Why Does Employment in All Major Sectors Move Together over the Business Cycle?*

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This version: February 2012

Abstract

In recessions, employment falls in all major sectors. Positive correlation of employment across sectors is a puzzle, because a standard two-sector business-cycle model driven by aggregate productivity shocks predicts negative correlation of total hours of work in the consumption-goods sector and the investment-goods sector. I start from the observation that most of the variability of total hours worked takes the form of variations in the number of workers. Hours per employed worker is only a secondary source of variation. The extensive margin is therefore critical in understanding the positive correlation of sectoral labor market variables, yet neglected by existing studies. This paper advances the literature on cross-sectoral correlation of employment by making unemployment an explicit feature of the model. I construct a two sector model with search and matching friction, capital adjustment costs, and partial wage stickiness. The model explains the positive cross-sectoral correlation through movements of workers in both sectors into and out of unemployment.

Keywords: Business cycles, sectoral employment correlations, unemployment volatility, search and matching, wage rigidity

JEL classification: E20, E24, E32, J64

1 Introduction

It is well established that total labor input in the US economy is positively correlated across sectors over the business cycle.¹ The positive cross-sectoral correlation is a key characteristic of business cycle data that also appears in the definition of a recession set by the Business Cycle Dating Committee of the National Bureau of Economic Research:

*I would like to thank Bob Hall, Nir Jaimovich, and Pete Klenow for their encouragement and comments. I have also benefited from discussions with Manuel Amador, Max Floetotto, Zvi Hercowitz, Dotan Persitz, Monika Piazzesi, Oren Rigbi, Martin Schneider, Henry Siu, Josie Smith, and seminar participants in the Stanford Macro Bag Lunch, CREI, the IMF, University of Washington, and UBC. I gratefully acknowledge financial support by the Kohlhaugen Fellowship through a grant to the Stanford Institute for Economic Policy Research.

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¹Christiano and Fitzgerald (1998), Rebelo (2005), Cassou and Vazquez (2011).

“...a recession is a broad contraction of the economy, not confined to one sector...”

The positive correlation of total hours stems from positive cross-sector correlations of both the extensive and the intensive margins. In this paper I show that the correlations between the *number of workers* across different sectors are slightly higher than the correlations across sectoral *hours per worker*. Moreover, the variability of the number of workers in a sector is on average more than 3 times as high as the variability of average hours worked per worker. Taken together, these two facts imply that the extensive margin is the critical margin in understanding the positive sectoral correlation of labor input.

Motivated by this observation, I construct a two sector business cycle model that explicitly characterizes the behavior of sectoral employment. The key feature of the model is a search and matching friction in the style of Diamond (1982), Mortensen (1982), and Pissarides (1985) (DMP). The model generates substantially higher correlations of labor market variables relative to a frictionless model when incorporating two additional elements: (1) capital adjustment costs, and (2) partial wage stickiness.

The model explains the positive correlation of employment in the two sectors through a number of key properties. First is the incorporation of an additional sector, where unemployed workers look for jobs. Absent this third, countercyclical sector, workers mainly switch between sectors, generating a negative correlation of labor across sectors. In a model with explicit treatment of unemployment the value of working in production sectors is higher during high productivity periods, and, by the principles of the DMP class of models, individuals move from unemployment into market activity in both production sectors. This process is further enhanced when wages are sticky, as firms allocate more resources towards recruiting. The second property is the fact that search and matching friction imposes a cost on reallocation of workers across sectors, delaying the flow of workers from the consumption-goods sector to the investment-goods sector during high productivity periods. Finally, capital adjustment costs impose a cost on reallocation of capital, and, because of complementarity in production, weaken the incentives to reallocate labor from the consumption sector to the investment sector. In what follows, I describe these mechanisms in more detail.

I start by investigating a two-sector economy with labor markets that obey the principles of the baseline search and matching model by Mortensen and Pissarides (1994). The

model predicts slightly higher correlation of total hours of work across the two sectors than does a frictionless model. However, total hours worked and the number of workers in the consumption-goods sector are negatively correlated with the same variables in the investment-goods sector. The intuition is that in high-productivity periods, households want to smooth consumption over time and increase the capital stock. This results in reallocation of resources from the consumption sector to the investment sector. In the model, reallocation is achieved through a vigorous response of vacancies in the investment sector, while vacancies in the consumption sector hardly respond. Because the consumption sector is larger, aggregate vacancies and unemployment have low variability.

The behavior of unemployment, aggregate vacancies, and consumption sector vacancies mimics a well-known shortcoming of one sector search and matching models, known as the unemployment volatility puzzle. In an influential paper, Shimer (2005) shows that the Mortensen-Pissarides (1994) model with conventional parameter values predicts very low volatility of vacancies and unemployment, while in the data these variables are very volatile.² The results of this paper suggest that in the context of the two sector model, the main source of the aggregate volatility puzzle is the lack of responsiveness of vacancies in the consumption sector.

A literature stimulated by Shimer’s paper has introduced a new concept of wage stickiness suited to the search and matching model. Hall (2005) calls this “equilibrium wage stickiness” because, in a bargaining setup, a sticky wage that remains within the bargaining set of the worker and employer is an economic equilibrium. A key insight of this literature is that when wages are sticky, employers do not fully compensate their workers for a gain in productivity, effectively increasing the employers’ share of the match surplus.³ Employers respond by putting more resources into recruiting workers, which causes a tightening of the labor market with lower unemployment.

Motivated by these findings, I incorporate partial wage stickiness into the model. Partial wage stickiness generates positive cross-sectoral correlation of total hours, vacancies, and employment. All sectors tighten when an increase in productivity raises the payoff to employers from recruiting.

²Shimer (2010) consider the conditions under which vacancies in the model are neutral to productivity shocks.

³See, for example, Hall and Milgrom (2008), Gertler and Trigari (2009), Rudanko (2010), and the discussions in Hall (2009) and Brügemann and Moscarini (2010).

This paper advances the literature on sectoral labor “co-movement puzzle” in real business cycles (RBC) models. In sharp contrast to the data, a two sector RBC model driven only by aggregate total factor productivity (TFP) shocks, generates strong *negative* correlation of total hours worked in the two sectors. In a frictionless model, the desire to smooth consumption induces households to increase their demand for investment goods, leading to an expansion of total hours worked in the investment-goods sector coupled with a contraction of total hours worked in the consumption-goods sector. Output in the consumption sector increases due to higher productivity.⁴

Several studies have proposed mechanisms that resolve the “co-movement” puzzle.⁵ While successful in generating positive cross-sector correlations of total hours, these models neglect the distinction between the extensive and intensive margins. Within my model, I show examples of mechanisms that give rise to positive correlation of sectoral *total hours* worked yet fail to achieve positive correlation of the sectoral *number of workers*.

The model is mostly related to Benhabib, Rogerson, and Wright (1991), who use home production to show the importance of a third, counter-cyclical sector to generating positive co-movement. However, the evidence shows that the movements between market work and search for market work, on the one hand, and tasks at home, on the other, are quite small over the business cycle. There is therefore a strong case for including an explicit treatment of unemployment in a model of sector employment rather than time spend at home. Some aspects of this paper are related to DiCecio (2009), who emphasizes nominal wage stickiness in generating positive correlation between total hours worked in the consumption sector and output. His model does not distinguish between the extensive and intensive margins. Moreover, I show that in the context of my model wage stickiness is insufficient to generate positive cross-sector correlation of employment. Finally, the results suggest a link between the unemployment volatility puzzle and the “co-movement” puzzle. Both wage stickiness and an alternative calibration along the lines suggested by Hagedorn and Manovskii (2008) go a long way towards resolving the two puzzles.

⁴See Christiano and Fitzgerald (1998) for more details.

⁵Benhabib, Rogerson, and Wright (1991), Hornstein and Praschnik (1997), Huffman and Wynne (1999), Boldrin, Christiano, and Fisher (2001), Jaimovich and Rebelo (2009), DiCecio (2009)

The remainder of the paper is organized as follows. In section 2 I document facts on cross-sectoral correlations of labor market variables. In section 3 I describe the two-sector model. The calibration is discussed in section 4. In section 5 I analyze a simulation results, and conclusions are in section 6.

