Contingent Convertible Bonds and Capital Structure Decisions*

Boris Albul[†]

Dwight M. Jaffee[‡]

Alexei Tchisty[§]

November 18, 2012

Abstract

This paper provides a formal model of contingent convertible bonds (CCBs), a new instrument offering potential value as a component of corporate capital structures for all types of firms, as well as being considered for the reform of prudential bank regulation following the recent financial crisis. CCBs are debt instruments that automatically convert to equity if and when the issuing firm or bank reaches a specified level of financial distress. CCBs have the potential to avoid bank bailouts of the type that occurred during the subprime mortgage crisis when banks could not raise sufficient new capital and bank regulators feared the consequences if systemically important banks failed. While qualitative discussions of CCBs are available in the literature, this is the first paper to develop a formal model of their properties. The paper provides analytic propositions concerning CCB attributes and develops implications for structuring CCBs to maximize their general benefits for corporations and their specific benefits for prudential bank regulation.

Keywords: Contingent Convertible Bond, Banking Regulation, Subprime Mortgage Crisis, Structural Model, Corporate Finance

1 Introduction

This paper provides a formal model of contingent convertible bonds (CCBs), a new instrument offering potential value as a component of corporate capital structures for all types of firms, as well as being considered for the reform of prudential bank regulation following the recent financial crisis. CCBs are debt instruments that automatically convert to equity if and when the issuing firm or bank reaches a specified level of financial distress. While qualitative discussions of CCBs are available in the literature, this is the first paper to

*We thank Anat Admati, Robert Anderson, Mark Flannery and Larry Wall as well as seminar participants at the University of California at Berkeley and the Federal Reserve Banks of Atlanta, New York, and San Francisco for their helpful comments.

[†]Federal Reserve Bank of Boston, boris.albul@bos.frb.org

[‡]Haas School of Business, University of California at Berkeley, jaffee@haas.berkeley.edu

[§]Haas School of Business, University of California at Berkeley, tchisty@haas.berkeley.edu

develop a complete and formal model of their properties¹. The paper provides analytic propositions concerning CCB attributes and develops implications for structuring CCBs to maximize their general benefits for corporations and their specific benefits for prudential bank regulation.

CCBs are receiving attention as a new instrument for prudential banking regulation because they have the potential to avoid the bank bailouts that occurred during the subprime mortgage crisis when banks could not raise sufficient new capital and bank regulators feared the consequences if systemically important banks failed². A more standard proposal for bank regulatory reform is to raise capital requirements since, if set high enough, they can achieve any desired level of bank safety. Very high capital ratios, however, impose significant costs on banks and thereby inhibit financial intermediation; and/or the capital requirements will be circumvented through regulatory arbitrage³. There have also been proposals to focus a component of the capital requirements on systemic risk (Adrian and Brunnermeier (2009)), or to prohibit banks outright from risky activities that are not fundamental to their role as financial intermediation (Volcker (2010)). While these proposals could well improve prudential bank regulation, they do not directly address the issue of how distressed banks can raise new capital in order to preclude the need for government bailouts.

In this setting, CCBs have been proposed by academics (Flannery (2002, 2009a, 2009b), Duffie (2009), Squam Lake Working Group on Financial Regulation (2009), and McDonald (2010)) and endorsed for further study by bank regulators (Bernanke (2009), Dudley (2009), and Flaherty (2010))⁴. Furthermore, both the House and Senate 2010 financial reform bills require studies of CCBs for regulatory applications

¹Raviv (2004) analyzes CCBs using a contingent claims approach, while Pennacchi (2010) analyzes CCBs using a jump process to create default. Neither paper includes tax shield benefits and Pennacchi (2010) has no bankruptcy costs. In contrast, we adapt a Leland (1994) model that integrates the tax shield benefit and bankruptcy cost effects of debt. This allows us to analyze the impact of various CCB contract terms for a range of issues including regulatory benefits, tax shield costs, and incentives for risk shifting and equity price manipulation.

²The bank bailouts during the subprime crisis reflect a failure of the regulatory principles created under the 1991 Federal Deposit Insurance Corporation Improvement Act (FDICIA) and specifically of its requirement that bank regulators take "prompt corrective action" (PCA) in response to declining bank capital ratios. Eisenbeis and Wall (2002) provide a detailed discussion of FDICIA and its PCA requirements. As examples of the PCA requirements, "significantly undercapitalized" banks are to raise new equity or promptly merge into a well capitalized bank, and "critically undercapitalized" banks are to be placed under a receiver within 90 days of attaining that status. The subprime crisis also revealed that subordinated debt holders failed to "discipline" the banks, while the government bank bailouts protected these debt holders from the major losses they would have otherwise faced in a bankruptcy.

³High capital requirements limit the use of debt tax shields, impose a high cost on existing shareholders when raising new capital due to the debt overhang problem, and accentuate important principal-agent inefficiencies within the banks; see Kashyap, Rajan, and Stein (2008), Squam Lake Working Group on Financial Regulation (2009), Dudley (2009), and Flannery (2009) for further discussion of these issues.

⁴There have also been proposals for contingent capital instruments that are not bonds. Kashyap, Rajan, and Stein (2008) propose an insurance contract that provides banks with capital when certain triggering events occur and Zingales and Hart (2009) focus on the use of credit swaps. Wall (2009) provides a survey of this evolving literature.

and provide regulatory approval for their use⁵. The Financial Stability Board of the G20 and the Bank for International Settlement are also studying CCBs. In fact, Lloyd's bank issued the first £7 billion (\$11.6 billion) CCBs (CoCo bonds)⁶ in 2009.

CCBs initially enter a bank's capital structure as debt instruments, thus providing the debt-instrument benefits of a tax shield and a control on principal-agent conflicts between bank management and shareholders. If and when the bank reaches the specified degree of financial distress, however, the debt is automatically converted to equity. The conversion recapitalizes the bank without requiring any ex-post action by banks to raise new equity or the government to bail them out. The automatic recapitalization feature of CCBs thus offers a relatively low-cost mechanism to avoid the costs that otherwise arise with the threatened bankruptcy of systemically important banking firms.

The existing CCB proposals-see especially Flannery (2009a) and McDonald (2010)-provide a list of issues that must be settled in formulating any specific plan for implementation:

- The trigger must be designed to avoid accounting manipulation, and the resulting conversion of CCB to equity must be automatic and inviolable. In fact, an accounting trigger in the Lloyd's bank 2009 CCB issue has already raised serious concern; see Duffie (2009). Most proposals instead recommend a trigger based on a market measure of each bank's solvency⁷. We model the case when equity value is used as a trigger. We also analyze the issue of market manipulation of the equity value that may arise with a market-value trigger.
- The CCB to equity conversion terms applied after the trigger is activated must be specified. A key question is how the value of the equity shares received at conversion compare to the value of the converting bonds; see Flannery (2009a and 2009b) and McDonald (2010). We consider the general case in which the ratio of the equity conversion value to the CCB face value is a contract parameter (λ) to be chosen. Among other effects, we analyze the impact this contract parameter may have on

⁵The House bill is 111th Congress, first session, H.R. 4173. The Senate bill is 111th Congress, second session, S.3217.

⁶Source: Financial Times from November 5, 2009.

⁷The Squam Lake Working Group and McDonald proposals require two triggers to be activated before conversion occurs. One trigger is based on each bank's own financial condition while the second trigger is based on an aggregate measure of banking system distress. This means that individual banks can become insolvent prior to CCB conversion if the aggregate trigger is not activated. For this reason, Flannery (2009a) argues for a single, bank-based, trigger. In this paper, we formally model only this single trigger case.

the incentive to manipulate the market value of the bank's equity shares. In all cases, however, we assume the CCB to equity conversion has no income tax consequences.

- The CCB contract could impose a dynamic sequence in which specified amounts of CCB convert at different thresholds. Flannery (2009a), furthermore, proposes a regulatory requirement whereby converted CCB must be promptly replaced in a bank's capital structure. While we comment on the possible advantages of such dynamic contract features, our formal model covers only the case of a one-time and complete conversion.
- The adoption of CCBs by banks could be voluntary or a required component of their capital requirements⁸. We consider both possibilities.

The key contribution of the current paper is to provide a formal financial model in which the effects of alternative CCB contract provisions can be analytically evaluated. We develop closed form solutions for CCB value by adapting the Leland (1994) model⁹. Our results apply equally well to the addition of CCBs to the capital structure of corporations generally, as well as for their specific application as a tool for prudential bank regulation. We make three assumptions throughout the paper regarding a firm's use of CCBs:

- 1) The firm is allowed a tax deduction on its CCB interest payments as long as the security remains outstanding as a bond. This would be the likely case for banks if CCBs were to become a formal and established component of prudential banking regulation. At the same time, this means that the public cost of the CCB tax shield must be included when evaluating the possible role of CCBs for prudential bank regulation. For corporations more generally, we acknowledge that the tax deductibility of CCB interest payment will likely require further IRS rulings, including possible legal challenges and new legislative actions.

⁸For example, Flannery (2009a) provides an illustrative example in which banks are required to choose between satisfying their capital requirements by (i) holding equity equal to 6% of an asset aggregate or (ii) holding equity equal to 4% of the asset aggregate and CCBs equal to 4% of the asset aggregate. This suggests a regulatory tradeoff in which 4 percentage points of CCBs are the equivalent of 2 percentage points of equity.

⁹The Leland model has been successfully applied in recent studies of other fixed-income debt security innovations, although none analyzes the case of a bond conversion triggered by financial distress. Bhanot and Mello (2006) study corporate debt that includes a rating trigger such that a rating downgrade requires the equity holders to compensate the bondholders with early debt redemption or other benefits. Manso, Strulovici, and Tchistyi (2009) study a class of debt obligations where the required interest payments depend on some measure of the borrower's performance. This could include the extreme case in which the debt interest coupons reach zero at some level of financial distress. This case would provide some of the same benefits in reducing or eliminating bankruptcy costs as provided by CCBs in the current paper.

- 2) In all cases, we assume that adding CCBs to a firm's capital structure has no impact on the level of the firm's asset holdings (A). In other words, we assume the addition of CCBs must take the form of either a CCB for equity swap (with the CCB proceeds paid out as a dividend to equity holders) or as a CCB for straight debt swap (with the CCB proceeds used to retire existing straight debt). The CCB for straight debt swap appears to be the most important case for regulatory applications, while we acknowledge that the future study of assets effects could be important for more general corporate finance applications.
- 3) Our analysis is carried out under a No Prior Default Condition - Condition 1 - that requires the CCB conversion to equity to occur at a time prior to any possible default by the firm on its straight debt. This condition constrains the set of feasible CCB contracts that are considered in our analysis. This constraint is a necessary, and sensible, requirement if CCBs are to have the desired property of reducing the bankruptcy costs associated with a bond default.

The following is a summary of our main questions and results. We first consider a firm that has a new opportunity to include CCBs within its existing capital structure in a setting where no regulatory restrictions are imposed on CCB issuance (with the exception of the contract constraints created by the No Prior Default Condition 1 as just described).

Q1. Will a firm include CCBs in its capital structure if it is freely allowed to do so?

A1. A firm will always gain from including CCB in its capital structure as a result of the tax shield benefit. This is true whether or not the firm also includes straight debt in its optimal capital structure; in fact, the optimal amount of straight debt is unaffected by the addition of CCB. Given that total assets are unchanged by assumption, in effect the CCB are being swapped for, i.e. replacing, equity in the capital structure. Since the asset-value default threshold on any existing straight debt is unchanged, adding CCB in this manner provides no benefits for regulatory safety, while taxpayers pay the cost of the additional tax shield. The addition of CCB in this form may also magnify the firm's incentive for asset substitution (to expand its asset risk).

We thus next consider a firm that operates under the regulatory constraint that it may issue CCB only as a part of a swap that retires an equal amount of straight debt. This constraint is implicit or explicit in

various proposals to use CCB for prudential bank regulation; see Flannery (2009a). The result depends on whether the firm is creating a *de novo* optimal capital structure or is adding CCB to an already existing capital structure.

Q2. Will a firm add CCBs to a *de novo* optimal capital structure, assuming it faces the regulatory constraint that the CCB can only replace a part of what would have been the optimal amount of straight debt?

A2. A bank creating a *de novo* capital structure under the regulatory constraint will always include at least a small amount of CCB in its optimal capital structure. The reduction in expected bankruptcy costs ensures a net gain, even if the tax shield benefits are reduced, at least for small additions of CCB. The addition of CCBs also has the effect of reducing the incentive for asset substitution. The bottom line is that CCBs in this form provide an unambiguous benefit for regulatory safety.

Q3. Will a firm add CCBs to an existing capital structure, assuming it faces the regulatory constraint that the CCB can only be introduced as part of a swap for a part of the outstanding straight debt?

A3. Assuming the initial amount of straight debt equals or exceeds the optimal amount, the existing equity holders will not voluntarily enter into the proposed swap of CCB for straight debt. While the swap may increase the firm's value—the value of reduced bankruptcy costs may exceed any loss of tax shield benefits—the gain accrues only to the holders of the existing straight debt. This is thus a classic debt overhang problem in which the equity holders will not act to enhance the overall firm value. To be clear, this result depends in part on our assumption that the straight debt has the form of a consol with indefinite maturity. If the straight debt has finite maturities, then the CCB could be swapped only for maturing debt, thus reducing the debt overhang cost.

Q4. How can CCBs be designed to provide a useful regulatory instrument for expanding the safety and soundness of banks that are acknowledged to be too big to fail (TBTF)?

A4. We assume a TBTF bank is one for which its straight debt is risk free because the bond holders correctly assume they will be protected from any potential insolvency. We also assume a regulatory limitation on the amount of debt such a bank may issue. Under this limitation, a CCB for straight debt swap reduces the value of the government subsidy because it reduces the expected cost of bondholder bailouts. While this

has a taxpayer benefit, the equity holders of such a bank would not voluntarily participate in such a swap.

