investment in financial assets and hours worked to maximize lifetime utility according to:

$$\max E_o \sum_{t=0}^{\infty} \beta^t \left[\frac{\left(C_t^{\mu} \left(1-l_t\right)^{1-\mu} \right)}{1-\gamma} \right]^{1-\gamma}$$

C_t denotes consumption, l_t represents ours worked, the parameter $\beta \in (0,1)$ is the discount factor, γ is the coefficient of relative risk aversion and μ the share of consumption in utility. The form of the budget constraint depends on what we assume about the degree of completeness in international financial markets. Under all specifications, however, we assume that households can trade shares in domestic mutual funds. Letting φ_t be the shares on the domestic mutual fund held by domestic household, the total amount of resources spent to accumulate shares is $\varphi_{t+1}\left(N_{D,t}+N_{e,t}\right)\tilde{v}_t$, where $\left(N_{D,t}+N_{e,t}\right)$ is the number of firms owned by the mutual fund and $\tilde{v}_t$ its average value in units of the home final good. For share holdings of φ_{t+1} in the next period, the household receives dividends $\tilde{d}_{t+1}$ from a fraction $(1-\delta)$ of the $\left(N_{D,t}+N_{e,t}\right)$ firms in the mutual fund, and has the option to sell his shares at a value of $\tilde{v}_{t+1}N_{D,t+1}$[12].

Financial Autarky

Here we assume that households do not have access to financial markets in the other country, thus being restricted to trade shares and domestic one-period risk-free bonds that pay an interest r_t^B. The budget constraint, in units of the home final good, is:

$$C_t+K_{t+1}+\varphi_{t+1}\tilde{v}_t\left(N_{D,t}+N_{e,t}\right)+B_{t+1}=(1+r_t^B)B_t+(1-\delta_k+r_t^k)K_t+(\tilde{d}_t+\tilde{v}_t)\varphi_t N_{D,t}+w_t l_t \quad (18)$$

where investment in physical capital is given by $I_t = K_{t+1}-(1-\delta_k)K_{t-1}$. The same budget constraint holds for the foreign household.

International Bond Trading

In this environment we allow households to trade one-period uncontingent bonds across countries, but we restrict the stock markets to be of exclusive access to domestic residents. The idea is to expand the risk sharing opportunities but still avoid any state-contingency in the payments of the assets that are traded internationally. Therefore, we assume that bonds give a risk free interest of r_t^B and $r_t^{B,*}$ in each country. It is a well established property of this financial structure that asset holdings are indeterminate in steady state. We avoid this problem by assuming a small quadratic adjustment cost function that recovers the determinacy of bond holdings and the stationarity of

[12]Recall, from equation (14), that $N_{D,t+1}=(1-\delta)\left(N_{D,t}+N_{e,t}\right)$

the model[13]. The steady state level of bond holdings, though, will not change relative to the financial autarky environment because in a symmetric steady state (the one we will focus on in this paper) the current account is equal to zero. The budget constraint for the household looks as follows:

$$\begin{aligned} & B_{t+1} + Q_t B_{*,t+1} + \frac{\eta}{2}\left(B_{t+1}^2 + Q B_{*,t+1}^2\right) + C_t + K_{t+1} + \varphi_{t+1}\tilde{v}_t\left(N_{D,t} + N_{e,t}\right) \\ = \; & (1-\delta_k + r_t^k)K_t + (\tilde{d}_t + \tilde{v}_t)\varphi_t N_{D,t} + w_t l_t + \left(1 + r_t^B\right) B_t + \left(1 + r_t^{B,*}\right) Q_t B_{*,t} + T \end{aligned} \tag{19}$$

where $B_{*,t}$ denotes domestic holdings of foreign bonds in period t measured in units of the foreign final good, η is a parameter of the adjustment cost function and T is the lump sum rebate of the collected resources from the adjustment costs technology.

Complete Markets

Also, we consider a case in which there exists a complete set of state contingent one-period Arrow securities available for trading among the households. A state in this economy is a vector of realizations of nature in each country and we denote it with $s_t = \left\{s_t^h, s_t^f\right\}$. The history up to and including date t of such realizations is given by s^t. We will be more specific about the stochastic properties of the model in the quantitative section of the paper. Continuing with the notation, we refer to $q(s_{t+1}, s^t)$ as the price of an Arrow security in units of the home final good at date t and history s^t that pays a unit of home consumption goods in $t+1$, and state s_{t+1}; and with $B(s_{t+1}, s^t)$ to the holdings of such assets by the home households. The notation extends to the foreign country in the same fashion it did under the international bond trading specification:

$$\begin{aligned} & C_t + K_{t+1} + \sum_{s^{t+1}} \left[q(s_{t+1}, s^t)B(s_{t+1}, s^t) + Qq^*(s_{t+1}, s^t)B_*(s_{t+1}, s^t)\right] + \varphi_{t+1}\tilde{v}_t\left(N_{D,t} + N_{e,t}\right) \\ = \; & (1-\delta_k + r_t^k)K_t + (\tilde{d}_t + \tilde{v}_t)\varphi_t N_{D,t} + B(s_t, s^{t-1}) + QB_*(s_t, s^{t-1}) \end{aligned}$$

The solution to the household's optimization problem is characterized by the standard labor-leisure and Euler equations, and the risk sharing equation for the case of complete markets. These are described in more detail in the appendix. Here we restrict attention to the first order condition for φ_{t+1} because it gives an expression that determines the average value of the firms, $\tilde{v}$, to be considered by the potential entrants:

$$\tilde{v}_t U_{c_t}(C_t, l_t) = \beta\left(1-\delta\right) E_t\left[U_{c_{t+1}}(C_{t+1}, l_{t+1})\left(\tilde{v}_{t+1} + \tilde{d}_{t+1}\right)\right] \tag{20}$$

[13] See Schmitt-Grohe and Uribe (2003) for a discussion on different ways to induce stationarity in international business cycle models

Iterating this equation forward and appealing to a no-bubble condition we have that the average value of the firm is determined by the discounted expected average profits, using the stochastic factor of the households and subject to the probability of survival of the firm.

3.4 Definition of Competitive Equilibrium

A *competitive equilibrium* in this economy is (the same elements apply to the foreign country): *a)* a set of consumption, labor, investment in physical capital and investment in financial asset choices, *b)* a set of price rules, factor demands and export decisions, *c*) demand functions for intermediate goods, *d)* a mass of new entrants N_e, *e)* a cutoff productivity z_x, *f)* a set of iceberg, entry and fixed export costs and *g)* a vector of asset prices, wages and interest rates such that: 1) given *g and d*, *a* solves household 's maximization problem; 2) given *c, e, f* and *g*; *b* solves the problem of the producers of intermediate goods and *e* satisfies the zero cutoff profit condition, 3) given *f* and *g*, *d* satisfies the free entry condition; and 4) markets clear.

Market clearing requires wages and the real rate of return on capital to be such that factor demands in production, entry and exporting equal supply. Moreover, the alternative financial structures we assume impose their own set of equilibrium conditions.

Financial Autarky:

The rate of return on domestic assets, r_t^B and $r_t^{*,B}$, must be such that $B_t = B_{*,t}^* = 0$, $\varphi_{t+1} = \varphi_t = 1$, and there is balanced trade

$$QN_{x,t}\left(\tilde{\rho}_{x,t}\right)^{1-\theta} Y_t^{*,c} = N_{x,t}^* \left(\tilde{\rho}_{x,t}^*\right)^{1-\theta} Y_t^C \tag{21}$$

International Bond Trading

Here, market clearing requires $B_t + B_{*,t} = 0$ and $B_t^* + B_{*,t}^* = 0$. The countries can accumulate net foreign assets in equilibrium, the evolution of which is given by:

$$B_{t+1} + Q_t B_{*,t+1} = (1 + r_t^B)B_t + Q_t(1 + r_t^{*,B})B_{*,t} + (w_t L_t + r_t^k K_t + N_{D,t}\tilde{d}_t - N_{e,t}\tilde{v}_t - C_t - I_t)$$

$$B_{t+1}^* + Q_t B_{*,t+1}^* = (1 + r_t^B)B_t^* + Q_t(1 + r_t^{*,B})B_{*,t}^* + (w_t^* L_t^* + r_t^{k,*} K_t^* + N_{D,t}^*\tilde{d}_t^* - N_{e,t}^*\tilde{v}_t^* - C_t^* - I_t^*)$$

where the foreign country's condition has been expressed in units of the final good in the home economy.

Complete Markets

Under complete markets the equilibrium condition for Arrow securities is implied by the risk sharing condition:

$$\frac{U_c(C,l)}{U_{c^*}(C^*,l^*)} = Q^{-1} \tag{22}$$

4 The Price Index in the Model and its Counterpart in the Data.

A well established property of CES production functions, which we adopted here for the production of final goods, is that they exhibit "love for variety". This means that even if physical quantities of intermediate goods do not change, aggregate output goes up if the range if available varieties expands. This property becomes problematic at the stage of measuring aggregate prices, since the love for variety manifests in the theoretical price index as well, questioning its applicability for the comparison of the model's predictions with the data. To see this more clearly, consider the expression for the price index in the model:

$$P_t = \left[N_{D,t}^{\frac{1-\rho}{1-\theta}} \tilde{p}_t^{1-\rho} + \left(N_{x,t}^*\right)^{\frac{1-\rho}{1-\theta}} \left(\tilde{p}_{x,t}^*\right)^{1-\rho} \right]^{\frac{1}{1-\rho}}$$

As the economy experiences entry both to domestic and foreign markets, aggregate prices P_t could fluctuate just for this reason, even if average prices remained constant. There is several empirical evidence that this gains from variety go mostly unmeasured in CPIs, as documented in Broda and Weinstein (2006). In order to get around this issue, we follow GM and AC a construct a price index where average prices are adequately weighted, by weights that add up to one. In particular, let $\tilde{P}_t = N_t^{\frac{1}{\rho-1}} P_t$, where $N_t = N_{D,t}^{\frac{1-\rho}{1-\theta}} + \left(N_{x,t}^*\right)^{\frac{1-\rho}{1-\theta}}$. Then,

$$\tilde{P}_t = \left[\frac{N_{D,t}^{\frac{1-\rho}{1-\theta}}}{N_t} \tilde{p}_t^{1-\rho} + \frac{\left(N_{x,t}^*\right)^{\frac{1-\rho}{1-\theta}}}{N_t} \left(\tilde{p}_{x,t}^*\right)^{1-\rho} \right]^{\frac{1}{1-\rho}} \tag{23}$$

This variety-adjusted price index looks more like CPIs in the data but it is still subject to the criticism that it updates the share of domestic and exported goods every period, while updates in the consumption basket are done sporadically by statistical agencies. Furthermore, even for constant share of domestic and foreign goods, the average price of the latter fluctuates in the model in response to changes in the productivity cut-off of exporters, leading to what could be associated with product substitution or quality adjustment in the data. Assessing the extent to which these features are captured in CPIs is critical for the interpretation of our results

According to the Handbook of Methods from the Bureau of Labor and Statistics (BLS)[14], the

[14] Handbook of Methods, Chapter 17. The Consumer Price Index. Last update available: 06/2007.

construction of the CPI has suffered several improvements in the recent years regarding the frequency of adjustment of base years and the consideration of product substitutions and quality adjustments. The consumption pattern of US households is identified from the Consumption Expenditure survey (CE), which since 1998 has increased its periodicity from a 10 to a 2 years lag. Furthermore, BLS employees are equipped with a replacement procedure to deal with products that seem to have been substituted out of the sample. Some of these instructions consist on sampling a new product whose qualities are closest to the original one, and treat it as a quality adjusted version of the lost item[15]. We view the more frequent realization of CE surveys as a tendency to improve on the frequency of updating of expenditure shares, and the quality adjustment as a data-counterpart of what in the model would be changes in average export prices due to variations in the average productivity of exporters. Nonetheless, one can think of $\tilde{P}$ as an upper bound on the degree of sophistication with which prices are aggregated. For this reason, we also keep track of a version of it where aggregate prices are measured with fixed expenditure shares and constant productivity of exporters, establishing a lower bound to the complexity of the price index.