2 Documenting Sectoral Co-movement of Workers, Hours, and Vacancies

The two production sectors in this paper comprises consumption and investment sectors. I obtain data counterparts by assigning the available major sectors employment, hours, and vacancies data to “consumption” and “investment”. I classify the sectoral data according to the use table of the BEA 2002 benchmark input-output tables. The use table allows to calculate the share of final sectoral output that is used for private consumption and private investment. I consider a sector a consumption (investment) sector if the share of output used for consumption (investment) is greater.⁶

The first column of Table 1 presents the correlation coefficients of the cyclical components of total hours worked by production workers in the aggregate non-farm private sector, the consumption sector, and the investment sector. The correlation coefficients are above 0.96, indicating a strong sectoral co-movement pattern. The correlations associated with the extensive margin (the number of workers) and the intensive margin (hours per worker) are depicted in the next two columns of the Table 1. Positive correlation is a characteristic of both margins, and the co-movement pattern is slightly stronger for the extensive margin.⁷

Figure 1 plots the cyclical component of both margins of labor input by sector. Not surprisingly, the pattern of strong positive co-movement is visible: employment and hours

⁶The difference in shares is significant for all sectors except professional and business services, for which the shares are very close to 0.5. I divide this sector equally between consumption and investment. Following this procedure, I classify natural resources and mining, construction, durable goods manufacturing, and a half of professional and business services as investment, and the rest as consumption. There are a number of alternative classifications in different studies. One traditional way is to classify durable goods manufacturing as investment and non-durable goods manufacturing and services as consumption (DiCecio (2009)). Huffman and Wynne (1999) follow a similar procedure to the one I use, but consider intermediate goods use as investment. Harrison (2003) and Floetotto, Jaimovich, and Pruitt (2009) use the proportional weights implied by the use tables. The different methods result in different sector sizes, but do not significantly alter the co-movement results.

⁷Stock and Watson (1999) document positive correlation of the number of workers in various sectors with output. Huffman and Wynne (1999) document positive correlations of output with hours per worker in consumption and investment sectors.

Table 1: Business Cycle Statistics: U.S. Data

	Total Hours	Workers	Hours per Worker	Vacancies
<i>Correlations</i>				
Aggregate , Consumption sector	0.988	0.988	0.929	0.995
Aggregate , Investment sector	0.993	0.990	0.950	0.956
Consumption sector , Investment sector	0.968	0.959	0.808	0.923
<i>Standard Deviation Relative to Output</i>				
Aggregate	1.257	1.068	0.277	11.302
Consumption sector	0.819	0.731	0.196	10.669
Investment sector	2.476	2.157	0.447	15.080

Notes: Top: correlation coefficients between the aggregate non-farm private sector, the consumption sector, and the investment sector, with respect to four variables: total hours of work, the number of workers, hours per worker, and vacancies. Bottom: standard deviations of total hours, number of worker, hours per worker, and vacancies in the aggregate non-farm private sector, the consumption sector, and the investment sector. Quarterly averages of monthly data. All magnitudes refer to the cyclical component measure as deviation of log of a variable from HP trend (smoothing parameter 1,600). Sources: Current employment statistics (CES) 1964:Q1 - 2009:Q3; For vacancies: Job Openings and Labor Turnover Survey (JOLTS) 2001:Q1 - 2009:Q3.

in the two sectors are synchronized. Yet, the two right plots in Figure 1 and the standard deviations reported in Tabel 1 suggest differences in variability. In both the consumption and investment sectors the extensive margin is substantially more variable than the intensive margin.⁸ These facts indicate that the observed sectoral correlation of total hours worked actually stems from positive correlation and variability of the extensive margin.

The empirical moments in Table 2 lead to a similar conclusion using data on the 11 major private non-farm sectors.⁹ Panel A presents the correlation matrix of the cyclical component of the number of workers in the different sectors. Panel B presents the correlation matrix of the cyclical component of average hours worked in the major sector. The correlations are higher for the extensive margin in almost all cases. The average correlation of the sectoral number of workers with the aggregate number of workers is 0.76, and the median pairwise correlation is 0.58. The comparable magnitudes for the average hours per worker are lower

⁸A number of studies describe a similar fact regarding the aggregate economy. See, for example, Stock and Watson (1999), Shimer (2009), Shimer (2010), Hall (2009).

⁹Christiano and Fitzgerald (1998) and Rebelo (2005) show that total hours co-movement is also apparent at a more disaggregated level of the manufacturing sector. Cassou and Vazquez (2011) provide a more detailed analysis of the extensive margin in the major private sectors, finding similar results.

Figure 1: US Business Cycle Date 1964Q1:2009Q3

Notes: Top left: cyclical component of the extensive margin by sector; Bottom left: cyclical component of the intensive margin by sector; Top right: cyclical component of both margins - consumption sector; Bottom right: cyclical component of both margins - investment sector. % deviations from HP trend (smoothing parameter 1,600). Source: Current Employment Statistics (CES).

(0.58 and 0.28, respectively). In addition, the volatility of the extensive margin is higher than the volatility of the intensive margin in *all* sectors, as appears in Panel C.

In search and matching models firms recruit new workers by maintaining vacancies. The last column of Table 1 shows the correlation coefficients of the cyclical components of sectoral and aggregate vacancies. All correlations are above 0.9, indicating a strong positive co-movement pattern, similar to the other labor market variables. The statistical moments in Table 1 also show that aggregate and sectoral vacancies are very volatile; the standard deviation of the cyclical component of sectoral and aggregate vacancies is more than 10 times as high as that of real GDP.¹⁰

¹⁰JOLTS data is only available starting December 2000. For the aggregate economy, the Help Wanted Index (HWI) is available for a longer time period. The high variability of aggregate vacancies remains when using the HWI. The standard deviation of the cyclical component of HWI is almost 9 times as high as real GDP

Table 2: US Business Cycle Date 1964Q1:2009Q3

Panel A: Correlation Matrix: Number of Workers												
	Aggregate Private Sector	Mining and Logging	Construction	Durables	Professional and Business Services	Nondurables	Trade, Transportation, and Utilities	Information	Finance	Education and Health	Leisure and Hospitality	Other Services
Aggregate Private Sector	1											
Mining and Logging	0.297	1										
Construction	0.917	0.184	1									
Durables	0.965	0.313	0.835	1								
Professional and Business Services	0.858	0.371	0.754	0.820	1							
Nondurables	0.887	0.154	0.776	0.897	0.728	1						
Trade, Transportation, and Utilities	0.956	0.268	0.905	0.886	0.878	0.800	1					
Information	0.810	0.349	0.677	0.796	0.659	0.705	0.737	1				
Finance	0.682	0.123	0.752	0.566	0.627	0.536	0.660	0.432	1			
Education and Health	0.493	0.274	0.333	0.439	0.276	0.352	0.436	0.430	0.406	1		
Leisure and Hospitality	0.870	0.222	0.811	0.817	0.770	0.729	0.847	0.712	0.583	0.312	1	
Other Services	0.679	0.349	0.545	0.581	0.625	0.489	0.679	0.531	0.538	0.745	0.585	1
Panel B: Correlation Matrix: Hours per Worker												
Aggregate Private Sector	1											
Mining and Logging	0.373	1										
Construction	0.464	0.336	1									
Durables	0.908	0.279	0.327	1								
Professional and Business Services	0.662	0.122	0.335	0.494	1							
Nondurables	0.836	0.230	0.366	0.843	0.526	1						
Trade, Transportation, and Utilities	0.776	0.205	0.199	0.700	0.436	0.620	1					
Information	0.680	0.218	0.207	0.590	0.397	0.521	0.450	1				
Finance	0.334	0.242	0.117	0.192	0.242	0.114	0.199	0.456	1			
Education and Health	0.365	0.044	0.064	0.208	0.559	0.215	0.344	0.156	0.205	1		
Leisure and Hospitality	0.640	0.078	0.302	0.526	0.584	0.498	0.680	0.277	0.161	0.374	1	
Other Services	0.351	0.053	0.246	0.106	0.653	0.244	0.140	0.231	0.171	0.626	0.359	1
Panel C: Ratio of Standard Deviations (Hours to Workers)												
Std. Deviation of Number of Wokers divided by Std. Deviation of Hours per Worker	3.861	3.731	5.618	3.759	6.289	2.188	4.292	4.629	4.255	2.506	2.444	3.086

