Accounting for the Cyclical Dynamics of Income Shares

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Abstract

Over the business cycle, labor’s share of output is negatively but weakly correlated with output, and it lags output by about four quarters. Profits’ share is strongly pro-cyclical, it neither leads nor lags output, and its volatility is about four times that of output. Despite the importance of understanding the dynamics of income shares for understanding aggregate technology and the degree of competition in factor markets, macroeconomics lacks models that can account for those dynamics. This paper constructs a model that can replicate those facts. We introduce costly entry of firms in a model with frictional labor markets and find that there is a link between the ability of the model to replicate income shares’ dynamics and the ability of the model to amplify and propagate shocks. That link is a countercyclical real interest rate, a well-known fact in US data but a feature that models of aggregate fluctuations have had difficulty achieving.

JEL codes: E3, E25, J3, E24

Keywords: Labor’s Share, Frictional Labor Market, Firm Entry

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1 Introduction

Most research in aggregate fluctuations has assumed factor income shares to be constant at all frequencies due to particular assumptions about technology and about the degree of competition in markets for goods and factors of production. The assumption of constant shares does not pass a test of casually inspecting the data, let alone analyzing it with sophisticated statistical tools. Understanding the time series behavior of income shares is critical for understanding the structure of aggregate technology and the behavior of factor markets. Despite their importance, macroeconomics lacks models that can quantitatively match income shares’ time series facts. This paper makes two contributions. First, it shows that existing models with time-varying income shares cannot account for their dynamics. Second, it constructs a model which can replicate, if not for all, many of the properties that describe income shares’ behavior.

Over the post-war period, the labor’s share correlation with output is negative but weak; it lags output by about four quarters; and is smoother than output. On the other hand, the profits’ share is strongly pro-cyclical; it neither leads nor lags output; and its volatility is about four times that of output. As labor is arguably the most important factor of production, these facts prompted earlier work to deviate from a Walrasian labor market. It did so by specifying contractual arrangements between employers and employees that broke the link between wages and the marginal product of labor. The goal was to match properties of labor’s share over the business cycle. Examples of this line of work include Boldrin and Horvath (1995), Gomme and Greenwood (1995), and Danthine and Donaldson (1992).

Dispensing with a Walrasian framework is a characteristic of a literature that features search and matching frictions in labor markets (e.g. Pissarides (1985)), and it is within this framework that our research should be placed.\footnote{An alternative approach to study time-varying income shares is to introduce that time variation exogenously. This exogeneity still allows the researcher to analyze joint dynamics of those income shares with endogenous variables; output, for example. This is done by Young (2004).} Although this literature
has claimed success in matching some labor market business cycle moments, we show that the dynamics of income shares are completely at odds with the data. We link this failure to the typical assumption of free entry of firms which leads to the asset value of a vacant position to be exactly zero at all frequencies. That free entry implies a value of a vacant position equal to zero can be easily seen from the textbook model of search and matching, for example Pissarides (2000). This model features firms posting vacancies in order to get matched to workers who are searching for jobs. If entry is free and any firm who wishes to do so can pay a vacancy-posting cost and wait for this position to get filled, the present value of such a vacancy must be zero in equilibrium. If it were positive, firms would continue to post vacancies, lowering the probability that a given vacancy gets filled until its present value reached zero.

We construct an environment in which the present value of a vacant position is always positive and endogenously varying over the business cycle. A vacancy has positive asset value because firms need to incur in entry costs before they are allowed to post a vacancy, hire workers, and begin production. The equilibrium value of a vacancy is equal to the sunk cost, so that firms are indifferent between entering or staying out of the market. This equilibrium asset value is also time-varying. The reason is that entrants rent factors of production to pay for the sunk cost and the efficiency of these factors is affected by the same shocks that generate aggregate fluctuations. As the prices and quantities of these factors vary with aggregate conditions, so do the expenditures that entrants undertake. In equilibrium, these expenditures must equal the capital value of a vacancy.

To some extent, our economy resembles a two-sector environment. The first sector produces goods and services that households consume and the second sector produces services that entrants need to purchase. These purchases, in turn, allow entrants to access the goods producing sector and profit from the sales of those goods. However, those two sectors compete for the same factors of production causing the dynamics of entrants to influence the dynamics of the demand, and hence prices, of those factors.
Because of specific modeling assumptions we describe later, it is the behavior of these prices that determine the dynamics of the value of a vacancy. We show that a reasonable parameterization of our environment can match the joint dynamics of labor’s share, profits’ share, and output. The model is consistent with the lagging behavior of the labor’s share, its weak correlation with output, and the hump-shaped response of the labor’s share to a shock to productivity. It is also consistent with the strong correlation of output with profits’ share and its lack of leading or lagging behavior. Matching the dynamics of income shares would be a Pyrrhic victory if it came at the expense of matching other business cycle moments. We show this is not the case. On the contrary, the very mechanism that improves the dynamics of income shares helps to amplify and propagate productivity shocks to the labor market and the dynamics of investment and consumption are similar to existing models. Of course, there are some dimensions along which our environment performs worse and we report and discuss those as well. For instance, it is difficult to match the magnitude of empirical impulse response of labor’s share to a productivity shock and at the same time match the lower - relative to output - volatility of labor’s share

We show that these results depend on one equilibrium outcome: that the value of a vacancy is countercyclical. In fact, parameterizations of our economy that yield a procyclical capital value of a vacancy are inconsistent with the dynamics of income shares and they feature virtually no amplification of shocks. Unfortunately, there are no good empirical counterparts to the asset value of a vacancy. Fortunately, in our model the dynamics of the value of a vacancy are determined essentially by the dynamics of the real interest rate. However imperfect, we do have measures of real interest rates, and we show that the real interest rate is indeed countercyclical. The negative correlation between the real rate and output, and the fact that most models of economic fluctuations can not replicate it, has been reported by King and Rebelo (1999). We provide alternative measures of the real rate and confirm previous findings. We emphasize that despite this counter-cyclicality of real rates, the dynamics of aggregate investment are
almost identical to those of existing models.

2 The Model Economy

2.1 Environment

Our economy is populated by a large extended household comprised of a continuum of members of total mass equal to $\tilde{N}$ and an infinite mass of firms.

Members in the household can either be employed or unemployed. Unemployed agents receive an unemployment benefit while they search for jobs with the hope of finding a job opportunity. This opportunity will allow them to enter into a relationship with a firm, to negotiate a contract that stipulates the retribution for their services, and to produce output during the following period. A fraction $N_t$ of employed agents works and gets paid the negotiated wage. Members of the household have preferences over a sequence of a composite of goods over time, $\{C_t\}_{t=0}^\infty$. The per-period utility function is of the relative risk aversion class. The household’s (expected) discounted lifetime utility as of time 0 is given by,

$$E_0 \sum_{t=0}^{\infty} \beta^t \left[ \frac{C_t^{1-\sigma}}{1-\sigma} \right],$$

where $\beta \in (0, 1)$ is the discount factor and $\sigma > 0$ is the coefficient of relative risk aversion. We assume that each firm produces a differentiated commodity. At each point in time, there is a subset of goods $X_t \subseteq X$ available to consumers and the composite good is made up of commodities from that subset. The available set is time-varying as not all firms will produce every period. To aggregate over the different commodities, we use a Dixit and Stiglitz (1977) aggregator:

$$C_t = \left( \int_{x \in X_t} [c_t(x)]^{\frac{1}{1-\gamma}} \cdot dx \right)^{\frac{\gamma}{1-\gamma}},$$
where $\gamma > 1$ is the symmetric elasticity of substitution between commodities. If $p_t(x)$ is the price\(^2\) of product $x$, then the level of $c_t(x)$ chosen to minimize the cost of acquiring $C_t$ given prices $\{p_t(x)\}$ for all $x$ is:

$$c_t(x) = \left(\frac{p_t(x)}{P_t}\right)^{-\gamma} C_t,$$  \hspace{1cm} (3)

where $P_t$ is the cost of acquiring one unit of the composite good, or the price index\(^3\):

$$P_t = \left(\int_{x \in X_t} [p_t(x)]^{1-\gamma} dx\right)^{\frac{1}{1-\gamma}}.$$  

Each firm uses capital and one unit of labor to produce its commodity. The job market in our economy is characterized by the existence of search and matching frictions (see Rogerson et al. (2005) for a survey of this literature). In order to hire a worker, a firm must post a vacancy and undertake a recruiting expense of $\omega$ per vacancy posted. Firms and potential workers match in a labor market according to a constant-returns-to-scale matching technology $M(\bar{N} - N, V)$ given by:

$$M(\bar{N} - N, V) = \frac{(\bar{N} - N)V}{((\bar{N} - N)^{\xi} + V)^{\xi}},$$  \hspace{1cm} (4)

This matching function takes as inputs the total number of unemployed individuals who are searching, $\bar{N} - N$, and the total number of vacancies posted by firms, $V$. The output is a number of matches $M$. Denoting by $\theta$ the vacancies to unemployment ratio

\(^2\)As the subset of goods changes over time it is more convenient to express this price in terms of “money” than to use any of the consumption goods as the numeraire. This is done for convenience only, and this “money” acts as a unit of account and it is not valued for facilitating trades or for any other quality.

