In for a Penny, In for a 100 Billion Pounds: 
Quantifying the Welfare Benefits from Debt Forgiveness

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VERY PRELIMINARY!

ABSTRACT

Since 1989, creditor countries have provided debt relief to developing countries worth more than 100 billion US dollars. Prominent lobby groups are campaigning for a further 400 billion US dollars in debt relief to be provided in the near future. How much could developing country’s gain from debt relief? How costly is it to provide debt relief? Would debt relief increase social welfare? And if so, to which countries should it be most urgently directed? In this paper, we develop a framework for measuring the marginal welfare gain from debt relief and indicate when this marginal measure can be used to estimate the total welfare benefit of debt relief. We then apply this framework to data on the debts of 72 developing countries to form an estimate of the global social welfare benefits of debt forgiveness.

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1 Introduction

Developing country debt relief to date has been substantial. Since 1956, the set of official creditors represented by the Paris Club have awarded debt relief on debts with a face value of 560 billion US dollars\(^1\). Between 1989 and 2010, creditors have forgiven developing country debt with a face value of more than 300 billion US dollars (World Bank 2012). Depetris and Kraay (2005) estimate that the present value of the debt forgiveness granted to low income countries since 1989 totalled 100 billion US dollars as of 2003, with an additional 24 billion of debt relief coming through the rescheduling of debt repayments. Further debt relief is likely, with official debt relief under the Highly Indebted Poor Country (HIPC) Initiative and Multilateral Debt Relief Initiative (MDRI) continuing, and with prominent lobby groups campaigning for a further 400 billion US dollars in debt relief (Jubilee Debt Campaign 2010).

Is further debt relief a good idea? How much could a developing country gain from debt relief? Or, equivalently: How costly is debt service to a developing country? Conversely: How costly is it to provide debt relief? Would debt relief increase social welfare? And if so, to which countries should it be most urgently directed? In this paper, we develop a framework for measuring the marginal welfare gain from debt relief and indicate when this marginal measure can be used to estimate the total welfare benefit of debt relief. We then apply this framework to data on the debts of 72 developing countries to form an estimate of the global social welfare benefits of debt forgiveness.

We distinguish between the efficiency and redistributive aspects of debt forgiveness. If international capital markets worked perfectly, then all people in all nations would value resources on the margin at exactly the same rate and there could be no efficiency gain from debt relief. However, as international capital markets are imperfect, agents value resources differently, and the reallocation of resources implied by debt forgiveness can result in efficiency improvements. Our method first combines estimates of differences in these marginal valuations with assumptions about the way in which debt forgiveness affects the operation of international capital markets to calculate the marginal welfare gain from debt forgiveness, and derives conditions under which this is likely to provide a good approximation to the

total welfare benefit of debt forgiveness. For borrowing constrained countries, the efficiency gains are typically negative: countries are borrowing constrained precisely because they value future resources at a lower rate than creditors, and hence redistributing resources in the future from creditors to debtors reduces efficiency and, everything else equal, also welfare. However, these efficiency gains and losses are likely to be small compared to the potential social welfare gains from redistributing resources from rich creditor countries to poor debtor countries, although redistributive gains are typically very sensitive to the assumed form of the social welfare function. The second part of our method combines various assumptions about the form of this social welfare function to measure the combined welfare benefits of debt forgiveness.

We find that, as expected, the pure efficiency gains from a debt forgiveness program applied to all 72 countries in our sample are negative: most countries are borrowing constrained, and hence value future resources relatively less than creditors, hence producing small welfare gains from debt relief. In aggregate as of the end of 2006, debt forgiveness for the 72 countries in our sample would have cost 817 billion US dollars and produced welfare increases of 551 billion US dollars. Nonetheless, there are efficiency gains from providing debt forgiveness to some low income sub-Saharan African economies including the Cote d’Ivoire and the Central African Republic. Once the possibility of social welfare improving redistribution is introduced, even under relatively conservative assumptions, the social welfare consequences of debt forgiveness are enormous. As a metric for understanding the magnitude of these gains, we find that in order for debt relief to have a neutral effect on social welfare, the weight attached to the welfare of individuals living in the poorest countries on earth (where per-capita consumption levels are on the order of 1% of the levels observed in rich countries) would have to be roughly two one-thousandths the weight attached to individuals in rich countries.

A number of researchers have attempted to estimate the cost of the observed debt relief provided by creditors. Estimates based on the face value of the debt forgiven or treated are problematic because they do not capture debt relief provided by face-value-neutral reschedulings of debt repayments (Daesking and Powell 1999, Depetris and Kraay 2005). Likewise, as emphasized by Dias, Richmond, and Wright (2011), face values can be a misleading indicator
of overall indebtedness. This has led to the development of discounted present value measures of debt relief (for example, Depetris and Kraay 2005, 2006). However, it is not always obvious what the appropriate rate of discount is and authors commonly use a constant rate over time and across countries. Like us, Sturzenegger and Zettelmeyer (2008) and Cruces and Trebesch (2010) provide estimates of the cost of debt relief to creditors using, at each point in time, a country, time and maturity specific discount rate. Unlike us, their focus is on estimating the cost of actual debt relief provided by commercial creditors, whereas we study both the prospective costs and benefits associated with potential future debt relief by both official and commercial creditors.

