Campaign Finance in U.S. House Elections*

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Abstract

This paper models the dynamics of fundraising, campaign spending, and accumulation of war chest in the context of U.S. House elections. We structurally estimate the model using campaign finance data and vote share data from 1984 to 2004. A salient feature of U.S. Congressional Elections is that any campaign money that was not used in previous elections can be carried over to the next election with possible deterrence effects. In our counterfactual experiment, we analyze the effect of publicly financing challenger campaigns which is designed to create a more level playing field. We find, however, that the intended effect of such campaign finance reforms are often off-set by increased spending of incumbents.

1 Introduction

The relationship between campaign spending and competitiveness of elections is a topic which has attracted a lot of attention among voters, policy makers, and the media. This is natural given the fundamental role elections play in ensuring that voter preferences are reflected in policy. To the extent that campaign fundraising and spending impede fair electoral competition, it has important implications for democracy. Understanding the relationship between campaign finance and electoral competition is also important for an informed debate on campaign finance reform.

In this paper we study the effect of campaign spending on election outcomes and the effect of campaign war chests on candidate entry by estimating a dynamic game using data from U.S. House elections. An important feature of campaign financing in Congressional elections is that candidates can carry over unspent money from past elections for use in

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future elections. These savings, called war chest, influence the entry decisions of potential challengers in addition to providing candidates with a source of funds to tap during tough future elections.¹ In this paper, we construct a model of challenger entry, campaign spending and savings that explicitly account for these forces and structurally estimate the primitives of the model. Our model builds on the model of campaign spending previously considered in Erikson and Palfrey (2000). The key component of the Erikson and Palfrey model is the trade-off between the cost of raising money and the benefit of spending. We extend their model in multiple directions by introducing an entry decision and a savings decision while allowing for unobserved candidate-specific persistent heterogeneity in quality. Allowing for heterogeneity in candidate quality enables us to capture how war chest impacts the distribution of the quality of potential challengers that one faces in future elections.

In our counterfactual experiment, we analyze the effect of various campaign finance reforms (e.g., partial public financing of campaign expenditures) that reduce the cost of fundraising for challengers (e.g., through partial public financing). Often, one of the important rationales for campaign finance reforms is to alleviate the spending imbalance between the incumbents and challengers in order to create a more level playing field between the incumbents and challengers.² We find, however, that the intended effect of such campaign finance reforms are often off-set by increased spending of incumbents.

In terms of identification, one of the key challenges of our paper is to identify the effect of campaign spending on vote shares while accounting for possible endogeneity of spending – that is, dealing with the correlation between campaign spending and the unobserved quality of the candidates. The importance of controlling for unobserved candidate quality has been recognized since the pioneering work of Jacobson (1976) where he finds that a simple regression of incumbent vote share on incumbent spending yields a *negative* coefficient on incumbent spending. This is in large part, due to the fact that incumbents spend large sums of money only when they face a high quality challenger. This paper proposes an identification strategy based on a control function approach which has several advantages over other alternatives that have been proposed.

Our identification strategy is closely related to Olley and Pakes (1996), which is a paper that uses a control function approach to estimate unobserved firm level productivity. First, we show that the incumbent’s policy function for periods in which the incumbent is uncontested can be inverted to back out the quality of the incumbent as a function of observed state variables and actions. Second, we show that the expected quality of potential challengers can be controlled for by conditioning on two sufficient statistics – the expected number of potential challengers and the probability of entry. We can then identify the

¹About 15%? of U.S. House elections go uncontested.

²Gross, D. and R. Goidel, *The States of Campaign Finance Reform*

effect of spending on vote shares by using residual variation in the incumbent’s war chest and other state variables that change the marginal cost of spending for the incumbent.

The major advantage of our identification strategy is that we can actually recover the quality – vote getting ability – of the candidates paralleling the productivity estimates in Olley and Pakes (1996).³ The quality measures that we recover through our procedure have an intuitive meaning: Given two candidates with quality q_1 and q_2 ($q_1 \geq q_2$), the candidate whose quality is q_1 obtains $(q_1 - q_2)$ higher vote share on average than the candidate with quality q_2 . Given that controlling for candidate quality is an important issue in many contexts beyond campaign financing, our identification strategy may prove useful for the analysis of other related questions in Congressional elections.

Another advantage of our identification strategy is that we can account for selection bias that arises due to the fact that the set of contested elections is not a random sample, but rather an outcome of an entry decision of the potential entrants. This means that as a consequence of optimal behavior by potential entrants, the distribution of challenger quality conditional on entry will depend on variables such as district demographic characteristics and incumbent characteristics (which are often used as control variables in the vote share regression).⁴ More simply put, even if the distribution from which challenger quality is drawn is uncorrelated with control variables such as district demographic characteristics ex-ante, they will be correlated conditional on entry. A string of papers in the literature examining the effect of campaign expenditure on vote shares have tried to deal with endogeneity by finding instruments for candidate spending.⁵ However, this is clearly not enough: identification requires finding instruments for all control variables – not just for campaign spending – to the extent that challenger entry decisions depend on them. This issue seems to have been overlooked in the literature.

Results + CF..For example, Jacobson (1976), which is one of the pioneering papers of the literature on campaign financing, finds a *negative* correlation between incumbent vote share and incumbent spending, and attributes this to the endogeneity of spending.

Literature This paper is related to the large empirical literature that studies the effect of campaign expenditure on election outcomes. For the purpose of putting our paper in context, it is useful to group previous work into three depending on how each paper deals

³Olley Pakes (1996) backs out the unobserved productivity of firms from firm investment behavior. Firm productivity in the model of Olley Pakes (1996) corresponds to candidate quality in our model.

⁴As an example, consider the number of terms the incumbent has served and how this variable affects the vote share. If low quality challengers are deterred and only high quality challengers enter when the tenure of the incumbent is long, then the tenure of the incumbent and the quality of the challenger will be positively correlated, conditional on entry. This will negatively bias the effect of incumbent tenure on vote share.

⁵To my knowledge, papers that use IV in this literature seem to instrument for spending, but not other variables. See, for e.g., Jacobson (1978), Green and Krasno (1988), Gerber (1998), Cox and Thies (2000), Lau and Pomper (2002), and Benoit and Marsh (2008).

with candidate heterogeneity. The first group consists of papers that take the view that candidate quality is a deterministic function by observables (e.g., past political career and previous occupation). While these observables may capture some of the heterogeneity in candidate quality, there are many aspects of candidates that are hard to quantify, such as (perceived) sincerity, honesty, etc., that voters may care about. Given that these hard-to-quantify characteristics are often as important as quantifiable variables, it is not clear that this approach can deal with candidate quality in a convincing way.

The second group of papers uses IV to instrument for endogenous variables. Some of the instruments that have been used in the literature include challenger wealth (Gerber, 1998) and previous incumbent spending (Green and Krasno, 1988).⁶ Whether these instruments are valid is often questionable. However, even if we leave that issue aside, there is another problem with this line of work, which is the selection bias arising from endogenous candidate entry that we described in the Introduction. The existing papers seemed to have ignored this issue. Once we account for endogenous candidate entry, almost all variables that affect the election outcome become endogenous. Given this fact, it seems more appropriate to use a control function approach based on an explicit model of entry rather than seek to instrument for all of the explanatory variables. >SEE COX and KATZ 2002 and discussion of it in Stone Fulton, Maestas, Maisel.

The third group of papers uses identification strategies that do not fall into either of the first two categories. Most notable among them are Levitt (1994) and Erikson and Palfrey (2000). Levitt (1994) is a paper that looks at repeat challengers and uses pairs of elections in which the same two candidates contended for a seat to difference out candidate fixed effects. Erikson and Palfrey (2000) is a paper that models the fundraising and the spending of the candidates and identifies instances in which the endogeneity problem is less severe. Instead of directly estimating the model, they run a regression of the vote share on spending using a subset of the elections that were predicted to be less susceptible to endogeneity bias. Our paper builds on this line of work by directly estimating a game of electoral competition which allows us to recover candidate quality measures and conduct various counterfactual policy experiments.

Our paper is also related to the literature that examines deterrence of entrants: In particular, deterrence of high quality challengers through the build up of war chest.⁷ Many papers have examined this issue, and the results reported in the previous literature are mixed.⁸ On the one hand, there is a lot of anecdotal evidence that potential candidates

⁶See Strattman (2005) for a brief survey.

⁷See, for example, Green and Krasno (1988), Box-Steffensmeier (1996), Goodliffe (2001), Ansolabehere and Snyder (2000), Stone, Maisel, and Maestas (2004) and Goodliffe (2011)

⁸See Goodliffe (2011) for a good summary of the findings reported in the previous literature.

are dissuaded from running for office because of incumbent war chest.⁹ Also, papers that use survey responses of potential challengers (i.e., individuals who are identified as being strong potential challengers) report results that are consistent with the view that strong challengers are deterred by incumbent war chest: For example, Maisel and Stone (1997) and Stone, Maisel, and Maestas (2004) find that “strategic resources and ability” of the incumbent, such as ability to raise money, deter potential entrants from running for office – while finding evidence that public financing of challengers or free TV/radio ads and postage to challengers would significantly improve their chances of running.

In contrast to the papers that find evidence that war chest deters entry, there are papers that find evidence to the contrary. For example, Green and Kranso (1988), Ansolabehere and Snyder (2000), and Goodliffe (2001) report little to no effect of incumbent war chest on the entry decisions of potential entrants or the quality of the entrants who decide to enter. The conflicting results reported in the literature seems to stem, at least partly, from the differences in how incumbent and challenger quality are defined and controlled for. We contribute to this literature by providing a way to control for heterogeneity that does not rely on coarse measures of candidate quality.

Lastly, our paper is related to the literature on estimation of dynamic games in industrial organization (e.g., Ericson, and Pakes, 1995 and Bajari, Benkard and Levin 2009). In particular, our identification strategy for recovering candidate quality parallels techniques often used in the firm productivity literature (e.g., Olley and Pakes 1996, Levinsohn and Petrin 2003, Akerberg, Caves and Fraser 2011).

(Abramowitz (1991), Samuels (2001), and Moon

War chest on entry and challenger quality.

Olley Pakes, Ericson Pakes.

STRATEGIC COMPLEMENTARITY. MAYBE SHOW SOME REDUCED FORM EVIDENCE? Talk about IV too. IV is problematic because we need to instrument for the seemingly exogenous variables as well because of selection, i.e. that q_I and ten_I will be negatively correlated, conditional on $\chi = 1$. To the extent that strategic complementarities or substitutabilities exist, evaluation of counterfactual policy without properly accounting for it may be problematic. Two State variables (marginal cost shifter). No randomness in actions for inversion.

We present the model in section 2 and explain our data in 3. We discuss our identification in section 4 and report our main results in section 5. In Section 6, we present our counterfactual experiments and in section 7, we conclude.

⁹See, for example, Cobb (1988: 16), and Salant (1996). Good discussion of the previous literature can be found in Goodliffe (2004, 2005, 2009)

2 Model

Overview We consider an extension of the campaign spending game of Erikson and Palfrey (2000) by adding dynamics to their model. The stage game of the model comprises of three sub-periods, the first in which potential challengers are drawn and entry decisions are made, the second in which candidates decide on how much to spend, save, and raise, and the third in which the winner of the election becomes the incumbent and decides to run for re-election. Our model captures the two potential channels through which campaign financing affects electoral competition; i.e. the direct effect through campaign spending and the indirect effect through war chest.

