How Well Did Social Security Mitigate the Effects of the Great Recession?

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Abstract

This paper studies the effectiveness of the social security program as an insurance mechanism against unanticipated catastrophic shocks to the household balance sheets. Our framework is an overlapping generations (OLG) life cycle model with uninsurable idiosyncratic earnings risk, lifetime uncertainty, endogenous labor supply and endogenously determined retirement. To analyze the efficacy of the social security in mitigating welfare losses during times of severe and unforeseen economic distress, we consider a social security system that mimics that of the U.S. economy. The calibrated model is used to study the effects of a one-time, unanticipated, large wealth shock on the welfare of households in economies with and without social security. While massive in both economies, we find that the welfare losses for older individuals due to the shock are partially mitigated by the presence of the social security program. This insurance channel is particularly important for older individuals who have already retired because they are unable to rebuild their wealth by working. Although we find that social security is particularly effective in mitigating the losses over the transition for individuals who are old at the time of the shock, the program does not come without welfare costs in the steady state. We estimate that, in the steady state economy with social security, the welfare is lower by the equivalent of 9.5 percent of total expected lifetime consumption than in the economy without the social security program.

JEL: E21, D91, and H55

Key Words: Social Security, Recessions, Overlapping Generations.

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1 Introduction

Established in the wake of the Great Depression, the social security program aims to provide valuable consumption insurance at old age. That said, the program is not without costs: according to the Congressional Budget Office, social security is the single largest federal program, with outlays representing roughly one-fifth of total federal expenditures in 2012. Both against the background of the fiscal consolidation debate and in light of the recent Great Recession, this paper seeks to advance current understanding of the role of the large-scale program as an insurance mechanism against unanticipated catastrophic shocks to household balance sheets.

A number of studies have examined the welfare implications of the social security program - we will overview the main findings below. In this paper, we build on findings in Glover et al. (2012) who study the implications of the Great Recession on welfare of U.S. households. Empirically, the authors find that the recession had a pronounced effect on household balance sheet: even after two years of recovery, the average household’s wealth in 2010 was over 20 percent below its corresponding pre-recession value in 2007. Furthermore, in an overlapping generations model (OLG), the authors show that a large shock to household wealth induces sizable welfare losses for individuals who are older at the time of the shock. In contrast, the authors find that cohorts who were younger at the time of the shock experience almost no change in their lifetime welfare. One reason for this discrepancy is that older agents who are retired rely on their savings to fund their current consumption. In contrast, younger agents, who are wealth-poor at the beginning of their lifetime (and are thus largely unaffected by the wealth shock), have many periods to re-accumulate wealth before they will begin to consume their savings at old age.

Motivated by the aforementioned results, this paper studies the efficacy of the social security program in mitigating welfare losses during times of severe and unforeseen economic distress. In particular, we quantify the role of the social security program as a vehicle of intergenerational risk-sharing and consumption insurance across different age cohorts when a sizable, adverse and unanticipated shock is introduced in the economy. Our framework is an OLG model that includes idiosyncratic earnings risk, endogenous labor supply, and endogenous retirement decisions, and it also incorporates realistic features of the U.S. social security system. The model is calibrated to match the salient features of the U.S. cross-sectional data as well as the key macroeconomic aggregates.

We conduct the quantitative assessment in two steps. First, we examine the distributional effects on welfare by age and ability over the transition after a one-time shock to the baseline economy with the social security program is introduced. Similar to the Great Recession, the shock causes a twenty percent loss of physical wealth.\(^1\) Similarly to Glover et al. (2012) and somewhat unsurprisingly, we find that the welfare losses are much larger for individuals who are old at the time of the shock as compared to individuals who are young.

Second, in order to assess how effective social security is at sharing the risk of such a shock between generations, we examine the welfare losses after the large shock is introduced in a counterfactual economy without a social security program. During the transition, we define the welfare loss as a constant percentage increase in per-period future consumption following the shock necessary to restore an individual’s expected future utility to that in the steady state.\(^2\) Zooming in on the group of agents who are most vulnerable to the effects of the shock, we find that the differences in welfare loss for agents close to or in retirement between the model with and without social security are massive. In particular, cohorts who are older than sixty five at the time of the shock suffer much larger welfare losses in the model with no social security than in the model baseline. We find that the additional average

\(^1\)In this very preliminary version of the paper, the Great Recession is modeled in a very stylized way and in a much more parsimonious manner than in Glover et al. (2012).

\(^2\)In a steady state, the welfare loss is defined in terms of the standard consumption equivalent variation (CEV) where the loss is defined as the constant percentage increase in per-period consumption that makes a newborn agent indifferent between being born into the two respective steady-state economies (for details, see, for example, D. and J. (May).)
welfare lost for these cohorts due to the absence of income insurance from a social security program is equivalent to 7.9 percent of future consumption. In contrast, the welfare lost by young individuals is roughly equivalent between the two economies. As discussed, young households hold relatively low levels of assets and are therefore less vulnerable to the wealth shock’s effects. Overall, across all the living cohorts, the average welfare loss is larger in the model without social security by an equivalent of 1.5 percent of future consumption than in the baseline model with the social security program.

As expected, the welfare benefits from the inter-generational risk sharing do not come without a cost in the steady state. In addition to social security taxes exacerbating liquidity constraints for younger households, social security benefits “crowd out” private savings. We find that in the steady state, the social security program causes a decrease in welfare equivalent, defined in the standard way, of approximately 9.5 percent of total lifetime consumption compared to the model without a social security program.

A number of additional findings emerge from our analysis. The detailed structure of the social security system in our model that closely replicates the current U.S. system combined with the large degree of heterogeneity across individual households allow us to examine the behavioral response of households to the large shock along several margins. In particular, our model allows us to compare changes in the consumption, saving and labor supply decisions alongside with the changes to the decision to retire when an adverse shock hits between the economy with and without social security. Perhaps most interestingly, in both economies, households tend to delay their retirement in response to the shock. Moreover, we find that this change in retirement behavior is even more pronounced in the economy where the social security program is absent.

Our work is related to three strands of the literature. The first strand focuses on the welfare consequences of the Great Recession. Most closely related to our work, Glover et al. (2012) use a calibrated OLG model with aggregate risk (but no idiosyncratic risk, endogenous labor supply or social security) to quantify how welfare costs of severe recessions are distributed across different age groups. Similar to this paper, the authors find that the generations that are older at the time of the downturn fare much worse than the cohorts that were younger at the time of the adverse shock. This paper builds on some of the mechanisms in Glover et al. (2012), but advances the research agenda by further exploring how effective the social security program is at mitigating these effects across different cohorts.

The second strand tries to measure the long-run implications on welfare of a social security program. These works try to weigh the relative benefit from providing partial insurance against risks for which no market option exists against the welfare costs of distorting an individual’s incentives to work and save. The seminal studies that largely focus on the benefit of providing intra-generational insurance for idiosyncratic earnings- and mortality risks include Auerbach and Kotlikoff (1987), Hubbard and Judd (1987), Hubbard (1988), Imrohoroglu et al. (1995), Storesletten et al. (1998), and Hong and Rios-Rull (2007). By and large, these studies tend to find that, primarily when the general equilibrium effects are accounted for, the benefits from the social security system providing insurance against intra-generational risks are not large enough to justify the welfare costs.3

More recently, Krueger and Kubler (2006) examine the welfare implications of social security in a steady-state model with both idiosyncratic risk and a moderate level of aggregate risk. The authors find that even with the additional benefit from providing inter-generational risk sharing against the aggregate risk, the negative effect on welfare from “crowding out” of private capital leads social security to not be a Pareto improvement. Similar to this paper, we aim to examine the welfare consequences of the current social security program. However, our study is different in that it focuses on the welfare implications of the social security program over the transitional period after a large shock to the household balance sheet, as opposed to focusing on the long-run welfare effects of the program.