Q5. May CCBs create an incentive for market manipulation?

A5. CCB may potentially create an incentive for either the CCB holders or the bank's equity holders to manipulate the bank's stock price to a lower value in order to force a CCB for equity conversion. The incentive for CCB holders to manipulate the equity price exists only if the ratio of equity conversion value to CCB value (λ in the model) is sufficiently high to make the conversion profitable for the CCB holders. The incentive for bank equity holders to manipulate the equity price exists, comparably, only if the ratio of equity conversion value to CCB value (λ) is sufficiently low to make the forced conversion profitable for the equity holders.

Q6. May restrictions on CCB contract and issuance terms be useful in maximizing the regulatory benefits of bank safety?

A6. The regulatory benefits of CCB issuance will generally depend on the CCB contract and issuance terms. Perhaps most importantly, the regulatory benefits vanish if banks simply substitute CCBs for capital, leaving the amount of straight debt unchanged. It is thus essential to require CCB issuance to substitute for straight debt (and not for equity). In addition, the higher the threshold for the conversion trigger the greater the regulatory safety benefits. The conversion ratio of equity for CCBs may also determine the incentive for CCB holders or equity holders to manipulate the stock price.

The structure of the paper is as follows. Part 2 develops the formal model. Part 3 applies the model to determine the role CCBs play in a bank's optimal capital structure. Part 4 analyzes bank issuance of CCBs when regulators require that the CCBs provide a net addition to bank safety. Part 5 applies the model to the role of CCBs when banks are too big to fail (TBTF). Part 6 provides our discussion of market manipulation involving CCBs. Part 7 provides a summary and policy conclusions.

2 Model

We use the capital structure model of Leland (1994) to analyze CCBs. In this model a firm has productive assets that generate after-tax cash flows with the following dynamics

$$\frac{d\delta_t}{\delta_t} = \mu dt + \sigma dB_t^Q, \quad (1)$$

where μ and σ are constant, and B^Q defines a standard Brownian motion under the risk-neutral measure. By assumption, the risk-free rate, r , is such that $\mu < r$, and the tax rate $\theta \in (0, 1)$. At any time t , the market value of assets, A_t is defined as the expected present value of all future cash flows,

$$A_t = \mathbb{E}^Q \left[\int_t^\infty e^{-r(s-t)} \delta_s ds \right] = \frac{\delta_t}{r - \mu}.$$

The dynamics for A_t are: $dA_t = \mu A_t dt + \sigma A_t dB_t^Q$.

The capital structure of the firm includes equity and a straight bond. The bond pays a tax-deductible coupon c_b , continually in time, until default. At default, fraction $\alpha \in [0, 1]$ of assets is lost.

Equity holders operate the firm. At current time t , the liquidation policy of the firm maximizes equity value:

$$W(A_t; c_b) \equiv \sup_{\tau \in \mathcal{T}} \mathbb{E}^Q \left[\int_t^\tau e^{-r(s-t)} (\delta_s - (1 - \theta)c_b) ds \right]$$

where $\mathcal{T}$ is the set of stopping times and $(\delta_s - (1 - \theta)c_b)$ is the after-tax dividend at time s , $t \leq s \leq \tau$. The optimal liquidation time is the first time $\tau(A_B) = \inf\{s : A_s \leq A_B\}$ that the asset level falls to some sufficiently low boundary $A_B > 0$. Leland (1994) shows that, at any time $t < \tau(A_B)$, the optimal default boundary

$$A_B = \beta(1 - \theta)c_b, \quad (2)$$

where $\beta = \frac{\gamma}{r(1+\gamma)}$ and $\gamma = \frac{1}{\sigma^2} \left[\left(\mu + \frac{\sigma^2}{2} \right) - \sqrt{\left(\mu - \frac{\sigma^2}{2} \right)^2 + 2r\sigma^2} \right]$. A_B does not depend on A_t and increases in

c_b . Equity holders liquidate the firm the first time equity value drops to zero.

We allow the firm to add a CCB to a capital structure that includes equity and a straight bond. The CCB pays a tax-deductible coupon c_c , continually in time, until conversion. The bond *fully* converts into equity when asset value falls to a pre-determined level A_C . The time of conversion is denoted by $\tau(A_C) = \inf\{s : A_s \leq A_C\}$. At $\tau(A_C)$, CCB holders receive equity, valued at its market price, in the amount of $(\lambda \frac{c_c}{r})$. Coefficient λ is the CCB contract term that determines the ratio of equity value to the face value of the bond, $\frac{c_c}{r}$. When $\lambda = 1$, equity value equals the face value of the bond. When $\lambda < 1$ ($\lambda > 1$), equity value is at a discount (premium) to the face value.

We require that the following condition always holds.

Condition 1 (No Default Before Conversion): c_b, c_c, λ and A_C are such, that the firm does not default before conversion at any $A_t \geq A_C$.

Before conversion, equity holders' value and their decision to liquidate depend on the characteristics of both bonds, and so does Condition 1. One interpretation of this condition is that the CCB issue is relatively small. The CCB claim on the firm's assets does not affect equity value to the extent that equity holders would want to default before conversion – equity value remains positive. Economically, a small CCB issue can be justified by either potential limits imposed by regulators or the market capacity to absorb this kind of debt.

When Condition 1 is violated and the firm is liquidated before conversion, at each time $s \geq t$ before default, equity holders receive a dividend in the amount of $(\delta_s - (c_b + c_c)(1 - \theta))$. The firm optimally defaults when equity value drops to zero at $\tilde{A}_B = \beta(1 - \theta)(c_b + c_c)$, based on equation (2). The default boundary is the same whether the capital structure of the firm includes a straight bond paying coupon c_b and a CCB paying coupon c_c , or whether it includes only a straight bond paying $(c_b + c_c)$. If Condition 1 is violated, from the point of view of default risk, issuing a CCB becomes redundant.

Lemma 1. *If the capital structure of the firm includes equity, a straight bond and a CCB, the optimal default boundary, A_B , is determined only by the size of the straight debt coupon, c_b , and is given by equation (2).*

Proof. Based on Condition 1, there is no default at conversion. After conversion, the maximum-valuation

problem of equity holders is the same as in Leland (1994), when the capital structure includes only equity and a straight bond paying coupon c_b . Hence, the same A_B . $\square$

We derive closed-form solutions for the values of various claims on the firm's assets prior to conversion.

Proposition 1. *If the capital structure of the firm includes equity, a straight bond and a CCB, then, for any $t \leq \tau(A_C)$,*

(i) *firm value*

$$G(A_t; c_b, c_c) = A_t + \frac{\theta c_b}{r} \left(1 - \left(\frac{A_t}{A_B} \right)^{-\gamma} \right) + \frac{\theta c_c}{r} \left(1 - \left(\frac{A_t}{A_C} \right)^{-\gamma} \right) - \alpha A_B \left(\frac{A_t}{A_B} \right)^{-\gamma}$$

(ii) *equity value*

$$\begin{aligned} W(A_t; c_b, c_c) = & A_t - \frac{c_b(1-\theta)}{r} \left(1 - \left(\frac{A_t}{A_B} \right)^{-\gamma} \right) - \frac{c_c(1-\theta)}{r} \left(1 - \left(\frac{A_t}{A_C} \right)^{-\gamma} \right) - \\ & A_B \left(\frac{A_t}{A_B} \right)^{-\gamma} - \left(\lambda \frac{c_c}{r} \right) \left(\frac{A_t}{A_C} \right)^{-\gamma} \end{aligned}$$

(iii) *straight bond value*

$$U^B(A_t; c_b, c_c) = \frac{c_b}{r} \left(1 - \left(\frac{A_t}{A_B} \right)^{-\gamma} \right) + \left(\frac{A_t}{A_B} \right)^{-\gamma} (1 - \alpha) A_B,$$

(iv) *CCB value*

$$U^C(A_t; c_c) = \frac{c_c}{r} \left(1 - \left(\frac{A_t}{A_C} \right)^{-\gamma} \right) + \left(\frac{A_t}{A_C} \right)^{-\gamma} \left(\lambda \frac{c_c}{r} \right)$$

(v) *tax savings*

$$TB(A_t; c_b, c_c) = \frac{\theta c_b}{r} \left(1 - \left(\frac{A_t}{A_B} \right)^{-\gamma} \right) + \frac{\theta c_c}{r} \left(1 - \left(\frac{A_t}{A_C} \right)^{-\gamma} \right)$$

(vi) *bankruptcy costs*

$$BC(A_t; c_b, c_c) = \alpha A_B \left(\frac{A_t}{A_B} \right)^{-\gamma}$$

It follows from Proposition 1 that the values of straight debt and bankruptcy costs are not affected by the presence of the CCB. Tax savings include

1. savings associated with the straight bond

$$TB^B(A_t; c_b, c_c) = \frac{\theta c_b}{r} \left(1 - \left(\frac{A_t}{A_B} \right)^{-\gamma} \right)$$

2. and the ones associated with the CCB

$$TB^C(A_t; c_b, c_c) = \frac{\theta c_c}{r} \left(1 - \left(\frac{A_t}{A_C} \right)^{-\gamma} \right). \quad (3)$$

Figure 1 plots equity value as a function of asset value. Before conversion, equity value is computed based on the closed-form solution from Proposition 1. After conversion, it is based on the solution that corresponds to the case, when the capital structure of the firm includes only equity and straight debt.¹⁰ It represents the value of new equity holders (the former CCB holders) and the value of old equity holders (the ones that ran the firm solely before conversion). In Subfigure 1-(a), equity value monotonically declines as A_t approaches the conversion boundary from the right-hand side and increases by the amount of $\left(\lambda \frac{c_c}{r} \right) = \9 at A_C . The value of equity is always positive before conversion. Condition 1 holds.

In Subfigure 1-(b), equity value is also positive before conversion, and Condition 1 holds. The key difference from the previous subfigure is that, before conversion, equity value as a function of asset value is non-monotonic. As A_t approaches A_C from the right-hand side, equity value, first, decreases and, then, increases. The intuition is as follows. First, when A_t is high and relatively far from A_C , changes in asset value have a dominant effect on equity value. As asset value declines, so does equity value. Second, as A_t continues to decline and gets closer to A_C , equity holders are facing a higher probability of eliminating the

¹⁰See equation (13) in Leland (1994).

Figure 1: **Equity value as a function of asset value, when Condition 1 holds.**

The lines plot equity value, W , for a range of asset value realizations, A_t , before (*solid line*) and after (*dashed line*) CCB conversion. By assumption, $r = 5\%$, $\mu = 1\%$, $\sigma = 15\%$, $\theta = 35\%$, $\alpha = 30\%$ and $c_b = \$5$. Based on (2), $A_B = \$43.7$.

(a) Monotonicity before conversion. $c_c = \$0.5$, $\lambda = 0.9$, $A_C = \$70$.

(b) Non-monotonicity before conversion. $c_c = \$3.5$, $\lambda = 0.05$, $A_C = \$70$.

Figure 2: **Value of expected payoffs to equity holders as a function of asset value, when Condition 1 is violated.**

The lines plot the expected present value of payoffs to equity holders, W , for a range of asset value realizations, A_t , before and after CCB conversion. By assumption, $r = 5\%$, $\mu = 1\%$, $\sigma = 15\%$, $\theta = 35\%$, $\alpha = 30\%$ and $c_b = \$5$. Based on (2), $A_B = \$43.7$.

(a) **Monotonicity before conversion.** $c_c = \$0.5$, $\lambda = 0.9$, $A_C = \$60$.

(b) **Non-monotonicity before conversion** $c_c = \$3.5$, $\lambda = 0.05$, $A_C = \$60$.

liability to pay coupon c_c . Since $\lambda = 0.1$ is small, the value that equity holders are giving up at conversion, $(\lambda \frac{c_c}{r}) = \3.5 , is relatively low compared to the face value of the bond, $\frac{c_c}{r} = \$70$. The positive effect from potentially not having to make coupon payments to CCB holders dominates the negative effect due to lower asset value realizations and a higher probability of having to give up value at conversion. As asset value approaches A_C , equity value increases.

In Figure 2 Condition 1 is violated. The value of expected payoffs to equity holders based on Proposition 1 becomes negative before conversion and the firm defaults prematurely. In Subfigure 2-(a) the value of expected payoffs monotonically declines in asset value until default and is negative at and immediately prior to A_C . In Subfigure 2-(b) the value is non-monotonic in A_t . It is positive at and immediately before conversion but is negative prior to that.

The issue of non-monotonicity of equity value in asset value becomes important when addressing CCB implementation. Working with asset values is convenient when performing analytical calculations. However, since equity prices are observable while asset values are not, in practice a CCB conversion rule based on equity value is more relevant. When equity price, $W(A_t; c_b, c_c)$, is strictly increasing in A_t before conversion, there is a one-to-one correspondence between equity and assets values. A_C and $W_C = W(A_C; c_b, c_c)$ become interchangeable, and the conversion rule can be rephrased – the CCB converts into equity when equity price drops to W_C . In the future, when necessary, we will explicitly avoid non-monotonicity of equity value in A_t by imposing the following condition.

Condition 2 (Monotonicity of Equity Value): c_b, c_c, A_C and λ are such, that equity value, $W(A_t; c_b, c_c)$, is strictly increasing in asset value, A_t , before conversion at any $A_t \geq A_C$.

Although Condition 2 further limits the parameter space defined by Condition 1, it is not too restrictive for practical purposes. As in the example of Subfigure 1-(b), it is violated when the amount of equity CCB holders receive at conversion is substantially smaller than the face value of the bond. As we argue in Section 6, CCBs with sufficiently low conversion ratios create an incentive for equity holders to expedite conversion by artificially driving the stock price down and, therefore, should be restricted by regulators.

