Lagos-Wright vs. Cash-in-Advance: Government Policy Response to War-Expenditure Shocks

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Abstract

I pit the Lagos-Wright micro founded model of money with the reduced-form cash-in-advance model in terms of their predictions for the response of government policy to war-expenditure shocks. The Lagos-Wright model performs well qualitative and quantitatively. Depending on specific assumptions about preferences, the cash-in-advance model either has some qualitative or quantitative shortcomings.

Keywords: government policy, lack of commitment, war shocks, micro founded models of money.

JEL classification: E13, E52, E62, E63, H60

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1 Introduction

There are currently two alternatives to modeling monetary economies. On the one hand, there are reduced-form models that *assume* agents' demand for fiat money. On the other hand, there are models that are explicit about the frictions that give rise to a demand for a medium of exchange. The most widely adopted framework in the reduced-form approach is the cash-in-advance (CIA) model due to Clower (1967) and later popularized by Lucas (1980) and Lucas and Stokey (1987).[1] The modern micro founded approach, which can be traced as far back as Jones (1976), has seen rapid development in recent time, following the work by Kiyotaki and Wright (1989). Current state-of-the-art models mostly build on the framework proposed by Lagos and Wright (2005) (LW).[2]

Arguably, micro founded models of money carry a theoretical advantage by virtue of presenting an explicit and logically consistent environment. Its advocates (e.g., see Wallace, 1998, 2001, Shi, 2006 and Williamson and Wright, 2010) argue that many key results and interesting insights are missed by adopting a reduced-form approach. Furthermore, policy recommendations may be sensitive to the details of the environment and, in this sense, micro founded models of money address a version of the *Lucas critique* (Lucas, 1976). However, at the end of the day, many—if not all—results derived from explicitly micro founded models can be replicated in comparatively simpler reduced-form models under suitable modifications. In addition, it is not always the case that richer environments are more empirically plausible. E.g., within the context of the LW framework, Martin (2010) shows that a variant with competitive markets explains actual U.S. government policy better than a variant with bilateral trade and bargaining.

One could (and possibly should) wonder about the empirical merits of the models representing each approach. In this paper, I evaluate the empirical plausibility of the LW and CIA models by studying the response of government policy to wartime expenditure shocks.[3] The advantage of using war episodes as natural experiments for this purpose is that they provide a set of well-established and quantitatively significant facts. To model government policy, I consider a benevolent government that cannot commit to future policy choices and uses money, nominal bonds and distortionary taxes to finance the provision of a valued public good, following my previous work in Martin (2009, 2010, 2011).

I find that the LW model matches all (identified) stylized facts of U.S. wartime policy and performs well quantitatively. The performance of the CIA model depends critically on our assumptions about preferences. For a benchmark case—which assumes linear utility in the "credit" good and shares some key analytical properties with the LW model—the CIA model provides results similar to the LW model, but gets one fact wrong: it predicts a decrease rather than an increase in real GDP during wartime. I then consider alternative preference specifications that allow the CIA model to match all qualitative facts. However, in all these cases the quantitative performance is not as good as the LW model. Specifically, the CIA model significantly underpredicts the response of the primary deficit and real GDP.

[1] Alternative reduced-form monetary frameworks are due to Baumol (1952), Tobin (1956), Sidrauski (1967) and Brock (1974).

[2] See Shi (1997) for an alternative model.

[3] In a different context, Aruoba and Schorfheide (2010) evaluate the empirical performance of a variant of the LW model by comparing it to a money-in-the-utility function model (MIU) and a vector autoregression (VAR). In their case, the LW model performs the worst.

The paper is organized as follows. Section 2 presents the two models and characterizes government policy for each case. Section 3 conducts the numerical evaluation. Section 4 concludes.

2 Models

This section characterizes government policy in the presence of expenditure shocks for the LW and CIA frameworks. Both these models of government policy have been analyzed extensively elsewhere, so the exposition here skips some details of the derivation.[4]

Common to both economies is the existence of a continuum of infinitely lived agents and a benevolent government that supplies a valued public good g.[5] To finance its expenditure, the government may use proportional labor taxes τ, print fiat money at rate μ and issue one-period nominal bonds, which are redeemable in fiat money. Agents derive utility from the public good according to $\psi v(g)$, where v is twice continuously differentiable, with $v_g > 0 > v_{gg}$, and ψ is a random variable following a Markov-process. Let Ψ be the (compact) set of all possible realizations of ψ and use $E[. \mid \psi]$ to denote the conditional expectation of future variables given the current state ψ.

All nominal variables—except for bond prices—are normalized by the aggregate money stock. Thus, today's aggregate money supply is equal to 1 and tomorrow's is $1 + \mu$. The government budget constraint is

$$1 + B + pg = p\tau n + (1 + \mu)(1 + qB'), \tag{1}$$

where B is the current aggregate bond-money ratio, p is the—normalized—market price of output, n is labor (as specified by each model) and q is the discount price of a one-period bond that earns a unit of fiat money. "Primes" denote variables evaluated in the following period. Thus, B' is tomorrow's aggregate bond-money ratio.

The government lacks the ability to commit to future policy choices and announces policy $\{B', \mu, \tau, g\}$ at the beginning of each period, after observing the contemporaneous realization of ψ. Let $\Gamma \in [-1, \bar{B}]$ be the set of possible debt levels, where $\bar{B}$ is large enough so that it does not constraint government behavior.

2.1 Lagos-Wright

In each period, two competitive markets open in sequence: a "day" and a "night" market. At the beginning of the period, agents receive an idiosyncratic shock that determines their role in the day market. With probability $\eta \in (0, 1)$ an agent wants to consume but cannot produce the (perishable) day-good, x, while with probability $1 - \eta$ an agent can produce but does not want consume. A *consumer* derives utility $u(x)$, where u is twice continuously differentiable, with $u_x > 0 > u_{xx}$. A *producer* incurs in utility cost x.[6] Agents lack commitment and are anonymous, in the sense that

[4]See Martin (2009, 2010, 2011).

[5]The assumption that g is valued is made for analytical convenience. We can conduct the same type of analysis using exogenous expenditure.

[6]The disutility from producing in the day can be generally written as a convex function of x. A linear specification reduces the number of parameters that we need to calibrate and makes the comparison to a CIA model more straightforward.

their trading histories are unobservable. Thus, credit transactions between agents are not possible. Since there is a lack of double coincidence of wants problem, some medium of exchange is essential for trade to occur.[7]

Suppose there exist financial institutions ("banks") endowed with a technology that allows them to record financial (but not trade) histories at zero cost, as in Berentsen, Camera and Waller (2007).[8] Banks cannot issue their own notes, nor can they provide third-party verification for government bonds in transactions between agents. At the beginning of each day, sellers can deposit their money holdings at banks, and buyers can borrow money from banks. Deposits d and loans l mature at night. Perfect competition in the banking sector implies that the deposit and loan interest rates are equal. Let $i \geq 0$ be the bank nominal interest rate. Assume perfect enforcement and no borrowing constraints in financial markets.

At night, all agents can produce and consume the (perishable) night-good, c. The production technology is assumed to be linear in hours worked, n. Utility from consumption is given by $U(c)$, where U is twice continuously differentiable, with $U_c > 0 > U_{cc}$. Disutility from labor is given by αn, where n is hours worked and $\alpha > 0$.

The government announces period policy at the beginning of the day, before agents' idiosyncratic shocks are realized. The government only actively participates in the night-market, i.e., taxes are levied on hours worked at night and open market operations are conducted in the night market. As in Aruoba and Chugh (2010), Berentsen and Waller (2008) and Martin (2011), public bonds are book-entries in the government's record. Since bonds are not physical objects and the government does not participate in the day market (i.e., cannot intermediate or provide third-party verification, which also prevents banks from intermediating debt), bonds are not used as a medium of exchange in the day market and thus, fiat money is essential.

An agent starts the period with individual money balances m and government bonds b. The ex-ante value for an agent that enters the day market is $V(m, b, B, \psi) = \eta V^c(m, b, B, \psi) + (1 - \eta)V^p(m, b, B, \psi)$, where $\{B, \psi\}$ is the aggregate state, and V^c and V^p are the values of being a consumer and a producer in the day market, respectively. Since bonds and bank claims are redeemed in fiat money at par, the composition of an agent's nominal portfolio at the beginning of the night is irrelevant. Note that bank claims at the beginning of the night market may be positive (deposits) or negative (loans) and include the accrued interest. Let $W(z, B, s)$ be the value for an agent that starts the night market with z net nominal assets. To simplify exposition, let us omit the dependence of prices and policy from aggregate state variables.

The problem of a consumer in the day is

$$V^c(m, b, B, \psi) = \max_{x,l} u(x) + W(m + b - \tilde{p}x - il, B, \psi).$$

subject to $m + l - \tilde{p}x \geq 0$, where $\tilde{p}$ is the normalized price of the day-good.

[7] See Kocherlakota (1998), Wallace (2001) and Shi (2006).