Notes: Major Sectors Correlations of the number of workers (the extensive margin) and hours per worker (the intensive margin) (1964:Q1 - 2009:Q3). Panel A: Correlation matrix of the number of workers in major sectors of the private economy. Panel B: Correlation matrix of hours per worker in major sectors of the private economy. Panel C: Standard deviation of hours per worker divided by standard deviation of the number of workers. In All sectors the number of workers is more volatile than the hours per worker. All magnitudes refer to the cyclical component measured as deviation of log of a variable from HP trend (smoothing parameter 1,600). Source: Current Employment Statistics (CES).

3 A Two Sector Model

I present a two sector model that consists of a number of key elements. There are two production sectors: consumption and investment. Hiring workers is subject to a search and matching friction as in Pissarides (1985), and Mortensen and Pissarides (1994). An unemployed person can search for a job in both sectors and only unemployed individuals are allowed to search for new jobs. The job destruction rate is exogenous and constant in each period. Households provide perfect insurance to their members in the sense of equal marginal utility of consumption. Firms make sectoral vacancy creation and investment decisions, where investment is subject to adjustment costs. The focus of this paper is understanding the effects of aggregate productivity shocks on sectoral variables. Therefore I follow the RBC literature by assuming an AR(1) Total Factor Productivity (TFP) process, identical across sectors, as the single driving force of the model economy. I present the setup of the consumption sector in detail. The setup of

the investment sector is analogous and omitted for brevity.

3.1 The Matching Process

Let n_c and n_i denote the current period measures of employment in the consumption (c) and investment (i) sectors, respectively. Denote the vacancies maintained by firms in the two sectors by v_c and v_i . A constant returns to scale Cobb-Douglas matching technology determines the number of new hires for each sector in each period. Assuming a constant sectoral exogenous separation probabilities π_c and π_i , employment in the two sectors evolves according to

$$n'_c = (1 - \pi_c)n_c + \chi_c (v_c)^{\gamma_c} (1 - n_c - n_i)^{1-\gamma_c} \quad (1)$$

$$n'_i = (1 - \pi_i)n_i + \chi_i (v_i)^{\gamma_i} (1 - n_c - n_i)^{1-\gamma_i} \quad (2)$$

where χ_c and χ_i are the matching efficiency parameters, and γ_c and γ_i represent the elasticity of the number of new sectoral matches with respect to sectoral vacancies.¹¹ The matching function implies that the probability that an unemployed worker finds a job in the consumption sector is $\phi_c = \left[\frac{v_c}{1-n_c-n_i} \right]^{\gamma_c}$, and the probability of filling a vacancy in the sector is $q_c = \left[\frac{v_c}{1-n_c-n_i} \right]^{\gamma_c-1}$. Analogous terms apply to the investment sector. Each unemployed person can search for a job in both sectors, and only the unemployed can search for jobs.¹²

3.2 Households

Each member of the household can be either unemployed (work zero hours), or employed in one of the two sectors. I assume full insurance within the household in the sense of equal marginal utility of consumption across individuals members.¹³ Hence, consumption (c) and hours worked (h) may differ across members, based on their employment status and the sector in which they are employed. Each employed individual receives the sectoral hourly wage w_c

¹¹Shimer (2007) shows that while not constant, the effect of varying separation rates is second order relative to the volatility of the job finding rate.

¹²A version of the model with exogenous probability of searching on the job does not yield significantly different results.

¹³Blundell, Pistaferri, and Preston (2008) find evidence for full insurance of transitory shocks for most households. The discussion in Hall (2009) also supports the view of substantial insurance among family members. In addition, this assumption results in a simplification of the analysis, as apparent in Merz (1995), Andolfatto (1996), and Hall (2009).

or w_i , while the unemployed receive constant unemployment benefits b . Households spend P_c per unit of consumption good. Households have access to a complete set of contingent claims B . I assume that the set of assets includes shares of firms, as households own firms. $Q(A^{t+1})$ denotes the vector of prices of Arrow securities that provide a unit of consumption for each possible productivity level A in the next period.¹⁴ Households consider the job finding probability in each sector (ϕ_c, ϕ_i) as exogenous, and discount future utility with a discount factor β . The households dynamic program is therefore

$$V(B, n_c, n_i, A) = \max_{c_c, c_i, c_u, B'} \{n_c U(c_c, h_c) + n_i U(c_i, h_i) + (1 - n_c - n_i)U(c_u, 0) + \beta \mathbb{E}V(B', n'_c, n'_i, A')\} \quad (3)$$

Subject to a budget constraint and a labor evolution equation for each sector:¹⁵

$$P_c(n_c c_c + n_i c_i + (1 - n_c - n_i) c_u) + \sum_{A'} Q(A') B' \leq w_c n_c h_c + w_i n_i h_i + b(1 - n_c - n_i) + B - T \quad (4)$$

$$n'_c = (1 - \pi_c) n_c + \phi_c (1 - n_c - n_i) \quad (5)$$

$$n'_i = (1 - \pi_i) n_i + \phi_i (1 - n_c - n_i) \quad (6)$$

Denoting the Lagrange multiplier on the budget constraint by λ and using the envelope condition with respect to current assets and the optimality condition with respect to next period's assets, the Euler equation that is used to price assets in the economy is

$$Q_t(A') = \beta Pr[A'|A] \frac{\lambda'}{\lambda} \quad (7)$$

Upon matching, the worker bargains with the firm over wages and hours. Agreement

¹⁴This is a technical simplification that makes Q the stochastic discount factor that discounts firms' profits. See Shimer (2010) for an example of explicit derivation of the dynamic program from a sequential problem.

¹⁵I assume that unemployment benefits are financed by a lump-sum tax T in each period.

determines the individual's consumption, compensation, and working hours according to their equilibrium values. Disagreement implies that the worker remains unemployed, receives unemployment benefits, continues to search for a job in both sectors, and have a different level of consumption. The difference between agreement and disagreement is captured exactly by marginal value of a job to the household, expressed as the envelope condition with respect to n_c .

$$V_{n_c} = U(c_c, h_c) - U(c_u, 0) + \lambda(w_c h_c - b + P_c c_u - P_c c_c) + (1 - \pi_c)\beta \mathbb{E}V'_{n_c} - \phi_c \beta \mathbb{E}V'_{n_c} - \phi_i \beta \mathbb{E}V'_{n_i} \quad (8)$$

The marginal values consist of utility differences $U(c_c, h_c) - U(c_u, 0)$, compensation differences $(w_c h_c - b)$, and consumption expenditure differences $(P_c c_u - P_c c_c)$ between being employed in the consumption sector and being unemployed. The continuation value from a job, assuming no separation is $(1 - \pi_c)\beta \mathbb{E}V'_{n_c}$. The last two terms, $\phi_c \beta \mathbb{E}V'_{n_c}$ and $\phi_i \beta \mathbb{E}V'_{n_i}$, represent the opportunity cost of being employed. Specifically, this cost is the probability to find a different job either in the consumption sector or the investment sector.