A small number of researchers have examined the effect of observed debt relief on the receiving country. Arslanalp and Henry (2006) study the effect of debt relief provided under the Brady Plan for a set of middle income countries and find that the Brady Plan led to significant increases in the stock markets of these countries. Depetris and Kraay (2005) study the impact of debt relief on 62 low-income country and find little evidence that debt relief has improved the public spending, policy choices, or growth rates of the targeted countries. Perhaps the closest paper to ours is by Sturzenegger and Zettelmeyer (2006) who attempt to estimate the differential impact of commercial debt restructurings on creditors and debtors for six middle income countries using differences between the market rates that prevailed immediately following a debt restructuring, and the rates that they estimate would have prevailed in normal times. The theory we present below, however, suggests that debt relief should be computed using discount rates that reflect economic conditions at the time the debt relief is provided, and we apply this theory to analyze the effects of the prospective future forgiveness of both official and commercial debts for over one hundred countries.

Finally, this paper can be viewed as a companion to our previous paper (Dias, Richmond, and Wright 2011) in which we point to problems in the way debt statistics are traditionally reported. In that paper we propose a number of alternative methods to illustrate and correct some of the problems associated with existing measures of indebtedness, and argue that the “best” measure of debt relief will probably vary according to the question of interest for which it is constructed. In this paper we derive the optimal measure of indebtedness and debt stocks to use when the question of interest concerns the welfare consequences of debt
forgiveness.

The rest of this paper is organized as follows. Section 2 develops, using a series of simple two-period examples, the main themes that come out of our theoretical analysis and introduce our measures of the welfare gain to a country from debt relief. Section 3 then establishes that the basic results from these examples generalize to environments with multiple time periods, richer structures of uncertainty, and nominal debt. Section 4 introduces our data sources, and Section 5 presents our results on the market cost of, and the country valuation of, debt forgiveness. Section 6 turns to an examination of the effect of debt relief on social welfare once the possibility of welfare gains from redistribution are allowed. Section 7 concludes while an appendix reports further details of our empirical methods.

2 Theoretical Examples

We begin with a number of simple two-period examples designed to illustrate the main points of the theory. In the first example, markets are imperfect as a result of a borrowing constraint, and debt forgiveness is assumed to have no effect on this borrowing constraint. In the second, debt forgiveness does affect the constraint and so the welfare benefits include the gain from a relaxation of the constraint. Whereas in the first two examples, borrowing constraints were imposed exogenously, in the third example market imperfections are derived endogenously from the presence of sovereign risk. These results are then generalized in the appendix.

2.A Example 1: A Constraint on New Borrowing

We begin with a simple example in which the quantity of international borrowing is constrained. Consider an economy without uncertainty that lasts for two periods. A country is represented by an agent with preferences over an aggregate consumption good in each period given by

\[ u(c_1) + \beta u(c_2), \]

where \( u \) is the utility function, and \( \beta \) is the discount factor.
for some $u$ concave and strictly increasing. The country receives income the income stream $(y_1, y_2)$, begins with foreign assets (or debts) $a_1^{*A}$ and $a_2^{*A}$ that pay off in periods 1 and 2 respectively, and may choose in period 1 to accumulate at price $q$ more foreign assets, $a_2$, that pay off in period 2 subject to a borrowing constraint

$$a_2 \geq a_2.$$  

The flow budget constraints facing the agent are given by

$$c_1 + qa_2 \leq y_1 + a_1^{*A},$$

$$c_2 \leq y_2 + a_2^{*A} + a_2.$$  

We want to know the change in the country’s welfare, as given by the representative agent, from a change the initial asset/debt position of the country from $(a_1^{*A}, a_2^{*A})$ to $(a_1^{*B}, a_2^{*B})$. The change could involve debt forgiveness, in which debts are reset to lower values ($a_1^{*A} < a_1^{*B}$), a debt reprofiling, in which the timing of debts repayments are changed without reducing the market value of the debt, or some combination of the two. We refer to the union of these possibilities as debt relief, but stress that more general changes in asset/debt structure are possible.

We can derive a bound on the change in welfare as follows. Let $V$ denote the optimum value function for the country. Then by the concavity of the period utility function $u$, this change in welfare can be represented by

$$V(a_1^{*B}, a_2^{*B}) - V(a_1^{*A}, a_2^{*A}) \leq u'(c_1^B)(c_1^B - c_1^A) + \beta u'(c_2^A)(c_2^B - c_2^A),$$

after taking a first order Taylor series approximation to the value function around the optimal consumption choices associated $(a_1^A, a_2^A)$ which we denote $(c_1^A, c_2^A)$.

As $u$ is strictly increasing, the flow budget constraints hold with equality and hence

$$c_1^B - c_1^A = (a_1^{*B} - a_1^{*A}) - q(a_2^B - a_2^A),$$

$$c_2^B - c_2^A = (a_2^{*B} - a_2^{*A}) + (a_1^B - a_1^A).$$
Hence, after dividing the expression for the change in welfare by $u'(c_1^A)$ in order to express the change in welfare in period 1 consumption units, we obtain

$$V (a_1^{*B}, a_2^{*B}) - V (a_1^{*A}, a_2^{*A}) \leq (a_1^{*B} - a_1^{*A}) + p^A (a_2^{*B} - a_2^{*A}) - (q - p^A) (a_2^B - a_2^A),$$

(1)

where we have denoted by

$$p^A = \beta \frac{u'(c_2^A)}{u'(c_1^A)},$$

which we refer to the period 1 country price of a unit of consumption in period 2, to be distinguished from the market price $q$.

If we knew the preferences of the representative agent, we could solve for the optimal choices of the country before and after the change in initial debts and compute the right hand side of (1) exactly. In general, however, the result will be sensitive to the exact specification of preferences. Fortunately, it is possible to say a few things about the change in welfare without making any further assumptions on preferences.