[8] Alternatively, we could assume that this type of financial institutions do not exist and/or that the day market is decentralized and subject to trading frictions. Martin (2010) shows that all these alternative variants of the LW framework display very similar—both qualitative and quantitative—government policy response to expenditure shocks. The particular variant adopted here has the advantage of being analytically the most similar to a CIA model. In addition, this variant eliminates the quantitative role of η, which facilitates comparison with CIA models.

Let κ be an individual producer's output of the day-good. The problem of a producer is

$$V^p(m, b, B, \psi) = \max_{\kappa, d} -\kappa + W(m + b + \tilde{p}\kappa + id, B, \psi)$$

subject to $m - d \geq 0$.

The problem of an agent at night is

$$W(z, B, \psi) = \max_{c,n,m',b'} U(c) - \alpha n + \psi v(g) + \beta E[V(m', b', B', \psi') \mid \psi].$$

subject to $pc + (1 + \mu)(m' + qb') = p(1 - \tau)n + z$.

The derivation of the monetary equilibrium is standard and thus, omitted. See Martin (2010). A key result in the Lagos-Wright framework is that, in a symmetric equilibrium, all agents exit the night market with the same money and bond balances, i.e., $m' = 1$ and $b' = B'$. The resource constraint in the day, $\eta x = (1 - \eta)\kappa$ and night, $c + g = n$, can be used to remove κ and n from the problem. Equilibrium in the day-financial market implies $d = 1$ and $l = \frac{\eta}{1-\eta}$.

A standard procedure is to use the equilibrium conditions to write policy variables and prices in terms of allocations. In a monetary equilibrium we get

$$\mu = \frac{\beta E[u'_x x' \mid \psi]}{x} - 1 \quad (2)$$

$$\tau = 1 - \frac{\alpha}{U_c} \quad (3)$$

$$\tilde{p} = \frac{1}{\eta x} \quad (4)$$

$$p = \frac{U_c}{\eta x} \quad (5)$$

$$q = \frac{1}{E[u'_x \mid \psi]}. \quad (6)$$

There is an additional restriction, $u_x - 1 \geq 0$, which comes from the requirement that the Lagrange multiplier associated with the day-consumer's budget constraint be non-negative. In equilibrium, it translates into a requirement that the nominal interest rate be non-negative. Martin (2011) shows that this constraint does not bind for any $B \geq -1$, so we can ignore it when we write the government's problem.

Using conditions (2)—(6), the government budget constraint (1) becomes

$$U_c c - \alpha(c + g) + \beta\eta E[x'(u'_x - 1) \mid \psi] + \beta\eta E[x' \mid \psi](1 + B') - \eta x(1 + B) = 0. \quad (7)$$

Note that x' is implemented by the government tomorrow, depending on the inherited level of debt and the realization of the aggregate shock. Thus, let $x' = \mathcal{X}(B', \psi')$, where $\mathcal{X}(B, \psi)$ is the policy that the current government anticipates its future-self to follow.

Given the perception that future governments implement $\mathcal{X}(B, \psi)$, the problem of the current government is

$$\mathcal{V}(B, \psi) = \max_{B',x,c,g} \eta(u(x) - x) + U(c) - \alpha(c + g) + \psi v(g) + \beta E[\mathcal{V}(B', \psi') \mid \psi]$$

subject to (7).[9] Let λ be the Lagrange multiplier associated with the government budget constraint. The first-order conditions imply

$$\begin{aligned} E\left[x'(\lambda-\lambda')+\lambda(u'_x+u'_{xx}x'+B')\mathcal{X}'_B \mid \psi\right] &= 0 \\ u_x-1-\lambda(1+B) &= 0 \\ U_c-\alpha+\lambda(U_c+U_{cc}c-\alpha) &= 0 \\ -\alpha+\psi v_g-\lambda\alpha &= 0. \end{aligned}$$

Note the presence of the derivative of the equilibrium function $\mathcal{X}(B,\psi)$ in the first equation above. This reflects the time-consistency problem, as the government tomorrow will not internalize how its actions affected current behavior—see Martin (2011) for further analysis. From the second equation, we can verify that if $\lambda>0$ then $u_x-1\geq 0$ does not bind for all $B\geq -1$, as stated. From the fourth equation above, we get $\lambda=\frac{\psi v_g}{\alpha}-1$. Thus,

$$E\left[x'(\psi v_g-\psi' v'_g)+(\psi v_g-\alpha)(u'_x+u'_{xx}x'+B')\mathcal{X}'_B \mid \psi\right] = 0 \tag{8}$$

$$\alpha(u_x-1)-(\psi v_g-\alpha)(1+B) = 0 \tag{9}$$

$$\psi v_g(U_c-\alpha)+(\psi v_g-\alpha)U_{cc}c = 0. \tag{10}$$

Definition 1 *A Markov-perfect equilibrium in the LW-economy is a set of functions* $\{\mathcal{B}(B), \mathcal{X}(B), \mathcal{C}(B), \mathcal{G}(B)\}$ *that solve* (7)—(10) *for all* $B\in\Gamma$ *and* $\psi\in\Psi$.

Equation (8) features the main mechanism in the model. The term $(\psi v_g-\psi' v'_g)$ is the standard trade-off between distortions today and tomorrow. The government has an incentive to smooth these distortions over time, i.e., minimize this wedge. The term $(u'_x+u'_{xx}x'+B')\mathcal{X}'_B$ appears due to the lack of commitment friction and reflects how changes in future policy affect current decisions. Changing the debt level today affects future (monetary) policy and this, in turn affects the current government budget constraint through two channels: the current demand for money and the financial cost of debt. If these effects amount to an overall relaxation (tightening) of the budget constraint, then the government has an incentive to increase (decrease) debt.[10]

2.2 Cash-in-advance

Consider a variant of the cash-credit good model of Lucas and Stokey (1983) and Lucas and Stokey (1987), as recently analyzed by Martin (2009) in the context of a model of government policy. In contrast to the LW model, there is only one production sector. Labor n produces output through a linear technology, which can be transformed one-to-one into a "cash" good x, a "credit" good c and the public good g. Cash-goods can only be purchased with fiat money balances brought from the previous period, as in Svensson (1985). Let p be the market price of output in terms of fiat money, normalized by the aggregate money stock.

The cash-in-advance constraint is typically motivated by assuming that households are composed of a producer-shopper pair. The producer stays at home, while the shopper takes the household's money holdings to buy goods from other producers. The cash-credit variant allows for only

[9]Note that in the government's objective, we simplify the expected day-utility, $\eta u(x)-(1-\eta)\kappa$ by using the day-market clearing condition, $\eta x=(1-\eta)\kappa$.

[10]See Martin (2010, 2011) for further explanation and analysis of this mechanism.

a subset of consumption goods to be necessarily bought with fiat money. One way to rationalize this variant is to assume that producers know some of the shoppers and are willing to sell on credit, payable at the end of period. When the shopper is unknown, sellers request immediate settlement in currency. The "Sevensson" timing of the cash-in-advance constraint can be obtained by assuming that the goods and asset markets open in sequence, with the goods market opening first.

Assume preferences are separable.[11] Specifically, the agent derives utility $u(x)$ from the cash-good, $U(c)$ from the credit good and $h(\ell)$ from leisure, where $\ell = 1 - n$. Assume $u_x > 0 > u_{xx}$, $U_c > 0 \geq U_{cc}$ and $h_\ell > 0 > h_{\ell\ell}$. Note that we are allowing for linear utility in the credit good. Let $V(m, b, B, \psi)$ be the value function associated with starting the period with money balances m, bonds b and aggregate state $\{B, \psi\}$.

The problem of the agent is

$$V(m, b, B, \psi) = \max_{m', b', x, c, n} u(x) + U(c) + h(1-n) + \psi v(g) + \beta E[V(m', b', B', \psi') \mid \psi]$$

subject to

$$\begin{aligned} px + pc + (1+\mu)(m' + qb') &= (1-\tau)pn + m + b \\ px &\leq m, \end{aligned}$$

where the first constraint above is the agent's budget constraint and the second is the cash-in-advance constraint.

In a monetary equilibrium, $m' = 1$, $b' = B'$ and the cash-in-advance constraint holds with equality. The resource constraint is $x + c + g = n$. Using the first-order conditions of the agent's problem, we can write prices and policy variables as functions of allocations. We get

$$\mu = \frac{\beta u_x' x'}{u_c x} - 1 \quad (11)$$

$$\tau = 1 - \frac{u_\ell}{u_c} \quad (12)$$

$$p = \frac{1}{x} \quad (13)$$

$$q = \frac{u_c'}{u_x'} \quad (14)$$

There is also the restriction $u_x - U_c \geq 0$, which derives from the requirement that the Lagrange multiplier of the cash-in-advance constraint be non-negative. In contrast with the LW environment, this constraint may bind here for values of debt larger than -1.

