Government Policy with Time Inconsistent Voters

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Abstract

Behavioral economics presents a “paternalistic” rationale for government intervention. Current literature focuses on benevolent government. This paper introduces politicians who may indulge/exploit these behavioral biases. We present an analysis of the novel features that arise when the political process is populated by voters who may be time inconsistent, a’la Phelps and Polak (1968) and Laibson (1997). Time inconsistent voters exhibit demand for commitment. We show that electorally accountable politicians may choose policies that interfere with individuals’ desire to commit, and that government may not be very effective in satisfying the demand for commitment.

1 Introduction

An important and influential approach to government policy has grown out of the field of behavioral economics.\footnote{Camerer et al (2004) contains a number of “second generation” contributions to behavioral economics. Frederick et al. (2002) surveys the experimental evidence on time discounting. Della Vigna (2009) surveys evidence from the field. See also Thaler and Sunstein (2009).} A number of contributors to this area (e.g., Camerer, Lowenstein, O’Donoughue, Rabin, and Thaler), argue that some form of government policy interventions can be justified by “paternalistic attitudes” even in cases outside the realm of the textbook approach to public policy, i.e., even absent externalities, public goods, and asymmetric information. In this context, a paternalistic government is viewed as a benevolent planner who designs policy to help agents make better decisions for the agent’s own interests.\footnote{This form of paternalism is controversial, partly because it drastically deviates from standard normative economics. For contrasting points of view on this issue, see various contributions in the edited volume by Caplin and Schotter (2008).}

This paper presents a simple model, where we consider what happens when, instead of a benevolent planner, the political process determines the design of policy. Our approach
can essentially be viewed as “behavioral economics meets political economy.” We follow the public choice tradition of considering how public policy is determined in an environment where there is no social planner: politicians have selfish objectives such as gaining re-election, or obtaining rents from lobbyists. In environments where voters suffer from behavioral biases, will politicians seeking election exploit/indulge the voters’ behavioral distortions? Are behavioral distortions amenable to aggregation into collective action? What are the implications for the constitutional scope of government activity?

These questions can in principle be addressed in several environments, depending on the specific behavioral distortion, or the political process under consideration. For instance, it would seem fruitful to introduce political economy considerations in economies populated by a variety of ‘behavioral agents,’ that is, agents suffering from distortions in beliefs, framing, and a variety of other biases that have been considered in the literature arguing in favor of paternalistic policies. Even relatively noninvasive policies, like those called “nudges” by Sunstein and Thaler (2009), seem fitting for this kind of analysis. We expect to investigate some of these issues in the future. This paper, however, focuses on the widely studied case of time inconsistency: agents have preferences that display present-bias or quasi-hyperbolic discounting a’ la Phelps-Polak (1968) and Laibson (1997). It is well known that these preferences can lead to reversals that are not consistent with the standard models of exponential discounting. Some of the issues related to time inconsistent preferences that we highlight can be represented as problems of self-control, and can also be studied in models of temptation and self-control a’ la Gul-Pesendorfer (2001, 2004, 2007). Initial analysis suggests that new and interesting phenomena arise in these environments, but in this paper we focus on the quasi-hyperbolic model.

Self-control problems can lead to procrastination (doing things too late), preproperation (doing things too early) (see O’Donoghue-Rabin 1999), insufficient savings for retirement (Laibson, Repetto, and Tobacman 1997), harmful obesity and addictions (Gul-Pesendorfer 2007, O’Donoghue-Rabin 2000), etc.. These self control problems also identify a demand for commitment (rehab clinics, illiquid assets with costly withdrawal, etc.) that cannot arise with exponential discounting.

We ask how government policy responds to voters’ time inconsistency, and how government intervention affects and responds to the demand for commitment. As a starting point in this paper, we assume the simplest and most standard political structure: Downsian competition among two office-seeking candidates.\(^3\)

We address the issue of **fiscal irresponsibility and public debt.** Some of the behav-

\(^3\)Our results are independent of candidates’ time preferences.
ioral economics literature focuses on inefficiently low savings under *laissez faire*. In order to understand how government policy may affect national savings, it is important to understand how political incentives for deficits are affected by voters’ time inconsistency. To study this issue, we embed politically determined government transfers in a highly stylized consumption-savings problem. We endow agents with ample commitment options by means of investment in illiquid assets. Agents use illiquid assets to constrain their future selves’ consumption plans. The environment is designed to ensure that, absent government intervention, agents can guarantee their commitment path of consumption. We introduce government intervention by allowing candidates to offer deficit-financed transfers to voters, subject to a maximal debt constraint. We show that for moderate debt constraints, in equilibrium, candidates choose the maximal debt, but voters are able to undo this by rebalancing their portfolios ex ante: a modified Ricardian equivalence result. When debt limits are high, however, government debt completely undermines individuals’ ability to commit. We then introduce distortions induced by government debt, and show that when the marginal distortions are not too high relative to the present bias of the decisive voter, equilibrium debt can still be high, leading to high total distortions. The logic is the following. Because debt is determined by voters’ collective choices, individual saving decisions in prior periods have no impact on debt. In a three period economy, for instance, each individual voter has a private incentive to try to undo expected second period debt by an appropriate mix of liquid and illiquid assets, saving less for period 2 and more for period 3. But this individual optimization will, in the aggregate, generate demand for transfers in the period 2, leading to voting for even higher debt. Thus, portfolio decisions in period 1 produce collective demand for debt in the second period, even if this is distortionary. Note that this vicious cycle is limited in the context of an individual being offered liquid assets to undo his prior commitments (as, for instance, in Gottlieb 2008), because such an individual can affect his own choices in the second period, while a voter cannot affect aggregate choices in subsequent periods. This analysis offers a new rationale for balanced budget rules in constitutions.

Our paper also contributes to the political economy literature on government debt. We offer a novel explanation of debt in an environment where previously investigated forces determining debt are inoperative. This approach can potentially be useful for thinking about austerity plans that may be suggested by analysts or international organizations such as the IMF to remedy unsustainable fiscal situations. For instance, in some contexts, one can borrow insights from behavioral economics such as the idea of “save more tomorrow” proposed by Thaler and Benartzi (2004) to argue that delayed austerity may be precisely the kind of reform that may be required, instead of being condemned as “timid reforms.”
2 Related Literature

Some authors (Benjamin and Laibson 2003, Caplan 2007, Glaeser 2006, Rizzo and Whitman 2009 a, b) have informally made the point that when government is not run by a benevolent social planner but by politicians influenced by voting decisions, it is not clear that government intervention is beneficial. In fact, Glaeser and Caplan explicitly make the case that, if voters are boundedly rational, then the case for limited government may be even stronger than in standard models. None of these papers considers time inconsistent agents. Bendor et al. (2011) present models based on aspiration-based learning to examine a wide variety of political phenomena.

Our paper is also related to the literature on the political economy of government debt. Some of the paper in this literature explain debt as the outcome of a struggle between different groups in the population who want to gain more control over resources. The reason debt is accumulated is that the group that is in power today may not be in power tomorrow and debt is a way to take advantage of this temporary power. Thus, for instance Cukierman and Meltzer (1989) and Song, Storesletten, and Zilibotti (2010) argue that debt is a tool to redistribute resources across generations. Persson and Svensson (1989), Alesina and Tabellini (1990), and Tabellini and Alesina (1990) argue that debt is a way to tie the hands of future governments which have different preferences from the current one. In Tabellini and Alesina (1990) voters choose the composition of public spending in an environment where the median voter theorem applies. If the median voter remains the same in both periods the equilibrium involves budget balance. If the median voter tomorrow has different preferences, the current median voter may choose to run a budget deficit to take advantage of his temporary power and tie the hands of the future government. The equilibrium may also involve a budget surplus because there is an "insurance" component that links the two periods as well: a surplus tends to equalize the median voter’s utility in the two periods. Tabellini and Alesina give conditions such that deficits will be incurred and show that increased polarization leads to larger deficits. Battaglini and Coate (2008) present a dynamic model of taxation and debt, where a rich policy space is considered within a legislative bargaining environment. Velasco (1996). He discusses a model where government resources are a “common property” out of which interest groups can finance their own consumption. Deficits arise in his model because of a dynamic “common pools” problem. Lizzeri (1999) presents a model of debt as a tool of redistributive politics.\footnote{Tabellini (1991) also illustrates how debt and social security differ as distributional instruments in an overlapping generations environment.}

In all these models voters are time consistent. Krusell, Kurusçu, and Smith (2002, 2010)
examine government policy for agents who suffer self-control problems. Krusell, Kurusçu, and Smith (2002) consider a neoclassical growth model with quasi-hyperbolic consumers. They show that, when government is benevolent but cannot commit, decentralized allocations are Pareto superior. This is due to a general equilibrium effect of savings that exacerbates an under-saving problem. Benabou and Tirole (2006) discuss how endogenously biased beliefs that are chosen by individuals for self-motivation can generate a belief in a just (unjust) world and ultimately affect redistributive politics.