In the quantitative part of the paper, we take our benchmark definition of the real exchange rate to be:

$$RER = \frac{\left[\frac{\left(N^*_{D,t}\right)^{\frac{1-\rho}{1-\theta}}}{N^*_t}\left(\tilde{p}^*_t\right)^{1-\rho} + \frac{N_{x,t}^{\frac{1-\rho}{1-\theta}}}{N^*_t}\tilde{p}_{x,t}^{1-\rho}\right]^{\frac{1}{1-\rho}}}{\left[\frac{N_{D,t}^{\frac{1-\rho}{1-\theta}}}{N_t}\tilde{p}_t^{1-\rho} + \frac{\left(N^*_{x,t}\right)^{\frac{1-\rho}{1-\theta}}}{N_t}\left(\tilde{p}^*_{x,t}\right)^{1-\rho}\right]^{\frac{1}{1-\rho}}}$$

which is constructed using $\tilde{P}$ and $\tilde{P}^*$. Substituting average prices according to the optimal pricing rule of the firms, we can write RER as:

$$RER = \frac{\left[\frac{\left(N^*_{D,t}\right)^{\frac{1-\rho}{1-\theta}}}{N^*_t}\left(TOE\right)^{1-\rho} + \frac{N_{x,t}^{\frac{1-\rho}{1-\theta}}}{N^*_t}\left(\frac{\tau\tilde{z}_D}{\tilde{z}_{x,t}}\right)^{1-\rho}\right]^{\frac{1}{1-\rho}}}{\left[\frac{N_{D,t}^{\frac{1-\rho}{1-\theta}}}{N_t} + \frac{\left(N^*_{x,t}\right)^{\frac{1-\rho}{1-\theta}}}{N_t}\left(TOE\frac{\tau\tilde{z}_D}{\tilde{z}^*_{x,t}}\right)^{1-\rho}\right]^{\frac{1}{1-\rho}}}$$

where TOE stands for terms of efficiency, and is defined as

$$TOE = \left(\frac{R^*_t}{R_t}\right)^{\alpha}\left(\frac{W^*_t}{W_t}\right)^{1-\alpha}\frac{Z_t}{Z^*_t} \tag{24}$$

This expression captures the evolution of relative effective marginal costs of production in each country, and would be the sole source of fluctuation in the real exchange rate if we did not account

[15] For a full description of procedure for product substitution see Handbook of Methods, Chapter 17, pages 22-23.

for changes in the share of domestic and foreign producers and if we kept the average productivity of exporters constant[16]. Therefore, we can go from the upper to the lower complexity bound of the real exchange rate by fixing the weights and the average productivity of exporters at their steady state level, and letting RER be entirely determined by the evolution of TOE.

The endogenous determination of the number and productivity of exporters also raises concerns about the computation of export and import prices, with which we construct the terms of trade. In the model, this is given by

$$TOT = \frac{P_x^*}{P_x} = \left(\frac{N_{x,t}}{N_{x,t}^*}\right)^{\frac{1}{\theta-1}} TOE \frac{\tilde{z}_{X,t}}{\tilde{z}_{X,t}^*} \tag{25}$$

As it happened with RER, one can make the claim that this way of measuring TOT is an upper bound for its data counterpart, as it requires that variations in the number and quality of exporters are captured in import and export price indexes[17]. Again, if we wanted these margins to go unmeasured in the model, we could fix the number and the average productivity of exporters at their steady state values, and let the dynamics of TOT be determined by TOE.

In the next section we discuss the quantitative implications of having entry and an endogeneous export decision in the standard international business cycle model. We will pay particular attention to the dynamics of RER and TOT and, given our discussion in this section, highlight the differences in the results depending on how we approach the measurement of international relative prices.

5 Quantitative Analysis

5.1 Basic calibration

We start by choosing benchmark values for the set of relevant parameters trying to match features of the US economy at a quarterly frequency. We follow the standard choices in the literature for the discount factor of households (β) of 0.99, for the intertemporal elasticity of substitution (γ) of 2 and set μ, which governs the share of consumption in the households' utility, to target a steady-state level of 1/3 for hours worked. We also follow the usual choice for the depreciation rate of the capital stock (δ_K) of 0.025 and we pick α equal to 0.36 in order to have a capital income share

[16]It is straightforward to show that if we fix the ratios $\frac{N_D^{\frac{1-\rho}{1-\theta}}}{N_x}$, $\frac{(N_x^*)^{\frac{1-\rho}{1-\theta}}}{N_x}$ (the same for the foreign country), and if we treat $\tilde{z}_x$ and $\tilde{z}_x^*$ as constant, then the log linearized expression for the RER boils down to the same formula we would get in a standard BKK model with intermediate inputs and CES technology for final goods (hat variables denote log-deviation from steady state): $R\hat{E}R = (2s_D - 1)\,T\hat{O}E$, where s_D is the share of domestic varieties in expenditure.

[17]Nakamura and Steinsson (2009) document a measureament bias in export and import prices because of product replacement.

close to 0.3[18]. For many of the other parameter values we follow closely GM's calibration in order to highlight the difference arising from the introduction of invesment and endogeneous labor. In addition, we devote section 5 and 6 to discuss the results of the model when we let the paramaters governing entry and exporting decisions to change.

Iceberg costs are set to $\tau = 1.3$, as in Obstfeld and Rogoff (2001) and GM, and readjust it in our various experiments to maintain a constant import share on GDP. For the fixed cost of exporting, fx, we choose it so as to match a steady state fraction of exporting firms of 21%, in line with the empirical findings for the US by Bernard et al. (2003). As long as we are not interested in a specific target for the mass of domestic firms in steady state, we have one degree of freedom at choosing f_e, so we normalize to be equal to1[19]. In our sensitivity analysis we will let the fraction of exporters to vary from 10% to 100%. Following Bernard et al. (2004), we choose the Pareto shape parameter κ to be 3.4 in order to reproduce in the model the standard deviation of log sales of 1.67 in US plants.

For our benchmark case we make the elasticity of substitution among domestic varieties (θ) equal to the elasticity of foreign and local goods (ρ) and set it to 3.8. A value of theta of 3.8 implies a mark-up of 35.7% relative to marginal costs, which is in the range of 3% to 70% of different empirical studies as documented by Schmitt-Grohe (1997). The standard choice in the international literature is a mark up lower than 20%, nonetheless, in models with a fixed number of varieties as pointed out by GM, firms don't pay sunk entry costs, which creates a gap between average and marginal costs as it is the case here. Hence, our benchmark case delivers a lower mark-up over average costs than over marginal costs, so we believe that this number seems reasonable for the baseline representation.

In the sensitivity analysis we vary θ and ρ and allow for lower mark-ups and higher complementarity between foreign and domestic goods. As we will see in the final section, changing these parameters has relevant implications for the strength of some of the mechanisms at work in the model. The death parameter (δ) is set in our benchmark specification at 0.025, which is consistent with an exogenous destruction rate of 10% in annual terms, as in GM and other papers with entry such as Restuccia and Rogerson (2008) , and implies also a steady-state share of labor accounted by exporting firms of 32%, which is close to the numbers for firms that trade reported by Bernard et al. (2005) [20], and a 10.25% of labor accounted by entrants. In our sensitivity analysis, we let the destruction rate to vary from 0.001 to 0.11, which implies a range of labor shares by new firms

[18]In our model α is not the capital income share given the presence of profits

[19]In the appendix we show that only the ratio $\frac{f_x}{f_e}$ matters for the calibration of the steady-state share of exporting firms.

[20]They report that in the US the labor share accounted by firms that import was 33% during 2000. For the same period, firms that trade accounted for 41.9% and firms that export participated with 39.4% of total employment in the private sector. The same stastistic for firms that both export and import is 30.4%.

from 0.5% to 10.25%.[21]

The summary of the calibrated parameters is on the next table:

Parameter Values

Symbol	Value	Description
β	0.99	Subjective discount factor
γ	2	Inverse of intertemporal elasticity of substitution
μ	0.38	Consumption share -to match $l = 1/3$
α	0.36	To match capital income of share
δ_K	0.025	Depreciation rate
δ	0.025	Exogenous death rate
τ	1.3	Iceberg costs
κ	3.4	Local parameter of the Pareto Distribution
f_e	1	Entry cost parameter - normalized
f_x	0.008	Fix exporting costs parameter -to match $N_X/N_D = 21\%$
η	0.0025	Adjustment cost of bond holdings
$z_{\min}$	1	Productivity parameter

Regarding the specification of the two stochastic processes, we follow closely BKK. The source of uncertainty is coming from aggregate productivity shocks for both the domestic and the foreign country. Using the estimates by BKK for Europe and the US, we allow for a small, positive spill-over effect between the two countries and some persistence to the productivity shock. The stochastic process can be written in the following way:

$$\begin{bmatrix} Z_t \\ Z_t^* \end{bmatrix} = \begin{bmatrix} .906 & .088 \\ .088 & .906 \end{bmatrix} \begin{bmatrix} Z_{t-1} \\ Z_{t-1}^* \end{bmatrix} + \begin{bmatrix} v_t \\ v_t^* \end{bmatrix}$$

The standard deviations and the correlation of the innovation is .00852 and .258 as suggested by BKK .

5.2 Baseline Results

5.2.1 Business Cycle Moments

[21] Although the data is clear about the fraction of US establishments that export, it is less so regarding the size of entering establishments, showing values for the fraction of labor accounted by entrants that go from 3% to 8%, depending on the data referring to firms or establishment. This is why we allow for a comprehensive range of values. Results on this exercise are presented in section 5.

Our benchmark environment contemplates that households can trade bonds across borders. The reason we picked bond trading as the base scenario is threefold: First, it allows us to compare our results with GM, so we can idenfiy the effects of adding investment and endogeneous labor keeping the same financial structure. Second, because it allows us to confront the results of the model with recent papers in the literature that have departed from the complete markets assumption such as Kehoe and Perri (2002) and, finally, because works in the exisitng literature such as Baxter and Cruccini (1995) have proved that aggregate dynamics are similar for the complete markets and the bond trading cases under relative low persitance parameterizations of the aggregate shock. In any case, we devote a whole section later in the paper to verifiy this claim under the three different financial structures.

We report in table 1 the main statistics of the simulated model at the business cycle frequency. In addition, we compare the statistics of our model with results coming from our replications of GM and BKK with the same paramater values[22]. Under this parameterization, our model can account for a volatility of output of 1.44%, closer to the 1.67% reported in the data[23], and higher than the other two. The high volatility of output in response to transitory shocks in aggregate productivity is in its most part a result of the combination of all margins of actions that agents have in this economy. Higher productivity in the home country leads to an increase in labor supply , higher accumulation of physical capital, larger creation of new firms in the domestic market and endogenous recomposition of the tradable basket of goods. All these margins allow the model to have a high degree of flexibility absent in models with a fixed number of firms, no exporting decisions, exogenous labor supply or no capital accumulation.

The model delivers reasonable numbers for the volatility of consumption, investment and hours worked relative to the volatility of output. Moreover, the model predicts a volatility of net exports as fraction of GDP of 0.4, much in line with what is observed in the data and improving on the standard international model that has found difficult to deliver the right dose of trade balance volatility. On this front, our model delivers a more sensible number that GM as adding physical capital introduces a new source for volatility of the trade balance as the home economy builds up capital to raise the profile of future consumption. Also note that relative to the BKK benchmark, our model predicts a volatility of hours worked closer to the data, mainly because entry puts and additional pressure on the domestic labor market.

For the baseline case, the model forecasts a volatility of the number of new firms of 3.8% relative to output. As standard models have a fixed number of entry our benchmark reference is the GM model, which under our calibration renders a volatility of entry of 3.9% relative to GDP. There is no simple comparison with real data for this metric, but in order to have a first approximation we

[22]For the GM replication we make the capital stock to be constant and equal to one and labor to be exogeneous. For the BKK replication we replicate their two sector model under our calibration and with international bond trading.

[23]Data is from Heathcote and Perri (2002), except for the Correlation between CH/CF and Terms of Trade, taken from Corsetti, Dedola and Leduc (2007). Volatility of entry is from our own calculations.

