Financial Expertise and Asset Prices*

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Abstract

This paper studies the effects of the joint distribution of the stock financial expertise and financial wealth on asset prices. By modeling financial expertise as a stock, we are able to incorporate economic ideas from capital theory as well as industrial organization into a model with slow moving capital. We aim to explain the persistence of risky arbitrage opportunities by modeling the entry and investment decisions of “financial experts”. Our theory also naturally yields size and performance distributions for experts, and we will use empirical distributions from the hedge fund industry to help to calibrate our model.

Key Words: financial expertise, slow moving capital, risky arbitrage, industry equilibrium.

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1 Introduction

Complex hedge fund strategies require specific investments. These investments can include the development of highly skilled human capital, such as trained financial engineers or computer scientists, as well as investments in relationships with broker dealers or other reputation capital, and the acquisition of financial data and information technology. This paper aims to understand the effects of such accumulated, specific capital on equilibrium asset prices. In particular, we are interested in explaining how prices for complex assets can experience persistent deviations from measures of their fundamental value, and why entry by new investors does not work to quickly eliminate such apparent profitable risky arbitrage opportunities.

We develop a quantitative model of the joint dynamics of financial wealth, and financial expertise. In our model, financial expertise is a durable, capital-like good, which enables its investor owners to achieve a better Sharpe ratio. When “experts” in a complex asset class experience a negative shock to their financial wealth, the price of the asset declines and expected returns increase. However, entry by new investors is not a foregone conclusion because the accumulated expertise of the incumbents acts as a barrier to entry. As a result, risky arbitrage opportunities can appear and persist in equilibrium. A by-product of our modeling strategy is that we construct an industry equilibrium model of financial experts. We plan to calibrate this model to match the size distribution of hedge funds, and potentially to some of the observed relationships between fund growth and returns.

In the recent financial crisis, the wealth of financial experts deteriorated. Contemporaneously, several examples of relatively persistent arbitrage opportunities appeared. Figure 1 displays the persistent deviations from covered interest rate parity between the dollar and the pound which appeared during 2007 and 2008, and remained through 2010. Other examples include the large and persistent corporate-bond CDS basis discussed in Duffie (2010), the persistent increase in the TIPS-Treasury Bond spread documented in Fleckenstein et al. (Forthcoming), and the deviations from triangular arbitrage parity and American depositary receipt parity discussed in Pasquariello (2013). Fleckenstein (2012) documents that fixed income
hedge fund returns predict arbitrage opportunities in nominal bonds vs. inflation indexed bonds globally. Pedersen et al. (2007) present evidence from earlier periods which links the net worth of hedge funds to returns to merger arbitrage, and convertible bond arbitrage.

It is realistic to assume that some markets for financial arbitrage are segmented, however clearly entry is possible. Following the traditional literature on industrial organization and industry equilibrium, we assume that investors must pay an entry cost to acquire “financial expertise” and to then participate in risky financial arbitrage. The question is then, why, when risky arbitrage spreads widen, do we not observe more entry, less exit, and “fast moving capital” to take advantage of the greater profit opportunities? That is, all else equal wider spreads should lead to more entry, but in practice we do not seem to observe this. Our work aims to explain entry and exit dynamics in order to shed light on the dynamics of risky arbitrage opportunities after shocks to financial wealth and the fundamental driving processes for risky asset returns.

We plan to use the CISDM Hedge Fund Database in order to calibrate our model and to examine its quantitative implications. We would like to examine the empirical moments for the distributions of fund size, age, performance, and expenditures across investment strategies. We will integrate this data with data on apparent persistent risky arbitrage opportunities during the 2008 financial crisis, as well as during earlier periods such as the 2005 period of convertible bond fund redemptions, and the aftermath of the 1998 financial crisis in Asia.

The outcome of our research will lend insight into the dynamics of asset prices during crises, and during periods of financial innovation. Thus, our model can potentially be used to examine the effects of asset purchase programs aimed at supporting declining asset prices. It can also be used to consider the effects of regulation on hedge fund industry dynamics and asset prices.
2 Literature

Our paper is related to the literature on slow moving capital, including Duffie (2010), Acharya et al. (2009), Duffie and Strulovici (2012). An early contribution which links arbitrageur wealth to asset prices in the context of segmented markets is Vayanos and Gromb (2012). Recently, Glode et al. (2012) developed a model of financial expertise as an arms race. Although their focus on the microfoundations of the dynamics of expertise is different, our work is related in that they are the first to consider the effect of the stock of financial expertise on asset prices.

3 Model

3.1 Model Set Up

We develop a Bewley (1986) model of the asset management industry that draws on Hopenhayn (1992)'s model of entry and exit.

Elements

Agents There are a continuum of agents of measure one. Agents have CRRA utility over consumption each period. These agents can either be experts or non-experts. Experts accumulate expertise via an exogenous process, and must pay a fixed cost each period to maintain this expertise.