3 A Model of Fiscal Irresponsibility

2.1 economy

We first consider a particularly stark three period model to highlight the basic forces at play. We then extend the model in a number of directions to show robustness and to consider additional phenomena.

There is a measure 1 of voters who live for three periods. To make things particularly simple, assume that in period 1 voters have an endowment $e$ from which to finance consumption over three periods, and no endowment is available in the other two periods. As in Laibson (1997), preferences over consumption sequence $c_1, c_2, c_3$ are given by

$$U_1(c_1, c_2, c_3) = u(c_1) + \beta \delta u(c_2) + \beta \delta^2 u(c_3),$$

$$U_2(c_2, c_3) = u(c_2) + \beta \delta u(c_3),$$

$$U(c_3) = u(c_3),$$

where $u$ is a continuous and strictly concave utility function. Because we want to study collective action, it is natural to allow for the possibility of heterogeneity in the present-bias parameter $\beta$; we assume that $\beta$ is distributed in the population according to a continuous strictly increasing cumulative distribution function $G$ on $[\underline{\beta}, \overline{\beta}]$, $0 \leq \underline{\beta} \leq \overline{\beta} \leq 1$, with median $\beta_M$. For expositional simplicity, assume that $\delta = 1$.

It is well-known that an agent with these preferences suffers a time inconsistency. Let $c^*_1(\beta)$, $c^*_2(\beta)$, and $c^*_3(\beta)$ denote the optimal consumption sequence with commitment in period 1, while $c^U_1(\beta)$, $c^U_2(\beta)$, and $c^U_3(\beta)$ denote the optimal consumption sequence without commitment in period 1. Finally, denote by $s^U_1(\beta)$ and $s^U_2(\beta)$ savings absent commitment. For any $\beta < 1$, and the commitment consumption sequence $c^*_1(\beta)$, $c^*_2(\beta)$, $c^*_3(\beta)$, in period 2, the consumer of type $\beta$ would like to transfer resources from the third period to the second

\[^5\text{We discuss additional sources of heterogeneity later.}\]
to obtain a consumption that is strictly higher than $c_2^*(\beta)$. This is because the optimal solution for the period 1 self requires

$$u'(c_2^*(\beta)) = u'(c_3^*(\beta))$$

which implies that

$$u'(c_2^*(\beta)) > \beta u'(c_3^*(\beta)).$$

Some of our results are much easier to state when we can order the second period consumption of agents with different $\beta$'s. For this purpose, we assume throughout most of the paper that utilities are such that $c_2^{U}(\beta) > c_2(\beta)$ and that $u'' < 0$. Stating our results under the complementary assumption complicates the statements but does not change our analysis qualitatively.\footnote{The assumption that $u'' < 0$ assures that $c_2^{U}(\beta)$ is monotonic in $\beta$, as the lemma below illustrates. In the presence of uncertainty, the assumption that $u'' > 0$ is commonly assumed (when agents are standard exponential discounter) in order to assure precautionary saving behavior. Introducing (individual or aggregate) uncertainty in our setting does not change the underlying forces described by our analysis.}

Under our assumptions, the committed and uncommitted consumption plans are monotonic in $\beta$. That is,

**Lemma 1** Second period consumption under commitment and absent commitment $c_2^*(\beta)$ and $c_2^{U}(\beta)$ are both monotonically increasing in $\beta$.\footnote{We consider the effects of first period elections later.}

We assume that at time 1 voters can choose to invest in liquid or illiquid assets and that all liquid and illiquid assets have the same exogenous rate of return of zero. We relax this assumption later which will allow us to consider additional issues but does not undermine our main results.

Illiquid assets are two-period securities that cannot be sold in period 2. Liquid assets are one period securities. Absent government intervention in period 2, by appropriate choice of the mix of liquid and illiquid assets, a voter can commit to any desired consumption stream for periods 2 and 3. The interplay between voters desire to commit in period 1 and government decisions in period 2 is a key effect in our model. We could allow for imperfect commitment and for endogenous commitment possibilities without major changes in the results. We will come back later to the consequences of allowing the degree of liquidity to be a government choice, as is the case with retirement assets for instance.

### 2.2 Polity

We now introduce a government that takes actions in periods 2 and 3.\footnote{We consider the effects of first period elections later.} There are two candidates running for office. Candidates are office motivated, receiving some positive payoffs
from each electoral victory.\footnote{It is natural to begin the analysis with standard “Downsian” candidates. We will revisit the question of ideologically motivated or virtuous candidates below.}

It will be clear that candidates’ time preferences play no role. There are simple majoritarian elections in periods 2 and 3. In period 2, each candidate offers a platform given by \((y, t)\) where \(y\) is a per capita transfer and \(t\) is a lump-sum tax. Let \(d = y - t\) denote per capita government debt in period 2. As a tie-breaking rule, we assume that whenever individuals are indifferent between the two candidates, they vote for either with equal probability.\footnote{This is akin to assuming that agents have lexicographic preferences that: a. respond to policy first, and upon indifference, to the identity of the candidate; and b. are uniformly distributed with respect to the preferred candidate.}

In what follows, we first consider a benchmark in which debt and taxation are non distortionary and then move on to the more interesting case of distortions. Debt is financed by foreign lenders at zero interest rate (tantamount to assuming a small open economy),\footnote{Studying interest rate determination in a closed economy can be allowed without destroying the main issues that arise in our model. General equilibrium effects are subtle, however, when agents are quasi-hyperbolic (see Krussel et al. 2002).} to be repaid by third period revenues raised by lump-sum taxes.\footnote{For most of our analysis, period 3 elections are vacuous. As mentioned above, we return to the effects of additional periods in Section 7.} We assume that there is a constitutionally imposed borrowing limit on the government; let \(\bar{d}\) denote the per capita value of this limit.

In period 1 a voter of type \(\beta\), predicting equilibrium per capita debt levels of \(d\), chooses savings intended for period 2, \(s_{12}\) and period 3, and \(s_{13}\) to solve

\[
\max_{s_{12}, s_{13}} u(c_1) + \beta u(s_{12} + d - s_{23}) + \beta u(s_{13} + s_{23} - d).
\]

In period 2 a voter of type \(\beta\) chooses savings \(s_{23}\) to solve

\[
\max_{s_{23}} u(s_{12} + d - s_{23}) + \beta u(s_{13} + s_{23} - d).
\]

The resulting optimal consumption sequence is denoted \(c_1(\beta; d), c_2(\beta; d), c_3(\beta; d)\).

### 4 Ricardian Equivalence with Time Inconsistent Voters

In this section we analyze the effects of government debt on private consumption levels. As it turns out, the magnitude of the cap on government debt is crucial: Small caps allow agents
to maintain their optimal commitment levels, while larger caps lead to distorted levels of consumption.

4.1 The Effects of Debt Ceilings

We start by showing that in our benchmark scenario without distortions there are strong electoral pressures to accumulate government debt, but, when the cap on debt $\bar{d}$ is small, in equilibrium, sophisticated voters who have access to ample commitment possibilities are able to undo the effects of debt. Consequently, government actions do not distort consumption choices relative to the commitment outcome.

**Proposition 1 (Small Debt – Ricardian Equivalence)** Assume $\bar{d} \leq \min_{\beta \in [\beta, \beta]} c^*_2(\beta)$. Then, in equilibrium both candidates offer platforms that require the government to run a deficit equal to $\bar{d}$. The equilibrium sequence of consumption choices for each voter of type $\beta$ are unaffected and are equal to $(c^*_1(\beta), c^*_2(\beta), c^*_3(\beta))$.