3One exception is Imrohoroglu et al. (2003), which find that if preferences are time-inconsistent then the benefits of social security outweigh the costs.
The final strand of the literature examines the effect on the economy of reforming the current social security program. Examples of these studies include: Olovsson (2010), Imrohoroglu and Kitao (2012), Kitao (2012), Huggett and Parra (2010), and Huggett and Ventura (1999). Amongst these papers, Olovsson (2010) examines the welfare gains of a social security program that efficiently shares aggregate risks between generations. The author finds that although agents would prefer to be born into these more efficient programs, the welfare costs during the transition outweigh the benefits for living agents. In the spirit of Olovsson (2010), we solve and document the welfare effects on all the living individuals during a transitional period. However, instead of exploring the dynamics along the transitional path after a change in the social security program, this paper studies how the economy evolves after there is a large shock to aggregate wealth.

This paper is organized as follows: Section 2 discusses the empirical data surrounding wealth and the Great Recession while Section 3 introduces the computational model. Section 4 presents the competitive equilibrium. Section 5 describes the functional forms and calibration parameters. Section 6 reports the results of the computational experiment. Section 7 concludes.

2 Data [To be Completed]

The analysis of the data is too preliminary to present at this point. We intend to present results from the Survey of Consumer Finances (SCF) that recount the effects of the Great Recession on the households' balance sheet. Moreover, we plan to use additional data sources, such as the the Survey of Income and Program Participation (SIPP) and the Panel Study of Income Dynamics (PSID), to document changes in the labor supply and retirement decisions of households both during the Great Recession and along the life cycle. Some of these stylized facts have already been demonstrated previously; two examples are Glover et al. (2012) and Erosa et al. (2011).

3 The Model

Our framework is an Aiyagari-Bewley-Huggett style economy with overlapping generations of heterogenous households. Households derive utility from nondurable consumption and leisure. Households supply labor elastically and receive an idiosyncratic uninsurable stream of earnings that is governed by shocks to household productivity as well as by the dynamics of the market wage. Households make joint decisions about their consumption of nondurable goods, labor supply and savings. Idiosyncratic earnings shocks can be partially insured through precautionary holdings of a single asset in the economy and through labor supply decisions. Retired households receive payments from a PAY-GO social security system wherein retirees’ benefits are paid for through income taxation of working-age individuals. The system of social security payments provides another margin of consumption insurance at old age. An important feature of this model is that households choose the age at which they retire, taking into consideration realistic features of the U.S. social security system such as the stylized system of social security payments that includes penalties for early retirement or credits for retiring past the normal retirement age.

3.1 Demographics

Time is assumed to be discrete, and the model period is equal to one year. In each period, the economy is populated by \( N \) overlapping generations of individuals of ages \( j = 20, 21, \ldots, J \), who face life-span uncertainty until the maximum possible age \( J \). The size of each new cohort grows at a constant rate \( n \). Mortality risk rises with age, with the conditional probability of survival from age \( j \) to age \( j + 1 \) being denoted \( \Psi_j \) where \( \Psi_J = 0 \). Annuity markets do not exits to insure life-span uncertainty and households are assumed
to have no bequest motive. In spirit of Conesa et al. (2009), accidental bequests arising due to the presence of mortality risk are distributed equally amongst the living in the form of transfers $T_{rt}$.

Agents work until their choose to retire at an endogenously determined age $j = R$. Endogenous retirement is an important extension of many existing models used to study the social security program. In the model, upon reaching the minimum retirement age $j = R$, a household chooses every period whether to retire or not. We assume that the binary decision to retire (i.e., $I = \{0, 1\}$ where $I = 1$ denotes the event of retirement) is irreversible, making retirement a self-absorbing state.$^4$

### 3.2 Endowments and preferences

Each period $t$, a household is endowed with one unit of time that can be used for leisure or market work. Household labor earnings are given by $y_t = w_t \omega_t h_t$, where $w_t$ represents a wage rate per efficiency unit of labor, $h_t$ is the fraction of the time endowment spent on labor market activities, and where the idiosyncratic labor productivity $\omega_t$ follows:

$$\log \omega_t = \theta_j + \alpha + \epsilon_t + v_t.$$  

(1)

In this specification based on and estimated by Kaplan (2012) on the PSID data, $\theta_j$ governs the average age-profile of wages (or age-specific human capital), $\alpha \sim NID(0, \sigma^2_{\alpha})$ is an individual-specific fixed effect (or ability) that is observed at birth and stays fixed for an agent over the life cycle, $\epsilon_t \sim NID(0, \sigma^2_{\epsilon})$ is a transitory shock to productivity received every period, and $v_t$ is a persistent shock, also received each period, which follows a first-order autoregressive process:

$$\epsilon_t = \rho \epsilon_{t-1} + \psi_t \sim NID(0, \sigma^2_{\epsilon}) \text{ and } \epsilon_1 = 0.$$  

(2)

Household preferences over the stream of consumption, $c$, and labor supply, $h$, over the life cycle are governed by a time-separable utility function

$$E_0 \sum_{j=0}^{J} \beta^j U(c_j, h_j),$$  

(3)

where $\beta$ is the discount factor and where the expectation is taken with respect to the life-span uncertainty and the idiosyncratic labor productivity process.

### 3.3 Market structure

The markets are incomplete and households cannot fully insure against the idiosyncratic labor productivity and mortality risks by trading state-contingent assets. They can, however, partially self-insure these risks by accumulating precautionary asset holdings, $a_t$. The stock of assets earns a market return $r_t$. We assume that households enter the economy with no assets and are not allowed to borrow against future income, so that $a_0 = 0$ and $a_t \geq 0$ for all $t$.

### 3.4 Technology

Firms are perfectly competitive with constant returns to scale production technology. Aggregate technology is represented by a Cobb-Douglas production function of the form $Y = F(K, N) = K^\zeta N^{1-\zeta}$, where $K$ and $N$ are aggregate capital and labor (measured

$^4$ Cahill et al. (2011) demonstrate that few people who retire re-enter the labor force. Furthermore, Coile and Levine (2006) find that the boom and bust cycle of the stock market in 2001 did not have a statistically significant effect on the rate of re-entry of retirees back into the labor force.
in efficiency units) and $\zeta$ is the capital share of output. Capital depreciates at a constant rate $\delta \in (0, 1)$. The firms rent capital and hire labor from households in competitive markets, where factor prices $r_t$ and $w_t$ are equated to their marginal productivity. The aggregate resource constraint is:

$$C_t + K_{t+1} - (1 - \delta)K_t + G_t \leq K_t^{\zeta}N_t^{1-\zeta},$$

where, in addition to the above described variables, $C_t$ and $G_t$ represent aggregate household and government consumption, respectively.