Using conditions (11)—(14), the government budget constraint (1) becomes

$$U_c(x+c) - h_\ell(x+c+g) + \beta E[x'(u_x' - U_c') \mid \psi] + \beta E[U_c' x' \mid \psi](1+B') - U_c x(1+B) = 0. \quad (15)$$

Note that the equation above features both x' and c', which are implemented by the government tomorrow. Let $x' = \mathcal{X}(B', \psi')$ and $c' = \mathcal{C}(B', \psi')$ be the policies that the current government anticipates its future-self to follow.

[11] As argued in the calibration section, this assumption is critical for matching the fact that primary deficits are decreasing in debt.

Given the perception that future governments implement $\mathcal{X}(B,\psi)$ and $\mathcal{C}(B,\psi)$, the problem of the current government is

$$\mathcal{V}(B,\psi) = \max_{B',x,c,g} \; u(x) + U(c) + h(1-x-c-g) + \psi v(g) + \beta E[\mathcal{V}(B',\psi') \mid \psi]$$

subject to (15) and $u_x - U_c \geq 0$. Let λ and ζ be the Lagrange multipliers associated with the government budget and non-negativity constraints, respectively. The first-order conditions imply

$$E\left[u_c' x'(\lambda - \lambda') + \lambda\{(u_x' + u_{xx}' x' + U_c' B')\mathcal{X}_B' + U_{cc}' x' B' \mathcal{C}_B'\} \mid \psi\right] = 0 \quad (16)$$

$$u_x - h_\ell + \lambda\{-h_\ell + h_{\ell\ell}(x+c+g) - U_c B\} + \zeta u_{xx} = 0 \quad (17)$$

$$U_c - h_\ell + \lambda\{U_c + U_{cc}(c - xB) - h_\ell + h_{\ell\ell}(x+c+g)\} - \zeta U_{cc} = 0 \quad (18)$$

$$-h_\ell + \psi v_g + \lambda\{-h_\ell + h_{\ell\ell}(x+c+g)\} = 0. \quad (19)$$

As shown in (16), the time-consistency problem generally derives from policy channels affecting both cash and credit goods. In contrast, the government in the LW model was only affected by the response of future policy in the allocation of the day-good. In spite of these differences, the mechanism that drives the change in debt is essentially the same in both models. On the one hand, there is an incentive to smooth distortions over time; on the other hand, how future governments react to changes in inherited debt, affects the current incentives of the government to issue debt.

From (17) and (18) we get

$$u_x - U_c + \zeta(u_{xx} + U_{cc}) = \lambda\{U_c(1+B) + U_{cc}(c - xB)\}. \quad (20)$$

Given $\lambda > 0$ and since $u_{xx} + U_{cc} < 0$, we get $\zeta = 0$ (i.e., $u_x - U_c \geq 0$ does not bind) if and only if $U_c(1+B) + U_{cc}(c - xB) \geq 0$. Note that if $U(c)$ is linear, then $\zeta = 0$ for all $B \geq -1$, as in the LW economy. In the numerical analysis below, we will use this property to settle on a benchmark calibration.

Since we need to allow for the possibility that the non-negativity constraint binds, consider the following equilibrium definition, which includes the equilibrium functions for the Lagrange multipliers, $\Lambda(B)$ and $\mathcal{Z}(B)$.

Definition 2 *A Markov-perfect equilibrium in the CIA-economy is a set of functions* $\{\mathcal{B}(B), \mathcal{X}(B), \mathcal{C}(B), \mathcal{G}(B), \Lambda(B), \mathcal{Z}(B)\}$ *that solve* (15)—(19), *with* $\Lambda(B) \geq 0$, $\mathcal{Z}(B)(u_x - U_c) = 0$, $\mathcal{Z}(B) \geq 0$, $u_x - U_c \geq 0$, *for all* $B \in \Gamma$ *and* $\psi \in \Psi$.

One property of these models is that the incentives to inflate increase with the level of debt. These can be easily seen in the right-hand-side of (20), where the direct effect of higher debt is to slacken the non-negativity constraint. In the numerical section, we will target positive inflation in the long-run (i.e., $\mu > 0$), which by (11) implies the non-negativity constraint will not bind in steady state. Thus, consider a parameterization for which there exists a set $\hat{\Gamma} \subset \Gamma$ such that $\mathcal{Z}(B) = 0$ for all $B \in \hat{\Gamma}$. Then, use (19) to solve for λ. A Markov-perfect equilibrium is then a set of functions $\{\mathcal{B}(B), \mathcal{X}(B), \mathcal{C}(B), \mathcal{G}(B)\}$ that solve (15)—(18) for all $B \in \hat{\Gamma}$ and $\psi \in \Psi$.

3 Numerical Analysis

3.1 Calibration

Consider the following functional forms:

$$\begin{aligned} u(x) &= \varphi \frac{x^{1-\sigma}-1}{1-\sigma} \\ U(c) &= \gamma \frac{c^{1-\rho}-1}{1-\rho} \\ h(\ell) &= \alpha \ln \ell \\ v(g) &= \ln g. \end{aligned}$$

The calibration approach follows Martin (2010), which calibrates several variants of the LW model. Target statistics are taken from 1962-2006 averages for the U.S. economy. Period length is set to a year. Government in the model corresponds to the federal government. The calibration targets are: debt over GDP, annual inflation, interest payment over GDP, outlays (excluding interest) over GDP and revenues over GDP. Inflation is measured from the CPI, while the rest of the variables are taken from the Congressional Budget Office. Government debt is defined as debt held by the public, excluding holdings by the Federal Reserve system.

For debt over GDP use $\frac{B^*(1+\mu^*)}{Y^*}$, where Y^* is steady state nominal GDP normalized by the aggregate money stock (and equal to velocity of circulation by the "equation of exchange"); note the use of $B^*(1+\mu^*)$ instead of just B^* since debt is measured at the end of the period in the data. In the LW economy, $Y^* = \eta\tilde{p}^*x^* + p^*(c^*+g^*) = 1+p^*n^*$; in the CIA economy, $Y^* = p^*(x^*+c^*+g^*) = p^*n^*$. Let π^* be steady state annual inflation, which in both models is equal to μ^*. Interest payments over GDP are defined as $\frac{B^*(1+\mu^*)(1-q^*)}{Y^*}$. Given that debt over GDP is already targeted, this implies a target for the nominal interest rate i^*, where $i^* = \frac{1}{q^*} - 1$. Interest payments are 2.1% of GDP in the data, which implies a target nominal interest rate of 7.3% annual. Outlays and revenues are defined as $\frac{p^*g^*}{Y^*}$ and $\frac{p^*\tau^*n^*}{Y^*}$, respectively. Note that this calibration approach implies the same values for B^* and Y^* in both models. Table 1 displays the target statistics.

Table 1: Target statistics

$\frac{B^*(1+\mu^*)}{Y^*}$	π^*	i^*	$\frac{p^*\tau^*n^*}{Y^*}$	$\frac{p^*g^*}{Y^*}$
0.308	0.044	0.073	0.182	0.182

One of the challenges in models where the government lacks commitment is the presence of the derivatives of equilibrium functions in the government's first-order conditions—see (8) and (16). However, as shown in Martin (2010) and Martin (2011), the steady state of the LW model can be solved locally. To see this, note that (8) in a distortionary steady state (i.e., where $\lambda^* > 0$ which implies $\psi v_g^* - \alpha > 0$) becomes $(u_x^* + u_{xx}^* x^* + B^*)\mathcal{X}_B^* = 0$. Martin (2011) shows that there exists a unique steady state with the property $B^* > -1$, $\mathcal{X}_B^* < 0$ and $\mu^* > \beta - 1$. All other steady states (there is a continuum of them) feature $B^* < -1$, $\mathcal{X}_B^* = 0$ and $\mu^* = \beta - 1$ (i.e., the Friedman rule).

We focus on the former steady state, $B^* > -1$, since we are conducting positive analysis, where long-run debt and inflation are positive. Thus, given the above functional forms, we get

$$B^* = \frac{\sigma - 1}{x^{*\sigma}}. \tag{21}$$

A steady state in the LW economy is an allocation $\{B^*, x^*, c^*, g^*\}$ satisfying (7), (9), (10) and (21).

Similarly, Martin (2009) shows that the CIA model admits a local solution if we assume $U(c)$ is linear. Thus, assume $\rho = 0$, i.e., $U(c) = \gamma c$. As mentioned above, this assumption implies the non-negativity constraint never binds (i.e., $\zeta = 0$). From equation (16) the term multiplying $\mathcal{C}'_B$ cancels. Thus, in steady state, focusing again on the case $\lambda^* > 0$ and $\mathcal{X}^*_B < 0$, we get

$$B^* = \frac{\sigma - 1}{\gamma x^{*\sigma}}, \tag{22}$$

which is remarkably similar to (21).[12] A steady state in the CIA economy with $U(c) = \gamma c$, is an allocation $\{B^*, x^*, c^*, g^*\}$ satisfying (15), (17), (18) and (22), where $\lambda^* = (1 - x^* - c^* - g^*)(\frac{1 - x^* - c^* - g^*}{\alpha g^*} - 1)$ and $\zeta^* = 0$.