**Proof.** Assume by way of contradiction that in equilibrium, a deficit $d < \bar{d}$ is implemented. If this is the expected deficit, the optimal mix of savings in liquid and illiquid assets for a voter of type $\beta$ in period 1 implement the commitment sequence of consumption $c^*_1(\beta), c^*_2(\beta), c^*_3(\beta)$ by choosing $s_{12}(\beta) = c^*_2(\beta) - d$, and $s_{13}(\beta) = c^*_3(\beta) + d$. This is feasible since $d < \bar{d}$. But, by definition of $c^*_2(\beta), c^*_3(\beta), u'(c^*_2(\beta)) > \beta u'(c^*_3(\beta))$, and hence, all voters would vote for a candidate who offered a slightly higher deficit. Thus, the only deficit that can be an equilibrium is $\bar{d}$. Given a deficit of $\bar{d}$, in period 1, each voter chooses $s_{12}(\beta) = c^*_2(\beta) - \bar{d}$. Given these savings choices, none of the voters would vote for a candidate that offered a lower deficit in the second period, proving that debt and this sequence of consumption do constitute an equilibrium. ■

Intuitively, when the debt limit is low, voters can anticipate government debt and reduce savings intended for period 2 to restore the desired (commitment) sequence of consumption. In a sense, the low debt cap provides a form of commitment by the government not to succumb to individuals’ revised preferences in later periods.\footnote{Notice that this result relies on the fact that agents foresee perfectly their susceptibility to temptations. We relax this assumption of perfect foresight in the following section.} Because of this saving behavior in period 1, given any anticipated debt level in the feasible range, all voters would like even higher debt in order to consume more in period 2 (they are endogenously liquidity constrained in period 2).

This result shows that, when there are no distortions and consumers have ample commitment possibilities, Ricardian equivalence can still obtain even with voters who are not...
time consistent. However, in contrast with the standard construction for Ricardian equivalence, where government debt induces contemporaneous and offsetting increases in private savings, in our setting, government debt must be offset by prior period savings. In fact, in the range of debt discussed in Proposition 1 a surprise change in debt in period 2 would lead to changes in consumption of the same sign: a reduction in debt would reduce consumption because voters are endogenously liquidity constrained; an increase in debt would increase consumption because voters give in to temptation to consume a larger amount.\footnote{With a longer horizon, the offsetting savings must still come from one period before: they cannot come from earlier periods or they would upset commitments.}

We now consider the case in which debt limits are intermediate. In this range of debt limits, the government can effectively provide commitment abilities to only a fraction of the population, those with high desired levels of commitment consumption. Namely, when the cap is $d$, all consumers with preference parameter $\beta$ for whom $d \leq c^*_2(\beta)$, are able to modify their period 1 savings to take into account the foreseen debt levels and implement their commitment levels of consumption. However, individuals with low levels of $\beta$ lose their commitment ability and end up consuming more than their optimal commitment consumption levels. Formally, let $\beta_L(d)$ be such that $c^U_2(\beta_L) = d$ and $\beta_H(d)$ denote the value corresponding to $c^*_2(\beta_H) = d$.\footnote{Note that $c^U_2(\beta)$ and $c^*_2(\beta)$ are monotonic in $\beta$ and so these values are well-defined. If instantaneous utilities varied in time this would no longer necessarily be the case.}

**Proposition 2 (Intermediate Debt – Partial Equivalence)** Assume $\min_{\beta \in [\beta, \beta]} c^*_2(\beta) < d < \max_{\beta \in [\beta, \beta]} c^U_2(\beta)$. Then, in equilibrium both candidates offer platforms with debt equal to $d$. Equilibrium consumption choice in period 2 is given by

$$c_2(\beta) = \begin{cases} c^U_2(\beta) & \beta \leq \beta_L(d) \\ d & \beta_L(d) \leq \beta < \beta_H(d) \\ c^*_2(\beta) & \beta \geq \beta_H(d) \end{cases}$$

**Proof.** Assume by way of contradiction that in equilibrium, a deficit $d < \overline{d}$ is implemented. It follows that $\beta_L(d) > \beta_L(\overline{d})$ and $\beta_H(d) < \beta_H(\overline{d})$. The optimal mix of savings in liquid and illiquid assets for a voter of type $\beta \geq \beta_H(d)$ in period 1 is such that $s_{12}(\beta) = c^*_2(\beta) - d^*$, and $s_{13}(\beta) = c^*_{3}(\beta) + d$. This is feasible since $d < \overline{d}$.

Optimal savings for period 2 for agents with $\beta < \beta_H(d)$ are $s_{12} = 0$. Agents such that $\beta_L(d) < \beta < \beta_H(d)$ consume $d$, while agents with $\beta \leq \beta_L(d^*)$ consume $c^U_2(\beta)$ and save $d - c^U_2(\beta)$. But, with this choice of period 2 consumption, all voters with preference
parameters $\beta > \beta_L (d)$ would vote for a candidate who offered a slightly higher deficit, while the remaining voters would be indifferent. Given our tie-breaking rule, a deviating candidate who offered a slightly higher deficit, proving the only deficit that can be an equilibrium is $\overline{d}$.

Given a deficit of $\overline{d}$, behavior follows as above. Given these savings choices, all voters who find themselves constrained in period 2 (those such that $\beta > \beta_L (\overline{d})$) would vote against a candidate that offered a lower deficit in the second period. Those with $\beta < \beta_L (\overline{d})$ would be indifferent for slightly lower debt levels, and opposed to substantially lower debt levels (below $c_2^U (\beta)$). Given our tie-breaking rule, therefore, a deviation to lower debt levels would not pay.

In contrast with Proposition 1, equilibrium debt is sufficiently high that some voters are no longer able to obtain their commitment consumption sequence. Those whose commitment consumption is lowest find themselves completely unable to commit, while those whose commitment consumption is intermediate find themselves only partially able to commit. Given any candidate equilibrium debt, all individuals who are constrained desire higher debt in period 2, driving candidate incentives to offer maximal debt.

Clearly, in the scenario depicted in Proposition 2, debt is no longer neutral, but we postpone the discussion of lack of neutrality.

We now turn to environments in which the government is less constrained in its capacity to generate deficits. Namely, situations in which the cap $\overline{d}$ is large. In such economies, the government can no longer commit not to indulge agents’ period 2 preferences and consumption is distorted relative to the optimal commitment levels for all agents. Formally,

**Proposition 3 (Large Debt – Equivalence Breakdown)** Assume $\overline{d} \geq \max_{\beta \in [\underline{d}, \overline{d}]} c_2^U (\beta)$. Then, there are multiple equilibria consistent with any debt level between $\max_{\beta \in [\underline{d}, \overline{d}]} c_2^U (\beta)$ and $\overline{d}$. In any equilibrium, the sequence of consumption is given by $(c_1^U (\beta), c_2^U (\beta), c_3^U (\beta))$. 

**Proof.** We first show that the claimed outcomes are part of an equilibrium. Given any candidate equilibrium debt $d^* \geq \max_{\beta \in [\underline{d}, \overline{d}]} c_2^U (\beta)$ that is expected by voters in period 1, an optimal policy by a voter of type $\beta$ in period 1 is $s_{12} = 0, s_{13} (\beta) = c_3^U (\beta) - (d^* - c_2^U (\beta))$. In addition, given $d^*$, in equilibrium $s_{2, 3} (\beta) = d^* - c_2^U (\beta)$ is to be saved from period 2 to period 3 by type $\beta$. Given this policy, by definition of $c_2^U (\beta), c_3^U (\beta)$, we have

$$u' (c_2^U (\beta)) = \beta u' (c_3^U (\beta))$$

giving no incentive to any period-2 self to change its savings plan away from $s_{2, 3}$. Suppose now that the period-1 self were to change (e.g., increase) $s_{13}$. Then, the period-2 self would
make an offsetting change (reduction) in $s_{23}$ to restore period 2 optimality. Thus, the period-1 self has no incentive to deviate.\textsuperscript{15}

Given these policies for the voters, consider a deviation to $d < d^*$ in period 2. As long as the deviation is small ($d \geq \max c_2^U(\beta)$), all voters are indifferent (they can just make an offsetting reduction in $s_{23}$ to restore the desired consumption sequence). If the deviation is large ($d < \max c_2^U(\beta)$), then, while some voters may still be indifferent (if $d > \min c_2^U(\beta)$), there is a positive mass of voters who can no longer make such offsetting reduction in $s_{23}$. All these voters would vote against a candidate offering such a deviation. A deviation to $d > d^*$ would leave all voters indifferent because they can make offsetting changes in $s_{23}$.

