Comparing General and Partial Equilibrium Approaches to the Study of Real Business Cycles

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Abstract

The international financial crises and deepening world-wide recession remind us that the economic fates of nations are tightly linked. How do different models perform in explaining correlated business cycles? This paper compares the performance of two types of international real business cycle models, namely two-country dynamic stochastic general equilibrium (DSGE) and small open economy (SOE) models in explaining the shock transmission mechanisms across countries, using 68 countries’ output and consumption data from the Penn World Table (PWT). In particular, we put our emphasis on the two types of models that have both permanent and transitory components of productivity shocks. In the two-country DSGE model, comovement of business cycles among different countries can be captured by correlated permanent and transitory components of productivities across countries. On the other hand, in the SOE model, comovement of economies can only be modeled through the channel of world interest rate shock. Our evidence shows that the interest rate shock has very limited impact on output growth. Therefore, the two-country DSGE model outperforms the SOE in capturing the effects of the permanent shocks originated from foreign countries, especially for the developed countries, which are highly integrated with each other.
1 Introduction

The variability of business cycles differs dramatically across the nations of the world. Figure 1 presents the standard deviation of income and consumption growth across the broadest possible sample: the 161 nations found in the PWT tables data over the period 1971 to 2005. Countries are ordered from the most to least volatile based on the standard deviation of their real GDP growth. They range in variability from an astounding 27.5% in Lebanon to a mere 1.88% in the Netherlands; the median country is Samoa (5.16%). Perhaps not surprisingly, the United States has one of the least volatile business cycles, ranking 149th. OECD nations occupy 16 of the 20 least volatile positions in the ranking. The often-cited business cycle fact that the ratio of the standard deviation of consumption growth relative to output growth is much less than one in the United States is a feat achieved by only 11 of the 161 nations in the PWT (the line at the bottom of the figure presents this ratio by country along with the U.S. benchmark value of 0.69). The median volatility ratio is 1.18. Nations with more GDP volatility tend to have more consumption volatility: the correlation of output and consumption volatility across nations is 0.76. An important goal for business cycle research is to explain this business cycle heterogeneity.

One possible approach is to consider the economic interactions of countries in general equilibrium. This was the approach originally taken by Backus, Kehoe and Kydland (1992) and Baxter and Crucini (1993). Another possible approach is to consider each country in isolate as a small open economy in a partial equilibrium setting. This was the approach originally taken by Mendoza (1991). We refer to these approaches as the dynamic stochastic general equilibrium (DSGE) approach and the dynamic stochastic partial equilibrium (DSPE) approach, respectively. Many papers have been written following one of these approaches, but there has been virtually no discussion of the trade-offs between the two. The general equilibrium approach has the advantage of determining both quantities and prices. However the challenge in matching both dimensions is often unmet leaving the models open to criticism and mistrust (see, for example, the six puzzles paper of Obstfeld and Rogoff (2000)). The partial equilibrium approach appears to enjoy more empirical success, but at
the expense of not identifying the underlying foreign shocks. Put differently, the treatment of an endogenous variable such as the world interest rate as an exogenous stochastic process is a reduced form and thus subject to the Lucas (1972) critique. The two approaches also typically embody different underlying risk sharing assumptions. The partial equilibrium model assumes individuals pool risks within countries, but not across them, while most general equilibrium models start from the premise that idiosyncratic risks are fully pooled everywhere. It seems important, then, to consider the general implications of these modeling choices for the risk sharing mechanisms that they imply.\footnote{To emphasize this point, reading the international business cycle literature one gets the impression that autarky is a better approach to modeling large open economies than is the DSGE approach (see Heathcote and Perri (2002)) while the small open economy approach with an elastic supply of international credit better matches the business cycles of the typical small open economy. Ironically large open economies seem open enough to drive small open economy business cycles, leading to a modeling quandry of sorts.}

The goal of this paper is to compare and contrast the general equilibrium and partial equilibrium approaches to modeling international business cycle transmission in the context of the business cycle patterns found in Figure 1. The model of Baxter and Crucini (1995) is used for the DSGE model because as size of one of the two countries converges toward zero, it collapses to the DSPE model of Mendoza (1991) when the exogenous interest rate process is correctly specified. For tractability we restrict our attention to home and foreign total factor productivity (TFP) shocks as the driving variables. In the general equilibrium model these two exogenous variables determine the evolution of the world interest rate. In the partial equilibrium model the interest rate is modeled as an autoregressive process as Mendoza originally did. We then conduct a number of variance decompositions of output and consumption growth, by country, into the exogenous sources. Comparing these across the general and partial equilibrium approach reveals the important trade-offs that exist across the two modeling approaches.

Recognizing the importance of permanent and transitory shocks emphasized in the emerging market context by Aguiar and Gopinath (2007) a rich stochastic process for productivity is developed. Productivity in each country is a combination of permanent and transitory (but persistent) components unique to the nation and shared with the large industrialized group.
Apart from the fact that agents trade only in commodities and one-period non-contingent bonds, financial frictions are absent from the analysis. Adding financial frictions such as an interest rate spread that varies with the movements in national debt and productivity as in Uribe and Yue (2006), is an obvious and fruitful next step to the exercises conducted here. These frictions are ignored here to sharpen the focus on the role of home and foreign productivity in determining a single world real interest rate. Gauging the trade-offs of general equilibrium and partial equilibrium approaches in the more sophisticated setting of heterogenous interest rates is beyond the scope of this paper.