3 Optimal Capital Structure Decisions

The main question we ask in this section is whether the firm would add a CCB to a *de novo* capital structure and whether there are important policy implications associated with CCB issuance.

3.1 *De Novo* Capital Structure with No Constraints

Consider that, at time t , the firm has no debt but plans to leverage up by choosing between two options. Equity holders can either issue an optimal amount of straight debt as in Leland (1994), or they can fix the size of a CCB ex-ante and solve for a new optimal amount of straight debt by maximizing firm value in the presence of the CCB. In the second case, the assumption is that the CCB parameters and the resulting optimal straight debt coupon satisfy Condition 1. Which of the two capital structures would the equity holders prefer?

Proposition 2. *If an unlevered firm chooses to leverage up by issuing a fixed-size CCB and an optimal amount of straight debt, so that Condition 1 does not bind, then, compared to the optimal capital structure that does not allow for CCBs ($c_c = 0$),*

(i) *optimal straight debt coupon is the same*

$$c_b^* = \frac{A_t}{\beta(1-\theta)} \left(\frac{\theta}{r} \right)^{\frac{1}{\gamma}} \left[(\gamma+1) \left(\frac{\theta}{r} + \alpha\beta(1-\theta) \right) \right]^{-\frac{1}{\gamma}}$$

(ii) *firm value is higher by the amount of tax savings associated with c_c*

$$G(A_t; c_b^*, c_c) = G(A_t; c_b^*, 0) + TB^C(A_t; c_b^*, c_c)$$

(iii) *adjusted for tax benefits, equity is crowded out by the CCB one-to-one*

$$W(A_t; c_b^*, c_c) = W(A_t; c_b^*, 0) - U^C(A_t; c_b^*, c_c) + TB^C(A_t; c_b^*, c_c)$$

(iv) *tax savings are higher by the amount of savings associated with coupon c_c*

$$TB(A_t; c_b^*, c_c) = TB(A_t; c_b^*, 0) + TB^C(A_t; c_b^*, c_c)$$

(v) *straight debt value and bankruptcy costs are the same*

$$U^B(A_t; c_b^*, c_c) = U^B(A_t; c_b^*, 0),$$

$$BC(A_t; c_b^*, c_c) = BC(A_t; c_b^*, 0).$$

The intuition behind the above results is as follows. The firm does not default before conversion due to Condition 1. After conversion, equity holders' value-maximization problem is the same as if the CCB was not part of the capital structure. This leads to the same optimal default boundary. Since the CCB does not change the timing of default, equity holders issue the same optimal amount of straight debt as in the case when CCBs are not allowed (item (v)). Straight debt coupon (item (i)) and bankruptcy costs (item (v)) are also the same. Tax benefits (item (iv)) and firm value (item (ii)) increase by the amount of tax savings associated with coupon c_c . Finally, since operational cash flows and the straight debt coupon are the same, making payments to CCB holders reduces equity value. Adjusted for CCB tax savings, equity value decreases by a dollar for each dollar of contingent capital debt (item (iii)).

Equity holders have an incentive to issue contingent capital debt – they increase firm value by taking advantage of additional tax benefits. The new type of debt crowds out equity without reducing the amount of straight debt. Proposition 2 suggests that, if not regulated, CCBs will result in higher leverage, higher tax subsidies and the same the level of default risk.

3.2 *De Novo* Capital Structure with Limits on Total Amount of Debt

We consider a regulatory limit on the amount of debt that the firm is allowed to raise. As before, at time t , the firm is unlevered but can either issue an optimal amount of straight debt only, $U^B(A_t; c_b^*, 0)$, or choose a fixed-size CCB, $U^C(A_t; \bar{c}_b, c_c)$, and a new optimal amount of straight debt, $U^B(A_t; \bar{c}_b, c_c)$, subject to satisfy-

ing Condition 1. The difference from Section 3.1 is that the total amount of debt in the second case can not exceed the amount of straight debt in the first one. Will the firm still chose to issue contingent capital debt?

The limit on the total amount of debt leads to the following formal constraint:

$$U^B(A_t; \bar{c}_b, c_c) + U^C(A_t; \bar{c}_b, c_c) = U^B(A_t; c_b^*, 0). \quad (4)$$

The same amount of contingent capital debt can be issued with different coupons and conversion-triggering asset levels. The firm, for instance, can pick A_C , use the solution for the value of CCB from Proposition 1 and find c_c based on (4):

$$c_c = \frac{(\hat{c}_b - \bar{c}_b) \left(1 - \left(\frac{A_t}{\bar{A}_B}\right)^{-\gamma}\right)}{1 - (1 - \lambda) \left(\frac{A_t}{A_C}\right)^{-\gamma}}$$

The assumption is that $\hat{c}_b$, c_c , A_C and λ satisfy Condition 1.

Proposition 3. *A capital structure that includes an optimal amount of straight debt only compares to the one that includes a fixed-size CCB and an optimal amount of straight debt subject to regulatory constraint 4 in the following way:*

(a) *the difference in firm values for the two capital structures equals the difference in equity values,*

$$\begin{aligned} G(A_t, \bar{A}_B; \bar{c}_b, c_c) - G(A_t, A_B^*; c_b^*, 0) &= W(A_t, \bar{A}_B; \bar{c}_b, c_c) - W(A_t, A_B^*; c_b^*, 0) = \\ (\theta + \alpha - \theta\alpha) \left(\left(\frac{A_t}{A_B^*}\right)^{-\gamma} A_B^* - \left(\frac{A_t}{\bar{A}_B}\right)^{-\gamma} \bar{A}_B \right) &- \theta \left(\frac{A_t}{A_C}\right)^{-\gamma} \left(\lambda \frac{c_c}{r}\right) \end{aligned} \quad (5)$$

(b) *if the CCB coupon c_c is sufficiently small, the total value of the firm is higher in the presence of the CCB: there exists $\bar{c}_1$ such that $G(A_t, \bar{A}_B; \bar{c}_b, c_c) > G(A_t, A_B^*; c_b^*, 0)$ for any $c_c \in (0, \bar{c}_1)$*

(c) *the expected cost of bankruptcy is lower in the presence of the CCB, $BC(A_t, \bar{A}_B; \bar{c}_b) < BC(A_t, A_B^*; c_b^*)$.*

The key result is that equity holders gain from replacing a small amount of straight debt with a CCB. The intuition is that even though the tax savings associated with coupon payments decrease due to $\tau(A_C) <$

$\tau(A_B^*)$, the firm benefits from reducing its bankruptcy costs due to a smaller amount of risky straight debt. For small amounts of CCB the benefits exceed lost tax savings.

The amount of CCB that the firm can issue is set by regulators exogenously, via constraint (4). Therefore, for the firm to be willing to replace straight debt with CCB, regulators need know how to set the constraint so that c_c does not exceed $\bar{c}_1$.

The main economic result is that equity holders are willing to issue a small CCB and replace a portion of straight debt under regulatory condition 4. By doing so they could reduce default risk without increasing the total leverage and raising tax subsidies.

4 Partially Replacing Existing Straight Debt with a CCB

We continue with the case when CCB replaces a portion of already existing (not necessarily in the optimal amount) straight debt. Assume that at time t the capital structure of the firm consists of equity and straight debt paying coupon $\hat{c}_b$ (not necessarily equal to c_b^*). The firm wants to issue CCB and swap it for a portion of straight debt in order to reduce $\hat{c}_b$ to $\bar{c}_b$, where $\bar{c}_b < \hat{c}_b$. Once the announcement is made, the market value of straight debt, that is still paying $\hat{c}_b$, will rise from $U^B(A_t, \hat{A}_B; \hat{c}_b, 0)$ to $U^B(A_t, \bar{A}_B; \hat{c}_b, 0)$ to reflect a lower default boundary due to a lesser amount of straight debt after the swap. For the straight debt holders to be indifferent between exchanging their holdings for CCB and continuing to hold straight debt the following budget equation should be true

$$U^B(A_t, \bar{A}_B; \hat{c}_b, 0) = U^C(A_t, \bar{A}_B; \bar{c}_b, c_c) + U^B(A_t, \bar{A}_B; \bar{c}_b, c_c). \quad (6)$$

The updated value of existing straight debt *post* announcement should equal the value of CCB plus the value of straight debt that remains after the swap.

Coupon $\bar{c}_b$ is set exogenously and, as before, the same amount of CCB can be issued with different coupons and conversion-triggering asset levels. The firm, for example, could pick $\bar{c}_b$ and A_C first and then

find c_c by solving (6) as shown below.

$$\begin{aligned}
U^C(A_t, \bar{A}_B; \bar{c}_b, c_c) &= U^B(A_t, \bar{A}_B; \hat{c}_b, 0) - U^B(A_t, \bar{A}_B; \bar{c}_b, c_c) \\
\frac{c_c}{r} \left(1 - \left(\frac{A_t}{A_C} \right)^{-\gamma} \right) + \left(\frac{A_t}{A_C} \right)^{-\gamma} \left(\lambda \frac{c_c}{r} \right) &= \frac{\hat{c}_b}{r} \left(1 - \left(\frac{A_t}{\bar{A}_B} \right)^{-\gamma} \right) + \left(\frac{A_t}{\bar{A}_B} \right)^{-\gamma} (1 - \alpha) \bar{A}_B - \\
&\quad \frac{\bar{c}_b}{r} \left(1 - \left(\frac{A_t}{\bar{A}_B} \right)^{-\gamma} \right) - \left(\frac{A_t}{\bar{A}_B} \right)^{-\gamma} (1 - \alpha) \bar{A}_B \\
\frac{c_c}{r} \left(1 - \left(\frac{A_t}{A_C} \right)^{-\gamma} \right) + \left(\frac{A_t}{A_C} \right)^{-\gamma} \left(\lambda \frac{c_c}{r} \right) &= \frac{(\hat{c}_b - \bar{c}_b)}{r} \left(1 - \left(\frac{A_t}{\bar{A}_B} \right)^{-\gamma} \right) \\
c_c &= \frac{(\hat{c}_b - \bar{c}_b) \left(1 - \left(\frac{A_t}{\bar{A}_B} \right)^{-\gamma} \right)}{1 - (1 - \lambda) \left(\frac{A_t}{A_C} \right)^{-\gamma}}
\end{aligned} \tag{7}$$

We analyze if equity holders would be willing to replace some of the existing straight debt with CCB and what effect this replacement would have on the total value of the firm.

Proposition 4. *If a leveraged firm with a capital structure that includes equity and straight debt replaces a portion of straight debt with CCB then*

(i) *the value of equity decreases, $W(A_t, \bar{A}_B; \bar{c}_b, c_c) - W(A_t, \hat{A}_B; \hat{c}_b, 0) < 0$*

(ii) *the change in the total value of the firm is such that*

$$\begin{aligned}
(a) \ G(A_t, \bar{A}_B; \bar{c}_b, c_c) - G(A_t, \hat{A}_B; \hat{c}_b, 0) &= \frac{\hat{c}_b \theta}{r} \left(\left(\frac{A_t}{\hat{A}_B} \right)^{-\gamma} - \left(\frac{A_t}{\bar{A}_B} \right)^{-\gamma} \right) + \alpha \hat{A}_B \left(\frac{A_t}{\hat{A}_B} \right)^{-\gamma} - \\
&\quad \alpha \bar{A}_B \left(\frac{A_t}{\bar{A}_B} \right)^{-\gamma} - \theta \left(\frac{A_t}{A_C} \right)^{-\gamma} \left(\lambda \frac{c_c}{r} \right) \\
(b) \ G(A_t, \bar{A}_B; \bar{c}_b, c_c) - G(A_t, \hat{A}_B; \hat{c}_b, 0) &> W(A_t, \bar{A}_B; \bar{c}_b, c_c) - W(A_t, \hat{A}_B; \hat{c}_b, 0)
\end{aligned}$$

(c) *if $\hat{c}_b \geq c_b^*$ and $\lambda \geq 2 - \left(\frac{A_t}{A_C} \right)^\gamma$, then there exists $\bar{c}_1$ such that $G(A_t, \bar{A}_B; \bar{c}_b, c_c) > G(A_t, \hat{A}_B; \hat{c}_b, 0)$ for $c_c \in (0, \bar{c}_1)$*

(iii) *the cost of bankruptcy decreases, $BC(A_t, \bar{A}_B; \bar{c}_b) < BC(A_t, \hat{A}_B; \hat{c}_b)$.*

If the firm is leveraged optimally or over-leveraged compared to its optimal capital structure ($\hat{c}_b \geq c_b^*$) it could benefit in terms of its *total value* from replacing a certain amount ($c_c \in (0, \bar{c}_1)$) of straight debt with

CCB (issued with $\lambda \geq 2 - \left(\frac{A_t}{A_C}\right)^\gamma$ ¹¹). There are two forces at work here. First, as before, replacing straight debt with CCB pushes the tax savings down, but the firm benefits from reducing the cost of bankruptcy. For certain amounts of CCB the benefits will dominate the lost tax savings.

Second, although debt becomes less risky due to $\bar{A}_B < \hat{A}_B$ the total amount of debt increases by the difference between the value of straight debt post announcement, $U^B(A_t, \bar{A}_B; \hat{c}_b, 0)$, and the value of straight debt pre announcement, $U^B(A_t, \hat{A}_B; \hat{c}_b, 0)$. By increasing the total amount of debt while reducing the cost of bankruptcy the firm benefits from relatively higher tax savings¹². The presence of these relative benefits is independent of the amount of CCB. The new tax savings for the firm, though, might (if not compensated by the reduction in tax savings due to the use of CCB) translate into additional social costs in the form of extra tax subsidies. As the amount of CCB debt increases ($c_c > \bar{c}_1$) lost tax benefits become larger and can turn all the gains, including the ones from lower bankruptcy costs and the additional tax savings, into losses.