Consider now a candidate equilibrium deficit $d^* < \max c_2^U(\beta)$. Such an expected deficit would constrain period 2 consumption for a positive mass of voters, leading to positive support in period 2 for a candidate offering $d > d^*$. \hfill \blacksquare

This result shows that, even when there are no distortions, if constraints on government action are loose, then government policy is distortionary, because it interferes with individuals’ ability to commit. Debt allows the government to undo the private commitments chosen by the voters in the prior period. Thus, the government acts as an enabler of the voters, substituting fiscal irresponsibility for private irresponsibility. Private commitments are not sufficient to induce consumption commitments: state commitment (such as tighter balanced budget constitutions) are essential. All period 1 selves (corresponding to all $\beta$’s) are made better off by tighter limits on $D$ that restore commitment abilities for individuals.\textsuperscript{16}

The previous results show that, with time inconsistent voters, inefficiencies may arise due to fiscal irresponsibility of elected governments despite no distortions from government debt. These results may seem closely related to the inefficiency of competitive credit markets when consumers are time inconsistent: even if consumers can buy illiquid assets to attempt to commit to a future consumption path, intermediaries such as credit card companies have the incentive to enter the market, leading to an undoing of commitment.\textsuperscript{17} However, the force underlying these results is quite different, and can lead to more dramatic inefficiencies.

\textsuperscript{15}There are multiple ways for the period 1 self to implement the uncommitted sequence, involving increasing $s_{12}$ and $s_{23}$ by the same amounts with offsetting reductions to $s_{13}$. All these are weakly dominated by the proposed sequence in the text.

\textsuperscript{16}The two previous results characterize the equilibrium when either $D$ is relatively low or relatively high. In the first case, commitment is unaffected for all $\beta$’s. In the second case, commitment is destroyed for all $\beta$’s. There is another case to consider where $D$ is intermediate. This is more complex and involves distinguishing agents whose commitment is fully destroyed, partially destroyed, or unaffected. In the interest of space, we do not present this case formally.

\textsuperscript{17}This point has been made by a number of authors. Gottlieb (2008) provides a detailed analysis of the effects of competition in markets with time inconsistent consumers.
In order to see this we must move to a world with distortions.

5 The Effects of Distortions

In the environment considered up to now, debt was not directly distortionary: the distortions only came from the effect of debt on individuals’ private commitments.

We now consider the case in which government debt can be directly distortionary. There are a number of ways in which this can happen. For instance, debt could interfere with optimal smoothing of tax distortions, or because the small open economy assumption is violated, and debt has general equilibrium effects, or because the rate at which resources can be borrowed from abroad is high relative to the citizen’s discount rate.

In this initial analysis we assume a simple distortion: for every dollar raised in period 3 to transfer resources to period 2, a fraction \( \eta \) is destroyed. Thus, a per capita debt of \( d \) to be paid in period 3 only yields \( d (1 - \eta) \) in period 2. This is a reduced form way to capture distortions that could come from a variety of sources, like, for instance, distortionary taxes.\(^{18}\)

Given savings from period 1 of \( s_{12} \) and \( s_{13} \), in period 2, a voter of type \( \beta \) would choose debt to maximize \( u (s_{12} + d (1 - \eta)) + \beta u (s_{13} - d) \). The first order condition is \( u' (c_2) (1 - \eta) = \beta u' (c_3) \). In contrast, the analogous first order condition evaluated in period 1 is \( u' (s_{12}) = u' (s_{13}) \). It follows that for any individual with preference parameter \( \beta < (1 - \eta) \), period-2 self wants to transfer resources from the third to the second period at the commitment solution. The mass of agents with \( \beta < (1 - \eta) \) therefore play an important role in the subsequent analysis.\(^{19}\)

The definition of optimal consumption levels now involves a subtlety that was absent in the case of no distortions: while in the previous analysis the optimal consumption sequences with and without commitment were independent of government debt (as long as debt was relatively small), this is no longer the case when there are distortions, because debt destroys wealth. Let \( c_1^* (\beta; d) \), \( c_2^* (\beta; d) \), and \( c_3^* (\beta; d) \) be the commitment sequence of consumption given debt \( d \). Namely, \( c_1^* (\beta; d) \), \( c_2^* (\beta; d) \), and \( c_3^* (\beta; d) \) is the solution of the following problem:

\[
\max \{u (c_1) + \beta (u (c_2) + u (c_3))\}
\]
\[
s.t. \quad c_1 + c_2 + c_3 = e - \eta d.
\]

\(^{18}\)Of course, there is no particular reason to expect these distortions to be proportional. This is assumed mainly for convenience. The qualitative analysis of this section does not depend on this assumption.

\(^{19}\)Note that we are implicitly assuming here that, with no government debt, consumers still face a rate of return of 1 on both the liquid and the illiquid assets. Our analysis is qualitatively unchanged if we assume that the rate a consumer at time \( t = 2 \) faces on borrowing from private markets is \( \frac{1}{1-\eta} \), that is, equal to the rate faced by the government when issuing public debt; See the next section.
Analogously, let $c_1^U(\beta; d), c_2^U(\beta; d)$, and $c_3^U(\beta; d)$ be the corresponding quantities without commitment.

For expository simplicity, we assume that utilities are such that both the commitment and the no-commitment consumption sequences are continuous in the debt level $d$. By construction, they are all decreasing functions of $d$.

We first consider the case where all agents have the same $\beta$, then move on to consider the effects of heterogeneity.

The behavior of equilibrium debt and consumption is divided into three regions depending on the debt limit. In order to determine the limits of these regions we need to define two values of debt that we call $d^*$ and $d^{**}$.

Define $d^*$ as the solution of $c_2^U(\beta; d^*) = d^*$.

We now need to introduce an artificial constrained maximization problem for a voter of type $\beta$.

\[
\max u(c_1) + \beta [u(c_2) + u(c_3)] \\
\text{s.t. } u'(c_2) = \frac{\beta}{1-\eta} u'(c_3), \\
c_1 + c_2 + c_3 = e - d\eta. 
\]

(1)

The first constraint is a “relaxed” commitment constraint, where resources transferred between period 3 and 2 suffered a unit loss of $\eta$. This will be the relevant constraint in determining debt in the second period. The larger the distortion $\eta$, the more relaxed this constraint is. The second constraint reflects the loss of resources due to the distortion. Denote by $V^u(\beta, \eta)$ be the value of this problem and by $c_1^n(\beta, d), c_2^n(\beta, d), c_3^n(\beta, d)$ the consumption sequence that solves the problem. We now define $d^{**}$ by the solution of $d^{**} = c_3^n(\beta, d^{**})$.\(^{20}\)

Analogously to the case of no distortions, we state the next result for the case in which $d^* < d^{**}$.

**Proposition 4 (Distortionary Equilibrium Debt)**

1. If $\beta > 1 - \eta$, then in equilibrium there is no debt, and consumption is given by $c_1^*(\beta), c_2^*(\beta), c_3^*(\beta)$.

2. Assume that $\beta < 1 - \eta$. If $\overline{d} \leq d^*$, then equilibrium debt is given by $\overline{d}$, and consumption is given by $c_1^*(\beta; \overline{d}), c_2^*(\beta; \overline{d}), c_3^*(\beta; \overline{d})$. If $d^* < \overline{d} \leq d^{**}$, then equilibrium debt is given by $\overline{d}$ and period 2 consumption is given by $c_2 = \overline{d}$. If $\overline{d} > d^{**}$, then debt is given by $d^{**}$ and period 2 consumption is given by $c_2 = d^{**}$.

\(^{20}\)Note that the intermediate value theorem assures that such $d^*$ and $d^{**}$ always exist since $c_2^*(\beta^*; 0), c_2^U(\beta^*; 0) > 0$, and both $c_2^*(\beta^*; 0)$ and $c_2^U(\beta^*; 0)$ are continuous and bounded.
Proof. Consider first the case in which $\beta > 1 - \eta$. We first show that there is an equilibrium with zero debt. Given an expected second period debt of zero, in period 1 voters choose the mix of liquid and illiquid assets $s_{12} = c_2^*(\beta)$ and $s_{13} = c_3^*(\beta)$ that implements the commitment consumption sequence $c_1^*(\beta), c_2^*(\beta), c_3^*(\beta)$. Given this mix of savings, $u'(c_2^*(\beta)) = u'(c_3^*(\beta))$. Thus, if $\beta > 1 - \eta$ $u'(c_2^*(\beta)) < \frac{\beta}{1-\eta}u'(c_3^*(\beta))$ and no voter has an incentive to vote for positive debt. Consider now any level of expected debt, $d$. The mix of savings has to be such that $u'(s_{12} + d) \leq u'(s_{13} + s_{23} - d)$. But then $u'(s_{12} + d) < \frac{\beta}{1-\eta}u'(s_{13} + s_{23} - d)$, inducing all voters to vote to reduce the debt.