Turning to the details and beginning with the general equilibrium approach, the strategy used to exploit a tractable two-country model in the context of an N-country empirical investigation is to fix a large region and rotate each small open economy through the simulations keeping the parameters of the stochastic TFP process of the large region fixed. The large region is an aggregate of the G-7 plus Australia, leaving 60 small open economies to pair with this single large region at each iteration. Each small open economy is assumed to have a country-specific component to their productivity process, which combined with their relative economic sizes makes their business cycles heterogeneous relative to each another. Figure 2 shows that the sum of the GDP of the G-8 countries is larger than the aggregate GDP of the rest 60 countries. Since the large region has a disproportionate influence over the world interest rate the simulation method is designed to approximate the quantitative implications of an N-country general equilibrium approach without the exploding the state-space of the model. We calibrate the parameters for each country pair so that the second moments and correlation coefficients of aggregate variables generated by the model match their counterparts in the data. Next, each country is modelled as a small open economy using the DSPE approach, again allowing for both permanent and transitory components in home and foreign TFP, and now with an exogenous world interest rate shock.

Using stochastic simulations of each estimated model we conduct variance decompositions of output growth and the consumption growth by exogenous source of variation. In

\footnote{See also, Garcia-Cicco, Pancrazi, and Uribe (2011) who emphasize that the permanent shock plays an insignificant role in the SOE when financial frictions are incorporated.}
the general equilibrium model the variance of each macroeconomic aggregate is allocated to
four shocks: the permanent and transitory components of TFP in each of the two countries.
In the partial equilibrium model the variance is allocated to the permanent and transitory
components of home TFP and to the exogenous world interest rate. As such, the exercise
thus allows us to gauge how much of the general equilibrium impact of the large region on
the business cycle of the small open economy is captured by the interest rate process.

Our paper makes three contributions to the international business cycle literature. The
first contribution of our study is that it covers a large set of countries (68) and a substantial
period of history, 35 years. We discover that the persistence of the TFP shocks in developed
and developing countries are significantly different, which is consistent with the findings by
Aguiar and Gopinath (2007) who used a more limited cross-section. The second contribution
is to provide a quantitative comparison of a two-country general equilibrium model with a
small open economy model. We find that the partial equilibrium model and the general
equilibrium model generate very similar variance decompositions when the shock processes
are properly specified for both models. However, proper specification of the shock processes
virtually presupposes the general equilibrium model as a starting point. This is because
it is not possible to identify the shocks to the small open economy without knowing the
structural relationship between the two economies. Consequently the sources of variance of
business cycles in small open economies is mis-specified in the partial equilibrium approach.
The third contribution is that we contrast the impact of oil price changes on the production
of oil net exporters and net importers, given we have OPEC countries in our dataset. We
find that the fluctuations in the relative price of oil contributes significantly to the business
cycles of most economies. In particular, the oil price is procyclical (countercyclical) for net
oil exporters (importers).

2 International Business Cycles

The data panel is drawn from the Penn World Tables (PWT) 6.2. The PWTs provide pur-
chasing power parity and national income accounts converted to international prices for 188
countries from 1950 to 2005. We use GDP as the output measure and private consumption as the consumption measure. Based on data availability, the final panel contains 68 countries and the sample runs annually from 1970-2005. Among the 68 countries in the panel, 26 countries are developed countries and 42 countries are developing countries and based on the classification found in the International Monetary Fund’s World Economic Outlook Report, April 2010.

One of the greatest challenges in addressing the business cycle heterogeneity of Figure 1 is the curse of dimensionality. Obviously, any attempt at modeling the aggregates or shocks using a standard unrestricted VAR model is hopeless given the number of countries involved. Fortunately, Kose, Otrok and Whiteman (2003) introduced a Bayesian dynamic factor model to help overcome this challenge. In the dynamic factor model, a common world factor accounts for the comovement of the business cycles of all the countries, and thus significantly reduces the number of papers to be estimated.

Based on the KOW approach, we estimate the following dynamic factor model:

$$
\Delta z_{j,t} = \begin{bmatrix}
\Delta y_{j,t} \\
\Delta c_{j,t} \\
\Delta i_{j,t}
\end{bmatrix} = \\
\begin{bmatrix}
a_{j,y} & b_{j,y} & c_{j,y} & \varepsilon_{j,t,y} \\
a_{j,c} & b_{j,c} & c_{j,c} & \varepsilon_{j,t,c} \\
a_{j,s} & b_{j,s} & c_{j,s} & \varepsilon_{j,t,i}
\end{bmatrix}
\begin{bmatrix}
y_{j,t} \\
c_{j,t} \\
i_{j,t}
\end{bmatrix} + \\
\begin{bmatrix}
f_{t, \text{world}} \\
f_{t, \text{country}}
\end{bmatrix}
$$

where the $j$ denotes the country. The data vector, $\Delta z_{j,t}$, contains the growth rate of real GDP, consumption and investment. As in KOW, the world factor and the country factor both follow an AR(3) process. The factor loadings on the world and country-specific factors are country-specific. The issues regarding identification and the method of estimation are elegantly described in Otrok and Whiteman (1998). KOW also included a third factor, a

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3The 68 countries include: United Arab Emirates, Argentina, Austria, Belgium, Bangladesh, Bolivia, Brazil, Switzerland, Chile, China, Cote d’Ivoire, Cameroon, Colombia, Costa Rica, Denmark, Dominican Republic, Ecuador, Spain, Finland, Greece, Guatemala, Hong Kong, Honduras, Indonesia, India, Ireland, Iran, Iraq, Iceland, Jamaica, Kenya, Korea, Kuwait, Libya, Sri Lanka, Luxembourg, Morocco, Mexico, Malaysia, Nigeria, Netherlands, Norway, New Zealand, Pakistan, Panama, Peru, Philippines, Portugal, Paraguay, Qatar, Saudi Arabia, Senegal, Singapore, El Salvador, Sweden, Thailand, Uruguay, Venezuela, South Africa, Zimbabwe, Australia, Canada, France, Germany, Italy, Japan, United Kindom, and United States.
regional (geographic) factor, but they concluded that the regional factor explains only a small fraction of the variation of each variable. Therefore, we exclude the regional factor in our model. Moreover, since the regional factor is orthogonal to the world factor and the country specific factor, excluding the regional factor wouldn’t affect our estimation of the world factor.