### 3.5 Government policy

The government partakes in three activities. First, the government distributes accidental bequests of the deceased agents in a form of lump-sum transfers, $T_{r_t}$, to the living.\(^5\) Second, the government collects a proportional social security tax, $\tau_{sst}$, on pre-tax labor income of working-age individuals (up to an allowable taxable maximum $\overline{y}$) to finance social security payments, $b_{sst}$, for retired workers (for details, see the section below).\(^6\) Third, government consumes in an unproductive sector. Following Conesa et al. (2009), Kitao (2012) and Imrohoroglu et al. (1995), the government consumption, $G_t$, is exogenously determined, and is modeled as proportional to the total output in the steady state economy, so that $G_t = \phi Y_t$. The government uses income tax revenue to finance its spending in the unproductive sector. In the spirit of many studies including Conesa and Krueger (2006), the government taxes each individual’s taxable income according to a progressive labor income tax schedule, $T(\overline{y}_t)$, where the taxable income, $\overline{y}_t$, is defined as:

$$\overline{y}_t = y_t + r_t (T_{r_t} + a_t) - 0.5 \tau_{sst}^{\overline{y}} \min\{y_t, \overline{y}_t\},$$

and where, as is the the case under current U.S. tax law, part of the pre-tax labor income, $y_t$, that is accounted for by the employer’s contributions to social security, $0.5 \tau_{sst}^{\overline{y}} \min\{y_t, \overline{y}_t\}$, is not taxable.

### 3.6 Social security program

We model the system of social security payments, $b_{sst}$, to mimic the U.S. system in several key respects. In the U.S., social security benefits for retired workers (paid out at and after the endogenously determined retirement period $R$) are based on each worker’s average level of earnings calculated over the highest 35 years of earnings.\(^7\) A baseline benefit formula is then applied to each worker’s average level of labor earnings to calculate the pre-adjustment social security benefit.\(^8\) The benefit formula is designed to ensure that the Social Security system is progressive, with the replacement rate being inversely related to past earnings. The baseline benefit calculation is governed by two cut-off points (also known as “bend points”) which jointly determine the degree of progressivity of the social security system. The third, implicit bend point is the cutoff on social security benefits and contributions. The cutoff limits the annual amount of earnings subject to taxation for a given year by determining $\overline{y}_t$, but also applies when those

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\(^5\)By the timing convention, the transfers are received at the beginning of the period before agent’s idiosyncratic labor productivity status is revealed.

\(^6\)Currently we use a value for $\overline{y}$ when calculating the social security tax such that it is never binding with regards to taxable income. We plan on evaluating the effect of this assumption in future work.

\(^7\)These earnings are expressed as workers’ average indexed monthly earnings (AIME), indexed to present by wage growth. We abstract from wage indexation in the model and, for computational simplicity, base the social security payment $b_{sst}$ on the average life cycle earnings with some adjustments described later in this section in Equation 8.

\(^8\)The monthly social security benefit is called primary insurance amount (PIA). Once annualized, the PIA corresponds to the model baseline retirement benefit $b_{sst}^{base}$. In general, the PIA is the benefit a person would receive if she elects to begin receiving retirement benefits at her normal retirement age (NRA).
earnings are used in a benefit computation. Finally, the social security system makes various adjustments to the baseline benefit amount, such as permanent percentage reductions for early retirement and permanent percentage credits for retirement past the normal retirement age (NRA).9

To model the U.S. social security system, we proceed in three steps. First, following Huggett and Parra (2010) and Kitao (2012), we calculate the model analog of each worker’s average level of labor earnings over the working life cycle. At every age, the total accumulated earnings follow the law of motion:

\[
x_{j+1} = \begin{cases} 
   \min\{y_j, \bar{y}\} + (j-1)x_j & \text{if } j \leq 35, \\
   \max\{x_j, \min\{y_j, \bar{y}\} + (j-1)x_j\} & \text{if } 35 < j < R, \\
   x_j & \text{if } j \geq R,
\end{cases}
\]

where \(x_j\) is the accounting variable capturing the equally-weighted average of earnings before the retirement age \(R\); and \(\bar{y}\) is the maximum allowable level of labor earnings subject to the social security tax that corresponds to the benefit-contribution cap. Moreover, to infuse an additional degree of realism while maintaining the model’s tractability, we extend on the specification in Huggett and Parra (2010) by introducing a rule to ensure that the total accumulated labor earnings, \(x_j\), accrued over the working life cycle and used in the benefit calculation cannot fall below their previously realized level, \(x_{j-1}\), after 35 working periods.10 Finally, since agents are not allowed to work during their retirement, which is assumed to be a self-absorbing state, \(x_j\) becomes constant at \(j = R\).

Second, the pre-adjustment social security benefit, \(b^M_{base}\), for each retiree is calculated using a convex, piecewise-linear function of average past earnings observed at retirement age, \(x_R\), so that the marginal benefit rate varies over three levels of taxable income:

\[
\tau_1 \text{ for } 0 \leq x_R < b_1 \\
\tau_2 \text{ for } b_1 \leq x_R < b_2 \\
\tau_3 \text{ for } b_2 \leq x_R < b_3,
\]

where \(\{b_1, b_2, b_3 = \bar{y}\}\) are the two bend points plus the benefit-contribution cut-off point, and where \(\tau_1, \tau_2, \tau_3\) represent the marginal replacement rates in the progressive social security payment schedule associated with the respective bend points.

Finally, adjustments for early and late retirement are calculated. In the U.S., workers can begin receiving permanently reduced monthly retirement benefits after reaching the early retirement age, \(R\).11 The size of the reduction varies with the months out of labor force between the time at which worker retired and her NRA.12 Conversely, when an individual retires after reaching the NRA, the social security benefit payments are increased by a fixed permanent proportion for every year spent working between the NRA and the maximum age cap \(\bar{R}\) for which the credit is available.13 As a result, the total social security benefit \(b_{ss}\) obtained by

\[\text{under the current law, the age at which a worker becomes eligible for full Social Security retirement benefits – the NRA – depends on the worker’s year of birth. For people born before 1938, the NRA is 65. For slightly younger workers, it increases by two months per birth year, reaching 66 for people born in 1943. The NRA remains at 66 for workers born between 1944 and 1954 and then begins to increase in two-month increments again, reaching 67 for workers born in 1960 or later.}

\[\text{computing the social security benefit over the highest 35 years of earnings would render the model intractable, as it would require tracking each period’s earnings as part of the model’s state space.}

\[\text{in the u.s., the minimum retirement age at which social security benefits become available is set at 62. in the data, more than two-thirds of the workers began receiving social security retirement benefits before their normal retirement age. the majority of those early recipients began collecting benefits at age 62. source: social security administration, annual statistical supplement, 2000, p. 240.}

\[\text{a benefit is reduced 5/9 of one percent for each month before normal retirement age, up to 36 months. if the number of months exceeds 36, then the benefit is further reduced 5/12 of one percent per month.}

\[\text{the delayed benefit retirement varies with year of birth, but reaches 2/3 of 1 percent of the benefit for every month delayed.} \]
the retiree is defined as:

$$b_\text{ss} = \begin{cases} (1 - n \kappa_1(n)) b^\text{base}_\text{ss} & \text{if } R \leq R < \text{NRA} \\ (1 - n \kappa_2(n)) b^\text{base}_\text{ss} & \text{if } \text{NRA} \leq R < \overline{R}, \end{cases}$$

where $n = (\text{NRA} - R)$ represents the years of early (delayed) retirement over which the penalty (credit) is accrued; and where $\kappa_1(n)$ and $\kappa_2(n)$ represent functions of yearly rates for early (delayed) retirement penalty (credit), respectively.