Consider now the case in which $\beta < 1 - \eta$. Given any $\bar{d} < d^*$, and any expected $d \leq \bar{d}$, optimal savings in period 2 $s_{23}$ are zero and $s_{12}, s_{13}$ are such that $u'(s_{12} + d) = u'(s_{13} - d)$. Thus, $u'(s_{12} + d) > \frac{\beta}{1-\eta}u'(s_{13} - d)$ and, if $d < \bar{d}$ voters would vote to increase the debt. Thus, in this scenario equilibrium debt must be $\bar{d}$ and consumption must be given by $c_1^*(\beta; \bar{d}), c_2^*(\beta; \bar{d}), c_3^*(\beta; \bar{d})$. If $d^* < \bar{d} \leq d^{**}$, then, by the same reasoning, equilibrium debt must be at least $d^*$. But then, by the definition of $d^*$, debt is higher than second period commitment consumption, and optimal savings are at a corner: $s_{12} = s_{23} = 0$, implying that $c_2 = d$. Because $d < d^{**}$, we then have that $\frac{\beta}{1-\eta}u'(c_3) < u'(c_2) < u'(c_3)$. This implies that voters vote for higher deficit unless $d = \bar{d}$. Finally, If $\bar{d} \geq d > d^{**}$, then by the definition of $d^{**}$ $u'(d) < \frac{\beta}{1-\eta}u'(c_3)$, so voters would vote to reduce the deficit. This proves that, for any $\bar{d} \geq d^{**}$ equilibrium debt is given by $d^{**}$. □

This result says that debt accumulation can result in very large distortions in a world where voters are time inconsistent. The logic is the following. Because debt is determined by voters’ collective choices, individual saving decisions in prior periods have no impact on debt: voters have an incentive to try to undo expected second period debt by optimizing their mix of liquid-illiquid assets by saving less for period 2 and more for period 3. But, when the debt ceiling is not too low, this individual optimization will, in the aggregate, generate demand for transfers in the second period, leading to voting for a positive debt. Thus, savings decisions in period 1 generate their own demand for debt in the second period, even if this is distortionary.

We now consider what happens when agents are heterogeneous in their present-bias parameter $\beta$. Let $\beta^*$ be such that $G(1-\eta) - (\beta^*) = 1/2$. That is, half the population has preferences that are between $\beta^*$ and $1-\eta$. Figure 1 depicts the shape of the commitment and the no-commitment consumption levels in period 2 as a function of preferences for a particular debt level.

The agent of type $\beta^*$ turns out to be the pivotal agent for determining debt in this environment. We can now define $d^*(\beta^*)$ and $d^{**}(\beta^*)$ as the solutions of $d^* = c_2^*(\beta^*, d^*)$ and
Figure 1: Consumption Patterns for a Given Debt Level

\[ d^{**} = c^2_2 (\beta^*, d^{**}). \]
1. If $\beta_M > 1 - \eta$, then in equilibrium there is no debt, and consumption is given by $c^*_1(\beta), c^*_2(\beta), c^*_3(\beta)$.

2. Assume that $\beta_M < 1 - \eta$. If $\bar{d} \leq d^{**}(\beta^*)$, then equilibrium debt is given by $\bar{d}$. If $\bar{d} > d^{**}(\beta^*)$, then debt is given by $d^{**}(\beta^*)$. For any equilibrium debt level $d$, individual consumption for an agent of preference parameter $\beta$, period-2 consumption level in equilibrium is given by:

$$c_2(\beta; d) = \begin{cases} 
  c^U_2(\beta; d) & \beta \leq \beta_L(d) \\
  d & \beta_L(d) \leq \beta < \beta_H(d) \\
  c^*_2(\beta; d) & \beta \geq \beta_H(d)
\end{cases}$$

With respect to the distribution of preferences, notice that a shift in distribution changes the debt structure in the economy only when it modifies the preferences $\beta^*$ of the ‘pivotal agent’. As $\beta^*$ increases, $c^*_2(\beta^*; d)$ and $c^U_2(\beta^*; d)$ increase for all $d$, and therefore both $d^*$ and $d^{**}$ increase.

We say $G'$ is a median preserving spread of $G$ if both share the same median $\beta_M$ and for any $\beta < \beta_M$, $G'(\beta) \geq G(\beta)$, while for any $\beta > \beta_M$, $G'(\beta) \leq G(\beta)$. Intuitively, this implies that, under $G'$, more weight is put on more extremal values of $\beta$ (see Malamud and Trojani (2009) for applications to a variety of other economic phenomena).

The above discussion then implies the following corollary.

**Corollary 1 (Distributional Shifts)**

1. Assume $G(1 - \eta) = G'(1 - \eta)$. If $G'$ First Order Stochastically Dominates $G$, and the corresponding medians $\beta_M, \beta'_M < 1 - \eta$, then equilibrium debt under $G'$ is (weakly) higher than that under $G$.

2. If $G'$ is a Median Preserving Spread of $G$, then equilibrium debt under $G'$ is (weakly) lower than that under $G$.

### 6 Institutions, Welfare, and Policy

We now evaluate how welfare in the equilibrium allocation presented in Proposition 4 compares with several alternative benchmarks/policies. We consider: 1. Tighter debt limits; 2. Private debt incurred via market intermediaries; 3. Policies determined by a social planner without commitment; and 4. Banning of illiquid assets.
Case 1 is fairly straightforward in principle: it is clear that in our context, tighter debt limits operate by eliminating access to distortionary debt while maintaining agents’ access to illiquid assets. Thus, this should increase welfare since it allows agents to maintain their full commitment consumption patterns without destroying wealth through debt distortions. The analysis is made more complex when we consider heterogeneous agents, and intermediate debt limits, as well as constitutional means that can work to impose such debt limits without being overturned in the future.\(^{21}\)

A natural alternative baseline for comparison is that in which agents have no access to illiquid assets. In other words, agents have no commitment power. This can arise whenever, say, agents have access to personal credit cards (with a rate of return of 1) that allow them to undo any commitment plan they’ve entered in earlier periods. Alternatively, whenever agents have access to illiquid assets and debt is non-distortionary and has no limit, effectively agents are tied to uncommitted consumption paths. The comparison with such environments is less straightforward since it presents a trade-off. On the one hand, debt allows for some level of commitment when illiquid assets are available. On the other hand, it entails a wealth loss. In order to highlight the effects of the magnitude of distortions on welfare levels, we focus first on homogeneous populations. That is, we assume all agents’ preference parameter is given by \(\beta\) (in particular, the effective ‘median’ preference parameter \(\beta^*\) coincides with \(\beta\) as well). Furthermore, we consider an economy in which, indeed, there is no debt limit. In such an economy, where there are no distortions, i.e., \(\eta = 0\), the no commitment solution is implemented.

**Proposition 5 (Welfare Effects of Distortions)** For any homogeneous population of preference parameter \(\beta\), the equilibrium with positive distortions \(0 < \eta < 1 - \beta\) leads to lower welfare for all three selves than the equilibrium with \(\eta = 0\). If \(\eta > 1 - \beta\), then welfare is higher than for any \(\eta < 1 - \beta\).

The proof of this proposition is in the Appendix. As mentioned above, there are two contrasting effects of positive distortions. On the negative side, given that there is debt in equilibrium, the presence of distortions causes wealth destruction. On the positive side, distortions relax the commitment constraint in the artificial maximization that determines equilibrium debt. In fact, when \(\eta\) is very high (\(\eta > 1 - \beta\)), then distortions serve as a full commitment device, because voters do not vote for positive deficits in the second period in equilibrium. The proposition shows that the negative effect dominates.

