Public Sector Employment in an Equilibrium Search and Matching Model (Work in Progress)

Jim Albrecht,¹ Lucas Navarro,² and Susan Vroman³

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¹Georgetown University and IZA

²ILADES, Universidad Alberto Hurtado, Santiago, Chile

³Georgetown University and IZA

Introduction

- ▶ The public sector accounts for a substantial fraction of employment in both developed and developing countries.
- ▶ There is a public-sector wage premium in many countries both in the raw data and after controlling for observables and endogenous sector choice.
- ▶ Wages in the public sector tend to be more compressed than in the private sector. The public-sector premium is higher at lower quantiles, and there is a negative public-sector premium at higher quantiles in some countries.

Questions

- ▶ How do the private- and public-sector labor markets interact?
- ▶ What types of workers tend to work in the public sector and in the private sector?
- ▶ How do the size of the public sector and the way that wages are set in that sector affect the overall unemployment rate and the distributions of productivity and wages?

Related Literature

- ▶ A natural approach to these questions is to incorporate public-sector employment into an equilibrium search and matching model. Surprisingly, there are few papers that do this.
- ▶ Burdett (2010) - Adds a public sector to the Burdett-Mortensen (IER 1998) model of on-the-job search.
- ▶ Quadrini and Trigari (ScanJE 2007) – Pissarides (2000) model in which private-sector productivity varies stochastically over time.
- ▶ Neither paper (nor any others that we know about) allows for worker heterogeneity.

Model - Basic Ingredients

- ▶ Worker heterogeneity – $Y \sim F(y)$ interpreted as human capital – from ANV (EJ 2009)
- ▶ Match-specific productivity – $X \sim G_s(x|y)$, $s = \{p, g\}$ with a first-order stochastic dominance assumption, i.e., $y' > y \Rightarrow G_s(x|y') < G_s(x|y)$. A similar idea (with a 2-point distribution for Y) can be found in Dolado, Jansen and Jimeno (BEJournal Macro 2007)
- ▶ The public sector posts v_g vacancies (exogenous). We formulate the hiring decision as a threshold rule, $x \geq \xi_g(y)$, and assume that wages are set according to an exogenous rule, $w_g(x, y)$.

Model - Assumptions

- ▶ Random search – only by the unemployed, no on-the-job search
- ▶ The rate at which unemployed workers meet prospective employers is given by an exogenous contact function, $m(\theta)$, where $\theta = (v_p + v_g)/u$. The fraction of those contacts that are with private-sector vacancies is $\phi = v_p/(v_p + v_g)$. The rate at which prospective employers meet job seekers is $m(\theta)/\theta$
- ▶ Labor market tightness is endogenous – there is a free-entry condition for private-sector vacancy creation

Model - Assumptions

- ▶ Not every contact leads to a match. We derive a reservation acceptance threshold for private-sector jobs for type- y workers. Public sector matches form only if $x \geq \xi_g(y)$
- ▶ Private-sector wages are determined by Nash bargaining; public-sector wages are given by $w_g(x, y)$
- ▶ Steady-state analysis

Value Functions - Workers

- ▶ $U(y)$, $N_p(x, y)$, and $N_g(x, y)$ are the values for a worker of type y associated with unemployment and with private-sector and public-sector jobs with match-specific productivity x .

$$\begin{aligned} rU(y) = & z + \phi m(\theta) E \max[N_p(x, y) - U(y), 0] \\ & + (1 - \phi) m(\theta) E \max[N_g(x, y) - U(y), 0] \end{aligned}$$

$$rN_p(x, y) = w_p(x, y) + \delta_p(U(y) - N_p(x, y))$$

$$rN_g(x, y) = w_g(x, y) + \delta_g(U(y) - N_g(x, y))$$

Value Functions - Firms

- ▶ $J(x, y)$ is the value associated with a private-sector job filled by a worker of type y with match-specific productivity x . V is the value of posting a private-sector vacancy

$$rJ(x, y) = x - w_p(x, y) + \delta_p(V - J(x, y))$$

$$rV = -c + \frac{m(\theta)}{\theta} E \max[J(x, y) - V, 0]$$

- ▶ The expectation is taken with respect to the joint distribution of (X, Y) across the unemployed

Wages

- ▶ Free entry $\Rightarrow V = 0$ in steady state. With $V = 0$, the Nash bargaining solution with exogenous worker share β gives

$$w_p(x, y) = \beta x + (1 - \beta)rU(y)$$

- ▶ Again, $w_g(x, y)$ is exogenous

Reservation Thresholds

- ▶ $R_p(y)$ is the value of x such that $N_p(x, y) = U(y)$. Using $N_p(R_p(y), y) = U(y)$, the Nash bargaining solution implies $R_p(y) = rU(y)$
- ▶ The reservation threshold for a public-sector job is $R_g(y) = \zeta_g(y)$. This is equivalent to assuming that $N_g(\zeta_g(y), y) \geq U(y)$

More on Reservation Thresholds

- To derive a recursion for $R_p(y)$, we rewrite our expression for $rU(y)$. This gives

$$\begin{aligned} R_p(y) = & z + \phi m(\theta) \frac{\beta}{r + \delta_p} \int_{R_p(y)}^{\bar{x}} (1 - G_p(x|y)) dx \\ & + (1 - \phi) m(\theta) \frac{1}{r + \delta_g} \int_{\xi_g(y)}^{\bar{x}} (w_g(x, y) - R_p(y)) dG_g(x|y) \end{aligned}$$

