Information Processing and Limited Liability*

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Abstract

Decision-makers often face limited liability and thus know that their loss will be bounded. We study how limited liability affects the behavior of an agent who chooses how much information to acquire and process in order to take a good decision. We find that an agent facing limited liability processes less information than an agent with unlimited liability. The informational gap between the two agents is larger in bad times than in good times and when information is more costly to process.

Keywords: limited liability, information acquisition, rational inattention. (*JEL*: D23, D83, E60.)

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In many situations an economic decision-maker has to take an action but is uncertain about the best possible action. The decision-maker can reduce his or her uncertainty but in general will not do so completely, because acquiring and processing information is costly. The decision-maker then has to decide how much information to acquire and process, trading off the expected benefit of uncertainty reduction against the cost of information acquisition and processing. In this paper we are interested in how limited liability affects the information choice of the decision-maker. We are motivated by the fact that decisions about how carefully to attend to information are often made by agents who face limited liability and thus know that their loss will be bounded. For example, think of a manager learning about returns on subprime mortgages or sovereign bonds as a prerequisite for choosing a portfolio of assets for a financial corporation.

We model an agent who chooses how much information to process about his or her optimal action. The agent's optimal action depends on a random fundamental. The agent has prior beliefs about the fundamental. In addition, the agent can acquire a signal about the fundamental. The agent chooses the properties of this signal subject to the constraint that more informative signals are more costly.

The model predicts that limited liability reduces the agent's incentive to process information. Hence the agent chooses to be less informed compared to an otherwise identical agent facing unlimited liability. The informational gap between the two agents is larger in "bad times" than in "good times." Furthermore, the informational gap between the two agents is larger when it is more costly to acquire and process information.

This paper belongs to the literature on rational inattention building on Sims (2003), in the sense that we model information acquisition and processing as uncertainty reduction, where uncertainty is measured by entropy.¹

1 Model

We study the decision problem of a single agent who decides how much information to acquire and process in order to take a good decision. The agent has to take an action $a \in \mathbb{R}$. The agent's payoff

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¹See Maćkowiak and Wiederholt (2011) for a list of other references in this literature.

depends on the action a and the fundamental z . The agent's payoff function is

$$U(a, z) = \max \{u(a, z), 0\}.$$

The function $u(a, z)$ would be the payoff function with unlimited liability. The max operator formalizes the notion of limited liability. The function $u(a, z)$ is quadratic, concave in the first argument, and has a nonzero cross-derivative. The agent is uncertain about the payoff-maximizing action because the agent is uncertain about the fundamental z . The agent has a normal prior $z \sim N(\mu_z, \sigma_z^2)$. The agent can acquire a signal s about the fundamental z before taking an action. The agent chooses the properties of this signal. Timing is as follows: (1) The agent chooses the properties of the signal. This decision problem is stated formally below. (2) The agent observes a realization of the signal and takes the action $a \in \mathbb{R}$ that maximizes $E[U(a, z) | s]$.

For tractability, we impose a restriction on the signal and a restriction on the payoff function. We assume that the signal and the fundamental have a bivariate normal distribution. That is, the signal has the form

$$s = z + \varepsilon,$$

where the noise ε is independent of the fundamental z and $\varepsilon \sim N(0, \tau^{-1})$. The agent chooses the precision of the signal $\tau \in \mathbb{R}_+$ but takes as given that the signal and the fundamental have a bivariate normal distribution. Next, since $u(a, z)$ is a quadratic function, we have

$$u(a, z) = u(a^*, z) - \varphi(a - a^*)^2,$$

where $a^* = \alpha + \beta z$ denotes the payoff-maximizing action at the fundamental z , and α , β and φ are constants with $\varphi > 0$ and $\beta \neq 0$. For tractability, we assume that $u(a^*, z)$ is independent of z and thus equals some constant $\bar{u}$. The interesting case is $\bar{u} > 0$ and we focus on this case. Furthermore, for ease of exposition, we assume that $a^* = z$. The last equation then becomes

$$u(a, z) = \bar{u} - \varphi(a - z)^2.$$

These restrictions on the signal and the payoff function allow us to derive simple, transparent results concerning how limited liability affects the amount of information processed about the fundamental.