Table 1: Business Cycle Statistics

	Data	Benchmark	GM(replic)	BKK(replic)
Volatilities (% Std Dev)				
Gdp	1.67	1.44	0.94	1.42
Exports	3.94	1.17	0.78	1.81
Trade Balance/Gdp	0.45	0.40	0.12	0.64
Standard Deviation relative to Gdp				
Consumption	0.81	0.52	0.68	0.49
Investment	2.84	2.45	-	4.06
Hours Worked	0.66	0.62	-	0.40
Number of Firms	0.48	0.28	0.29	-
Number of Exporters	-	0.80	0.85	-
Entry	3.77	3.78	3.86	-
International Correlations				
Gdp	0.58	0.01	0.42	0.10
Consumption	0.36	0.79	0.96	0.79
Investment	0.30	-0.86	-	-0.70
Hours Worked	0.42	-0.58	-	-0.54
Number of Firms	na	-0.13	-0.30	-
Entry	na	-0.30	-0.53	-
Other Correlations				
Ratio Cons vs RER	-0.35	-0.87	-0.60	0.76
Ratio Cons vs TOE	-	0.97	-0.45	0.76
Ratio Cons vs TOT	-0.74	0.08	0.22	0.76

compute the business cycle volatility of new establishments based on the new dataset provided by the U.S. Census Bureau[24]. We compute the relative volatility of entry to GDP in annual terms and for that purpose use the hp-filter of the log of the two series[25]. According to the numbers of the Business Dynamic Statistics (BDS) and our calculations, new establishments have a cycle volatility of 3.8%, which is exactly what the model predicts. Note that in our model, entry is by far the most volatile aggregate variable. This is explained by the fact that upon shock, the economy devotes a significant amount of resources in order to expand the set of varieties as firm creation works as a saving decive for households to smooth over time the positive wealth effect associated to the shock. The data seems to confirm this high elasticity of entry relative to cycle, but as a robustness excercise we explore in the next section an alternative specification of entry costs that allows us to control directly for this elasticity and analyze what are the predictions of the model when entry is less volatile.

The models also predicts a correlation between the ratio of relative consumption $\frac{C_H}{C_F}$ and the real exchange rate of -0.87, reverting the so-called Backus-Smith puzzle, present in the standard model. Although departing from complete markets and making labor endogenous is a first step

[24]The Business Dynamic Statistics database is available on the US Census Bureu website. It has information at the firm and establishment level for the period 1977-2005. It has information on exit and entry of establishments, job creation and destruction at the firm level and other firm size statistics by age, sector and state. See the Data Appendix for a more detailed description of the dataset and our calculations

[25]We use $\lambda = 6.25$ as suggested by Ravn and Uhlig (2002) for detrending annual data.

to move away from a perfect positive correlation between the ratio of consumption and the RER, what lies behind the predicted negative sign is the appreciation of the RER after the shock. This appreciation is caused the recomposition of the consumption basket at home in favor of higher-priced foreign goods (firms with lower productivity) and an increase in the home bias of the consumption basket, which translates into higher aggregate prices as domestic goods are more expensive on average than their imported analogous. We will discuss this in more depth in the next section.

Despite some of these nice features, the baseline case does a poor job at predicting international correlations. The output-consumption anomaly documented originally by BKK is present in this model. Cross-country consumption correlation is 0.79 while output correlation is 0.01. Investment and labor are negatively correlated in the two countries, -0.86 and -0.58, respectively, as the shock induces to shift production from the foreign to the domestic economy. As we mentioned earlier, the positive cross-country correlation of output present in the GM model is a result coming from the absence of investment in that model. If there is some degree of risk sharing and investment is introduced, the positive output commovement turns negative as the shocks induces a production reallocation towards the more productivity economy. For this very same reason, our model exhibits a strong negative correlation of the factors of production. This negative correlation is also shared by entry, which has commoves negatively between the two countries. We don't have data on international firm dynamics, but this is one dimension in which our model can be tested.

5.2.2 Impulse-Responses

Figure 1 presents the responses of the key variables of the model to a transitory positive shock of one standard deviation on domestic aggregate productivity. Because of the shock, the home market becomes more attractive as the value of domestic firms increases because of higher current and expected future profits, alluring prospective entrants to start a business. New firms join the pool of domestic producers by paying the sunk entry costs denominated in labo units but given the time-to-build assumption they turn operative only the next period. Hence, at the moment of the shock, all new output comes from the intensive margin as existing firms scale up their size, but as time passes the extensive margin carries on with the positive effect of the shock. The mass of domestic producers, N_D, which is a state variable fixed at the moment of the shock, increases over time as a result of the positive entry. The fact that new firms pay entry costs, diverts some labor away from current production and put some pressure on domestic wages, otherwise absent. Despite higher demand for domestic inputs by existing and prospective firms, the home country is relative more efficient as a result of the positive shock, which explains why the terms of efficiency (TOE) [26] increases after the shock and slowly returns to its steady state value.

From the household point of view, the positive productivity shock generates a wealth effect

[26] Recall that the Terms of Efficiency are defined as the ratio of marginal costs of producers abroad relative to home with the same firm-specific productivity level. See equation (24)

that translates into higher consumption. In the international bond trading case, there exists some risk sharing between the two countries, so consumption in the foreign country also increases, although to a less extent. In addition, higher consumption abroad is a result of the propagation effect caused by the possibility of trade. Higher demand in the home market for both domestic and imported varieties, leads to an increase in the real value of foreign goods that drives output of the foreign country upwards, despite the lower use of inputs by foreign intermediate producers . The asymmetric response of consumption in the two countries reflects, then, the risk sharing options available to households and the reallocation of production towards the most efficient economy.

In order to smooth consumption, households invest in physical capital and finance the creation of new firms. Firm creation can be seen as saving device that delivers higher output in the future due to a larger number of varieties at the current cost of the labor disutility associated to the creation of a new business and the foregone opportunity to scale up production in existing firms. Therefore, entry and investment follow a similar pattern as the two react favorably to the shock. Nevertheless, their paths are no perfectly synchronized as the two are not perfect substitutes. Their relative costs are different: while entry requires of labor, investment is in terms of units of the final good. Their relative price is also different. The value of entry is determined by the expected profit of domestic firms. This asymmetry can be seen from the fact that in the foreign country the capital stock declines for about 10 quarters while entry falls only for few periods. The reason why the mass of firms recovers faster than the capital stock is that the value of foreign firms increases over time, as foreign firms can tap some profits from the home country. Higher demand for imported goods in the home country outweighs the worsening of their relative efficiency and as a result more foreign producers have access to local markets and higher profits.

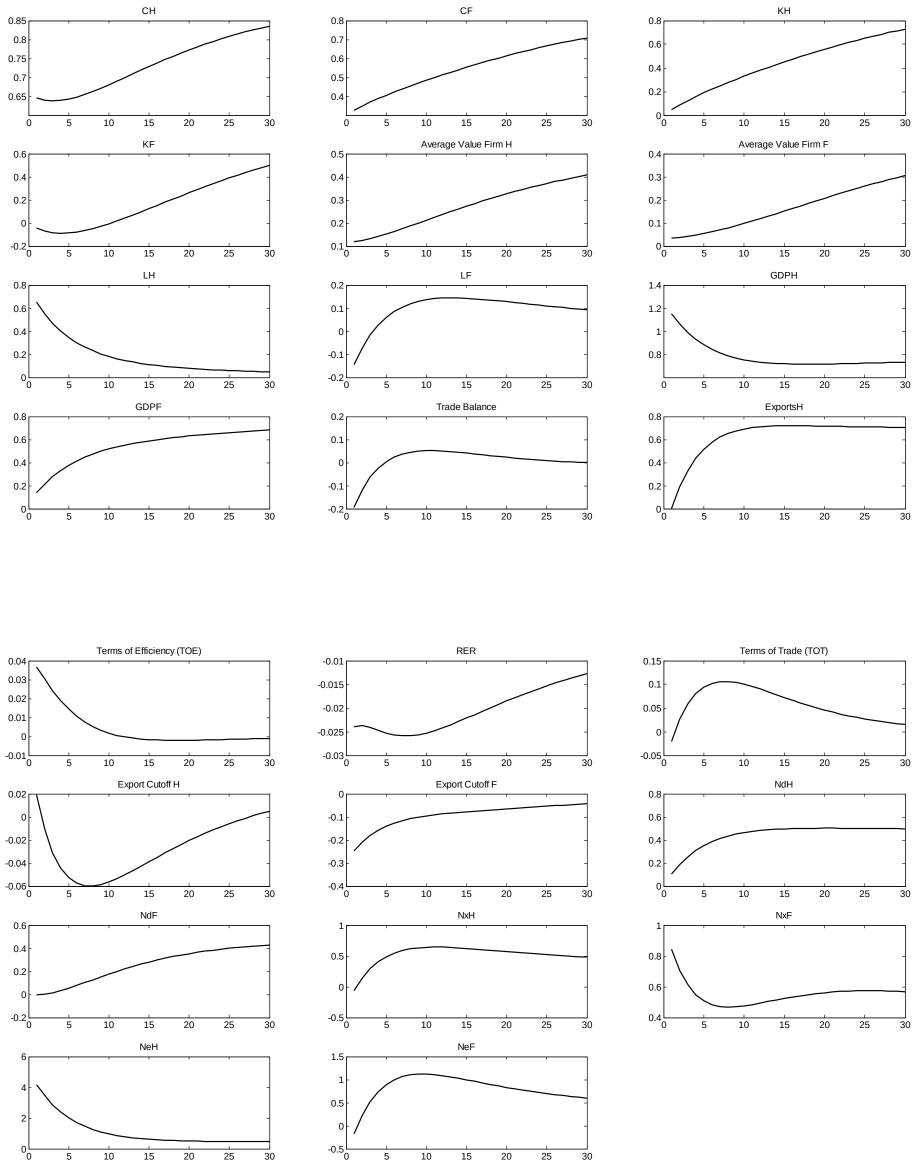

Figure 1. Impulse Responses to a Home Productivity Shock (log-deviations from steady-state)

Because of higher demand in the home country, the export threshold in the foreign country falls. Despite that TOE is increasing, more foreign firms manage to export. As a result of this, less productive foreign firms have access to consumers in the home country, which leads to an increase in the mass of foreign exporters. On contrary, relative lower demand abroad induces local producers to serve mainly domestic markets making the export threshold at home to increase for a few periods after the shock. The initial increase in the home threshold and the fall in its foreign counterpart, leads to a decline in the mass of domestic relative to foreign exporters and leads to an appreciation of the RER as the domestic consumption basket includes now a relative higher proportion of less productive foreign firms.

In addition to this force, the relative differences between entry abroad and at home induces the RER to appreciate, as in the GM model. The larger the entry in the home country, the more the bias of domestic households towards goods produce locally. As domestic varieties are more expensive than imported goods, on average, a higher intensity of local goods in the consumption basket translates into higher aggregate prices in the home country and a lower real exchange rate. It happens that for our baseline parameterization these two channels: 1) relative changes in the export threshold and 2) changes in the share of domestic goods in the consumption basket, both in support on an *appreciation* of the RER, offset 3) the effect of higher terms of efficiency (higher TOE), which is in favor of a *depreciation*. As a result, the correlation between the ratio of consumption abroad and at home is positively correlated with TOE and negatively correlated with the real exchange rate. Here our previous discussion on how prices are measured plays a key role for the assessment of the performance of the model. If one assumes that price indexes and hence the real exchange rate, as measured in the data, don't reflect any of the composition effects implied by channels 1) and 2), then the model cannot account for the negative correlation between CH/CF. In this case, TOE will be the relevant measure for the real exchange rate and the model will predict a depreciation of it after the shock, just as in BKK.

Note that our model delivers a different prediction for TOE than the GM model does. Absent endogeneous labor and investment, TOE appreciates as a result of the shock. When labor is in fixed supply the domestic labor market can only adjust through higher wages in response to higher demand by existing firms and potential entrants. Furthermore, without physical capital the economy can only rely on entry to smooth consumption over time, hence creation of new firms is stronger adding even more pressure on domestic wages. This is why in the GM model, the relative price of labor abroad relative to home appreciates. Adding elastic labor supply and investment helps to ease off the pressure on domestic wages and prevent the TOE to appreciate.

6 The Role of Entry and Fixed Costs of Exporting

Quantities and prices behave differently in the model once we allow for endogenous entry to domestic and foreign markets. In this section we address the issue of how these two channels interact in shaping the dynamics of aggregate variables.

There are two dimensions of the entry and export decisions that we could experiment with to identify the role of each of them in the behavior of quantities and prices. One refers to their steady state participation, namely the steady state size of new entrants and the steady state fraction of exporting firms. Controlling these shares and considering the two limiting cases where the number of domestic producers is fixed or all firms in the economy export allow us to identify the marginal contribution of each channel. These exercise stands as particularly relevant in light of the opposing results found in GM and AC, the former stressing the role of entry, and the latter diminishing the relevance of sunk and fixed exporting costs.