Expert Trading Technology These experts can invest in a risky trading strategy, as well as a riskless asset. The riskless asset delivers a fixed net return equal to $r_f$. $I$ denotes the total amount of wealth invested in the risky technology. We use $\alpha(I)$ to denote the alpha-generating production function that is specific to this risky trading technology. There are decreasing returns to scale: $\alpha'(I) < 0$. $e$ denotes the (agent specific) stock of financial expertise for expert investors. The risky trading technology delivers the following stochastic returns:

$$r_{t+1} = r_f + \alpha(I_t) + \epsilon_{t+1}\sigma(e_t),$$

where $\epsilon_{t+1}$ are shocks to the returns.
where $\epsilon$ is drawn from a standard normal distribution. Hence, the conditional Sharpe ratio from operating this risky trading technology is given by:

$$\frac{\alpha(I_t)}{\sigma(e_t)},$$

which increases in the agent’s stock of financial expertise $e$, and decreases in the total amount of financial wealth $I$ allocated to the risky asset. We only allow for long positions in the risky trading strategy.

**Non-Expert Trading Technology** Non-experts can invest only in the risk free asset. Non-experts can enter, and become experts by paying a fixed cost to initialize their stock of financial expertise. Experts must pay a maintenance cost to maintain their expertise, so that sufficiently low expertise leads to exit.

Agents solve the following recursive problem:

$$v(t, w, e) = \max_{x,n}\{v_x(t, w, e), v_n(w)\} \tag{1}$$

where $t$ is type expert ($x$) or non-expert ($n$), $w$ is financial wealth, $e$ is financial expertise.

For agents which choose to be experts, we have:

$$v_x(t, w, e) = \max_{i, \theta} \frac{c^{1-\gamma}}{1-\gamma} + \beta E \left[ pv(t', w', e') + (1 - p) \frac{w'^{1-\gamma}}{1-\gamma} \right] \tag{2}$$

subject to

$$c = w - i - f_{xx} \mathbb{1}_{t=x} - f_{nx} \mathbb{1}_{t=n}$$

$$w' = i(1 + \theta r' + (1 - \theta)r_f)$$

$$r' = r_f + \alpha(I) + e' \sigma(e)$$

$$e' = \xi + \rho e + \eta$$

$$t' = x$$
where \( I \) is an indicator function and \( f_{xx} \) is the maintenance cost for current experts and \( f_{nx} \) is the entry cost for new experts, and where \( f_{nx} \gg f_{xx} \). Also, \( \rho < 1 \) and \( \xi \geq \xi \) where \( \xi \) is the initial level of expertise, which applies to new experts. Finally, \( \alpha'(I) < 0 \) and \( \sigma'(e) < 0 \), where \( I \) is the aggregate financial wealth allocated to the risky asset. These assumptions imply that investors’ Sharpe ratios are increasing in their stock of financial expertise, but decreasing in the aggregate amount of financial wealth allocated to the risky asset. We assume that there is an exogenous death probability of experts, \( p \), to ensure a bounded stationary equilibrium. If an agent receives a death shock, she dies at the end of the current period and consumes all of her wealth.

For agents which choose to be non-experts in the following period, we have:

\[
v_n(w) = \max_c c^{1-\gamma} + \beta E \left[ pv(t', w', e') + (1 - p) \frac{w'^{1-\gamma}}{1-\gamma} \right]
\]

subject to

\[
\begin{align*}
c &= w - i \\
w' &= i(1 + r_f) \\
e' &= \xi \\
t' &= n
\end{align*}
\]

Again, the death probability \( p \) ensures bounded wealth in the stationary equilibrium.

### 3.2 Model with Homogeneity

We first consider a model where both the maintenance cost and entry cost are proportional to wealth. Similar to the standard portfolio choice problem, the value functions are homogenous in wealth.
Proposition 1  The value functions are determined by

\[ v_e(t, w, e) = \frac{w^{1-\gamma}}{1-\gamma} f(t,e), \]
\[ v_n(t, w) = \frac{w^{1-\gamma}}{1-\gamma} g(t), \]

and the optimal policy functions are

\[ i^e(t, w, e) = x(t, e) w, \]
\[ i^n(t, w) = y(t) w, \]
\[ c^e(t, w, e) = (1 - x(t, e) - f_e) w, \]
\[ c^n(t, w) = (1 - y(t)) w, \]

where \( x(t, e), y, f(t, e), g(t, e) \) and portfolio choice \( \theta(e) \), are determined by the following conditions,

\[ x^{\gamma} (1 - x - f_e)^{-\gamma} = \beta E \left[ (p \min \{ f(e'), g(e') \} + (1-p)) \right] E \left[ (1 + r_f + \theta r_s')^{1-\gamma} \right], \]
\[ E \left[ (1 + r_f + \theta r_s')^{-\gamma} r_s' \right] = 0, \]
\[ y^{\gamma} (1 - y)^{-\gamma} = \beta^\gamma E \left[ (p \min \{ f(e'), g(e') \} + (1-p)) \right], \]