Table 2 displays the parameters that match the calibration targets for each model. As we can see β, σ and ψ are the same in both models. The discount factor is the same since it targets the nominal interest rate, given the target for inflation. The value for ψ is normalized to 1. The curvature of the $u(x)$ function, σ, is the same given the analytical similarities between the two models—see (21) and (22)—and is another reason for the choice of LW variant with banks. In this sense, if we had dispensed with the existence of banks, the value for σ in the LW model would have been significantly higher.

Table 2: Benchmark calibration

Parameters	LW	CIA
α	4.1722	0.5342
β	0.9728	0.9728
γ	1.0000	5.9982
ρ	8.1879	0.0000
σ	2.5084	2.5084
φ	10.7525	0.0674
ψ	1.0000	1.0000
η	0.5000	—

There is one parameter which is idiosyncratic to the LW model: η, the measure of day-consumers, which is arbitrarily set at 0.5. The specific value of η turns out not to matter (again, this would not be the case in a variant without banks); the simulations conducted below are virtually identical for any value of η (with φ correspondingly recalibrated to match target statistics).

[12] This is one of the reasons why the LW variant with banks appears more suitable than others. Without financial intermediaries in the day-market, the equation for long-run debt in the LW model would be $B^* = \frac{\eta(\sigma-1)}{x^{*\sigma}} - 1 + \eta$. See Martin (2010).

The values of α, γ and φ are quite different between the two models (note that γ has been normalized to 1 in the LW model); these parameters weight the contribution of each good to utility and reflect the (very) different objective functions of agents in the two economies.

Figure 1 shows select equilibrium policy variables for different levels of debt, for the benchmark calibration with no uncertainty. The Markov-perfect equilibrium for each monetary economy is solved following the global method described in the appendix. As we can see, the benchmark calibration implies very similar policies in the two models. Both the money growth rate and tax revenue over GDP are increasing in debt, whereas expenditure over GDP is relatively flat. One implication is that the primary deficit (expenditure minus tax revenue) is decreasing in debt, a feature of the U.S. data, as shown in Bohn, 1998. In the case of the CIA model, a decreasing primary deficit justifies the assumption of separable preferences. Indeed, as shown in Martin (2009), if we assume non-separable utility between the cash and credit goods, then these goods need to be (strong) complements to match observed debt over GDP; however, this implies tax revenue is decreasing in debt and thus, primary deficit increasing rather than decreasing in debt. A related consequence is a steep money growth rate function (since revenue increases with debt, inflation is needed to finance the higher financial burden), which implies unrealistically high inflation in response to expenditure shocks. Here, assuming separable preferences resolves these problems, as shown in Figure 1 and the numerical analysis below.

A salient quantitative difference between the two monetary economies is the steepness of real GDP in terms of debt. In both cases, real GDP is decreasing in debt, as the higher financial burden implies larger distortions and thus, lower output. However, this effect is significantly more pronounced in the case of the LW model.

3.2 Stylized facts of wartime U.S. government policy

Figure 2 displays select policy variables for the U.S. between 1791 and 2006.[13] Periods with temporarily high defense outlays due to wars are highlighted: the War of 1812, the Mexican-American War, the Civil War, World War I, World War II, the Korean War, the Vietnam War and the Afghanistan—Iraq wars.[14]

Let us focus on the three episodes which involved a sizable increase in government expenditure: the Civil War and the two World Wars. These episodes posses several stylized facts, which will guide our evaluation of model performance. First, these wars were financed with a mix of debt, revenue and inflation.[15] Goldin (1980) estimates that the contribution of debt and seignorage to war financing was 91% for the Civil War, 76% for World War I and 59% for World War II, with the rest being financed with contemporaneous taxes. Second, the increase in debt is large and persistent.[16] In fact, debt displays the largest and most persistent effect from war among all policy

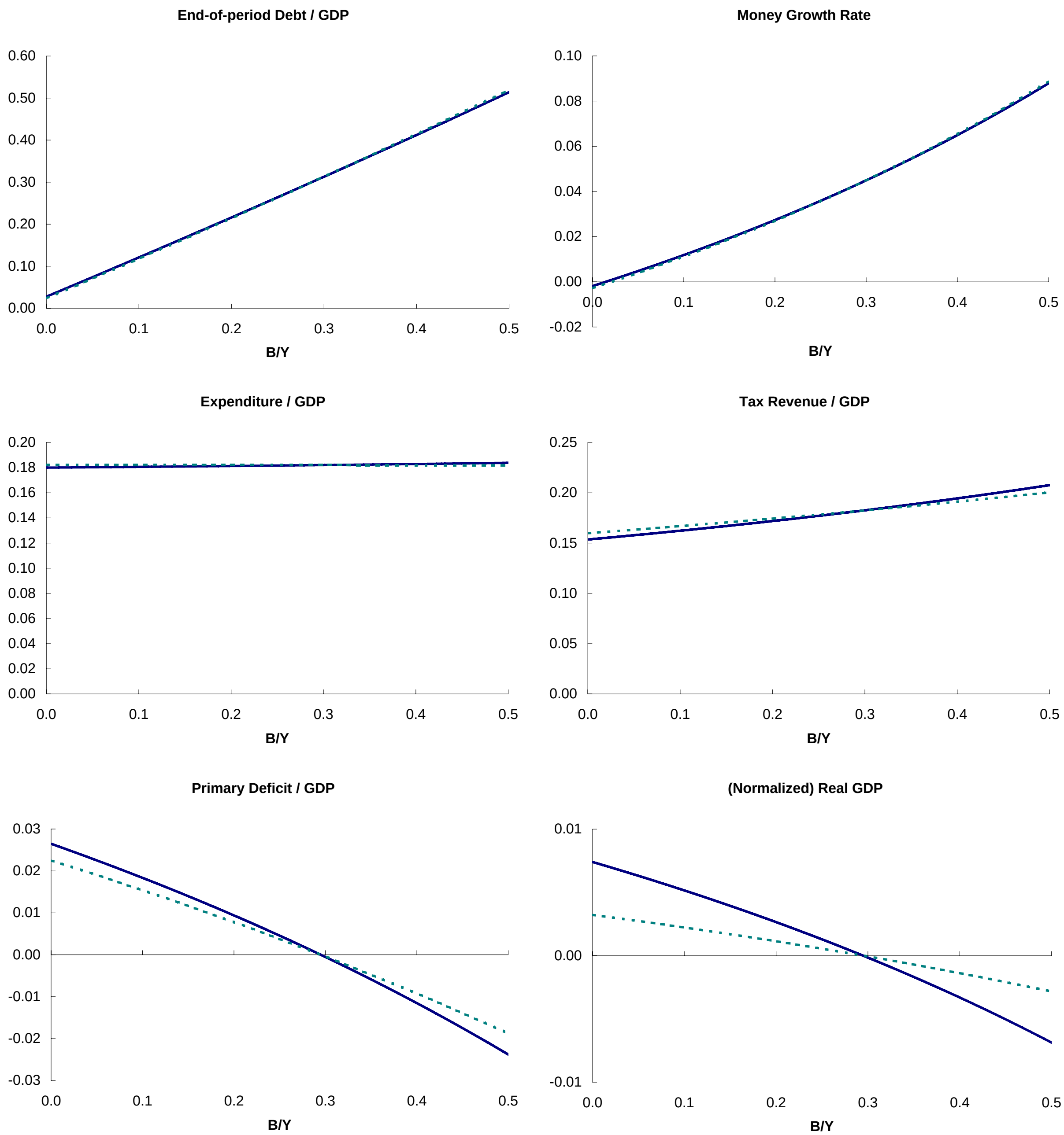
[13]The data is taken from Wallis (2006*a*), Wallis (2006*b*), Lindert and Sutch (2006), Johnston and Williamson (2008), the Bureau of Labor Statistics and the Office of Management and Budget, as described in Martin (2009).

[14]Note that for the Vietnam War, only the period 1967-1969 is highlighted as it is during these years that defense expenditure temporarily increase. Except for this brief period, defense expenditure over GDP actually decreased steadily throughout the 60s and 70s. The wars in Afghanistan and Iraq which started in 2001 and 2003, respectively, implied a small but steady increase in defense outlays—less than 1 percentage point in terms of GDP by 2006. Most of the increase in total government expenditure in the early 2000s is due to non-defense items.

[15]The Korean War, in contrast, was almost exclusively financed with contemporaneous taxes—see Ohanian (1997).

[16]See also Marcet and Scott (2009) for evidence on the persistence of debt relative to other policy variables.