KOW also use the Penn World Tables data. However, our data is different from KOW’s in several aspects. First, our data source is PWT version 6.3, while KOW’s is PWT version 5.5. Second, we have 68 countries in our sample, while KOW used 60 countries. Our sample consists of 59 countries from KOW’s sample and adds China and 8 OPEC countries (Libya, Nigeria, Iran, Iraq, Kuwait, Qatar, Saudi Arabia, and United Arab Emirates). Third our sample period is from 1971 to 2005, while the sample period of KOW’s data is from 1961 to 1990.

Beginning with Hamilton(1983), a number of authors have stressed the importance of oil price shocks during the sample period used here. Oil prices affect the macroeconomy through many different channels. Backus and Crucini (2000) developed a three-region model in which two regions trade manufactured goods (as in the original Backus, Kehoe and Kydland (1995) paper) and a third region produced oil. A supply reduction by the oil producing region is transmitted to oil importing nations through higher input costs in the two manufacturing regions, mimicking the textbook treatment of an oil price shocks as a ‘supply shock.’ In terms of measured productivity it is not unreasonable to assume the following structure: \( \ln A_{jt} = \zeta_j \ln p_{jt}^O + \ln X_{jt}, \) where \( p_{jt}^O \) is the relative price of oil (i.e. the price of oil imports relative to the domestic GDP deflator) and \( X_{jt} \) represents other factors that determine measured productivity (including true TFP shocks). The parameter \( \zeta_j \) captures the magnitude and possibly the sign of the impact of the oil price shock. That is, \( \zeta_j \) is expected to be negative for the net oil importers and positive for net oil exporters, especially the OPEC countries. Consider the following linear regression model:

\[
\Delta y_{jt} = \alpha_1 \cdot \Delta p_{jt}^O + \alpha_2 \cdot I_j \cdot \Delta p_{jt}^O + \beta_{1t} \cdot D_t + \beta_{2j} \cdot D_j + \varepsilon_{jt}^y \tag{1}
\]
where $\Delta y_{jt}$ is the output growth rate and $p^O_{jt} = \ln(P^o_{jt}S_{jt}/P_{jt})$ is the relative price of oil in terms of the domestic CPI. That is, $P^o_{jt}$ is the world oil price in US dollars, $S_{jt}$ is the nominal exchange rate between the U.S. and country $j$ and $P_{jt}$ is the consumer price index of country $j$. $I_j$ is a dummy variable equal to 1 if country $j$ is a net oil exporter and zero otherwise. Finally, $D_t$ and $D_j$ are time and country dummy variable to capture the year fixed-effect and the country fixed-effect respectively. Thus the impact effect of a change in the relative price of oil on output growth, after controlling for time and country-effects is $\Delta y_{jt}/\Delta p^O_{jt} = \alpha_1$, for net oil importers and $\Delta y_{jt}/\Delta p^O_{jt} = \alpha_1 + \alpha_2$ for the net oil exporters.

$\text{Table 1 – Oil Price and Income Growth}$

<table>
<thead>
<tr>
<th></th>
<th>Coef.</th>
<th>Std. Err.</th>
<th>P-Value</th>
</tr>
</thead>
<tbody>
<tr>
<td>$\alpha_1$</td>
<td>-0.033</td>
<td>0.016</td>
<td>0.043</td>
</tr>
<tr>
<td>$\alpha_2$</td>
<td>0.037</td>
<td>0.018</td>
<td>0.035</td>
</tr>
<tr>
<td>$R^2$</td>
<td></td>
<td>0.128</td>
<td></td>
</tr>
</tbody>
</table>

The regression result is shown in Table 1. We can see that the value of coefficient $\alpha_1$ is -0.033. This value is in line with Hamilton (2008)'s finding using the post-war data of the US. The value of coefficient $\alpha_2$ is 0.037. Therefore, we have $\alpha_1 < 0$, and $\alpha_1 + \alpha_2 > 0$. In other words, an increase in $\Delta p^O_{jt}$ will have a negative effect on $\Delta y_{jt}$ of the oil net importers, and a positive effect on $\Delta y_{jt}$ of the net oil exporters. This finding is consistent with our intuition that high oil prices will impede output growth of the net oil importers, while facilitate the output growth of the net oil exporters.

Returning to the factor model, Figure 3, presents three series: i) the world factor from the original KOW paper (1960 to 1989); ii) the world factor estimated using the model above and our panel (1971 to 2005) and iii) the year fixed-effects estimate in the OLS regression above. First, we find that the estimated world factor is very robust to the inclusion of China and OPEC nations as well as a significant shift in the sample period of estimation. The second finding is that the year-fixed effects from the OLS regression with oil prices produces a similar trend to the world factor estimated from the full Bayesian dynamic factor model.
In summary, we have documented significant business cycle heterogeneity in terms of the volatility of GDP and consumption growth rates across countries. Despite this heterogeneity there remains a significant world business cycle and an important factor in the business cycle heterogeneity aside from productivity variation itself may be asymmetric responses to equilibrium movements in the relative price of oil across net exporters and net importers. With these facts in the background we turn, now, to a description of the model and how the stochastic properties of national and G-8 aggregate productivity shocks are estimated.

3 Models

The two country DSGE model developed by Baxter and Crucini (1995) has been a workhorse in the international real business cycle literature. This single-good, single-asset two-country general equilibrium model features trade in goods and a single non-contingent bond with the two countries potentially differing in relative economic size. Their model is a natural benchmark to compare with the standard small open economy model because as the size of one of the two countries converges to zero, the world interest rate becomes exogenous to the smaller of the two countries. This does not mean, however, that the joint stochastic process of domestic productivity and the world interest rate of the small open economy may be specified in an ad hoc fashion. Quite the contrary, the solution to the DGSE model is needed to determine precisely the shock process that mimics the general equilibrium solution. An important goal of our work is to see how closely the typical specification of the DSPE model mimics the true business cycle dynamics of the DSGE model.