3.3 Firms

A firm produces either the consumption or the investment good, pays compensation to employees, and makes investment (I) and recruiting decisions (v). The cost per vacancy (a) is in terms of the good produced and constant over time.¹⁶ Firms take the (sectoral) job filling rates q_c , q_i as given. Firms are owned by households and discount future values according to the stochastic discount factor $Q_t(A^{t+1}) \equiv Q'$. Investment is subject to quadratic capital adjustment costs. Denoting total factor productivity by A , the price the investment good by P_i , sectoral capital stocks by k_c and k_i , and depreciation of capital by δ , the consumption

¹⁶Another way to model the recruiting cost is assuming that both types of firms pay a constant amount in terms of consumption. A third way is to assume that the firm must allocate employees from production to recruiting. The results of the model are essentially the same regardless of the specific choice.

firm's dynamic program is:

$$J(k_c, n_c, n_i, A) = \max_{I_c, v_c} \{ P_c A F(k_c, n_c h_c) - w_c n_c h_c - a P_c v_c - P_i I_c - \frac{\Lambda}{2} k_c \left(\frac{I_c}{k_c} - \delta \right)^2 + \mathbb{E} [Q' J(k'_c, n'_c, n'_i, A')] \} \quad (9)$$

s.t.

$$k'_c = (1 - \delta)k_c + I_c \quad (10)$$

$$n'_c = (1 - \pi_c)n_c + q_c v_c \quad (11)$$

The set of first order conditions with respect to investment and capital result in

$$\mu_c = P_i + \Lambda \left(\frac{I_c}{k_c} - \delta \right) \quad (12)$$

$$\mu_c = \mathbb{E} \left[Q' \left(P'_c A' F'_{k_c} + \frac{\Lambda}{2} \left(\frac{I'_c}{k'_c} - \delta \right)^2 + \mu'_c - \delta P'_i \right) \right] \quad (13)$$

where μ_c is the Lagrange multiplier associated with the capital evolution equation, and F_{k_c} denotes the marginal product of capital in the consumption sector. The second equation is an Euler equation that incorporates capital adjustment costs.

The envelope condition with respect to n_c represent the surplus gained from hiring an additional worker at the equilibrium wage and hours worked. This marginal value consists of increased production, the compensation paid, and continuation value conditional on no separation.

$$J_{n_c} = P_c A F_{n_c} - w_c h_c + (1 - \pi_c) \mathbb{E} [Q' J'_{n_c}] \quad (14)$$

The optimality conditions with respect to vacancies sets the level of sectoral vacancies according to the zero profit condition:

$$P_c a = q_c \mathbb{E} [Q' J'_{n_c}] \quad (15)$$

The zero profit condition implies that firms create vacancies up to the point where the marginal cost of posting a vacancy equals the probability of filling the vacancy multiplied by the value of a match to the employer.

3.4 Wages and Hours

Upon matching, the employer and employee enter a Nash bargaining process to determine hours worked and the hourly wage. Denoting the employer's bargaining power by τ , and the marginal value of a job to the employer by J_{n_c} the Nash bargaining problem is the solution to

$$\max_{w,h} V_{n_c}^{1-\tau} J_{n_c}^\tau \quad (16)$$

Nash bargaining has two implications. First, hours per worker are set efficiently hence the total match surplus in terms of consumption units ($J_n + \frac{V_n}{\lambda}$) is maximized. A second implication is division of the surplus according to a constant share rule. The resulting sectoral hours worked and hourly wage are:

$$w_c h_c = (1 - \tau) \left[P_c A F_{n_c} + a P_c \frac{v_c}{1 - n_i - n_c} + a P_i \frac{v_i}{1 - n_i - n_c} \right] + \tau \left[\frac{U(c_u, 0) - U(c_c, h_c)}{\lambda} + P_c (c_c - c_u + b) \right] \quad (17)$$

$$- \frac{U_h(c_c, h_c)}{U_c(c_c, h_c)} = P_c A F_{H_c} \quad (18)$$

where $H_c = n_c h_c$ denotes total hours of work in the sector. Detailed derivation of equations (17) and (18) is provided in the appendix.

A number of factors affect compensation to employees. First is the marginal revenue product of an additional employee ($P_c A F_{n_c}$). The next two terms reflect an increase in wages when the labor market is tighter and it is easier for searchers to find jobs. The bottom part reflects the fact that households are compensated for the utility and consumption expenditure differences that arise from agreeing to take the job rather than continue searching, as well as the forgone unemployment benefits. In DMP models, this value is the flow value of unemployment. These components are weighted by the bargaining power parameter τ .

When modelling wage stickiness, I modify the compensation equation (17) such that the bargaining power of the employer is pro-cyclical, an increasing function in labor market tightness. The intuition is as follows: the Nash bargained wage is typically some weighted average of the worker's and the employers reservation wages. When productivity is higher, both reservation wages increase, and so is the wage. However, if the wage is rigid, it will be

closer to the *worker's* reservation wage during high productivity periods, effectively increasing the employer's bargaining share. Similarly, the employer's bargaining share decreases during low productivity periods. The relationship between wage stickiness and pro-cyclical employer's bargaining power has also been established in a number of studies that model wage setting under different assumptions.¹⁷

An important feature of the functional form that I use is the fact that the bargaining power is always between 0 and 1. Therefore, the resulting wage is guaranteed to remain within the bargaining set, and the (sticky) wages are always consistent with the concept of equilibrium.¹⁸

The employers' bargaining power function is

$$\tau(x) = \frac{\tau_0}{\tau_0 + (1 - \tau_0) \left(\frac{x^{ss}}{x}\right)^\eta} \quad (19)$$

where $x = \frac{v_e + v_i}{1 - n_c - n_i}$ is the aggregate labor market tightness, and x^{ss} is the steady state level. When $\eta = 0$ the bargaining power is constant, as standard in DMP models. When $\eta > 0$ the employers bargaining power is higher whenever the labor market is tighter than steady state ($x > x^{ss}$). Higher η implies that the bargaining power is more sensitive to changes in tightness, resulting in *lower* wage variability.

4 Calibration

I describe the functional forms and parameter values chosen for simulations of the baseline and some alternative models, mostly relying on evidence in existing studies. A time period is set to one month. Table 3 summarizes the calibration of parameters.

¹⁷Examples include the alternating offer bargaining in Hall and Milgrom (2008), the staggered Nash bargain in Gertler and Trigari (2009), long term contracting consider by Rudanko (2010), and the endogenous wage rigidity in Gali and van Rens (2010). Brugemann and Moscarini (2010) discuss the concept of rent rigidity that is similar to the modification proposed in this paper.

¹⁸See Hall (2005) for discussion of "equilibrium sticky wages".

4.1 Utility Function

I choose the utility function suggested by Shimer (2009)

$$U(c, h) = \frac{c^{1-\sigma} \left(1 + (\sigma - 1) \frac{\theta\psi}{1+\psi} h^{1+\frac{1}{\psi}} \right)^\sigma - 1}{1 - \sigma} \quad (20)$$

This utility function allows for hours consumption complementarity whenever $\sigma > 1$. The evidence described in Hall and Milgrom (2008), Shimer (2009), and Hall (2009) suggest a difference in consumption between employed and unemployed of about 15 percent. Targeting this ratio in steady state implies $\sigma = 1.486$. The Frisch elasticity of labor supply is determined by the parameter ψ . As Hall and Milgrom (2008) I calibrate the labor supply elasticity to 1 as a compromise between the lower elasticity found for prime-age men (0.7) and the higher one found for women and younger men (above 1). The parameter θ is set to target average hours worked per worker at 0.3. Finally, I set the discount factor $\beta = 0.996$ reflecting a quarterly interest rate of about 1 percent.¹⁹

4.2 Matching and Labor Market Parameters

I calibrate the elasticity of new matches with respect to vacancies $\gamma = 0.5$. This corresponds to the upper end of the range of estimates suggested by Petrongolo and Pissarides (2001).²⁰ I set the matching efficiency parameter $\chi = 0.3915$ to target a steady state unemployment of 5.9%. According to Silva and Toledo (2009) the recruiting cost involved in posting a vacancy is around 4.3 percent of *quarterly* pay. I follow their calibration and calibrate the recruiting cost $a = 0.12$, around 12 percent of *monthly* pay. Following Hall and Milgrom (2008), I calibrate the unemployment benefits parameter to reflect about 25 percent of average compensation ($b = 0.25$).²¹ The separation rate π is set to 4.2 percent per month according to the average separation rate in JOLTS. Finally, $\tau_0 = \frac{1}{2}$, and $\eta = 0$, reflecting equal sharing of match surplus between the employer and the worker and no wage stickiness.