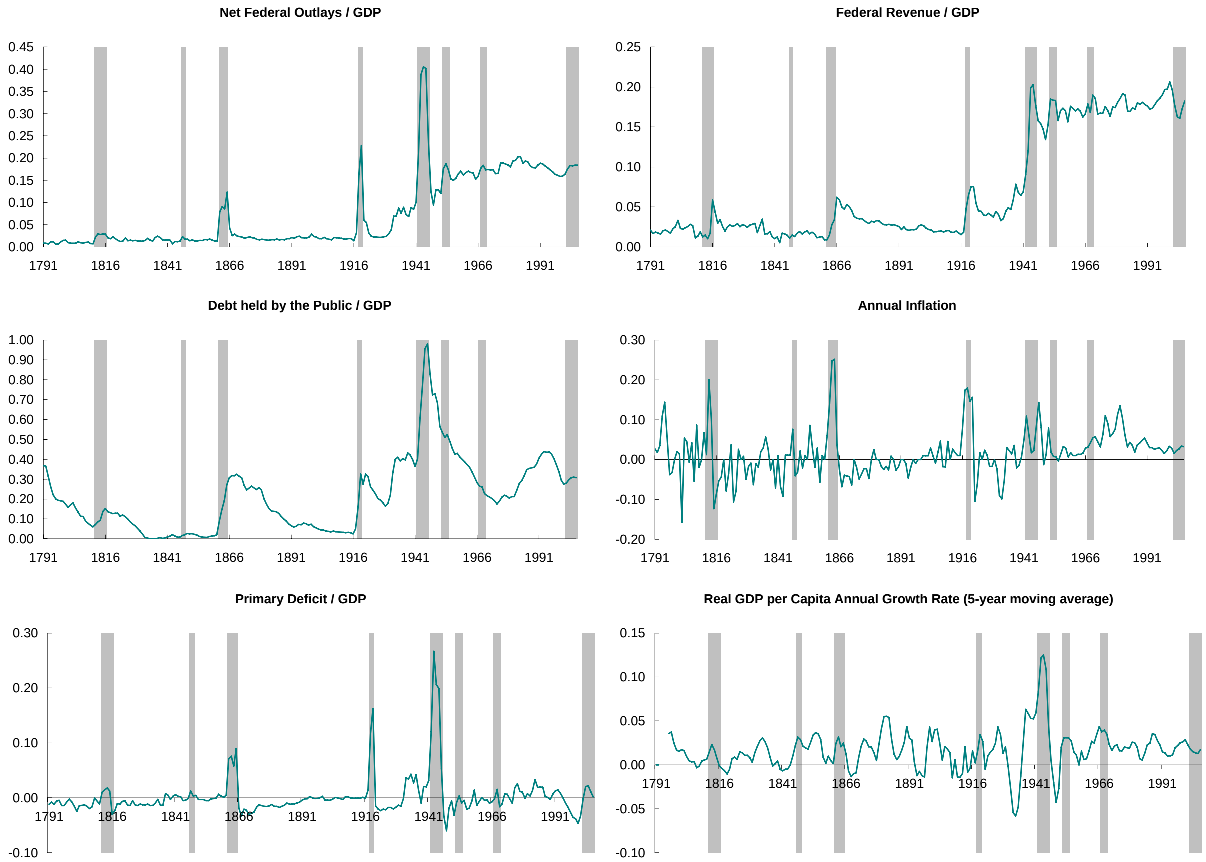
Figure 1: Non-stochastic equilibrium policies



Note: solid and dashed lines correspond to LW and CIA models, respectively; real GDP is normalized by its steady state value.

variables. Third, as a consequence of the behavior of taxes and debt, there is sharp increase in the primary deficit during wartime, followed by a post-war surplus; this implies tax revenue remains at high levels longer than expenditure. Fourth, inflation partly finances the temporary increase in expenditure and may also be used to reduce accumulated debt. Ohanian (1998) identifies the

Figure 2: U.S. government policy: 1791-2006



Note: periods with temporarily high defense outlays due to wars are highlighted.

recurring pattern of significant inflation financing of war expenditure. Note that World War II displays a significant post-war inflation (1946-1948) which, according to Ohanian's calculations, implemented a repudiation of debt equivalent to about 40% of GNP. Fifth, real GDP increases during wars.[17] For example, the average real GDP per capita annual growth rate was 1.4% between 1791 and 1929; this figure was about 2.5% during the Civil War and World War I. Real GDP per capita grew 2.7% annually on average between 1933 and 2006; during World War II, growth was 10.9% annual.

Table 3 focuses on the three main wars discussed above and presents an alternative way to analyze policy response to war-shocks. For each policy variable, the table displays the difference between the peak value during wartime and the average of the preceding five years. For GDP per capita, three measures are presented, each for a different de-trending method: linear, HP-filter and HP-filter with the smoothing parameter suggested by Ravn and Uhlig (2002).

[17] Ohanian (1997) documents this fact for World War II and the Korean War.

Table 3: War peak vs 5-year pre-war average

	Civil War	World War I	World War II
Policy variable			
Outlays / GDP	0.108	0.208	0.322
Revenue / GDP	0.021	0.048	0.135
Deficit / GDP	0.087	0.160	0.252
Debt / GDP	0.258	0.292	0.579
Inflation	0.243	0.122	0.097
Detrended GDP per capita			
Linear	0.075	0.060	0.500
HP-filter	0.096	0.071	0.279
HP-filter (Ravn-Uhlig)	0.063	0.058	0.137

Note: for each variable, the table displays peak value during wartime minus the average of the 5 preceding years; "HP-filter" uses a smoothing parameter equal to 100*; "HP-filter (Ravn-Uhlig) uses a smoothing parameter equal to* 6.25*.*

3.3 Benchmark simulation

In this section, we simulate the response of government policy to a war-expenditure shock in the LW and CIA models. Assume $\Psi = \{\psi_L, \psi_H\}$, where $1 = \psi_L < \psi_H$; the low value corresponds to peacetime and the high value to wartime. For the LW-economy set $\psi_H = 2$ which implies a wartime expenditure of about 31% of GDP, which is roughly the average during War World II; to obtain a similar increase in the CIA-economy, set $\psi_H = 1.7$. Let $\Pi(\psi_i \mid \psi_j) = Prob(\psi' = \psi_i \mid \psi = \psi_j)$ for $i, j = \{L, H\}$. The conditional probabilities of war and peace are hard to measure since we only observe realized outcomes. I take the values used in Martin (2009), which imply an unconditional probability of 9% for war, with an average duration of 4.5 years. Thus, let $\Pi(\psi_L \mid \psi_L) = 0.9775$ and $\Pi(\psi_H \mid \psi_H) = 0.7778$.

The models are solved globally, using the method described in the appendix. The numerical approximation provides equilibrium functions $\{\mathcal{B}^i(B), \mathcal{X}^i(B), \mathcal{C}^i(B), \mathcal{G}^i(B)\}$, $i = \{L, H\}$, from which the state-contingent policy variables can be constructed.

Next, I simulate the economy for 200 periods, labeled -99 to 100, starting at the non-stochastic steady state, B^*. The economy stays in peacetime during 100 periods. Note that some variables converge to values that differ somewhat from those corresponding to a non-stochastic steady state, since during peacetime the government is expecting at some point to be hit by an expenditure shock. In period 1 the economy is hit by a war shock which last for 5 periods (like World War II) and then goes back to peacetime for the remaining periods. The series for real GDP is normalized by its long-run value during peacetime, so that it is equal to zero for both models, right before the war shock is realized.

Figure 3 shows the simulated policy response to the war shock described above, for the LW and CIA models, displaying periods -10 to 40. As we can see, in the periods leading up to the war shock, debt over GDP is significantly lower than in the non-stochastic steady state. The reason for this is that, in the stochastic economy, the government has less incentives to build up debt during peacetime since it expects to issue a significant amount during wartime.[18] In other words, the lower average debt is a direct consequence of distortion smoothing. The lower inflation is related to the lower debt, since there is less to service.

As we can see in Figure 3, the LW model matches all the stylized facts enumerated above, whereas the CIA model matches all but one. First, both models have the government use a mix of instruments to finance the war. This result extends the findings in Martin (2009) to the LW model. Second, the increase in debt is large and persistent, more so than any other policy variable. Third, there is a significant increase in the primary deficit, followed by a post-war surplus. Fourth, inflation increases during wartime to finance expenditure and in the post-war period helps reduce the burden of accumulated debt. Where the two models differ is in the fifth fact: the LW features a significant increase in real GDP, whereas the CIA model displays a (small) reduction in real GDP.

The quantitative increase of real GDP in the LW model provides an adequate fit to the data. The initial increase in real GDP per capita is about 9% in the Civil War and World War I, and 15% in World War II; the LW model displays a 16% increase. Where the model does not perform too well is in the subsequent increase in real GDP during World War II, which continued growing significantly for three more years. This is not surprising given that we are not modeling the increase in productivity experienced during this war episode—see McGrattan and Ohanian (2008).