Timeline At each period, $t = 1, 2, \dots, \infty$, an election takes place. The time between the periods is two years as Congressional elections take place every two years. An election is either “open” or “non-open” at the beginning of the period, depending on whether there is a sitting incumbent or not. We discuss non-open elections first. The stage game for non-open elections is as follows:

1. State variables (which we describe in the next subsection) evolve from previous values to current values. Nature draws the quality of the potential challengers, $q_{C,1}, q_{C,2}, \dots, q_{C,N}$, from F_{q_C} , where N is the number of challengers. The potential challengers are drawn from the opposite party of the incumbent (i.e., Republican challengers are drawn if the incumbent is a Democrat and vice versa). Each potential challenger decides whether to enter or not by comparing the value of entering and staying out. Upon entry, challengers pay an entry cost κ .
- 2(a) If exactly one challenger enters,
 - The incumbent observes the quality of the challenger (and vice versa). Then the incumbent and the challenger simultaneously make decisions regarding how much money to spend, to raise and to save.
 - The respective vote shares of the candidates are determined as a function of spending, quality, other characteristics of the candidates and a random shock.
- 2(b) If M ($1 < M \leq N$) challengers enter,
 - A Primary takes place that determines which of the challengers becomes the Party nominee. Challenger n with quality $q_{C,n}$ becomes the winner of the Primary with probability $\pi(q_{C,n}, \mathbf{q}_{C,-n})$ if the quality of the challengers that enter are given by $\mathbf{q}_C = (q_{C,1}, q_{C,2}, \dots, q_{C,M})$, where $\mathbf{q}_{C,-n} = (q_{C,1}, \dots, q_{C,n-1}, q_{C,n+1}, \dots, q_{C,M})$

Figure 1: Timing of the Stage Game.

- We go to Case 2(a).

2(c) If no challenger enters,

- The incumbent decides how much to spend, raise and save, and the incumbent becomes the winner with probability 1.

3. The winner receives benefit B from being in office. At the end of the period, the winner decides to retire or run for reelection. Conditional on not retiring, the winner then becomes the incumbent next period. The amount of money she saved from the previous period is carried over. We assume that once a candidate loses an election, she will not run for office again.¹⁰

The timeline described above is for non-open races (i.e., elections with a sitting incumbent), but the sequence of events is similar for open races (i.e., an election with no incumbent) as well. The only difference is that for open races, the challengers are drawn from both parties instead of just one in step 1. Figure 1 describes the sequence of events of the stage game.[Because almost all open races are contested by both we assume that the seat is contested]

Our model is a dynamic extension of the model of campaign spending of Erikson and Palfrey (2000): In fact, their model corresponds to stage game 2(a) of our model. As in their model, we assume that candidates observe each other's quality before they decide how much to spend and save. This is important, since this is exactly what gives rise to the

¹⁰Whenever an incumbent loses a seat and runs for an election later on, we treat her as a new entrant. This is a rare event, however.

endogeneity of spending, which has been shown to be important in the literature.¹¹ The entry model, on the other hand, bears resemblance to the model of Ericson and Pakes (1995) often used to study industry dynamics.

State Variables The vector of state variables at the beginning of the stage game are $\mathbf{s}_t = \{ten_{I,t}, w_{I,t}, X_t, q_I, D_I\}$, where $ten_{I,t}$ denotes the tenure of the incumbent, $w_{I,t}$ is the incumbent war chest, X_t is a vector of demographic characteristics of the Congressional District, q_I is the incumbent quality, and D_I is a dummy variable which takes the value 1 if the incumbent is a Democrat and -1 for a Republican. The tenure of the incumbent ten_I evolves from the previous period as $ten_{I,t} = ten_{I,t-1} + 1$, deterministically. The incumbent war chest, $w_{I,t}$ is determined as a function of how much the incumbent has saved from past periods. The transition of the demographic characteristics, X_t , is assumed to follow an exogenous first-order Markov process.

The incumbent quality, q_I , is a variable which captures the incumbent's ability to get votes. We assume that q_I is constant over time. While this is certainly restrictive, note that we are *not* ruling out the possibility of a deterministic evolution in q_I . Any deterministic trend will be accounted for through ten_I : Hence our definition of q_I can be interpreted as the detrended quality. Also, the main reason for restricting q_I to be fixed is because of data issues. Estimating the model with randomly evolving q_I is very data intensive, although it is conceptually straightforward to allow for q_I to be stochastic. We will discuss in detail how to extend our model to incorporate stochastic q_I at the end of Section XYZ.

Incumbent payoffs and the value function We now specify the primitives of the model in more detail. We start with the period utility, u_I , of the incumbent. Dropping the time subscript for notational simplicity, we specify u_I as follows:

$$u_I = B \cdot P_w(d_I, d_C, q_I, q_C, ten_I, D_I, X) - C_I(w'_I + d_I - w_I; q_I) + H_I(d_I). \quad (1)$$

B is the benefit from holding office. It is multiplied by P_w , the probability of winning. We write P_w as a function of the campaign spending of the incumbent, d_I and the spending of the challenger d_C , as well as the incumbent's quality q_I , challenger's quality q_C , the tenure of the incumbent, ten_I , the Party of the incumbent, D_I , and the demographic characteristics of the Congressional District, X .¹²

The second term of equation (1), C_I , captures the costs the incumbent incurs from raising money. If we denote by fr_I , the amount of money raised by the incumbent, then fr_I can be written as $fr_I = (w'_I + d_I - w_I)$. In equation (1), fr_I has been replaced with

¹¹See discussion in Jacobson (1978), Green and Krasno (1988), and Gerber (1998), for example.

¹²We assume that $d_I, d_C \in \mathbb{R}^+$, $q_I, q_C \in Q \subset \mathbb{R}$ and $x \in X \subset \mathbb{R}$ for some countable set Q and X

$(w'_I + d_I - w_I)$.¹³ We let C_I depend on q_I and assume that C_I is decreasing in q_I : That is, higher quality candidates have lower cost of raising money. The last term, H_I captures the consumption value of spending. Campaign spending is sometimes used in ways that seems to benefit the candidates directly, such as for the purchase of personal articles and for debt repayments to the candidate.¹⁴ H_I is intended to capture this.

Next, we specify the vote share function as follows:

$$vote_I = \beta_I d_I + \beta_C d_C + q_I + q_C + \beta_{ten} ten_I + (2D_I - 1)\beta_X X + \varepsilon,$$

where $vote_I$ denotes the vote share of the incumbent and ε is an error term that is assumed to follow a Normal distribution with mean equal to 0.5 and variance equal to σ_ε^2 . The vote share is expressed as a function of spending, quality, and other control variables. The linear specification of the vote share function has been used extensively in the previous literature and we adopt the same specification for comparability.¹⁵ Note that since the probability of winning is equivalent to obtaining at least 50% the votes, the specification of the vote share function gives rise to an expression for P_w as follows:

$$P_w = \Phi \left(\frac{1}{\sigma_\varepsilon} (\beta_I d_I + \beta_C d_C + q_I + q_C + \beta_{ten} ten_I + (2D_I - 1)\beta_X X) \right), \quad (2)$$

where Φ is the c.d.f. of a standard Normal.

We now consider the interim value function of the incumbent in (non-open) contested elections whose own quality is q_I and faces a challenger whose quality is q_C (corresponds to 2-(a) in the timeline). The state variables for the interrim value function are $\{\mathbf{s}, q_C\} = \{ten_{I,t}, w_{I,t}, X_t, q_I, D_I, q_C\}$. For a given strategy of the challenger, the interim value function v_I^e , can be expressed as follows:

$$v_I^e(\mathbf{s}, q_C) = \max_{w'_I \geq 0, d_I \geq 0} u_I + P_w(d_I, d_C, q_C, q_I) \delta E_{\mathbf{s}'}[V_I(\mathbf{s}')],$$

where the incumbent's choice variables are the amount of savings $w'_I \in \mathbb{R}^+$ and the amount of spending $d_I \in \mathbb{R}^+$. We note that these control variables are (in part) functions of q_I and q_C because q_I , and q_C are observed to the candidates at the time they decide on d_I and w'_I . This is an important feature of the model, allowing the spending variables to be correlated with quality. The first term of the right hand side corresponds to the period utility described above and the second term of the expression is the continuation value. The continuation value is the expectation of the next period value function ($E_{\mathbf{s}'}[V_I(\mathbf{s}')]])$

¹³We let w'_I , d_I and fr_I to take any non-negative real value, but for purely technical reasons we assume that the amount of war chest, w_I lies in a countable set.

¹⁴in instances where the candidate has put her own money into the campaign

¹⁵Include Citation

multiplied by the probability of winning (P_w) and discounted by δ . The expectation of the next period value function is taken with respect X' , the realization of X next period, and r , the rate of return on w'_I .¹⁶ Note finally that V_I is *not* expressed as a function of q_C , because V_I is the ex-ante value function of the incumbent, i.e., the continuation value before potential challengers make their entry decisions.

Before we write out the expression for the continuation value V_I , let us consider the incumbent's problem when she is uncontested (i.e., 2-(c) in the timeline). Let $\tilde{u}_I$ be the period utility of the incumbent when she is uncontested:

$$\tilde{u}_I = B - \tilde{C}_I(w'_I + d_I - w_I; q_I) + \tilde{H}_I(d_I).$$

Note that this expression is obtained by replacing P_w with 1, C_I with $\tilde{C}_I$ and H_I with $\tilde{H}_I$ in equation (1). $\tilde{C}_I$ and $\tilde{H}_I$ capture the cost of raising money and the benefit of spending money in uncontested elections which may be different from C_I and H_I . P_w is replaced with 1 because the uncontested incumbent wins with probability one. The value function of the incumbent when she is uncontested is as follows:

$$v_I^{ne}(\mathbf{s}) = \max_{w'_I \geq 0, d_I \geq 0} B - \tilde{C}_I(w'_I + d_I - w_I, q_I) + \tilde{H}_I(d_I) + \delta E_{\mathbf{s}'}[V_I(\mathbf{s}')].$$

We now discuss the continuation value of the incumbent, V_I . The expression for $V_I(\mathbf{s})$ is as follows:

$$\begin{aligned} V_I(\mathbf{s}) &= (1 - \lambda)P_e(\mathbf{s})v_I^{ne}(\mathbf{s}) \\ &+ (1 - \lambda)(1 - P_e(\mathbf{s})) \int_{q_C} v_I^e(\mathbf{s}) dF_{q_C}(t) | \{\mathbf{X}' * \mathbf{1} \geq 1\}, \end{aligned}$$

where λ is the probability that the incumbent retires, an event which we assume as exogenous.¹⁷ In practice, we let λ to be a function of ten_I . The expression on the right hand side is multiplied by $(1 - \lambda)$ because it is the probability that the incumbent decides to run for reelection. [think about normalization of retiring and continuation value when losing] Note that V_I is the sum of two parts: The first term is the value of the incumbent

¹⁶ $r(w'_I)$ is basically w'_I multiplied by the interest rate: But as we mentioned before, we made the assumption that w_I takes values in a countable set W whereas w'_I can take any non-negative real value. To make things consistent, we assume that $r(w'_I)$ is a random variable that takes values in a countable set W . An example would be a mapping r that takes w'_I to $\lfloor rw'_I \rfloor$ or $\lfloor rw'_I \rfloor + 1$ ($\lfloor x \rfloor \equiv \max\{y \in \mathbb{N} | y \leq x\}$) where $\Pr((w'_I) = \lfloor rw'_I \rfloor) = rw'_I - \lfloor rw'_I \rfloor$ and $\Pr((w'_I) = \lfloor rw'_I \rfloor + 1) = 1 - \Pr((w'_I) = \lfloor rw'_I \rfloor)$.

¹⁷ Some retirements (e.g., due to health reasons) can be considered as exogenous, but others are clearly not. While we can incorporate endogenous retirement in our model, it comes at a high estimation cost. Diermeier Kean and Merlo (2008) is a paper that studies the career concern of House Representatives with endogenous retirement choice. One of their conclusions is "...that the selectivity bias induced by politicians' decisions whether to run for reelection is actually rather modest." (p.349)

when she is uncontested, and the second part is the value of the incumbent when she is contested. P_e and $1 - P_e$ are the respective probabilities of those events. Note also that in the second term of the expression, we need to take the expectation of v_I^e with respect to the distribution of q_C conditional on challenger entry. The conditional distribution is denoted as $F_{q_C}(t)|\{\chi' * \mathbf{1} \geq 1\}$, where χ is a vector of indicator variables, each element of which corresponds to an entry decision of a challenger.

The probability of being uncontested, $P_e(\mathbf{s})$, and the distribution of challenger quality conditional on entry, $F_{q_C}(t)|\{\chi' * \mathbf{1} \geq 1\}$, are obviously equilibrium objects, because entry is a result of optimal decisions by the potential challengers. As a result, $P_e(\mathbf{s})$ and $F_{q_C}(t)|\{\chi' * \mathbf{1} \geq 1\}$ are both endogenously determined as a function of variables (such as q_I and w_I) that affect the decisions of potential entrants. We describe the challengers' entry decision below.