Proof. See Appendix. ■

Given the value functions, expert exits the market iff

\[ \frac{f(e)}{1-\gamma} < \frac{g(e)}{1-\gamma}. \]

For potential entrants, they become experts iff

\[ \frac{f(e)(1-f_{xx})^{1-\gamma}}{1-\gamma} \geq \frac{g(e)}{1-\gamma}. \]
3.3 Recursive Equilibrium

A stationary industry recursive equilibrium consists of aggregate demand of risky asset, $I$, pricing function, $\{\alpha (I), \sigma (I)\}$, entry and exit decisions, $\{\mathbb{I}_{\text{entry}} (w, e), \mathbb{I}_{\text{exit}} (w, e)\}$, optimal policy functions $\{i^e(t, w, e), i^a(t, w), c^e(t, w, e), c^a(t, w), \theta (t, w, e)\}$, and an stationary evolution of industry dynamics, $\Omega (t, w, e)$, such that

1. Expert optimality: Given the Sharpe ratio, $\frac{\alpha (I)}{\sigma (e)}$, on risky asset, expert chooses the optimal policy functions $\{i^e(t, w, e), c^e(t, w, e), \theta (t, w, e)\}$ that solves the Bellman equation 2.

2. Non-expert optimality: Given the Sharpe ratio, $\frac{\alpha (I)}{\sigma (e)}$, on risky asset, non-expert choose the optimal policy functions $\{i^e(t, w, e), i^a(t, w), c^e(t, w, e), c^a(t, w), \theta (t, w, e)\}$ that solves the Bellman equation 3.

3. Entry and exit: The optimal entry and exit decisions, $\{\mathbb{I}_{\text{entry}} (w, e), \mathbb{I}_{\text{exit}} (w, e)\}$, solves the equation 1.

4. Market clearing: The Sharpe Ratio on the risky asset reflects the investment decisions of all financial experts,

$$I = \int \int \int \theta (t, w, e) i^e(t, w, e) dtdwde$$

5. $\Omega (t, w, e)$ is a stationary distribution, given the entry and exit decisions.

4 Comparative Statistics

In this section, we characterize experts’ and non-experts’ behaviors and industry dynamics. In particular, we start with the model where the maintenance cost and entry cost are proportional to wealth as the benchmark. We describe the invariant distribution of wealth and expertise. We assume that $\alpha = \frac{1}{1+\psi I}$, and $\sigma = \frac{1}{1+\phi e}$. We parameterize the model assuming that one period is one year, and the discount factor is 0.96 with implied risk-free interest rate 4.17%. All parameter values are summarized in Table 1.
4.1 Homogeneity Case

Figures 2 and 3 show the policy functions and the evolution of mean wealth as the survival probability varies from 1 to 0.9. Due to the linear return technology, without a positive death probability, agents' wealth can grow unboundedly.

Figure 4 plots the policy functions of experts and non-experts, as well as the wealth distribution in the stationary equilibrium. First, agents with high expertise invest more on risky asset; second, investment decisions, as a fraction of total wealth, are decreasing function of expertise (a wealth effect). And the entry rate is roughly 5.5% each period. With a higher variance of returns on risky asset, shown in Figure 5, agents invest less in the risky asset, and there is a lower fraction of experts, and the entry and exit rates are larger.

The homogeneous model enables us to get a closed form solution. However, only the equilibrium $\alpha$ depends on aggregate wealth. If there is a large (exogenous) shock to agents' wealth, it will introduce a higher excess return. Since the entry and exit decision are purely determined by the expertise level, this homogenous model implies more entry after a one-time shock to wealth. Figure 6 plots the impulse response function with a 10% temporary drop in wealth. A drop in wealth indicates a decline in the aggregate investment on risky asset, which creates a temporary increase in excess returns. Given a higher excess return, there is more entry and less exit. The general equilibrium effect implies that the percentage increase in excess returns is smaller that the percentage change in aggregate wealth, since people will reallocate some of their investment in the riskless asset to risky asset. The arbitrage opportunities diminish rapidly after four periods.

4.2 Non-Homogeneity Case

In this section, we consider a model in which every policy function depends both on expertise level and on wealth. In this case, we no longer have a closed form solution. We use discretize value function iteration to solve the model. We choose a large enough state space for wealth such that there is no mass point near the wealth boundary in the stationary distribution.
We numerically solve the model and then simulate the model with 200 periods until reach the stationary distribution. The top panel of Figure 7 graphs the evolution of mean of wealth and stationary wealth distributions. It shows that the wealth is more concentrated at the bottom and the distribution is a decreasing function of wealth. The bottom part of Figure 7 plots portfolio choices and investment choices of agents as functions of expertise level and wealth. Agent with a higher expertise invests more on risky asset.