Another important aspect of entry and the export decision is their elasticity with respect to aggregate productivity shocks. How responsive do entry and the extensive margin of exports have to be in order to non-trivially impact on the dynamics of aggregate variables? The benchmark model, for instance, predicted a volatility for the number of entering firms that is 3.8 times that of GDP, making it one of the most volatile variables in the cycle. This fact motivates us to study the role that the volatility of the two entry margins might have in making them relevant. We do so in the model by specifying entry and fixed export costs functions that explicitly control for the elasticities of the number of entrants and the fraction of exports relative to a productivity shock , without affecting the steady state of these two variables. The latter is an essential feature of the experiment, since it allows us to identify the role of volatilities separately from the volatility effect implied by different steady state sizes

6.1 Changes in the Steady State Share of Entry

The level of entry in the symmetric steady state of the model is a fraction of the steady state mass of domestic producers, determined by the exogenous death probability

$$N_e^{ss} = \frac{\delta}{1-\delta} N_D^{ss}$$

An implication of the common distribution of productivities within entrants and existing firms is that the average size of entering firms is identical to that of incumbents, implying that $\frac{\delta}{1-\delta}$ also determines the share of labor accounted for by new establishments. Under our benchmark calibration, this share is 10.25% annually, attained with a quarterly death rate δ of 2.5%. Since we consider this value as an upper bound for the parameter, we experiment with death rates that start there, and go down to 0[27]. Throughout these cases, we keep active the extensive margin of trade, leaving the calibration of the share of exporting firms at the benchmark 21% level.

[27] This is a limiting case where there is no exit of firms, so there cannot be entry. In this version of the model, we fix the number of firms, and eliminate the free entry condition from the set of equilibrium equations

Business cycle statistics of a relevant subset of aggregate variables are illustrated in figure 2. The left panel contains the volatility of investment, exports and the trade balance over GDP for each value of the death shock. A striking flat pattern for the volatility of these macroeconomic aggregates arises for all positive values of the exit shock. This suggests that, conditional on positive entry, the steady state size of entering firms does not seem to be relevant for the short run performance of investment, exports and the trade balance[28]; allowing the model to accommodate a wide range of calibrated values for δ without severely affecting quantities.

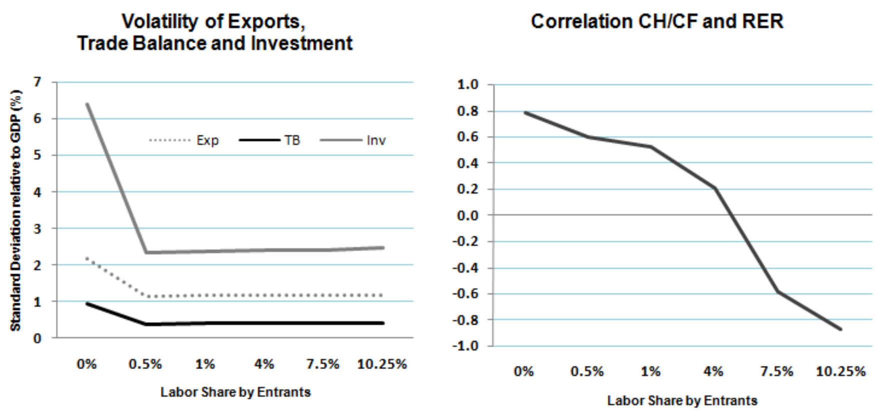

Figure 2.

The smooth response, however, is broken once we consider the case of no entry, identified with $\delta = 0$ in the graph. There, investment, exports and the trade balance are substantially more volatile than in all other cases, and more so than in the standard BKK counterpart[29]. Recall that the adjustment of exports at the extensive margin is still operative, which is now the sole difference between this version of our model and BKK. To further explore a potential non-trivial interaction between entry, investment and the exporting decision, signalled by the overshoot in the volatilities, we reproduce the impulse response function for entry, investment, exports and the trade balance in figure 3. The picture shows a strikingly different short run response in these variables. Investment increases roughly 4 times as much in the no-entry scenario, leading to similar order of magnitudes in the differential response of exports and the trade balance. Absent entry, the home country responds to a positive aggregate productivity shock increasing its capital stock, in order to smooth out the effect of the shock. Investment goods, in turn, are produced in the final goods sector combining domestic and, importantly, foreign inputs. The foreign country can meet the increased

[28]Other non-reported macroeconomic aggregates display a similar pattern

[29]BKK statistics for a comparable calibration gives a volatility of investment and exports of around 4 and 1.8 times that of GDP respectively, and a volatility of net exports in the order of 0.64

foreign demand both through the intensive and the extensive margin. This latter feature, which results from the heterogeneity in firm's productivity and the fixed exporting costs, gives an additional boost to domestic invesment, exports and the trade balance, and enhances the volatility[30]. When investment is interacted with entry, they behave as *substitutes* in serving as consumption smoothing devices. Households create more firms than they build capital when having the two options available, which dampens the volatility of investment. In addition, since entry costs are in labor units, the creation of establishments does not spill into a higher demand for foreign inputs, reducing the volatility of exports and the trade balance[31].

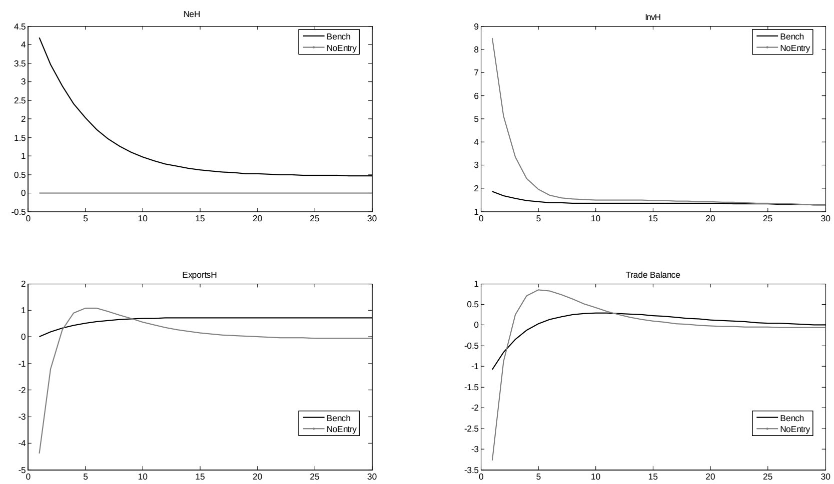

Figure 3. Impulse Responses to a Home Productivity Shock (log-deviations of steady-state)

[30] Although our interest here is in assessing changes in volatilities against variations in the environment, the high levels in the case of no-entry can be partly attributed to the high elasticity of substitution that we work with here, which we borrowed from the trade literature. In this regard, our contribution from understanding the relationship between entry and investment would have been to reconcile working high estimate for the elasticity of substitution and still get plausible values for the volatilities of aggregate quantities

[31] Certainly, had entry costs being measured in units of the final good, it would have also been the case that entry translates into higher demand for foreign inputs, higher volatility of exports and the trade balance. In Fattal-Jaef and Lopez (2009) we explore the connection between entry and investment, and its dependence on the units of measurement of entry costs; in the context of a closed economy, where the relationship can be characterized more sharply. We find that measuring entry costs in units of the final good leads to indeterminacy under most reasonable parameterizations, if the production technology in the final goods sector displays love for variety, as it does in this paper. When entry costs are measured in labor units, the equilibrium is determinate and predicts that entry and investment are substitutes, as in the open economy setting of this paper.

The right panel in figure 2 illustrates the behavior of the correlation between the ratio of home and foreign consumption and the real exchange rate[32]. Even though the amount of entry proved not to be particularly relevant for quantities, it has strong implications for the dynamics of relative prices. As soon as the share of labor accounted for by entrants falls below 4%, the correlation between RER and the ratio of consumptions goes back to positive, in accordance with the Backus and Smith puzzle. Recall that variations in the sign of this correlation are mainly driven by the cyclical behavior of the RER, since $\frac{C_H}{C_L}$ increases upon shock in most specifications of two-country models. Thus the extent to which entry can revert the dynamics of the RER, compared with the BKK benchmark, depends critically on the exogenous exit rate. Again, an interesting case to consider is the one where the number of firms is fixed, which corresponds to $\delta = 0$ in the right panel. This is as close as we can get to AC without explicitely having sunk export costs in the model. The correlation between the RER and the ratio of home to foreign consumption is 0.8, almost as high as in the BKK benchmark. Despite our model not having the rich structure of trade costs that AC have, we share their finding that these costs, on their own, are not enough to deliver big quantitative departures from the BKK model.

6.2 Changes in the Steady State Fraction of Exporting Firms

The share of exporting firms in this model is given by $\frac{Nx}{N_D} = 1 - G(z_x)$, which uniquely pins down the export cutoff z_x from the Pareto cumulative distribution function. Parameter values have to be chosen so that this threshold satisfies the zero exports profit condition, for which we re-calibrate the fixed exporting cost. We also re-calibrate the iceberg cost, so that the steady state share of imports in GDP remains constant throughout all experiments.

The left panel in figure 3 shows the volatilities of investment, exports and the trade balance against steady state fractions of exporting firms that range from 10% to 100%. As before, quantities do not display big changes in all cases where only a subset of firms export, showing a mild increasing pattern. Although not reported in the graph[33], the volatility of the number of exporting establishments, which proxies for the extensive margin of trade, increases slightly as we increase the share of exporters, in response to the export productivity cutoffs shifting to regions of the Pareto distribution with higher concentration of firms. Therefore, minor changes in the export cutoff moves a bigger mass of firms in an out of the tradable sector, driving up the volatility of the number of exporters and, with it, the volatility of exports, investment and the trade balance. Quantitatively, though, this effect is faily small.

[32]In this section, we work with the definition of Price Indices that contemplates changes in the expenditure share and the average productivity of exporters: see equation (23)

[33]See Table 5 in the Appendix

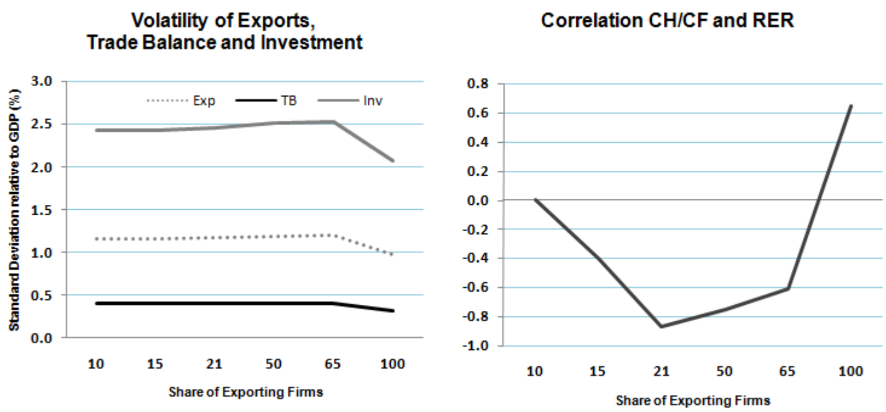

Figure 4.

The picture looks different in the limiting case where all firms export[34]. There, the foreign country can only serve the increased demand for imports by the home economy scaling up the production of existing firms. This effect is partially offset by an increase in the marginal cost of production in the foreign country, which limits the overall expansion in exports. Therefore, exports cannot adjust as strongly as they did when there was an additional inflow of firms entering export status, thereby reducing the volatility of aggregate variables.

International relative prices are very sensitive to the steady state fraction of exporting firms (see figure 4). The correlation between the ratio of home to foreign consumption and the RER displays a U-shaped pattern, being close to zero for very low values of the steady state share, turning negative for a range of values that covers our benchmark calibration of 21%, and becoming positive in the limiting case were all firms export. This picture highlights the role of the endogenous determination of the average productivity of exporters in shaping the dynamics of the RER.

There are two channels through which the endogenous export decision impacts the RER. There is a direct effect, resulting from its determination of the average productivity of exporters, which translates directly into the RER through average import prices. And there is an indirect effect, following from switches in expenditure form imported to domestic goods. The export decision determines the productivity advantage of exporters and thus the extent to which countries' aggregate prices are increased by switching expenditure away from foreign inputs.