\(^3\) $P$ can be obtained by solving the consumer expenditure minimization problem for constructing one unit of composite good:

$$P = \min \int_{x \in X_t} p(x) c(x) dx,$$

$$s.t. \quad C = \left(\int_{x \in X_t} [c(x)]^{\frac{1}{\gamma}} dx\right)^{\frac{1}{1-\gamma}} = 1.$$
\( \frac{V}{N-N} \), the probabilities that a vacancy gets filled, \( q_t \), and that a worker finds a job, \( f_t \) are given by\(^4\),

\[
q_t = \frac{M(N-N,V)}{V} = \frac{1}{(1 + \theta_t^t)^{\frac{1}{\theta_t^t}}},
\]

\[
f_t = \frac{M(N-N,V)}{N-N} = \frac{\theta_t}{(1 + \theta_t^t)^{\frac{1}{\theta_t^t}}}.\]

A match between a firm and a worker results in a wage contract that specifies a wage \( w_t(x) \) paid in exchange of labor services. We assume that firms and workers split the surplus from their relationship according to a Nash bargaining rule. We will be more specific about this rule below after we have fixed some notation regarding workers’ and firms’ value functions. The relationship between a firm and a worker can break either because the firm exogenously ends production – which happens with probability \( \tau \), or for any other reason – which happens at rate \( s \).

Firms need to pay a sunk cost to begin the goods production process.\(^5\) Opening a firm or starting a new product variety needs \( y^E \) effective units of capital, i.e. \( y^E = Z_t K_t^E \).

We assume the productivity process \( Z_t \) is first-order Markov. Denoting by \( r_t \) the rental rate of capital and noting that one unit of capital produces \( Z_t \) units of the composite good, the sunk cost of entry is \( \frac{r_t y_t^E}{Z_t} \) or \( r_t K_t^E \) (in units of the composite consumption good). We denote the number of entrants –the number of firms that pay the sunk cost – by \( N_{E,t} \).

Let us now describe the technology for producing the differentiated commodity, which, as the reader may recall, involves capital and labor. Denoting the firm’s output of the differentiated product \( x \) by \( y_t^c(x) \), we can formally describe that technology as,

\[
y_t^c(x) = Z_t l_t(x)^{1-\alpha} (K_t^c(x))^\alpha
\]

\(^4\)We depart from the more frequent Cobb-Douglas specification for the matching function in order to bound the job-finding and vacancy-filling probabilities to be between 0 and 1. This functional form was chosen by Den Haan, Ramey, and Watson (2000).

\(^5\)Our approach for modeling firm entry follows Bilbiie, Ghironi, and Melitz (2006).
where $Z_t$ is the same random productivity process that determines the efficiency of capital when paying for the sunk cost, and $l_t(x)$ is the amount of labor employed by the firm, which is one if the firm produces and zero otherwise. The firm charges a price equal to $\rho_t(x)$ and its profits are given by $\pi_t(x) = \rho_t(x)y_t^e(x) - w_t(x) - r_tK_t^c(x)$.

Finally, the government plays a very limited role in our economy. Its task is solely to tax the household a lump-sum quantity and rebate it to the unemployed in the form of a benefit.

2.2 Optimization and Equilibrium

We restrict ourselves to a symmetric equilibrium in which all goods-producing firms charge equal prices, $\rho_t(x) = \rho_t$; demand one unit of labor which gets paid the same wage $w_t(x) = w_t$; and produce the same amount of output, $y_t^e(x) = y_t^e$. Given the CES structure of the consumption aggregate, the relative price $\rho_t$ that firms charge is given by $^{6}N_t^{\gamma-1}$ and the per-firm profit is given by, $\pi_t = \rho_t y_t^c - w_t - r_tK_t^c$. The relevant state vector for the firm is the quadruplet $(K_t, N_t, V_t, Z_t)'$ with $K_t = N_{E,t}K_t^E + N_tK_t^c$. To save on notation, we write down value functions without being specific about their dependence on the state vector.

Households own a diversified portfolio of firms and as a result firms discount expected future flows taking into account the household’s inter-temporal condition. Consequently, a firm’s appropriate discount factor between periods $t$ and $t+1$ is,

$$\Delta_{t+1} = \beta \left( \frac{C_{t+1}}{C_t} \right)^{-\sigma}.$$  \hspace{1cm} (8)

Let $Q_t$ denote the capital value of a vacancy and $J_t$ denote the capital value of a filled

Given that $p_t(x) = \rho_t$ and $\rho_t = \frac{p_t}{p_t} = \frac{p_t}{\left( \int_{x \in X_t} [p_t]^{1-\gamma} dx \right)^{1-\gamma}}$, implies that $\rho_t = \frac{p_t}{p_t \left( \int_{x \in X_t} dx \right)^{1-\gamma}}$ and as a result, $\rho_t = \left( \int_{x \in X_t} dx \right)^{1-\gamma} = N_t^{\gamma-1}$, as $N_t$ is the both the fraction of firms producing as well as the number of workers in the goods-producing sector by our assumption of one job per firm.
job. The following two recursive relationships must be satisfied:

\[ Q_t = -\omega + (1 - \tau) E_t \Delta_{t+1} [q_t J_{t+1} + (1 - q_t) Q_{t+1}] \], \hspace{1cm} \text{(9)}

\[ J_t = \pi_t + (1 - \tau) E_t \Delta_{t+1} [(1 - s) J_{t+1} + s Q_{t+1}] \]. \hspace{1cm} \text{(10)}

Equation (9) states that the value of a vacancy (once the entry decision has been made) is the difference between two objects. First, the expected value of entering the labor market and trying to match with a worker. This matching happens with probability \( q_t \), as long as the firm survives for one period, which happens with probability \( 1 - \tau \). Second, the vacancy cost \( \omega \).

The interpretation of equation (10) is analogous: the value of a filled job is the profit flow \( \pi \) plus the expected continuation value of the relationship between the firm and the worker. Conditional on the firm’s survival, the relationship ends with probability \( s \) and continues with probability \( 1 - s \).

In equilibrium, the entry of firms occurs until the value of a vacancy is equal to the sunk cost,

\[ Q_t = r_t K_t^E \] \hspace{1cm} \text{(11)}

Due to entry costs, vacant jobs have positive value in equilibrium which in turn leads firms to repost vacancies following separations. The following two equations give the laws of motion for the stock of employment and vacancies:

\[ N_{t+1} = (1 - \tau) [(1 - s) N_t + f_t (\bar{N} - N_t)], \hspace{1cm} \text{(12)}\]

\[ V_{t+1} = (1 - \tau) [(1 - q_t) V_t + s N_t] + N_{E,t}. \hspace{1cm} \text{(13)}\]

Employment at time \( t + 1 \) is the sum of matches \( (1 - s) N_t \) that were not destroyed either by the death of a firm or any other form of separation, and the newly-formed matches \( f_t (\bar{N} - N_t) \) from a previous pool of unemployed people. The total number of vacancies in the economy, given by equation (13), is equal to vacancies that did
not get filled in the current period, $(1 - q_t) V_t$ plus the number of separated matches $s N_t$. Of course, we need to include the fraction of firms which continue operating for at least one more period. Finally, we need to add to that total the number of newly created firms $N_{E,t}$, each of which posts a vacancy. Both employment and vacancies are predetermined variables.