\(^{21}\)For instance, the U.S. federal government does have a debt limit, but this was increased 74 times since 1962, and 10 times during the last decade.
6.1 Social Planner without Commitment

In the environment we study, voters are time inconsistent while the politicians simply pursue election in each period. As an alternative, consider a situation in which a time-inconsistent social planner, sharing the population preference parameter $\beta$, determines consumption allocations. It is easy to see that the allocation with such a social planner is given by $c_1^0(\beta, 0), c_2^0(\beta, 0), c_3^0(\beta, 0)$, namely by the solution of the maximization problem given in (1). Therefore, first-period welfare is increasing in $\eta$ since higher distortions lessen the commitment constraint.\footnote{Krusell et al. (2002) show that in an economy with capital accumulation there is an additional issue in contrasting a decentralized economy and a social planner without commitment. Specifically, the social planner takes into account the fact that, while individuals take the returns to savings as given, the social planner takes into account the fact that, with decreasing returns, increased aggregate savings reduce returns to capital accumulation. This leads to even worse undersaving with a social planner.} This is clearly in stark contrast with the result in Proposition 5. Thus, in our setting, there is an interesting non monotonicity in the effect of government intervention: moderate government intervention in the form of democratically elected politicians offering deficit financed transfers lead to worse outcomes than either decentralized allocations or fully centralized allocations. This is in principle informative about the debate over libertarian paternalism.

6.2 Credit Cards and Private Debt

Consider now an environment in which there is no government. However, individuals can borrow on the private market from intermediaries such as credit card companies. The model is otherwise the same as in Section 3. For the purposes of comparison with our analysis of government debt, assume that credit card companies charge a proportional fee $\eta$ for every dollar borrowed in the second period. This could be due to markups in an imperfectly competitive credit market or to costs born by credit card companies.

Proposition 6 (Equilibrium with Credit Cards) For any $\eta > 0$, agents make portfolio decisions in period 1 that ensure that no transactions take place with credit card companies. For any $\beta$, the equilibrium consumption sequence is given by $c_1^1(\beta, 0), c_2^1(\beta, 0), c_3^1(\beta, 0)$. In equilibrium welfare of all period 1 selves is increasing in $\eta$.

The logic of this result is the following. The availability of credit in the second period limits the commitment possibilities for time-inconsistent agents. However, sophisticated agents anticipate this issue and take appropriate steps to counteract this temptation. Every
consumption profile that is attainable via positive debt with credit cards is also attainable with an appropriate mix of liquid-illiquid assets. Thus, with positive distortions it cannot be the to ever end up with positive credit card debt. Agents internalize the commitment constraint in period 2 and ‘give up on commitment’ just enough that they do not waste resources by dealing with credit card companies. Clearly first period welfare is increasing in \( \eta \) because higher \( \eta \) relaxes the commitment constraint.

This result provides a stark contrast with Proposition 5, and the key difference is the fact that public debt is a result of collective action, so individuals have a private incentive to undo public debt.

We do not claim that this is a realistic model of credit card debt with or without time inconsistency.\(^{23}\) The point of this stark model is to draw an important contrast between private and public debt.

### 6.3 Period 1 Financial Structure

We now consider the socially optimal mix of liquid and illiquid assets when government is fiscally irresponsible. A common argument in the behavioral literature is that, in environments with time-inconsistent agents, an efficiency enhancing paternalistic policy is to subsidize or otherwise promote the existence of illiquid commitment assets. Our results suggest that, in evaluating such policies, it would be useful to consider how this affects the political economy of debt. Specifically, in our model, an implication of Proposition 5 is that welfare is higher for all selves when illiquid assets are banned or taxed, rather than subsidized.

### 6.4 Debt Limits

Suppose now that debt limits are finite. For sufficiently low debt limits \((d \leq d^*)\), agents are able to implement their full commitment consumption plan with a wealth reduced by the frictions due to the debt implemented. Denote the resulting (indirect) utility of each agent with commitment and no debt by \(V^C(\beta)\), with no commitment and no debt by \(V^U(\beta)\), and with commitment and debt level \(d\) by \(V^C(\beta; d)\). Notice that \(V^C(\beta; d)\) is decreasing in \(d\) as increases in \(d\) are tantamount to decreases in wealth. Furthermore, \(V^C(\beta; 0) = V^C(\beta) > V^U(\beta)\).

For intermediate levels of debt, \(d^* < d \leq d^{*}\), agents can implement ‘partial commitment’. In this region, as the debt limit increases there are two effects on agents’ equilibrium utility.

\(^{23}\)See Angeletos et al. (2001) for a model with coexistence of credit card debt and investment in illiquid assets.
The direct effect is that present for lower levels of debt as well, lowering the effective wealth of the agent. The indirect effect pertains to the agents’ decline in commitment ability. Since both these effects work in tandem, the indirect utility decreases in this region as well (with greater slope around $d^*$).

Finally, for large levels of debt limits, $d > d^{**}$, equilibrium debt is fixed at $d^{**}$ and agents cannot commit. The resulting (indirect) utility is given by $V_U(\beta; d^{**})$. Since the debt in this region implies a loss of wealth without enabling agents to commit, $V_U(\beta; d^{**}) > V_U(\beta)$.

Figure 2 summarizes this discussion. As a corollary, it follows that for sufficiently small debt limits, welfare is higher in our setting relative to the no commitment case, while for sufficiently high debt limits, welfare is lower in our setting. That is:

\textbf{Corollary} There exists $\bar{d} \in (0, d^{**})$ such that for all $d < \bar{d}$, equilibrium welfare exceeds that generated in an economy without illiquid assets. For all $d > \bar{d}$, welfare in our setting is lower than that generated in an economy without illiquid assets.

6.5 Revisiting Heterogeneity

We now go back to the case of a heterogeneous population of agents, with a non-trivial distribution of preference parameters. For a given equilibrium level of debt $d > 0$, for suf-
sufficiently low preference parameters $\beta, d$ is too large to allow for any level of commitment. These agents lose wealth due to the debt distortions, but gain no commitment value. In particular, their indirect utility is lower relative to the world in which no illiquid assets are available. At the other extreme, consider agents who are nearly time consistent, with preference parameter $\beta$ close to 1. These agents suffer very minor commitment issues. Therefore, the loss of commitment is preferable to them then the loss of wealth due to distortionary debt. Formally, we get the following proposition:

**Proposition 7 (Preference Parameters and Welfare)** For any debt level $d > 0$, there exist $\beta_L, \beta_H \in (0, 1)$ such that all agents with preference parameters $[0, \beta_L] \cup [\beta_H, 1]$ are weakly worse off by the introduction of illiquid assets.

Note that the proposition implies a spread of the distribution of preferences that maintains the pivotal preference parameter $\beta^*$, and therefore the equilibrium level of debt, reduces welfare relative to the world in which no illiquid assets are available.

Throughout the paper we assume that voters act under a pre-determined debt ceiling. Nonetheless, the debt limit itself is conceivably determined through a political process much like the one we study. Suppose then that in period 1 agents vote on the debt limit that would affect the debt imposed in period 2 as in the model studied thus far. In such a setting, all agents would favor low debt limits in period 1. In fact, since illiquid assets allow agents to commit without experiencing the loss of wealth that results from distortionary debt, equilibrium would entail a debt limit fixed at zero. Of course, if agents could vote again on the debt limit in period 2 (prior to determining the debt level itself, as in the model studied thus far), they would collectively choose a positive debt limit and consumption would be distorted (relative to the commitment paths). This suggests the importance of timing in constitutional reform. Since most amendments take a substantial amount of time to pass, changes in debt limits are likely to occur a significant time prior to the ‘temptation’ of consumption. Even if multiple elections occurred over such amendments, it would be difficult to achieve a super-majority to agree over time on an increase on the debt limit itself (as pointed above, early in the process, one would expect voters to reject debt limit increases).

### 7 Concluding Remarks and Extensions

We introduced a political process determining fiscal policy when voters are time inconsistent. Several messages arise from our analysis. First, absent distortions, as long as debt limits are low enough, the availability of illiquid assets makes debt irrelevant for ultimate consumption.
levels since agents can adjust incoming debt income by an appropriate ex-ante allocation of liquid and illiquid assets. In particular, there is a Ricardian equivalence of sort. When debt limits are high, agents’ ability to commit is impaired. That is, electorally accountable politicians ultimately choose policies that interfere with individuals’ ex-ante desire to commit. When debt is distortionary some of these effects are accentuated since debt entails an effective loss of wealth. In fact, we show that there can be a substantial loss in welfare relative to the case in a world without any ability to commit and without debt. The paper highlights the importance of analyzing the political process when contemplating enlarging the menus of policies directed at enhancing the welfare of ‘behavioral’ electorates. Indeed, when focusing on time inconsistency, the underlying message of our paper is that governments may not be very effective in satisfying the demand for commitment.