Although the total value of the firm could increase, equity holders will not replace voluntarily any amount of existing straight debt with CCB as their value decreases. All the potential gains in the total value of the firm plus a portion the value of equity are passed on to debt holders. The observed effect is due to debt overhang inefficiency.

The key economic result of Section 4 is that if the firm decided to partially replaces existing straight debt with CCB the total value of the firm would increase while bankruptcy costs together with the total amount of risky straight debt would decrease. Equity holders, however, will never initiate this kind of debt replacement on their own due to debt overhang inefficiency.

5 TBTF Firms

In this section we look at firms that are 'too big' for the government to let them fail as they pose systemic risk. In our model the government bails out a TBTF firm by assuming control over its assets and taking over its obligation to make payments to straight debt holders at the point of bankruptcy. We study the possible

¹¹Note, that $2 - \left(\frac{A_t}{A_C}\right)^\gamma \leq 1$ for $A_t \geq A_C$.

¹²Based on this, one would expect that, everything else being equal, $\bar{c}_1 > \bar{c}_1$.

reduction in the government's TBTF subsidy that may be achieved by requiring partial replacement of straight debt with CCB in the capital structure of a TBTF firm.

Consider a firm with a capital structure that includes equity and straight debt, paying coupon c_b . In the structural model we have used so far the firm reaches bankruptcy when the value of assets, A_t , hits the default boundary level, A_B , for the first time. At that point, if the government decides to step in to prevent bankruptcy, it will obtain assets worth A_B and an obligation to pay c_b forever with the risk-free value of $\frac{c_b}{r}$. Therefore, the value of the government subsidy at the time of bankruptcy is $\frac{c_b}{r} - A_B$.

Given (A.1), at any time t before bankruptcy, the value of subsidy is¹³

$$S(A_t; c_b, 0) = \left(\frac{c_b}{r} - A_B \right) \left(\frac{A_t}{A_B} \right)^{-\gamma}. \quad (8)$$

By definition, a government subsidy prevents the firm from going into default. Therefore, it eliminates bankruptcy costs, $BC(A_t; c_b) = 0$, and makes straight debt default-free, $U^B(A_t; c_b, 0) = \frac{c_b}{r}$.

The optimal time to default $\tau(A_B)$ solves the maximum-equity-valuation problem of equity holders. A government subsidy kicks in at time $\tau(A_B)$ and covers only straight debt obligations. Therefore, it affects neither the timing of default nor the value of $A_B = \beta(1 - \theta)c_b$. This implies that, provided that the capital structure does not change, the value of equity remains the same. The government guarantee benefits only the debt holders and does not subsidize equity.

Based on Lemma 1, the time of default and the value of assets at the time of default do not depend on whether the capital structure of the firm includes equity and straight debt or equity, *the same amount of* straight debt and CCB. Therefore, given (8), for a fixed amount of straight debt, c_b , the value of subsidy is the same whether CCB is present or not,

$$S(A_t; c_b, c_c) = S(A_t; c_b, 0). \quad (9)$$

Proposition 5. *Let a firm have a capital structure that includes equity and straight debt, paying coupon c_b . If at any time t the government issues a guarantee for the straight debt of the firm, then*

¹³We return to our initial notations. The default boundary is tied to the corresponding coupon on straight debt based on (2).

1. the larger is the amount of outstanding straight debt the larger is the subsidy, $\frac{dS(A_t; c_b, 0)}{dc_b} > 0$

2. the total value of the firm will increase to

$$G(A_t; c_b, 0) = A_t + \frac{\theta c_b}{r} \left(1 - \left(\frac{A_t}{A_B} \right)^{-\gamma} \right) + \left(\frac{c_b}{r} - A_B \right) \left(\frac{A_t}{A_B} \right)^{-\gamma}. \quad (10)$$

5.1 Replacing Straight Debt with CCB

Assume that a TBTF firm is currently unlevered but is considering leveraging up. We try to understand how the value of government subsidy would depend on whether the firm chooses to issue straight debt or both straight debt and CCB under a regulatory limit for the total amount of debt.

Based on equation (10), the total value of the firm strictly increases in the size of government subsidy¹⁴, which, based on Proposition 5 (part 1), increases with the amount of straight debt. Therefore, equity holders of an unlevered firm would try to issue as much straight debt as possible, collect the proceeds as dividends and default immediately after the issuance¹⁵. Knowing this, the government could set limits on how much debt a TBTF firm could issue. It could set a maximum straight debt coupon c_b^g for the capital structure that includes equity and straight debt. It could also offer the firm an alternative to issue straight debt with coupon $\bar{c}_b$ and CCB with coupon c_c such that the total amount of debt is fixed

$$U^B(A_t; c_b^g, 0) = U^C(A_t; \bar{c}_b, c_c) + U^B(A_t; \bar{c}_b, c_c). \quad (11)$$

In the presence of a government guarantee, straight debt is risk-free. Therefore, given the closed-form solution for $U^C(A_t; \bar{c}_b, c_c)$ from Proposition 1, equation(11) can be re-written as

$$\frac{c_b^g}{r} = \frac{c_c}{r} \left(1 - \left(\frac{A_t}{A_C} \right)^{-\gamma} \right) + \lambda \frac{c_c}{r} \left(\frac{A_t}{A_C} \right)^{-\gamma} + \frac{\bar{c}_b}{r}.$$

¹⁴The last term on the right-hand side of equation (10) equals the value of subsidy from equation (8).

¹⁵Although the market would know that the firm is going default, due to the presence of the government guarantee, the firm would still be able to sell this debt.

This leads to

$$\bar{c}_b = c_b^g - c_c \left(1 - (1 - \lambda) \left(\frac{A_t}{A_C} \right)^{-\gamma} \right). \quad (12)$$

Note, that $c_b^g > \bar{c}_b$. Based on equation (9) and Proposition 5 (part 1),

$$S(A_t; c_b^g, 0) > S(A_t; \bar{c}_b, c_c). \quad (13)$$

Replacing straight debt with CCB reduces the value of subsidy. Also, based on equation (12), the higher is c_c the lower is $\bar{c}_b$. The larger is the portion of CCB, the smaller is the amount of the remaining straight debt and, therefore, the lower is the value of the government subsidy.

A government subsidy eliminates bankruptcy costs, $BC(A_t; \hat{c}_b, 0) = 0$, and the total value of the firm for the case when it does not issue CCB is

$$G(A_t; c_b^g, 0) = A_t + TB(A_t; c_b^g, 0) + S(A_t; c_b^g, 0).$$

Equivalently, the total value of the firm when it does issue CCB is

$$G(A_t; \bar{c}_b, c_c) = A_t + TB(A_t; \bar{c}_b, c_c) + S(A_t; \bar{c}_b, c_c).$$

Consider the difference

$$G(A_t; c_b^g, 0) - G(A_t; \bar{c}_b, c_c) = TB(A_t; c_b^g, 0) - TB(A_t; \bar{c}_b, c_c) + S(A_t; c_b^g, 0) - S(A_t; \bar{c}_b, c_c). \quad (14)$$

Based on equation (13), the last difference on the right-hand side of the above equation is strictly positive, $S(A_t; c_b^g, 0) - S(A_t; \bar{c}_b, c_c) > 0$. In the appendix (see proof of Proposition 6) we also show that $G(A_t; c_b^g, 0) - G(A_t; \bar{c}_b, c_c) > 0$. The reduction in the value of government subsidy is the main driver behind the reduction in the total value of the firm when straight debt is replaced with CCB.

The above arguments can be summarized in a proposition.

Proposition 6. *Replacing straight debt with CCB in a new capital structure of an unlevered TBTF firm subject to a regulatory limit on how much debt it is allowed to issue*

1. *reduces the value of the government subsidy, $S(A_t; c_b^g, 0) > S(A_t; \bar{c}_b, c_c)$*
2. *reduces the total value of the firm (i.e., the gains of equity holders from leveraging up the firm), $G(A_t; c_b^g, 0) > G(A_t; \bar{c}_b, c_c)$.*

We started this subsection by considering an unlevered TBTF firm. Had the firm been leveraged with straight debt, replacing its existing straight debt with CCB would have led to results similar to the ones captured in Proposition 6.

The key observation is that, due to the government subsidy, the straight debt of a TBTF firm is risk-free, so, when the firm announces the swap of a portion of straight debt for CCB, the straight debt does not appreciate. In other words, there is no debt overhang effect similar to the one we observed in section 4. Therefore, the regulatory constraint (11) is the same whether the firm replaces existing or to-be-newly-issued straight debt with CCB.

Proposition 7. *Replacing straight debt with CCB in a capital structure of a TBTF firm that includes equity and straight debt*

1. *reduces the value of the government subsidy*
2. *reduces the value of equity holders.*

Propositions 6 and 7 suggest that equity holders of a TBTF firm will oppose issuing CCB.

6 Multiple Equilibrium Equity Prices and Market Manipulations

Our goal in this section is to present two equity-value issues that may arise when firms use CCB. First, we show the existence of multiple equilibrium equity prices, and, second, we look at the incentives of market participants to manipulate the equity market. We distinguish between manipulations by CCB holders and the ones by equity holders.

We assume Condition 2 and, therefore, the value of equity is strictly increasing in the value of assets before conversion. This allows us to formulate a conversion rule for CCB directly in terms of equity value - CCB converts into equity when the equity value drops to $W_C = W(A_C; c_b, c_c)$. W_C corresponds to A_C in the earlier analysis.

6.1 Two Equilibrium Equity Prices

We start by showing the existence of two different equilibrium prices for the equity of a firm that issues straight debt, equity and CCB.

As it is captured in Figure 3, consider three time instances t , t_+ and t_{++} , where $t < t_+ < t_{++}$.

At time t the market value of assets, A_t , is uncertain. The uncertainty is resolved at t_{++} when A_t takes either the value of A_H with probability p or the value of A_L with probability $(1 - p)$. A_H , A_L and A_C are such that $A_L < A_C < A_H$.

Figure 3: **Equity price, asset value and CCB conversion decisions.**

The sequence of possible events is as follows. The market value of equity, W_t , is observed at time t . If $W_t \leq W_C$, then at time t_+ CCB converts into equity. Otherwise, there is no conversion at t_+ . The realized market value of assets, A_t , is observed at t_{++} . If the realization is A_L and there was no conversion at t_+ CCB converts into equity at time t_{++} . Otherwise, there is no conversion at t_{++} .

Consider the case when there is no conversion (i.e., $W_t > W_C$) at time t_+ . If the realization of A_t at time t_{++} is A_H , then the value of old equity at time t_{++} is $W(A_H; c_b, c_c)$. This value reflects the fact that, due to $A_C < A_H$, A_H does not trigger conversion at t_{++} . There are no new equity holders.

On the other hand, if the realization of A_t at time t_{++} is A_L , then the value of old equity at time t_{++} is $W(A_L; c_b, 0) - \lambda \frac{c_c}{r}$. In this case, due to $A_L < A_C$, A_L does trigger conversion at t_{++} . The value of old equity equals the value of total (old and new) equity post conversion minus the value of new equity issued to replace CCB.

We denote the value of old (observed) equity at time t_+ when there is no conversion at t_+ by $\bar{W}_t$. It can be calculated as the expected value of old equity at time t_{++} :

$$\bar{W}_t = pW(A_H; c_b, c_c) + (1 - p) \left(W(A_L; c_b, 0) - \lambda \frac{c_c}{r} \right). \quad (15)$$

Now consider the case when at time t_+ CCB does convert into equity (i.e., $W_t \leq W_C$). We denote the value of old (observed) equity at time t_+ when there is conversion at t_+ by $\hat{W}_t$. Then, the value of total (old and new) equity post conversion at time t_+ is $\hat{W}_t + \lambda \frac{c_c}{r}$. The values of total equity at time t_{++} for A_H and A_L are $W(A_H; c_b, 0)$ and $W(A_L; c_b, 0)$, correspondingly. And, the value of total equity at time t_+ is its expected value at time t_{++} :

$$\hat{W}_t + \lambda \frac{c_c}{r} = pW(A_H; c_b, 0) + (1 - p)W(A_L; c_b, 0).$$

This leads to

$$\hat{W}_t = pW(A_H; c_b, 0) + (1 - p)W(A_L; c_b, 0) - \lambda \frac{c_c}{r}. \quad (16)$$

Based on (15), (16) and Proposition 1, the difference in the observed values of equity for the two cases is

$$\begin{aligned} \bar{W}_t - \hat{W}_t &= p(W(A_H; c_b, c_c) - W(A_H; c_b, 0)) - \frac{\lambda c_c}{r} - (1 - p) \frac{\lambda c_c}{r} \\ &= p \left(-\frac{c_c(1 - \theta)}{r} \left(1 - \left(\frac{A_H}{A_C} \right)^{-\gamma} \right) - \frac{\lambda c_c}{r} \left(\frac{A_H}{A_C} \right)^{-\gamma} + \frac{\lambda c_c}{r} \right) \\ &= p \frac{c_c}{r} \left(1 - \left(\frac{A_H}{A_C} \right)^{-\gamma} \right) (\lambda + \theta - 1). \end{aligned}$$

If $(\lambda + \theta) > 1$, $\bar{W}_t > \hat{W}_t$.

We proved the following theorem.

Theorem 1. *If $(\lambda + \theta) > 1$, then there could be two market equilibria*

1. *a relatively high equity price at time t and no conversion of CCB into equity at time t_+*
2. *and, a relatively low equity price at time t and conversion of CCB into equity at time t_+ .*

Note that the proof above is based on the assumption that W_C is such that $\hat{W}_t < W_C < \bar{W}_t$. Since $\bar{W}_t$ is strictly higher than $\hat{W}_t$, we can find values of W_C that satisfy the above condition by picking certain values for c_c , p , etc.¹⁶

The intuition behind Theorem 1 is as follows. Early conversion leads to a guaranteed loss of tax benefits associated with CCB. This leads to a lower value of equity, $\hat{W}_t$. If, on the other hand, the debt is not converted before the uncertainty about the value of assets is resolved, the tax benefits are lost only with probability $(1 - p)$. This corresponds to a higher value of equity, $\bar{W}_t$.