The household’s problem is relatively straightforward. Given its current period resources, it chooses consumption and investment to maximize the expected discounted value of lifetime utility. In addition to wage income and unemployment benefits, the household gets interest from renting capital as well as a pay-out from its diversified ownership stake in firms. The aggregate dividends firms pay out equal to $d_t = N_t \pi_t - \omega V_t - Q_t N_{E,t}$. Finally, the household also gets taxed a lump-sum amount $T_t$ which the government uses to finance the unemployment benefit program. Denoting by $W_t$ the household’s value function at time $t$, the optimization problem can be expressed as:

$$W_t = \max_{C_t, I_t} C_t^{1-\sigma} + \beta E_t W_{t+1}$$

subject to

$$C_t + I_t = b (\bar{N} - N_t) + w_t N_t + r_t K_t + d_t - T_t,$$

$$K_{t+1} = (1 - \delta) K_t + I_t.$$  

The optimal inter-temporal condition is:

$$\beta E_t \left[ \left( \frac{C_{t+1}}{C_t} \right)^{1-\sigma} (r_{t+1} + 1 - \delta) \right] = 1$$

As was mentioned in the previous section, wages for the employed workers are the result of Nash bargaining between each worker-firm pair. The surplus of the match for the household is captured by the change in welfare derived from having a marginal
unemployed person employed. This change is given by \( \frac{\partial W_t}{\partial N_t} \) which in units of the consumption good is \( \frac{\partial W_t}{\partial N_t} C_t^\sigma \). The surplus for the firm is given by \( J_t - Q_t \), the difference between the value of a filled job and the value of a vacancy. The Nash bargaining solution when the firm’s bargaining parameter is given by \( \phi \) satisfies the following surplus-splitting rule:

\[
\frac{J_t - Q_t}{1 - \phi} = \frac{C_t^\sigma \frac{\partial W_t}{\partial N_t}}{\phi},
\]

which yields the following equation for wages:

\[
w_t = (1 - \phi)b + \phi(\rho_t Z_t - r_t K_t^C + \omega) - \phi(1 - \theta_t)(\omega + Q_t - (1 - \tau)E_t \beta \Delta_{t+1} Q_{t+1}) \tag{19}
\]

To better understand the analysis on the dynamics of income shares that follows, let us define here these shares. Total output \( y_t \) can be decomposed in three elements: payments to capital, to labor, and to equity-holders. As a result we can re-write it as,

\[
y_t = r_t K_t + N_t w_t + N_t \pi_t \tag{7}
\]

Labor’s share is then defined as \( \frac{w_t N_t}{y_t} \) and profits’ share is defined as \( \frac{N_t \pi_t}{y_t} \).

We can now describe a symmetric equilibrium for our economy. It is a sequence of prices \( \rho_t, w_t, r_t \); a sequence of aggregate quantities \( K_t, C_t, N_t, V_t, N_{E,t}, \pi_t \); and a sequence of value functions \( Q_t, J_t, W_t \) such that for any time period \( t \), the following conditions hold:

1. *(Household Optimization)* Given prices \( \rho, w, r \) the household’s optimization results in decision rules for \( C_t \) and \( I_t \) and the value function \( W_t \).

2. *(Factor Market Clearing)* The interest rate \( r_t \) equates the capital demanded by new entrants \( N_E \) and current producers \( N_t \) to that supplied by the household, and the

\[\text{To be clear about how we reach this expression, recall that profits are defined by } \pi_t = \rho_t y_t^C - r_t K_t^C - w_t. \text{ Total output } y_t \text{ is defined as the sum of output in the two sectors: } y_t = r_t K_t^E N_t^E + \rho_t y_t^C N_t. \text{ Simple substitution yields } y_t = r_t K_t^E N_t^E + N_t (\pi_t + r_t K_t^C + w_t) = r_t K_t + N_t \pi_t + N_t w_t.\]
wage $w$ satisfies the Nash bargaining solution given by equation (19).

3. *(Goods Market Clearing)* \( C_t + I_t + \omega V_t = \rho_t y_t N_t \).

4. *(Firm’s Optimization)* Given the demand for a differentiated commodity given by equation (3), \( \rho_t \) is the profit-maximizing price for the monopolist. Aggregate labor demand and vacancies posted by all firms, \( N_{Et} \), \( N_t \) and \( V_t \) satisfy equations (12) and (13), and the vacancy and filled position values satisfy equations (9) and (10).

5. *(Entry Condition)* \( Q_t = r_t K_t^E \).

6. *(Government)* The government satisfies its budget constraint: \( b(\bar{N} - N_t) = T \).

### 2.3 Calibration

We calibrate the model to the monthly frequency by assigning values to parameters, so that steady-state moments in the model match those observed in U.S. data. The risk aversion coefficient \( \sigma \) is set to 1.5 which is well within the range of values typically used in studies of aggregate fluctuations. The discount factor \( \beta \) is set to 0.99\(^\frac{1}{3}\) which implies a steady-state interest rate equal to 4.1% per annum.

We assume that the productivity process \( Z_t \) follows an AR(1) process with persistence parameter \( \rho_z \) and a zero-mean normally distributed shock with variance \( \sigma_e^2 \). We set \( \rho_z = 0.964 \) and \( \sigma_e = 0.0052 \) which are consistent with the cyclical persistence and variance in the observed Solow residual.\(^8\) Lacking direct evidence on a reasonable value for the workers’ bargaining parameter \( \phi \), we set it equal to 0.5 to make our results comparable to the existing literature (e.g. Shimer (2005)).

\(^8\)We are being somewhat loose here. In the presence of monopolistic competition, variations in the Solow residual cannot be directly associated with productivity of factors of production. The computation of the Solow residual assumes perfect competition and only then can that association be made. For an extensive discussion, see Hornstein (1993).
We calibrate the exit probability $\tau$ and the separation rate $s$ following a procedure similar to that used by Den Haan et al. (2000). Let $\Sigma$ be the total job separation rate caused either by a firm’s death or by any other cause. The rate at which firms exit the market and do not repost vacancies is $\tau$, while $(1 - \tau)s$ is the rate at which workers separate from firms but where firms repost vacancies immediately after. Hence, $\Sigma = \tau + (1 - \tau)s$. The fraction of vacancies that are reposted right after separations is then $\frac{(1-\tau)s}{\Sigma}$. Denote this quantity by $\Omega$. Note also that $\Sigma N$ gives the total flow out of employment, and as a result, $\Omega q \Sigma N$ gives the total number of posted vacancies filled. If we subtract the number of posted vacancies filled from the total flow out of employment, we get the steady-state mass of jobs that is destroyed permanently: $\Sigma N - \Sigma N \Omega q = \Sigma N (1 - \Omega q)$. In a steady state, job destruction must equal job creation. The empirical evidence described by Shimer (2005) sets $\Sigma$ equal to 0.1 at the quarterly frequency which implies $1 - (1 - 0.1)^{1/3} = 0.035$ at the monthly frequency. Therefore,

$$\Sigma = (1 - \tau)s + \tau = 0.035 \quad (20)$$

Davis et al. (1996) report that the job-creation-to-employment ratio in the manufacturing sector is 0.052 quarterly, which implies a value of 0.018 at the monthly frequency. Given a value of $q = 0.802$ per month,

$$\frac{\text{Job Creation}}{\text{Employment}} = \frac{\Sigma N (1 - \Omega q)}{N} = 0.018 \quad (21)$$

From equations (20) and (21) we can solve for $s = 0.021$ and $\tau = 0.014$.

Consistent with estimates reported by Basu and Fernald (1997), we set $\gamma = 11$, which implies a markup of 10 percent. Changing the total mass of workers $\bar{N}$ only amounts to changing the levels, i.e., the scale of output and the mass of employment, etc., but the unit-free ratios, e.g., unemployment rate, v-u ratio, and consumption-output ratio etc., are not affected. Therefore, a choice of $\bar{N}$ does not affect any of the
second moments and the impulse responses. We just pick $\bar{N} > 1$ so that the monopolist’s price is larger than the resulting price if markets are competitive, which is given by $\lim_{\gamma \to \infty} N_1^{1/\gamma - 1} = 1$.