To better understand how these effects work, lets compare the benchmark economy, where only a 21% of the firms in the economy export in the steady state, with one where all firms sell their products overseas. The *direct effect* in the benchmark calibration acts in favor of a RER appreciation,

[34]Recall that in this extreme we are still allowing for new firms to enter the domestic market.

as the average productivity of foreign exporters decreases relative to home[35]. This force, however, is turned off when all firms in the economy export. The *indirect effect* also makes the RER to fall in the benchmark scenario. There, entry of new firms make the home country switch expenditure away from foreign varieties towards domestic ones, which are produced by less productive firms and offered at higher prices. When all firms export, there is no productivity advantage in foreign exporters, thus switching expenditure towards domestic goods no longer incresases aggregate domestic prices. On the contrary, it reduces it, since the home country now economizes on the iceberg cost. As a result, the forces that made the RER appreciate after a positive productivity shock in the benchmark economy are absent in this limiting case, justifying that the correlation with the ratio of consumptions is back to positive.

Putting the results from this and the previous sections together it follows that neither entry nor the fixed exporting cost can, on their own, revert the standard forces that drive the correlation between the RER and the ratio of consumptions in the BKK model. The two margins are *complements* with each other in turning the correlation negative. Furthermore, on the quantities side, the co-existence of entry to domestic *and* foreign markets proved vital in keeping the volatilities of aggregate quantities closer to the data. This result enhances the value of having introduced physical capital accumulation to the model, as it depends crucially on the dampening effect of entry on investment that we have identified in this paper

6.3 The Elasticities of Entry and the Number of Exporters

We now focus on the role of the degree of responsiveness in entry and the fraction of exporters to aggregate productivity shocks. The question is if, conditional on steady state sizes that fall in the favorable range for the relevance of the entry choices, there is also an additional requirement on their volatilities, which is the observable measure for what we here refer to as the elasticity with respect to aggregate shocks.

Consider the following specifications of the entry and fixed exporting cost functions:

$$\begin{aligned}\hat{f}_e &= f_e + \gamma_e \left[\exp\left(N_{D,t} - N_D\right) - 1\right] \\ \hat{f}_x &= f_x + \gamma_x \left[\exp\left(z_{X,t} - z_X\right) - 1\right]\end{aligned}$$

Variables without subscripts refer to steady state values. N_D is the steady state number of domestic producers, and z_X is the steady state export productivity cutoff, which pins down the fraction of exporting firms.

The entry cost is now an increasing function of the *change* in the aggregate number of domestic

[35] Recall that individual prices depend negatively on productivity. Thus, when the average productivity of foreign exporters falls relative to home, the price of imported varieties increases at home relative to the foreign country, appreciating the real exchange rate.

producers, introducing a negative externality on the decision of future entrants[36]. The strength of this externality is determined by the parameter γ_e. Notice that the steady state formulation of the cost functions is identical to the benchmark one, ensuring that the steady state in this economy remains unchanged. In the same fashion, the fixed exporting cost is an increasing function of the change in the *fraction* of firms that export[37], determined here by deviations in the export productivity cutoff. The parameter under controll is now γ_x. An additional nice feature of these cost functions is that it allows to converge smoothly to the BKK model when it the limit we make γ_x and γ_e sufficiently high, so there is a constant number of firms and an a steady fraction of exporters[38].

6.3.1 The Elasticity of Entry

We start studying the role of the volatility of entry. We fix γ_x at 0, and move γ_e from 0 to a level that is high enough to keep the number of producers constant[39].

Figure 5 illustrates the volatilities of investment, exports and the trade balance on the left panel; and the volatility of entry and the number of exporters on the right one. Figure 6 depicts the correlation between the RER and the ratio of home to foreign consumption.

[36] The firms that generated the negative externality on today's potential entrants are those who entered the economy in the previous period because of the one-period lag in entry.

[37] If the fixed export cost were a function of deviation in the *number* (rather than the fraction) of exporters , then we would make the entry externality spill into the export decision as well. This is because the number of exporters is determined by both the productivity threshold and the overall number of firms. Then, when new firms enter the economy, everything else equal, the number of exporter increases, increasing with it the exporting cost.

[38] We could not converge smoothly to BKK with variations in the steady state shares since there was a discontinuity in the type of models we are solving. If we wanted to make the number of firms and the fraction of exporters constant, we had to do it eliminating the free entry condition and the zero export cutoff profit equation from the model.

[39] This does not mean that there will be no entry in the economy, since some firms are exiting. What it says is that there will be zero *net entry* in every period.

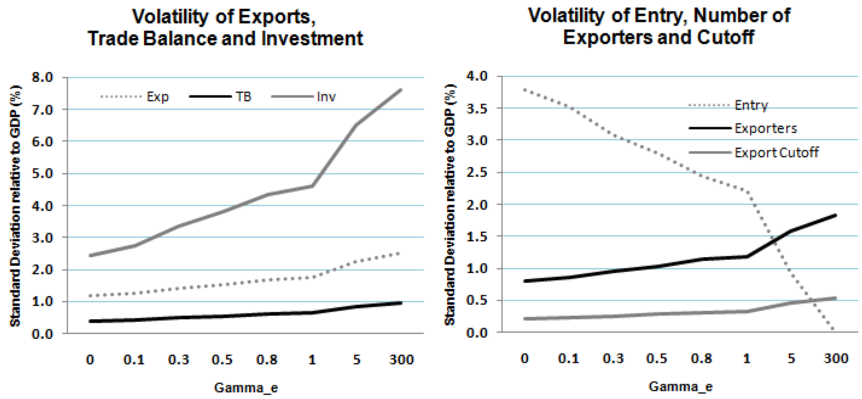

Figure 5.

A clear pattern emerges in the two panels of figure 5. As we dampen the volatility of entry, the volatilities of investment, exports and the trade balance go up sharply, at the same time that the volatility of the number of exporters increases. Entry has to be highly volatile, as much as 2 times the volatility of output, for the volatility of quantities to fall within plausible values. For instance, when $\gamma_e = 0.5$, the volatility of investment is close to 4 times the volatility of output (a reasonable upper bound for this statistic); while the volatility of entry is still in the order of 2.5 times that of GDP. The graph reinforces the idea that entry and investment behave as substitutes upon productivity shocks, and that the household is levying on entry most of the burden of consumption smoothing. As the entry externality makes the relative price of entry higher, the incentives are reversed and investment becomes more volatile[40].

[40] This is as far as we can go in characterizing the relationship between entry and investment in this rich open economy setting. We make further progress analytically in Fattal-Jaef and Lopez (2009), where we work with a simplified closed economy environment

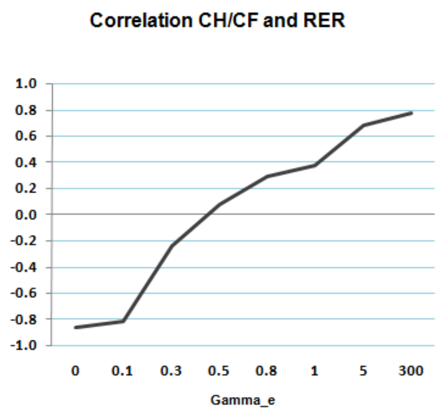

Figure 6.

The volatility in exports and the trade balance mirrors that of investment. Since the final good requires foreign intermediate inputs, the increased volatility in the home country's demand for imported varieties maps into a higher volatility of exports, the trade balance and the number of exporters.

Regarding the role of the volatility of entry on the dynamics of the RER, and its correlation with the ratio of consumptions, figure 6 shows that entry has to be fairly volatile for this correlation to fall in the negative region. For relative high values of γ_e, the correlation is back to positive. The driving mechanism here is that by dampening the elasticity of entry we are weakening the expenditure switching effect on the RER, which as we discussed earlier acts in favor of the RER appreciation. The less the economy adjusts the number of firms through entry, the less it reallocates expenditure towards domestic, relatively more expensive, varieties. In the limit, when net entry has been reduced to zero ($\gamma_e = 300$ on the graph), the correlation is as high as 0.8, consistent with the number we found earlier when we shut down entry setting the exogenous death rate equal to 0.

6.3.2 The Elasticity of the Share of Exporting Firms

We now experiment with different elasticities for the fraction of exporting firms, keeping constant the elasticity of entry at the benchmark value ($\gamma_e = 0$). Results are in figures 7 and 8. A salient feature of the graphs is how little it takes to make the extensive margin irresponsive to shocks. As soon as γ_x becomes positive, the volatility of the export cutoff falls almost to zero [41].

[41] Notice that although the fraction of exporters is not adjusting, the extensive margin of trade is not turned off. Because of entry, even for a fixed *share* of exporters, the *number* is varying.

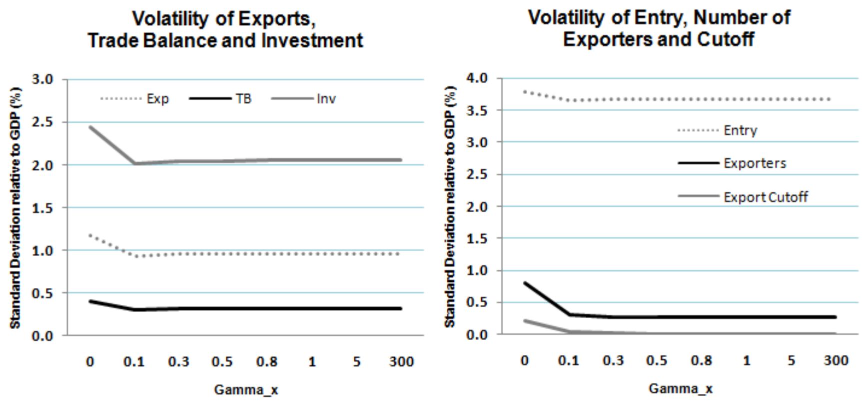

Figure 1: Figure 7.

Regarding quantities, the right panel allows us to verify a result we found earlier: adjustments in the fraction of exporting firms expand the volatilities of aggregate quantities. The volatility of investment relative to GDP, for example, drops from 2.5 to 2, while the relative volatility in exports and the trade balance falls by similar amounts.

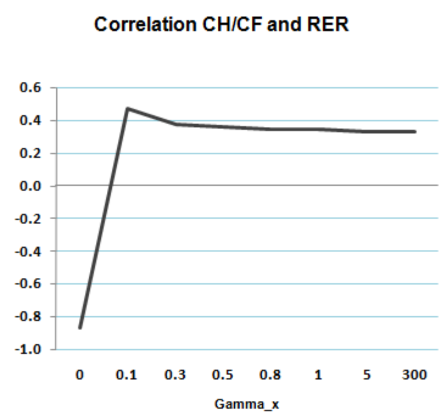

Figure 8.

The graph for the correlation between the RER and $\frac{C_H}{C_F}$ is also illustrative of the importance of fluctuations in the fraction of exportes for the dynamics of relative prices. When these adjustments become negligible, the correlation turns counterfactually positive. Once again we can explain this result looking at the *direct* and *indirect* effects of the export cutoff on the RER[42]. The *indirect effect* is still pushing towards a RER appreciation. The *direct effect*, on the contrary, is (almost) gone . When the export productivity cutoff does not react to aggregate shocks, the model loses a force that made the RER appreciate in the benchmark calibration. Overall, the correlation increases to roughly 0.4, much higher than the -0.87 of the benchmark model, but lower than the 0.8 in the situation where all firms export.

6.4 Discussion

The main message from our analysis is that allowing for entry *and* incorporating fixed costs of exporting are essential for the model to have distinguishable quantitative properties from BKK. Moreover, the number of producers and the share of exporters must be volatile enough for the mechanisms to work.

To some extent, our results reconcile the findings in GM and AC. The former highlights the relevance of entry as a key force driving the novel RER dynamics, while the latter undermines the aggregate effects of featuring sunk exporting costs with a constant number of firms. We share with AC the view that opening an extensive margin of trade alone does not noticeably affect the dynamics of aggregate variables, while we share with GM that letting entry be a volatile variable in the business cycle improves the fit of the model remarkably.

7 Sensitivity Analysis

7.1 The Role of the Financial Structure.

Although the response of the economy facing different financial structures is well understood for the BKK benchmark, there is no such knowledge for economies that feature heterogeneous firms, entry and endogenous export decisions. Heathcote and Perri (2002), for instance, find that economies endowed with complete financial markets or restricted to trade a single uncontingent bond yield similar quantitative results, while assuming financial autarky improves the fit of the model in terms of prices, worsening it in terms of quantities. Kehoe and Perri (2002) endogenously restrict risk sharing across countries by assuming limited enforceability of contracts, finding that such institutional arrangement can revert the incentives to shift capital to the more productive economy, thereby reversing the negative cross country correlation of investment and hours worked that the standard model delivers. Our goal in this section is to document the impor-

[42]Note that here we have turned off any variation in the fraction of exporters, but still have that not every producer exports. This is crucial, for it means that when the home economy switches expenditure from foreign to domestic varieties, it is actually facing higher prices, given that there is a productivity disadvantage in domestic producers relative to foreign exporters. This is different from section 5.2 where we made all firms to export.