We now state the agent's information choice problem. The agent decides how much information to process before taking the action. The agent aims to maximize the expected payoff minus the cost

of processing information. Processing more information is formalized as receiving a more precise signal. The agent anticipates that for a given realization of the signal he or she will take the best action given his or her posterior. The agent also understands that there is limited liability. The cost of a signal is assumed to be linear in the amount of information contained in the signal about the fundamental. Formally, the agent's decision problem is to choose the precision of the signal $\tau \in \mathbb{R}_+$ so as to maximize

$$E \left[\max_{a \in \mathbb{R}} E \left[\max \left\{ \bar{u} - \varphi(a - z)^2, 0 \right\} | s \right] \right] - \lambda I(z; s) \quad (1)$$

subject to

$$s = z + \varepsilon, \quad (2)$$

and

$$I(z; s) = \frac{1}{2} \log_2 \left(\frac{\sigma_z^2}{\sigma_{z|s}^2} \right), \quad (3)$$

where $z \sim N(\mu_z, \sigma_z^2)$, $\varepsilon \sim N(0, \tau^{-1})$, and z and ε are independent. The constant $\lambda > 0$ is the marginal cost of information. The term $I(z; s)$ quantifies the amount of information contained in the signal about the fundamental. Following Sims (2003), we quantify information as reduction in uncertainty, where uncertainty is measured by entropy. Recall that the signal and the fundamental have a bivariate normal distribution. The amount of information contained in the signal about the fundamental is then given by equation (3) where $\sigma_{z|s}^2$ denotes the conditional variance of the fundamental given the signal. The inner max operator in (1) formalizes limited liability. The middle max operator in (1) is the assumption that the agent takes the best action given his or her posterior. The outer max operator is the assumption that the agent chooses the optimal precision of the signal given the payoff function and the cost of information.

2 Solution

To solve the decision problem (1)-(3), we apply the three max operators. First, the expected payoff associated with action a after the agent has received signal s equals

$$E[U(a, z) | s] = \int_{-\infty}^{\infty} \max \left\{ \bar{u} - \varphi(a - z)^2, 0 \right\} f(z|s) dz,$$

where $f(z|s)$ denotes the conditional density of the fundamental given the signal. Note that limited liability kicks in if and only if the distance between the actual action a and the payoff-maximizing action z is at least Δ , where

$$\Delta = \sqrt{\frac{\bar{u}}{\varphi}}.$$

Thus,

$$E[U(a, z) | s] = \int_{a-\Delta}^{a+\Delta} [\bar{u} - \varphi(a - z)^2] f(z|s) dz.$$

Second, to find the best action after the agent has received the signal, set the partial derivative of $E[U(a, z) | s]$ with respect to a equal to zero. Applying the Leibniz integral rule and using the definition of Δ yields the first-order condition

$$\frac{\partial E[U(a, z) | s]}{\partial a} = -\varphi 2 \int_{a-\Delta}^{a+\Delta} (a - z) f(z|s) dz = 0.$$

The unique action satisfying this first-order condition is the conditional mean of the payoff-maximizing action

$$a = \mu_{z|s}.$$

Furthermore, at this action $\partial^2 E[U(a, z) | s] / \partial^2 a < 0$. Hence, the best action given the agent's posterior is the conditional mean of the payoff-maximizing action. Certainty equivalence holds. The maximum expected payoff therefore equals

$$\max_{a \in \mathbb{R}} E[U(a, z) | s] = \int_{\mu_{z|s}-\Delta}^{\mu_{z|s}+\Delta} [\bar{u} - \varphi(\mu_{z|s} - z)^2] f(z|s) dz.$$

Note that in our model limited liability does not affect the best action given the agent's posterior. Certainty equivalence holds without and with limited liability. The reasons are the form of the objective and the symmetry of the conditional distribution of z given s . However, limited liability does affect the expected payoff associated with this action. To see this, rewrite the last equation as

$$\max_{a \in \mathbb{R}} E[U(a, z) | s] = \bar{u} - \varphi \sigma_{z|s}^2 - 2 \int_{-\infty}^{\mu_{z|s}-\Delta} [\bar{u} - \varphi(\mu_{z|s} - z)^2] f(z|s) dz. \quad (4)$$

The first term on the right-hand side of (4) is the expected payoff if there were unlimited liability. The second term is the expected benefit from limited liability.