3.1 Preferences and Technology

Individuals in each country have Cobb-Douglas preferences over consumption and leisure

\[ U(C_{jt}, L_{jt}) = \beta^t \frac{1}{1-\sigma} [C^\theta_{jt}L^{1-\theta}_{jt}]^{1-\sigma}, \]  

(2)

where parameter \( \theta \in (0, 1) \), and the intertemporal elasticity of substitution is \( 1/\sigma \).
All countries produce final goods using capital and labor. The production function is Cobb-Douglas and each country experiences stochastic fluctuations in the level of factor productivity, $A_{jt}$,

$$Y_{jt} = A_{jt}K_{jt}^{1-\alpha}N_{jt}^\alpha.$$  \hfill (3)

The stochastic processes for productivity will involve permanent and transitory components each potentially with a component common across nations and unique to the nation. The processes are described in more detail and estimated in the next section.

The capital stock in each country, depreciates at the rate $\delta$ and is costly to adjust:

$$K_{jt+1} = (1-\delta)K_{jt} + \phi(I_{jt}/K_{jt})K_{jt},$$  \hfill (4)

where $\phi(.)$ is the adjustment cost function. As in Baxter and Crucini (1995), adjustment costs have the following properties: i) at the steady-state, $\phi(I/K) = I/K$ and $\phi'(I/K) = 1$ so that in the deterministic solution to the model the steady state with and without adjustment costs are the same and ii) the elasticity of the investment-capital ratio with respect to Tobin’s Q is $\eta = -(\phi'/\phi'') \div (i/k) = 15$.

### 3.2 Closing the Models

Following Baxter and Crucini (1995), the two country DSGE model is closed by imposing one intertemporal budget constraint and world goods market clearing. The intertemporal budget constraint is:

$$B_{jt} + Y_{jt} - C_{jt} - I_{jt} - B_{jt+1}P_{t}^{B} = 0$$  \hfill (5)

where $B_{jt+1}$ denotes the quantity of bonds purchased in period $t$ by country $j$. $P_t^B$ is the price of a bond purchased in period $t$ and maturing in period $t + 1$. The bond is not state-contingent, it pays one physical unit of output in all states of the world. Implicitly this defines, $r_t$, the real rate of return for the bond (i.e., $P_t^B \equiv (1 + r_t)^{-1} < 1$). The price of this bond is endogenous in the two-country equilibrium model, determined by the market-clearing condition in the world bond market.
The world goods market clearing condition is:

\[
\frac{1}{\sum_{j=0}^{1}} \pi_j (Y_{jt} - C_{jt} - I_{jt}) = 0, \tag{6}
\]

where \(\pi_j\) denotes the fraction of world GDP produced by country \(j\). These weights are necessary because the quantities in the constraint are in domestic per capita terms. In our applications, \(j = 0\), will be an aggregate of eight large industrialized countries while \(j = 1\) will be a particular small open economy.

The small open economy is closed with an intertemporal budget constraint identical to (5) with the discount rate following an exogenous stochastic process describe below. In addition, the following boundary condition is imposed:

\[
\lim_{t \to \infty} \beta^t p_{jt} B_{jt+1} = 0,
\]

where \(p_{jt}\) is the multiplier on the intertemporal budget constraint of small open economy \(j\).

### 3.3 Parameterization

All of the parameters except those governing the stochastic processes are set to common values across nations. Table 2 presents the calibrated parameter values used in our model.

<table>
<thead>
<tr>
<th>Parameter</th>
<th>Value</th>
</tr>
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<tbody>
<tr>
<td>(\beta)</td>
<td>0.954</td>
</tr>
<tr>
<td>(\sigma)</td>
<td>2</td>
</tr>
<tr>
<td>(\theta)</td>
<td>0.233</td>
</tr>
<tr>
<td>(\alpha)</td>
<td>0.58</td>
</tr>
<tr>
<td>(\delta)</td>
<td>0.1</td>
</tr>
</tbody>
</table>
The value of $\beta$ is set to be 0.954, so that the annual real interest rate is 6.5%. The parameter of relative risk aversion $\sigma$ is 2 and labor’s share $\alpha$ in the production function is 0.58. In the Cobb-Douglas preferences, the consumption share expenditure is: $\theta = 0.233$. The depreciation rate of capital, $\delta$, is assigned a value of 0.10.

4 Results

Both the DSGE and DSPE versions of the model are driven by productivity shocks. It is well-known that there is a close correspondence between the productivity shocks one feeds into a IRBC model and the path of GDP that results. Put differently, with a large physical capital stock subject to adjustment costs, the internal propagation mechanisms of the basic neoclassical model typically account for a small part of output variance. This turns out to be quite useful for our purposes. Since the focus is on comparing two variants of IRBC models, it makes sense to match the observable properties of macroeconomics as closely as possible in choosing the stochastic productivity processes so that differences across the two models are clearly identified as differences in the model structure and not the model fit.

That said, it is not at all obvious what productivity processes are consistent with macroeconomic fluctuations in the large PWT cross-section, given the model. The indirect inference about TFP is a useful by-product of the quantitative exercises we conduct. We begin by describing the stochastic productivity specification used in the equilibrium model and the international business cycle moments matched to estimate parameters of that process. Next we conduct variance decompositions in the two-country general equilibrium model and the small open economy model to convey the trade-offs that exist in taking one approach or the other.

4.1 International productivity

The existing international business cycle literature emphasizes two key properties of total factor productivity. The first could be described as relating to the broader issue of tech-
The notion that technical advances in one country spillover to others with a lag. Backus, Kehoe and Kydland (1992) modeled spillovers using a two-country VAR model under the null hypothesis that the level of total factor productivity is trend stationary. Thus, the off-diagonal element in their first-order autoregressive VAR captured the rate of productivity convergence. Baxter and Crucini (1995) allowed for non-stationary productivity variation and conducted co-integration tests between U.S. and Europe and U.S. and Canada. The persistence of the productivity gap was shown to be an essential ingredient in assessing the form of incomplete markets they modeled, which carries over to the current paper. If the country-specific component of productivity variation has a large permanent component, the wealth effects are significant and the lack of ex ante risk-sharing has significant consequences for business cycle dynamics and welfare. The recent literature has gravitated toward the view that productivity shocks are permanent which seems more consistent with the notion of technological adoption producing ever increasing productivity at an uncertain rate of progress. Since this literature is newly emerging and quite empirically demanding it should come as no surprise that the jury is still out on international productivity convergence, even among the industrialized world. Drawing on this literature, we consider the most flexible specification of productivity that the data allow.