We are left with six parameters to calibrate: $(b, y^E, \delta, \omega, \xi, \alpha)$. To do so, we choose six additional moments that the model needs to match in its steady-state. Based on his own calculations, Shimer (2005) documents that the monthly job finding rate is 0.45. Blanchard and Diamond (1989) argue that vacancy postings have an average of 3 weeks, which implies that the vacancy filling rate is $1 - (1 - 1/3)^4 = 0.802$ per month. Note that the steady state value of market tightness can be written as $\theta = \frac{\ell}{q} = 0.56$. We choose to match the aggregate capital to aggregate output ratio and we set it to a value of 36, which implies a value of 3 at the annual frequency. We set total recruiting costs as a fraction of GDP, given by $\omega V / Y$, to 0.015. A controversial choice is that of the value of the unemployment benefit $b$. Much of the literature argues that the value of non work activities is far below what workers produce on the job. However, calibrations such as Hagedorn and Manovskii’s (2007) claim much success in terms of the cyclical properties of the model when the outside option for workers is very close to their productivity. Under the interpretation of $b$ as purely monetary unemployment benefits, we set $b$ so that the steady state replacement ratio $b / w$ is 0.42 as in Shimer (2005) and Gertler and Trigari (2006). Finally, an additional moment we want to match is the steady-state value for labor’s share, which is 0.60 for the sample we consider. Concluding, to assign values to the vector of parameters $(b, y^E, \delta, \omega, \xi, \alpha)$, we choose the following six moments: $f = 0.45, \theta = 0.56, \omega V / Y = 0.015, K / Y = 36, b / w = 0.42$, and $wN / Y$.

We summarize our parameterization in Table 2.1.

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As in the steady-state the number of entrants $N^E$ is small, one needs a high value of $y^E$ in order to match the value for the capital-to-output ratio found in the data.
Table 2.1: Summary of Parameterization

<table>
<thead>
<tr>
<th>Parameter</th>
<th>Value</th>
<th>Target/Source</th>
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<tr>
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</tr>
<tr>
<td>$\beta$</td>
<td>$(0.99)^{0.5}$</td>
<td>$r = 4.1%$</td>
</tr>
<tr>
<td>$\delta$</td>
<td>0.002</td>
<td>$K/Y = 36$</td>
</tr>
<tr>
<td>$b$</td>
<td>0.380</td>
<td>$b/w = 0.42$</td>
</tr>
<tr>
<td>$y^E$</td>
<td>$3,140^9$</td>
<td>$wV/Y = 0.015$</td>
</tr>
<tr>
<td>$\omega$</td>
<td>0.516</td>
<td>$f = 0.45$</td>
</tr>
<tr>
<td>$\xi$</td>
<td>1.551</td>
<td>$\theta = 0.561$</td>
</tr>
<tr>
<td>$\alpha$</td>
<td>0.02</td>
<td>$wN/Y = 0.60$</td>
</tr>
</tbody>
</table>

3 Results

3.1 The Cyclical Behavior of Labor’s and Corporate Profits Shares

Having assigned parameter values to the model, we solve it, simulate it, and judge its implications against U.S. data. Our solution technique is standard: we approximate the true solution by a first order expansion around the model’s deterministic steady-state. Since the calibration is done at the monthly frequency, we transform the model’s output by aggregating its “monthly” data into “quarterly” data by taking three-month averages. We transform the model’s output and U.S. data in the same way: we detrend them by taking logs and applying a Hodrick-Prescott filter.\(^{10}\)

To construct the labor’s and profit shares we proceed as follows. To measure the share of income that goes to labor we add total private salaries and wages, supplements to salaries and wages, and all of proprietor’s income. We divide this sum by national income. To compute profits’ shares we take corporate profits and we divide

\(^{10}\)The HP smoothing parameter we use is 1,600 - a standard choice when using quarterly data.
them by national income. Our sample starts in 1951 (1st quarter) and ends in 2003 (fourth quarter).

Figure 1 shows correlations between real GDP and the labor’s share with different leads and lags for the U.S. economy. Correlations are not strong - the maximum is about 0.40 - and the contemporaneous correlation is (significantly) smaller than the correlation between output and the labor’s share led four periods. Consequently, labor’s share lags real GDP because its correlation coefficient with output is highest after 4 quarters. Labor’s share is countercyclical, but very weakly so: the contemporaneous correlation is -0.28, and the 5th percentile for the sample distribution of that correlation is -0.15. Figure 2 shows a similar picture for the (corporate) profits’ share which shows stronger cyclical dynamics than the labor’s share, it is procyclical, and shows no leading or lagging pattern.

![Figure 1: Correlations between Real Gross Domestic Product and (led and lagged) Labor’s Share of Real GDP - US Data 1951-2003.](image)

How well does the model of costly firm entry match these patterns compared to...
standard models where firm entry is free? Let’s begin with the labor’s share.\textsuperscript{11} Figure 3 displays the empirical cross-correlations - the same values as Figure 1 represented by the dotted and dash-dotted lines - along with the cross-correlations from the entry model - labeled “Model” in the figure and represented by the circled-thick line. The labor’s share in the model matches remarkably well the patterns observed in the data. The contemporaneous correlation is weak with a value of -0.15 and well within the error bounds provided for the empirical correlations.

The contemporaneous correlation is weak with a value of -0.15 and well within the error bounds provided for the empirical correlations.

\begin{figure}[h]
\centering
\includegraphics[width=\textwidth]{figure2.png}
\caption{Correlations between Real Gross Domestic Product and (led and lagged) Profits’ Share of Real GDP - US Data 1951-2003.}
\end{figure}

The correlations with one lead and one lag quantitatively match their empirical counterparts, and only correlations at higher leads and lags are somewhat stronger than those found in the data. Most importantly, the model gets the lagging pattern of the labor’s share right: after an increase in output, the labor’s share increases four quarters after without a large contemporaneous effect. Figure 4 shows the analog to

\textsuperscript{11}All results presented in the paper have CES preferences -see equation (2). This specification features constant markups. To introduce time-varying markups we changed the utility function to be of the translog type -see Feenstra (2003). The results are similar to the baseline CES case and they are available upon request.
Figure 3 but for the profits’ share instead of the labor’s share. The figure shows that the model with costly entry matches well the correlations at several leads and lags of the profits’ share and output. In fact, all correlations are within the error bounds constructed for the empirical point estimates. Before we explain the pattern of correlations in the model with costly entry, it is illustrative to compare it to a benchmark model: the model with free entry. Readers can think of this model as a version of the one solved in Shimer (2005) to which we add capital, we model it in discrete time, and to improve its fit of labor market business cycle dynamics, we calibrate it along the lines of Hagedorn and Manovskii (2009). Figure 5 adds to Figure 3 the patterns of correlations between output and labor’s share computed from the model with free entry. The figure shows how the labor’s share in the free entry model is strongly countercyclical. Although it might not be apparent just by glancing at the graph, the value of the contemporaneous correlation is -0.95, as opposed to -0.15 in the costly entry model and -0.20 in the data. These differences are large. The performance of the free entry model regarding the profits share is better, as one can see in Figure 6, but considerably worse than the Costly Entry model. In the free entry model, the strong cyclicality in the profits share, which is consistent with the data, comes at the expense of a strong cyclicality in the labor’s share, which is not. De-linking the cyclical dynamics of the two shares, in the sense of generating weak correlations between labor’s share and output and strong correlations between the profits’ share and output, is something the model we construct is able to achieve.

The strong countercyclicality of labor’s share the free entry model generates is caused by the relatively larger response of output to a rise in productivity. The reader should recall that matching frictions prevent employment from adjusting immediately to a productivity shock; this is a feature of all models we present in this paper. In the free entry model, both wages and output respond rapidly to a change in productivity but output responds relatively stronger. As a result, the labor share falls sharply

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12We describe with more detail the structure and calibration of the free-entry model in the Appendix.
Figure 3: Correlations between Real Gross Domestic Product and (led and lagged) Labor’s Share of Real GDP - US Data 1951-2003 (dashed-dotted lines) and Costly Entry model (circled line)

(relative to its steady state value) but as employment rises in subsequent periods, the labor’s share rises as well. This explains the strong negative contemporaneous correlation and the mildly positive correlation of output with labor’s share led three or four quarters.