Third, we are now almost in the position to solve the decision problem (1)-(3). Before doing that, it is useful to study in some detail the expected benefit from limited liability. Standard formulas for the moments of a truncated normal distribution yield

$$-2 \int_{-\infty}^{\mu_{z|s}-\Delta} \left[\bar{u} - \varphi \left(\mu_{z|s} - z \right)^2 \right] f(z|s) dz = -2\Phi \left(-\frac{\Delta}{\sigma_{z|s}} \right) \left[\bar{u} - \varphi \sigma_{z|s}^2 \left(1 + \frac{\Delta}{\sigma_{z|s}} \frac{\phi \left(-\frac{\Delta}{\sigma_{z|s}} \right)}{\Phi \left(-\frac{\Delta}{\sigma_{z|s}} \right)} \right) \right], \quad (5)$$

where $\phi(\cdot)$ and $\Phi(\cdot)$ denote the pdf and cdf of the standard normal distribution. It follows that the expected benefit from limited liability depends on the conditional variance of the payoff-maximizing action but not on the conditional mean of the payoff-maximizing action. Thus, without loss in generality, we can set $\mu_{z|s} = 0$ when we study the expected benefit from limited liability. Moreover, equation (5) can be used to compute the expected benefit from limited liability without numerical integration. Next, it is useful to study several derivatives of the expected benefit from limited liability. The derivative of the expected benefit from limited liability with respect to $\sigma_{z|s}^2$ equals

$$-2 \int_{-\infty}^{-\Delta} [\bar{u} - \varphi z^2] \frac{\partial f(z|s)}{\partial \sigma_{z|s}^2} dz. \quad (6)$$

This expression is always strictly positive. The term in square brackets is strictly negative for all $z \in (-\infty, -\Delta)$, and the derivative of the density function with respect to its variance is strictly positive for all $z \in (-\infty, -\sigma_{z|s})$. Thus, expression (6) is strictly positive when $-\Delta \leq -\sigma_{z|s}$. Furthermore, taking the derivative of expression (6) with respect to $\bar{u}$ (and taking into account that Δ depends on $\bar{u}$) yields

$$-2 \int_{-\infty}^{-\Delta} \frac{\partial f(z|s)}{\partial \sigma_{z|s}^2} dz. \quad (7)$$

This expression is strictly negative. Lowering $\bar{u}$ (and thereby reducing Δ) increases expression (6). Thus, expression (6) is strictly positive also when $-\Delta \in (-\sigma_{z|s}, 0]$. In summary, the derivative of the expected benefit from limited liability with respect to $\sigma_{z|s}^2$ is always strictly positive, and this derivative is larger when $\bar{u}$ is lower.

Finally, we turn to the information choice problem (1)-(3). Using equations (4)-(5) and noting that choosing $\tau \in \mathbb{R}_+$ is equivalent to choosing $\sigma_{z|s}^2 \in (0, \sigma_z^2]$ one can write the decision problem

(1)-(3) as

$$\max_{\sigma_{z|s}^2 \in (0, \sigma_z^2]} \left\{ \bar{u} - \varphi \sigma_{z|s}^2 + B(\bar{u}, \varphi, \sigma_{z|s}^2) - \lambda \frac{1}{2} \log_2 \left(\frac{\sigma_z^2}{\sigma_{z|s}^2} \right) \right\}, \quad (8)$$

where $B(\bar{u}, \varphi, \sigma_{z|s}^2)$ denotes the expected benefit from limited liability which is given by equation (5). One can also state the decision problem (8) in terms of uncertainty reduction. Defining

$$\kappa = \frac{1}{2} \log_2 \left(\frac{\sigma_z^2}{\sigma_{z|s}^2} \right)$$

the problem can be stated as

$$\max_{\kappa \in \mathbb{R}_+} \left\{ \bar{u} - \varphi \sigma_z^2 2^{-2\kappa} + B(\bar{u}, \varphi, \sigma_z^2 2^{-2\kappa}) - \lambda \kappa \right\}. \quad (9)$$

The first-order condition for this problem is

$$\varphi \sigma_z^2 2^{-2\kappa} - \frac{\partial B(\bar{u}, \varphi, \sigma_{z|s}^2)}{\partial \sigma_{z|s}^2} \sigma_z^2 2^{-2\kappa} = \frac{\lambda}{2 \ln(2)}. \quad (10)$$

In the following, let κ_U^* and κ_L^* denote the solution without limited liability and with limited liability, respectively. Without limited liability the solution would be

$$\kappa_U^* = \begin{cases} \frac{1}{2} \log_2 \left(\frac{\varphi \sigma_z^2 2 \ln(2)}{\lambda} \right) & \text{if } \frac{\varphi \sigma_z^2 2 \ln(2)}{\lambda} \geq 1 \\ 0 & \text{otherwise} \end{cases}. \quad (11)$$