Specifically, the logarithm of total factor productivity in country includes four components:

\[ \ln A_{jt} = (\omega_j^P \ln A_{P0t} + \omega_j^T \ln A_{T0t}) + (\ln A_{Pjt} + \ln A_{Tjt}), \]

where \( j = 0 \) is the G-8 index, \( j > 0 \) are the remaining 60 nations in our panel. The factor loadings \( \omega_j^P \) and \( \omega_j^T \) are the sources of common productivity movement across country \( j \) and the G-8 aggregate.

In each case, the permanent components evolve as random walks,

\[ \ln A_{Pjt} = \ln A_{Pj|t-1} + \ln \varepsilon_{jt}. \]
whereas the transitory components are AR(1) processes

\[ \ln A^T_{jt} = \rho \ln A^T_{j,t-1} + \ln \varepsilon^T_{jt}. \]  

(9)

The innovations to the permanent and transitory components of TFP, \( \varepsilon^P_{jt}, \varepsilon^T_{jt} \), are i.i.d. draws from normal distributions with mean zero. The variance of the innovations varies across countries: \( \varepsilon^T_{jt} \sim N(0, v^T_j \sigma^T_0) \), \( \varepsilon^P_{jt} \sim N(0, v^P_j \sigma^P_0) \), \( \varepsilon^T_{0t} \sim N(0, \sigma^T_0) \), and \( \varepsilon^P_{0t} \sim N(0, \sigma^P_0) \), where \( \sigma^T_0 \) and \( \sigma^P_0 \) are the standard deviations of the innovations to the transitory and permanent components of TFP in the aggregated G-8 region. Thus, \( v^P_j \) and \( v^T_j \), are the relative standard deviations of the corresponding innovations in the other 60 small open economies.

4.1.1 Productivity of the G-8

We calculate the weighted sum of annual output and consumption of the G-8 countries, which are denoted as \( y_0, c_0 \), and compute the second moments of GDP growth, consumption growth and the logarithm of the savings are, denoted \( \Delta y_0, \Delta c_0, \) and \( y_0 - c_0 \), respectively. The calibration strategy is to choose the appropriate value of \( (\rho_0, \sigma^P_0, \sigma^T_0) \), so that the variance of \( (\Delta y_0, \Delta c_0, y_0 - c_0) \) generate by the simulated model match their counterparts in the G-8 aggregate data. As the assumption underlying our approach is that the G-8 is by far the largest region and idiosyncratic movements in individual productivity of the G-60 have no discernible impact on the G-8, the calibration here is done in a closed economy (effectively setting \( \pi = 1 \) in the model).

The model is simulated 2,700 times with a range of the persistence parameter, \( \rho_0 \), restricted to the closed interval \([0.40, 0.95]\). The range of the innovation standard deviations of \( \sigma^P_0 \) and \( \sigma^T_0 \) are restricted to the closed interval \([0.006, 0.02]\). The outcome of the moment-matching exercise is that the three parameters, \( (\tilde{\rho}_0, \tilde{\sigma}^P_0, \tilde{\sigma}^T_0) \) for the G-8 are equal to \((0.85, 1.1, 1.2)\). Table 3 compares the second moment properties of the data to the model simulation. The difference between the moments from the data and from the model is less than 10%.
Table 3 – Matching second moments of G-8 aggregate

<table>
<thead>
<tr>
<th>Standard deviation of:</th>
<th>Data</th>
<th>Model</th>
</tr>
</thead>
<tbody>
<tr>
<td>GDP growth ($\Delta y_0$)</td>
<td>1.80</td>
<td>1.94</td>
</tr>
<tr>
<td>Consumption growth ($\Delta c_0$)</td>
<td>1.28</td>
<td>1.15</td>
</tr>
<tr>
<td>Inverse log savings ratio ($y_0 - c_0$)</td>
<td>1.44</td>
<td>1.32</td>
</tr>
</tbody>
</table>

G-8 productivity parameters

- $\hat{\rho}_0 = 0.85$
- $\hat{\sigma}_0^p = 1.1$
- $\hat{\sigma}_0^T = 1.2$

4.1.2 Productivity of small open economies in general equilibrium

To calibrate the productivity processes in the small open economies in the general equilibrium model, the two-country DSGE model is used (see Appendix A). The persistence of the transitory component of TFP is set equal to its G-8 counterpart, $\rho_j = 0.85$ $\forall j$. While this choice is based on maintaining some aspects of symmetry across countries, it turns out this is equivalent to a quarterly persistence of 0.96 and thus consistent with the findings of Aguiar and Gopinath (2007). They estimated persistence of their transitory component of productivity at the quarterly frequency of 0.97 for Canada and 0.95 for Mexico, respectively. Moreover, they find this value is close to what the persistence of transitory component of productivity equals for a number of other developed countries as well.

Turning to the innovation variances of the components of productivity, the approach taken is as follows. Each of the 60 small open economies is taken in turn and combined with the G-8 and treated as the large and small open economies that populate the Baxter-Crucini two-country general equilibrium model. The model is simulated setting the relative size of the small country based on the fraction of world GDP it produces on average over the sample period of observation. The parameter settings of the persistence and innovation
standard deviations in the of the G-8 forcing processes are maintained at the values of Table 1 no matter what country is paired with the G-8 aggregate. The open economy model is then simulated and a range of values for the relative innovation variance of the permanent and transitory shock to the small country is applied with the goal of matching: i) the variance of GDP and consumption growth of the country in question and ii) the correlation of GDP growth and consumption growth between the G-8 and the small open economy. The innovation standard deviations for the transitory and permanent components of the small open economy’s productivity, $v_j^T$ and $v_j^P$, are restricted to lie in the closed interval $[0.1, 15]$, with grid points at spaces of 0.1. The interval for the factor loadings on the permanent and transitory productivity of the G-8, $\omega_j^P$ and $\omega_j^T$, is similarly diffuse $[-15, 15]$.