Challenger payoffs and value function We start by describing the model of the challenger once the challenger has become the Party nominee (corresponds to 2-(b) in the timeline) and work our way backwards. The interim value function of the entrant who has won the Primary can be written as:

$$\begin{aligned} v_C^e(\mathbf{s}, q_C) = & \max_{w'_C \geq 0, d_C \geq 0} B \cdot (1 - P_w(d_I, d_C, q_C, q_I)) - C_C(w'_C + d_C, q_C) \\ & + H_C(d_C) + \delta(1 - P_w(d_I, d_C, q_C, q_I))E_{\mathbf{s}'}[V_I(\mathbf{s}')], \end{aligned}$$

where the choice variables are the amount of spending, d_C , and the amount of savings, w'_C . Note first that the probability of winning is now $(1 - P_w)$ and that the continuation value of the challenger is V_I , which is the same continuation value we defined in the previous subsection for the incumbent. The continuation value is the same as the incumbent because the challenger becomes the incumbent next period if he wins. The arguments of V_I are the quality of the challenger, (who becomes the incumbent) q_C , war chest w'_C , tenure ($= 1$), the Party of the challenger, $-D_I$, and X' . Note that v_C^e depends on w_I and incumbent tenure, ten_I , but not on challenger war chest as the challenger typically starts out with no money from previous elections.¹⁸ Finally, we subscript the functions $C(\cdot)$ and $H(\cdot)$ by C and allow them to be potentially different from $C_I(\cdot)$ and $H_I(\cdot)$.

We now discuss the challenger's ex-ante value function (corresponds to 2-(a) in the timeline). KOKOARA. Let N denote the number of potential challengers and let $q_{C,1}, q_{C,2}, \dots, q_{C,N}$ denote the quality of the potential challengers drawn from F_{q_C} . We assume that the number of potential challengers, N , is a constant (non-random). Actually, this is without loss of

¹⁸ Almost all challengers start with zero war chest. The exceptions are those who challenge repeatedly, which are rare. See Table xyz

generality under some conditions on F_N , as we discuss later. Also, N can depend on the state variables, but we suppress the dependence for now. Now, let $p(q_C)$ be the probability that a challenger with quality equal to q_C wins the Primary. $p(q_C)$ is only a function of q_C and not the quality of other challengers because we assume that each challenger must make an entry decision before learning the identity of the other challengers. The ex-ante value function of the challenger is then

$$V_C(\mathbf{s}, q_C) = \max\{p(q_C)v_C^e(\mathbf{s}, q_C) - \kappa, R\}.$$

where v_C^e denotes the interim value of entering which we defined above, κ is the fixed cost of entry, and R is the reservation value.¹⁹ We note that $p(q_C)$ is an equilibrium object and we will describe below how it is determined. The entry decision of the challenger, which we denote as χ , is as follows:

$$\chi = \begin{cases} 1: & \text{if } p(q_C)v_C^e - \kappa > R \\ [0, 1]: & \text{if } p(q_C)v_C^e - \kappa = R \\ 0: & \text{if } p(q_C)v_C^e - \kappa < R \end{cases}.$$

If the value of entry, v_C^e , is increasing in q_C , the entry decision takes a cut-off form. The entry decision can therefore be expressed alternatively as follows:

$$\chi = \begin{cases} 1: & \text{if } q_C > \bar{q}_C \\ [0, 1]: & \text{if } q_C = \bar{q}_C \\ 0: & \text{if } q_C < \bar{q}_C \end{cases}.$$

where $\bar{q}_C$ is defined implicitly as $p(\bar{q}_C)v_C^e(\mathbf{s}, \bar{q}_C) - \kappa = R$. $\bar{q}_C$ is the type of the challenger that is indifferent between entering and not entering. If we exogenously fix v_C^e to be a function that is increasing in q_C , then there exists a symmetric pure strategy Nash equilibrium in cut-off strategies. When we consider the entry game of the challengers, we will always focus on this Nash equilibrium,

Entry Probability and the Conditional Distribution of q_C Now we describe how the probability of winning the Primary, $p(q_C)$, the probability that there is at least one entrant in the Primary, $P_e(\mathbf{s})$, and the distribution of q_C conditional on entry, $F_{q_C}|\{\chi' * \mathbf{1} \geq 1\}$, are determined. Recall that each potential entrant decides whether to enter or not by comparing his own quality q_C with the threshold $\bar{q}_C$. The probability that there are M ($\leq N$) entrants ($M - 1$ other potential entrants) is then given by $\text{Bin}(N, M - 1; 1 - F_{q_C}(\bar{q}_C))$, where

¹⁹We have implicitly assumed that R is non-random, but this is not necessary. Nothing will change if we let R to depend on q_C as $R(q_C)$ as long as $p(q_C)v_C^e(s, q_C)$ increases faster than $R(q_C)$ as a function of q_C (Note that this is trivially true if R is a constant).

$\text{Bin}(n_1, n_2; p)$ is the probability that we have n_2 successes out of n_1 trials with success rate p . Let $\pi(q_{C,n}, \mathbf{q}_{C,-n})$ denote the probability that challenger n wins the Primary when the quality of challenger n is given by $q_{C,n}$ and the opponents by $\mathbf{q}_{C,-n}$, where $\mathbf{q}_C = (q_{C,n}, \mathbf{q}_{C,-n})$ is a $M \times 1$ vector and we assume that π is increasing in $q_{C,n}$ and symmetric in $\mathbf{q}_{C,-n}$. Then,

$$\Pr(\text{Win Primary} | M \text{ total entrants}) = \frac{\int_{\bar{q}_C}^{+\infty} \cdots \int_{\bar{q}_C}^{+\infty} \pi(q_{C,n}, \mathbf{q}_{C,-n}) dF_{\mathbf{q}_{C,-n}}}{(1 - F_{q_C}(\bar{q}_C))^{M-1}},$$

where $F_{\mathbf{q}_{C,-n}}$ is the distribution of $\mathbf{q}_{C,-n}$ and the area of integration is between $\bar{q}_C$ and $+\infty$. Hence the probability of winning the Primary, $p(q_C)$, has the following expression:

$$p(q_C) = \sum_{M=1}^N \text{Bin}(N, M; 1 - F_{q_C}(\bar{q}_C)) \frac{\int_{\bar{q}_C}^{+\infty} \cdots \int_{\bar{q}_C}^{+\infty} \pi(q_{C,n}, \mathbf{q}_{C,-n}) dF_{\mathbf{q}_{C,-n}}}{(1 - F_{q_C}(\bar{q}_C))^{M-1}}$$

and P_e can be expressed as

$$P_e = \sum_{M=1}^N \text{Bin}(N, M; 1 - F_{q_C}(\bar{q}_C)).$$

Likewise, the expression for the distribution of q_C conditional on the event of challenger entry is as follows:

$$F_{q_C}(t) | \{\chi' * \mathbf{1} \geq 1\} = \sum_{M=1}^N \text{Bin}(N, M; 1 - F_{q_C}(\bar{q}_C)) \frac{\int_{\bar{q}_C}^t \int_{\bar{q}_C}^{+\infty} \cdots \int_{\bar{q}_C}^{+\infty} M \pi(q_{C,1}, \mathbf{q}_{C,-1}) dF_{\mathbf{q}_C}}{(1 - F_{q_C}(\bar{q}_C))^M}.$$

Note that this expression depends only on $\bar{q}_C$, and N and moreover, that state variables such as $(q_I, w_I, \text{ten}_I, x)$ affect $F_{q_C}(t) | \{\chi' * \mathbf{1} \geq 1\}$ only through $\bar{q}_C$ and N . In this sense, $\bar{q}_C$ and N are sufficient statistics for the conditional distribution. Now using the fact that $\Pr(\{\chi' * \mathbf{1} \geq 1\})$ can be expressed as

$$\Pr(\{\chi' * \mathbf{1} \geq 1\}) = 1 - \text{Bin}(N, 0; F_{q_C}(\bar{q}_C)) = 1 - F_{q_C}(\bar{q}_C)^N$$

and that the expected number of candidates in the Primary, $E[M]$, can be written as

$$E[M] = \sum_{M=1}^N M \cdot \text{Bin}(N, M; 1 - F_{q_C}(\bar{q}_C)) = N(1 - F_{q_C}(\bar{q}_C))$$

we can express $\bar{q}_C$ and N as functions of $\Pr(\{\chi' * \mathbf{1} \geq 1\})$ and $E[M]$. This implies that $E[M]$, and $\Pr(\{\chi' * \mathbf{1} \geq 1\})$ are also sufficient statistics for $F_{q_C}(t) | \{\chi' * \mathbf{1} \geq 1\}$. This fact is helpful in our identification.

Lastly, we make a few remarks concerning our model. First, we described the model assuming that the number of potential entrants, N , to be non-random. This is actually without loss of generality in many cases: As it will become clear below, our identification strategy will only rely on our ability to express the conditional distribution $F_{q_C}(t)|\{\chi' * \mathbf{1} \geq 1\}$ with easily estimable objects such as $E[M]$ and $\Pr(\{\chi' * \mathbf{1} \geq 1\})$. In this respect, even if N is stochastic, as long as $E[M]$ and $\Pr(\{\chi' * \mathbf{1} \geq 1\})$ are sufficient statistics – and it is for many distributions including the geometric and negative binomial (with a fixed failure number) – the identification and estimation procedure that we describe below will not change at all. It is easy to show that in this case, $E[M]$ and $\Pr(\{\chi' * \mathbf{1} \geq 1\})$ are sufficient statistics.

One drawback of our entry model is that we cannot capture dynamic decision making of potential entrants.²⁰ We discuss possible extensions that accommodate this in footnote xyz. (inversion with two state variables—but require estimation only using the period immediate after. not slight modification of ashyworth? gowrisankaran? merlo kean diermeier) ENTRANTS: MORE NATURAL TO TALK ABOUT A POTENTIAL ENTRANT DECIDING “WHEN” TO ENTER. COMPUTE OPTION VALUE OF WAITING ETC. DISCUSS.

Open Races We have now finished describing the set up for non-open elections. For open elections, we draw the potential challengers from both parties. The entry decisions in open elections are analogous to the challengers’ entry decisions described earlier. The interim value function of one of the open-seat challengers, v_O , has the following expression:

$$v_O(q_O, q_{-O}, x, D_O) = \max_{w'_O, d_O} B \cdot \tilde{P}_w(d_O, d_{-O}, q_O, q_{-O}) - C_O(w'_O + d_O, q_O) + H_O(d_O) + \delta \tilde{P}_w(d_O, d_{-O}, q_O, q_{-O}) E_{r, X'}[V_I(q_O, (1+r)w'_O, 1, X', D_O)]$$

where the variables subscripted by O pertain to the challenger’s own actions and characteristics, while variables subscripted by $-O$ reference those of the opponent. $\tilde{P}_w$ is the probability of winning for this challenger. Similarly as before, the expression for $\tilde{P}_w$ is as follows:

$$\tilde{P}_w = \Phi \left(\frac{1}{\tilde{\sigma}_\varepsilon} (\beta_O d_O + \beta_O d_{-O} + q_O + q_{-O} + (2D_O - 1)\beta_X X) \right).$$

Equilibrium Formally, the players of the game for non-open elections are the incumbent and an infinite sequence of potential challengers. Similarly, for open elections, the players are the two initial open-seat challengers and an infinite sequence of challengers. The strategies of the game are how much to spend, save, raise, and the entry decisions

²⁰Banks Kieweit

of challengers as functions of the state variables. The solution concept we use is anonymous stationary Markov Perfect Equilibria (e.g. Maskin Tirole). In equilibrium, the policy functions of each player are best responses to the policy functions of other players. The anonymity simply restricts the policy functions of potential challengers to all be the same. Under a technical assumption on the size of the state space,²¹ there exists an equilibrium (possibly mixed) of this game (Whitt).