First, if the borrowing constraint binds at $A$ then from the first order necessary conditions for the country we have $p^A < q$. That is, the country values resources on the margin at a rate less than the market (and hence would like to borrow more). But as $a_2^B \geq a_2^A = a_2$, since the borrowing constraint is binding at $A$, this means we can bound the change in welfare by the change in the country value of the initial assets/debts of the country

$$V (a_1^{*B}, a_2^{*B}) - V (a_1^{*A}, a_2^{*A}) \leq (a_1^{*B} - a_1^{*A}) + p^A (a_2^{*B} - a_2^{*A}).$$

(2)

Second, note that if the borrowing constraint is not binding at $A$, then $p^A = q$ and hence

$$V (a_1^{*B}, a_2^{*B}) - V (a_1^{*A}, a_2^{*A}) \leq (a_1^{*B} - a_1^{*A}) + q (a_2^{*B} - a_2^{*A}).$$

That is, if all agents (country and creditors) value resources on the margin at the same rate, the change in welfare is bounded above by (and for small changes in initial assets, equal to)
the change in the *market value* of the initial assets/debts of the country.

Some intuition for this result can be derived from Figure 1 which plots the budget set and indifference curves associated with a country that is borrowing constrained at point $A$. The initial endowment point $E^A$ is defined for income and initial assets $(y_1 + a_1^A, y_2 + a_2^A)$, and as drawn represents a country with debts coming due in both periods. The budget constraint has slope equal to the negative of the gross international interest rate, $-1/q$, and becomes vertical at the borrowing constraint. If the country's debts are forgiven (so that $a_1^B = a_2^B = 0$) the endowment point moves to $E^B$ and the budget set moves accordingly. As drawn the country is still borrowing constrained after debt forgiveness and consumes at point $B$.

There are a number of different approaches to the measurement of welfare changes in the presence of quantity constraints or other nonlinearities in budget sets. One approach, which originated in response to war time rationing, follows Rothbarth (1941) and postulates a set of “virtual prices” and incomes at which the country would choose to consume its rationed quantities in the absence of a borrowing constraint. In our example, these virtual prices are given by what we called the country price $p^A$, which also determines the slope $(-1/p^A)$ of the virtual budget constraint through $A$ that is tangent to the initial indifference curve corresponding to utility level $V(a_1^A, a_2^A)$.

The change in welfare from debt forgiveness could then be calculated as an equivalent variation; specifically, we can measure the amount of virtual income necessary to leave the country as well off as if the country had been granted debt forgiveness. In Figure 1, this amount, in period 1 consumption units, is represented by the horizontal difference between the initial virtual budget constraint through $A$ and the virtual budget constraint that is tangent to the indifference curve corresponding to utility level $V(a_1^B, a_2^B)$. This calculation, however, requires knowledge of a country’s preferences. However, without any information on preferences, an upper bound on this equivalent variation can be found by measuring the horizontal difference between the virtual budget constraint through $A$ and the one through $B$. It is this country valuation of debt forgiveness that we derived above as an upper bound to the welfare benefit of debt forgiveness.

It is worth stressing a number of points about this measure. First, it is an upper bound
on the change in welfare. Thus, a negative value for this measure implies that welfare has certainly declined. However, a positive value need not indicate that welfare has increased. In our empirical work below, we measure the welfare consequences of pure debt forgiveness in which we know that welfare would increase, and our aim is only to quantify the size of this change.

This of course begs a second question: are these bounds are likely to be reasonably tight? There are two points in the analysis at which approximations are used that prevent the country value of debt relief being an exact indicator of welfare. The first is the initial linear approximation to the change in welfare. The approximation here is likely to be good, and hence the upper bound is likely to be reasonably tight, when debt relief is small in scale. We argue that this is likely to be true for many developing countries where the stock of debt represents only a modest fraction of the flow of one years income. The second approximation arises when we approximate the change in the country’s quantity of borrowing induced by the debt relief. This approximation is likely to be good whenever the country is likely to remain borrowing constrained following debt relief, which we argue is also plausible.

Third, the interpretation of the welfare results we produce is subject to the same criticisms that have been leveled against the practice of debt relief in the past. The first is in practice the resources obtained from debt relief may not be distributed equitably within society; this is something we ignore in our representative agent framework. The second is that we ignore some incentive problems associated with debt relief. One example concerns dynamic consistency: in order to encourage appropriate future borrowing, the grant of debt forgiveness should be a one-off measure, but this may not be credible. As another example, if the decision maker for the country is not benevolent and can take many actions that are detrimental to the population, the conditioning of debt relief on these actions may produce greater welfare gains that unconditional debt relief. Finally, it is important to note that our model does allow for one common criticism of debt relief calculations: that they do not take into account the possibility that the country may return to high indebtedness levels by borrowing in the future.

Fourth and finally, the calculation is robust to a number of other changes in the environment. In the appendix, we show that this results can be extended to an environment
with uncertainty and many periods, to debts denominated in nominal terms and/or in a foreign currency, as well as the presence of savings constraints as well as borrowing constraints. The calculation is, however, sensitive to the precise form of the borrowing constraint or other market imperfection that prevents the country from equating its marginal rate of substitution to the international interest rate. The following examples, which cover cases of default risk as well as borrowing constraints, illustrate this issue and show that a closely related calculation suffices to bound welfare changes in these cases.