Table 4 – Small open economy productivity parameters

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<tr>
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<tr>
<td></td>
<td>1/3 Median</td>
<td>2/3</td>
<td>1/3 Median</td>
<td>2/3</td>
<td></td>
</tr>
<tr>
<td>$\omega_j^P$</td>
<td>0</td>
<td>0.5</td>
<td>1.5</td>
<td>2</td>
<td>2</td>
</tr>
<tr>
<td>$\omega_j^T$</td>
<td>0</td>
<td>0</td>
<td>0.5</td>
<td>0</td>
<td>0</td>
</tr>
<tr>
<td>$v_j^P$</td>
<td>2.1</td>
<td>2.9</td>
<td>3.9</td>
<td>0.7</td>
<td>1.0</td>
</tr>
<tr>
<td>$v_j^T$</td>
<td>1.2</td>
<td>2.0</td>
<td>2.7</td>
<td>0.7</td>
<td>1.3</td>
</tr>
</tbody>
</table>

Table 4 reports the median parameters and points in the distribution of the values of the calibrated parameters ($\omega_j^P, \omega_j^T, v_j^P, v_j^T$) for developing and developed countries. Four patterns emerge. First, the factor loadings on the permanent component of the G-8 aggregate productivity shock ($\omega_j^P$) is larger for developed countries than developing countries. For the median developed country, the value of $\omega_j^P$ is 2, while for the median developing country, the median value is actually 0. This finding is consistent with our intuition, as we normally consider that the developed countries are more technologically integrated with the G-8 than are the developing countries. The parameterization, does, however suggest a magnification effect of G-8 productivity on productivity variation of the small open developed nations.

Second, the stationary component of the productivity of the small economies appears
unrelated to its counterpart in the G-8 aggregate. This is true for both developed and
developing countries. The median for either group is $\omega_j^T$. This fact shows that the comov-
ment of productivities across countries are mainly driven by the comovement of permanent
component, rather than the transitory component. If the transitory component of produc-
tivity is a proxy for policy changes and other shocks, it may not be surprising that they are
idiosyncratic to the nations involved.

Third, for small economies, especially developing countries, the relative standard devia-
tion of the innovations to the permanent component exceeds that of the transitory compo-
nent in almost every case. Since these are relative to the G-8 values, it means the permanent
innovations are considerably elevated relative to the transitory component as we move from
the G-8 to the emerging market economies. In fact, the median developing country has an
innovation standard deviation 2.4 (1.6) times that of the G-8 aggregate for the permanent
(transitory) component of national TFP. Recall that for the G-8 aggregate, the standard
deviations of the permanent and transitory innovations were quite similar, 1.1 and 1.2 per-
cent, respectively. Thus the median small country has permanent innovations dominating
transitory ones by a factor of about 1.38. Thus small countries are subject to more produc-
tivity variation and, in particular, more permanent variation. This is important to recognize
because, according to the simple permanent income model, the wealth effect of a unit inno-
vation to the permanent component shock is much greater that of the transitory shock. The
complete distribution of the four parameters across the small open economies are presented
in Figure 4 and Figure 5.

The presence of a permanent component in productivity in the small open economies
that is shared with the G-8 region (i.e. the positive factor loading $\omega_j^P$) is a key facet
of the international business cycle model in terms of overcoming the comovement puzzle.
That is, the fact that under complete international risk-sharing consumption correlations
should be close to one while output correlations are often negative whereas empirically (in
industrialized nations) the typically pair-wise correlation of output growth and consumption
growth across countries is in the neighborhood of 0.25. Incomplete markets with a rich mix
of permanent and transitory and common and idiosyncratic shocks allows a much closer
match of theory to data.

Table 5 – Correlation of Output Growth

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<td>Median</td>
<td>2/3</td>
<td>1/3</td>
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<tr>
<td>$\rho(\Delta y_{GS}, \Delta y_j)$</td>
<td>0.10</td>
<td>0.15</td>
<td>0.25</td>
<td>0.40</td>
</tr>
<tr>
<td>$\rho(\Delta c_{GS}, \Delta c_j)$</td>
<td>-0.08</td>
<td>0.03</td>
<td>0.10</td>
<td>0.24</td>
</tr>
<tr>
<td>$\Delta \rho_{M,D}^y/\rho_M^y$</td>
<td>-0.04</td>
<td>0</td>
<td>0.06</td>
<td>0.02</td>
</tr>
<tr>
<td>$\Delta \rho_{M,D}^c/\rho_M^c$</td>
<td>-0.12</td>
<td>-0.03</td>
<td>0.03</td>
<td>-0.09</td>
</tr>
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</table>

Table 5 shows the cross-country distribution of the correlation of output growth and consumption growth between the G-8 region and small open economies. We have three findings. First, the output growth of developed countries are much more strongly correlated with the output growth of the G-8 region than developing countries. The median value of the correlation coefficient of output growth is 0.46 for developed countries, and 0.15 for developing countries. Second, consumptions are also positively correlated between the small open economies and the G-8 region for most economies, but are less positively correlated than output. The median consumption correlation coefficient for the developing countries is 0.03 for developing countries, and 0.33 for developed countries. The entire cross-country distribution of the international correlations are presented in Figure 6. As is evident the consumption correlations are not only lower on average than the output correlations, they have less of a central tendency across countries.