Table 2: Financial Structure

	Financial Structure		
	Complete Markets	Bond	Financial Aut.
Volatilities (% Std Dev)			
Gdp	1.52	1.44	1.42
Exports	1.89	1.17	0.75
Trade Balance/Gdp	0.75	0.40	0.00
Std Dev relative to Gdp			
Consumption	0.48	0.52	0.54
Investment	3.83	2.45	1.11
Hours Worked	0.71	0.62	0.61
Number of Firms	0.31	0.28	0.28
Number of Exporters	1.22	0.80	0.51
Entry	4.10	3.78	3.70
International Correlations			
Gdp	-0.09	0.01	0.05
Consumption	0.92	0.79	0.74
Investment	-0.95	-0.86	-0.32
Hours Worked	-0.71	-0.58	-0.55
Other Correlations			
Ratio Cons vs RER	-0.66	-0.87	0.56
Ratio Cons vs TOE	-0.13	0.97	0.87
Ratio Cons vs TOT	-0.49	0.08	0.91

tance of the financial structure for the behavior of micro-level variables, and its repercussion on the aggregate economy. To this end, we follow the Heathcote and Perri (2002) approach and consider three alternative specifications: complete markets, international bond trading[43] and financial autarky[44].

We start looking at business cycle statistics and impulse response functions for the economy with complete markets, reported in table II and figure 9**.** Consistent with earlier findings, aggregate variables are more volatile, and international correlations are amplified. The volatility of entry and the number of exporters are also higher, motivated by the incentives for the foreign country to shift production and production units to the home country, being able to receive the beneifts of its higher productivity by trading state contingent claims.

Before studying the implications of complete markets for the dynamics of the real exchange rate, recall that this is tightly connected with the dynamics of relative consumption through the risk-sharing condition, applicable to this market arrangement

$$\frac{U_c(C,l)}{U_{c^*}(C^*,l^*)} = Q^{-1}$$

We argued in a previous section, however, that this measure of the RER is not the adequate one

[43]Our benchmark specification in previous sections.

[44]In section 2, where we described the model, we specified how each financial structure enters into the budget constraint of the households, and fits into the definition of the equilibrium

when comparing the quantitative predicions of the model with the data. There, we proposed an alternative price index that, when used to calculate the RER, breaks any theoretical linkeage with the dynamics of relative consumptions.

The table shows that the correlation between the ratio of home to foreign consumption and the RER remains negative, consistent with the sign we observe for this correlation in the data, and opposite to would be obtained in the BKK model. What is more surprising is that even under our conservative measure of the RER (proxied by TOE), the correlation is negative (-0.13), distinguishing the complete markets economy from the one with international bond trading, where taking TOE as the definition of real exchange rate rendered a high and positive correlation. Moreover, the sign of the correlation between relative consumption and the terms of trade (TOT) also turns negative (-0.49), as in the data.

The figure with impulse response functions might help gather intuition about what is driving the results. As a first impression, notice that micro-level variables (the export cutoffs, the number of new and exiting firms, and the measure of exporters) display great sensitivity to the financial structure, that translates into the aggregate variables that depend directly on them, such as the real exchange rate, the terms of trade and the terms of efficiency. The latter, in particular, behaves very different under complete markets than under bond trading or financial autarky. When markets are complete, TOE depreciates only slightly in the first periods, and keeps depreciating before it goes back to the steady state. The reason why the initial depreciation is smaller is that facing a positive aggregate productivity shock, the home country increases investment and the entry of new firms more strongly, exercising an additional pressure on the market for factors of production. Although entry and investment serve the same purpose upon shock, once the stock of firms has gone up the two complement each other, as once entrants become operative they demand more capital. For this reason, TOE depreciates even less a couple of periods after the shock. Thereafter, since the economy has accumulated a lot more capital stock at home than abroad, the rental rate of capital falls markedly in the former relative to the latter, reflecting in the sharp depreciation we observe in TOE about 10 periods after the shock. This dynamic is accompanied by the usual increase in $\frac{C}{C^*}$ that takes place after positive productivity shocks to the home economy[45], explaining the negative correlation between the two.

[45]We did not report the impulse response for $\frac{C}{C^*}$, but it is well understood that, even under complete markets, this variable increases after positive productivity shocks.

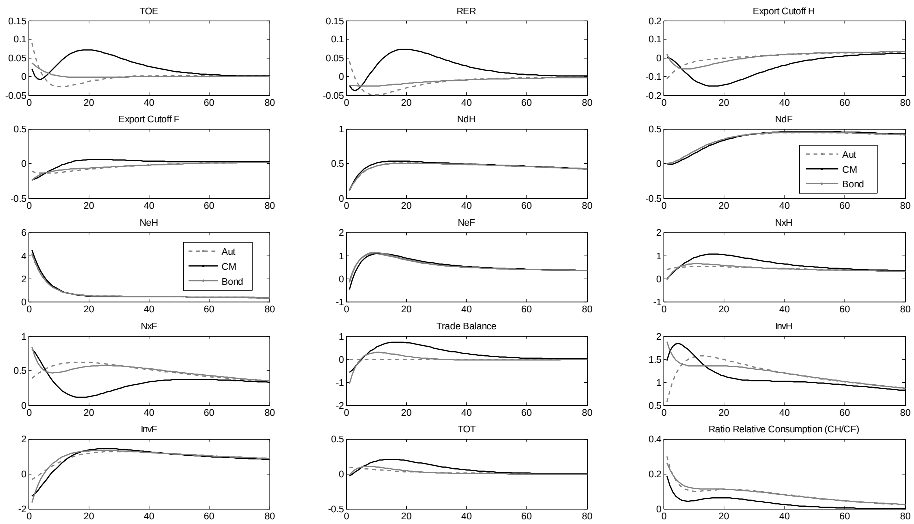

Figure 9. Impulse Responses under different Financial Structures

The differential response in TOE is also a key ingredient explaining the dynamics of TOT. Replacing $\frac{N_x}{N_x^*} = \left(\frac{z_x^*}{z_x}\frac{N_D}{N_D^*}\right)^k$ in equation (25), we get that the terms of trade are given by[46]:

$$TOT = \frac{P_x^*}{P_x} = \left(\frac{N_{D,t}}{N_{D,t}^*}\right)^{\frac{1}{\theta-1}} TOE \left(\frac{z_{X,t}^*}{z_{X,t}}\right)^{\frac{k-(\theta-1)}{(\theta-1)}} \tag{26}$$

Notice that the measurement concerns that we raised about aggregate price indexes apply to import and export prices as well, since they also depend on variables that go unmeasured in import/export price data. To avoid this, we can follow what we did with the real exchange rate and look at a conservative measure of the terms of trade, defining it as equal to the terms of efficiency (TOE). In this case, as it happens in the standard BKK model with constant number of identical firms, all correlations with respect to RER and TOT are identical and given by the correlation with respect to TOE.

The short run behavior of TOT is quite similar in the complete markets and international bond trading settings, starting to differentiate approximately 10 periods under the shock. As the acceler-

[46]Recall, from the calibration, that $k = 3.4$ and $\theta = 3.8$, so $k - (\theta - 1)$ is greater than zero.

ated increase in investment in the home country dissipates, the foreign exporters see the demand for their goods cut back, and face relatively higher marginal costs. Since these effects are substantially stronger under complete markets, the medium term export productivity cutoff increases abroad before it returns to the steady state level, while it falls in the home country. The evolution of the ratio of export productivity thresholds, then, adds to the dynamics of TOE in shaping the behavior of TOT across these financial structures[47].

It is less clear how the dynamics of the TOT just described for the complete markets case match up with the negative correlation between this and the relative consumptions. The confusion follows from the fact that although different in magnitudes, the direction of the changes are the same as in the bond trading setting, but there the correlation was virtually zero. For this particular result it turns relevant to consider the implications of the available risk sharing opportunities for the evolution of the ratio of consumption, which we have been leaving in the background so far. We can see in the impulse response functions how the state contingent claims allow for a better sharing of risk across countries, as dictated by the smaller increase in the ratio of home to foreign consumption. It is the differential response of this variable, that contributes in explaining the negative correlation with the TOT.

The effect of precluding all trades in international financial markets is also consistent with previous findings for standard BKK models**.** Output, consumption and investment are less volatile; international correlations of investment and hours worked are higher, but still counterfactually negative; and the excess of correlation between home and foreign consumption over each country's GDP is reduced. Forbidden any borrowing in financial markets, investment and entry have a weaker response to positive shocks in the home economy, thus making the *terms of efficiency* going up by a higher amount. In addition, the milder increase in the home country's aggregate demand (and a milder fall in foreign's aggregate demand) combined with the sharper depreciation in terms of efficiency translate into a smaller drop in foreign country's export productivity threshold, and in a reduction of home country's threshold. As a result, the average productivity of foreign exporters decreases relative to the home's, but less so than in the other two cases; so the real exchange rate depreciates during the first periods. Since the ratio of consumption is going up, these dynamics for the real exchange rate map into a positive correlation with the ratio of consumptions.

Although the behavior of aggregate variables under the three financial structure is in line with previous research, the differential behavior at the macro level feeds back into the microeconomic underpinnings of the model, making it deliver substantially different responses in micro-level variables that shape the dynamics of the RER and the terms of trade. An important contribution of this section is to have shown that, if complete markets were to be a better representation of international financial markets, allowing for entry to both domestic and foreign markets would

[47]The effects coming from $\frac{N_D}{N_D^*}$ are negligible, as we can see from the impulse response functions for the number of domestic and foreign producers

Table 3: Elasticity of Substitution between Foreing and Domestic Goods

	ρ					
	1.5	2	2.5	3	4	5
Volatilities (% Std Dev)						
Gdp	1.33	1.36	1.39	1.41	1.45	1.49
Exports	0.68	0.78	0.88	0.98	1.22	1.58
Trade Balance/Gdp	0.13	0.19	0.25	0.31	0.43	0.62
Std Dev relative to Gdp						
Consumption	0.53	0.52	0.52	0.52	0.52	0.52
Investment	1.41	1.72	1.94	2.13	2.54	3.16
Hours Worked	0.63	0.64	0.63	0.64	0.62	0.60
Number of Firms	0.30	0.30	0.29	0.29	0.28	0.27
Number of Exporters	0.51	0.57	0.63	0.68	0.83	1.07
Entry	3.94	3.92	3.85	3.79	3.76	3.67
International Correlations						
Gdp	0.08	0.06	0.04	0.03	0.01	-0.02
Consumption	0.80	0.81	0.80	0.80	0.78	0.76
Investment	-0.53	-0.70	-0.77	-0.82	-0.88	-0.92
Hours Worked	-0.53	-0.55	-0.57	-0.57	-0.59	-0.59
Other Correlations						
Ratio Cons vs RER	0.88	0.60	-0.40	-0.97	-0.80	-0.41
Ratio Cons vs TOE	0.97	0.98	0.99	1.00	0.94	0.68
Ratio Cons vs TOT	0.81	0.70	0.54	0.36	0.02	-0.20

improve the predictions of the standard BKK model regarding the dynamics of the RER and the TOT, regardless of measurement issues in the price index.

7.2 The degree of substitution between local and foreign varieties (ρ).

The double nested CES technology we assumed for the production of the final good allows to vary the degree of substitution between domestic and foreign varieties without affecting the level of mark-up of intermediate producers. In table IX , we present the main statistics for cases where ρ varies from 1.5 to 5. The higher the complementarity between local and foreign goods, the less volatility in output, net exports and investment. A high complementarity prevents the reallocation of the factors from the less productive country to the most efficient one, dampening the overall volatility of the model. In addition, as we get closer to a case where the shares of domestic and imported goods are constant, countries exhibit a higher positive correlation of outputs and its turn a less negative correlation of production inputs, meaning lower negative cross-correlations of investment and hours worked. The high degree of complementarity also eliminates the effect of consumption switching towards domestic goods that was critical for getting in the baseline model an appreciation of the real exchange rate. Additionally, the degree of substitution has an effect on the behavior of the export thresholds. For low values of ρ, home producers find easier to export to the foreign country, as the complementarity makes the increase in foreign consumption to require of imported goods. As long as the export threshold for foreign producers continue to drop, and even more, the changes in the two thresholds continues to be supportive of an appreciation of the real exchange rate but it weakens as we increase the degree complementarity. Therefore, for ρ lower than 2.5, the model looses its ability to predict a negative correlation between the ratio of consumption and the real exchange rate.