To understand the dynamics in the costly-entry model it is useful to remember that output $y_t$ is equal to,

$$\rho_t y_t^c N_t + r_t K_t^E N_t^E = \rho_t y_t^c N_t + Q_t N_t^E.$$ 

In other words, total output is the sum of income in two “sectors”: the commodity producing sector, $\rho_t y_t^c N_t$; and the “start-up” sector, $r_t K_t^E N_t^E$. The joint dynamics of both sectors determine the dynamics of total output. Suppose there is positive productivity shock. An immediate response is a drop in $K_t^E$, the “per-start-up” amount of capital, as $y_t^E$ is constant. The number of entrants $N_t^E$ rises, as the present value of profits is now higher. The remaining key variable determining the behavior of output in the “start-up” sector is therefore the interest rate, $r_t$. The equilibrium interest rate is
determined by the relative demand and supply of capital in the two sectors. Total capital, $K_t = N^E_t K^E_t + N^K_t$, is a predetermined variable but the economy can reallocate it intra-temporally between the two sectors. The technology of the goods producing sector being Cobb-Douglas forces interest rates to rise in the face of a positive technology shock. This is a standard result in models of economic fluctuations with Cobb-Douglas technology and the culprit for the strong procyclicality of real interest rates in the real business cycle literature. What happens in the “start-up” sector? Because $K^E_t$ falls when $Z_t$ rises, the demand for capital by any given entrant is lower, forcing interest rates to drop. In summary, the behavior of interest rates in the face of an increase in productivity is the result of two counteracting forces. On the one hand, technology in the goods producing sector pulls interest rates upward when productivity rises, but on the other hand, it lowers the amount of capital an entrant needs, lowering the demand for capital and pulling rates downward. Using the calibration described previously interest rates are countercyclical. This drop in interest rates is responsible for the more muted response of output (relative to that of wages) in the costly entry model. In turn, this also dampens the negative response of labor’s share to an increase in productivity.

Figure 4: Correlations between Real Gross Domestic Product and (led and lagged) Profits’ Share of Real GDP - US Data 1951-2003 (dashed-dotted lines) and Costly Entry model (circled line).
The evolution of interest rates helps explain the more muted response of output in the costly entry model. However, this factor is only part of the story when it comes to explaining the different dynamics of the labor’s share in the two models. The existence of entry costs may, depending on parameter values, make the response of employment (and wages) to persist over time. This persistence is due to resources in the start-up sector competing with those in the goods producing sector. As a result, entrants may find it optimal to delay their entrance so more capital can be used for producing goods when productivity is high. This delayed response raises wages and employment for several quarters raising the numerator in the expression for labor’s share and that explains the lagging behavior, that is, the high positive correlation between output and labor’s share led four quarters.

![Correlation of Labor's Share (t+j) and Real GDP (t): Model vs. Data](image)

Figure 5: Correlations between Real Gross Domestic Product and (led and lagged) Labor’s Share of Real GDP - US Data 1951-2003 (dashed-dotted lines), Costly Entry model (circled line) and Free Entry model (squared line).

So far we have seen that the joint dynamics of firm entry, the asset value of a vacant position, and the interest rates are important for understanding the dynamics of income shares. In truth, distinguishing between the dynamics of the value of a vacancy and interest rates in the Costly Entry models is not necessary. Recall that the value of a vacancy $Q_t$ is equal to $r_t K^E_t$. In both Costly Entry models $K^E_t$ displays exactly the same
dynamics - again, because $y^E$ is a constant and $Z_t$ is exogenous. So the behavior of $Q$ is essentially driven by the behavior of interest rates, $r$. But let us return to understanding the dynamics of income shares by showing through a different channel that it is indeed the joint dynamics of entry and real interest rates that are crucial for the weak low contemporaneous correlation between output and the labor’s share and the lagging pattern of the cross-correlations.

This different channel is running an experiment that involves making firm entry less attractive by lowering the efficiency of the matching technology. This efficiency is represented by $\xi$, which we set now to a value of 0.38 - it was 1.551 before. This has two effects on firm entry. First, it lowers the steady-state value of entrants. As matching becomes more difficult, the probability of matching to a worker decreases. This decrease lowers the prospects of making any profits, leading to a lower level of entrants in equilibrium. Second, firm entry is less persistent. To understand this second effect, assume first a positive shock to productivity from the steady state. If matching efficiency is low, firms need to enter relatively early to be able to match with workers and still take advantage of the higher productivity. Consequently, firm entry concentrates in the first few periods after the shock. The concentration of entrants early on, results in a relatively high demand for capital in the “start-up” sector which prevents interest rates from falling. As a result, the response of output is closer to that of the Free Entry model; response that we know implies a larger drop in labor’s share. Because entry is not delayed, the response of wages and employment is not as persistent either. Therefore, the numerator of the labor’s share does not rise too much in subsequent periods reducing the correlation between output and labor’s share led four quarters. The disappearance of the lagging behavior and the appearance of a strong countercyclicality of labor’s share are apparent in Figure 7. That figure shows that the contemporaneous correlation between the labor’s share and output is close to -1. Profits shares also display similar dynamics to the free entry model - see Figure 8. It is worth re-emphasizing that these differences arise because of changing dynamics in firm entry, interest rates,
and the asset value of a vacant position. They do not arise because of differences in the level of sunk costs $y^E$ which has remained at the value calibrated in the previous section throughout the exercise of lowering $\xi$.

![Correlation of Profits’ Share (t+j) and Real GDP (t): Model vs. Data](image1)

Figure 6: Correlations between Real Gross Domestic Product and (led and lagged) Profits’ Share of Real GDP - US Data 1951-2003 (dashed-dotted lines), costly entry model (circled line) and free entry model (squared line).

![Correlation of Labor’s Share (t+j) and Real GDP (t): Entry Models](image2)

Figure 7: Correlations between Real Gross Domestic Product (output) and (led and lagged) Labor’s Share of output - costly entry model (circled line) and Low $\xi$ model (squared line).

The previous figures, and the intuition behind them, make clear that the behavior
of entrants, the behavior of the value of a vacancy, and the dynamics of interest rates are important for understanding the dynamics of income shares. To further validate our model we need to bring more evidence to the table. In particular, we show that the dynamics of real interest rates in the data are consistent with the Costly Entry model at the expense of the other two. To get an empirical counterpart to interest rates in the theoretical models, we first obtain quarterly measures of nominal yields from corporate bonds (Baa-rated). We restrict the sample to be the same as that used to compute correlations of income shares: 1951:Q1-2003:Q1. To transform those nominal yields into real yields, we subtract the inflation rate for that quarter (annualized, because yields are annualized as well).\footnote{We take current inflation to be a reasonable forecast of inflation in the next three months. In the short-run this “random-walk” forecast works remarkably well (see Stock and Watson (1999b)).} The first row of Table 3.1 shows the correlation at the quarterly frequency of real interest rates and output in the data and in the three model economies.\footnote{We de-trend real interest rates, both in the data and in the model economies, by computing the percentage deviation relative to steady state.} The correlation between the real interest rate and output in the data is -0.348.\footnote{Stock and Watson (1999a) using expected inflation from a VAR and the yield in T-bills, report a correlation of -0.35.} Both the Free Entry model and the Costly Entry model with low $\xi$ feature...
Table 3.1: Correlations between $y_t$ and $r_t$, $Q_t$ and $N^E_t$: Data vs. Models

<table>
<thead>
<tr>
<th></th>
<th>US Data</th>
<th>Costly Entry</th>
<th>Costly Entry (low $\xi$)</th>
<th>Free Entry</th>
</tr>
</thead>
<tbody>
<tr>
<td>Corr($y, r$)</td>
<td>-0.348</td>
<td>-0.272</td>
<td>0.982</td>
<td>0.972</td>
</tr>
<tr>
<td>Corr($y, Q$)</td>
<td>N/A</td>
<td>-0.664</td>
<td>0.795</td>
<td>N/A</td>
</tr>
<tr>
<td>Corr($y, N^E$)</td>
<td>0.510</td>
<td>0.983</td>
<td>0.992</td>
<td>N/A</td>
</tr>
</tbody>
</table>

pro-cyclical interest rates with correlations close to 1 (0.972 and 0.982 respectively). On the other hand, the Costly Entry model features countercyclical interest rates. The correlation of these with output has a value of -0.272, remarkably close to that observed in the data.\(^{16}\)

Given the correlations of interest rates and output in the two Costly Entry models, it is not surprising that the correlation between the value of a vacancy is negative in the Costly Entry model and positive in the Costly Entry model with low $\xi$. Given the tight link in the model between $Q_t$ and $r_t$, even though we lack empirical measures of the value of a vacancy, through the lens of the model the real interest rate is an excellent proxy. This proxy strengthens the hypothesis that the Costly Entry model is a good representation of the data.