### 3.7 Timing convention

At the beginning of the period, uncertainty about early death is revealed to all agents. The living agents receive transfers from accidental bequests and on asset holdings from the previous period. Household that previously retired receive the social security benefit and interest on their accumulated asset holdings, pay off their tax liabilities and make their consumption-saving decision. For households that have not retired in previous periods, the labor productivity status is revealed. Households eligible for retirement then determine whether to retire or work. Working households supply labor and capital to the firm and production takes place. The working households next receive factor income, pay off their tax liabilities and make the consumption-saving decision. If the household chooses to retire then they receive their social security benefit.

### 3.8 Dynamic program of a of a previously working household

A household that was working in the previous period and is indexed by type $(a_t, x_t, \alpha, z_t, y_t, j)$ solves the dynamic program:

$$V_t(a, x, \alpha, z, v, j) = \begin{cases} \max_{c, c', h} U(c, h) + \beta s_j E V_{t+1}(a', x', \alpha, z', v', j + 1) & \text{if } j \leq R, \\ \max_{c, c', h, l} U(c, h) + \beta s_j E V_{t+1}(a', x', \alpha, z', v', j + 1) & \text{if } R < j \leq \overline{R}, \end{cases}$$

subject to

$$c + c' = (1 + r)(Tr + a) + y - \tau ss \min\{y, \overline{y}\} \quad \text{if } I = 0,$$

$$c + c' = (1 + r)(Tr + a) + y - \tau ss - b^\text{ss} \quad \text{if } I = 1.$$  

by choosing consumption, $c$, savings, $c'$, and time spent working, $h'$. The accounting variable $x$ is the average lifetime labor earnings as of period $t$ and follows the law of motion specified in equation 6. Households earn interest income $r(Tr + a)$ on the lump-sum transfer $Tr$ from accidental bequests and on asset holdings from the previous period, $a$. $y$ represents the pre-tax labor income of the working households and is described in Section 3.2. $\overline{y}$ defines the taxable income on which the income tax, $T$, is paid, and follows the process in equation 5. Finally, $\tau ss$ is the social security tax rate that is applied to the pre-tax labor income, $y$, up to an allowable taxable maximum, $\overline{y}$. As in the U.S. system, households of age $j < \overline{R}$ are not eligible for social security benefits and as, such, are not allowed the decision to permanently retire. Instead, households who decide not to participate in the labor market prior to reaching the minimum retirement age $j = \overline{R}$ can do so by choosing zero labor hours (i.e., $h = 0$). Upon reaching the minimum retirement age, households make a permanent decision to retire, with $I = 1$ describing the event of retirement ($I = 0$ otherwise). Finally, households are forced into a mandatory retirement after reaching age $\overline{R}$.

(or 8 percent annualized) for individuals born in 1943 or later. Source: Jonathan F. Pingle, Social Security’s Delayed Retirement Credit and the Labor Supply of Older Men, 2006. No credit is given after age 69.
3.9 Dynamic program of a previously retired household

Upon reaching the minimum retirement age $R$, households are allowed to retire permanently. Retired households receive a constant stream of social security payments whose size is determined by the level of the average life cycle labor earnings observed at the retirement period, $x_R$. Retired households are unaffected by labor productivity shocks observed post-retirement, as the option to work in retirement is not allowed in the model. As such, a retired household indexed by type $(a_t, x_R, \alpha, z_t, v_t, j)$ solves the dynamic program:

$$V_t(a, x_R, \alpha, z, v, j) = \max_{c, a'} \left( c \cdot U(c, h) + \beta \sum_{t+1}^{\infty} \mathbb{E} V_{t+1}(a', x_R, \alpha, z, v', j+1) \right),$$

subject to

$$c + a' = (1+r)(Tr + a) + b^{ss} - T(\hat{y}),$$

by choosing consumption, $c$, and savings, $a'$. Similarly to non-retired households, retirees earn interest income $r(Tr + a)$ on the transfer, $Tr$, and their existing asset holdings, $a$, but also receive the social security payment, $b^{ss}$.

4 Equilibrium

Let $a \in R_+, x \in R_+, \mu \in E = \{\mu_1, \mu_2, \ldots, \mu_N\}$, $j \in J = \{1, 2, \ldots, J\}$, and let $S = R_+ \times R_+ \times E \times J$. Let $B(R_+)$ be the Borel $\sigma$–algebra of $R_+$ and $P(E), P(J)$ the power sets of $E$ and $J$, respectively. Let $\Theta = B(R_+ \times B(R_+) \times P(E) \times P(J)$, and let $M$ be the set of all finite measures over the measurable space $(S, \Theta)$.

**Definition.** Given a sequence of social security payment functions $\{SS_t\}^{\infty}_{t=1}$, and initial conditions capital, $K_1$, and the measure of agents of type $(a_t, x_t, \mu_t, j_t)$ at time $t$, $\Omega_t$, a competitive equilibrium is a sequence of individual functions for the households $\{v_t, c_t, a'_t, h_t\}^{\infty}_{t=1}$, sequence of production plans for firms $\{N_t, K_t\}^{\infty}_{t=1}$, prices $\{w_t, r_t\}^{\infty}_{t=1}$, government policies for the social security system, $\{\tau^{ss}_t, \delta^{ss}_t\}^{\infty}_{t=1}$, and income taxation functions, $\{T_t\}^{\infty}_{t=1}$, a sequence of transfers $\{Tr_t\}^{\infty}_{t=1}$, and a sequence of measures $\{\Omega_t\}^{\infty}_{t=1}$, $\Omega_t \in M$, such that for all $t$, the following hold.

1. Given prices, policies, transfers, and initial conditions, $v_t$ is the solution to the dynamic programming problem in equations 8 - 12, with $c_t, a'_t$, and $l_t$ as associated policy functions.

2. The prices $w_t$ and $r_t$ satisfy

$$r_t = \zeta \left( \frac{N_t}{K_t} \right)^{1-\zeta} - \delta$$

$$w_t = (1-\zeta) \left( \frac{N_t}{K_t} \right)^{\zeta}.$$

3. The social security policies satisfy:

$$\int \omega w_T \tau_{ss} \Omega_t(i, a, \mu, j) = \int b^{ss} I \Omega_t(i, a, \mu, j).$$

4. Transfers are given by:

$$Tr_t = \int (1-\Psi) a \Omega_t(i, a, \mu, j).$$

5. Government budget balance:

$$G = \int T^r[r(a + Tr) + (1-\tau_{ss})w_0 h] \Omega_t(i, a, \mu, j).$$
6. Market clearing:

\[ K = \int a \Omega_t(i, a, \mu, j), \quad N = \int \omega_j \Omega_t(i, a, \mu, j) \text{ and} \]

\[ \int c \Omega_t(i, a, \mu, j) + \int a \Omega_t(i, a, \mu, j) + G = K^{\frac{\epsilon}{\gamma}N^{1-\frac{\epsilon}{\gamma}}} + (1-\delta)K. \]

**Definition.** We define a steady state equilibrium as a competitive equilibrium where \( \Omega_t \) is constant and all aggregate variables grow at the same rate as population.