Here, we make one remark concerning equilibria. In general, equilibrium exists in mixed strategies but not necessarily in pure strategies in our setting. But one thing to note is that there always exists an equilibrium in which the incumbent plays pure strategies in uncontested races. This is because the problem of the incumbent in uncontested periods can be thought of as a single agent decision problem, in the sense that changing the incumbent's strategy in uncontested periods does not alter the challengers' best response. Infact, if we take an equilibrium in which the incumbent plays a mixed strategy in uncontested periods, and then replaced that strategy with an alternative one in which the incumbent plays a particular action (in the support of the mixed strategy) with probability one (and keep everything else the same), the resulting profile of strategies will still remain an equilibrium. Henceforth, we assume that the incumbent plays pure strategies in uncontested periods. This is somewhat important, because we will use the policy function of the incumbent in uncontested races to invert out q_I . Aside from this however, we do not have to assume that players play pure strategies. Our identification and estimation results are valid under both mixed strategy and pure strategy equilibria. We follow an approach that is similar to Bajari Benkard Levin in estimating some of the parameters, but our approach does not require the estimation of the policy function itself: It only requires estimation of the projection of the policy function on observable states. Whether the randomness in the observed actions comes from mixing or from the randomness in the realizations of unobserved states is irrelevant for our approach.²²

3 Data

We now explain the data. We use data on the amount of campaign contributions, candidate spending, and savings of U.S. House candidates from 1984 to 2008. This data comes from the Federal Election Commission (FEC). The FEC is a government organization established by the Federal Election Campaign Act of 1971 that compiles data on contributions and spending of candidates who run in federal elections. Candidates are required to disclose

²¹The state space needs to be countable.

²²The last remark concerns uniqueness of equilibria. As we do not solve for the MPE for each parameter, our estimates are going to be consistent in the presence of multiple equilibria as long as the data is generated by the same equilibrium across districts.

to the FEC the amount they raise and spend.²³ We augment this data with data on electoral returns, candidate characteristics and demographic characteristics of Congressional Districts. The electoral returns and candidate characteristics were collected from the database of the CQ Press and the demographic characteristics were collected from the Census and the Bureau of Labor Statistics.

Our sample was created by first dropping House elections in Louisiana and those in Texas in years 1996 and 2006 which were deemed unconstitutional in the Supreme Court ruling.²⁴ We also dropped special elections held outside of the regular election cycle, elections that occur right after the special election, instances in which two incumbents ran against each other and open-seat elections in which there was no contender from one of the major parties. FRACTION OF EACH OF THESE ELECTIONS THAT WERE DROPPED. Some samples were also dropped because of missing data. We end up with a base panel of 4,180 contested non-open elections, 806 uncontested non-open elections and 481 open elections. We then use the two-party vote share to construct our vote share variable and normalize the campaign finance data to 1984 dollars in order to create our spending, fund raising and savings variables.^{25,26}

We end up with a balanced panel of 367 Congressional Districts each with 11 elections for a total of 4037 races. Summary statistics are displayed in Table 1 and Table 2. Among 4037 races, there were 389 open races, or about 9.6% of the total. The number of uncontested races was 549 or 13.6% of the total and 15.0% of non-open races. The rest of the races (3099 or 76.8%) involved an incumbent and a challenger.

Table 1 displays summary statistics of key incumbent campaign funding variables. Incumbent spending is about 43% lower (466.3 to 263.8) in uncontested races than in contested races, and the increase in war chest (=End cash–Beginning cash) is about 125% higher (80.3 to 35.7) in uncontested races. Mean war chest is 12% higher (119.6 to 107.2) in uncontested

²³Federal Election Campaign Act, Chapter 14, Subsec. 1, Sec. 434-(a). The requirement applies to all candidates seeking a seat in the House of Representatives who raise or spend more than \$5,000. See also FEC Campaign Guide.

²⁴Elections in Louisiana employ a run-off system unlike any other State in the United States. The Congressional Elections that were affected by the Supreme Court ruling are TX03, TX05, TX06, TX07, TX08, TX09, TX18, TX22, TX24, TX25, TX26, TX29 and TX30 in 1996 and TX15, TX21, TX23, TX25, and TX28 in 2006.

²⁵We ignore third party candidates. While we acknowledge that this is a shortcoming, strong third party candidates are very rare in U.S. Congressional Elections. In fact, once we reclassify candidates who nominally run as an independent, but as a de facto candidate for one of the major parties, no third party candidate appears as a winner in any election in the sample. (Sometimes, a candidate who is clearly a candidate of one of the two major parties does not officially appear on the ballot as such for bureaucratic reasons. These candidates are de facto Republicans or Democrats, and appear as Republicans or Democrats in subsequent elections) Moreover, no third party candidate receives more than a third of the total vote during the sample period.

²⁶An example include Congressman Bernard Sanders from Vermont who is an independent, but has voted with the Democrats for organizational and committee assignment purposes. (need source..) Another example is Congressman Virgil Goode from Virginia who changed party affiliations during the middle of his career.

	Full Sample	Contested	Uncontested
Incumbent Spending (\$1000)	435.8 (326.5)	466.3 (335.9)	263.8 (192.2)
Incumbent Beginning Cash (\$1000)	109.1 (167.0)	107.2 (169.4)	119.6 (152.4)
(End cash)-(beginning cash) (\$1000)	42.4 (158.3)	35.7 (122.2)	80.3 (28.4)
Incumbent vote share	-	0.663 (0.095)	-
N	3648	3099	549

Table 1: Discriptive Statistics of Vote Share and Spending

	Full Sample	Open Seat	Run Against Incumbent
Challenmger Spending (\$1000)	217.6 (342.2)	493.1 (436.7)	148.4 (273.6)
Challenger Beginning Cash (\$1000)	0.5 (4.2)	0.5 (4.2)	0.6 (4.1)
Challenger End Cash	3.6 (14.1)	9.9 (24.7)	2.1 (9.1)
Challenger Vote Share	-	-	0.337
	-	-	0.095
Winning Vote Share	0.653 (0.093)	0.600 (0.085)	0.666 (0.091)
N	3877	778	3099

Table 2: Discriptive Statistics of Vote Share and Spending

racers than in contested races. The average vote share of the incumbent is 66.3%.

Table 2 displays corresponding statistics for challengers. Average challengers spending is about half of incumbents, but open seat challengers spending is more than three times than the average challenger who competes against an incumbent. Note that almost all challengers start with no beginning cash, and few candidates save for the next period. The average vote share of the challenger is 65%, but open seats are more closely contested with an average winning vote share of 60% compared to 67%.

4 Identification and Estimation

In this section, we discuss identification and estimation of the key components of our model. We first show how to deal with unobserved candidate quality and the endogeneity issues that arise. In particualr, we provide one way of identifying the marginal effect of challenger

and incumbent spending on the vote shares that properly accounts for the endogeneity. We then show how the other primitives of the model such as C and H etc., can be identified using the first order conditions that equate the marginal benefit/cost of spending, saving and fundraising.

4.1 First Step: Vote production function

4.1.1 Overview

Our approach builds on ideas pioneered by Olley and Pakes and then subsequently developed by Levinsohn, and Petrin and Akerberg, Caves and Fraser. As with Olley Pakes, there are two sources of endogeneity that need to be considered in our setting: One concerning sample selection and the other concerning simultaneity. The sample selection problem arises due to the fact that which elections get contested and which remain uncontested is not selected at random: Rather, it is an outcome of an entry decision of the potential entrants. This means that as a consequence of optimal behavior by potential entrants, the distribution of challenger quality conditional on entry is going to change as the characteristics of the incumbents change. To give a concrete example, suppose that the average quality of the challenger is lower when the incumbent is less established (perhaps because even low quality challengers may find it worthwhile to enter, thinking that there is a chance of winning against the incumbent), so that quality of the challenger, q_C , and incumbent tenure, ten_I , are positively correlated conditional on challenger entry.²⁷ If we then run OLS of incumbent vote shares on ten_I on the subset of contested samples there will be a positive bias on the coefficient of ten_I . Regression analysis that only uses contested elections without accounting for selection is thus problematic. This issue is similar to the problem that arises when an unbalanced panel is artificially restricted to be a “balanced panel” (see Olley Pakes for a detailed discussion). Silmilar to Olley and Pakes, we will use the probability of entering, or the “propensity score” as a control function to deal with the selection problem.²⁸

The second source of endogeneity is simultaneity. The simultaneity problem in our context is that unobserved quality of the candidates, q_I and q_C are correlated with actions d_I , and d_C , in equation XYZ. Similar to Olley and Pakes, we deal with one part of the endogeneity – correlation of d_I and d_C with q_I – by solving out q_I as a function of observable actions and states through the inversion of the policy function. We deal with the other part

²⁷We stress that the model can easily generate positive or negative correlation between q_C and ten_I . If the number of entrants, $N(q_I, w_I, ten_I, x, D_I)$, is negatively correlated with ten_I , there will be a negative correlation between q_C and ten_I conditional on entry. On the other hand, if $N(q_I, w_I, ten_I, x, D_I)$ has zero correlation (or positive correlation) with ten_I , then there will be a positive correlation between q_C and ten_I . This is because v_C^e is (most likely) going to be decreasing in ten_I , which implies that $\bar{q}_C$ is going to be increasing in ten_I , which in turn implies that $E[q_C | \mathbf{x}' * \mathbf{1} \geq 1]$ is going to be increasing in ten_I .

²⁸Robinson and Rubin

of the endogeneity – correlation of d_I and d_C with q_C – by using the war chest, w_I as an instrument. Although w_I is correlated with q_C conditional on entry, (hence it does not satisfy the IV assumption directly) it turns out that q_C is mean independent of w_I conditional on $E[M]$ (expected number of challengers in the Primary), and $\Pr(\{\chi' * \mathbf{1} \geq 1\})$ (Probability of challenger entry). This means that once we include $E[M]$ and $\Pr(\{\chi' * \mathbf{1} \geq 1\})$ in the equation XYZ as auxiliary regressors, w_I is going to be mean independent of the residual error term, allowing us to proceed by instrumenting d_I and d_C with w_I .

Before we lay out the steps in detail, we note the two main technical differences between our setup and OP. The first main difference is that we have two unobservables q_I and q_C as opposed to just one (productivity) in OP. This makes controlling for sample-selection harder in our setting than in OP, as the propensity score cannot be directly estimated. It also requires additional work in order to deal with endogeneity, since the policy function inversion takes care of just one part of endogeneity. The second main difference is that we treat the unobservables q_I as fixed ($q_{I,t} = q_{I,t+1}$) unlike in OP where they are allowed to change over time. Allowing the unobservables to change over time in our framework is conceptually straightforward and we will discuss how to do so at the end of this section. But as it will be clear in our discussion there, the data requirement needed to estimate time varying unobservables is quite demanding: It requires estimating the model on a very particular subset of the full sample. For this reason, we will maintain the assumption of fixed unobservables in our actual estimation.

4.1.2 Sample Selection

We discuss how to deal with sample selection first, under the assumption of fixed q_I . From the previous section, recall that the distribution of q_C conditional on entry has the following expression.

$$F_{q_C}(t) | \{\chi' * \mathbf{1} \geq 1\} = \sum_{M=1}^N \text{Bin}(N, M; 1 - F_{q_C}(\bar{q}_C)) \frac{\int_{\bar{q}_C}^t \int_{\bar{q}_C}^{+\infty} \cdots \int_{\bar{q}_C}^{+\infty} M \pi(q_{C,1}, \mathbf{q}_{C,-1}) d\mathbf{F}_{\mathbf{q}_C}}{(1 - F_{q_C}(\bar{q}_C))^M}.$$

Note that this expression depends on $(q_I, w_I, \text{ten}_I, X)$ only through $\bar{q}_C(q_I, w_I, \text{ten}_I, x)$, and $N(q_I, w_I, \text{ten}_I, x)$. Recall from the discussion in the previous section that there is a one-to-one and onto mapping from $(\bar{q}_C, N)$ to $(\Pr(\{\chi' * \mathbf{1} \geq 1\}), E[M])$, which implies that

$$\begin{aligned} E[q_C | \{\chi' * \mathbf{1} \geq 1\}, q_I, w_I, \text{ten}_I, x] &= E[q_C | \Pr(\{\chi' * \mathbf{1} \geq 1\}), E[M]] \\ &\equiv g(\Pr(\{\chi' * \mathbf{1} \geq 1\}), E[M]). \end{aligned}$$

The function g is a complicated equilibrium object the exact shape of which depends on unknown model parameters. From our perspective, however, the important point is that

the conditional expectation of q_C given entry can be expressed *only* as functions of $P_e = \Pr(\{\chi' * \mathbf{1} \geq 1\})$ and $E[M]$ (as opposed to all of the conditioning variables, q_I, w_I, ten_I, X). Hence, if we knew P_e and $E[M]$, we can control for the selection bias by including a nonparametric function of P_e and $E[M]$ in equation XXX.²⁹ To see the intuition for how this works, suppose again that the average quality of the challenger is lower when the incumbent is less established so that q_C and ten_I are positively correlated conditional on entry, as in the previous example. Then variation in ten_I will have a direct effect on the vote share as well as an indirect effect through the change in $E[q_C | \{\chi' * \mathbf{1} \geq 1\}, q_I, w_I, ten_I, x]$. By including $g(\Pr(\{\chi' * \mathbf{1} \geq 1\}), E[M])$ in equation XXX, we can control for the indirect effect and identify just the direct effect.