Finally, to the best of our knowledge there are no good measures of firm entry. We have taken the one used by Bilbiie, Ghironi and Melitz (2005); they report a correlation of 0.510 with output (see their Figure 2). However, even if good measures of entry existed, this variable does not allow to distinguish among the two Costly Entry models. The reason is firm entry is procyclical and similar in magnitude in both models.

We now provide more evidence supporting the Costly Entry environment. We do so by looking at yet another feature of the data that existing models have difficulty replicating. Ríos-Rull and Choi (2008), building on the empirical analysis of Ríos-Rull and Santaeulalia-Llopis (2008) note that an empirical regularity of the labor’s share is at odds with US data in a large class of models. This regularity pertains to the response

\(^{16}\)Gomme, Ravikumar, and Rupert (2010) find a weaker, but negative nevertheless (-0.13), correlation between output and after-tax returns in the SP500 stock market index.
of the labor’s share to a productivity shock. To replicate their results, we construct a series for the Solow residual, we use our measure of labor’s share described above and we follow their same methodology for computing that response. Consequently, we fit a bivariate VAR to these two series and we identify a “fundamental” innovation to the technology process by assuming that the labor’s share does not affect technology contemporaneously. Specifically we assume that the “structural” representation of the reduced-form VAR takes the following form,

\[
lsht = \alpha_0 + \beta_0 z_t + \alpha_{11} lsht-1 + \alpha_{12} z_{t-1} + \epsilon_{lsht,t}
\]

\[
z_t = \alpha_1 + \alpha_{21} lsht_{t-1} + \alpha_{22} z_{t-1} + \epsilon_{z,t}
\]

Figure 9 shows the response of labor’s share to a one-standard deviation innovation to the technology process. The labor’s share falls contemporaneously and starts rising one quarter after the shock. The rise continues for about five years, after which the labor’s share slowly returns to its steady-state level.

![Empirical Impulse Response of Labor’s Share to a 1 s.d. Tech. Shock](image.png)

Figure 9: Response of the labor’s share to a one standard deviation (orthogonalized) innovation to technology.
Besides the “over-shooting” property, the two most noticeable features are the magnitude of the rise (significantly above the steady-state level) and the persistence of the response. In their quest for models that can match this feature of the data, Ríos-Rull and Choi (2008) focus on a family of search and matching models of the labor market in which wages are the result of Nash bargaining. The standard (i.e. Pissarides (1985) or Shimer (2005)) model matches only the initial drop. After the first period, the response of the labor’s share is rather muted when compared to the data. It never rises much above its steady state level and it displays virtually no persistence. This is true even when the model is calibrated to match the volatility of the vacancies-to-unemployment ratio observed in the data. As our framework belongs to the same family of search and matching models we perform a similar analysis.

Figure 10 displays the response of the labor’s share in the same three models discussed above: the costly-entry model, the free entry model, and the costly-entry with low matching efficiency. We compute the quarterly response as the three-month average of the original monthly response. The first model, labeled “Model” is the costly-

![Diagram](model_impulse_responses.png)

Figure 10: Response of the labor’s share to a one standard deviation (orthogonalized) innovation to technology in our three model economies.
entry model parameterized according to Section 2.3. The second, labeled “Model: low $\xi$, is the costly-entry model with a lower matching efficiency obtained through a lower value for $\xi$. Finally, the third model is the free entry model described briefly above and in more detail in the Appendix. The costly-entry model features a labor’s share that initially falls, rising significantly above its steady state value until its peak five years after the impulse. It also persists at above-steady-state levels for a long period of time. The intuition behind these results follows from the general discussion of the cyclical dynamics of income shares. The sunk costs of entry introduce sluggishness in the decision of firms, generating persistence and helping to achieve the longer duration in the response to a shock. The labor’s share rises because output in the setup-sector falls initially, relative to its long-run value. Consistent with the plots shown for the correlations of labor’s share and output, the responses in the free entry model and the costly entry model with low matching efficiency are close, and do not match the data well.

3.2 Other Business Cycle Dynamics

After explaining the mechanism by which the costly entry model generates dynamics of income shares that roughly match those of US data, we now turn to show that improvement does not come at the expense of many other business cycle dynamics. Of course, there are dimensions along which the costly entry models under-performs the free entry model and we describe those as well. Table 3.2 shows some statistics for our sample of U.S. data for some selected quantities. We focus on consumption ($C$), investment ($I$), unemployment ($U$), vacancies ($V$), the vacancies-to-unemployment ratio ($V/U$), GDP ($Y$), consumption ($C$) and total factor productivity ($Z$). Regarding labor market variables, the two most salient features are: the high volatility of unemployment, vacancies, and the vacancies-to-unemployment ratio; and the strong negative correlation between unemployment and vacancies. The first of the two has been the object of a large literature spawned by Shimer’s (2005) study, who showed a large dis-
Table 3.2: Summary Statistics, Quarterly U.S. Data, 1951:1 to 2003:4

<table>
<thead>
<tr>
<th></th>
<th>C</th>
<th>I</th>
<th>Y</th>
<th>Z</th>
<th>V</th>
<th>U</th>
<th>V/U</th>
</tr>
</thead>
<tbody>
<tr>
<td>Std. Dev (rel. to output)</td>
<td>0.526</td>
<td>2.984</td>
<td>1.00</td>
<td>0.590</td>
<td>8.635</td>
<td>7.778</td>
<td>16.073</td>
</tr>
<tr>
<td>Autocorr</td>
<td>0.851</td>
<td>0.888</td>
<td>0.838</td>
<td>0.742</td>
<td>0.904</td>
<td>0.869</td>
<td>0.895</td>
</tr>
<tr>
<td>Corr. Matrix</td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>C</td>
<td>1.000</td>
<td>0.692</td>
<td>0.780</td>
<td>0.195</td>
<td>0.728</td>
<td>-0.649</td>
<td>0.705</td>
</tr>
<tr>
<td>I</td>
<td>1.000</td>
<td>0.865</td>
<td>0.360</td>
<td>0.832</td>
<td>-0.727</td>
<td>0.799</td>
<td></td>
</tr>
<tr>
<td>Y</td>
<td>1.000</td>
<td>0.541</td>
<td>0.900</td>
<td>-0.837</td>
<td>0.888</td>
<td></td>
<td></td>
</tr>
<tr>
<td>Z</td>
<td>1.000</td>
<td>0.309</td>
<td>-0.152</td>
<td>0.239</td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>V</td>
<td>1.000</td>
<td>-0.918</td>
<td>0.981</td>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>U</td>
<td>1.000</td>
<td>-0.977</td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>V/U</td>
<td>1.000</td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
</tbody>
</table>

crepancy between the data and traditional models of search and matching in labor markets (e.g. Pissarides (1985)). Subsequently, work by Hagedorn and Manovskii (2009) has shown that an appropriate calibration of the model yields a volatility between the vacancies-to-unemployment ratio and output close to that observed in the data. However, their calibration is still subject to criticism based on the large magnitude of the response of unemployment to minor changes in the level of unemployment benefits (see Hornstein et al. (2005)). Other important facts about labor markets are the weak correlation between vacancies and productivity, the positive correlation between vacancies and output, and the negative correlation between unemployment and output. Other well-known business cycle facts are the lower volatility of consumption and the higher volatility of investment, relative to that of output. Both are highly procyclical.