## 5 Calibration

### 5.1 Demographics, endowments and preferences

There are 80 overlapping generations of individuals of ages \( j = 20, \ldots, 100 \). We follow Conesa et al. (2009) and Kitao (2012) in setting the population growth rate, \( n \), to 1.1 percent, to match the growth rate in the U.S. economy. The conditional survival probabilities \( \{\Psi_j\}_{j=20}^{100} \) are derived from the U.S. life tables (Bell and Miller (2002)).

Following Huggett and Parra (2010), the process for the idiosyncratic labor productivity, \( \omega \), is calibrated based on the estimates from the PSID data in Kaplan (2012).\(^{14}\) The deterministic labor profile, \( \exp^{\theta_j} \), is shown in Figure 5.1. The profile is smoothed by fitting a quadratic function in age, normalized such that the value equal one when an agent enters the economy, and extended to cover ages 20 through 69 which we define as the last period in which households are assumed to participate in the labor activities (\( R \)).\(^{15}\) The permanent, persistent and transitory idiosyncratic shocks to individual’s productivity are distributed normal with a mean of zero. The remaining parameters are also set in accordance with the estimates in Kaplan (2012): \( \rho = 0.958, \sigma_\delta = 0.065, \sigma_\nu = 0.017 \) and \( \sigma_\epsilon = 0.081 \). We discretize all three of the shocks in order to solve the model. We use two states to represent the transitory and permanent shocks and five states for the persistent shock. For expositional convenience, we refer to the two different states of the permanent shock as high and low ability types.

![Figure 1: \( \exp^{\theta_j} \)](image)

We model the preferences as additively separable between consumption and labor:

\[ U(c_t, n_t) = c_t^{1-\gamma} \left( \frac{1}{1-\gamma} - \chi_1 + \frac{1}{1+\delta} - \chi_2 \right). \tag{13} \]

\(^{14}\)For details on estimation of this process, see Appendix E in Kaplan (2012).

\(^{15}\)The estimates in Kaplan (2012) are available for ages 25-65.
with $\gamma > 0$, $\sigma > 0$, $\chi_1 > 0$ and $\chi_2 > 0$. The constant relative risk aversion preferences over consumption are standard and are characterized by the risk aversion coefficient, which determines household desire to smooth consumption across time and states. The existing estimates of $\gamma$ typically ranging between 1 and 3; in this paper, we set $\gamma = 2$. The parameter $\sigma$ represents the Frisch labor supply elasticity on the intensive margin. Past microeconometric studies estimate the Frisch elasticity to be between 0 and 0.5 (see, for example, Kaplan (2012), Altonji (1986), MaCurdy (1981), Domeij and Floden (2006) or Browning et al. (1999). However, more recent research shows that these estimates may be biased downward (see Imai and Keane (2004), Domeij and Floden (2006), Pistaferri (2003), Chetty (2009), and Contreras and Sinclair (2008)). As such, we calibrate $\sigma$ at $\frac{1}{2}$ – the upper range of the available estimates. The scaling constant $\chi_1$ is calibrated such that on average agents work one third of their endowment prior to the normal retirement age. Additionally, the fixed cost of working $\chi_2$ is set such that seventy percent of individuals retire by the normal retirement age which matches the data. The fixed cost $\chi_2$ implies that the disutility from working discontinuously increases when an agent goes from zero to positive hours worked. Finally, in order to characterize the household preferences described in Equation 3, we calibrate the discount factor $\beta$ to 0.996 such that the capital-to-output ratio matches U.S. data of 2.7.

### 5.2 Social Security

For simplicity, we set the NRA at 66, irrespective of the calendar year in which an agent was born. Following the current U.S. social security system, the early retirement age $R$ is set at 62 while the maximum age over which delay retirement credits can be accrued is set at 69. As discussed above, it is assumed that at age 70 no agent in the economy works. The early retirement percentage penalty, $\kappa_1$ is based on the actual value in the U.S. Social Security system and is set at 6.7 percent ($\kappa_{1a}$) for the first three years of early retirement and is set at 5 percent ($\kappa_{1b}$) for prior years. The delayed retirement credit, $\kappa_2$ is set at 8 percent per annum. The marginal replacement rates in the progressive social security payment schedule ($\tau_{r1}$, $\tau_{r2}$, $\tau_{r3}$) are also set at their actual respective values of 0.9, 0.32 and 0.15. Finally, we follow Huggett and Parra (2010) in setting the bend points and the maximum earnings $y_{max}$ equal to the actual multiples of mean earnings used in the U.S. social security system so that the bend points $b_1$ and $b_2$ occur at 0.21 and 1.29 times average earnings in the economy. The third implicit bend point $b_3$ and the maximum taxable earnings by social security, $y_{max}$ are equal to 2.42 times average earnings in the economy.

### 5.3 Government

We set the government spending in the unproductive sector to 17 percent of GDP in the steady state, so that $\phi = 0.17$. We follow a host of literature (two examples include Conesa et al. (2009) and Imrohoroglu and Kitao (2012)), in using the three parameter tax function from Gouveia and Strauss (1994) to calculate the income taxes over the taxable income for each individual.

$$T(\tilde{y}_t; Y_0, Y_1, Y_2) = Y_0(\tilde{y}_t - (\tilde{y}_t - Y_1 - Y_2)^{\frac{1}{\tau}}),$$

\[ (14) \]

\[ ^{16}\text{We use this utility function that is additively separable in labor and consumption since Peterman (2013) and Conesa et al. (2009) demonstrate that when leisure, instead of labor, enters the utility function that the Frisch labor supply elasticity on the intensive margin is not constant over the lifetime and can affect the economy.} \]

\[ ^{17}\text{We note that estimates of the Frisch elasticity from simulated data in this model would be larger than 0.5 due to changes in the wages affecting an individuals P.I.A. However, Peterman (2012) demonstrates that the increase in elasticity from endogenously determined social security benefits is small.} \]