2.B Example 2: A Constraint on Total Borrowing

In the first example, the borrowing constraint affected only the new borrowing $a_2$ and was not directly affected by debt relief. As a consequence, the resulting welfare gain from debt relief can be thought of as a pure resource contribution of debt relief. In general, however, borrowing constraints may adjust to the debt relief which creates additional welfare gains and losses. In this example, we consider a case in which the total stock of debt is constrained, and hence where debt forgiveness results in a one-for-one loosening of borrowing constraints.
Specifically, consider an economy identical to the one in example 1 but for the change in the borrowing constraint. The value function for the country satisfies

\[ V(a_1^*, a_2^*) = \max_{c_1, c_2, a_2} u(c_1) + \beta u(c_2), \]

subject to the flow budget constraints

\[ c_1 + qa_2 \leq y_1 + a_1^*, \]
\[ c_2 \leq y_2 + a_2^* + a_2, \]

and the new borrowing constraint

\[ a_2^* + a_2 \geq a_2. \]

The same process as in example 1 gives rise to the same equation (1). The difference occurs when we approximate the change in new borrowing \( a_2^B - a_2^A \) using the borrowing constraint. With this change, we now obtain an upper bound for the change in welfare of

\[ \frac{V(a_1^B, a_2^B) - V(a_1^A, a_2^A)}{u'(c_1^A)} \leq (a_1^B - a_1^A) + q(a_2^B - a_2^A). \]

That is, in this two period example, the change in welfare is bounded above by the market value of the debt relief. This result does not, however, generalize to more than two periods: in general, the change in welfare is once again bounded above by the country value of the extra resources that can be obtained in each period as a result of debt relief. Whereas in the first example debt relief did not affect borrowing constraints and the extra resources available in period 1 were equal to the debt relief in period 1, here the agent receives the extra resources in period 1 and can borrow more today against the extra resources available in period 2 at rate \( q \). As this total amount is all available in the first period, it is not discounted by country prices when computing the period 1 consumption value of the change in welfare.

More intuition for this result can be obtained from looking at Figure 2. In this case, although the endowment point moves by \((a_1^B - a_1^A, a_2^B - a_2^A)\) from point \( E^A \) to \( E^B \) as
before, the kink in the budget set moves by \((a_1^* - a_1^A) + q (a_2^* - a_2^A), 0)\) from point \(A\) to point \(D\) as the country is now able to consume more today both because of extra current period resources and because they can borrow more against the extra resources now available tomorrow. It is the country value of this shift in the kink of the budget set that constitutes our upper bound for the welfare gains from debt relief.

In the appendix, we show how to derive analogous results for cases in which the borrowing constraint responds linearly, but not one-for-one, with debt relief. We also show how similar results can be derived for non-linear borrowing and savings constraints of the form

\[ g(a_2^*) \geq a_2 \geq f(a_2^*), \]

for some \(g\) concave and some \(f\) convex.

The next two examples establish similar results when the non-linearity of the budget set arises from the risk of sovereign default.
2.C Example 3: Default Risk on New Borrowing

In the above examples, the borrowing constraints were imposed exogenously. In this example, we show how to derive similar results in models where limitations on access to international capital markets are determined endogenously by the risk of sovereign default. Two technical complications arise. First, the value function in models of sovereign debt is not always differentiable as a result of the discrete choice to default or repay. We consider a variant on a standard sovereign default model in which the only penalty for default is a stochastic loss of output. This formulation also generates a smooth value function which is necessary when we take a first order approximation to the value function. Second, the value function may not be concave, and hence it is not always possible to show that our measure is an upper bound on welfare changes, although it remains locally valid. To emphasize this, we examine differential changes in initial assets or debts, instead of discrete changes.

Consider a world that last for two periods. We solve the model recursively. The country enters the second period with assets or debts contracted at the beginning of time $a_2^*$ as well as new loans contracted in period 1 $a_2$. If the country defaults, they pay a resource cost $D$, a random variable that is realized at the start of the second period. For simplicity, all other aspects of the model are deterministic (this is relaxed in the appendix). Hence, in period two if the country does not default on $b_2$ it receives

$$u(y_2 + a_2^* + a_2),$$

while if it does default it receives

$$u(y_2 + a_2^* - D).$$

Note that the country can only default on new debt, and so forgiveness of debts coming due in period 2 will not affect the price of borrowing in period 1. Hence, this model is closest to example 1 studied above. We interpret these debts as owed to official creditors who have greater powers to enforce repayment.

Obviously the country will default iff $D < -a_2$ which occurs with probability $\pi(a_2)$. 
Risk neutral competitive intermediaries will therefore lend to the country at bond price

\[ q(a_2) = \frac{1 - \pi(a_2)}{1 + r}, \]

where \( r \) is the international interest rate.

The value to the country from entering period 2 with these assets/debts, and prior to the resolution of uncertainty, is given by

\[ W(a^*_2, a_2) = \pi(a_2) E\left[u(y_2 + a^*_2 - D) | D < -a_2\right] + (1 - \pi(a_2)) u(y_2 + a^*_2 + a_2), \]

which has

\[ \frac{\partial W}{\partial a^*_2} = \pi(a_2) E\left[u'(y_2 + a^*_2 - D) | D < -a_2\right] + (1 - \pi(a_2)) u'(y_2 + a^*_2 + a_2) \]

\[ = E\left[u'(c_2)\right]. \]

The first period problem of the country is

\[ V(a^*_1, a^*_2) = \max_{c, a_2} u(c) + \beta W(a^*_2, a_2), \]

subject to

\[ c + q(a_2) a_2 \leq y_1 + a^*_1, \]

the first order conditions of which are

\[ u'(c) = \lambda \]

\[ \beta \frac{\partial W}{\partial a_2} = \lambda \left[q(a_2) + a_2 \frac{\partial q}{\partial a_2}\right]. \]