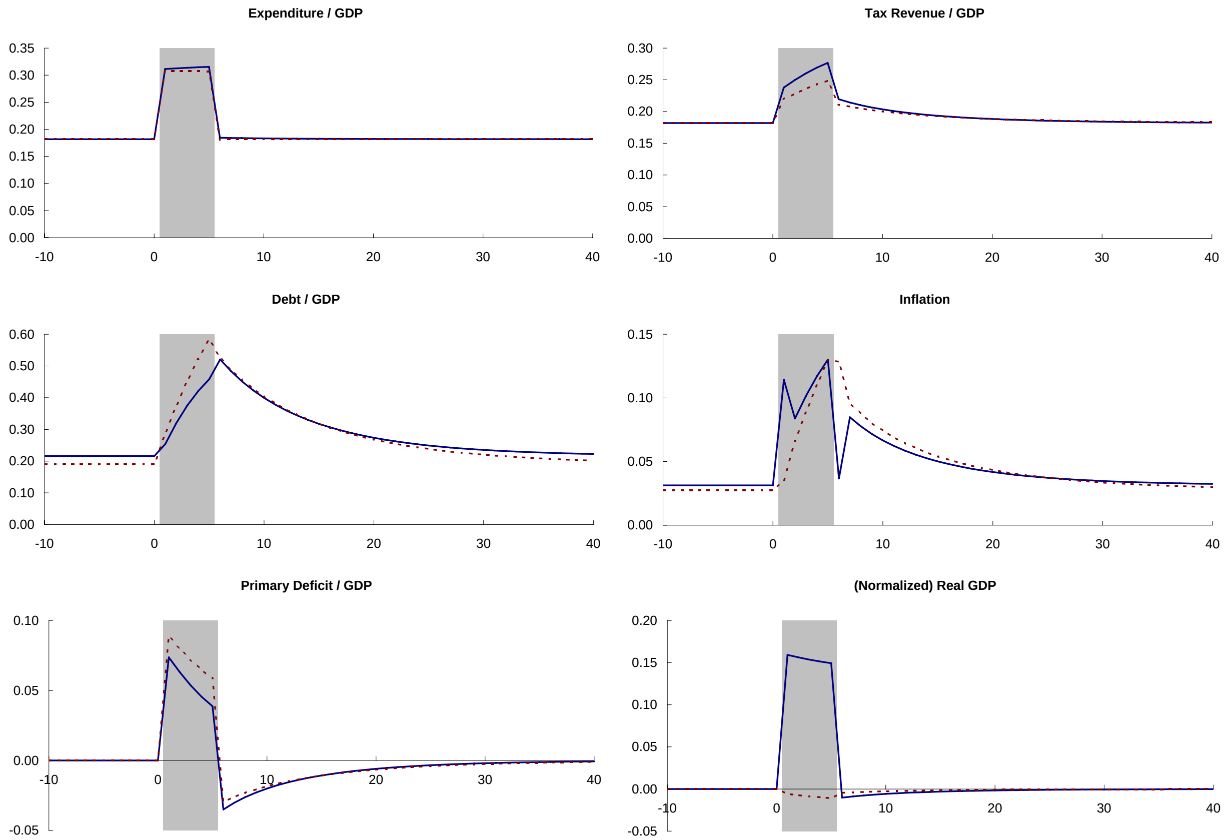
What accounts for the difference in GDP response between models is the linear term in preferences. In the case of the LW model, the linearity in night-labor disutility absorbs the expenditure shock, which results in a small decrease in private consumption (both day and night goods) and a significant increase in night labor; thus, real GDP increases. In the CIA model, the linearity in the credit good absorbs the expenditure shock, which results in minor decreases in cash-good consumption and labor, and a significant reduction in credit-good consumption; thus, real GDP drops. The counterfactual behavior of real GDP in the CIA model can be corrected by relaxing the linearity assumption in credit-good utility; however, as shown below, this correction introduces some quantitative problems.

Besides the behavior of real GDP, there are some significant quantitative differences between the LW and CIA models. First, the CIA model relies more on deficit financing, i.e., less on tax revenue and more on debt. The other difference stems from the behavior of inflation: in the CIA model, inflation is single peaked, whereas in the LW model, inflation is saw-shaped. Note, however, that in both models the money growth rate is single-peaked (and quite similar quantitatively); thus, the difference in observed inflation comes from differences in the behavior of money demand (which is idiosyncratic to each model) rather than supply. The LW model appears more attuned with the data; in particular, note the saw-shaped pattern of inflation in World War II and the post war peak; also note the inflation-deflation cycles in previous war episodes—see Figure 2.

If there is one significant fault in both models is that they do not seem to rely enough on deficit (debt) financing. E.g., in the two World Wars, the primary deficit over GDP increased well above 10%, whereas in both models this variable peaks below this mark. As we shall see in the next section, one can correct this by modeling a gradual increase in expenditure. Despite this shortcoming, the quantitative behavior of debt, inflation and the deficit in both models appears overall empirically plausible.

[18]This result is quite general. E.g., see Lucas (1986).

Figure 3: Simulated government response to a war shock



Note: solid and dashed lines correspond to LW and CIA models, respectively; periods with wartime expenditure are highlighted.

For the CIA model, the calibration approach adopted here is a significant improvement over the one used in Martin (2009). Compared to the World War II-type shock analyzed in that paper, the behavior of inflation and tax revenue is more empirically plausible here. First, inflation does not climb to implausibly high numbers; second, revenue does not exhibit a sharp post-war slump. As argued above, the better fit is due to the assumption of separable preferences between cash and credit goods, which allows for the primary deficit to be *ceteris paribus* decreasing in debt. This result also dispels the notion that the CIA model features inherently higher inflation volatility than the LW model.[19] Instead, the behavior of inflation in the CIA model depends critically on the specific details of the calibration.

3.4 Gradual increase in expenditure

In this section, we analyze policy response when expenditure increases gradually, rather than abruptly, which is a more realistic depiction of war episodes. For example, during World War II, federal outlays over GDP first climbed from an average of 8% to about 20%, then stayed at about 40%

[19]See Aruoba and Chugh (2008) for a treatment of this point.

for three years, then came down to a bit above 20% and finally settled around 12% until the Korean War. To model this type of behavior, let $\Psi = \{\psi_L, \psi_M, \psi_H\}$, where $1 = \psi_L < \psi_M < \psi_H$. We are going to keep to unconditional probabilities of war and peace the same, so let $\Pi(\psi_L \mid \psi_L) = 0.9775$, same as in benchmark, with $\Pi(\psi_M \mid \psi_L) = \Pi(\psi_H \mid \psi_L) = 1 - \Pi(\psi_L \mid \psi_L)/2$; let $\Pi(\psi_M \mid \psi_M) = \Pi(\psi_H \mid \psi_M) = \Pi(\psi_H \mid \psi_H) = \Pi(\psi_M \mid \psi_H) = 0.3889$ and $\Pi(\psi_L \mid \psi_M) = \Pi(\psi_L \mid \psi_H) = 0.2222$. For the LW economy, let $\psi_M = 1.5$ and $\psi_H = 3$; for the CIA model let $\psi_M = 1.4$ and $\psi_H = 2.25$. This parameterization implies expenditure over GDP climbs to 25% and 40% in states M and H, respectively. The economy is simulated as before, only now $\psi = \psi_M$ in periods 1 and 5, $\psi = \psi_H$ in periods 2, 3 and 4, and $\psi = \psi_L$ in all other periods.

Figure 4 shows the policy response when war shocks feature a gradual increase in expenditure. As we can see, this case maintains all qualitative properties of the benchmark simulation. The difference between cases is quantitative. Even though average wartime expenditure is only slightly larger (31% of GDP in the benchmark case, 35% of GDP in the gradual case), all variables exhibit a significantly larger increase. In particular, the deficit increases to more empirically plausible levels, correcting an important shortcoming of the benchmark simulation. For the LW model, we see that real GDP now increases for two consecutive periods: when ψ increases to ψ_M and then again, when it increases to ψ_H.