We now consider a number of natural extensions of our basic model.

7.1 Elections in Period 1

The model studied so far allowed for government actions and elections in periods 2 and 3. We now extend the model to consider elections in period 1 as well. The objective of this extension is to evaluate whether collective action in period 1 could effectively satisfy the demand for commitment agents display in period 1 or at least limit the distortions associated with debt accumulation in period 2.

The economic environment is the same as the one assumed in previous sections. There are two candidates running for office, both in period 1 and in period 2. The candidates are office motivated. The policy space is extended to allow candidates to offer a transfer $y_1$ and a lump-sum tax $t_1$ in period 1, as well as a transfer $y_2$ and tax $t_2$ in period 2 (elections in period 3 are redundant as before). Deficit financing is allowed. Tax collection in any period carries distortions of a unit loss $\eta > 0$ for every unit collected.

For this robustness check, we focus on the case of high debt limit, $\overline{d} \geq d^{**}$, so that equilibrium debt is given by $d^{**}$. Notice that when debt is high, more agents could be expected to suffer from an inability to commit, so potentially period 1 elections could be useful.

By taxing themselves in period 1 and investing the proceeds in the liquid asset agents can effectively commit resources for consumption in period 2 and hence reduce debt accumulation. On the other hand, if the proceeds of taxes carried to period 2 are smaller than $d^{**}$, in per-capita terms, a strict majority of agents in period 2 will support a positive debt level so as to increase consumption in period 2. Let $t = t_1 - y_1$ denote per-capita taxes in period 1 and $d = y_2 - t_2$ denote debt in period 2. It turns out that even though by taxing
themselves in period 1 agents can indeed limit debt accumulation in period 2, this strategy simply shifts some of the repayment of debt from period 3 to period 1, but does not alter total distortions and has no ultimate effects on consumption profiles.

**Proposition 8** In the economy with elections in every period and with high debt limit, \( \overline{d} \geq d^{**} \), the set of equilibria is characterized by pairs of period 1 taxes and period 2 debt of the form \((t, d)\) such that \( t \in [0, d^{**}] \) and \( d = d^{**} - t \); total distortions and consumption profiles are unchanged for all agents relative to the case in which elections take place only in period 2.\(^{24}\)

### 7.2 Arbitrary Number of Periods

We now study an economy that may last for more than 3 periods, say \( T > 3 \) periods. We assume that voters can choose to invest in liquid or illiquid assets. An illiquid asset with maturity \( m \), acquired in period \( \tau \), pays off in period \( \tau + m \) and cannot be sold before then. We assume that in any period \( \tau = 1, \ldots, T - 2 \) illiquid assets are available with any maturity \( m \) between 2 and \( T - \tau - 1 \). A liquid asset is instead by definition an asset with maturity 1. We assume that liquid assets are available in any period \( \tau = 1, \ldots, T - 1 \).

Absent government intervention, by appropriate choice of the mix of liquid and illiquid assets with different maturities, a voter can commit to any desired consumption stream. We study, however, an economy in which elections occur in any period \( \tau \geq 2 \) (though period \( T \) elections are vacuous).\(^{25}\) Elections at any time \( \tau \) involve two office motivated candidates, as in previous sections, offering a platform given by \((y_{\tau}, t_{\tau})\), where \( y_{\tau} \) is a per capita transfer and \( t_{\tau} \) is a lump-sum tax. Let \( d_{\tau} = y_{\tau} - t_{\tau} \) denote the change in per capita government debt in period \( \tau \). Consider for simplicity the case in which tax collection carries no distortions.\(^{26}\)

The characterization of equilibrium debt and consumption in this economy follows closely the one we obtained in Proposition 1-3 for the economy with 3 periods. Details are complicated, however, since the equilibrium is non stationary. In particular, \( d_{\tau} \) is typically not constant and hence the debt limit, \( \overline{d} \), might be binding in some periods (and for some agents) and not others. If \( \overline{d} \) is large enough, so that it never binds, equilibrium consumption will coincide with the optimal consumption sequence without commitment, \( \tilde{c}_{\tau,T}^{U}(\beta) \), for \( \tau = 1, 2, \ldots, T \).

\(^{24}\)Only if the distortion on taxes at \( t = 1 \) were smaller than the distortion on taxes at \( t = 3 \) election in period 1 would help reducing debt in period 2 and hence distortions in period 3.

\(^{25}\)Elections in period 1 can be added along the lines of the previous section, without any qualitative impacts.

\(^{26}\)The extension to the case with distortions is conceptually straightforward, though it requires a substantial notational buildup.
At equilibrium, debt is accumulated in any period $\tau$, that is, $d_{\tau} > 0$ and is repaid at time $T$ with the proceeds of the agents’ dynamic portfolio strategies that include trading of liquid and illiquid assets of different maturities.
8 Appendix

Proof of Lemma 1

The commitment solution satisfies
\[ u'(c_1^*(\beta)) = \beta u'(c_2^*(\beta)) = \beta u'(c_3^*(\beta)). \]
Since \( u \) is strictly concave, \( c_2^*(\beta) = c_3^*(\beta) \), and the budget constraint implies that as \( \beta \) increases, \( c_2^*(\beta) = c_3^*(\beta) \) increase as well.

For any wealth \( e_2 \) left at period 2, denote by \( c_2^{U}(\beta; e_2) \) and \( c_3^{U}(\beta; e_2) \) the uncommitted solution. Optimization requires:
\[ u'(c_2^{U}(\beta; e_2)) = \beta u'(c_3^{U}(\beta; e_2)) \]
and so \( c_2^{U}(\beta; e_2) \) and \( c_3^{U}(\beta; e_2) \) are increasing in \( e_2 \). Furthermore, for any given wealth \( e_2 \), \( c_2^{U}(\beta; e_2) \) is decreasing in \( \beta \). Consider period 1 self’s constraint:
\[ u'(c_1^{U}(\beta)) = \beta \left[ u'(c_2^{U}(\beta; e_2)) + u'(c_3^{U}(\beta; e_2)) \right], \]
where \( e_2 = e_1 - c_1^{U}(\beta) \).

The derivative of the right hand side with respect to \( \beta \) is:
\[
\begin{align*}
&u'(c_2^{U}(\beta; e_2)) + u'(c_3^{U}(\beta; e_2)) + \beta \left[ u''(c_2^{U}(\beta; e_2)) \frac{\partial c_2^{U}(\beta; e_2)}{\partial \beta} + u''(c_3^{U}(\beta; e_2)) \frac{\partial c_3^{U}(\beta; e_2)}{\partial \beta} \right] = \\
&= u'(c_2^{U}(\beta; e_2)) + u'(c_3^{U}(\beta; e_2)) + \beta \frac{\partial c_2^{U}(\beta; e_2)}{\partial \beta} \left[ u''(c_2^{U}(\beta; e_2)) - u''(c_3^{U}(\beta; e_2)) \right].
\end{align*}
\]
Since \( c_2^{U}(\beta; e_2) > c_3^{U}(\beta; e_2) \), \( u'' < 0 \), and \( c_2^{U}(\beta; e_2) \) is decreasing in \( \beta \), the above expression is positive. Period 1 self’s constraint then implies that \( c_1^{U}(\beta) \) is decreasing in \( \beta \) and \( c_2^{U}(\beta) \) is therefore increasing in \( \beta \).

Proof of Proposition 5

Similar arguments to those of Lemma 1 imply that for any level of debt \( d \), \( c_2^*(\beta; d) \) and \( c_2^{U}(\beta; d) \) are increasing in \( \beta \).