The change in welfare is

\[ dV = u'(c) dc + \beta \left[ \frac{\partial W}{\partial a^*_2} da^*_2 + \frac{\partial W}{\partial a_2} da_2 \right], \]
which after substituting for the FOCs and rearranging becomes
\[
\frac{dV}{\lambda} = dc + \left[ q(a_2) + a_2 \frac{\partial q}{\partial a_2} \right] da_2 + \beta \frac{1}{\lambda} \frac{\partial W}{\partial a_2^*} da_2^*.
\]

Differentiating the budget constraint and substituting we get
\[
\frac{dV}{\lambda} = da_1^* + \beta \frac{1}{\lambda} \frac{\partial W}{\partial a_2^*} da_2^*.
\]

Substituting for the derivative of \( W \) we then get
\[
\frac{dV}{\lambda} = da_1^* + \frac{E \left[ u'(c_2) \right]}{\lambda \cdot u'(c)} da_2^*
\]
\[
= da_1^* + pda_2^*;
\]

that is, our internal valuation of debt forgiveness continues to be a useful indicator of the welfare effects of debt forgiveness when borrowing is constrained by the risk of sovereign default.

If the country can default on both initial debts \( a_2^* \) and newly issued debts \( a_2 \) in period 2, the analysis mimics that of example 2 with
\[
\frac{dV}{\lambda} = da_1^* + qda_2^*.
\]

## 3 Data Sources

To implement our measures above, we need data on three objects. First, we need data on the expected future cashflows associated with the outstanding stock of sovereign debt at the point in time that we want to evaluate the benefits of future debt relief. Second, we need data on the market price or discount rates associated with these debts. Third, we need data to estimate the marginal country valuation of debt relief. We describe our data sources in turn.
3.A Debt Stocks and Flows

The primary source of statistics on external sovereign debt is the World Bank’s *Global Development Finance* (GDF) publication and which are derived from the World Bank’s *Debtor Reporting System* (DRS). The DRS has been in existence since 1951 and records detailed information at the level of an individual loan for external borrowing. All countries that receive a World Bank loan consent in the loan or credit agreement to provide information on their external debt. The details of the reporting procedures are described in World Bank (2000).

One of the purposes of the DRS database is to generate projections of future debt service obligations of a country under various assumptions. Towards this end, the DRS records the years-to-maturity, interest rate, currency of denomination, and grace period of each debt contract at each point in time. Such detailed data are only collected for long term debts (debts with a maturity at issue in excess of one year), and therefore all the results below correspond to such long term debt. Based on these data, and combined with forecasts for the paths of future interest rates (for floating rate debt) and exchange rates, it can be used to generate projections of debt service denominated in US dollars. We restrict attention to sovereign debts that are either owed by the public sector of the country, or are owed by private sector borrowers but are guaranteed by the public sector of the country (public and publicly guaranteed).

Data on individual loans is confidential and direct access to the DRS is restricted. The data reported below is derived from an unpublished dataset constructed by World Bank staff at our request. The World Bank ensured the confidentiality of the loan level data by aggregating data across multiple loans. To preserve comparability with existing publicly available World Bank external debt statistics, we use the same interest rate and exchange rate assumptions that were used in compiling the GDF.

Our data on cashflows begin in 1980 and end in 2007, and for each year we generate

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2 Statistics on external debt are also available from the Joint External Debt Hub (JEDH), which is jointly maintained by the Bank for International Settlements (BIS), the International Monetary Fund (IMF), the Organization for Economic Cooperation and Development (OECD), and the World Bank (WB), and combines data from the DRS with data from creditor and market sources.
projected cashflows over a forty year time horizon. To preserve comparability with GDF data, we denote the sum of cashflows from year \( t \) onwards as the stock of debt as of the \textit{end of year} \( t - 1 \). Hence, our data series begin in 1979 and end in 2006.

### 3.B Market Discount Rates

Ideally, we would like to infer discount rates from market prices of debts (or equivalently, simply calculate the market value of a portfolio of debts). In practice, there are very few developing countries for which liquid debt markets exist. Even for countries where liquid debt markets exist, prices only begin to become available in the 1990s, and many debts owed by the country – including official debts, most bank loans, and project credits – are not traded at all. In the absence of market prices, there is considerable debate as to the appropriate discount rate to use when calculating indebtedness. Although one could in principal use a different discount rate at each date for each different debt issued by different countries in different currencies and coming due at different maturities, most official organizations and researchers discount at a constant rate. For example, the International Monetary Fund (2004) used a 7.5% discount rate in its analysis of debt sustainability, the Development Assistance Committee (DAC) uses a constant 10% market rate (Dikhanov 2006), while Easterly (2001, 2002) relies on the (constant) time series average of LIBOR rates. Likewise, when valuing settlement offers in the context of a debt restructuring, Depetris and Kraay (2005) use a 7.25% rate while in a survey Andritzky (2006) finds that the most frequently applied approach is a constant discount factor around 10%. The World Bank (various) is an exception in using discount rates that vary over time with maturity and currency of issue (although not across countries).

In this paper we experiment with a number of different methods for calculating market discount rates and compare them with the alternative of a constant discount rate. Our benchmark discount rates are derived from data on yields for high yield US Corporations. For sovereign countries that are rated, we use the Standard and Poors concordance to map a sovereign rating into a corporate rating. For unrated sovereigns, we assign a rating based on the ratings produced by Institutional Investor. For countries without an institutional investor
rating, we infer a rating from countries in the same region that are comparable in per capita income, level of indebtedness, and nominal GDP. Once we have ratings for each country at each point in time, we then discount using the appropriate corporate yield. The appendix provides more detail on the process for assigning ratings to countries.