Table 3.3 displays the models’ standard deviations for some selected variables for three theoretical economies. The first two, labeled the Costly Entry and the Free Entry models, are the same economies as those described in previous sections. The third economy is a Free Entry model calibrated along the lines of Shimer (2005). More specifically, this model has a lower value of unemployment - relative to being employed - and a higher bargaining power for workers.\(^\text{17}\) Compared to the other two models,

\(^{17}\)In the Appendix we describe the calibration of both Free Entry economies.
Table 3.3: Standard Deviations

<table>
<thead>
<tr>
<th></th>
<th>Cost. Entry</th>
<th>Free Entry</th>
<th>Free Entry (low $b/w$ and high $\phi$)</th>
</tr>
</thead>
<tbody>
<tr>
<td>$U$</td>
<td>2.872</td>
<td>6.736</td>
<td>1.245</td>
</tr>
<tr>
<td>$V$</td>
<td>1.636</td>
<td>5.988</td>
<td>0.742</td>
</tr>
<tr>
<td>$V/U$</td>
<td>4.659</td>
<td>15.847</td>
<td>1.683</td>
</tr>
<tr>
<td>$C$</td>
<td>1.223</td>
<td>0.219</td>
<td>0.220</td>
</tr>
<tr>
<td>$I$</td>
<td>4.570</td>
<td>4.413</td>
<td>4.739</td>
</tr>
<tr>
<td>$Y$</td>
<td>1.000</td>
<td>1.000</td>
<td>1.000</td>
</tr>
</tbody>
</table>

the Free Entry model (middle column) matches the data quite well. The degree of amplification is large and as result standard deviations are close to those observed in the data. Nevertheless, in the Costly Entry model the increases in the standard deviation, relative to the Free Entry model, are not trivial. Vacancies, unemployment, and the vacancies-to-unemployment ratio all display volatilities that are almost three times those shown by the Free Entry model with the standard calibration.\(^{18}\)

To understand the mechanism of amplification in the Costly Entry model, it is useful to step back to a standard version of the Pissarides (1985) model. For concreteness, let’s take the one solved by Shimer (2005): let us abstract from capital, firm entry, and revert to continuous time. Firms and workers search for jobs and the output of a worker if matched with a firm is $Z$, an exogenous value for productivity that follows a given stochastic process. When workers are looking for a job they receive unemployment benefits $b$ and the matching mechanism is identical to that of the Costly Entry model. The dynamics of this economy are represented by the following four equations (where $r$ denotes the discount rate instead of the interest rate and $\theta$ the vacancies-to-unemployment ratio):

\(^{18}\)Perhaps this is the correct place to emphasize that most results in this paper do not obtain if we specify a time-varying vacancy posting cost $\omega$. That specification is not equivalent to the “Costly Entry” model. However, the time-varying posting cost -if it is countercyclical - causes the volatility of labor market variables to increase, but the persistent dynamics that characterize the Costly Entry model are absent. It is precisely those dynamics that improve the behavior of income shares and that behavior is our main object of study.
\[ rQ = -\omega + q(\theta) (J - Q), \]
\[ rJ = Z - w - s(J - Q), \]
\[ rW_u = b + f(\theta) (W_e - W_u), \]
\[ rW_e = w - s(W_e - W_u). \]

The notation of the value functions follows the Costly Entry model: \( J \) is the value of a filled job, \( Q \) is the value of a vacancy, \( W_e \) is the value of being employed, and \( W_u \) is the value of being unemployed. To understand the high volatility of labor market variables it is illustrative to calculate the elasticity of \( \theta \) - the vacancies-to-unemployment ratio - to changes in the net labor productivity \( Z - b \). Assuming free entry yields \( Q = 0 \) in equilibrium, and the elasticity under that case is given by:

\[
\frac{\varepsilon_{\theta|Z-b}}{(r+s)(1-\eta(\theta)) + \phi f'(\theta)} = \frac{r + s + \phi f(\theta)}{(r+s)(1-\eta(\theta)) + \phi f(\theta)}
\]

where \( \eta(\theta) \in [0, 1] \) is the elasticity of \( f(\theta) \) with respect to \( \theta \). How does this elasticity change if we assume \( Q > 0 \) and if \( Q \) is cyclical and varies with \( Z \)? One can show that its value is given by the relatively more complicated expression:

\[
\frac{\varepsilon_{\theta|Z-b}}{(r+s)(1-\eta(\theta)) + \phi f'(\theta)} = \frac{(r + s + \phi f(\theta)) \Psi + (1 - \phi) rq(\theta) Q \Psi}{(r+s)(1-\eta(\theta)) + \phi f(\theta)}
\]

where \( \Psi = 1 - (Z - b + \omega) rQ'(Z) / (\omega + rQ) \). If \( Q'(Z) < 0 \) - that is, if \( Q \) is countercyclical or responds negatively to changes in \( Z \), the \( \Psi > 1 \). Therefore, the elasticity with \( Q > 0 \) is larger than when \( Q = 0 \).

\[ ^{19} \text{The equilibrium condition that pins down the value of } \theta \text{ is given by:} \]
\[ \frac{r + s}{q(\theta)} + \phi \theta = (1 - \phi) \frac{Z - b - rQ}{\omega + rQ}. \]

From this expression - once we have set \( Q = 0 \) - it is easy to find the derivative of \( \theta \) with respect to \( Z - b \).
Table 3.4: Cross Correlations: Costly Entry Model

<table>
<thead>
<tr>
<th></th>
<th>C</th>
<th>I</th>
<th>V/U</th>
<th>U</th>
<th>V</th>
<th>Y</th>
<th>Z</th>
</tr>
</thead>
<tbody>
<tr>
<td><strong>C</strong></td>
<td>1.000</td>
<td>0.846</td>
<td>0.686</td>
<td>-0.878</td>
<td>0.627</td>
<td>0.891</td>
<td>0.823</td>
</tr>
<tr>
<td><strong>I</strong></td>
<td></td>
<td>1.000</td>
<td>-0.167</td>
<td>0.345</td>
<td>0.146</td>
<td>0.983</td>
<td>0.945</td>
</tr>
<tr>
<td><strong>V/U</strong></td>
<td>1.000</td>
<td></td>
<td>-0.882</td>
<td>0.907</td>
<td>-0.005</td>
<td>0.138</td>
<td></td>
</tr>
<tr>
<td><strong>U</strong></td>
<td></td>
<td></td>
<td>1.000</td>
<td>-0.838</td>
<td>0.185</td>
<td>0.039</td>
<td></td>
</tr>
<tr>
<td><strong>V</strong></td>
<td></td>
<td></td>
<td></td>
<td>1.000</td>
<td>0.317</td>
<td>0.455</td>
<td></td>
</tr>
<tr>
<td><strong>Y</strong></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td>1.000</td>
<td>0.990</td>
<td></td>
</tr>
<tr>
<td><strong>Z</strong></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td>1.000</td>
<td></td>
</tr>
</tbody>
</table>

Table 3.5: Cross Correlations: Free Entry Model

<table>
<thead>
<tr>
<th></th>
<th>C</th>
<th>I</th>
<th>V/U</th>
<th>U</th>
<th>V</th>
<th>Y</th>
<th>Z</th>
</tr>
</thead>
<tbody>
<tr>
<td><strong>C</strong></td>
<td>1.000</td>
<td>0.949</td>
<td>0.814</td>
<td>-0.735</td>
<td>0.760</td>
<td>0.979</td>
<td>0.856</td>
</tr>
<tr>
<td><strong>I</strong></td>
<td></td>
<td>1.000</td>
<td>0.791</td>
<td>-0.889</td>
<td>0.795</td>
<td>0.993</td>
<td>0.982</td>
</tr>
<tr>
<td><strong>V/U</strong></td>
<td>1.000</td>
<td></td>
<td>-0.623</td>
<td>0.710</td>
<td>0.785</td>
<td>0.775</td>
<td></td>
</tr>
<tr>
<td><strong>U</strong></td>
<td></td>
<td></td>
<td>1.000</td>
<td>-0.511</td>
<td>-0.899</td>
<td>-0.843</td>
<td></td>
</tr>
<tr>
<td><strong>V</strong></td>
<td></td>
<td></td>
<td></td>
<td>1.000</td>
<td>0.810</td>
<td>0.876</td>
<td></td>
</tr>
<tr>
<td><strong>Y</strong></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td>1.000</td>
<td>0.987</td>
<td></td>
</tr>
<tr>
<td><strong>Z</strong></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td>1.000</td>
<td></td>
</tr>
</tbody>
</table>

Readers may think about our Costly Entry model as one in which $Q(Z)$ is an endogenous and complicated object. Its cyclical dynamics are jointly determined with the volume of entrants in the market and the behavior of interest rates. We do not display standard deviations from the Costly Entry model with low matching efficiency $\xi$, but the volatility of labor market variables in that model is smaller than in the Free Entry model - even the one calibrated with a high $b/w$. It turns out that interest rates are procyclical, making $Q$ procyclical as well. Risking being repetitive, the value of a vacancy $Q$ is countercyclical in the Costly Entry model because interest rates are countercyclical. Therefore, the mechanism that amplifies the volatility of labor market variables is tightly linked to the mechanism that improves the dynamics of income shares.