- Given θ and ϕ , the reservation thresholds are uniquely determined

Free Entry

- ▶ The free-entry condition for private-sector vacancy creation is

$$c = \frac{m(\theta)}{\theta} E \max[J(x, y), 0]$$

Let $f_u(y)$ be the density of y across the unemployed. Substituting for $J(x, y)$ gives

$$c = \frac{m(\theta)}{\theta} \left(\frac{1 - \beta}{r + \delta_p} \right) \int_{\underline{y}}^{\bar{y}} \int_{R_p(y)}^{\bar{x}} (1 - G_p(x|y)) dx f_u(y) dy$$

Steady-State Conditions

- By Bayes Law

$$f_u(y) = \frac{u(y)f(y)}{u}$$

where $u(y)$ is the type- y unemployment rate and $u = \int u(y)f(y)dy$ is the overall unemployment rate.

- The type-specific unemployment rates are derived from steady-state conditions – the flow of type- y workers from unemployment to private-sector employment must equal the flow in the reverse direction and similarly for transitions between unemployment and public-sector employment.

$$u(y) = \frac{\delta_g \delta_p}{\delta_g \delta_p + \delta_g \phi m(\theta)(1 - G_p(R_p(y)|y)) + \delta_p(1 - \phi)m(\theta)(1 - G_g(R_g(y)|y))}$$

A Final Unknown

The final unknown is ϕ , the fraction of vacancies that are posted by private-sector firms. Since $v_p + v_g = \theta u$,

$$\phi = \frac{v_p}{v_p + v_g} \Rightarrow \phi = \frac{\theta u - v_g}{\theta u}$$

Equilibrium

- ▶ An equilibrium is a function, $R_p(y)$, and a pair of scalars, θ and ϕ , that satisfy the relevant equations (private-sector matches are formed iff there is positive surplus, free-entry, steady-state)
- ▶ Equilibrium always exists. This can be seen from the free-entry condition:

$$c = \frac{m(\theta)}{\theta} \left(\frac{1 - \beta}{r + \delta_p} \right) \int_{\underline{y}}^{\bar{y}} \int_{R_p(y)}^{\bar{x}} (1 - G_p(x|y)) dx f_u(y) dy$$

- ▶ The RHS of this equation can be written as a function of θ alone. It is continuous in θ , converges to ∞ as $\theta \rightarrow 0$, and converges to zero as $\theta \rightarrow \infty$. Once we solve for θ , we can recover the other equilibrium objects.
- ▶ Note that we do not claim uniqueness

Wage Distributions

- ▶ Let $H_s(w)$ denote the distribution function of wages paid in sector s . We can develop expressions for $H_p(w)$ and $H_g(w)$ as follows. Consider first the distribution of private-sector wages across workers of type y , say $H_p(w|y)$.

$$H_p(w|y) = 0 \text{ for } w < w_p(R_p(y), y).$$

For $w < w_p(R_p(y), y)$, we have

$$\begin{aligned} H_p(w|y) &= P[w_p(R_p(y), y) \leq w_p(X, y) \leq w|y] \\ &= P[R_p(y) \leq \beta X + (1 - \beta)R_p(y) \leq w|y] \\ &= P[R_p(y) \leq X \leq \frac{w - (1 - \beta)R_p(y)}{\beta}|y] \\ &= G_p\left(\left(\frac{w - (1 - \beta)R_p(y)}{\beta}\right)|y\right) - G_p(R_p(y)|y) \end{aligned}$$

Wage Distributions

- ▶ The unconditional distribution of private-sector wages is

$$H_p(w) = \int_{\underline{y}}^{\bar{y}} H_p(w|y) f_p(y) dy,$$

where

$$f_p(y) = \frac{n_p(y)f(y)}{n_p}$$

is the density of Y across workers employed in the private sector. The private-sector employment rate for type- y workers, $n_p(y)$, is derived from steady-state conditions.

- ▶ The same approach can be used to find the distribution of public-sector wages.

What's Next?

- Specify functional forms for $F(y)$ and $G_s(x|y)$

$$Y \sim \text{unif}[0, 1]$$

$$X \sim \text{unif}[y, y + 1] \quad \ln X \sim N(y, \sigma^2)$$

- Specify functional forms for $\xi_g(y)$ and $w_g(x, y)$

$$\xi_g(y) = \alpha + R_p(y)$$

$$w_g(x, y) = \gamma x + (1 - \gamma)rU(y) = \gamma x + (1 - \gamma)R_p(y)$$

- Calibrate

- Run numerical policy experiments – change v_g , $\xi_g(y)$ and $w_g(x, y)$

- ▶ Assume $Y \sim \text{unif}[0, 1]$ and $X \sim \text{unif}[y, y + 1]$.
- ▶ Assumed parameter values:

r	0.012
z	0.3
c	0.3
β	0.75
$m(\theta)$	$3\theta^{0.25}$
δ_p	0.1
δ_g	0.05
v_g	0.0125
α	0
γ	0.5

- ▶ Results: $\theta = 0.95$, $u = 0.094$, $\phi = 0.86$