6.2 Equity Market Manipulations

We continue with analyzing the conditions under which market participants might be willing to manipulate the equity market.

6.2.1 Manipulation by CCB Holders

We start with the case when CCB holders attempt the manipulation.

The motivation is as follows. Market participants might profit by buying CCB when the stock price of the firm is above the conversion-triggering level W_C , driving the price down (by spreading negative news, short selling equity, etc.) in order to trigger conversion and then selling the equity obtained as the result of conversion when the price corrects.

¹⁶One can try to specify more explicitly the corresponding values (or ranges of values) for these parameters by using (15), (16) and Proposition 1.

Assume that, as in Section 6.1, at time t the market value of assets, A_t , is uncertain. At some future time the uncertainty is resolved and A_t can take the value of A_H with probability p or A_L with probability $(1 - p)$. p reflects correct beliefs about realizations A_H and A_L . Also, A_H and A_L are such that, when the true value of A_t is realized, only A_L triggers conversion (i.e., $A_L < A_C < A_H$).

As before, we also assume that the market value of equity is observed before the uncertainty about A_t is resolved.

We model CCB holder manipulations as actions that drive down the equity price by convincing the market that the probability of A_t reaching A_H is p' and the probability of A_t reaching A_L is $(1 - p')$, where $p' < p$.

Assume that, if the market believes that the probability of A_t reaching A_H is p (and the probability of A_t reaching A_L is $(1 - p)$), the value of equity is above W_C and, therefore, there is no conversion. On the other hand, if the market believes that the probability is p' , CCB does convert into equity.

In expectation the *post* manipulation (i.e., after the stock price is driven down, CCB is converted into equity and the correct belief p is restored) value of equity is

$$\tilde{W}_t = pW(A_H; c_b, 0) + (1 - p)W(A_L; c_b, 0).$$

When the market is manipulated, at the point of conversion, the value of equity is

$$\tilde{\tilde{W}}_t = p'W(A_H; c_b, 0) + (1 - p')W(A_L; c_b, 0).$$

CCB holders receive equity in the amount of $\lambda \frac{c_c}{r}$. As the market belief corrects, the value of equity changes from $\tilde{\tilde{W}}_t$ to $\tilde{W}_t$. Therefore, the expected value of the payoff to (former) CCB holders after the market corrects is

$$\Pi'_t = \lambda \frac{c_c}{r} \frac{pW(A_H; c_b, 0) - (1 - p)W(A_L; c_b, 0)}{p'W(A_H; c_b, 0) - (1 - p')W(A_L; c_b, 0)}.$$

If there is no manipulation that triggers conversion, the expected value of the payoffs to CCB holders is

$$\Pi_t = pU^C(A_H; c_b, c_c) + (1-p)\lambda \frac{c_c}{r}.$$

Consider the difference in these two values

$$\Pi'_t - \Pi_t = \lambda \frac{c_c}{r} \frac{(p-p')(W(A_H; c_b, 0) - W(A_L; c_b, 0))}{p'W(A_H; c_b, 0) - (1-p')W(A_L; c_b, 0)} - p \left(U^C(A_H; c_b, c_c) - \lambda \frac{c_c}{r} \right).$$

By using the closed-form solution for $U^C(A_H; c_b, c_c)$ from Proposition 1 and rearranging terms, we get

$$\begin{aligned} \Pi'_t - \Pi_t &= \lambda \frac{c_c}{r} \frac{(p-p')(W(A_H; c_b, 0) - W(A_L; c_b, 0))}{p'W(A_H; c_b, 0) - (1-p')W(A_L; c_b, 0)} - p(1-\lambda) \frac{c_c}{r} \left(1 - \left(\frac{A_H}{A_C} \right)^{-\gamma} \right) \\ &= \lambda \frac{c_c}{r} \frac{p-p'}{p' + \frac{W(A_L; c_b, 0)}{W(A_H; c_b, 0) - W(A_L; c_b, 0)}} - p(1-\lambda) \frac{c_c}{r} \left(1 - \left(\frac{A_H}{A_C} \right)^{-\gamma} \right). \end{aligned} \quad (17)$$

It's easy to see that if $\lambda = 0$, based on equation (17), $\Pi'_t < \Pi_t$ and, therefore, CCB holders do not have an incentive to manipulate the market.

Also from (17), $\Pi'_t - \Pi_t$ is strictly increasing in λ and the value of λ for which the difference in the two payoffs is zero is

$$\lambda^* = \frac{p \left(1 - \left(\frac{A_H}{A_C} \right)^{-\gamma} \right)}{\frac{p-p'}{p' + \frac{W(A_L; c_b, 0)}{W(A_H; c_b, 0) - W(A_L; c_b, 0)}} + p \left(1 - \left(\frac{A_H}{A_C} \right)^{-\gamma} \right)}. \quad (18)$$

Clearly, $\lambda^* > 0$. Also, since $\frac{p-p'}{p' + \frac{W(A_L; c_b, 0)}{W(A_H; c_b, 0) - W(A_L; c_b, 0)}} > 0$, $\lambda^* < 1$.

The above arguments are summarized in Theorem 2.

Theorem 2. *There exists $\lambda^* \in (0, 1)$ such that $\Pi_t - \Pi'_t = 0$ and*

1. *if $\lambda \leq \lambda^*$ CCB holders will not manipulate the equity market*
2. *and, if $\lambda > \lambda^*$ CCB holders will manipulate the equity market.*

The intuition for why small values of λ should prevent manipulation is as follows. At conversion, CCB

holders give up a stream of future coupon payments for the value of $\lambda \frac{c_c}{r}$. Small values of λ mean that, even after we account for the appreciation of equity post conversion, the value CCB holders receive is too small compared to the value of future coupon payments they need to give up. Therefore, CCB holders will not try to force conversion.

Based on equation (18), there are two major drivers behind the value of λ^* . The first one is the distance between the probabilities p and p' . The bigger is the difference $(p - p')$ the lower is λ^* . The interpretation is that the greater the benefit of a manipulation, the lower should the conversion ratio be in order to avoid manipulation.

The second driver is the difference between equity values for asset realizations A_H and A_L . Here, again, the bigger is the difference $(W(A_H; c_b, 0) - W(A_L; c_b, 0))$ the lower is λ^* . That is, the greater the possible asset range, the greater the benefit of a manipulation, and the lower should the conversion ratio be.

6.2.2 Manipulation by Equity Holders

We turn to the case when equity holders might attempt to manipulate the market.

The motivation is that equity holders might increase the value of their holdings by manipulating the equity price down to W_C , triggering conversion, and then correcting the market belief. The potential value increase arises if the obligation to pay the CCB holders c_c is removed through conversion at below the market value.

The value of (old) equity holders before they attempt to manipulate the market is $W(A_t; c_b, c_c)$. At the point of conversion the value of equity is $W(A_C; c_b, 0)$. As the market belief corrects, the value of equity rises to $W(A_t; c_b, 0)$. The new value of (old) equity holders equals the difference between $W(A_t; c_b, 0)$ and the value of (new) equity that belongs to former CCB holders, $\left(\lambda \frac{c_c}{r}\right) \frac{W(A_t; c_b, 0)}{W(A_C; c_b, 0)}$. (Old) Equity holders will not manipulate the market if

$$W(A_t; c_b, c_c) - \left[W(A_t; c_b, 0) - \lambda \frac{c_c}{r} \frac{W(A_t; c_b, 0)}{W(A_C; c_b, 0)} \right] \geq 0.$$

Based on Proposition 1

$$W(A_t; c_b, c_c) - \left[W(A_t; c_b, 0) - \lambda \frac{c_c}{r} \frac{W(A_t; c_b, 0)}{W(A_C; c_b, 0)} \right] = -\frac{c_c(1-\theta)}{r} \left(1 - \left(\frac{A_t}{A_C} \right)^{-\gamma} \right) - \frac{\lambda c_c}{r} \left(\frac{A_t}{A_C} \right)^{-\gamma} + \frac{\lambda c_c}{r} \frac{W(A_t; c_b, 0)}{W(A_C; c_b, 0)}. \quad (19)$$

Clearly, when $\lambda = 0$ equity holders will manipulate the market as the right-hand side of equation (19) is negative.

For $\theta = 0$ and $\lambda = 1$ the right-hand side of equation (19) is strictly positive:

$$-\frac{c_c}{r} \left(1 - \left(\frac{A_t}{A_C} \right)^{-\gamma} \right) - \frac{c_c}{r} \left(\frac{A_t}{A_C} \right)^{-\gamma} + \frac{c_c}{r} \frac{W(A_t; c_b, 0)}{W(A_C; c_b, 0)} = \frac{c_c}{r} \left(\frac{W(A_t; c_b, 0)}{W(A_C; c_b, 0)} - 1 \right) > 0.$$

As θ increases the right-hand side of equation 19 only becomes larger. Therefore, for any feasible θ , if $\lambda = 1$, the difference in the equity values is going to be positive. This means that no market manipulations will be taking place.

It is also clear that the right-hand side of equation (19) is strictly increasing in λ and the value of λ for which the difference in the equity values is zero is

$$\lambda^{**} = \frac{(1-\theta) \left(1 - \left(\frac{A_t}{A_C} \right)^{-\gamma} \right)}{\frac{W(A_t; c_b, 0)}{W(A_C; c_b, 0)} - \left(\frac{A_t}{A_C} \right)^{-\gamma}}. \quad (20)$$

For values of λ higher or equal to λ^{**} the right-hand side of equation (19) is going to be non-negative and equity holders will not have an incentive to manipulate the market. On the other hand, for values of λ lower than λ^{**} they will manipulate the market. Note, that λ^{**} is a decreasing function of the asset value A_t . Therefore, for the above observation to be true for any realization of A_t before conversion, we choose the highest

$$\lambda^{**} \leq \frac{(1-\theta) \left(1 - \left(\frac{A_t}{A_C} \right)^{-\gamma} \right)}{\frac{W(A_C; c_b, 0)}{W(A_C; c_b, 0)} - \left(\frac{A_t}{A_C} \right)^{-\gamma}} = (1-\theta). \quad (21)$$

We proved the following theorem.

Theorem 3. *If $\lambda \geq (1 - \theta)$ equity holders will not manipulate the equity market, and if $\lambda < (1 - \theta)$ equity holders will manipulate the equity market.*

It is intuitive that equity holders will not manipulate the market when λ values are above λ^{**} , since the cost of the obligation to pay c_c is 'too' high.

There maybe additional costs associated with manipulations (by both equity and CCB holders) that we did not take into account. These could include implementation costs, potential penalties, legal fees, etc. The additional costs would make manipulations harder to implement.

7 Summary and Policy Conclusions

This paper has provided a formal model of CCBs. The results of the formal model are summarized in Tables 1 and 2. Table 1 summarizes the primary effects of CCB issuance on firm and equity value as a function of the firm's capital structure status and any imposed constraints. Table 2 provides our primary results showing how the conversion ratio λ affects the incentives for CCB holders and equity holders to manipulate the firm's stock price in order to trigger conversion.

In terms of prudential bank regulation, we have shown that CCBs provide a new instrument that allows banks or firms to recapitalize in an automatic and dependable fashion whenever their capital reaches a distressed level. In other words, CCBs generally have the potential to provide most of the tax shield benefits of straight debt while providing the same protection as equity capital against bankruptcy costs. For CCBs to be effective in this role, however, it is important that the banks be required to substitute CCBs for straight debt, and not for equity, in their capital structure. The regulatory benefits of CCBs for bank safety also are greater the higher the trigger at which conversion occurs.

We conclude with comments on important topics for future research. We first comment on three extensions that would generalize assumptions in the current paper. One useful extension would fully determine the firm's optimal capital structure in the presence of CCBs. In particular, our analysis has been static in

Table 1: Effects of CCB issuance on the capital structure of the firm*

Firm	Constraint	Firm Value	Equity Holders' Value	Default Risk	Asset Substitution	Tax Savings	Other Effects	Firm Decision
Unleveraged	Sufficiently small amount of CCB	↑	↑	↔	↑	↑	n/c	Issue CCB on top of optimal amount of SD
Leveraged with SD	Sufficiently small amount of CCB	↑	↑	↔	↑	↑	n/c	Issue CCB on top of existing amount of SD
Unleveraged	Total amount of debt is fixed	↑	↑	↓	↓	~	n/c	Replace some SD with CCB
Leveraged	Total amount of debt is fixed	↑	↓	↓	↓	~	Debt overhang	Do not issue CCB
TBTF (Leveraged/Unleveraged)	Total amount of debt is fixed	↓	↓	↓	n/c	~	Reduced government subsidy	Do not issue CCB

*SD: straight debt; TBTF: Too-big-to-fail; n/c: not considered; ↑: increase; ↓: decrease; ↔: no change; ~: no effect or insignificant increase/decrease

Table 2: Incentives of CCB holders and equity holders to manipulate the stock market

Conversion Ratio	Action	Intuition
$0 < \lambda^* < \lambda$	CCB holders want to drive the stock price down to trigger conversion	If λ is high CCB holders receive a large amount of undervalued equity at conversion
$\lambda \leq \lambda^*$	CCB holders do not want to trigger conversion	If λ is low CCB holders are poorly compensated at conversion
$\lambda < 1 - \theta$	Equity holders want to drive the stock price down to trigger conversion	If λ is low equity holders can cheaply get rid of the obligation to pay c_c
$1 - \theta \leq \lambda$	Equity holders do not want to trigger conversion	If λ is high conversion is costly to equity holders

the sense that we assume the firm's entire CCB issue is converted into equity at a single point when the trigger is activated. We suspect, however, that the CCB benefits would expand further if the bonds could be converted in a sequence of triggers and/or that banks committed to issue new CCBs as soon as the existing bonds were converted. A second factor is that our analysis has assumed that both the CCBs and straight debt have an unlimited maturity in the fashion of a consol. We expect that an analysis with finite maturity bonds would find lower debt overhang costs of swapping CCBs for straight debt. A third extension would allow the geometric Brownian motion of asset dynamics to include jumps. This would have an impact on the valuation of all claims in the model. A related assumption is that we have not allowed the firm to use CCBs to purchase additional assets. While we do not expect this will change our basic results, this should be confirmed.