With limited liability we obtain the following three results. First, limited liability reduces equilibrium information processing. Formally, $\kappa_U^* > \kappa_L^*$ so long as $\kappa_U^* > 0$. The reason is that information processing decreases the expected benefit from limited liability. The second term on the left-hand side of (10) is strictly negative. Second, limited liability reduces equilibrium information processing by more when $\bar{u}$ is smaller. Formally, $\kappa_U^* - \kappa_L^*$ is larger when $\bar{u}$ is smaller so long as $\kappa_L^* > 0$. The reason is as follows. The expected benefit from limited liability is larger when $\bar{u}$ is smaller. More importantly, the derivative of the expected benefit from limited liability with respect to $\sigma_{z|s}^2$ is larger when $\bar{u}$ is smaller. See expression (7). Therefore, limited liability decreases the incentive to process information by more when $\bar{u}$ is smaller. Third, we verified numerically that limited liability reduces equilibrium information processing more strongly when the marginal cost of processing information is larger. Formally, κ_U^* is smaller, κ_L^* is smaller, and $\kappa_U^* - \kappa_L^*$ is larger when λ is larger so long as $\kappa_L^* > 0$. The intuition is simple. When information is more costly, agents make on average larger mistakes with unlimited liability, which increases the effect of limited liability.

3 Extension

So far we have assumed that the signal and the fundamental have a bivariate normal distribution. We showed that the action equals the conditional expectation of the fundamental given the signal. Hence the action and the fundamental also have a bivariate normal distribution. In this section, we relax the Gaussianity assumption and we let the agent solve for the optimal joint distribution of the action and the fundamental. For tractability, we discretize the joint distribution of the action and the fundamental. The agent solves

$$\max_{f(a,z)} \left[\sum_{j=1}^J \sum_{k=1}^K U(a_j, z_k) f(a_j, z_k) - \lambda I(a; z) \right] \quad (12)$$

subject to the constraint

$$\sum_{j=1}^J f(a_j, z_k) = g(z_k), \quad k = 1, \dots, K, \quad (13)$$

where $f(a, z)$ denotes the joint distribution of a and z , and $g(z)$ denotes the marginal distribution of z , which the agent takes as given. Here $g(z)$ is a discretized version of the normal distribution with mean μ_z and variance σ_z^2 . We solve the decision problem (12)-(13) numerically for different parameter values. We evaluate $I(a; z)$ based on the formula $I(a; z) = H(a) + H(z) - H(a, z)$, where H stands for entropy. We compare the solutions to the solutions of an identical problem with unlimited liability.²

We find that the three results emphasized in the previous section continue to hold after dropping the Gaussianity assumption: limited liability reduces equilibrium information processing; limited liability decreases equilibrium information processing by more when $\bar{u}$ is smaller; and limited liability reduces equilibrium information processing by more when the marginal cost of information is larger.

We study the shape of the optimal joint distribution of the action and the fundamental. We know that with unlimited liability the optimal distribution is Gaussian.³ The question we study, then, is: Does limited liability induce deviations from normality in the optimal distribution of

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²In the problem with unlimited liability we simply set $U(a, z) = u(a, z)$ in expression (12).

³With unlimited liability the payoff function is quadratic and, given a quadratic objective and a Gaussian fundamental, the optimal distribution of the action and the fundamental is Gaussian. See Maćkowiak and Wiederholt (2009).

the action and the fundamental? We find that the answer is yes. The deviations from normality are small when the marginal cost of information λ is near zero, but the deviations from normality become non-trivial for larger values of λ . To understand the nature of the deviations from normality it helps to focus on the optimal conditional distribution of a given z and to distinguish between high-probability values of z (close to μ_z) and low-probability values of z (far from μ_z). For high-probability values of z , we find that: (i) the conditional mean of a is a number between μ_z (the unconditional mean) and z , as under normality, (ii) the conditional variance of a is small, and (iii) the conditional distribution of a is bell-shaped but has positive excess kurtosis. For low-probability values of z , we find that: (i) the conditional mean of a approximately equals μ_z , (ii) the conditional variance of a is large, and (iii) the conditional distribution of a is approximately uniform (and thus has negative excess kurtosis close to that of the uniform distribution, -1.2). One way to summarize these findings is to say that limited liability induces the agent to: (1) add fat tails to the distribution of his or her action given high-probability realizations of the fundamental, and (2) learn little about the optimal action given low-probability realizations of the fundamental.

4 Conclusions

We study how limited liability affects the behavior of an agent who chooses how much information to process in order to take a good decision. We find that an agent facing limited liability processes less information than an agent with unlimited liability, particularly so when times are bad and when information is more costly to process.

References

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