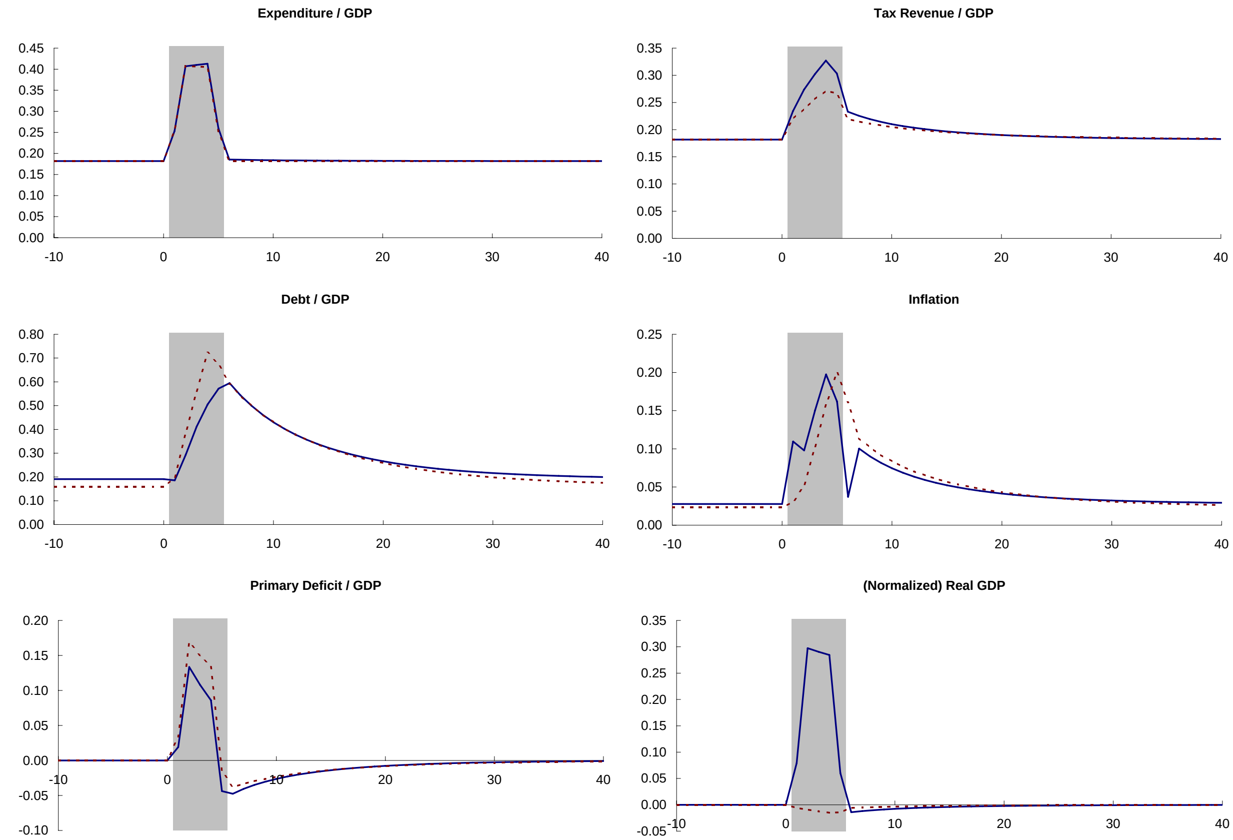
3.5 Alternative calibration for CIA economy

Perhaps the most important assumption for the CIA model in the benchmark calibration is linear utility in the credit good. Consider then setting $\rho = 2.0000$, and to match target statistics, let $\alpha = 3.7393$, $\gamma = 5.2329$, $\psi = 2.9979$ and $\sigma = 2.9884$. Note that β and φ are left at benchmark and that ψ is no longer normalized to 1. In a non-stochastic economy, the equilibrium policy functions are significantly different than benchmark—see Figure 5. Compared to the LW model, the money growth rate function in the CIA model is now much steeper, whereas tax revenue and the primary deficit are significantly flatter. The steepness of the money growth rate is a special concern as large changes in debt would imply (perhaps) implausibly large swings in inflation. Interestingly, real GDP in terms of debt is now very similar in the two models.

Figure 6 shows the policy response for the benchmark LW model and the CIA with non-linear credit-good utility, for the case with two expenditure states. We keep the same transition probabilities, but for the CIA model parameterize $\psi_H = 7.0451$ (i.e., 2.35 times $\psi_L = \psi$). Note that now the CIA model displays an increase in real GDP during wartime, thereby correcting the counterfactual implication of the benchmark calibration. The increase however, is not as quantitatively significant as the LW model. More importantly, the new calibration shifts the financing burden from deficit to taxes. In effect, the increase in debt is much lower than either the benchmark calibration or the LW model. Similarly, the wartime increase in the primary deficit is quantitatively small. Overall, an increase in ρ fixes the qualitative problems of the CIA model, but at the cost of making the quantitative behavior less empirically plausible. Further increasing ρ only exacerbates these problems. If we consider a gradual increase in expenditure, the differences between models become more pronounced.

Given the findings above, one possibility is that the linearity in labor is critical for the good performance of the LW model. To address this concern, consider a general specification for utility from leisure, $h(\ell) = \alpha \frac{\ell^{1-\chi}-1}{1-\chi}$. If we take the benchmark calibration, with $\rho = 0$ and $\chi = 1$, and

Figure 4: Simulated government response to a war shock with gradual increase in expenditure



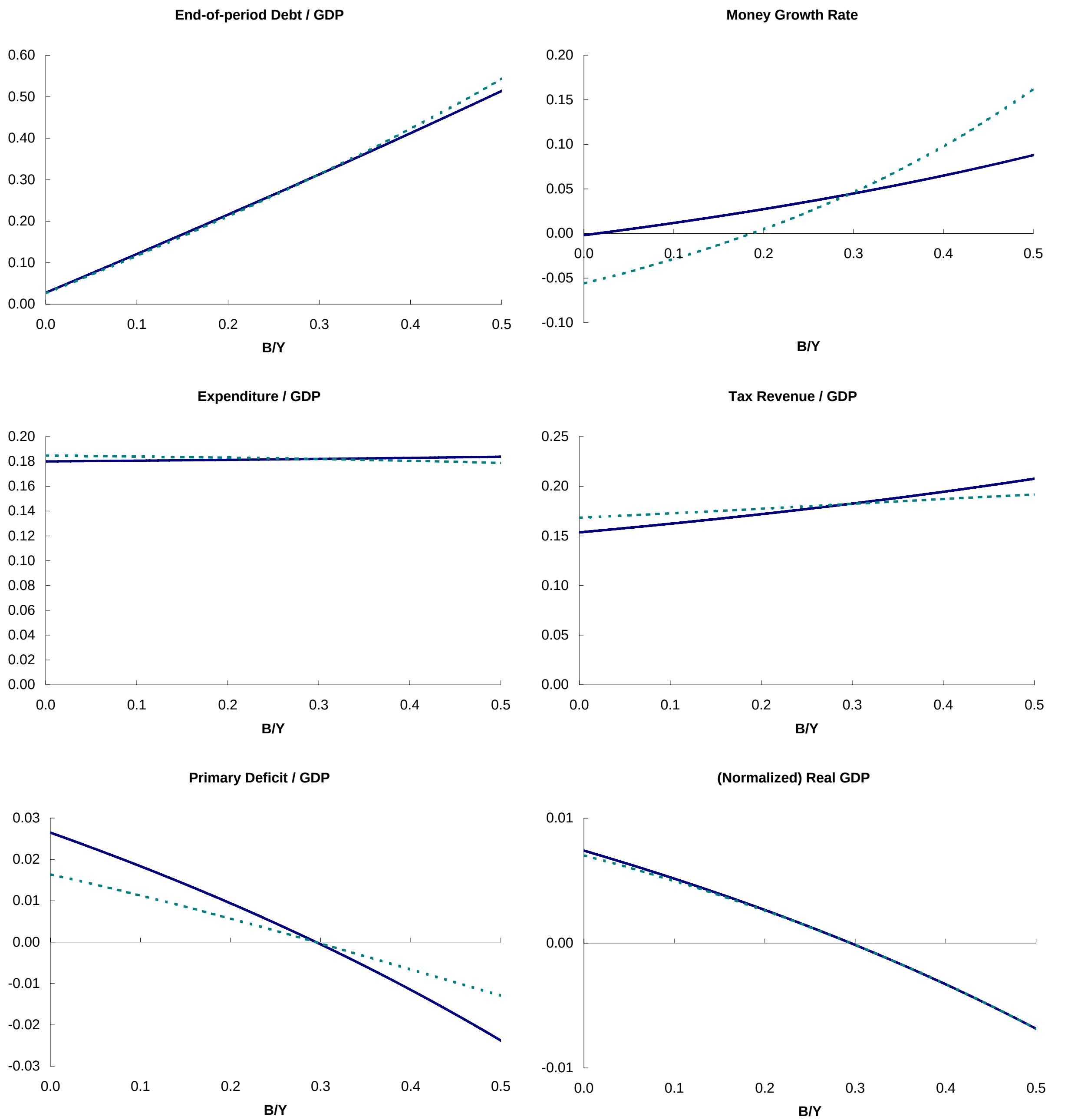
Note: solid and dashed lines correspond to LW and CIA models, respectively; periods with wartime expenditure are highlighted.

start decreasing χ (recalibrating all other parameters along the way to match target statistics), the results do not change significantly. If we take the alternative calibration, with $\rho = 2$ and $\chi = 1$, and make χ go towards zero (while recalibrating), several problems appear. First, as we decrease χ there is an even heavier reliance on tax rather than deficit financing of wartime expenditure, thus, worsening the problems described above. Second, as χ gets very close to zero (i.e., $h(\ell)$ approaches linear), the primary deficit is no longer decreasing in debt (it is first decreasing and then increasing) and when a war-expenditure shock hits, both debt and deficit decrease rather than increase. Overall, decreasing the curvature of leisure in the CIA model worsens its qualitative properties.[20]

Consider then increasing χ above 1 for the calibration with $\rho = 2$. In this case, the response of debt, the primary deficit and real GDP to a war-expenditure shock is larger than before, but the difference is small and not sufficient to overcome the model's quantitative shortcomings. For example, assuming $\chi = 10$ (with corresponding recalibration) we get the following results. Debt over GDP peaks at 10 percentage points lower in the CIA model than in the LW model. The primary deficit peaks at only 4.4% of GDP for the CIA model, whereas it peaks at 7.4% for the

[20] All calculations are omitted to save space but are available upon request.