To summarize, we find it is necessary to have both permanent and transitory productivity variation to match the business cycle movements of the small open economies in the panel. Moreover, the innovation variances of own-country productivity innovations are typically greater than that of the G-8 and quite diverse in the cross-section. One central tendency across both the developing and developed small open economies is the apparent lack of a role for a transitory component of the G-8 productivity to spillover to productivity in these countries. This seems plausible in the sense that the permanent component of productivity
variation is more likely to true productivity while the transitory component is quite plausibly standing in for other policy and non-policy shocks which are likely to both transitory (fiscal shocks, terms of trade shocks) and idiosyncratic to the nation.

4.1.3 Productivity of small open economies in partial equilibrium

In moving to the partial equilibrium model, we assume the domestic and world TFP shocks are exactly the same stochastic processes as estimated using the GE model simulations earlier as this represents our null model. It is important to note that this presents a much richer stochastic model of TFP for the small open economies than is typical in the existing literature. For example, Mendoza (1991) modeled Canadian TFP as a simple AR(1) process whereas in our specification Canada’s TFP will be the sum of the four components discussed above: permanent and transitory world productivity and permanent and transitory components of productivity unique to Canada.

Putting aside this issue, the key alteration in the move to partial equilibrium is the move from an endogenous equilibrium real interest rate to an exogenous real interest rate. In the two-country equilibrium model there are three state variables (the domestic and foreign capital stock and domestic bonds) and four shocks (each country’s permanent and transitory component of productivity). Thus, the linear approximate solution of the model would produce deviations of the equilibrium real interest rate that is a linear function of all seven variables. Moreover, since the state and exogenous variables all follow first-order autoregressions, the autoregressive part of the equilibrium real interest rate would be of order seven. The moving average component would be of order 4 (given the four unique underlying shocks). The interest then would be an ARMA (7,4). Each additional country one adds to the general equilibrium model adds 4 more lags and 2 more shocks. While the quantitative importance of these state and exogenous variables falls as the size of the countries added to the model falls, it become quite unwieldy even in the three country case. Since the point of the exercise is to explore potential errors of prediction or interpretation that arise when the interest rate is modeled in a more ad hoc fashion, we assume as Mendoza
(1991) did, that the discount rate follows an AR(1) process:

$$\ln P_t^B = \gamma \ln P_t^B + \ln \varepsilon_{jt},$$

(10)

where $0 < \gamma < 1$ denotes the persistence of the logarithm of the bond price, and $\varepsilon_{jt}$ is an iid draw from a normal distribution with zero mean and standard deviation $\sigma_{jB}^2$.

To parameterize the real interest rate we match the same second moments as before (namely, the variance of output and consumption growth), but use the partial equilibrium model of a small open economy as the simulation model. Table 6 shows the estimation results for the median and two points in the cross-sectional distribution of the 60 small countries. We observe three patterns in the table. First, the persistence of the bond price process is similar in both developing and developed countries. The median autocorrelation coefficient is 0.25 for developing countries and 0.23 for developed countries. Second, the median standard deviation of the innovation term is the same for developing and developed countries as well. This is reassuring since our approach presumes a single world real interest rate and full integration of international bond markets at that common rate. Obviously this is a very strong assumption, but at least the persistence and volatility of the real interest rate is similar across cases based on the simulation estimates. It is worth noting that the developing country group is more asymmetric than the developed country group as evident in the much higher persistence and innovation of the implied real interest rate in the tail of the distribution.

<table>
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<th>Table 6 – Bond Price Shock Parameters</th>
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<tr>
<td>Developing</td>
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<tr>
<td>$\gamma_j$</td>
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<tr>
<td>$\sigma_{jB}^2$</td>
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</table>
4.2 Variance decomposition

With the calibration of the DGSE and DGPE models complete, we are in a position to compute variance decompositions of consumption and output growth of each country into the underlying exogenous sources of variation using each modeling approach.

4.2.1 The two country general equilibrium model

We start our variance decomposition from the two-country DSGE model. Table 7 shows the variance decomposition of output growth. Comparing the contribution of different components to the cyclical variation in GDP growth between developing and developed countries, three patterns emerge. First, domestic productivity shocks dominate the business cycles in developing countries, while the productivity shocks of the large economy (G-8) dominate the business cycles of the developed countries. For the median developing country, a sum of 15.7% (not shown) output growth variation is due to the productivity shocks originating from the large economy, compared to a much higher proportion of 51.5% (not shown) for the developed countries.

Second, if we focus on the domestic productivity shocks, the permanent component is quite comparable in importance as the transitory component for the developing countries (33.1% versus 33.6% for the medians). By contrast, for the developed countries, the transitory component vastly outweighs the permanent component, 24.6% compared to 3.9%. This particular finding is of interest in light of AG, who state that permanent productivity shocks are much more important in explaining output variation in developing countries than in developed countries. Our general equilibrium results show this is true when conditioning on shocks of domestic origin (the first panel). When examining the productivity linkages between the G-8 and other developed countries, the permanent shock is very important, accounting for 47.5% of the variance for the median developed country.
Table 7 – Variance Decomposition of Output Growth in DSGE

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<td>1/3 Median</td>
<td>2/3</td>
<td>1/3 Median</td>
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</tr>
<tr>
<td>Domestic shocks</td>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>$\epsilon^P_j$</td>
<td>25.8</td>
<td>33.1</td>
<td>48.5</td>
<td>2.9</td>
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<tr>
<td>$\epsilon^T_j$</td>
<td>16.4</td>
<td>33.6</td>
<td>51.7</td>
<td>17.0</td>
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<tr>
<td>G-8 shocks</td>
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</tr>
<tr>
<td>$\epsilon^P_0$</td>
<td>4.4</td>
<td>8.8</td>
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<td>32.5</td>
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<tr>
<td>$\epsilon^T_0$</td>
<td>1.4</td>
<td>6.9</td>
<td>11.6</td>
<td>2.9</td>
</tr>
</tbody>
</table>

4.2.2 The small open economy model

The variation of output of small open economies is accounted by five different shocks in the small open economy model: domestic permanent and transitory TFP shocks, world permanent and transitory TFP shocks, and a world real interest rate shock. Figure 7 plots the total variance accounted for the shocks originating outside of the home country in both the DSGE model and the DSPE model. In the DSGE model we are adding up the impact of the $\epsilon^P_0$ and $\epsilon^T_0$ shocks, the permanent and transitory shocks to G-8 productivity. In the general equilibrium model these shocks affect the smaller economies directly through their role in directly altering productivity in the smaller economies and indirectly through the transmission of business cycles from the large to the small countries. In the DSPE model, we are adding up the impact of the $\epsilon^T_0$, $\epsilon^P_0$, and $\sigma^{PB}_j$ shocks.