\[ ^{19}\text{An alternative formulation that would induce agents to make decisions on the extensive margin is to include a non-linear mapping between hours and productivity (for example see Rogerson and Wallenius (2009)). Although both modeling options create an active extensive margin, we found that solving for a steady state when using a fixed cost was more stable with respect to initial guesses.} \]
<table>
<thead>
<tr>
<th>Parameter</th>
<th>Value</th>
<th>Source</th>
</tr>
</thead>
<tbody>
<tr>
<td><strong>Demographics:</strong></td>
<td></td>
<td></td>
</tr>
<tr>
<td>Normal Retirement Age: $j_{exog}$</td>
<td>66</td>
<td>By Assumption</td>
</tr>
<tr>
<td>Max Age: $J$</td>
<td>100</td>
<td>By Assumption</td>
</tr>
<tr>
<td>Surv. Prob: $\Psi_j$</td>
<td>Bell and Miller (2002)</td>
<td></td>
</tr>
<tr>
<td>Pop. Growth: $n$</td>
<td>1.1%</td>
<td>Conesa et al. (2009)</td>
</tr>
<tr>
<td><strong>Firm Parameters:</strong></td>
<td></td>
<td></td>
</tr>
<tr>
<td>$\zeta$</td>
<td>0.36</td>
<td>Data</td>
</tr>
<tr>
<td>$\delta$</td>
<td>8.33%</td>
<td>$\frac{1}{\gamma} = 25.5%$</td>
</tr>
<tr>
<td>$A$</td>
<td>1</td>
<td>Normalization</td>
</tr>
<tr>
<td><strong>Preference Parameters:</strong></td>
<td></td>
<td></td>
</tr>
<tr>
<td>Conditional Discount: $\beta$</td>
<td>0.996</td>
<td>$\frac{K}{\gamma} = 2.7$</td>
</tr>
<tr>
<td>Risk aversion: $\gamma$</td>
<td>2</td>
<td>Conesa et al. (2009)</td>
</tr>
<tr>
<td>Frisch Elasticity: $\sigma$</td>
<td>0.5</td>
<td>Intensive Frisch $= \frac{1}{\gamma}$</td>
</tr>
<tr>
<td>Disutility to Labor: $\chi_1$</td>
<td>41.9</td>
<td>Avg. $h_j = \frac{1}{\gamma}$</td>
</tr>
<tr>
<td>Fixed Cost to Working: $\chi_2$</td>
<td>0.725</td>
<td>70% retire by $J_{nr}$</td>
</tr>
<tr>
<td><strong>Productivity Parameters:</strong></td>
<td></td>
<td></td>
</tr>
<tr>
<td>Persistence Shock: $\sigma_\nu^2$</td>
<td>0.017</td>
<td>Kaplan (2012)</td>
</tr>
<tr>
<td>Persistence: $\rho$</td>
<td>0.958</td>
<td>Kaplan (2012)</td>
</tr>
<tr>
<td>Permanent Shock: $\sigma_\delta^2$</td>
<td>0.065</td>
<td>Kaplan (2012)</td>
</tr>
<tr>
<td>Transitory Shock: $\sigma_\varepsilon^2$</td>
<td>0.081</td>
<td>Kaplan (2012)</td>
</tr>
<tr>
<td><strong>Government Parameters:</strong></td>
<td></td>
<td></td>
</tr>
<tr>
<td>$\Upsilon_0$</td>
<td>0.258</td>
<td>Gouveia and Strauss (1994)</td>
</tr>
<tr>
<td>$\Upsilon_1$</td>
<td>0.768</td>
<td>Gouveia and Strauss (1994)</td>
</tr>
<tr>
<td>$\phi$</td>
<td>17%</td>
<td>Conesa et al. (2009)</td>
</tr>
<tr>
<td><strong>Social Security:</strong></td>
<td></td>
<td></td>
</tr>
<tr>
<td>$\kappa_{1a}$</td>
<td>6.7%</td>
<td>U.S. SS Program</td>
</tr>
<tr>
<td>$\kappa_{1b}$</td>
<td>5%</td>
<td>U.S. SS Program</td>
</tr>
<tr>
<td>$\kappa_2$</td>
<td>8%</td>
<td>U.S. SS Program</td>
</tr>
<tr>
<td>$\tau_{r1}$</td>
<td>90%</td>
<td>U.S. SS Program</td>
</tr>
<tr>
<td>$\tau_{r2}$</td>
<td>32%</td>
<td>U.S. SS Program</td>
</tr>
<tr>
<td>$\tau_{r3}$</td>
<td>15%</td>
<td>U.S. SS Program</td>
</tr>
<tr>
<td>$b_1$</td>
<td>$1.21 \times$ Avg Earnings</td>
<td>Huggett and Parra (2010)</td>
</tr>
<tr>
<td>$b_2$</td>
<td>$1.29 \times$ Avg Earnings</td>
<td>Huggett and Parra (2010)</td>
</tr>
<tr>
<td>$b_3$</td>
<td>$2.42 \times$ Avg Earnings</td>
<td>Huggett and Parra (2010)</td>
</tr>
<tr>
<td>$\tau_{ss}$</td>
<td>10.0%</td>
<td>Mrkt Clearing</td>
</tr>
</tbody>
</table>
We set these parameters to the estimates in Gouveia and Strauss (1994), and similarly to Conesa et al. (2009), we solve for the scaling factor $\Upsilon_2$ so that in the steady state the income taxes equals government spending. Finally, the social security tax, $\tau_{ss}$ is determined such that in steady state the program is balanced budget.\(^{20}\)

6 Results

6.1 Baseline model with social security system

We begin by examining the predictions of the model for the steady state economy with social security. Figure 2 shows the average baseline life cycle profiles across all households and across the different skill types (high and low). The generated average profiles of consumption and savings in Panels A and B follow expected patterns. Not surprisingly, the average consumption and savings of the high skill workers are superposed to those of the low skill types.

Average household labor supply, shown in Panel C, starts at 40 percent of the total labor endowment at age 20, and gradually decreases as the households age, reaching roughly 30 percent of the total labor endowment by age 65. The average profile across all households between ages 25 and 30 matches well the profile estimated on the PSID data by Erosa et al. (2011), although the decline in labor supply over the life cycle is perhaps more pronounced in the model than in the data. Additionally, very young households who are liquidity constrained in the model spend more time working than in the data. In the model, the positive wealth effect that labor supply has on household consumption and precautionary asset holdings more than offsets the disincentive to work associated with wages being relatively low at the beginning of the life cycle (for further discussion of this prediction, see Heathcote et al. (2010)). Moreover, neither the labor supply nor the decision to retire (see Table 2) are uniform across the skill types. In the model, the low skill workers on average both spend more time in the labor market and retire at older age (66 versus 63) than their high skill counterparts.\(^{21}\)

Table 2: Average Retirement Age In Steady State Economies

<table>
<thead>
<tr>
<th></th>
<th>Social Security</th>
<th>No Social Security</th>
</tr>
</thead>
<tbody>
<tr>
<td>All Households</td>
<td>64.8</td>
<td>68.1</td>
</tr>
<tr>
<td>Low Skill Types</td>
<td>66.3</td>
<td>69.2</td>
</tr>
<tr>
<td>High Skill Types</td>
<td>63.3</td>
<td>67</td>
</tr>
</tbody>
</table>

6.2 Counterfactual model with no social security

At this point, it is useful to compare the baseline model with the social security program against a counterfactual economy in which government-provided consumption insurance at old age is absent. Table 3 compares macroeconomic aggregates between the two models, while Figure 3 provides a comparison of average age-profiles of consumption, savings, and labor supply.

Consistent with findings in a number of existing studies discussed in the introduction, the economy with no social security is characterized by higher level of aggregate capital, as the social security program reduces the dependence of retirees on personal

\(^{20}\)Currently instead of capping the labor income that is taxed at $\tau_{ss}$ at $Y_{max}$ we subject all labor income to the social security tax.

\(^{21}\)Using the PSID data, Erosa et al. (2011) estimate that the annual labor supply of workers with only a high school diploma is lower than the labor supply of workers with a college degree. At the same time, using the same data source, Shourideh and Troshkin (2011) document that the more productive workers on average retire significantly sooner than their low productivity counterparts, with the average retirement age of 62.8 versus 69.5 (for details, see their Table 4).
Figure 2: **Steady State Life Cycle Profiles in Model with Social Security**

A: Consumption Profiles in Steady States

B: Savings Profiles in Steady States

C: Labor Profiles in Steady States

Note: These plots are the average values by age and ability type in all three models.
savings to fund their old-age consumption. The higher aggregate level of capital in the counterfactual economy with no social security is thus characterized by a higher wage rate and a lower rental rate relative to the baseline model.