In OP, it is possible to estimate the propensity $\Pr(\{\chi' * \mathbf{1} \geq 1\})$ and to use the estimated propensity to purge the selection bias. We still need an extra step to make this work in our set up because neither P_e nor $E[M]$ cannot be estimated directly: P_e and $E[M]$ are both functions of unobservable q_I . To deal with this issue as well as to account for the simultaneity bias, we next consider how to invert out q_I from the policy function of uncontested incumbents.

4.1.3 Simultaneity

Recall the problem of the incumbent when she is uncontested:

$$v_I^{ne}(\mathbf{s}) = \max_{w'_I \geq 0, d_I \geq 0} B - \tilde{C}_I(w'_I + d_I - w_I, q_I) + \tilde{H}_I(d_I) + \delta E_{r, X'}[V_I(\mathbf{s}')]. \quad (3)$$

We denote the incumbent's policy function as $w'_I = w'_I(\mathbf{s}) = w'_I(q_I, w_I, ten_I, X, D_I)$ and $d_I = d_I(\mathbf{s}) = d_I(q_I, w_I, ten_I, X, D_I)$. The policy functions can be viewed as mappings from q_I to (w'_I, d_I) , holding fixed the other state variables. If the mapping $q_I \mapsto (w'_I, d_I)$ is one-to-one (given w_I, ten_I, X and D_I) – and we will prove in the Appendix that it is for the functional form of $\tilde{C}_I$ and $\tilde{H}_I$ we use for our estimation – then we can uniquely solve for q_I using these policy functions as: $q_I = q_I(w'_I, d_I; w_I, ten_I, X, D_I) = q_I(s_{NC})$, where we denote by s_{NC} the vector of state variables and actions in the uncontested period. Inversion of the policy functions of the uncontested incumbent is attractive (compared to inverting the policy functions in contested periods) because it is relatively easy to show that the mapping $q_I \mapsto (w'_I, d_I)$ is one-to-one since we are dealing with just one unobservable variable (as opposed to two unobservables – q_I and q_C – for contested periods) and it does not suffer from collinearity issues that can arise in control function approaches. (See Akerberg Caves and Fraser).

²⁹If we did not have a sufficient statistic and if $E[q_C | \{\chi' * \mathbf{1} \geq 1\}, q_I, w_I, ten_I, x]$ depended instead on $\{q_I, w_I, ten_I, x\}$ separately, there is going to be an issue with multicollinearity.

The inversion of the policy function achieves two things. First, it enables us to express the propensity score P_e and the expected number of candidates in the Primary $E[M]$ as functions of (s_{NC}, w_I, ten_I, X) , which are observed, instead of $(q_I, w_I, ten_I, X, D_I)$, which are not. Recall from the previous subsection that because P_e and $E[M]$ are both functions of q_I , we could not directly estimate these functions. But we can now replace out q_I with s_{NC} , which allows us to estimate them as functions of observables, (s_{NC}, w_I, ten_I, X) , using a subset of observations for which (1) we observe the incumbent in more than two periods, and (2) there is a period in which the incumbent was uncontested. Once we estimate P_e and $E[M]$ as functions of observables, we can use the estimated P_e and $E[M]$ to control for selection bias. For the purpose of identification, we take P_e and $E[M]$ as known from now on.

The inversion of the policy function also solves one part of the endogeneity problem. Recall that d_I and d_C in equation (XXX) are correlated with q_I and q_C . But once we can express q_I as a function of observables, we can explicitly control for q_I . Note that since the policy inversion results in expressing q_I as a function of actions and state variables of a period in which the incumbent was uncontested, we do not have collinearity issues. The set of variables that control for q_I and the set of regressors in XXX are disjoint.

Lastly we discuss how to deal with the endogeneity of d_I and d_C with respect to q_C . The idea is to use w_I as an instrument. Consider the following projection of the vote shares on $\Omega \equiv \{s_{NC}, w_I, ten_I, X, \{\chi' * \mathbf{1} \geq 1\}\}$:

$$\begin{aligned} vote_I &= E[vote_I|\Omega] + (votes_I - E[votes_I|\Omega]) \\ &= E[\beta_I d_I - \beta_C d_C + \beta_{ten} ten_I + D_I \beta_X X + q_I - q_C + \varepsilon|\Omega] + \epsilon \\ &= \beta_I E[d_I|\Omega] - \beta_C E[d_C|\Omega] + \beta_{ten} ten_I + D_I \beta_X X + q_I(s_{NC}) - g(P_\varepsilon, E[M]) + \epsilon(4) \end{aligned}$$

where $\epsilon \equiv (votes_I - E[votes_I|\Omega])$ and $E[\epsilon|\Omega] = 0$ by construction. We have used the fact that $q_I = q_I(s_{NC})$, $E[\varepsilon|\Omega] = 0$, and $E[q_C|\Omega] = g(P_\varepsilon, E[M])$ to go from the second line of the expression to the third. Note that $g(P_\varepsilon, E[M])$ is a nonparametric function that controls for selection bias as we discussed above.

The conditioning set Ω in the above expression ?? corresponds to the information available to the incumbent before q_C is realized. Projecting the variables on Ω is attractive for three reasons. One reason is that all the variables that constitute Ω are observable to the researcher, so we can compute objects such as $E[d_I|\Omega]$ and $E[d_C|\Omega]$. Another reason is that all the variables that constitute Ω are predetermined in the sense that they are decided before the realization of q_C . This means that once we condition on Ω , and net out the selection term, $g(P_\varepsilon, E[M])$, we do not have to worry about the endogeneity of d_I and d_C with q_C . By construction, the error term ϵ is mean independent of all the regressors.

Lastly, Ω contains an excluded variable w_I , which is correlated with d_I and d_C . This means that w_I can be used as an instrument. Roughly speaking, our identification strategy is to first project the vote share on Ω and then look at how the projected variables, $E[d_C|\Omega]$ and $E[d_I|\Omega]$, affect $E[votes_I|\Omega]$. Thus, Ω is going to serve as instruments, similar to 2SLS. Equation 4 serves both as an identification equation and an estimation equation.

It is useful at this point to explain the variation in the data that allows us to identify the parameters of the vote share equation. Consider two incumbents, say I_1 and I_2 , who took similar actions in periods when they were uncontested.³⁰ The two incumbents can be thought of as having the same q_I . Take I_1 and I_2 and now look at periods in which they were contested. Suppose that the state variables for I_1 and I_2 in the contested period, $\Omega^i = \{s_{NC}^i, w_I^i, ten_I^i, \mathbf{x}^i, \{\mathbf{X}' * \mathbf{1} \geq 1\}\}_{i \in \{1,2\}}$ are such that the ex-ante probability of entry, P_ε , and the number of potential entrants, $E[M]$, are the same, so $P_\varepsilon^1 = P_\varepsilon^2$ and $E[M^1] = E[M^2]$. Note that the state variables themselves need not be identical in order for P_ε and $E[M]$ to be the same. For example, incumbent I_1 may have had a higher war chest than I_2 , ($w_I^1 > w_I^2$) but the other state variables, ($ten_I, \mathbf{x}$) may have been more favorable for incumbent I_2 , resulting in $P_\varepsilon^1 = P_\varepsilon^2$ and $E[M^1] = E[M^2]$.³¹ Given two sets of state variables Ω^1 and Ω^2 that induce the same P_ε and $E[M]$, consider computing $E[votes|\Omega^i]$, $E[d_I|\Omega^i]$, and $E[d_C|\Omega^i]$. These objects can be computed simply by pooling incumbents that are similar to I_1 or I_2 and taking the averages. For our purposes, therefore, $E[votes|\Omega^i]$, $E[d_I|\Omega^i]$, and $E[d_C|\Omega^i]$ are all identified. Then we obtain,

$$\begin{aligned} E[votes_I|\Omega^1] - E[votes_I|\Omega^2] &= \beta_I(E[d_I|\Omega^1] - E[d_I|\Omega^2]) - \beta_C(E[d_C|\Omega^1] - E[d_C|\Omega^2]) \\ &\quad + \beta_{ten}(ten_I^1 - ten_I^2) + \beta_X(D_I^1 X^1 - D_I^2 X^2), \end{aligned}$$

where we have used the fact that both I_1 and I_2 have the same quality and the face the same expected challenger quality. Hence we can identify the coefficients of (XYZ) by correlating the differences in the spending of the incumbents ($E[d_I|\Omega^1] - E[d_I|\Omega^2]$) and challengers ($E[d_C|\Omega^1] - E[d_C|\Omega^2]$) etc., to the differences in the vote shares. Basically, the preceeding argument exploits the variation in $\{w_I, ten_I, \mathbf{x}\}$ which moves d_I and d_C , but keeps P_ε and $E[M]$ constant.

³⁰More precisely, we need to incumbents who took similar actions when they were uncontested, and moreover, the state variables in the uncontested periods were simialr as well. Alternatively, we can think of the same incumbent in two different periods.

³¹As the number of state variables is larger than 2, the Jacobian of $\begin{pmatrix} P_\varepsilon \\ E[M] \end{pmatrix}$ with respect to (a subset of) the state variables, $J \equiv \frac{\partial}{\partial \{w_I, ten_I, \mathbf{x}\}} \begin{pmatrix} P_\varepsilon \\ E[M] \end{pmatrix}$, is going to have more columns than rows. This means that the column vectors of J are linearly dependent, implying that there will be a non-zero vector $\mathbf{z}$ that satisfies $J\mathbf{z} = \begin{pmatrix} 0 \\ 0 \end{pmatrix}$. Hence $\{w_I, ten_I, \mathbf{x}\}$ and $\{w_I, ten_I, \mathbf{x}\} + \mathbf{z}$ both lead to the same P_ε and $E[M]$.

Our argument for dealing with endogeneity can be summarized as follows. The problem that we want to solve is that d_I and d_C are correlated with the econometric error term, $q_I - q_C + \varepsilon$. We can deal with q_I by policy function inversion. The problem then is reduced to dealing with the correlation of d_I and d_C with q_C . Now, what we would like to do at this point is use the war chest, w_I , as an instrument: w_I is excluded from the vote share equation, and moreover it is correlated with d_I (through the cost function) and d_C (through equilibrium play). At first glance, this IV strategy may seem hard to implement because w_I is correlated with q_C , conditional on entry. But once $g(P_\varepsilon, E[M])$ is included as an auxiliary regressor, the sample selection in q_C that arises as a consequence of variation in w_I is also controlled for. This allows us to implement the IV strategy.

For estimation, we use a subset of the sample in which we see the same incumbent in at least two different periods, a period in which the incumbent is uncontested and a subsequent period in which the incumbent is contested. Specifically, we look for the first episode in which the incumbent is uncontested and use the actions and the state variables in that period as s_{NC} . We then look at subsequent periods during the incumbent's tenure and identify all periods in which the incumbent is contested. In our sample, it is often the case that for a given incumbent, there are multiple periods in which the incumbent is contested subsequent to the uncontested period. Moreover, it is also often the case that for a given Congressional District, there are more than one incumbent who is uncontested in one period, and is contested in a later period. What we do then, is to collect and stack the vote shares, spending, etc., as a vector for each Congressional District, which we take as our unit of observation. We then use Ai and Chen for estimation.