¹⁹The “naive” intertemporal elasticity of substitution is $\frac{-1}{\sigma}$. The own price Frisch elasticity of consumption demand is -0.75 on average in the steady state. See Trabandt and Uhlig (2009) for discussion of these elasticities and the relation to balanced growth.

²⁰Estimates that rely on JOLTS data typically yield higher elasticity, but given that JOLTS data cover a shorter and more recent sample, I choose 0.5 as a compromise between the two sets of evidence.

²¹Hall and Milgrom (2008) describe different findings regarding the replacement rate provided by unemployment benefits, ranging from 12 to 36 percent.

Table 3: Parameter Values: Baseline Model

Utility			
Curvature	σ	1.486	15% decline when unemployed
Elasticity of hours supply	ψ	1	Hall and Milgrom (2008)
Disutility of work	θ	8.07	avg. hours per worker = 0.3
Discount factor	β	0.9959	
Matching and Labor Market Parameters			
Elasticity of matches to vacancies (C)	γ_c	0.5	Compromise between JOLTS and
Elasticity of matches to vacancies (I)	γ_i	0.5	Petrongolo and Pissarides (2001)
Matching efficiency (C)	χ_c	0.3915	unemployment in steady state
Matching efficiency (I)	χ_i	0.3915	equals 5.9%
Unemployment benefits	b	0.25	25% of avg. compensation
Separation rate	π	0.042	JOLTS monthly average
Vacancy cost	a	0.12	Silva and Toledo (2007)
Employer's bargaining power	τ	0.5	
Technology and Shocks			
Capital share in production	α	0.33	
Capital depreciation rate	δ	0.0083	
Capital adjustment costs	Λ	6	see text
TFP AR parameter	ρ	0.98	Shimer (2009)
TFP shock std. deviation	σ_ϵ	0.005	Shimer (2009)

Notes: see section 4 for discussion.

4.3 Technology, Shocks, and Adjustment Costs

I assume an identical (across sectors) constant returns to scale Cobb-Douglas production technology $y = Ak^\alpha (nh)^{1-\alpha}$. I follow the standard RBC calibration with respect to shocks and technology. Hence I set $\alpha = 0.33$, the monthly capital depreciation rate $\delta = 0.083$, the AR(1) coefficient of the TFP process $\rho = 0.98$, and the standard deviation of TFP shocks $\sigma_\epsilon = 0.005$. In simulations with capital adjustment costs I set $\Lambda = 6$, consistent with Cooper and Haltiwanger (2006), who estimate a coefficient of slightly less than 0.5 using annual data and a model with only convex adjustment costs (as in my model). This estimate is also close to the higher range of estimates in Hall (2004).

5 Simulation Results

Based on the reported calibration, I log-linearize the model around the steady state, and simulate different specifications of the model. Each simulation includes 600 months, and repeated

Table 4: Model Simulation Results

	(1) Data	(2) RBC	(3) Baseline	(4) Sticky wages	(5) No Cap Adj. costs	(6) Hagedorn Manovskii (benefits)	(7) Hagedorn Manovskii (utility)	(8) No Income effect effect
Sectoral Labor Correlations								
<i>Total Hours</i>								
Corr(C sector, Aggregate)	0.99	-0.99	-0.58	0.86	0.55	0.91	0.83	0.98
Corr(I sector, Aggregate)	0.99	0.99	0.98	0.98	0.98	0.99	0.99	0.98
Corr(C sector, I sector)	0.97	-1.00	-0.73	0.75	0.39	0.87	0.73	0.91
<i>Workers</i>								
Corr(C sector, Aggregate)	0.99		-0.65	0.57	-0.09	0.35	0.41	-0.64
Corr(I sector, Aggregate)	0.99		0.86	0.97	0.95	0.97	0.97	0.88
Corr(C sector, I sector)	0.96		-0.94	0.42	-0.31	0.23	0.30	-0.91
<i>Hours per Worker</i>								
Corr(C sector, Aggregate)	0.93		0.82	0.43	0.69	0.39	0.59	0.99
Corr(I sector, Aggregate)	0.95		0.79	0.52	0.70	0.49	0.40	0.98
Corr(C sector, I sector)	0.81		0.30	-0.54	-0.03	-0.61	-0.50	0.95
<i>Vacancies (JOLTS)</i>								
Corr(C sector, Aggregate)	1.00		0.45	0.95	0.91	0.92	0.92	0.80
Corr(I sector, Aggregate)	0.96		0.71	0.99	0.99	0.99	0.99	0.81
Corr(C sector, I sector)	0.92		-0.31	0.93	0.85	0.87	0.88	0.30
Standard Deviation (relative to std. dev of output)								
Vacancies (Aggregate;Index)	8.79							
Vacancies (Aggregate;JOLTS)	11.30		0.74	6.56	6.50	6.42	6.84	0.83
Vacancies (Consumption;JOLTS)	10.67		0.60	5.94	5.60	5.71	6.10	0.55
Vacancies (Investment;JOLTS)	15.08		8.36	14.21	18.63	16.17	17.03	6.24
Unemployment	7.46		1.39	6.03	6.41	6.10	6.44	1.23

Notes: Top: Correlations of labor market variables for various specifications of the model. Bottom: Standard deviations of selected variables relative to the standard deviation of output. Results are based on quarterly averages of monthly simulations. Each simulation is 600 months long and repeated 500 times. Statistical moments are measured as deviations of log of a variable from HP trend (smoothing parameter 1600).

500 times. I report the results in Table 4, based on quarterly averages of monthly simulations. The top part describes the sectoral labor market correlations (total hours, number of workers, hours per worker, and vacancies). In the bottom part I report the standard deviations of selected variables relative to the standard deviation of output.²²

I start the analysis with a baseline model that includes search and matching friction and capital adjustment costs. The baseline model generates slightly higher correlations of sectoral total hours relative to a frictionless model. The baseline model is still at odds with the data

²²Consistent with the data moments, the results are based on quarterly averages of the monthly simulations, in logs and as deviations from HP trend (smoothing parameter 1,600).

due to negative total hours and extensive margin co-movement. The source of this shortcoming relies on the result that vacancies in the consumption sector hardly respond to productivity shocks, while vacancies in the investment sector demonstrate a substantial positive response, leading to a negative correlation of vacancies and employment across sectors. I then show that adding partial wage stickiness to the model substantially improves the results both for total hours and the extensive margin of employment. To isolate the role of capital adjustment costs I also consider a model without such costs. The conclusion is that both sticky wages and capital adjustment costs have a significant role, but neither is sufficient.²³

5.1 Baseline Model

The baseline model has matching friction with flexible (Nash) wages and capital adjustment costs (column 3 of Table 4). This model slightly improves over the frictionless model. The correlations of total hours in the consumption sector with total hours in the economy (-0.58) and with total hours in the investment sector (-0.73) are higher than the frictionless two sector RBC model (column 2). However, these correlations are still negative and the extensive margin correlations are even more negative. The correlations of the number of workers in the consumption sector with the aggregate number of workers and with the number of workers in the investment sector are -0.65 and -0.94, respectively.

The behavior of sectoral vacancies highlights two issues. First, vacancies in the consumption sector hardly respond. The ratio of standard deviation of consumption sector vacancies to output is 0.6, just over *one twentieth* of the magnitude in the data (10.67). In addition, the response of vacancies varies across sectors, and the correlation of vacancies in the consumption and investment sectors is -0.31. The solid line in Figure 2 plots the impulse response functions of sectoral vacancies to a positive productivity shock and leads to similar conclusion: the response of vacancies in the consumption sector is small, sometimes negative, and different from the response of vacancies in the investment sector.