The intertemporal Euler equation controls the slope of consumption profile over an agent’s lifetime.\textsuperscript{22} The lower return to savings in the counterfactual model with no social security leads the average consumption profile in the counterfactual economy with no social security to be less steep relative to the baseline. Moreover, in an economy with no social security, agents backload their labor supply to later years, in part because the lower return on capital reduces their incentives to amass large amount of savings for post-retirement consumption early in life.

Turning to the welfare results, our results confirm the findings in the existing studies (see, for example, Hong and Rios-Rull (2007), Hubbard and Judd (1987), Imrohoroglu et al. (1995), and Storesletten et al. (1998)): in a steady-state, agents prefer to live in an economy where a social security system is largely absent. The consumption equivalent variation (CEV), or the uniform percent increase in each periods consumption necessary to make agents on average indifferent between born into the economy without social security versus the economy with a social security, is estimated at 9.5 percent.

\begin{table}[h]
\centering
\caption{Aggregates in Steady States} \\
\begin{tabular}{lcc}
\hline
 Aggregate & Full S.S. & No S.S. \\
\hline
 Y & 1.14 & 1.3 \\
 K & 3.08 & 4.3 \\
 N & 0.65 & 0.67 \\
 w & 1.12 & 1.25 \\
 r & 0.05 & 0.03 \\
 Tr & 0.05 & 0.08 \\
 $\tau_{ss}$ & 0.1 & 0 \\
\hline
\end{tabular}
\end{table}

\textbf{6.3 Transitional dynamics}

Having discussed the steady state comparison of the baseline model against the counterfactual model with no social security, we next assess the effect of a large, unexpected, one-time shock to household wealth in the baseline- and counterfactual economies. The shock is introduced as a 20 percent unexpected reduction in household wealth in the economy at time zero.

Panel A in Figure 4 plots the evolution of the percentage differences in the baseline model’s aggregate capital, efficient labor, the wage and the return to capital relative to their respective steady state values over time. At time of the shock, the aggregate capital decreases by 20 percent by construction. In response, the rental rate increases by approximately 50 percent from its steady state value of 5 percent, while the efficient wage falls by 10 percent. Given that the labor supply margin is available (and in light of the fact that unemployment risk is not modeled in this version of the paper), agents who have not yet retired respond to the shock by increasing their labor supply to partly mitigate the effects of the shock on their consumption and asset holdings. Over time, as the economy sets on the transition path to the original steady state, the stock of capital gradually converges to its pre-shock level and the rental rate falls while the efficient wage rate rises.

\textsuperscript{22}The relationship is
\[
\left(\frac{c_{j+1}}{c_j}\right)^{\sigma_i} = \Psi_j \bar{\rho}_t, 
\]
where $\bar{\rho}_t$ is the marginal after-tax return on capital.
Figure 3: Life Cycle Profiles in Steady State

A: Consumption Profiles in Steady States

B: Savings Profiles in Steady States

C: Labor Profiles in Steady States

Note: These plots are the average values by age and ability type in both models.
Panel B in Figure 4 repeats the exercises in the model with no social security. Both qualitatively and quantitatively, the transitional dynamics of the macroeconomic aggregates are essentially identical between the baseline economy with social security and the counterfactual economy where the social security system is absent when expressed in terms of percentage differences from their respective steady-state values.

![Figure 4: Transition Of Aggregate Economic Variables](image)

A: Transition with Social Security
B: Transition without Social Security

### 6.4 Comparing welfare losses along the transition between economies with and without social security

During the transition, we define the welfare loss as a constant percentage increase in per-period future consumption following the shock necessary to restore an individual’s expected future utility to that in the steady state. Furthermore, we define the average welfare loss in each economy over the transition as the population-weighted mean of these values for each of the cohorts alive at the time of the shock. Table 4 describes the average welfare losses over the transitional path for each economy, and also reports the losses for select age groups. The average overall welfare loss is 5.5 percent in the model with no social security and 4 percent in the baseline model with social security. The resulting difference in the welfare loss (1.5 percent) highlights the ability of the social security program to partly mitigate the adverse effect of a large, unanticipated wealth shock on household welfare.

<table>
<thead>
<tr>
<th></th>
<th>Average</th>
<th>20-65</th>
<th>66-100</th>
<th>20-40</th>
<th>40-60</th>
<th>60-80</th>
<th>80-100</th>
</tr>
</thead>
<tbody>
<tr>
<td>Full SS</td>
<td>4</td>
<td>3.6</td>
<td>5.4</td>
<td>2.7</td>
<td>4.4</td>
<td>5.6</td>
<td>4.9</td>
</tr>
<tr>
<td>No SS</td>
<td>5.5</td>
<td>3.8</td>
<td>13.4</td>
<td>2.4</td>
<td>4.5</td>
<td>10.8</td>
<td>15.5</td>
</tr>
<tr>
<td>Difference</td>
<td>-1.5</td>
<td>-0.2</td>
<td>-7.9</td>
<td>0.2</td>
<td>-0.1</td>
<td>-5.1</td>
<td>-10.6</td>
</tr>
</tbody>
</table>

Panels A and B in Figure 5 plot the distributions of the transitional welfare losses by age at time of the shock for low and high ability types in the respective economies. As expected, the losses tend to be smaller for younger individuals: around 3.6 percent of expected future consumption in the baseline model with social security versus 3.8 percent in the counterfactual economy where social security is missing. At the same time, estimated at roughly 5.4 and 13.4 percent respectively, the welfare losses for older households are larger. The result is not surprising in light of the fact that it is the older households who have amassed a significant amount of savings over their life cycle and who, therefore, are affected the most by the catastrophic wealth shock. In addition, the
average additional welfare lost in the model without social security during the transition for agents older than 65 is the equivalent of 7.9 percent of future expected consumption.

Notably, the panels jointly demonstrate the important role that social security plays in mitigating the effects of the shock for older agents. In the baseline economy with no social security, the average welfare lost over the transition is hump-shaped in household age at the time of the shock, peaking approximately at age 70 when no agent in the economy is by assumption allowed to work. In marked contrast, the welfare losses increase monotonically with household age at the time of the shock in the counterfactual model with no social security. In our view, these results clearly demonstrate the key role of social security as an insurance mechanism for an old-age consumption at times of large and unforeseen economic distress.

Figure 5: Welfare Lost in Transition

A: CEV Lost (Full SS)

<table>
<thead>
<tr>
<th>Age at Shock</th>
<th>Avg.</th>
<th>Low</th>
<th>High</th>
</tr>
</thead>
<tbody>
<tr>
<td>20</td>
<td>30</td>
<td>40</td>
<td>50</td>
</tr>
<tr>
<td>60</td>
<td>70</td>
<td>80</td>
<td>90</td>
</tr>
</tbody>
</table>

B: CEV Lost (No SS)

<table>
<thead>
<tr>
<th>Age at Shock</th>
<th>Avg.</th>
<th>Low</th>
<th>High</th>
</tr>
</thead>
<tbody>
<tr>
<td>20</td>
<td>30</td>
<td>40</td>
<td>50</td>
</tr>
<tr>
<td>60</td>
<td>70</td>
<td>80</td>
<td>90</td>
</tr>
</tbody>
</table>

Note: These plots are the average welfare lost by type and age at the time of the shock.