Since the DGSE model is the null model, it gives the correct attribution of the variance to productivity in the G-8. In the developed country sub-sample, the misattribution of variance is relatively minor since the two lines track each other quite closely. In the developing country sub-sample are not necessarily larger on average, but they are clearly more concentrated in the cross section. For example, in the case of Kenya the partial equilibrium model attributes almost everything to the shocks internal to the country whereas the correct answer is the opposite with most of the variation coming from abroad.

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The more subtle problem with these comparisons has to do with the identification of the underlying sources of the shocks. Although the SOE model generates similar results to the DSGE model in variance decomposition, this was largely engineered by the fact that we built a rich stochastic structure where productivity in the small economies was in part determined by productivity in the G-8, through the factor loadings. This is rarely done in practice. Typically the only foreign variable entered into the state space of the small open economy model is the world interest rate. Unfortunately, the a simple first-order autoregressive model of the real interest rate is a poor proxy for the presence of foreign productivity in the list of exogenous variables. Table 8, for example shows a more disaggregated variance decomposition. According to the table, the real interest rate shock (bond price shock in our model) can only explain a negligible portion of output variation for both developed and developing countries in our SOE model. This is consistent with Mendoza’s (1991) study of Canada using a small open economy model.

This creates the illusion that Canadian productivity is driving the Canadian business cycle. Given that productivity and output are highly correlated even in open economies such as Canada, one might be led to further argue that modeling Canada as a closed economy would be a good idea since doing so gives rise to a similar variance of Canadian GDP and contribution of productivity to that variation. There are two problems with this interpretation, which Table 8 lays bare. First, the spillover from G-8 productivity to the small open economy is about as large as it was in the general equilibrium model. By specifying Canada’s productivity as partly originating in the G-8 group, a large ‘transmission’ effect found. In contrast, if we start with a small open economy model and proxy foreign productivity with the real interest rate, we learn very little about the true underlying origins of productivity changes or their international diffusion.

The other danger of modeling a country in partial equilibrium is that even the most careful measurement of productivity variation is likely to be contaminated by unmeasured variation in utilization of capital, labor hoarding and the impact of intermediate input variation, to name perhaps the most prominent. Consequently the movements in measured Canadian productivity are likely picking up equilibrium responses of Canadian consumers
and firms to the broader general equilibrium.

Put differently, if it is important to identify the sources of productivity movements and determinants of the real interest, our results point to the value of the general equilibrium approach. Given that the motivation of developing general equilibrium models is at least in part to identify the shocks from the endogenous choices, it appears that this is very difficult to do in a small open economy setting. Moreover, matching the within sample moments gives no guarantee that a policy change or structural economy change will be adequately described with the small open economy model fit to a historical interest rate and domestic productivity process.

Table 8 – Variance Decomposition of Output Growth in DSPE

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<td>2/3</td>
<td>1/3</td>
<td>Median</td>
<td>2/3</td>
</tr>
<tr>
<td>Domestic shocks</td>
<td>$\epsilon_j^p$</td>
<td>22.2</td>
<td>32.3</td>
<td>39.4</td>
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<td>7.4</td>
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<tr>
<td></td>
<td>$\epsilon_j^T$</td>
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<td>52.3</td>
<td>15.8</td>
<td>22.6</td>
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<td>Interest rate shocks</td>
<td>$\epsilon_j^{PB}$</td>
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<td>0.1</td>
<td>5.2</td>
<td>0.1</td>
<td>0.2</td>
</tr>
<tr>
<td></td>
<td>$\epsilon_0^p$</td>
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<td>3.8</td>
<td>9.8</td>
<td>23.9</td>
<td>42.1</td>
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<tr>
<td></td>
<td>$\epsilon_0^T$</td>
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<td>1.2</td>
<td>9.8</td>
<td>0.0</td>
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</tr>
</tbody>
</table>

5 Concluding remarks

In this paper we have compared the performance of one-country SOE model with the two-country DSGE model. We conduct variance decompositions for the 60 small economies under the two-country general equilibrium framework and one-country small open economy framework. We find that the limitation of the SOE model is that it cannot capture the properties of permanent TFP shock from the foreign countries. This is particularly true, for the small developed countries, whose economic behavior is heavily influenced by permanent
TFP shocks originating from the G-8, the SOE model tends to significantly underestimate the influences of the shocks originated from abroad.

References


Figure 1. International business cycles

- Std. dev. GDP (growth)
- Std. dev. CON (growth)
- $\sigma(\Delta c)/\sigma(\Delta y)$
- U.S. $\sigma(\Delta c)/\sigma(\Delta y)$
Figure 2. Relative economic size in 2005
Figure 3. World business cycles
Figure 4. Frequency distributions of standard deviation of productivity innovations relative to the G-8

**Permanent shocks** (developing countries)

**Transitory shocks** (developing countries)

**Permanent shocks** (developed countries)

**Transitory shocks** (developed countries)
Figure 5. Frequency distributions of factor loadings on G-8 productivity

Permanent component (developing countries)

Transitory component (developing countries)

Permanent component (developed countries)

Transitory component (developed countries)
Figure 6A - Small economy output correlations with the G-8

Figure 6B - Small economy consumption correlations with the G-8
**Figure 7A - Variance decomposition of output growth (developing countries)**

**Figure 7B - Variance decomposition of output growth (developed countries)**