We conclude with two topics for future research concerning the use of CCBs for prudential bank regulation. As one topic, Flannery (2009a) has suggested that banks be presented with the choice of raising their capital ratio by a given amount or of raising their capital ratio by a smaller amount as long as it is combined with a specified amount of CCBs. The regulatory parameters in such a menu determine the tradeoff between regulatory benefits and bank costs. A calibrated version of our model could potentially measure the terms of this tradeoff. A second regulatory topic concerns the amount of tax shield benefit

allowed CCB. As just one example, it would be useful to explore the effects of allowing full deduction for interest payments that correspond to the coupon on similar straight bank debt, but to exclude any part of the CCB coupon that represents compensation for the conversion risk.

Appendix: Proofs

Proof of Proposition 1. Based on Duffie (2001), for a given constant $K \in (0, A_t)$, the market value of a security that claims one unit of account at the hitting time $\tau(K) = \inf\{s : A_s \leq K\}$ is, at any $t < \tau(K)$,

$$E_t^Q \left[e^{-r(\tau(K)-t)} \right] = \left(\frac{A_t}{K} \right)^{-\gamma} \quad (\text{A.1})$$

We use this result to derive closed-form solutions for the values of straight debt, CCB, tax benefits and bankruptcy costs as the present values of the corresponding cash flows. Equation (A.1) is applied repeatedly with different values for K .

$$\begin{aligned} U^B(A_t; c_b, c_c) &= E_t^Q \left[\int_t^{\tau(A_B)} e^{-r(s-t)} c_b ds + e^{-r(\tau(A_B)-t)} (1-\alpha) A_B \right] \\ &= E_t^Q \left[\frac{c_b}{r} \left(1 - e^{-r(\tau(A_B)-t)} \right) + e^{-r(\tau(A_B)-t)} (1-\alpha) A_B \right] \\ &= \frac{c_b}{r} \left(1 - \left(\frac{A_t}{A_B} \right)^{-\gamma} \right) + \left(\frac{A_t}{A_B} \right)^{-\gamma} (1-\alpha) A_B. \\ U^C(A_t; c_c) &= E_t^Q \left[\int_t^{\tau(A_C)} e^{-r(s-t)} c_c ds + e^{-r(\tau(A_C)-t)} \left(\lambda \frac{c_c}{r} \right) \right] \\ &= E_t^Q \left[\frac{c_c}{r} \left(1 - e^{-r(\tau(A_C)-t)} \right) + e^{-r(\tau(A_C)-t)} \left(\lambda \frac{c_c}{r} \right) \right] \\ &= \frac{c_c}{r} \left(1 - \left(\frac{A_t}{A_C} \right)^{-\gamma} \right) + \left(\frac{A_t}{A_C} \right)^{-\gamma} \left(\lambda \frac{c_c}{r} \right). \\ TB(A_t; c_b, c_c) &= E_t^Q \left[\int_t^{\tau(A_B)} e^{-r(s-t)} \theta c_b ds + \int_t^{\tau(A_C)} e^{-r(u-t)} \theta c_c du \right] \\ &= \frac{\theta c_b}{r} \left(1 - \left(\frac{A_t}{A_B} \right)^{-\gamma} \right) + \frac{\theta c_c}{r} \left(1 - \left(\frac{A_t}{A_C} \right)^{-\gamma} \right). \\ BC(A_t; c_b) &= E^Q \left[\int_0^{\tau(A_B)} e^{-r(s-t)} \alpha A_B ds \right] = \alpha A_B \left(\frac{A_t}{A_B} \right)^{-\gamma}. \end{aligned}$$

At any time t before conversion, the following budget equation holds:

$$A_t + TB(A_t; c_b, c_c) = W(A_t; c_b, c_c) + U^B(A_t; c_b, c_c) + U^C(A_t; c_b, c_c) + BC(A_t; c_b, c_c),$$

Therefore,

$$\begin{aligned}
W(A_t; c_b, c_c) &= A_t + TB(A_t; c_b, c_c) - U^B(A_t; c_b, c_c) - U^C(A_t; c_c) - BC(A_t; c_b) \\
&= A_t + \frac{\theta c_b}{r} \left(1 - \left(\frac{A_t}{A_B} \right)^{-\gamma} \right) + \frac{\theta c_c}{r} \left(1 - \left(\frac{A_t}{A_C} \right)^{-\gamma} \right) \\
&\quad - \frac{c_b}{r} \left(1 - \left(\frac{A_t}{A_B} \right)^{-\gamma} \right) - \left(\frac{A_t}{A_B} \right)^{-\gamma} (1 - \alpha) A_B \\
&\quad - \frac{c_c}{r} \left(1 - \left(\frac{A_t}{A_C} \right)^{-\gamma} \right) - \left(\frac{A_t}{A_C} \right)^{-\gamma} \left(\lambda \frac{c_c}{r} \right) - \alpha A_B \left(\frac{A_t}{A_B} \right)^{-\gamma} \\
&= A_t + \frac{c_b(\theta - 1)}{r} \left(1 - \left(\frac{A_t}{A_B} \right)^{-\gamma} \right) + \frac{c_c(\theta - 1)}{r} \left(1 - \left(\frac{A_t}{A_C} \right)^{-\gamma} \right) - \\
&\quad A_B \left(\frac{A_t}{A_B} \right)^{-\gamma} - \left(\lambda \frac{c_c}{r} \right) \left(\frac{A_t}{A_C} \right)^{-\gamma}.
\end{aligned}$$

Finally,

$$\begin{aligned}
G(A_t; c_b, c_c) &= W(A_t; c_b, c_c) + U^B(A_t; c_b, c_c) + U^C(A_t; c_c) \\
&= A_t + \frac{\theta c_b}{r} \left(1 - \left(\frac{A_t}{A_B} \right)^{-\gamma} \right) + \frac{\theta c_c}{r} \left(1 - \left(\frac{A_t}{A_C} \right)^{-\gamma} \right) - \alpha A_B \left(\frac{A_t}{A_B} \right)^{-\gamma}.
\end{aligned}$$

□

Proof of Proposition 2. Equity holders maximize total firm value, $G(A_t; c_b, c_c)$,

$$\max_{c_b \geq 0} G(A_t; c_b, c_c) \equiv \max_{c_b \geq 0} [A_t + TB(A_t; c_b, c_c) - BC(A_t; c_b)].$$

Proposition 1 and equation (2) lead to:

$$\max_{c_b \geq 0} \left[\frac{\theta c_b}{r} \left(1 - \left(\frac{A_t}{\beta(1 - \theta)c_b} \right)^{-\gamma} \right) + \frac{\theta c_c}{r} \left(1 - \left(\frac{A_t}{A_C} \right)^{-\gamma} \right) - \alpha \beta (1 - \theta) c_b \left(\frac{A_t}{\beta(1 - \theta)c_b} \right)^{-\gamma} \right].$$

FOCs:

$$\begin{aligned}
& \left(\frac{\theta c_b}{r} \left(1 - \left(\frac{\beta(1-\theta)}{A_t} \right)^\gamma c_b^\gamma \right) + \frac{\theta c_c}{r} \left(1 - \left(\frac{A_C}{A_t} \right)^\gamma \right) - \alpha \frac{(\beta(1-\theta))^{\gamma+1}}{A_t^\gamma} c_b^{\gamma+1} \right)' = 0 \\
& \left(\frac{\theta c_b}{r} - \frac{\theta}{r} \left(\frac{\beta(1-\theta)}{A_t} \right)^\gamma c_b^{\gamma+1} + \frac{\theta c_c}{r} \left(1 - \left(\frac{A_C}{A_t} \right)^\gamma \right) - \alpha \frac{(\beta(1-\theta))^{\gamma+1}}{A_t^\gamma} c_b^{\gamma+1} \right)' = 0 \\
& \frac{\theta}{r} - \frac{\theta(\gamma+1)}{r} \left(\frac{\beta(1-\theta)}{A_t} \right)^\gamma c_b^\gamma - \alpha(\gamma+1) \frac{(\beta(1-\theta))^{\gamma+1}}{A_t^\gamma} c_b^\gamma = 0 \\
& c_b = \left[\frac{\theta}{r} (\gamma+1)^{-1} \left(\frac{\beta(1-\theta)}{A_t} \right)^{-\gamma} \left(\frac{\theta}{r} + \alpha \beta (1-\theta) \right)^{-1} \right]^{\frac{1}{\gamma}}.
\end{aligned}$$

Finally,

$$c_b^*(A_t; c_c) = \frac{A_t}{\beta(1-\theta)} \left(\frac{\theta}{r} \right)^{\frac{1}{\gamma}} \left[(\gamma+1) \left(\frac{\theta}{r} + \alpha \beta (1-\theta) \right) \right]^{-\frac{1}{\gamma}}.$$

One can repeat the above calculations for the case when CCBs are not allowed to show formally that

$$c_b^*(A_t; c_c) = c_b^*(A_t; 0).$$

One can closely follow the math in the proof of Proposition 1 and item (v) will follow.

As for item (iv), based on Proposition 1 and (3)

$$\begin{aligned}
TB(A_0; c_b^*, c_c) &= \frac{\theta c_b^*}{r} \left(1 - \left(\frac{A_0}{A_B} \right)^{-\gamma} \right) + \frac{\theta c_c}{r} \left(1 - \left(\frac{A_0}{A_C} \right)^{-\gamma} \right) \\
&= TB(A_0; c_b^*, 0) + TB^C(A_0; c_b^*, c_c).
\end{aligned}$$

By re-grouping the terms in the formula for the value of equity from Proposition 1, we get

$$\begin{aligned}
W(A_0; c_b^*, c_c) &= A_0 - \frac{c_b^*(1-\theta)}{r} \left(1 - \left(\frac{A_0}{A_B} \right)^{-\gamma} \right) - A_B \left(\frac{A_0}{A_B} \right)^{-\gamma} - \\
&\quad \left[\frac{c_c(1-\theta)}{r} \left(1 - \left(\frac{A_0}{A_C} \right)^{-\gamma} \right) + \left(\lambda \frac{c_c}{r} \right) \left(\frac{A_0}{A_C} \right)^{-\gamma} \right] + \\
&\quad \frac{\theta c_c}{r} \left(1 - \left(\frac{A_0}{A_C} \right)^{-\gamma} \right) \\
&= W(A_0; c_b^*, 0) - U^C(A_0; c_c) + TB^C(A_0; c_b^*, c_c)
\end{aligned}$$

as in item (ii).

Finally,

$$\begin{aligned}
G(A_0; c_b^*, c_c) &= A_0 + TB(A_0; c_b^*, c_c) - BC(A_t, c_b) \\
&= \left[A_0 + \frac{\theta c_b}{r} \left(1 - \left(\frac{A_t}{A_B} \right)^{-\gamma} \right) - \alpha A_B \left(\frac{A_t}{A_B} \right)^{-\gamma} \right] + \\
&\quad \frac{\theta c_c}{r} \left(1 - \left(\frac{A_t}{A_C} \right)^{-\gamma} \right), \\
&= G(A_0; c_b^*, 0) + TB^C(A_0; c_b^*, c_c).
\end{aligned}$$

This proves item (i). □

Proof of Proposition 3. Based on regulatory constraint (4)

$$\begin{aligned}
G(A_0, \bar{A}_B; \bar{c}_b, c_c) - G(A_0, A_B^*; c_b^*, 0) &= W(A_0, \bar{A}_B; \bar{c}, c_c) + U^C(A_0, \bar{A}_B; \bar{c}_b, c_c) + \\
&\quad U^B(A_0, \bar{A}_B; \bar{c}_b, c_c) - W(A_0, A_B^*; c_b^*, 0) - \\
&\quad U^B(A_0, A_B^*; c_b^*, 0) \\
&= W(A_0, \bar{A}_B; \bar{c}_b, c_c) - W(A_0, A_B^*; c_b^*, 0).
\end{aligned}$$

The change in the total value of the firm equals the change in the value of equity.