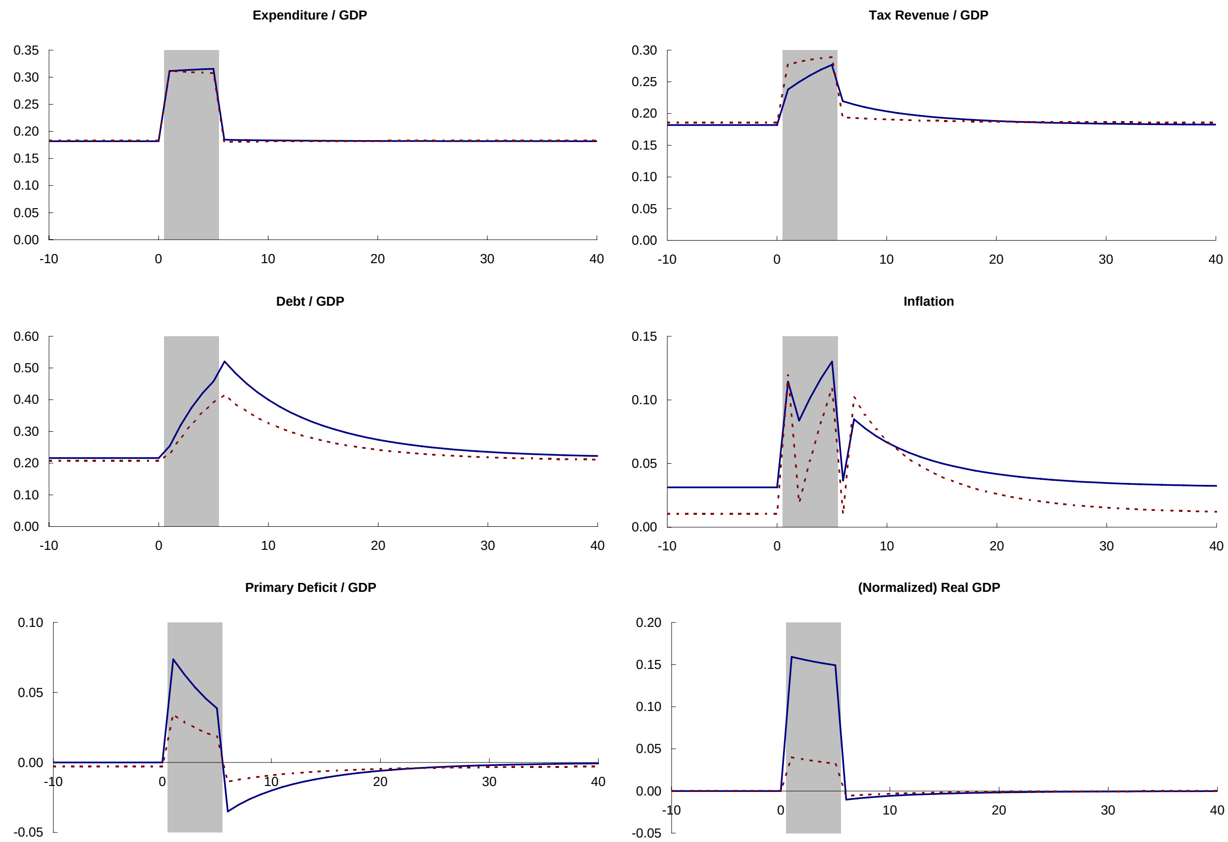
Figure 5: Non-stochastic equilibrium policies with non-linear credit-good utility



Note: solid and dashed lines correspond to LW and CIA models, respectively; real GDP is normalized by its steady state value.

LW model. Finally, real GDP increases by about 4.5% in the CIA model when the war shock hits; recall that this increase is about 16% for the LW model. Again, considering a gradual increase in expenditure does not alter the relative performance of the two models.

Figure 6: Simulated government response to a war shock with non-linear credit-good utility



Note: solid and dashed lines correspond to LW and CIA models, respectively; periods with wartime expenditure are highlighted.

4 Conclusions

In the preceding sections, I offered a different take on the ongoing debate about how to model monetary economies. Instead of arguing about the theoretical merits of the micro founded approach or how its qualitative findings can be easily replicated in simpler reduced-form models, I pitted two representative examples of each camp in terms of their implications for wartime government policy. The significant size and effects of wartime financing allowed us to separate these two otherwise similar analytical environments.

The LW model matches the stylized facts of wartime U.S. government policy with relative ease. Indeed, a standard calibration delivers good qualitative and quantitative results. In contrast, the CIA model either suffers from some qualitative or quantitative shortcomings, depending on the specific parameterization. Still, relative to previous work, the calibration approach proposed in this paper significantly improves the CIA model fit to the data.

Interestingly, one of the most distinctive assumptions of LW model, the linearity of night-labor, does not seem by itself to be driving the results. As argued, many of the implications of this assumption can be replicated in the CIA model by assuming a linear utility in the credit good.

However, such an assumption makes the CIA model fail to match an important stylized fact of wartime policy, namely the increase in output. Converging instead towards linear utility in leisure only worsens the CIA model's qualitative and quantitative performance.

Instead, the good performance of the LW model should be attributed to the joint assumption of linearity in night-labor *and* the two-sector (or market) structure. As becomes clear when contrasting the equations characterizing a monetary equilibrium for each case, the presence of two markets allows for a separation of the effects of monetary and fiscal policy, and this seems to be key for the results. The large increase in debt during wartime is accompanied by a significant increase in inflation to help serve it, but this only affects the day market. In the night market, distortions also increase due to the increase in taxes; however, the linear disutility of night-labor absorbs the shock and this allows real GDP to increase significantly. In contrast, in the CIA model there is a single production sector. Thus, we either get the wrong qualitative response in output (when credit-good consumption absorbs the shock due to linear utility) or a spread-out policy response with a counterfactually low increase in deficit and output.

It is of course debatable how hard the findings in this paper hit the scoreboard. It should be noted, however, that they do grant one previously unknown and empirically significant edge to the LW framework.

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Appendix

A Numerical approximation of equilibrium policy functions

The Markov-perfect equilibrium is approximated globally using a projection method with the following algorithm:

(i) Define a grid with 10 points over debt. Create the indexed functions $\mathcal{B}^i(B)$, $\mathcal{X}^i(B)$, $\mathcal{C}^i(B)$, and $\mathcal{G}^i(B)$, for $i = \{L, H\}$ or $i = \{L, M, H\}$, depending on the case. Set an initial guess. Note that if we are solving the case without expenditure shocks—see Figure 1—then policy functions need not be indexed as ψ is a constant.

(ii) Construct the following system of equations: for every point in the debt and ψ grids, evaluate equations characterizing the Markov-perfect equilibrium. Note that these equations contain policy functions (and their derivatives), evaluated tomorrow. Use cubic splines to interpolate between debt grid points and calculate the derivatives of policy functions.

(iii) Use a non-linear equations solver to solve the system in (ii). In the benchmark case, with $\{\psi_L, \psi_H\}$, there are 80 equations. The unknowns are the values of the policy functions at the grid points. In each step of the solver, the associated cubic splines need to be updated so that the interpolated evaluations of future choices are consistent with each new guess.

For the CIA model, when $\rho > 0$, we need to verify that the non-negativity constraint does not bind; if it does, make the appropriate modification to the system of equations. Alternatively, since the constraint only binds for low debt levels, set the lower bound on debt sufficiently high so that the constraint never binds. Note this can only be done if at the debt level for which $\mathcal{B}^L(B) = B$, the non-negativity constraint does not bind.

Since the LW model has two sectors, we need to construct measures of real GDP and the aggregate price level. In the model, real GDP is measured using the non-stochastic steady state as the base period for prices. Thus, let $y_t = \ln(\tilde{p}^* x_t + p^*(c_t + g_t))$ be the measure of log real GDP in the artificial economy. To calculate the inflation rate, define the aggregate (normalized) price level P as the weighted average of prices in the day and night markets. I.e., for any period t, let $P_t \equiv s_D \tilde{p}_t + s_N p_t$, where s_D and s_N are the expenditure shares for the day and night markets, respectively. Expenditure shares are constructed using the non-stochastic steady state statistics as the base period: $s_D \equiv \frac{\tilde{p}^* x^*}{Y^*}$ and $s_N \equiv \frac{p^*(c^*+g^*)}{Y^*}$. The inflation rate is defined as: $\pi_t \equiv \frac{P_t(1+\mu_{t-1})}{P_{t-1}} - 1$.