The insufficient response of consumption sector vacancies is similar to the results in Shimer (2005), who shows that the textbook one sector DMP model fails to generate the

²³In this respect, the results of this paper are complementary to the conclusions made by DiCecio (2009) regarding the importance of sticky nominal wages in generating pro-cyclicality of total hours worked in the consumption sector.

Figure 2: Response of Vacancies: Baseline and the Full Model

Notes: Response of vacancies in the Investment sector (top) and the Consumption sector (bottom) to a one standard deviation positive shock to TFP. The solid lines represent the baseline model. Dashed lines represent the model with pro-cyclical employer's bargaining power (wage stickiness) and capital adjustment costs.

observed volatilities of vacancies and unemployment. Using a slightly modified model, Shimer (2010) shows that vacancies may even be completely neutral to productivity changes in a one sector model. The consumption sector and aggregate vacancies in my model are clearly in line with Shimer's results. The relatively high volatility of investment sector vacancies is surprising given the standard structure of the search and matching model. An intuitive reason is related to the motivation to invest in order to smooth consumption. Shimer (2010) points out that the introduction of capital to the one sector model breaks the neutrality of vacancies but still results in a small response. The results of the baseline model are consistent with Shimer's result in the following way: neutrality breaks in the investment sector as this is a good time to invest. However, since the investment sector is small, the aggregate effect is small, and therefore aggregate vacancies and unemployment demonstrate insufficient variability in the model.

5.2 Partial Wage Stickiness

A natural extension of the model is the inclusion of partial wage stickiness. Hall (2005), Hall and Milgrom (2008), Gertler and Trigari (2009), and Hall (2009) show that in a one sector economy sticky wages make unemployment volatility higher and closer to the observed magnitudes in the data. A fairly intuitive explanation is that when wages are partially sticky, they do not fully respond to the increase in productivity, therefore employers' effective share of the match surplus is higher and their incentive to create more vacancies is stronger than the Nash case. Similarly, facing a negative productivity shock, employers pay a higher wage relative to Nash bargain, amplifying the negative vacancy response. To generate pro-cyclical employer's bargaining power I calibrate $\eta = 0.91$ in equation (19). The choice of η is consistent with standard deviation of wages that is roughly 55 percent of the standard deviation of output, within the range of evidence on volatility of hourly wage.²⁴ The variability of wages is lower than the wage variability for newly hired workers in Haefke, Sonntag, and van Rens (2008). However, their sample starts only in 1979 and they show that wages of both stayers and new hires are substantially more volatile in later periods starting 1984. As my data begins in 1964, the variability of wages seem to be consistent with the evidence. In addition, the elasticity of average wages with respect to TFP is approximately 0.85, well within the confidence interval around the point estimate in table 9 of Haefke, Sonntag, and van Rens (2008).

The results of the model with sticky wages are presented in column 4 of Table 4. The model performs better on a number of dimensions. The correlations of total hours in the consumption sector with total hours in the aggregate economy (0.86) and with total hours in the investment sector (0.75) are significantly higher and closer to their data counterparts. Vacancies across the two sectors are now highly correlated (0.93) leading to a substantial improvement in the extensive margin correlations. The correlation between the number of workers in the consumption sector and the number of workers in the aggregate economy and the investment sector are 0.57 and 0.42, respectively. Figure 2 illustrates the contribution of sticky wages. As expected, consumption sector vacancies are more responsive and more synchronized with investment sector vacancies, leading to higher correlation of sectoral employment. The negative correlation of hours per worker is a concern. However, the correlation magnitude is

²⁴See Stock and Watson (1999), and Haefke, Sonntag, and van Rens (2008).

Table 5: Cross-sector covariances

	Data	Sticky Wages
Total Hours	1.00	1.00
Workers	0.76	0.68
Hours per worker	0.04	-0.07

Notes: Cross-section covariance of workers and hours divided by cross-section covariance of total hours. In the data and the model the ratio of covariance of hours per worker to the covariance of total hours is close to zero, indicating that co-movement of hours has a secondary role in the data and in the model. All simulations are quarterly averages of monthly simulations. Each simulation is 600 months long and repeated 500 times. Statistical moments are measured as deviation of log of a variable from HP trend (smoothing parameter 1,600).

affected by a very low variability of hours worked in the model. Table 5 provides intuition—the ratio of *covariance* of hours per worker across sectors to the covariance of total hours of work is very close to zero both in the data (0.04) and according to the model (-0.07). The covariance of the extensive margin contributes significantly more to total hours covariance both in the data (0.76) and in the model (0.68). Finally (and not surprisingly), the model provides a better account of unemployment variability. Unemployment is 6 times as volatile as output, comparing to being just slightly more volatile than output in the baseline model.

The model is also informative about the role of sticky wages and the importance of extensive margin correlations. DiCecio (2009) argues that sticky wages is the key mechanism that generates sectoral labor co-movement. To assess this claim, consider a model with sticky wages and without capital adjustment costs. Judging by the total hours correlations in column 5 of Table 4, that argument appears plausible. Total hours worked in the consumption sector are positively correlated with total hours worked in the economy and the investment sector. However, the model fails to generate positive extensive margin correlations that are arguably at the heart of sectoral co-movement.

In addition, previous attempts to resolve the “co-movement puzzle” usually focus on generating positive correlation between total hours in the consumption sector and some aggregate variable, such as output or total hours worked.²⁵ The results in Table 4 illustrate that this is an easier task than generating positive correlations across the two sectors. A natural

²⁵See, for example, Benhabib, Rogerson, and Wright (1991), Hornstein and Praschnik (1997), Huffman and Wynne (1999), Boldrin, Christiano, and Fisher (2001), Jaimovich and Rebelo (2009), and DiCecio (2009)

Figure 3: Response of Vacancies (with and without capital adjustment costs)

Notes: Response of vacancies in the Investment sector (top) and the Consumption sector (bottom) to a one standard deviation positive shock to TFP. The solid lines represent the model with capital adjustment costs and sticky wages. The dashed lines represent the model with sticky wages but without capital adjustment costs.

explanation is composition – as the consumption sector is large, it accounts for a large proportion of aggregate labor input and production, and therefore increases the correlation between consumption sector variables and aggregate variables.

Further increases in the degree of wage stickiness can potentially resolve these issues, but at a price of reducing the variability of wages to just slightly above zero, in sharp contrast to the data. The sectoral vacancy responses in Figure 3 provide intuition. When the model includes sticky wages only, the increased incentives lead to more significant responses in both sectors, but the initial differences are still large. With capital adjustment costs the initial difference is smaller, leading to a higher correlations of sectoral vacancies and employment.

The set of findings leads to the conclusion that wage stickiness is indeed important in generating positive cross sector correlations, but it is insufficient. The effects of capital adjustment costs through complementarity play a significant role.

5.3 The Role of Capital Adjustment Costs

The dominant force that generates negative comovement in a frictionless model is the increased demand for investment goods, hence both capital and labor are expected to flow from the

consumption sector to the investment sector.²⁶ In a model with no restrictions on capital, capital indeed flows from the consumption to the investment sector through opposite responses of sectoral investments. The correlation between sectoral investments is -0.83. Capital and labor are complements in production, hence the fact the capital flows from the consumption to the investment sector induces a lower marginal product of labor and lower demand for labor in the consumption sector.

To fully understand the role of capital adjustment costs in the model examine Figure 4 that plots the response of sectoral capital stock under different assumptions. When aggregate capital is predetermined and can be freely reallocated across sectors, capital flows from the consumption sector to the investment sector immediately. The existence of adjustment costs slightly reduces the magnitude, but does not eliminate the capital flow. Similarly, when sectoral capital stocks are predetermined and there are no capital adjustment costs, the flow of capital from consumption to investment occurs through investment in the “I” sector, and disinvestment in the “C” sector. The only difference between the response under this assumption and the response under free reallocation is a delay of one period. Thus, only the combination of predetermined sectoral capital and capital adjustment cost imposes an actual cost on the reallocation process.²⁷ The model with capital adjustment costs makes sectoral investment and capital stocks positively correlated. As capital in both sectors increase, the relative differences of the marginal product of labor are smaller, leading to higher correlations of sectoral vacancies and employment.