8 Conclusion.

In this paper we examine the role of the extensive margin of production and exporting for the dynamics of aggregate variables in a two country model. We find that these two margins can bring improvements on the traditional international business cycle model, particularly for explaining the volatility of investment and net exports and for the dynamics of the real exchange rate. Regarding the former, we find that there is an important interaction between investment and entry over the business cycle, as the two can serve from the household point of view as savings devices to smooth the wealth effect caused by a positive productivity shock. We find that under our basic calibration, a model without entry delivers too high volatility of investment and net exports given our assumption of the relative high degree of substitution between domestic and imported goods. Entry dampens the volatility of investment and for this purpose mimics the effect of adjustment cost, but given that creation of new establishments require domestic labor, also has an effect on the labor market and on the dynamic of exports, that adjustments cost cannot generate along. Here, the interaction of the two extensive margins is critical and we found that a model that lacks either one cannot produce the novel dynamics of the full model. This conclusions comes from an exercise in which we turn off partially or totally each channel at the time. We do so with two different approaches: we change the steady- state fraction of entrants and the share of exporting firms and control for the elasticity of entry and exporting relative to productivity shocks.

Regarding the dynamics of the real exchange rate we found, conforming the results of GM, that the model predicts a negative correlation between relative consumptions and the real exchange rate, reverting the so-called Backus-Smith puzzle. This result is based on price indices that control for the gains from variety present in the theoretical price index. However, we acknowledge that the resulting measure of aggregate prices still contemplates changes in the share of expenditure in home and foreign goods and the average productivity of exporters, arguably absent in statistical agencies' CPI. In light of this concern, we provide a conservative measure of aggregate prices that neglects the direct effect of these margins on the RER, finding that this it resembles the one in standard BKK models.

To our knowledge we are the first in exploring the interaction between entry and exporting decisions in the usual international business cycle environment where capital and labor are endogenous and using the BDS data on creation of firms and the establishments at the business cycle frequency. Although our main focus here was on the relevance of entry and fixed costs of exporting for standard international business cycle statistics, our model can speak about other interesting features of the data as whether entry leads or lags the cycle, what is the correlation between entry and the number of exporters and the persistence in the number of producers, to name just a few. We view these as interesting economic issues, that should be addressed in future research.

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9 Appendix

9.1 Data

We use the Business Dynamics Statistics (BDS) from the US Census Bureau to calculate our statistics on entry of new firms and establishments. The BDS is annual and goes from 1977 to 2005. It covers all non-farm private establishments in all sectors of the economy and firms can be tracked by their size, age, initial state of location in a particular state. The data base contains information on entry, exit, job creation and job destruction at the establishment and firm level. Our numbers for labor share of new firms are taken directly from the dataset while we need some computations to calculate the labor share accounted by new establishments. The labor accounted by new firms averages 3.29% for the 1977 to the 2005 period. In the public available dataset there is not direct information of employment by new establishments. Hence, we impute how many employees are hired by new establishments of different sizes, assuming that a new establishment of a specific size will have the same labor share as existing establishments of the same size. According to our numbers, the average for this number over the period in reference is 8.39%. For our numbers regarding volatility of entry we took the aggregate series of new establishments and compute its cyclical deviations from a trend using the HP filter. The smoothing parameter we use is 6.5 a suggested by Ravn and Uhlig for annual data.

9.2 Tables

Table 4: Statistics for Different Steady-State Shares

	Labor Share of Entry in Steady State					
	0%	0.5%	1%	4%	7.5%	10.25%
Volatilities (% Std Dev)						
Gdp	1.32	1.47	1.47	1.46	1.45	1.44
Exports	2.18	1.14	1.17	1.19	1.17	1.17
Trade Balance/Gdp	0.95	0.36	0.38	0.40	0.40	0.40
Std Dev relative to Gdp						
Consumption	0.53	0.45	0.46	0.49	0.51	0.52
Investment	6.40	2.32	2.35	2.38	2.38	2.45
Hours Worked	0.39	0.70	0.67	0.64	0.62	0.62
Number of Firms	na	0.06	0.08	0.14	0.22	0.28
Number of Exporters	1.56	0.78	0.79	0.80	0.80	0.80
Entry	na	17.45	8.71	4.59	3.87	3.78
Other Correlations						
Ratio Cons vs RER	0.78	0.59	0.52	0.20	-0.59	-0.87
Ratio Cons vs TOE	0.97	0.96	0.95	0.93	0.94	0.97
Ratio Cons vs TOT	0.93	0.85	0.80	0.64	0.43	0.08

Table 5: Statistics for Different Fractions of Exporting Firms

	Fraction of Exporting firms (%)					
	10	15	21	50	65	100
Volatilities (% Std Dev)						
Gdp	1.47	1.46	1.44	1.39	1.37	1.48
Exports	1.15	1.16	1.17	1.19	1.20	0.97
Trade Balance/Gdp	0.40	0.40	0.40	0.40	0.40	0.31
Std Dev relative to Gdp						
Consumption	0.53	0.53	0.52	0.51	0.51	0.54
Investment	2.42	2.42	2.45	2.51	2.52	2.06
Hours Worked	0.61	0.62	0.62	0.65	0.66	0.61
Number of Firms	0.28	0.28	0.28	0.29	0.30	0.26
Number of Exporters	0.80	0.80	0.80	0.84	0.85	0.26
Entry	3.71	3.74	3.78	3.93	3.99	3.44
Other Correlations						
Ratio Cons vs RER	0.00	-0.40	-0.87	-0.75	-0.61	0.65
Ratio Cons vs TOE	0.98	0.97	0.97	0.90	0.86	0.81
Ratio Cons vs TOT	-0.01	0.03	0.08	0.29	0.39	0.65

Table 6: Statistics for Different Elasticities of Entry

	γ_e							
	0	0.1	0.3	0.5	0.8	1	5	300
Volatilities (% Std Dev)								
Gdp	1.44	1.43	1.41	1.39	1.38	1.37	1.31	1.28
Exports	1.17	1.25	1.41	1.53	1.66	1.74	2.24	2.52
Trade Balance/Gdp	0.40	0.44	0.51	0.57	0.63	0.66	0.86	0.97
Std Dev relative to Gdp								
Consumption	0.52	0.53	0.53	0.53	0.54	0.54	0.55	0.55
Investment	2.44	2.76	3.34	3.80	4.33	4.61	6.49	7.61
Hours Worked	0.62	0.61	0.55	0.54	0.50	0.48	0.39	0.33
Number of Firms	0.28	0.27	0.24	0.22	0.19	0.18	0.08	0.00
Number of Exporters	0.80	0.86	0.96	1.04	1.14	1.18	1.58	1.82
Entry	3.78	3.51	3.09	2.80	2.43	2.20	0.94	0.00
Other Correlations								
Ratio Cons vs RER	-0.87	-0.82	-0.24	0.08	0.29	0.37	0.68	0.78
Ratio Cons vs TOE	0.97	0.96	0.94	0.93	0.94	0.94	0.97	0.97
Ratio Cons vs TOT	0.08	0.19	0.35	0.45	0.55	0.60	0.79	0.86

Table 7: Different Entry Elasticities with Constant Fraction of Exporting Firms

	γ_e							
	0	0.1	0.3	0.5	0.8	1	5	300
Volatilities (% Std Dev)								
Gdp	1.48	1.47	1.45	1.43	1.41	1.40	1.34	1.31
Exports	0.95	0.99	1.06	1.11	1.17	1.21	1.45	1.61
Trade Balance/Gdp	0.31	0.33	0.35	0.38	0.40	0.42	0.51	0.56
Std Dev relative to Gdp								
Consumption	0.54	0.54	0.55	0.56	0.56	0.56	0.58	0.57
Investment	2.06	2.26	2.63	2.93	3.29	3.47	4.82	5.65
Hours Worked	0.61	0.57	0.54	0.52	0.49	0.47	0.36	0.30
Number of Firms	0.28	0.26	0.23	0.21	0.19	0.18	0.08	0.00
Number of Exporters	0.28	0.26	0.23	0.21	0.19	0.18	0.08	0.00
Entry	3.68	3.42	3.00	2.73	2.37	2.23	1.00	0.00
Other Correlations								
Ratio Cons vs RER	0.33	0.50	0.75	0.86	0.91	0.92	0.93	0.94
Ratio Cons vs TOE	0.86	0.95	0.96	0.95	0.93	0.93	0.93	0.94
Ratio Cons vs TOT	0.48	0.56	0.67	0.74	0.80	0.82	0.91	0.94

Table 8: Different Elasticities of Fraction of Exporters

	γ_x							
	0	0.1	0.3	0.5	0.8	1	5	300
Volatilities (% Std Dev)								
Gdp	1.44	1.49	1.49	1.49	1.48	1.48	1.48	1.48
Exports	1.17	0.92	0.95	0.95	0.95	0.95	0.95	0.95
Trade Balance/Gdp	0.40	0.30	0.31	0.31	0.31	0.31	0.31	0.31
Std Dev relative to Gdp								
Consumption	0.52	0.54	0.54	0.54	0.54	0.54	0.54	0.54
Investment	2.44	2.02	2.05	2.05	2.06	2.06	2.06	2.06
Hours Worked	0.62	0.60	0.60	0.60	0.61	0.61	0.61	0.61
Number of Firms	0.28	0.28	0.28	0.28	0.28	0.28	0.28	0.28
Number of Exporters	0.80	0.31	0.28	0.27	0.28	0.28	0.28	0.28
Entry	3.78	3.66	3.67	3.67	3.68	3.68	3.68	3.68
Other Correlations								
Ratio Cons vs RER	-0.87	0.47	0.37	0.35	0.34	0.34	0.33	0.33
Ratio Cons vs TOE	0.97	0.81	0.84	0.85	0.85	0.85	0.86	0.86
Ratio Cons vs TOT	0.08	0.49	0.45	0.45	0.44	0.44	0.44	0.43

Table 9: Different Elasticities of Fraction of Exporters with Constant Number of Firms

	γ_x						
	0	0.1	0.3	0.5	0.8	1	300
Volatilities (% Std Dev)							
Gdp	1.28	1.32	1.31	1.31	1.31	1.31	1.31
Exports	2.52	1.51	1.51	1.58	1.59	1.59	1.61
Trade Balance/Gdp	0.97	0.52	0.54	0.55	0.55	0.55	0.56
Std Dev relative to Gdp							
Consumption	0.55	0.58	0.58	0.58	0.58	0.58	0.57
Investment	7.61	5.46	5.60	5.61	5.63	5.63	5.65
Hours Worked	0.33	0.30	0.30	0.30	0.30	0.30	0.30
Number of Firms	0.00	0.00	0.00	0.00	0.00	0.00	0.00
Number of Exporters	1.82	0.24	0.24	0.04	0.03	0.02	0.00
Entry	0.00	0.00	0.00	0.00	0.00	0.00	0.00
Other Correlations							
Ratio Cons vs RER	0.78	0.96	0.94	0.94	0.94	0.94	0.94
Ratio Cons vs TOE	0.97	0.93	0.93	0.93	0.94	0.94	0.94
Ratio Cons vs TOT	0.86	0.95	0.94	0.94	0.94	0.94	0.94

10 Technical Appendix

10.1 Existence and Uniqueness of the Symmetric Steady State

We now determine the set of equations that characterize a symmetric non-stochastic steady state where $Z = Z^* = Q = 1$, and $Y^C = Y^{C,*} = Y$. We do this for the case of financial autarky only, although the steady state is the same across financial structures.