Finally, we discuss how our approach can be extended to accomodate time varying q_I . In order to do so, we have to first specify the evolution of q_I and also the timing of events, i.e. what the information set of the players are at each point in time. One natural setup is that (1) q_I evolves as an AR(1) process as $q_{I,t+1} = q_{I,t} + \xi_{t+1}$ and that (2) ξ_{t+1} is revealed before the candidates decide how much to spend, save, and raise, but after the challenger makes his entry decision (that is, after 1 and before 2 in the time line).³² This would be the case if the challenger makes the entry decision based on what he knows from the previous elections and learns the new incumbent quality $q_{I,t+1}$ only as he starts to compete for the seat. If the timing is such, then the probability of entry in period $t + 1$, is a function of $q_{I,t}$, w_I , ten_I , x : $P_\varepsilon = P_\varepsilon(q_{I,t}, w_I, ten_I, x, D_I)$ (note that we now have a time subscript on q_I). It is no longer the case that we can use s_{NC} from any period to substitute out q_I as q_I is not fixed. However it is still true that $q_{I,t}$ is a function of $s_{NC,t}$: That is, by only using a subset of the samples in which (1) the incumbent was uncontested and (2) the

³²What is important is that ξ_{t+1} is not observed to the entrant when he makes his entry decision. ξ_{t+1} can realize before the entry decision as long as this is only privately observed to the incumbent.

incumbent was contested in the following period, we can proceed just as before. The vote share equation that we would estimate is now

$$vote_{I,t} = \beta_I d_{I,t} + \beta_C d_{C,t} + \beta_{ten} ten_{I,t} + (2D_I - 1)\beta_x x_t + q_I(s_{NC,t-1}) + g(P_{\varepsilon,t}) + (q_{I,t} - q_I(s_{NC,t-1})) + (q_{C,t} - g(P_{\varepsilon,t})) + \varepsilon_t$$

We have added in time subscripts to empasize that we are using observables from period $t - 1$, $s_{NC,t-1}$, to control for $q_{I,t}$. The econometric error term is now $(q_{I,t} - q_I(s_{NC,t-1})) + (q_{C,t} - g(P_{\varepsilon,t})) + \varepsilon_t$, where $(q_{I,t} - q_I(s_{NC,t-1})) = \xi_t$. As ξ_t is not correlated with w_t , our approach would work for this case as well.

4.2 Second Step

We now consider identification and estimation of the cost function C_I and the function that accounts for the consumption value of spending, H_I . At the outset, we note that C_I and H_I are identified only relative to the benefit of holding office B so we normalize B to 1. We also take the discount factor δ as given.³³ As before, we first discuss our approach under the assumption that q_I is constant over time. We discuss how to relax this assumption at the end of Section 4.3.

4.2.1 Overview

We consider identification and estimation of model parameters other than the vote share function such as the cost function C_I and the function that accounts for the consumption value of spending, H_I . The following are the two key restrictions that identify C_I and H_I :

$$\frac{\partial}{\partial d_I} C_I(w'_I + d_I - w_I, q_I) = \frac{\beta_I}{\sigma_\varepsilon} \phi(K) \cdot (B + \delta E_{r,x'}[V_I]) + \frac{\partial}{\partial d_I} H_I(d_I),$$

$$\frac{\partial}{\partial w'_I} C_I(w'_I - w_I + d_I, q_I) = \delta \Phi(K) \frac{\partial}{\partial w'_I} E_{r,x'}[V_I].$$

These two equations are obtained by taking the derivative of the RHS of equation XXX and substituting P_{win} with $\Phi(K)$, where $K = \frac{1}{\sigma_\varepsilon}(\beta_I d_I + \beta_C d_C + q_I + q_C + \beta_{ten} ten_I + (2D_I - 1)\beta_x x - 0.5)$, and $\phi(\cdot)$ ($\Phi(\cdot)$) is the pdf (cdf) of the standard normal. The first expression equates the marginal cost of fundraising to the marginal benefit of spending and the second expression equates the marginal cost of fundraising to the marginal benefit of saving. The two first order conditions hold for all samples with d_I and w'_I strictly positive.

In exploiting these two equations, we first need to evaluate the continuation value, V_I ,

³³Identifying the discount factor in dynamic games is known to be very difficult. We follow the literature in taking δ as given. See Magmac and Theirry for a detailed discussion.

as well as its derivative, since they appear in both equations. Below, we show that we can directly compute the continuation value as a function of unknown parameters by simulation. Simulating the continuation value is an idea that was proposed by Hotz Miller, Sanders and Smith in the context of single decision problems and then extended to game settings by Bajari Benkard and Levin. The key to this approach is to (1) estimate the transition of the state variables (2) estimate the equilibrium policy function nonparametrically without solving for the model, and (3) for a given policy function (optimal or otherwise) and given the estimated transition from step (1), forward simulate the value function as a function of parameters. The parameters are then estimated by the restriction that the policy function estimated in step (2) attains a higher value than any other policy function at the true parameter. We adapt their approach in our context. In particular, instead of estimating the policy function in step (1), we estimate the projection of actions on observed states. If we were to observe all state variables, the two would be the same, but in our application, q_C is unobserved. It turns out that in our particular application, we only need the projection rather than the policy function itself. This will turn out to be convenient for accomodating mixed strategies as well. We also use the two first order conditions for our optimality condition rather than the inequality restriction in BBL. Below, we describe our approach in detail.

4.2.2 Estimation of the transition.

Transition of X and w' Estimate the transition probability of the observable state variables, i.e., the transition of demographic characteristics from x to x' , and estimating the transition of savings w'_t to next period war chest $w_{t+1} = (1 + r)w'_t$. We assume an exogenous first order Markov process for the evolution of these state variables.

Probability of winning in contested periods We also estimate the probability that the incumbent wins in contested periods given Ω , $E[1\{vote_I \geq 0.5\}|\Omega]$, where $\Omega \equiv \{s_{NC}, w_I, ten_I, X, \{\chi' * \mathbf{1} \geq 1\}\}$. Note that we are not conditioning on q_C , d_I , and d_C in taking this expectation.

4.2.3 Estimation of the projection of the policy function on observed states

The second step involves estimation of the conditional distribution of spending, saving, etc., given observed states. This step is very simialr to the estimation of the policy function in BBL. The difference is that because we do not observe the full set of variables on which players condition their actions, – we do not observe q_C – we estimate the conditional distribution of the actions given observed state variables. The relevant objects we estimate are

- The conditional distribution of incumbent spending in contested periods given observable states Ω denoted as $F_{d_I}(\cdot|\Omega)$. F_{d_I} is the projection of the equilibrium strategy $d_I(q_C, \mathbf{s})$ on Ω . This conditional distribution, as well everything listed below, can be estimated in a standard way by kernels or sieves. Also, recall that q_I can be expressed as a known function of observables as $q_I(s_{NC})$ from our earlier discussion, so we are treating it as known.
- The conditional distribution of incumbent savings in contested periods, given incumbent victory and Ω , denoted as $F_{w'_I}(\cdot|\Omega, Won)$.
- The conditional distribution of amount raised in contested periods denoted as $F_{fr_I}(\cdot|\Omega)$.
- The conditional distribution of own spending, savings and amount raised in uncontested periods, given observable states $\Omega^{ne} \equiv (s_{NC}, w_I, ten_I, X, D_I, \{\chi' * \mathbf{1} = 0\})$ denoted as $F_{d_I}(\cdot|\Omega^{ne}), F_{w'_I}(\cdot|\Omega^{ne}), F_{fr'_I}(\cdot|\Omega^{ne})$.

4.2.4 Computation of the continuation value

With these projections in hand, it is possible to compute the continuation value for a given profile of parameters by simulating draws from these distributions. We compute $E_{r,x}[V_I]$ (as a function of parameters that determine C_I, H_I etc.) as follows:

1. Randomly draw $X_{t+1}, w_{I,t+1}$ given X_t and $w_{I,t}$ and the transition matrix estimated in Sec. 4.2.2.
2. Draw a random variable U_{ent} from a uniform distribution. If U_{ent} is less than the probability of entry, i.e. $U_{ent} \leq P_e$, then there is entry (Recall that P_e is estimated in Sec 4.1.3). If $U_{ent} > P_e$, then there is no entry.
3. Depending on whether there is entry or not, draw $d_{I,t+1}$ and $fr_{I,t+1}$ from the respective conditional distributions estimated in Sec. 4.2.3. In case of entry, further draw a random variable U_{win} from a uniform distribution.
4. The period utility function is computed as $u_{I,t+1} = B - \tilde{C}_I(fr_{I,t+1}, q_I) + \tilde{H}_I(d_{I,t+1})$ in the case of no entry and either as $u_{I,t+1} = B - C_I(fr_{I,t+1}, q_I) + H_I(d_{I,t+1})$ or $u_{I,t+1} = -C_I(fr_{I,t+1}, q_I) + H_I(d_{I,t+1})$ depending on whether U_{win} is smaller or bigger than $E[1\{vote_I \geq 0.5\}|\Omega]$. A draw of U_{win} with a smaller value than $E[1\{vote_I \geq 0.5\}|\Omega]$ is interpreted as a victory for the incumbent while a draw with a larger value is interpreted as a victory for the entrant.
5. Terminate the process if the incumbent loses to the entrant. Otherwise, draw w'_I from $F_{w'_I}(\cdot|\Omega, Won)$. This determines the amount of savings. Finally, draw a uniformly

distributed random variable U_{retire} which determines whether the incumbent retires according to whether $U_{retire} \leq \lambda$ or not. λ is the probability of retiring.

6. If the incumbent does not retire, then go back to step 1 and repeat. Otherwise, terminate.
7. Repeat steps 1 through 6 and take the average.

Note that in computing the continuation value, $E_{r,x'}[V_I]$, we do not need to know the joint distribution of the actions, d_I , w'_I , and fr_I . Knowledge of the marginal distribution alone is enough to compute the continuation value. This follows from the additive separability of u_I and it greatly reduces the data requirement.

4.2.5 Optimality Condition

Once we are able to express $E_{r,x'}[V_I]$ (and its derivative) as a function of parameters, we can derive the restriction that we use for estimation. This is done by first solving for K for the subset of samples with d_I and w'_I strictly positive using the second first order equation as

$$K = \Phi^{-1} \left(\frac{\partial}{\partial w'_I} C_I \middle/ \delta \frac{\partial}{\partial w'_I} E_{r,x'}[V_I] \right).$$

We then plug this expression into the first equation. Note that the equation obtained in this way only includes observables and parameters. We can then pin down the parameters that determine the cost functions C_I , $\tilde{C}_I$, the functions that accounts for the consumption value of spending, H_I , $\tilde{H}_I$ as well as σ_ε . We do so by choosing the parameters that make the equation come as close to holding with equality as possible.

4.3 Third Step

In this subsection we discuss identification and estimation of the cost function (C_C) and the function that accounts for the consumption value of spending (H_C) for challengers as well as the conditional distribution of q_C , and q_O given entry, $F_{q_C}(\cdot | \chi' * \mathbf{1} \geq 1)$ and $F_{q_O}(\cdot | \chi' * \mathbf{1} \geq 1)$. First, note that identification of C_I , H_I and σ_ε from the previous discussion implies that we can back out K , and hence q_C , from equation XXX. This implies that the conditional distribution of q_C given entry, $F_{q_C}(\cdot | \chi = 1)$ is also identified. Now consider the challenger counterpart of the first order conditions (XXX),

$$\begin{aligned} \frac{\partial}{\partial d_I} C_C(w'_C + d_C, q_C) &= -\frac{\beta_C}{\sigma_\varepsilon} \phi(K) (B + \delta E_{r,x'}[V_I]) + \frac{\partial}{\partial d_C} H_C(d_C) \\ \frac{\partial}{\partial w'_C} C_C(w'_C + d_C, q_C) &= \delta(1 - \Phi(K)) \frac{\partial}{\partial w'_C} E_{r,x'}[V_I(q_C, r(w'_C, 1, x', 1 - D_I))] \end{aligned}$$

The parameters of the cost function are identified from the second equation because all the terms on the right hand side of the second equation have already been identified (K and q_C are identified from the previous steps).