Data on corporate bond yields come from a number of sources. For the period 1979-1987 we rely on Altman (1989) for data on average corporate yields by rating as well as reference US treasury yields. We calculate the spread between the US treasuries and rating specific corporate yields and apply that spread to our 10-yr year-end US Treasury yields. There was no liquid market for high yield corporates during this period so we must acknowledge that Altman is making a number of strong assumptions to get these estimates. For the period 1988-2007 we use proprietary index data from Barclays derived from Point(TM). This data consists of monthly corporate bond issuance weighted average yields by credit rating and maturity. In this exercise we use the 10-yr yield as our reference since the liquidity in these markets was limited early on. For both sets of data, to calculate the entire yield curve we applied the spread observed at the 10-yr point over US Treasuries in both levels and percentage to the entire US treasury curve that we had interpolated using a non-parametric cubic spline.

For comparison purposes, we also produce estimates of the market discount rate using the yield curve for US Treasuries, as well as discount rates that vary over time but not by country, over country and not by time, and constant discounting.

3.C Country Valuations

The model we use for the estimation of the country valuations of their debt stock, is one in which the country compares today’s consumption with tomorrow’s expected future consumption. Under standard assumptions for preferences, a consumer will tend to prefer saving today if it sees less consumption in the future than it has today or in case it sees it has more consumption in the future than it has today it will like to borrow. This model is known as the Consumer Capital Asset Pricing Model (CCAPM). When consumer preferences are of the CRRA type, then, the computation of the pricing kernel implied by this model requires
information on future real consumption in local currency units, price level, exchange rate and population.

The data on these four variables comes from various sources because it was not possible to find all the information in one single source. As the main source we used the World Development Indicators database from the World Bank: real consumption in local currency units is measured by the variable "Final consumption expenditure, etc."; price level is measured by the consumer price index; exchange rate is measured by the variable "Official exchange rate (LCU per USD)."; and population is measured by the variable "Population, total". Because we are interested in estimating the future path of a transformation of these variables, we need information on these for all countries in our sample for a time period that is longer than the one of our sample on debt stocks and flows. In particular, for estimation purposes, we need this data for the period 1960 to 2006 - at any given year we require to have 20 years of past data. Unfortunately, this information is not readily available in one single dataset and, in some cases, it does not exist at all. In order to compile as much of this information as possible we also used other alternative sources like the World Penn Tables dataset, the International Financial Statistics (IFS) database from the International Monetary Fund (IMF), various IMF country reports, etc. For some countries it was not possible to obtain all this data and for this reason we did not include such countries in our analysis. In Appendix X we describe how the implementation of the CCAPM valuation model was conducted.

The relative lack of consumption data for many countries means that we are only able to compute this measure for 72 countries (as compared to market value estimates for 100 countries for the entire sample period, and many more for subperiods).

4 Results: Efficiency

The theoretical results discussed above indicate that we can infer the existence of possible efficiency improving debt relief from differences in the country value and market value of a given stock of sovereign debt. We begin by discussing the market value of sovereign debt before turning to a comparison between market and country values.
4.A The Market Value of Sovereign Debt

Using our data from the Debtor Reporting System, we can compute a number of different measures of debt stocks. We are able to do this for 76 countries in our sample. To give a sense of the scale of sovereign debt, at the end of 2006 the total face value of the outstanding and disbursed sovereign debt for our set of countries was just under 1 trillion US dollars (951 billion, to be precise). This measure combines the face value of very different types of debt, including both loans issued at par and bonds issued at a discount, for which face values can be very different even when contracted cash flows are identical. An alternative “apples-to-apples” comparison treats the entire stock of a country’s debt as though it was made up of a series of zero-coupon bonds. This measure, referred to by Dias, Richmond and Wright (2011) as the zero-coupon equivalent face value of a stock of debts, is necessarily larger than the face value of these debts, and stood at 1.4 trillion US dollars at the end of 2006.
Measures of the discounted stock of developing country debt are naturally smaller. As described above, our benchmark estimates use discount rates derived from corporate bond yields that are time, country, and maturity varying; that is, for each country at each point in time, we construct a yield curve for the country’s debt. Discounting in this way yields a market value of debt of 827 billion US dollars at the end of 2006. If the debt of multilateral institutions is viewed as risk free, and are discounted at the US Treasury rate, this total rises to 863 billion. Suppressing the variation in discount rates by maturity results in very little difference in estimated market values (the stock rises from 827 billion to 830 billion at the end of 2006). Treating all developing country sovereign debt as risk free, and hence discounting using US treasuries produces a market value of 950 billion, while discounting at the commonly used 10 per-cent rate yields a market value of 700 billion.

Variation in these estimates of market values over time is plotted in Figure 3 where debt stocks are scaled by the level of gross national income (GNI) of these countries. At the start of the period, when US interest rates were high, the estimates computed using the conventional constant 10% discount rate are largest. In all other periods, the estimates computed assuming all debt is risk free are larger, with the difference on the order of 5% of GNI. All of the other series are quite similar. All series reach a peak in 1987 at the height of the 1980s debt crisis, and decline to levels around 10% of GNI by the end of the sample.