Assume first that \( \tilde{d} \leq d^* \). Suppose equilibrium debt is \( d < \tilde{d} \). Notice that monotonicity implies that \( c_2^*(\beta; d) \geq c_2^*(\beta^*; d) \geq d \) for all \( \beta \geq \beta^* \). Furthermore, by definition of \( d^* \) and continuity of \( c_2^*(\beta; d) \), for sufficiently small \( \varepsilon > 0 \), \( c_2^*(\beta; d) \geq d \), for all \( \beta \geq \beta^* - \varepsilon \). It follows that all agents with preference parameter \( \beta \in [\beta^* - \varepsilon, 1 - \eta) \) best respond by investing
in illiquid assets leaving them with period 2 wealth of $c^*_2(\beta; d) - d$. However, in period 2, these agents are keen to shift resources from period 3 to period 2, and would therefore prefer a slightly higher debt level. From the definition of $\beta^*$, there would therefore be a strict majority support for higher debt. In particular, the only candidate for equilibrium debt is $\bar{d}$. Now, when debt is expected to be $\bar{d}$, for sufficiently small $\varepsilon > 0$, any agent with preference parameter $\beta \in [\beta^* - \varepsilon, 1 - \eta)$ would best respond in period 1 by investing in illiquid assets so that $\max \{0, c^*_2(\beta; d) - d\}$ is left for the period 2 self. These agents, forming a strict majority, would oppose any ex-post reduction of debt in period 2. It follows that $\bar{d}$ constitutes the equilibrium debt level and the commitment consumption stream is implemented for the agent with preferences $\beta^*$.

Consider now the case $d^* < \bar{d} \leq d^{**}$. As above, for any $d < d^*$, when voters use best responses, there would be a strict majority support for an increase in debt in period 2. Suppose, then, that in equilibrium the debt is $d$, $d^* \leq d < \bar{d}$. For all $\beta \geq \beta^*$, monotonicity implies that $c^U_2(\beta; d) \geq c^U_2(\beta^*; d) \geq d$. Furthermore, by definition of $d^{**}$ and continuity of $c^U_2(\beta; d)$, for sufficiently small $\varepsilon > 0$, $c^U_2(\beta; d) \geq d$, for all $\beta \geq \beta^* - \varepsilon$. Therefore, any agent with preference parameter $\beta \in [\beta^* - \varepsilon, 1 - \eta)$ would best respond by investing in illiquid assets so that $\max \{0, c^*_2(\beta; d) - d\}$ is left for the period 2 self. In particular, all agents with preference parameter $\beta \in [\beta^* - \varepsilon, 1 - \eta)$, constituting a strict majority of agents, would support a higher debt in period 2. It follows that $\bar{d}$ is the only candidate for equilibrium debt. In fact, the above arguments suggest that whenever debt is expected to be $\bar{d}$, a strict majority of voters would oppose any ex-post reduction of debt in period 2. Therefore, in equilibrium, debt is given by $\bar{d}$, coinciding with the consumption level in period 2 of the agent with preference parameter $\beta^*$.

Finally, suppose that $\bar{d} > d^{**}$. As before, for any $d < d^{**}$, whenever voters best respond, there would be a strict majority support for an increase in debt in period 2. Assume equilibrium debt is $d > d^{**}$. As before, from monotonicity and continuity of $c^U_2(\beta; d)$, and the definition of $d^{**}$, there exists $\varepsilon > 0$ such that for all $\beta \leq \beta^* + \varepsilon$, $c^U_2(\beta; d) < d$. Since distortions imply that, in period 2, agents never desire a debt level that exceeds their intended consumption, it follows that all agents with preference parameters $[0, \beta^* + \varepsilon]$, a strict majority, would desire a lower debt level. Debt $d^{**}$ constitutes part of an equilibrium. Indeed, continuity and monotonicity imply that any debt $d > d^{**}$ would exceed the period-2 uncommitted consumption level for agents with preference parameter $[0, \beta^* + \varepsilon]$ for some $\varepsilon > 0$, and therefore be opposed by a strict majority. Similarly, any debt $d < d^{**}$ would imply that a strict majority of agents with preference parameter $[\beta^* - \varepsilon, 1 - \eta)$, for some $\varepsilon > 0$, that have used a best response, cannot afford their uncommitted consumption level in period 2, and therefore oppose a deviation from $d^{**}$ to $d$. The proposition’s claim then follows.
Proof of Proposition 6

Consider the following maximization problem:

\[
\max u(c_1) + \beta [u(c_2) + u(c_3)] \\
\text{s.t. } u'(c_2) = \frac{\beta}{1-\eta} u'(c_3) \\
c_1 + c_2 + c_3 = e - \eta c_2.
\]  

(2)

This problem corresponds to an agent who chooses the debt level and her consumption plan in tandem. In particular, it generates a higher overall utility (from period 1’s perspective) than that experienced by an agent who accepts the equilibrium level of debt as given and cannot alter it unilaterally, corresponding to the welfare levels for distortions \( \eta \). Furthermore, the two coincide when \( \eta = 0 \). We now show that the maximized objective of problem (2) is decreasing in \( \eta \). Indeed, suppose \( \eta_1 > \eta_2 \). Denote the solution of (2) for distortions \( \eta_1 \) by \((c_1, c_2, c_3)\). We now approximate a policy under distortions \( \eta_2 \) small enough that satisfies the constraints and generates a strictly higher value for the objective.

For \( \eta_2 \) close enough to \( \eta_1 \), there exists \( \varepsilon > 0 \), \( \varepsilon < c_3 \) such that

\[ u'(c_2) = \frac{\beta}{1-\eta_2} u'(c_3 - \varepsilon). \]

Therefore,

\[ u'(c_2) = \frac{\beta}{1-\eta_2} \left[ u'(c_3) - \varepsilon u''(c_3) + O(\varepsilon^2) \right]. \]

Since \((c_1, c_2, c_3)\) is a solution to the problem with distortions \( \eta_1 \), \( u'(c_2) = \frac{\beta}{1-\eta_1} u'(c_3) \). It follows that:

\[ \varepsilon = \frac{(\eta_2 - \eta_1) u'(c_2)}{\beta u''(c_3)} + O(\varepsilon^2). \]

Consider then the policy \((c_1 + \varepsilon + (\eta_1 - \eta_2) c_2, c_2, c_3 - \varepsilon)\) when the distortions are \( \eta_2 \). Notice that, by construction, this policy satisfies the two constraints in problem (2). The difference between the generated objective and the maximal value of the objective under distortions \( \eta_1 \) is then:

\[ \Delta = [u(c_1 + \varepsilon + (\eta_1 - \eta_2) c_2) - u(c_1)] + \beta [u(c_3 - \varepsilon) - u(c_3)]. \]

Using a first order approximation,

\[
\Delta = (\varepsilon + (\eta_1 - \eta_2) c_2) u'(c_1) - \beta \varepsilon u'(c_3) = \\
= (\eta_1 - \eta_2) c_2 u'(c_1) + \frac{(\eta_2 - \eta_1) u'(c_2) u'(c_1)}{u''(c_1)} - \frac{(\eta_2 - \eta_1) u'(c_2) u'(c_3)}{u''(c_3)} + O(\varepsilon^2) \\
= (\eta_1 - \eta_2) u'(c_2) \left[ \frac{u'(c_1) c_2}{u'(c_2)} - \frac{u'(c_1) - \beta u'(c_3)}{\beta u''(c_3)} \right] + O(\varepsilon^2).
\]

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Notice that the solution to problem (2) with distortions $\eta_1$ must satisfy

$$u'(c_1) = \beta [u'(c_2) + u'(c_3)]$$

and so:

$$\Delta = (\eta_1 - \eta_2) u'(c_2) \left[ \frac{u'(c_1)c_2}{u'(c_2)} - \frac{u'(c_2)}{u''(c_3)} \right] + O(\varepsilon^2),$$

which from concavity of the instantaneous utility $u$, is positive whenever $\eta_1$ and $\eta_2$ are close enough. In particular, the optimal solution for problem (2) with distortions $\eta_2$ must generate a strictly higher level of the objective function that the solution with distortions $\eta_1$. It follows that welfare in our distortion economy is lower under any $\eta > 0$ relative to the case of $\eta = 0$.

Proof of Proposition 7

For any debt level $d$, let $\beta^U$ be the maximal parameter $\beta$ such that uncommitted consumption level in period 2 with wealth $e - \eta d$ is lower than $d$. Since period 2 uncommitted consumption is monotonic in $\beta$ for all levels of wealth, all agents with parameter $\beta \leq \beta^U$ consume the uncommitted consumption profile with debt $d$, that generates lower utility than the uncommitted consumption profile absent distortionary debt. For agents with preference parameter $\beta = 1$, the uncommitted consumption path coincides with the full-commitment consumption path. In particular, for these agents, the introduction of distortionary debt entails a welfare reduction due to the effective loss of wealth. Continuity implies the claim of the proposition.
References