Denote $W(A_0, \bar{A}_B; \bar{c}_b, c_c) - W(A_0, A_B^*; c_b^*, 0)$ by ΔW . Based on budget equation (A.2)

$$\begin{aligned}
\Delta W &= A_0 + TB(A_0, \bar{A}_B; \bar{c}_b, c_c) - U^B(A_0, \bar{A}_B; \bar{c}_b, c_c) - U^C(A_0, \bar{A}_B; c_c) - BC(A_t, \bar{A}_B; \bar{c}_b) - \\
&\quad A_0 - TB(A_0, A_B^*; c_b^*, 0) + U^B(A_0, A_B^*; c_b^*, 0) + BC(A_0, A_B^*; c_b^*) \\
&= TB(A_0, \bar{A}_B; \bar{c}_b, c_c) - TB(A_0, A_B^*; c_b^*, 0) + BC(A_0, A_B^*; c_b^*) - BC(A_t, \bar{A}_B; \bar{c}_b).
\end{aligned}$$

Next, based on closed-form solutions from Proposition 1

$$\begin{aligned}\Delta W &= \frac{\theta \bar{c}_b}{r} \left(1 - \left(\frac{A_0}{\bar{A}_B} \right)^{-\gamma} \right) + \frac{\theta c_c}{r} \left(1 - \left(\frac{A_0}{A_C} \right)^{-\gamma} \right) - \frac{\theta c_b^*}{r} \left(1 - \left(\frac{A_0}{A_B^*} \right)^{-\gamma} \right) + \\ &\quad \alpha A_B^* \left(\frac{A_0}{A_B^*} \right)^{-\gamma} - \alpha \bar{A}_B \left(\frac{A_0}{\bar{A}_B} \right)^{-\gamma}.\end{aligned}$$

Add and subtract terms and use Proposition 1 again

$$\begin{aligned}\Delta W &= \theta \left\{ \frac{\bar{c}_b}{r} \left(1 - \left(\frac{A_0}{\bar{A}_B} \right)^{-\gamma} \right) + \left(\frac{A_0}{\bar{A}_B} \right)^{-\gamma} (1 - \alpha) \bar{A}_B + \frac{c_c}{r} \left(1 - \left(\frac{A_0}{A_C} \right)^{-\gamma} \right) + \right. \\ &\quad \left(\frac{A_0}{A_C} \right)^{-\gamma} \left(\lambda \frac{c_c}{r} \right) - \frac{c_b^*}{r} \left(1 - \left(\frac{A_0}{A_B^*} \right)^{-\gamma} \right) - \left(\frac{A_0}{A_B^*} \right)^{-\gamma} (1 - \alpha) A_B^* - \\ &\quad \left(\frac{A_0}{\bar{A}_B} \right)^{-\gamma} (1 - \alpha) \bar{A}_B - \left(\frac{A_0}{A_C} \right)^{-\gamma} \left(\lambda \frac{c_c}{r} \right) + \left(\frac{A_0}{A_B^*} \right)^{-\gamma} (1 - \alpha) A_B^* \} + \\ &\quad \alpha A_B^* \left(\frac{A_0}{A_B^*} \right)^{-\gamma} - \alpha \bar{A}_B \left(\frac{A_0}{\bar{A}_B} \right)^{-\gamma} \\ &= \theta \{ U^B(A_0, \bar{A}_B; \bar{c}_b, c_c) + U^C(A_0, \bar{A}_B; c_b, c_c) - U^B(A_0, A_B^*; c_b^*, 0) - \\ &\quad \left(\frac{A_0}{\bar{A}_B} \right)^{-\gamma} (1 - \alpha) \bar{A}_B - \left(\frac{A_0}{A_C} \right)^{-\gamma} \left(\lambda \frac{c_c}{r} \right) + \left(\frac{A_0}{A_B^*} \right)^{-\gamma} (1 - \alpha) A_B^* \} + \\ &\quad \alpha A_B^* \left(\frac{A_0}{A_B^*} \right)^{-\gamma} - \alpha \bar{A}_B \left(\frac{A_0}{\bar{A}_B} \right)^{-\gamma}.\end{aligned}$$

Based on (4) and by re-grouping terms

$$\begin{aligned}\Delta W &= -\theta \left(\frac{A_0}{\bar{A}_B} \right)^{-\gamma} (1 - \alpha) \bar{A}_B - \theta \left(\frac{A_0}{A_C} \right)^{-\gamma} \left(\lambda \frac{c_c}{r} \right) + \theta \left(\frac{A_0}{A_B^*} \right)^{-\gamma} (1 - \alpha) A_B^* + \\ &\quad \alpha A_B^* \left(\frac{A_0}{A_B^*} \right)^{-\gamma} - \alpha \bar{A}_B \left(\frac{A_0}{\bar{A}_B} \right)^{-\gamma}.\end{aligned}$$

Finally,

$$\begin{aligned}
\Delta W &= -(\theta(1-\alpha) + \alpha) \left(\frac{A_0}{\bar{A}_B} \right)^{-\gamma} \bar{A}_B + (\theta(1-\alpha) + \alpha) \left(\frac{A_0}{A_B^*} \right)^{-\gamma} A_B^* - \\
&\quad \theta \left(\frac{A_0}{A_C} \right)^{-\gamma} \left(\lambda \frac{c_c}{r} \right) \\
&= (\theta + \alpha - \theta\alpha) \left(\left(\frac{A_0}{A_B^*} \right)^{-\gamma} A_B^* - \left(\frac{A_0}{\bar{A}_B} \right)^{-\gamma} \bar{A}_B \right) - \theta \left(\frac{A_0}{A_C} \right)^{-\gamma} \left(\lambda \frac{c_c}{r} \right),
\end{aligned}$$

which completes the proof of the first part of the proposition.

Denote $G(A_0, \bar{A}_B; \bar{c}_b, c_c) - G(A_0, A_B^*; c_b^*, 0)$ by ΔG . We know that $\Delta G = \Delta W$ and, therefore,

$$\Delta G = (\theta + \alpha - \theta\alpha) \left(\left(\frac{A_0}{A_B^*} \right)^{-\gamma} A_B^* - \left(\frac{A_0}{\bar{A}_B} \right)^{-\gamma} \bar{A}_B \right) - \theta \left(\frac{A_0}{A_C} \right)^{-\gamma} \left(\lambda \frac{c_c}{r} \right).$$

This leads to

$$\frac{\partial \Delta G}{\partial \bar{c}_b} = -(\theta + \alpha - \theta\alpha)(\gamma + 1) \left(\frac{A_0}{\bar{A}_B} \right)^{-\gamma} \frac{\partial \bar{A}_B}{\partial \bar{c}_b} - \theta \left(\frac{A_0}{A_C} \right)^{-\gamma} \frac{\lambda}{r} \frac{\partial c_c}{\partial \bar{c}_b}. \quad (\text{A.2})$$

Based on (5)

$$\begin{aligned}
\frac{\partial c_c}{\partial \bar{c}_b} &= \frac{-\frac{\partial U^B(A_t, \bar{A}_B; \bar{c}_{b+1})}{\partial \bar{c}_b}}{\frac{1}{r} \left(1 - (1-\lambda) \left(\frac{A_t}{A_C} \right)^{-\gamma} \right)} \\
&= \frac{\frac{1}{r} \left(1 - \left(\frac{A_t}{\bar{A}_B} \right)^{-\gamma} \right) - \frac{\bar{c}_b}{r} \gamma \left(\frac{A_t}{\bar{A}_B} \right)^{-\gamma} \frac{1}{\bar{A}_B} \frac{\partial \bar{A}_B}{\partial \bar{c}_b} + (1-\alpha)(\gamma+1) \left(\frac{A_t}{\bar{A}_B} \right)^{-\gamma} \frac{\partial \bar{A}_B}{\partial \bar{c}_b}}{\frac{1}{r} \left(1 - (1-\lambda) \left(\frac{A_t}{A_C} \right)^{-\gamma} \right)}. \quad (\text{A.3})
\end{aligned}$$

Given that $\bar{A}_B = \beta(1 - \theta)\bar{c}_b$ and $\frac{\partial \bar{A}_B}{\partial \bar{c}_b} = \beta(1 - \theta)$ and by plugging (A.3) into (A.2), we get

$$\begin{aligned}
\frac{\partial \Delta G}{\partial \bar{c}_b} &= -(\theta + \alpha - \theta\alpha)(\gamma + 1) \left(\frac{A_t}{\bar{A}_B} \right)^{-\gamma} \beta(1 - \theta) + \\
&\quad \theta \left(\frac{A_t}{A_C} \right)^{-\gamma} \frac{\lambda \frac{1}{r} \left(1 - \left(\frac{A_t}{\bar{A}_B} \right)^{-\gamma} \right) - \frac{1}{r} \gamma \left(\frac{A_t}{\bar{A}_B} \right)^{-\gamma} + (1 - \alpha)(\gamma + 1) \left(\frac{A_t}{\bar{A}_B} \right)^{-\gamma} \beta(1 - \theta)}{\frac{1}{r} \left(1 - (1 - \lambda) \left(\frac{A_t}{A_C} \right)^{-\gamma} \right)} \\
&= -(\theta + \alpha - \theta\alpha)(\gamma + 1) \left(\frac{A_t}{\bar{A}_B} \right)^{-\gamma} \beta(1 - \theta) + \\
&\quad \frac{\theta \lambda \left[\frac{1}{r} \left(1 - \left(\frac{A_t}{\bar{A}_B} \right)^{-\gamma} \right) - \frac{1}{r} \gamma \left(\frac{A_t}{\bar{A}_B} \right)^{-\gamma} + (1 - \alpha)(\gamma + 1) \left(\frac{A_t}{\bar{A}_B} \right)^{-\gamma} \beta(1 - \theta) \right]}{\left(\frac{A_t}{A_C} \right)^{\gamma} - 1 + \lambda}.
\end{aligned}$$

Since $\frac{\lambda}{\left(\frac{A_t}{A_C} \right)^{\gamma} - 1 + \lambda} \leq 1$

$$\begin{aligned}
\frac{\partial \Delta G}{\partial \bar{c}_b} &\leq -(\theta + \alpha - \theta\alpha)(\gamma + 1) \left(\frac{A_t}{\bar{A}_B} \right)^{-\gamma} \beta(1 - \theta) + \\
&\quad \theta \left[\frac{1}{r} \left(1 - \left(\frac{A_t}{\bar{A}_B} \right)^{-\gamma} \right) - \frac{1}{r} \gamma \left(\frac{A_t}{\bar{A}_B} \right)^{-\gamma} + (1 - \alpha)(\gamma + 1) \left(\frac{A_t}{\bar{A}_B} \right)^{-\gamma} \beta(1 - \theta) \right] \\
&= - \left(\frac{A_t}{\bar{A}_B} \right)^{-\gamma} \left[(\gamma + 1) \beta(1 - \theta) \alpha + \frac{\theta \gamma}{r} \right] + \frac{\theta}{r} \left(1 - \left(\frac{A_t}{\bar{A}_B} \right)^{-\gamma} \right) \\
&= - \left(\frac{A_t}{\bar{A}_B} \right)^{-\gamma} \left[(\gamma + 1) \left(\alpha \beta(1 - \theta) + \frac{\theta}{r} \right) \right] + \frac{\theta}{r}.
\end{aligned}$$

Based on the above

$$\left. \frac{\partial \Delta G}{\partial \bar{c}_b} \right|_{\bar{c}_b = c_b^*} \leq - \left(\frac{A_t}{A_B^*} \right)^{-\gamma} \left[(\gamma + 1) \left(\alpha \beta(1 - \theta) + \frac{\theta}{r} \right) \right] + \frac{\theta}{r}. \quad (\text{A.4})$$

From (4)

$$\begin{aligned}
\frac{\beta(1 - \theta)c_b^*}{A_t} &= \left(\frac{\theta}{r} \right)^{\frac{1}{\gamma}} \left[(\gamma + 1) \left(\frac{\theta}{r} + \alpha \beta(1 - \theta) \right) \right]^{-\frac{1}{\gamma}} \\
\left(\frac{A_t}{A_B^*} \right)^{-\gamma} &= \left(\frac{\theta}{r} \right) \left[(\gamma + 1) \left(\frac{\theta}{r} + \alpha \beta(1 - \theta) \right) \right]^{-1}
\end{aligned}$$

By using this in (A.4)

$$\left. \frac{\partial \Delta G}{\partial \bar{c}_b} \right|_{\bar{c}_b = c_b^*} \leq -\frac{\theta}{r} + \frac{\theta}{r} = 0.$$

This means that there exists $\bar{c}_1$ such that, for any $c_c \in (0, \bar{c}_1)$, $\Delta G \leq 0$. Given that $G(A_0, A_B^*; c_b^*, 0)$ is fixed, for any $c_c \in (0, \bar{c}_1)$, $G(A_0, \bar{A}_B; \bar{c}_b, c_c) \geq G(A_0, A_B^*; c_b^*, 0)$. This proves the second part of the proposition.