4.B The Country Value of Sovereign Debt

As noted above, we focus for the time being on univariate forecasts of consumption growth in computing country discount rates. In future drafts of this paper we will introduce alternative forecasting methods making use of data on, among other things, income and interest rates. For most countries in our sample, consumption is close to a random walk, and so we focus on forecasts made under this assumption.

Figure 4 plots our two measures of the country value of debt: one corresponding to the assumption that debt forgiveness does not relax borrowing constraints (the borrowing constraint on the “flow” of new debt measure), and the second under the assumption that debt relief in one period relaxes the borrowing constraint in the previous period by the market
value of that debt (the borrowing constraint on the “stock” of debt measure). For comparison, we also plots our benchmark estimate of the market value of debt, which is also our estimate of the cost of providing debt relief. As can be seen in the Figure, the stock and flow measures are sufficiently close together that they are often indistinguishable. Both lie below the market value of debt, which was expected given that many countries in our sample would appear to be borrowing constrained. The differences between the two series was largest in the middle of the 1980s debt crisis, when many developing countries were in recession and hence valued future resources very little relative to present resources. The two lines grow close together at the turn of the millennium when the average developing country was the beneficiary of prosperous times.

There is, of course, a great deal of variation across countries. The next four Figures plot the evolution of our measures for four sample countries. In the first example we have Mexico which displays the pattern of a country that is, on average borrowing constrained in that its country values of debt relief lie below the estimated market values. Somewhat surprisingly, the stock based measure (which includes the added benefit of the effect of debt relief on welfare) lies below the flow based measure, which is possible if Mexico is savings constrained in some periods. In the second example – China – we see that country valuations
lie far below market valuations of China’s (modest) stock of foreign sovereign debt. This is as a result of high observed growth rates of consumption which is consistent with a strong demand for more borrowing and a relatively low margin value associated with future resources.

The next two examples are cases in which country values exceed market values for at least part of the period under study. In both cases – Burkina Faso and the Central African Republic – the countries had significant periods of consumption decline. The model interprets this as a country that is unable to save to smooth consumption, and hence places a very high value on future resources and hence a high value of debt relief. The Central African Republic is also an example when the stock measure lies significantly below the flow measure, which is to be expected for a savings constrained economy.
5 Results: Redistribution and Efficiency

5.A Methods

To estimate the effect of the redistribution of resources across countries on social welfare, it is necessary to specify how social welfare depends on the level of welfare in each individual country. There is a large literature devoted to discussions of how this should be done, which we lean on below.

We begin by assuming that there are \( n = 1, \ldots, N \) countries each with welfare levels \( V_n \). As above, we think of these \( V_n \) as the sum over all members of society of the welfare derived by each member of society. We then define a Bergson-Samuelson social welfare function of the form

\[
W = W \left( V_1, V_2, \ldots, V_N \right).
\]

(3)

We can differentiate to find that the change in social welfare is given by

\[
dW = \sum_{n=1}^{N} \frac{\partial W}{\partial V_n} dV_n
\]

\[
= \sum_{n=1}^{N} \frac{\partial W}{\partial V_n} \lambda_n \frac{dV_n}{\lambda_n},
\]

where \( \lambda_n \) is the marginal utility of a unit of period zero consumption per capita in country \( n \), and in our application \( dV_n/\lambda_n \) is our measure of the welfare change from the previous analysis of the value of debt forgiveness.

To estimate the change in social welfare, we need to place values on the social welfare weights defined as

\[
\frac{\partial W}{\partial V_n} \lambda_n.
\]

Different researchers have approached this in different ways. One approach, associated with Harberger (1971) and Parish (1976), assumes that the welfare weight is one for all agents. That is, “a dollar is a dollar” when it comes to adding up welfare changes. This corresponds
to our measures of the efficiency improvement associated with debt forgiveness which, as noted above, are typically negative for borrowing constrained countries.

As an alternative, we will focus on a weighted utilitarian social welfare function where the weights $\partial W/\partial V_n$ are constant, but may vary across countries. As a reference case we will report results for the case when all weights are equal, which we can then normalize to one. In such a case, the only variation across countries comes from the fact that consumption per capita levels are not equalized, so that redistribution towards poorer countries will tend to increase social welfare as these countries have a higher marginal utility of consumption.

The term $dW$ above represents a change in social welfare denominated in units of social welfare. We convert this into monetary units under the assumption that the creditor losses from debt forgiveness are distributed amongst all of the other countries in the world – the “rest of the world” or ROW – in proportion to their income levels, so that the relative weights in this welfare calculation are given by

$$\frac{\lambda_n}{\lambda_{ROW}} = \frac{u'(\frac{c_{n}}{N_n})}{u'(\frac{c_{ROW}}{N_{ROW}})} \equiv \theta_n.$$ 

If we also normalize the welfare weight for the rest of the world to 1, and denote the welfare weight of other countries by $\phi_n$, then our measure of the social welfare benefits of debt relief, in units of rest of the world consumption, are given by

$$\frac{dW}{\lambda_{ROW}} = \sum_{n=1}^{N} \phi_n \theta_n \frac{dV_n}{\lambda_n}$$

In what follows we first report results for the case in which $\phi_n = 1$ for all $n$ and hence redistribution contributes to welfare only through the $\theta_n$. As we will see, the observed heterogeneity in consumption per capita levels around the world imply that the welfare benefits of redistribution are vast. As an alternative, we will posit that the country weights increase with relative consumption (so that poorer countries have lower weights) by setting

$$\phi_n = \left( \frac{\frac{c_{n}}{N_n}}{\frac{c_{ROW}}{N_{ROW}}} \right)^{\alpha},$$
for some $\alpha > 0$ and compute that level $\alpha^*$ that sets the total social welfare benefit of debt forgiveness to zero. That is, we compute how much welfare weights would have to fall for low income countries in order to make debt forgiveness welfare neutral. The reader is then invited to assess whether or debt forgiveness is desirable based on their own assessment of whether welfare weights decline faster than $\alpha$ (in which case global debtor forgiveness is welfare reducing) or more slowly than $\alpha^*$ (in which case global debt forgiveness is social welfare increasing). We also present results for individual countries.