In the economy without social security all of the agent’s post-retirement consumption is financed by their savings. In contrast, in the model with social security, some of the the post-retirement consumption is financed by the social security benefit. The social security benefit acts as partial insurance for a retired agent’s consumption. Figure 6 highlights this point. The figure plots the percentage decline in consumption in the first period after the shock relative to the level of the consumption the agent would have received if not for the shock. The effect of the shock on immediate consumption is similar for agents still working in both models. In contrast, in the model without social security, retired agents make much more dramatic cuts to their immediate consumption following the shock compared to the model with social security.

6.5 The Effect of Shock on Average Lifecycle Profiles

Panels A-C in Figures 7-9 plots the average consumption, saving and labor profiles over the transition for an agent who was 35, 62, and 80 years of age at the time of the shock and lives in an economy with the social security program, and compares them to profiles of the identical, same-age agent in the steady state economy who has never experienced the shock. Figures 11-13 repeat the exercise for the counterfactual economy with no social security system. In the baseline economy with social security in Figure 7, an average agent who is 35 years of age loses 20 percent of her accumulated wealth at time of the shock. The timing of the shock corresponds with a drop in consumption and an increase in labor hours, as agents use the intensive labor margin to partially offset the effect of the shock. However, as Panel B demonstrate, young agents are able to re-accumulate their original asset positions relatively quickly: by the age of 60, the savings profile of a young agent who has lived through a severe recession looks essentially identical to one of the same agent who has never experienced the large and negative wealth shock.
Figure 6: **Percent of Consumption Lost in 1st Subsequent Period After Shock**

![Graph showing percent of consumption lost in 1st subsequent period after shock]

Note: This plot is the average percent drop in consumption due to the shock in the first subsequent period.

Turning to middle-aged individuals, Figure 8 plots the average life cycle profiles of an agent who was 62 at time of the shock. Unlike the younger cohorts, the agents at the brink of retirement are never able to recover the savings levels that they would have reached had they never lived through the recession. In an attempt to try to regain some of the wealth lost to the shock, these agents tend to delay retirement (see discussion below). Despite changes in the labor supply on the extensive margin, the wealth shock is associated with a persistently depressed consumption levels. Unsurprisingly, the effects of the shock are similar agents that are even older (see Figure 9).

Figure 10 shows the fraction of household who are retired when they reach the respective ages 62-70 as the economy sets on a transitional path to the original steady-state following the wealth shock. Focusing on Panel A, at steady state, about 30 percent of all agents who are 62 at time of the shock are retired. That said, 5 periods after the shock, only roughly 23 percent of all households who are age 62 at that point are retired. The large drop in the fraction of retired households in the economy following the shock demonstrates that pre-retirement households respond to the large loss in wealth by delaying their planned retirement. That said, the decision to postpone retirement is mostly in effect for households who have not reached the normal retirement age of 66. For older households, the decision to retire is largely unaffected by the size of the shock. This is likely in part because many of these older households have already entered retirement - a self-absorbing state in the model - by the time of the wealth shock and, by construction, cannot therefore re-enter the labor force after the adverse shock hits.

Finally, turning to the retirement decision by skill type (Panels B and C), our results indicate that the decision to postpone retirement in mostly in effect for high skill agents in the economy. In the steady-state model as in the data (see Shourideh and Troshkin (2011)), high skill workers in the model tend to retire sooner than their low skill counterparts. As a result, adjustments along the extensive margin are more likely for these agents than for the low skill workers who tend to retire at the normal retirement age or later irrespective of the effect of the shock.

Comparing Figures 7 - 9 with Figures 11 - 13, it clear that the shock affects the life cycle profiles in the two economies in a similar manner, although the magnitude of the effect is greater in the model without social security. Figure 15 shows the fraction of households who are retired when they reach the respective ages of 62-70 over the transitional path in the model without social security. Overall, the shapes of the plots in the model without socials security also look similar to those in the model with social security (previously shown in Figure 10). However, the fraction of households who choose to delay retirement in response to the shock is larger in the economy without social security.
Figure 7: Life Cycle Profiles in Transition (Full Social Security)

A: Consumption Profiles 35 at Shock
B: Capital Profiles 35 at Shock
C: Labor Profiles 35 at Shock

Note: These plots are the average profiles for high and low ability types for agents over the transition who were 35 at the time of the shock.
Figure 8: Life Cycle Profiles in Transition (Full Social Security)

A: Consumption Profiles 62 at Shock

B: Capital Profiles 62 at Shock

C: Labor Profiles 62 at Shock

Note: These plots are the average profiles for high and low ability types for agents over the transition who were 62 at the time of the shock.

Figure 9: Life Cycle Profiles in Transition (Full Social Security)

A: Consumption Profiles 80 at Shock

B: Capital Profiles 80 at Shock

Note: These plots are the average profiles for high and low ability types for agents over the transition who were 80 at the time of the shock.
7 Conclusion

This paper quantitatively assesses the ability of social security to lessen the welfare losses due to a large unexpected shock to household wealth. In particular, first, we examine the distributional effects on welfare by age and ability over the transition after a one-time unexpected shock to the household balance sheet in a baseline economy with a social security program. Second, we examine the welfare losses after a similar large shock is introduced in a counterfactual economy without a social security program. Generally, we find in both economies that the welfare losses during the transition tend to be smaller for younger individuals than older individuals. We also find that social security can be helpful in mitigating these welfare losses. Overall, across all the living cohorts, we find that the average welfare loss are larger in a model without social security by the equivalent of 1.5 percent of expected future consumption compared to a model with social security program. Moreover, we find that social security is particularly helpful to agents older than 65 at the time of the shock. Specifically, we find that for these individuals the average additional welfare lost is the equivalent of 7.9 percent of their future expected consumption when social security is absent.

The older agents in the model without social security are particularly vulnerable to the shock to their wealth because all of their consumption is funded directly out of their savings. In contrast, in the model with social security, some of these agents’ consumption is funded with social security benefits. Although we find that social security is particularly effective in mitigating the losses over the transition for individuals who are old at the time of the shock, the program does not come without some welfare cost. We estimate that, in the steady state economy with social security, the welfare is lower by the equivalent of 9.5 percent of total expected lifetime consumption than in the economy without the social security program.
Figure 11: Life Cycle Profiles in Transition (No Social Security)

A: Consumption Profiles 35 at Shock
B: Capital Profiles 35 at Shock
C: Labor Profiles 35 at Shock

Note: These plots are the average profiles for high and low ability types for agents over the transition who were 35 at the time of the shock.
Figure 12: **Life Cycle Profiles in Transition (No Social Security)**

A: Consumption Profiles 62 at Shock

B: Capital Profiles 62 at Shock

C: Labor Profiles 62 at Shock

Note: These plots are the average profiles for high and low ability types for agents over the transition who were 62 at the time of the shock.

Figure 13: **Life Cycle Profiles in Transition (No Social Security)**

A: Consumption Profiles 80 at Shock

B: Capital Profiles 80 at Shock

Note: These plots are the average profiles for high and low ability types for agents over the transition who were 80 at the time of the shock.
Figure 14: Percent Retired Over Transition Without Social Security

A: Percent Retired Over Transition

B: Percent of Low Retired Over Transition

C: Percent of High Retired Over Transition

Note: These plots are the percent of individuals retired at each age over the transition.
Figure 15: Percent Retired Over Transition Without Social Security

A: Percent Retired Over Transition

B: Percent of Low Retired Over Transition

C: Percent of High Retired Over Transition

Note: These plots are the percent of individuals retired at each age over the transition.
References