Tables 3.4 and 3.5 display correlations between some selected variables in the Costly Entry and the Free Entry models. Consumption and investment are procyclical in both models, with correlations well above 0.70. The correlation between vacancies and pro-
ductivity is weaker and closer to the data (0.309) in the Costly Entry model (0.455) than in the Free Entry model (0.876). However, we pay a price for this weaker correlation between vacancies and productivity in the form of an acyclical unemployment rate; surely the biggest drawback of our model. The reason for the acyclical unemployment rate is that vacancies are a predetermined variable and as a result the vacancies-to-unemployment ratio is not very cyclical, and therefore employment is not very cyclical. However, the Costly Entry model shows a stronger correlation between vacancies and unemployment - the slope of the Beveridge curve - than the model with Free Entry. The reason is that in the Costly Entry model vacancies do not react immediately to changes in productivity: existing firms that separate from their workers do no repost vacancies; entrants need to wait for one period and pay \( y^E \) before posting them. As a result vacancies and unemployment move closer to one-to-one than in the Free Entry environment, where vacancies immediately adjust to changes in productivity but vacancies do not. Finally, costly entry has an effect on the persistence of shocks over time. Table 3.6 displays the first four autocorrelations for the same variables as those in Tables 3.4 and 3.5. Even though investment is less persistent in the Costly Entry model, labor market variables - in particular vacancies and the vacancies-to-unemployment ratio - are more correlated over time.\(^{20}\) In particular, regarding the persistence of vacancy creation, these results confirm the findings of Fujita and Ramey (2007).

4 Final Remarks

We have constructed a quantitative model of the macroeconomy that is consistent with most income shares’ time series facts. The novel aspect of our environment relative to models with frictional labor markets is to assume costly entry by firms. This assumption introduces cyclical dynamics in the asset value of a vacant position; a value which in equilibrium has to equal the expenditures undertaken by firms to enter production.

\(^{20}\)Of course, the autocorrelation of shocks is the same across the two models.
markets. We show that for the model to account for income shares’ dynamics and to propagate and amplify productivity shocks, that asset value has to be negatively correlated with output over the business cycle. Or, what is equivalent in our framework, that interest rates have to be negatively correlated. This negative correlation of real interest rates and output has proved difficult to obtain in production economies.

Even though the framework can account for many time series facts regarding labor markets and income shares times, there is still work to be done. For instance, the labor’s share seems to have a high volatility at low-frequencies but a low volatility at high-frequencies. This translates in a response of that share to changes in productivity that remains above the steady state for many quarters, but a cyclical component that is smoother than output. These two facts are difficult to reconcile with the type of model we have presented and call for further research to account for low-frequency movements in income shares that are of a different nature than the high frequency movements observed between expansions and recessions.
References


A Appendix: The Free-Entry Model

For the sake of exposition we briefly describe the free-entry model (Pissarides (1985), Shimer (2005)). This model serves as a benchmark framework for many of the results in the text.

The economy is populated by a large household of measure one. Members of the household can be either employed or unemployed. Denote the fraction of those employed at time $t$ by $N_t$. The household’s preferences are given by,

$$E_0 \sum_{t=0}^{\infty} \beta^t \left[ \frac{(C_t)^{1-\sigma}}{1-\sigma} \right],$$

(22)

A general good $Y$ is produced using capital $K$ and labor $N$ by a single firm employing the following technology,

$$Y = ZK^\alpha N^{1-\alpha}$$

(23)

$Z$ represents technology (TFP) that evolves according to:

$$\log(Z_t) = \rho \log(Z_{t-1}) + \epsilon_t$$

(24)

The innovation $\epsilon_t$ is $i.i.d$. The labor market is characterized by search and matching frictions. Unemployed workers search and the firms post vacancies. It costs $\omega$ to post one vacancy and workers and firms match according to the following matching technology,

$$M(1 - N, V) = \frac{(1 - N)V}{((1 - N)^{\xi} + V^{\xi})^{\frac{1}{\xi}}}$$

(25)

The household owns shares in the firm obtaining profits equal to $\pi_t$. As a result, the firm discounts the future (between any period $t$ and $t+1$) using the intertemporal marginal rate of substitution of the household: $\beta \left( \frac{C_{t+1}}{C_t} \right)^{-\sigma}$. The equilibrium value of a vacancy is zero, as firms will enter until there are no gains to made by posting them. Denote by $q$ the probability that a firm fills a vacancy, by $s$ the rate at which existing
matches between workers and firms separate, and by \( J \) the capital value of a filled job. In equilibrium,
\[
\omega = q_t \beta \left( \frac{C_{t+1}}{C_t} \right)^{-\sigma} J_{t+1}
\]

(26)

and
\[
J_t = \pi_t + \beta \left( \frac{C_{t+1}}{C_t} \right)^{-\sigma} J_{t+1} (1 - \sigma)
\]

(27)

We assume wages are negotiated through Nash bargaining, in which firms have a bargaining weight equal to \( \phi \). Wages are the solution to the following surplus splitting rule,
\[
\frac{J_t}{1 - \phi} = \frac{C_t^\gamma \frac{\partial W_t}{\partial N_t}}{\phi}.
\]

(28)

The interpretation of this expression is analogous to that of the text. As vacancies have zero value in equilibrium the threat point for the firm is zero. For the representative household it is given by the marginal disutility of having one more member unemployed. Finally, the aggregate resource constraint equates total goods produced net of vacancy creation costs to the sum of investment and consumption:
\[
Y_t - \omega V_t = C_t + I_t.
\]

(29)

Table A.1 summarizes our parameterization of the free entry model. We keep the targets for the calibration the same as in the costly entry model to ease comparisons. The only exception to this approach is the target for the ratio of unemployment benefits to wages. This ratio is set to 0.426 in the costly entry model and to 0.95 in the free entry model. In the spirit of the parameterization used by Hagedorn and Manovskii (2010) we also decrease the bargaining weight \( \phi \) to a value of 0.1. It is both of these two features that are responsible for the large amplification mechanism. Table A.2 displays the parameterization of the model labeled “Free Entry with low \( b/w \) and high \( \phi \)”. This alternative parameterization is the result of setting \( \phi \) to 0.5 and calibrating \( b/w \) to be 0.426 and it is roughly consistent with the one used by Shimer (2005).
Table A.1: Summary of Parameterization (Free Entry)

<table>
<thead>
<tr>
<th>Parameter</th>
<th>Value</th>
</tr>
</thead>
<tbody>
<tr>
<td>$\sigma$</td>
<td>1.500</td>
</tr>
<tr>
<td>$\phi$</td>
<td>0.100</td>
</tr>
<tr>
<td>$\rho_z$</td>
<td>0.964</td>
</tr>
<tr>
<td>$\sigma_z$</td>
<td>0.005</td>
</tr>
<tr>
<td>$s$</td>
<td>0.019</td>
</tr>
<tr>
<td>$\beta$</td>
<td>$(0.99)^{1/3}$</td>
</tr>
<tr>
<td>$\delta$</td>
<td>0.005</td>
</tr>
<tr>
<td>$b$</td>
<td>3.016</td>
</tr>
<tr>
<td>$\omega$</td>
<td>2.410</td>
</tr>
<tr>
<td>$\xi$</td>
<td>1.172</td>
</tr>
<tr>
<td>$\alpha$</td>
<td>0.300</td>
</tr>
</tbody>
</table>

Table A.2: Summary of Parameterization (Free Entry with low $b/w$ and high $\phi$)

<table>
<thead>
<tr>
<th>Parameter</th>
<th>Value</th>
</tr>
</thead>
<tbody>
<tr>
<td>$\sigma$</td>
<td>1.500</td>
</tr>
<tr>
<td>$\phi$</td>
<td>0.5</td>
</tr>
<tr>
<td>$\rho_z$</td>
<td>0.964</td>
</tr>
<tr>
<td>$\sigma_z$</td>
<td>0.005</td>
</tr>
<tr>
<td>$s$</td>
<td>0.019</td>
</tr>
<tr>
<td>$\beta$</td>
<td>$(0.99)^{1/3}$</td>
</tr>
<tr>
<td>$\delta$</td>
<td>0.005</td>
</tr>
<tr>
<td>$b$</td>
<td>1.353</td>
</tr>
<tr>
<td>$\omega$</td>
<td>3.112</td>
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<tr>
<td>$\xi$</td>
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</tr>
<tr>
<td>$\alpha$</td>
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</tr>
</tbody>
</table>