Price Indexes:

From the equations for the price index we get:

$$1 = N_D^{\frac{1-\rho}{1-\theta}} \left[\left(\frac{\theta}{\theta-1} r^\alpha w^{1-\alpha} A \right)^{1-\rho} \left(\tilde{z}_d^{\rho-1} + \tau^{1-\rho} \left(\frac{N_x}{N_D} \right)^{\frac{1-\rho}{1-\theta}} \tilde{z}_x^{\rho-1} \right) \right]$$

It will be helpful in later calculations to rewrite the above as:

$$\left(\frac{\theta}{\theta-1} r^\alpha w^{1-\alpha} A \right) = N_D^{\frac{1}{\theta-1}} \left(\tilde{z}_d^{\rho-1} + \tau^{1-\rho} \left(\frac{N_x}{N_D} \right)^{\frac{1-\rho}{1-\theta}} \tilde{z}_x^{\rho-1} \right)^{\frac{1}{\rho-1}} \tag{27}$$

Total Average Profits

$$\tilde{d}_t = \frac{1}{\theta} \left(\frac{\theta}{\theta-1} r^\alpha w^{1-\alpha} A \right)^{1-\rho} N_D^{\frac{\theta-\rho}{1-\theta}} Y \left(\tilde{z}_d^{\rho-1} + \tau^{1-\rho} \left(\frac{N_x}{N_D} \right)^{\frac{1-\rho}{1-\theta}} \tilde{z}_x^{\rho-1} \right) - \frac{N_x}{N_D} r^\xi w^{1-\xi} B f_x \tag{28}$$

Zero-Cutoff Profit (ZCP)

$$\frac{1}{\theta}\left(\frac{\theta}{\theta-1}r^{\alpha}w^{1-\alpha}A\right)^{1-\rho}N_D^{\frac{\theta-\rho}{1-\theta}}\tau^{1-\rho}z_x^{\rho-1}\left(\frac{N_x}{N_D}\right)^{\frac{1-\rho}{1-\theta}}Y=r^{\xi}w^{1-\xi}Bf_x \tag{29}$$

Law of Motion for Number of Firms

$$N_D=\frac{(1-\delta)}{\delta}N_e \tag{30}$$

Interest Rates

$$\left(1+r^b\right)\beta=1 \tag{31}$$

$$r=r^b+\delta_k \tag{32}$$

Free Entry Condition (combined with Euler Equation for shares in Mutual Fund)

$$\tilde{v}=\frac{\beta(1-\delta)}{1-\beta(1-\delta)}\tilde{d}=r^{\xi}w^{1-\xi}Bf_e$$

Aggregate Accounting

$$C+\delta_k K+\tilde{v}N_e=wL+rK+\tilde{d}.N_D \tag{33}$$

Resource Constraint for Final Goods

It can be verified that, from our choice of the final good being the numeraire, taking the resource constraint for the final good and imposing symmetry in steady state gives us back the price equation.

Labor Leisure Condition

$$w(1-L)=C\frac{(1-\mu)}{\mu} \tag{34}$$

In addition to all these, we have the clearing conditions for the markets of labor and capital stock.

10.1.1 Suggested Algorithm to Solve for Steady State

As in shown in Melitz (2003), the characterization of the steady state symmetric equilibrium is mainly determined by the interaction of the free entry (FE) and the zero-cutoff profit (ZCP) condition. This is not going to change in our extension that includes endogenous labor and physical capital stock. For instance, combining the expression for $\left(\frac{\theta}{\theta-1}r^{\alpha}w^{1-\alpha}A\right)$ into the average total profits equation, we get:

$$\tilde{d}_t = \frac{1}{\theta} \frac{Y}{\left[N_D\left(\tilde{z}_d^{\rho-1} + \tau^{1-\rho}\left(\frac{N_x}{N_D}\right)^{\frac{1-\rho}{1-\theta}} \tilde{z}_x^{\rho-1}\right)\right]} \left(\tilde{z}_d^{\rho-1} + \tau^{1-\rho}\left(\frac{N_x}{N_D}\right)^{\frac{1-\rho}{1-\theta}} \tilde{z}_x^{\rho-1}\right) - \frac{N_x}{N_D} r^{\xi} w^{1-\xi} B f_x$$

Although a couple of terms could be cancelled here, we rather don't do it for the moment. Considering this in the free entry condition:

$$\frac{1}{\theta} \frac{\beta(1-\delta)\left(\tilde{z}_d^{\rho-1} + \tau^{1-\rho}\left(\frac{N_x}{N_D}\right)^{\frac{1-\rho}{1-\theta}} \tilde{z}_x^{\rho-1}\right) Y}{\left[1-\beta(1-\delta)\right]\left[N_D\left(\tilde{z}_d^{\rho-1} + \tau^{1-\rho}\left(\frac{N_x}{N_D}\right)^{\frac{1-\rho}{1-\theta}} \tilde{z}_x^{\rho-1}\right)\right]} - \frac{N_x}{N_D} r^{\xi} w^{1-\xi} B \frac{\beta(1-\delta)}{\left[1-\beta(1-\delta)\right]} f_x = r^{\xi} w^{1-\xi} B f_e$$

Dividing both sides by $r^{\xi} w^{1-\xi} B$ and defining[48]:

$$\Omega = \frac{Y}{\theta\left(r^{\xi} w^{1-\xi} B\right)\left[N_D\left(\tilde{z}_d^{\rho-1} + \tau^{1-\rho}\left(\frac{N_x}{N_D}\right)^{\frac{1-\rho}{1-\theta}} \tilde{z}_x^{\rho-1}\right)\right]}$$

we get:

$$\frac{\beta(1-\delta)}{\left[1-\beta(1-\delta)\right]} \Omega \left(\tilde{z}_d^{\rho-1} + \tau^{1-\rho}\left(\frac{N_x}{N_D}\right)^{\frac{1-\rho}{1-\theta}} \tilde{z}_x^{\rho-1}\right) - \frac{N_x}{N_D} \frac{\beta(1-\delta)}{\left[1-\beta(1-\delta)\right]} f_x = f_e \tag{FE}$$

In addition substitution of the expression for $\left(\frac{\theta}{\theta-1} r^{\alpha} w^{1-\alpha} A\right)$ in the zero-cutoff profit condition gives:

$$\frac{Y}{\theta\left(r^{\xi} w^{1-\xi} B\right)\left[N_D\left(\tilde{z}_d^{\rho-1} + \tau^{1-\rho}\left(\frac{N_x}{N_D}\right)^{\frac{1-\rho}{1-\theta}} \tilde{z}_x^{\rho-1}\right)\right]} \left(\frac{N_x}{N_D}\right)^{\frac{\theta-\rho}{1-\theta}} \tau^{1-\rho} z_x^{\rho-1} = f_x$$

which from our definition of Ω it becomes;

$$\Omega \tau^{1-\theta} \left(\frac{N_x}{N_D}\right)^{\frac{\theta-\rho}{1-\theta}} z_x^{\theta-1} = f_x \tag{ZCP}$$

Note that FE and ZCP, in combination with the equation for N_x/N_d and the Pareto property that $\tilde{z}_x = v z_x$; pin down[49] the cutoff productivity for exporters and the term Ω. We can recover all other variables once we've determined this 2.

[48] Notice how crucial it is, for the characterization of the symmetric steady state, that the fixed export cost and the entry cost are measured in the same units

[49] As a result of the simplifications generated by the Pareto parametrization, this system can be solved by hand.

10.2 Demand Functions for Labor and Capital Stock in Production

From the FOC of an intermediate good producer with respect to $l(z)$ and $k(z)$ we combine to obtain:

$$k(z) = \frac{w}{r^k}\frac{\alpha}{1-\alpha}l(z) \tag{A.1}$$

Substituting in the resource constraint for firm with productivity z, making use of the demand functions for the variety, the pricing rules, and the expression for the marginal cost, $A \equiv \left(\frac{1}{\alpha}\right)^{\alpha}\left(\frac{1}{1-\alpha}\right)^{1-\alpha}$, it results that:

$$\begin{aligned} l(z) \quad = \quad & \left[\frac{r^k\,(1-\alpha)}{w\alpha}\right]^{\alpha}\left[(r^k)^{\alpha}w^{1-\alpha}A\frac{\theta}{\theta-1}\right]^{-\theta}(Zz)^{\theta-1} \\ & \left[\left(\frac{P}{P_D}\right)^{\rho-\theta}Y^C + x(t)\tau^{1-\theta}\left(\frac{P^*}{P_X}\right)^{\rho-\theta}Q^{\theta}Y^{*,C}\right] \end{aligned} \tag{A.2}$$

Using this to recover $k(z)$ from A.1 we get:

$$\begin{aligned} k(z) \quad = \quad & \left[\frac{w\alpha}{r^k(1-\alpha)}\right]^{1-\alpha}\left[(r^k)^{\alpha}w^{1-\alpha}A\frac{\theta}{\theta-1}\right]^{-\theta}(Zz)^{\theta-1} \\ & \left[\left(\frac{P}{P_D}\right)^{\rho-\theta}Y^C + x(t)\tau^{1-\theta}\left(\frac{P^*}{P_X}\right)^{\rho-\theta}Q^{\theta}Y^{*,C}\right] \end{aligned} \tag{A.3}$$

Importantly, these expressions establish that it is also going to be true for labor and physical capital demands that the average demands are equal to the demands for the average firm. That is, the inclusion of capital into the model does not complicate the aggregation of variables across firms.

Making use of the statistics for average productivities, and aggregating across all domestic and exporting firms, this functions become;

$$L_p \quad = \quad N_{D,t}\left[\frac{r^k\,(1-\alpha)}{w\alpha}\right]^{\alpha}\left[(r^k)^{\alpha}w^{1-\alpha}A\frac{\theta}{\theta-1}\right]^{-\theta}Z^{\theta-1} \tag{35}$$

$$\left[\left(\frac{P}{P_D}\right)^{\rho-\theta}Y^C\tilde{z}_D^{\theta-1} + \frac{N_{x,t}}{N_{D,t}}\tau^{1-\theta}Q^{\theta}\left(\frac{P^*}{P_X}\right)^{\rho-\theta}Y^{*,C}\tilde{z}_{x,t}^{\theta-1}\right] \tag{36}$$

$$K_p = N_{D,t}\left[\frac{w\alpha}{r^k(1-\alpha)}\right]^{1-\alpha}\left[(r^k)^\alpha w^{1-\alpha}A\frac{\theta}{\theta-1}\right]^{-\theta}Z^{\theta-1} \quad (37)$$

$$\left[\left(\frac{P}{P_D}\right)^{\rho-\theta}Y^C\tilde{z}_D^{\theta-1}+\frac{N_{x,t}}{N_{D,t}}\tau^{1-\theta}Q^\theta\left(\frac{P^*}{P_X}\right)^{\rho-\theta}Y^{*,C}\tilde{z}_{x,t}^{\theta-1}\right] \quad (38)$$

10.3 First Order Condition to Household's Problem

The labor-leisure condition:

$$w_t\left(1-l_t\right)=C_t\frac{(1-\mu)}{\mu}$$

Financial Autarky:

The Euler equations for capital stock accumulation and bond holdings:

$$C_t^{-1}\left[C_t^\mu\left(1-l_t\right)^{1-\mu}\right]^{1-\gamma}=\beta E_t\left\{C_{t+1}^{-1}\left[C_{t+1}^\mu\left(1-l_{t+1}\right)^{1-\mu}\right]^{1-\gamma}\left(r_{t+1}^k+1-\delta_k\right)\right\}$$

$$C_t^{-1}\left[C_t^\mu\left(1-l_t\right)^{1-\mu}\right]^{1-\gamma}=\beta\left(1+r_{t+1}^b\right)E_t\left\{C_{t+1}^{-1}\left[C_{t+1}^\mu\left(1-l_{t+1}\right)^{1-\mu}\right]^{1-\gamma}\right\}$$

Similarly in the foreign country.

International Bond Trading

The Euler equation for the accumulation of bonds becomes, in the home country:

$$C_t^{-1}\left[C_t^\mu\left(1-l_t\right)^{1-\mu}\right]^{1-\gamma}\left(1+\eta B_{t+1}\right)=\beta\left(1+r_{t+1}^b\right)E_t\left\{C_{t+1}^{-1}\left[C_{t+1}^\mu\left(1-l_{t+1}\right)^{1-\mu}\right]^{1-\gamma}\right\}$$

$$C_t^{-1}\left[C_t^\mu\left(1-l_t\right)^{1-\mu}\right]^{1-\gamma}\left(1+\eta B_{*,t+1}\right)Q_t=\beta\left(1+r_{t+1}^{b,*}\right)E_t\left\{C_{t+1}^{-1}\left[C_{t+1}^\mu\left(1-l_{t+1}\right)^{1-\mu}\right]^{1-\gamma}Q_{t+1}\right\}$$

Similar equations hold in the foreign country. Recall that budget constraints were expressed in units of the final good in each country, so the real exchange rate will enter differently in the Euler equations at home and abroad in order to convert foreign payments in to domestic real units.

Complete Markets

Although the domestic risk free assets become redundant, the FOC defined in financial autarky still holds under complete markets. What is new under complete markets is that the trade of a complete set of Arrow securities allows for full insurance of all idiosyncratic risk:

$$\frac{U_c(C,l)}{U_{c^*}(C^*,l^*)} = Q^{-1}$$