\footnote{Since we are using the discretize value function, One issue is that some choices will lead to states “off grid”. Also, because we set an upper bound on wealth to avoid unnecessary extrapolation algorithm, agents will reduce investment on risky asset when close to the upper bound. However, this does not matter if we choose a large enough grids and make sure that these choices are mapped to the upper bound of the state space and have zero mass in the stationary equilibrium.}
References


Appendix

Proof to Proposition 1

We prove this Proposition by guess and verify. First, we rewrite the optimal decision problem as following.

For expert:

\[
v_e(t, w, e) = \max_{i, \theta} \left( w - i - f_{xx} w_{t=x} - f_{nx} w_{t=n} \right)^{1-\gamma} + \beta E \left[ pv(t', w', e') + (1 - p) \right]
\]

s.t.

\[
w' = i(1 + r_f + \theta r_s')
\]

\[
r_s' = \alpha(I) + e' \sigma(e)
\]

\[
e' = \xi + \rho e + \eta
\]

For non-experts:

\[
v_n(t, w, e) = \max_{i, \theta} \left( w - i \right)^{1-\gamma} + \beta E \left[ pv(t', w', e') + (1 - p) \right]
\]

s.t.

\[
w' = i(1 + r_f)
\]

\[
e' = \xi + \rho e + \eta
\]

and

\[
v(t, w, e) = \max \{ v_e(t, w, e), v_n(t, w, e) \}.
\]

Guess the value functions:

\[
v_e(t, w, e) = \frac{w^{1-\gamma}}{1 - \gamma} f(t, e), v_n = \frac{w^{1-\gamma}}{1 - \gamma} g(t, e),
\]

and the policy functions:

\[
i^e = x(t, w, e) w, i^n = y(t) w,
\]

we have

\[
w' = xw(1 + r_f + \theta r_s'),
\]

\[
\beta E \left[ (p \min \{ f(e'), g(e') \} + (1 - p)) w'^{-\gamma} (1 + r_f + \theta r_s') \right] = (1 - x - f_e)^{-\gamma} w^{-\gamma},
\]

\[
\beta E \left[ (p \min \{ f(e'), g(e') \} + (1 - p)) w'^{-\gamma} r_s' \right] = 0,
\]

\[
\beta E \left[ (p \min \{ f(e'), g(e') \} + (1 - p)) y^{-\gamma} w^{-\gamma} (1 + r_f)^{1-\gamma} \right] = (1 - y)^{-\gamma} w^{-\gamma},
\]

\[
g(e) = (1 - y)^{-\gamma},
\]

\[
f(e) = (1 - x - f_{xx} x_{t=x} - f_{nx} x_{t=n})^{-\gamma} (1 - f_e).
\]
Substitute all \( w' \) by \( xw(1 + r_f + \theta r'_s) \), we get

\[
E \left[ (1 + r_f + \theta r'_s)^{-\gamma} r'_s \right] = 0,
\]

\[
x^\gamma (1 - x - c) = \beta E \left[ \min \{ f(e'), g(e') \} + (1 - p) \right] E \left[ (1 + r_f + \theta r'_s)^{1-\gamma} \right],
\]

\[
y^\gamma (1 - y) = \beta E \left[ \min \{ f(e'), g(e') \} + (1 - p) \right].
\]

The first equation determines the optimal portfolio choice, which is independent of wealth. The second and the third equations solve the investment choice, and it is independent of wealth.
<table>
<thead>
<tr>
<th>Parameter</th>
<th>Symbol</th>
<th>Value</th>
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<tr>
<td>Discount factor</td>
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<td>Risk Aversion</td>
<td>$\gamma$</td>
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<td>Survival Probability</td>
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<td>Expertise shock drift</td>
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<td>Expertise shock drift Persistence</td>
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<td>Variance of risky asset return</td>
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<tr>
<td>Sensitivity of Excess return to aggregate investment</td>
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<tr>
<td>Sensitivity of variance of return to expertise</td>
<td>$\phi$</td>
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</tr>
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</table>

Table 1: Parameterizations

Figure 1: This figure shows persistent deviations from covered interest rate parity between the US and UK during and after the 2008 financial crisis. Data are in logs.
Figure 2: p=1
Figure 3: p=.9
Figure 4: $f_{xx}=0.005, f_{nx}=0.015, \sigma=0.1$
Figure 5: \( f_{xx}=0.005, f_{nx}=0.015, \sigma=0.2 \)
Figure 6: Impulse Response Function with 10% Drop in Wealth
Figure 7: Wealth distribution and portfolio choice as a function of wealth and expertise