5.4 An Alternative to Sticky Wages: The Hagedorn-Manovskii Calibration

The results thus far suggest that generating positive comovement of sectoral vacancies and employment is related to resolving the unemployment volatility puzzle. Following Shimer (2005), various extensions to the one sector DMP model have been proposed as resolutions to the unemployment volatility puzzle. I consider the alternative calibration suggested by Hagedorn and Manovskii (2008) – a solution to the unemployment volatility puzzle – and show

²⁶Huffman and Wynne (1999) show that sectoral investment flows are positively correlated with output at annual frequency.

²⁷Huffman and Wynne (1999) make a similar argument using a model with costs on intra-temporal capital reallocation. Their model does a good job in matching correlations of sectoral hours with output. As the baseline model results show, a model with only capital adjustment costs slightly increases the total hours correlations yet fails to generate positive total hours and extensive margin correlations.

Figure 4: Response of Sectoral Capital

Notes: Top figures represent models with no adjustment costs. The bottom figures represent models with adjustment costs. Solid lines indicate that aggregate capital is a state variable, hence capital can be reallocated without delay. Dashed lines indicate a model where sectoral capital stocks are predetermined, hence reallocation is delayed.

that it also increases the cross-sector correlations of vacancies and employment.

Hagedorn and Manovskii (2008) argue that the problem is not in the DMP model but in the way it is typically calibrated. They suggest an alternative calibration of two elements. First, increase the employers' bargaining power to $\tau = 0.95$. Then, the value of non-market activity, or the flow value of unemployment, should be much higher than the usual calibration.²⁸ The latter implies a small difference between the marginal product of an employee and the flow value of unemployment, leading to low intra-period profits from a job. Therefore, small changes in productivity generate significant percentage changes in profits. Combined with the increased bargaining power the incentives to post vacancies are much stronger for a given increase in productivity.

In my model, the flow value of unemployment is a composition of utility differences, consumption differences and the unemployment benefits b :

$$z_c = \frac{U(c_u, 0) - U(c_c, h_c)}{\lambda} + b + P_c c_c - P_c c_u \quad (21)$$

²⁸In most DMP models this is a single parameter z . Early models calibrated this parameter to 0.4, but recent evidence suggest that the magnitude should be higher at around 0.7. Hagedorn and Manovskii (2008) calibration suggest a magnitude of around 0.95.

My baseline calibration implies an average value of $z = 0.75$, in line with recent standard calibration of DMP models.²⁹ In order to increase the flow value of unemployment, there are two options. First, the unemployment compensation can be increased. Second, the utility function parameters, and the Frisch labor supply elasticity in particular, can be re-calibrated. To assess whether this calibration strategy can be used instead of sticky wages, I choose high values of unemployment such that the aggregate unemployment volatility is similar to the one implied by a model with sticky wages. The other elements of the model are unchanged.

The results in columns 6 and 7 of Table 4 show that positive co-movement can be achieved. However, there are a concerns with respect to the alternative calibration. First, increasing the flow value of unemployment through unemployment benefits b result in unemployment compensation that is around 48 percent of average compensation - higher then most existing estimates. Alternatively, re-calibrating the utility function implies a Frisch labor supply elasticity of around 2.75, a significantly higher elasticity than most existing estimates. The latter also results in relatively high volatility of hours per worker in the consumption sector. The ratio of standard deviations of hours per worker to the standard deviation of number of workers in the “C” sector is 0.65, while the corresponding ratio in the data is 0.27. I interpret these results as indicative that the two puzzles are indeed correlated. In addition, in the context of this model, wage stickiness is slightly preferable over the alternative calibration.

5.5 The Importance of Explicit Treatment of the Extensive Margin

The motivation for the model is the empirical observation that the key source of the cross-sector correlations of total hours worked is the variability of the extensive margin. In section 5.2 I illustrated how looking at total hours worked alone may lead to an incomplete interpretation regarding the mechanisms at work. In this section I provide another example for a mechanism—preferences with no income effect on labor supply—that generates strong positive cross-sector correlation of total hours worked, but a strong *negative* cross-sector correlation of the number of workers. Jaimovich and Rebelo (2009) show that using utility function with very low income effect on labor supply generates a strong positive correlation between total hours worked in the consumption and investment sectors. The reason is that during high productiv-

²⁹see, for instance, Hall and Milgrom (2008), Hall (2009), and Pissarides (2009).

ity periods households significantly increase their hours supply as there is only a substitution effect, without an offsetting income effect.

To assess the contribution of no income effect, I consider the utility function suggested by Greenwood, Hercowitz, and Huffman (1988)

$$U(c, h) = \ln \left(c - \frac{\theta}{1 + \frac{1}{\psi}} h^{1 + \frac{1}{\psi}} \right) \quad (22)$$

where ψ is again the Frisch elasticity of labor supply.

The results are reported in column 8 of Table 4. The correlation of total hours worked in the consumption sector with total hours worked in the investment sector is 0.91. However, the correlation of the number of workers in the consumption sector with the number of workers in the investment sector is -0.91. Consistent with intuition, households substantially increase hours supply in this model, thus the adjustment of labor input is achieved mostly by changing hours worked per worker, rather than the number workers. The standard deviation of hours worked per worker in the consumption sector is three times as high the the standard deviation of the number of workers—a sharp contrast to the data. This set of results provides another support for the need to explicitly model and analyze the behavior of extensive margin across sectors.

5.6 Heterogenous Matching Technology Parameters

A potential concern in the context of my model is whether the matching function parameters are different across sectors. To address this concern, I classify the sectoral JOLTS hirings and vacancies data to consumption and investment, and estimate

$$\begin{aligned} \ln(\phi_t^j) = & \underbrace{\text{const}}_{\substack{-0.251 \\ (0.051)}} + \underbrace{D^i}_{\substack{-0.376 \\ (0.108)}} + \\ & \underbrace{\gamma}_{\substack{0.727 \\ (0.046)}} \times \ln \left(\frac{v_{t-1}^j}{1 - n_t} \right) + \underbrace{\gamma \times D^i}_{\substack{-0.121 \\ 0.059}} \times \ln \left(\frac{v_{t-1}^j}{1 - n_t} \right) + \epsilon_{j,t} \end{aligned} \quad (23)$$

where D^i denotes a dummy variable for the investment sector.³⁰ The estimation results suggest that both the elasticity and the matching efficiency parameters are higher for the consumption sector relative to the investment sector. As the elasticity estimates are once again above the 0.3 - 0.5 range suggested by Petrongolo and Pissarides (2001), I scale the parameters down to set the weighted average of elasticities to 0.5, while keeping the ratio between the sectoral elasticities according to the estimation.³¹ I follow the same approach for the matching efficiency parameter in order to have the same steady state unemployment as in the baseline calibration (5.9 percent).

Relative to the symmetric calibration, the consumption sector total hours, vacancies, and number of workers are now more correlated with the investment sector and the aggregate economy. Figure 5 illustrates the intuition through vacancy responses. In all the models there exists a stronger incentive to post vacancies in the investment sector relative to the consumption sector. The fact that the elasticity of the matching function with respect to vacancies is higher in the consumption sector implies that the return to posting a vacancy is relatively higher in the consumption sector. The result is a narrower difference of the sectoral vacancy responses that leads to higher employment correlations.

³⁰The classification of sectoral vacancies is identical to the classification of employment variables that is described above.

³¹The resulting calibrated parameters are $\gamma_c = 0.51, \gamma_i = 0.834 \times \gamma_c = 0.425, \chi_c = 0.408$, and $\chi_i = 0.6 \times \chi_c = 0.281$

Figure 5: Response of Vacancies with non-identical sectoral matching functions

Notes: Response of vacancies in the Investment sector (top) and the Consumption sector (bottom) to a one standard deviation positive shock to TFP.