We now discuss identification of the conditional distribution of q_O . Note that even if we condition the sample on incumbents who started out their career as an open-seat challenger, and collect the values of q_I , this would not be enough to identify the distribution of q_O . This is because we can recover the value of q_I for only a subset of the population, i.e. challengers who (1) subsequently became incumbents and furthermore, (2) did not face any challenger in a subsequent election. In other words, what we observe is $f_{\mathbf{q}}(q_O, q_{O'}|E)$: The joint distribution of $(q_O, q_{O'})$ conditional on E , where E is the event that both (1) and (2) happens. Given that $F_{q_O}(\cdot|\chi' * \mathbf{1} \geq 1)$ is related to $f_{\mathbf{q}}(\cdot|E)$ as follows

$$f_{q_O}(q_O|\chi' * \mathbf{1} \geq 1) = \sqrt{f_{\mathbf{q}}(q_O, q_O|\chi' * \mathbf{1} \geq 1)} = \sqrt{\frac{f_{\mathbf{q}}(q_O, q_O|E) \Pr(E)}{\Pr(E|q_O, q_O)}},$$

we need to take account of $\Pr(E|q_O, q_O)$ in order to identify $f_{q_O}(q_O|\chi' * \mathbf{1} \geq 1)$. We show in the Appendix that we can simulate $\Pr(E|q_O, q_O)$ for given values of candidate quality pairs using the estimated parameters of the model. Hence $F_{q_O}(\cdot|\chi' * \mathbf{1} \geq 1)$ is also identified. can be done by simulation using the estimated the primitives of the model, We discuss this in more detail in the Appendix.

Identification of Parameters Related to the Entrant's Decision Recall that we have yet to discuss identification of parameters related to the entrant's decision and the primary such as R , κ , $p(q_C)$, $\pi(\mathbf{q}_C)$ and the unconditional distribution of q_C , F_{q_C} . The equation which relates these primitives to observable outcomes is the entrant's optimal decision rule:

$$\chi = \begin{cases} 1: & \text{if } v_C^e > \frac{R+\kappa}{p(q_C)} \\ [0, 1]: & \text{if } v_C^e = \frac{R+\kappa}{p(q_C)} \\ 0: & \text{if } v_C^e < \frac{R+\kappa}{p(q_C)} \end{cases}.$$

If we let $\underline{q}_C(\mathbf{s})$ denote the quality of the challenger who is just indifferent between entering and not entering, $\frac{R+\kappa}{p(\underline{q}_C)}$ is identified by the relationship $\frac{R+\kappa}{p(\underline{q}_C)} = v_C^e(\underline{q}_C, \mathbf{s})$. It is also clear that the As for R , κ , $p(q_C)$, $\pi(\mathbf{q}_C)$ and the unconditional distribution of q_C , F_{q_C} , they are not identified unless we impose additional assumptions. While this may seem problematic, there is a sense in which this is not a real limitationan unsatisfactory, this is not a big problem. Given that our

Now we discuss the Third Step. From the Second Step, we recover $F_{q_C, d_C, w'_C}|\varpi$, which allows us to estimate C_C , H_C etc. just as we did for C_I and H_I above. Also, we can

back out the distribution of q_C from $F_{q_C, d_C, w'_C} | \varpi$. This enables us to recover R . Finally, estimation of the distribution of q_O can be done without any modification.

5 Specification and Implementation

5.1 Specification

We specify the cost function of the incumbent C_I as a quadratic function $C_I(fr_I, q_I) = C_I^0(fr_I)^2 \exp(-C_I^1 q_I)$ and similarly for the cost functions of challengers and non contested incumbents, C_C , $\tilde{C}_I$ as $C_C(fr_C, q_C) = C_C^0(fr_C)^2 \exp(-C_C^1 q_C)$. We also let H_I ($\tilde{H}_I$, H_C , H_O) be quadratic $H_I(d_I) = H_I^0 d_I^2$ etc. As we have mentioned before, we assume a linear function for the vote production function,

$$\begin{aligned} vote_I &= \beta_I d_I + \beta_C d_C + q_I + q_C + \beta_x x + \beta_{ten} ten_I + \varepsilon, \\ vote_O &= \beta_O d_O + \beta_{-O} d_{-O} + q_O + q_{-O} + \beta_x x + \varepsilon. \end{aligned}$$

We include racial composition and unemployment for each Congressional District in x to control for demographic characteristics. As for the transition of x , we assume an exogenous AR(1) process as $x_{t+1} = \alpha x_t + \xi_{t+1}$ and $\xi_{t+1} \sim N(0, \sigma_\xi)$. With regard to the transition of w , we assume that $w_{t+1} = \lfloor r_t w'_t \rfloor + 1$ with probability $r_t w'_t - \lfloor r_t w'_t \rfloor$ and $w_{t+1} = \lfloor r_t w'_t \rfloor$ with probability $1 - (r_t w'_t - \lfloor r_t w'_t \rfloor)$ where r_t is the interest rate and $\lfloor t \rfloor = \max\{n \in \mathbb{N} | n \leq t\}$. In other words, w_{t+1} is either equal to the largest integer that is less than $r_t w_t$ or it is equal to the smallest integer that is bigger than $r_t w_t$ and the probability of the two events depend on how far $r_t w_t$ is from the two integers. We assume that $\delta = 0.9$ or annual discount rate of 0.95. Although nonparametric identification of the distribution of q_C and q_O is possible as we explained in the previous section, we assume a log-linear functional form for our estimation. A parametric specification allows for a more precise estimate of parameters with a moderate size sample.

<KORE ha results no tokorode, niko specification wo kaite, koreno setsumei wo surunokanaaa.>

Finally, we make one modification to the model we described in Section 2. As has been pointed out in the previous literature there seems to be challengers who run just for the sake of running without any regard to the likelihood of winning. Note that in this model, the challengers weigh the likelihood of winning in making their entry decision. While this seems like a good assumption for serious candidates, there are some candidates who seem to run only for the sake of running without giving any regard for the likelihood of winning. In order to model these candidates, we modify the model in our estimation to allow for the possibility of a challenger

5.2 Implementation

In the previous section, we described our estimation procedure in three steps. Conceptually, the first step precedes the second step and the second step precedes the third. However, in terms of estimation, it is convenient to turn the estimation into a two step procedure. In particular, we structure our estimation procedure so that results from Akerberg, Chen, and Hahn can be applied. In their paper, they show conditions under which semiparametric two-step estimation of dynamic models can be thought of as a parametric two-step estimation for inference purposes.

Consider the following two-step estimation procedure. The first step consists of estimating the probability of entry $P_\varepsilon = P_\varepsilon(s_{NC}, w_I, ten_I, x)$ (Section 4.1.3), estimating vote production function (Section 4.1.3), estimating the state transition (Section 4.2.2), and estimating the projection of the policy functions and the probability of winning (Section 4.2.3). First, estimating P_ε amounts to estimating the nonparametric regression $E[\chi|s_{NC}, w_I, ten_I, x]$. We use a second order polynomial in a probit estimation. Second, estimation of the vote production function involves estimating a partially linear model with endogenous regressors using conditional moment restrictions, $E[votes_I - \beta_I d_I - \beta_C d_C - \beta_x x - \beta_I ten_I - q_I(s_{NC}) - g(P_\varepsilon)|\Omega] = 0$, where $\Omega = \{s_{NC}, x, ten_I, w_I, \chi = 1\}$. We use a series estimator as in Ai and Chen. The idea of Ai and Chen is to first fix the parameters of the model, (i.e. β s, $q_I(\cdot)$, and $g(\cdot)$ in our model) and then project the residual ($votes_I - \beta_I d_I - \beta_C d_C - \beta_x x - \beta_I ten_I - q_I(s_{NC}) - g(P_\varepsilon)$) onto Ω . Series expansion is used for both $q_I(\cdot)$ and $g(\cdot)$ as well as for the basis functions of Ω . Then the parameters of the model and the coefficients on the basis functions are chosen jointly to make the projection be as close to 0 as possible. In practice, we use a second order polynomial to approximate $q_I(\cdot)$ and $g(\cdot)$ and also second order polynomials for the basis function. Third, as for the estimation of the state transition, this is a parametric problem as we have specified an AR(1) process with normal innovation. Fourth, in order to estimate the projection of the policy functions, we use an Hermite expansion of order 1. The probability of winning $\tilde{P}_w$ is estimated using a second order polynomial in a probit estimation.

Our second step of the two-step procedure consists of the optimality condition obtained from the incumbent first order condition (Section 4.2.5), the first order condition for the challengers (Section 4.3.1), the likelihood of challenger entry, and the likelihood of q_ε and q_O . As we explained in the previous subsection, we assume a parametric distribution for q_ε and q_O : This makes it possible to estimate the distribution of q_ε and q_O as well as R using likelihood-based methods. Note that the second step involves only estimation of finite-dimensional parameters.

Our two-step estimation procedure fits into the framework discussed in Akerberg, Chen and Hahn, where the first step involves semiparametric sieve M-estimation and the second

step involves estimation of finite-dimensional parameters from moment restrictions. The standard errors reported in the Results section are obtained using the results of Akerberg, Chen and Hahn.

6 Estimation:

6.1 Specification

6.2 Implementation:

7 Preliminary Results

7.1 Parameter Estimates

8 Counter Factual Analysis

9 Conclusion

10 Bibliography

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11 Appendix

Suppose First, consider simulating the continuation value $E_{r,x'}[V_I]$ as a function of the state variables ($q_{I,t}$, $w'_{I,t}$, $ten_{I,t}$, x_t , and D_I) for a given model parameter. Now we need to draw a new value of $q_{I,t}$ each period as $q_{I,t+1} = q_{I,t} + \xi_{t+1}$ but this is not enough. As C_I is not additive in fr_I and q_I , we need the joint distribution of (fr_I, q_I) , $F_{fr_I, q_I}(\cdot, \cdot | q_{I,t-1}, w_{I,t}, ten_{I,t}, x_t, D_I, \chi = 1)$, (and not just the marginals) to simulate the expected value of C_I .

In order to estimate F_{fr_I, q_I} , we first need to estimate the joint distribution of $(fr_{I,t}, w_{I,t+1}, q_{I,t+1})$ given $(q_{I,t-1}, w_{I,t}, ten_{I,t}, x_t, D_I, \chi_t = 1 \text{ and } \chi_{t+1} = 0)$ which we denote as $F_{fr_{I,t}, w_{I,t+1}, q_{I,t+1}}$. The reason why we do this is because $F_{fr_{I,t}, q_{I,t}}$ cannot be directly estimated while it is possible to estimate the latter: The realization of $q_{I,t}$ is unknown when $\chi_t = 1$, but the realization of $q_{I,t+1}$ can be backed out as $q_{I,t+1} = q_I(s_{NC,t+1})$ for a subset of observations with $\chi_{t+1} = 0$.

Note that we can express the density of $(fr_{I,t}, w_{I,t+1}, q_{I,t+1})$ using the density of $(fr_{I,t}, w_{I,t+1}, q_{I,t})$:

$$f_{fr_{I,t}, w_{I,t+1}, q_{I,t+1}}(fr_I, w_I, q_I) = \int_{t=-\infty}^{t=+\infty} f_{fr_{I,t}, w_{I,t+1}, q_{I,t}}(fr, w, t) \cdot E[P_e(t, w_{I,t+1}, ten_{I,t+1}, x_{t+1}, D_I)] f_{\xi}(q_I - t) dt, \quad (5)$$

where $f_{fr_{I,t},w_{I,t+1},q_{I,t}}$ is the distribution of $F_{fr_{I,t},w_{I,t+1},q_{I,t+1}}$, $f_{fr_{I,t},w_{I,t+1},q_{I,t}}$ is the distribution of $F_{fr_{I,t},w_{I,t+1},q_{I,t}}$ and f_{ξ} is the distribution of ξ . The left hand side density, $f_{fr_{I,t},w_{I,t+1},q_{I,t+1}}(fr_I, w_I, q_I)$, is given by integrating the product of $f_{fr_{I,t},w_{I,t+1},q_{I,t}}(fr_I, w_I, t)$ and $f_{\xi}(q_I - t)$, adjusting for the fact that $f_{fr_{I,t},w_{I,t+1},q_{I,t+1}}$ is conditioned on $\chi_{t+1} = 0$. The middle term on the right hand side of the equation, $E[P_e(t, w_{I,t+1}, ten_{I,t+1}, x_{t+1}, D_I)]$, is the probability that $\chi_{t+1} = 0$: It accounts for the fact that we only observe the realization of $q_{I,t+1}$ for which $\chi_{t+1} = 0$. Roughly speaking, the probability that we have $(fr_{I,t}, w_{I,t+1}, q_{I,t+1})$ equal to (fr_I, w_I, q_I) conditional on $\chi_{C,t+1} = 1$ is given by the product of the probability that $(fr_{I,t}, w_{I,t+1}, q_{I,t})$ is equal to (fr_I, w_I, t) and the probability that ξ is equal $q_I - t$, integrated over t , and adjusted for the probability of challenger entry.