5.B Results

We consider a program of total debt forgiveness as of end 2006 for all 72 countries for which we have complete data (the complete list of countries is contained in the appendix). The total market value of the debt of these countries, using our benchmark measure, was 817 billion US dollars. By contrast, our preferred estimate of the welfare gains of debt forgiveness, using the stock measure which allows for debt forgiveness to relax borrowing constraints, was 551 billion US dollars, implying that a broad based debt forgiveness program would lead to an efficiency reduction of 266 billion US dollars.

To examine the extent to which this is offset by the potential benefits from income redistribution, we begin by considering the pure utilitarian social welfare function in which the welfare of an individual in any one country contributes as much to social welfare as the welfare of an individual in any other country. Consistent with our construction of country values above, we assume that preferences take a isoelastic form and calibrate the intertemporal elasticity of substitution to $1/2$ (a coefficient of relative risk aversion of 2). This number is entirely standard, and suggests a relatively low degree of curvature in individuals preferences. As a consequence, the resulting welfare estimates should be conservative. Alternative calibrations, with lower intertemporal elasticities of substitution, or non-homothetic preferences (in particular, including a subsistence level of consumption which would, for poor countries, decrease their intertemporal elasticity of substitution beyond that for rich countries) would give larger numbers.

Under this pure utilitarian social welfare function, the welfare benefits of this debt
forgiveness program are enormous: at a cost of 817 billion dollars, debt forgiveness would result in a social welfare increase of 7.3 quadrillion dollars (that is, 7.3 million billion dollars). This should not be surprising; our sample includes some of the poorest countries in the world where consumption per-capita is on the order of 1% of the levels observed in the rich world. Given our assumption of a coefficient of relative risk aversion of 2, the pure utilitarian social welfare function weights the dollar contribution of debt forgiveness to welfare of these countries 10,000 times more highly than the welfare costs of providing the debt relief by rich countries.

This value for debt relief in aggregate is so large that it is difficult to comprehend. As an alternative metric on the size of the potential gains from debt relief, we ask the alternative question: how much less must the welfare of individuals living in developing countries be valued by society in order to balance the costs and benefits of debt relief? Towards an answer to this question, and as described above, we postulate that welfare weights increase with the prosperity of a country with a constant elasticity given by the parameter $\alpha$ introduced above, and solve for the level of $\alpha$ that makes debt relief welfare neutral. We find that this critical level $\alpha^* = 1.87$.

To put this number in perspective, it means that the welfare weight of a person living in a country with half the per-capita consumption of the rich world (or in our case, the average for the rest of the world) would be 27% of the welfare of a person in a rich country; with 20% the per-capita consumption, a 5% welfare weight; 10% per-capita consumption, a 1.4% welfare weight; and, for a level of per-capita consumption 1% of the level in the rest of the world, a 0.02% welfare weight. We invite the reader to determine for themselves whether this is a reasonable set of relative welfare weights. Netting out the effect of these declining weights with the high weight produced in the pure utilitarian case, the welfare of a country with 50% the per-capita consumption of the rich world would be 9.4% higher than for a rich country; 35% higher for a country with 10% the per-capita consumption level; and, 83% higher for a country with 1% the per-capita consumption level.

Even allowing for welfare weights to decline in this way, there are considerable differences in the net contributions of individual countries debt forgiveness to world welfare. Using these weights, debt forgiveness for middle income countries like Argentina, Brazil and Mexico
typically reduces social welfare (relative to the cost of providing this debt forgiveness, the return in terms of welfare would be -25%, -6% and -29% respectively). Conversely, the net contribution of debt forgiveness in the low income countries of sub-Saharan Africa to social welfare are typically 3 to 4 times larger than their cost.

6 Conclusions

In conclusion, we would like to stress a number of limitations to our analysis. First, we have focused entirely on representative agent models of international borrowing. As a consequence, we have nothing to say about the impact of inequalities within a country on the impact of debt relief, nor on the likelihood that the gains from debt forgiveness would be distributed equitably within society. Conceivably, if income is distributed inequitably and poorer citizens are more likely to receive the benefits of debt forgiveness (perhaps through more social programs financed by the country’s government) the gains from debt relief would be even larger than the ones we have estimated.

Second, and related to the first point, we have assumed that nations act benevolently with respect to their citizens. In practice, as has often been stressed by opponents of debt relief and forgiveness in the past, the proceeds from debt relief may be diverted to socially wasteful, or even socially deleterious, purposes.

Third, we have ignored the effect of debt forgiveness on the incentives of countries to both implement superior economic policies, as well as to borrow and repay appropriately in the future. On the former, it is conceivable that conditioning the availability of debt relief on government actions could produce outcomes that are superior to those offered by the sort of unconditional debt forgiveness program envisaged in this paper. On the latter, debt forgiveness today may be subject to a time inconsistency problem. If developing countries perceive that the probability of future debt forgiveness is high, they may be encouraged to borrow in greater amounts and default more frequently. These costs have not been included in our analysis, which implicitly assumes that the promise never to grant debt forgiveness in the future is credible.
References