6 Conclusion

In this paper I argue that most of the observed positive correlation of sectoral total hours worked originates in variability of the number of workers. I construct a two sector business cycle model that includes search and matching friction, capital adjustment costs, and partial wage stickiness. The model allows for the analysis of a richer set of labor market variables, unlike previous literature that focus only on total hours. Model simulations predict correlations of sectoral total hours, vacancies, and employment that are substantially closer to the data relative to the predictions of frictionless models.

I show that generating positive co-movement of the number of workers is linked to the solution to another shortcoming of search and matching models - insufficient responsiveness of vacancies and unemployment. In a baseline version of the model that includes search and matching friction and capital adjustment costs, vacancies in the consumption sector hardly respond. Adding partial wage stickiness to the model increases the responsiveness of vacancies in the consumption sector and induces stronger co-movement of sectoral vacancies and employment.

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A Deriving wages and hours worked

In this section I derive the hourly wage equation and the condition for hours worked. I follow closely the method in Shimer (2010), with some required adjustments for the current model.

The envelope condition for employment in the consumption sector is identical to equation (8). As explained in section 3.2, this is the marginal value that the households gains from having one more member working in the consumption sector. A similar term exists for the investment sector.

$$V_{n_c} = U(c_c, h_c) - U(c_u, 0) + \lambda(w_c h_c - b + P_c c_u - P_c c_c) + (1 - \pi_c)\beta \mathbf{E} V'_{n_c} - \phi_c \beta \mathbf{E} V'_{n_c} - \phi_i \beta \mathbf{E} V'_{n_i} \quad (\text{A.1})$$

As in section 2.4.2 of Shimer (2010), let $\tilde{V}_{n_c}$ denote the marginal value to a household with an equilibrium level of employment, of having one member working some level of hours $\tilde{h}$ at some wage $\tilde{w}$, rather than being unemployed.

$$\tilde{V}_{n_c} = U(c_c, \tilde{h}_c) - U(c_c, h_c) + \lambda(\tilde{w}_c \tilde{h}_c - w_c h_c) + V_{n_c} \quad (\text{A.2})$$

The marginal value to a firm employing a worker is explained in section 3.3, and expressed by:

$$J_{n_c} = AP_c F_{n_c} - w_c h_c + (1 - \pi_c) \mathbf{E} [Q' J'_{n_c}] \quad (\text{A.3})$$

Denoting total hours of work by $H_c = n_c h_c$, we can write the last equation as

$$J_{n_c} = AP_c F_{H_c} h_c - w_c h_c + (1 - \pi_c) \mathbf{E} [Q' J'_{n_c}] \quad (\text{A.4})$$

Let $\tilde{J}_{n_c}$ denote the marginal value to the firm of having a worker employed at an arbitrary wage $\tilde{w}_c$ and working an arbitrary number of hours $\tilde{h}_c$.

$$\tilde{J}_{n_c} = AP_c F_{H_c} (\tilde{h}_c - h_c) - (\tilde{w}_c \tilde{h}_c - w_c h_c) + J_{n_c} \quad (\text{A.5})$$

Assuming Nash bargaining, the equilibrium wage and hours must satisfy the Nash bargaining solution

$$(w_c, h_c) = \operatorname{argmax}_{\tilde{w}_c, \tilde{h}_c} \tilde{V}_{n_c}^{1-\tau} \tilde{J}_{n_c}^\tau \quad (\text{A.6})$$

The first order condition with respect to $\tilde{w}_c$ yields:

$$(1 - \tau) \tilde{V}_{n_c}^{-\tau} \tilde{J}_{n_c}^\tau (\lambda \tilde{h}_c) = \tau \tilde{V}_{n_c}^{1-\tau} \tilde{J}_{n_c}^{\tau-1} \tilde{h}_c \Rightarrow \tau \tilde{V}_{n_c} = (1 - \tau) \lambda \tilde{J}_{n_c} \quad (\text{A.7})$$

The first order condition with respect to $\tilde{h}_c$ yields:

$$\begin{aligned} \tau [AP_c F_{H_c} - \tilde{w}_c] \tilde{V}_{n_c}^{1-\tau} \tilde{J}_{n_c}^{\tau-1} &= - [U_{\tilde{h}_c} + \lambda \tilde{w}_c] (1 - \tau) \tilde{V}_{n_c}^{-\tau} \tilde{J}_{n_c}^\tau \\ \Rightarrow \\ \tau \tilde{V}_{n_c} [AP_c F_{H_c} - \tilde{w}_c] &= -(1 - \tau) \lambda \tilde{J}_{n_c} \left[\frac{U_{\tilde{h}_c}}{\lambda} + \tilde{w}_c \right] \\ \Rightarrow \\ AP_c F_{H_c} - \tilde{w}_c &= - \frac{U_{\tilde{h}_c}}{\lambda} - \tilde{w}_c \end{aligned} \quad (\text{A.8})$$

where in the last step I use the fact that $\tau \tilde{V}_{n_c} = (1 - \tau) \lambda \tilde{J}_{n_c}$.

The resulting condition $AP_c F_{H_c} = - \frac{U_{\tilde{h}_c}}{\lambda}$ is exactly equation (19) (taking into account the fact that $\lambda = U_c$).

To solve for the hourly wage, I use the fact that in equilibrium, $\tilde{V}_{n_c} = V_{n_c}$ and $\tilde{J}_{n_c} = J_{n_c}$. Substituting this in the condition $\tau \tilde{V}_{n_c} = (1 - \tau) \lambda \tilde{J}_{n_c}$, results in the following:

$$\begin{aligned} \tau \left(U(c_c, h_c) - U(c_u, 0) + \lambda(w_c h_c - b + P_c c_u - P_c c_c) + (1 - \pi_c) \beta \mathbf{E} V'_{n_c} - \phi_c \beta \mathbf{E} V'_{n_c} - \phi_i \beta \mathbf{E} V'_{n_i} \right) = \\ (1 - \tau) \lambda (AP_c F_{n_c} - w_c h_c + (1 - \pi_c) \mathbf{E} [Q' J'_{n_c}]) \end{aligned} \quad (\text{A.9})$$

To derive the hourly wage, I use the following steps:

1. On the right hand side, and using the definition of Q' : $(1 - \tau) \lambda (1 - \pi_c) \mathbf{E} [Q' J'_{n_c}] = (1 - \tau) \beta \mathbf{E} [\lambda' J'_{n_c}]$. As the first order condition with respect to w must hold every period, this implies that the last term on the right hand side of the equation cancels with $\tau(1 -$

$\pi_c)\beta\mathbf{E}V'_{n_c}$ on the left hand side.

2. Divide both sides of the (remaining) equation by λ .

3. Focus on the term $-\tau\phi_c\beta\mathbf{E}V'_{n_c}\frac{1}{\lambda}$.

(a) Using the bargaining first order condition, rewrite this term as $-(1-\tau)\phi_c\beta\mathbf{E}\left[\frac{\lambda'}{\lambda}J'_{n_c}\right] = -(1-\tau)\phi_c\mathbf{E}[Q'J'_{n_c}]$

(b) Using the matching function: $q_c\frac{v_c}{1-n_c-n_i} = \phi_c$. Therefore we can substitute for ϕ_c :
 $-(1-\tau)\phi_c\mathbf{E}[Q'J'_{n_c}] = -(1-\tau)q_c\frac{v_c}{1-n_c-n_i}\mathbf{E}[Q'J'_{n_c}]$

(c) Now note that the zero profit condition requires that $P_c a = q_c\mathbf{E}[Q'J'_{n_c}]$.

(d) Therefore rewrite the entire term as $aP_c\frac{v_c}{1-n_c-n_i}$

4. Repeat step 3 for the term $-\phi_i\beta\mathbf{E}V'_{n_i}$ to get $aP_i\frac{v_i}{1-n_c-n_i}$

5. Rearrange to get the hourly wage equation as in equation (18) in the main text.