Our problem can now be described as recovering $f_{fr_{I,t},w_{I,t+1},q_{I,t}}(fr, w, t)$ (from which we can recover $F_{fr_{I,t},q_{I,t}}$) from observables $f_{fr_{I,t},w_{I,t+1},q_{I,t+1}}$, $E[P_e(t, w_{I,t+1}, ten_{I,t+1}, x_{t+1}, D_I)]$ and $f_{\xi}(q_I - t)$. This problem turns out to be exactly analogous to estimation of the true density of a random variable in the presence of measurement error. Hence the results from measurement problems carry over to our problem. To see this, define a functional $T_{fr,w}$ for every fr and w as

$$T_{fr,w} \circ \varphi(q_I) = \int_{t=-\infty}^{t=+\infty} \varphi(t) E[P_e(t, w_{I,t+1}, ten_{I,t+1}, x_{t+1}, D_I)] f_{\xi}(q_I - t) dt.$$

Then the expression above can be written as $f_{fr_{I,t},w_{I,t+1},q_{I,t+1}} = T_{fr,w} \circ f_{fr_{I,t},w_{I,t+1},q_{I,t}}$. Note the analogy to measurement error problems where X is observed with measurement error as $Y = X + \varepsilon$ (Y is observed, X and ε are not observed, X and ε are independent and the distribution of ε is known up to finite dimensional parameter) and the task is to recover the distribution of X . The density of Y , f_Y can be expressed as $f_Y(y) = \int_{t=-\infty}^{t=+\infty} f_X(t) f_{\varepsilon}(y-t) dt$ where f_X and f_{ε} are the densities of X and ε . Identification of $f_{fr_{I,t},w_{I,t+1},q_{I,t}}$ can be shown just as in identification of f_X in measurement error problems. Estimating $f_{fr_{I,t},w_{I,t+1},q_{I,t}}$ is possible using Carroll and Hall, or Hall Horowitz etc.

$$v_I^{ne}(q_I, w_I, ten_I, x, D_I) = \max_{w'_I \geq 0, d_I \geq 0} B - \tilde{C}_I(w'_I + d_I - w_I, q_I) + \tilde{H}_I(d_I) + \delta E_{r,x'}[V_I(q_I, r(w'_I), (ten_I + 1), x', D_I)].$$

Means that if $d_I \neq 0$, then

$$\frac{\partial}{\partial d_I} \tilde{H}_I(d_I) - \frac{\partial}{\partial d_I} \tilde{C}_I(w'_I + d_I - w_I, q_I) = 0$$

If we assume that

$$\tilde{H}_I(x) = \theta_H x^\alpha$$

and

$$\tilde{C}_I(x) = \theta_C x^\beta e^{f(q_I)}$$

where $\beta > \alpha$, and $f(x)$ is decreasing in x , then

$$\alpha \theta_H d_I^{\alpha-1} = \beta \theta_C f r_I^{\beta-1} e^{f(q_I)}$$

which implies

$$e^{f(q_I)} = \frac{\alpha \theta_H d_I^{\alpha-1}}{\beta \theta_C f r_I^{\beta-1}}$$

or

$$\begin{aligned} f(q_I) &= C + (\alpha - 1) \log d_I - (\beta - 1) \log f r_I \\ \frac{\partial}{\partial d_I} C_I(w_I' + d_I - w_I, q_I) &= \frac{\beta_I}{\sigma_\varepsilon} \phi(K) \cdot (B + \delta E_{r,x'}[V_I]) + \frac{\partial}{\partial d_I} H_I(d_I), \\ \frac{\partial}{\partial w_I'} C_I(w_I' - w_I + d_I, q_I) &= \delta \Phi(K) \frac{\partial}{\partial w_I'} E_{r,x'}[V_I]. \end{aligned}$$

Furthermore, we assume that

$$\tilde{C}_C(x) = \text{cost_}c * \theta_C x^\beta e^{f(q_I)}$$

and

$$\tilde{C}_O(x) = \text{cost_}o * \theta_C x^\beta e^{f(q_I)}$$

with regard to the cost function. With regard to the benefit of spending, we assume that

$$\tilde{H}_C(x) = \text{ben_}c * \theta_H x^\alpha$$

and

$$\tilde{H}_O(x) = \text{ben_}o * \theta_H x^\alpha$$

Here, we make some remarks on the relationship between the equilibrium and the identification/estimation. Our first comment relates to the set of parameters that we consider in the identification and estimation. We have yet to specify the model we take to the data, but we note here that when we do specify the model, we will only consider the set of parameters for which the equilibrium value functions v_C^e , v_I^e , v_I^{ne} , etc. are monotone increasing in own quality.³⁴ The monotonicity of the value function is a natural restriction to impose,

³⁴We only rule out parameter values that yield no equilibrium with this property. As long as there is at

as higher levels of quality increases the probability of winning and decreases the cost of raising money. If for a given profile of parameters, there exists no equilibrium such that the value functions are monotone, then that particular profile of parameters is unlikely to be realistic. Therefore, we restrict our parameter space ex-ante to those with equilibrium value functions that are monotone in own quality. This restriction will be retained throughout.³⁵

12 Identification of F_{q_O}

We discuss the identification of the distribution of the quality of the open-seat challenger, F_{q_O} . The identification of F_{q_O} is not straight forward due to the fact that we only back out the realization of the quality of an open-seat challenger i , $q_{i,O}$, in the following two cases: (a) candidate i goes on to win the seat and subsequently becomes an uncontested incumbent, or (b) the opponent of i wins the election and becomes an uncontested incumbent down the road. This fact obviously creates nontrivial selection on the observed distribution of q_O .

In order to show that F_{q_O} is identified, first note that what we observe is $f_{\mathbf{q}}(q_1, q_2|E)$, where E is the event that $\mathbf{q}_O$ is observed. Note that what we want is the unconditional distribution $f_{\mathbf{q}}(\cdot)$, which is related to $f_{\mathbf{q}}(\cdot|E)$ as follows:

$$f_{\mathbf{q}}(\mathbf{t}) = \frac{f_{\mathbf{q}}(\mathbf{t}|E) \Pr(E)}{\Pr(E|\mathbf{t})}.$$

Note that $\Pr(E)$ and $f_{\mathbf{q}}(\mathbf{t}|E)$ are identified from the data. Note also that

$$\Pr(E|\mathbf{t}) = \int f(E|\mathbf{q} = \mathbf{t}, \mathbf{d}, \mathbf{w}') f(\mathbf{d}, \mathbf{w}'|\mathbf{t}).$$

Note that $f(E|\mathbf{q} = \mathbf{t}, \mathbf{d}, \mathbf{w}')$ is identified. This is the probability that at least one of the candidates will be uncontested later, conditional on $\mathbf{q} = \mathbf{t}, \mathbf{d}, \mathbf{w}'$. Once we have the pair (q, w') , we can simulate the incumbent's election history, from which we can identify and obtain the probability of being uncontested in a subsequent election. It is also easy to see that $f(\mathbf{d}, \mathbf{w}'|\mathbf{t})$ is identified: First note that

$$f(\mathbf{d}, \mathbf{w}'|\mathbf{t}) = \frac{f(\mathbf{d}, \mathbf{w}', \mathbf{t})}{\int f(\mathbf{d}, \mathbf{w}', \mathbf{t}) d\mathbf{d}d\mathbf{w}'},$$

Second, note that

$$f(\mathbf{d}, \mathbf{w}', \mathbf{t}) = \frac{f(\mathbf{d}, \mathbf{w}', \mathbf{t} | E) f(E)}{f(E | \mathbf{d}, \mathbf{w}', \mathbf{t})}.$$

least one equilibrium (in case there are multiple equilibria) where the value functions are monotone we do not rule the parameter out.

³⁵It is hard to analytically derive the set of parameters with this property, but it is possible to that guarantee the monotnicity of the value function, but this will be taken into account in the estimation.

Substituting out $f(\mathbf{d}, \mathbf{w}', \mathbf{t})$ using the previous expression, we obtain the following expression:

$$f(\mathbf{d}, \mathbf{w}'|\mathbf{t}) = \frac{f(\mathbf{d}, \mathbf{w}', \mathbf{t} | E)}{f(E | \mathbf{d}, \mathbf{w}', \mathbf{t}) \int \frac{f(\mathbf{d}, \mathbf{w}', \mathbf{t} | E)}{f(E | \mathbf{d}, \mathbf{w}', \mathbf{t})} d\mathbf{d}d\mathbf{w}'}.$$

Since everything which appears on the right hand side is identified, $f(\mathbf{d}, \mathbf{w}'|\mathbf{t})$ is identified. This means that $\Pr(E|\mathbf{t})$ and $f_{\mathbf{q}}(\mathbf{t})$ are identified as well. In practice, we are going to use

$$\Pr(E|\mathbf{t}) = \int f(E|\mathbf{q} = \mathbf{t}, \mathbf{d}, \mathbf{w}') f(\mathbf{d}, \mathbf{w}'|\mathbf{t}) \sim \int f(E|\mathbf{q} = \mathbf{t}, \mathbf{d}, \mathbf{w}') f(\mathbf{d}, \mathbf{w}'|\mathbf{t}, E)$$

The third term, $\int f(E|\mathbf{q} = \mathbf{t}, \mathbf{d}, \mathbf{w}') f(\mathbf{d}, \mathbf{w}'|\mathbf{t}, E)$ is easy to compute, since $f(\mathbf{d}, \mathbf{w}'|\mathbf{t}, E)$ is directly observable. Of course, $f(\mathbf{d}, \mathbf{w}'|\mathbf{t}) \neq f(\mathbf{d}, \mathbf{w}'|\mathbf{t}, E)$ in general: it is only equal under pure strategy.

Sufficient Statistic We show the claim for a geometric distribution, where the pdf is given by $p(1-p)^k$ for $k \in [0, 2, \dots, \infty]$. First, note that

$$\begin{aligned} E[M] &= \sum_{N=1}^{+\infty} p(1-p)^N N(1 - F_{q_C}(\bar{q}_C)) \\ &= (1 - F_{q_C}(\bar{q}_C)) \frac{1}{p}. \end{aligned}$$

Second, note that

$$\begin{aligned} \Pr(\{\boldsymbol{\chi}' * \mathbf{1} \geq 1\}) &= \sum_{N=1}^{+\infty} p(1-p)^N (1 - \text{Bin}(N, 0; F_{q_C}(\bar{q}_C))) = \sum_{N=1}^{+\infty} p(1-p)^N (1 - F_{q_C}(\bar{q}_C)^N) \\ &= 1 - \frac{p}{(1 - F_{q_C}(\bar{q}_C) + pF_{q_C}(\bar{q}_C))} \end{aligned}$$

It turns out that $p = 1 + \frac{1}{E[M] - \frac{1}{(1 - \Pr(\{\boldsymbol{\chi}' * \mathbf{1} \geq 1\})E[M])}}$ and $\bar{q}_C = F_{q_C}(-E[M] + \frac{1}{(1 - \Pr(\{\boldsymbol{\chi}' * \mathbf{1} \geq 1\})})^{-1})$. Since the distribution of q_C conditional on entry, $F_{q_C}|\{\boldsymbol{\chi}' * \mathbf{1} \geq 1\}$ is just a function of p and $\bar{q}_C$, it is also a function of $E[M]$ and $\Pr(\{\boldsymbol{\chi}' * \mathbf{1} \geq 1\})$.