As for the bankruptcy costs, based on equation (A.2), $A_B^* = \beta(1 - \theta)c_b^*$ and $\bar{A}_B = \beta(1 - \theta)\bar{c}_b$. Given the closed-form solutions from Proposition 1

$$\begin{aligned} BC(A_0, \bar{A}_B; \bar{c}_b) - BC(A_0, A_B^*; c_b^*) &= \alpha \bar{A}_B \left(\frac{A_0}{\bar{A}_B} \right)^{-\gamma} - \alpha A_B^* \left(\frac{A_0}{A_B^*} \right)^{-\gamma} \\ &= \alpha \bar{c}_b \beta (1 - \theta) \left(\frac{\bar{c}_b \beta (1 - \theta)}{A_0} \right)^\gamma - \\ &\quad \alpha c_b^* \beta (1 - \theta) \left(\frac{c_b^* \beta (1 - \theta)}{A_0} \right)^\gamma \\ &= (\bar{c}_b^{\gamma+1} - (c_b^*)^{\gamma+1}) \alpha \frac{(\beta(1 - \theta))^{\gamma+1}}{A_0^\gamma}. \end{aligned}$$

Since $\bar{c}_b < c_b^*$, the last term is strictly negative. Therefore, $BC(A_0, \bar{A}_B; \bar{c}_b) < BC(A_0, A_B^*; c_b^*)$. $\square$

Proof of Proposition 4. Denote $W(A_t, \bar{A}_B; \bar{c}_b, c_c) - W(A_t, \hat{A}_B; \hat{c}_b, 0)$ by $\Delta \hat{W}$. When the capital structure of the firm includes only equity and straight debt the closed-form solution for the value of equity is

$$W(A_t, \hat{A}_B; \hat{c}_b, 0) = A_t - \frac{\hat{c}_b(1 - \theta)}{r} \left(1 - \left(\frac{A_t}{\hat{A}_B} \right)^{-\gamma} \right) - \hat{A}_B \left(\frac{A_t}{\hat{A}_B} \right)^{-\gamma}. \quad (\text{A.5})$$

Based on (A.5) and the closed-form solution for $W(A_t, \bar{A}_B; \bar{c}_b, c_c)$ from Proposition 1

$$\begin{aligned}
\Delta \hat{W} &= A_t - \frac{\bar{c}_b(1-\theta)}{r} \left(1 - \left(\frac{A_t}{\bar{A}_B}\right)^{-\gamma}\right) - \frac{c_c(1-\theta)}{r} \left(1 - \left(\frac{A_t}{\bar{A}_C}\right)^{-\gamma}\right) - \bar{A}_B \left(\frac{A_t}{\bar{A}_B}\right)^{-\gamma} - \\
&\quad \left(\lambda \frac{c_c}{r}\right) \left(\frac{A_t}{\bar{A}_C}\right)^{-\gamma} - A_t + \frac{\hat{c}_b(1-\theta)}{r} \left(1 - \left(\frac{A_t}{\hat{A}_B}\right)^{-\gamma}\right) + \hat{A}_B \left(\frac{A_t}{\hat{A}_B}\right)^{-\gamma} \\
&= \frac{(\hat{c}_b - \bar{c}_b)(1-\theta)}{r} \left(1 - \left(\frac{A_t}{\bar{A}_B}\right)^{-\gamma}\right) - \frac{c_c(1-\theta)}{r} \left(1 - \left(\frac{A_t}{\bar{A}_C}\right)^{-\gamma}\right) - \\
&\quad \left(\lambda \frac{c_c}{r}\right) \left(\frac{A_t}{\bar{A}_C}\right)^{-\gamma} + \frac{\hat{c}_b(1-\theta)}{r} \left(\left(\frac{A_t}{\bar{A}_B}\right)^{-\gamma} - \left(\frac{A_t}{\hat{A}_B}\right)^{-\gamma}\right) + \hat{A}_B \left(\frac{A_t}{\hat{A}_B}\right)^{-\gamma} - \bar{A}_B \left(\frac{A_t}{\bar{A}_B}\right)^{-\gamma}.
\end{aligned}$$

Multiply both sides of (7) by $(1-\theta)$ and use the result to reduce the first three terms after the equal sign above to get

$$\begin{aligned}
\Delta \hat{W} &= -\theta \left(\lambda \frac{c_c}{r}\right) \left(\frac{A_t}{\bar{A}_C}\right)^{-\gamma} + \frac{\hat{c}_b(1-\theta)}{r} \left(\left(\frac{A_t}{\bar{A}_B}\right)^{-\gamma} - \left(\frac{A_t}{\hat{A}_B}\right)^{-\gamma}\right) + \\
&\quad \hat{A}_B \left(\frac{A_t}{\hat{A}_B}\right)^{-\gamma} - \bar{A}_B \left(\frac{A_t}{\bar{A}_B}\right)^{-\gamma}.
\end{aligned} \tag{A.6}$$

Continue with showing that $\Delta \hat{W} < 0$. From (A.6)

$$\begin{aligned}
\Delta \hat{W} &= \frac{\hat{c}_b(1-\theta)}{r} \left(\frac{A_t}{\bar{A}_B}\right)^{-\gamma} - \bar{A}_B \left(\frac{A_t}{\bar{A}_B}\right)^{-\gamma} - \\
&\quad \left(\frac{\hat{c}_b(1-\theta)}{r} \left(\frac{A_t}{\hat{A}_B}\right)^{-\gamma} - \left(\frac{A_t}{\hat{A}_B}\right)^{-\gamma}\right) - \theta \left(\lambda \frac{c_c}{r}\right) \left(\frac{A_t}{\bar{A}_C}\right)^{-\gamma} \\
&= -(H(\hat{A}_B) - H(\bar{A}_B)) - \theta \left(\lambda \frac{c_c}{r}\right) \left(\frac{A_t}{\bar{A}_C}\right)^{-\gamma},
\end{aligned} \tag{A.7}$$

where $H(X) \equiv \frac{\hat{c}_b(1-\theta)}{r} \left(\frac{A_t}{X}\right)^{-\gamma} - X \left(\frac{A_t}{X}\right)^{-\gamma}$. $H(X)$ is such that

$$\begin{aligned}
H'(X) &= \gamma \frac{\hat{c}_b(1-\theta)}{r} \left(\frac{A_t}{X}\right)^{-\gamma} \frac{1}{X} - (1+\gamma) \left(\frac{A_t}{X}\right)^{-\gamma} \\
&= \left(\frac{A_t}{X}\right)^{-\gamma} \frac{1}{X} \left(\gamma \frac{\hat{c}_b}{r} (1-\theta) - (1+\gamma)X\right).
\end{aligned}$$

Since $\hat{A}_B = \frac{\gamma(1-\theta)\hat{c}_b}{r(1+\gamma)}$

$$\begin{aligned} H'(\hat{A}_B) &= \left(\frac{A_t}{\hat{A}_B}\right)^{-\gamma} \frac{1}{\hat{A}_B} \left(\gamma \frac{\hat{c}_b}{r} (1-\theta) - (1+\gamma) \frac{\gamma(1-\theta)\hat{c}_b}{r(1+\gamma)} \right) \\ &= \left(\frac{A_t}{\hat{A}_B}\right)^{-\gamma} \frac{1}{\hat{A}_B} \left(\gamma \frac{\hat{c}_b}{r} (1-\theta) - (1-\theta) \gamma \frac{\hat{c}_b}{r} \right) \\ &= 0. \end{aligned}$$

It is also clear from above that if $0 < X < \hat{A}$, then $H'(X) > 0$. $H(X)$ is an increasing function of X on $(0, \hat{A}_B)$.

Since $0 < \bar{A}_B < \hat{A}_B$, $H(\hat{A}_B) > H(\bar{A}_B)$ and, based on (A.7), $\Delta \hat{W} < 0$. The value of equity always decreases.

This proves item (i).

Denote $G(A_t, \bar{A}_B; \bar{c}_b, c_c) - G(A_t, \hat{A}_B; \hat{c}_b, 0)$ by $\Delta \hat{G}$. When the capital structure of the firm includes only equity and straight debt the closed-form solution for the total value of the firm is

$$G(A_t, \hat{A}_B; \hat{c}_b, 0) = A_t + \frac{\hat{c}_b \theta}{r} \left(1 - \left(\frac{A_t}{\hat{A}_B} \right)^{-\gamma} \right) - \alpha \hat{A}_B \left(\frac{A_t}{\hat{A}_B} \right)^{-\gamma}. \quad (\text{A.8})$$

Given (A.8) and the closed-form solution for $G(A_t, \bar{A}_B; \bar{c}_b, c_c)$ from Proposition 1

$$\begin{aligned} \Delta \hat{G} &= A_t + \frac{\theta \bar{c}_b}{r} \left(1 - \left(\frac{A_t}{\bar{A}_B} \right)^{-\gamma} \right) + \frac{\theta c_c}{r} \left(1 - \left(\frac{A_t}{\bar{A}_C} \right)^{-\gamma} \right) - \alpha \bar{A}_B \left(\frac{A_t}{\bar{A}_B} \right)^{-\gamma} - \\ &\quad A_t - \frac{\hat{c}_b \theta}{r} \left(1 - \left(\frac{A_t}{\hat{A}_B} \right)^{-\gamma} \right) + \alpha \hat{A}_B \left(\frac{A_t}{\hat{A}_B} \right)^{-\gamma} \\ &= \frac{\theta \bar{c}_b}{r} \left(1 - \left(\frac{A_t}{\bar{A}_B} \right)^{-\gamma} \right) + \frac{\theta c_c}{r} \left(1 - \left(\frac{A_t}{\bar{A}_C} \right)^{-\gamma} \right) - \alpha \bar{A}_B \left(\frac{A_t}{\bar{A}_B} \right)^{-\gamma} - \\ &\quad \frac{\hat{c}_b \theta}{r} \left(1 - \left(\frac{A_t}{\hat{A}_B} \right)^{-\gamma} \right) + \alpha \hat{A}_B \left(\frac{A_t}{\hat{A}_B} \right)^{-\gamma}. \end{aligned}$$

Multiply both sides of (7) by θ and replace $\frac{\theta c_c}{r} \left(1 - \left(\frac{A_t}{A_C}\right)^{-\gamma}\right)$ above to get

$$\begin{aligned}
\Delta \hat{G} &= \frac{\theta \bar{c}_b}{r} \left(1 - \left(\frac{A_t}{\bar{A}_B}\right)^{-\gamma}\right) + \frac{\theta(\hat{c}_b - \bar{c}_b)}{r} \left(1 - \left(\frac{A_t}{\bar{A}_B}\right)^{-\gamma}\right) - \theta \left(\frac{A_t}{A_C}\right)^{-\gamma} \left(\lambda \frac{c_c}{r}\right) - \\
&\quad \frac{\hat{c}_b \theta}{r} \left(1 - \left(\frac{A_t}{\hat{A}_B}\right)^{-\gamma}\right) + \alpha \left(\hat{A}_B \left(\frac{A_t}{\hat{A}_B}\right)^{-\gamma} - \bar{A}_B \left(\frac{A_t}{\bar{A}_B}\right)^{-\gamma}\right) \\
&= \frac{\hat{c}_b \theta}{r} \left(\left(\frac{A_t}{\hat{A}_B}\right)^{-\gamma} - \left(\frac{A_t}{\bar{A}_B}\right)^{-\gamma}\right) + \alpha \left(\hat{A}_B \left(\frac{A_t}{\hat{A}_B}\right)^{-\gamma} - \bar{A}_B \left(\frac{A_t}{\bar{A}_B}\right)^{-\gamma}\right) - \\
&\quad \theta \left(\frac{A_t}{A_C}\right)^{-\gamma} \left(\lambda \frac{c_c}{r}\right).
\end{aligned}$$

This proves the first part of item (ii) of the proposition.

We continue with showing that $\Delta \hat{G} > \Delta \hat{W}$. Based on (A.6) and the above result

$$\begin{aligned}
\Delta \hat{G} - \Delta \hat{W} &= \frac{\hat{c}_b}{r} \left(\left(\frac{A_t}{\hat{A}_B}\right)^{-\gamma} - \left(\frac{A_t}{\bar{A}_B}\right)^{-\gamma}\right) - (1 - \alpha) \left(\hat{A}_B \left(\frac{A_t}{\hat{A}_B}\right)^{-\gamma} - \bar{A}_B \left(\frac{A_t}{\bar{A}_B}\right)^{-\gamma}\right) \\
&= \frac{\hat{c}_b}{r} \left(\frac{A_t}{\hat{A}_B}\right)^{-\gamma} - (1 - \alpha) \hat{A}_B \left(\frac{A_t}{\hat{A}_B}\right)^{-\gamma} - \\
&\quad \left(\frac{\hat{c}_b}{r} \left(\frac{A_t}{\bar{A}_B}\right)^{-\gamma} - (1 - \alpha) \bar{A}_B \left(\frac{A_t}{\bar{A}_B}\right)^{-\gamma}\right) \\
&= F(\hat{A}_B) - F(\bar{A}_B),
\end{aligned} \tag{A.9}$$

where $F(X) \equiv \frac{\hat{c}_b}{r} \left(\frac{A_t}{X}\right)^{-\gamma} - (1 - \alpha) X \left(\frac{A_t}{X}\right)^{-\gamma}$. $F(X)$ is such that

$$\begin{aligned}
F'(X) &= \gamma \frac{\hat{c}_b}{r} \left(\frac{A_t}{X}\right)^{-\gamma} \frac{1}{X} - (1 - \alpha)(1 + \gamma) \left(\frac{A_t}{X}\right)^{-\gamma} \\
&= \left(\frac{A_t}{X}\right)^{-\gamma} \frac{1}{X} \left(\gamma \frac{\hat{c}_b}{r} - (1 - \alpha)(1 + \gamma) X\right).
\end{aligned}$$

Note, that $\hat{A}_B = \frac{\gamma(1-\theta)\hat{c}_b}{r(1+\gamma)}$ and, therefore,

$$\begin{aligned} F'(\hat{A}_B) &= \left(\frac{A_t}{\hat{A}_B}\right)^{-\gamma} \frac{1}{\hat{A}_B} \left(\gamma \frac{\hat{c}_b}{r} - (1-\alpha)(1+\gamma) \frac{\gamma(1-\theta)\hat{c}_b}{r(1+\gamma)} \right) \\ &= \left(\frac{A_t}{\hat{A}_B}\right)^{-\gamma} \frac{1}{\hat{A}_B} \left(\gamma \frac{\hat{c}_b}{r} - (1-\alpha)(1-\theta) \gamma \frac{\hat{c}_b}{r} \right) \\ &= \left(\frac{A_t}{\hat{A}_B}\right)^{-\gamma} \frac{1}{\hat{A}_B} \gamma \frac{\hat{c}_b}{r} (1 - (1-\alpha)(1-\theta)). \end{aligned}$$

By assumption, $\alpha \in [0, 1]$ and $\theta \in (0, 1)$, so $(1 - (1-\alpha)(1-\theta)) > 0$. It follows that $F'(\hat{A}_B) > 0$ for all $0 < X \leq \hat{A}_B$, and, since $0 < \bar{A}_B < \hat{A}_B$, $F(\hat{A}_B) > F(\bar{A}_B)$. Finally, based on (A.9), $\Delta\hat{G} > \Delta\hat{W}$.

We continue with proving the last statement of item (ii).

$$\begin{aligned} \frac{\partial \Delta\hat{G}}{\partial \bar{c}_b} &= \left(-\gamma \left(\frac{A_t}{\bar{A}_B}\right)^{-\gamma} \frac{1}{\bar{A}_B} \frac{\hat{c}_b \theta}{r} - \alpha(1+\gamma) \left(\frac{A_t}{\bar{A}_B}\right)^{-\gamma} \right) \frac{\partial \bar{A}_B}{\partial \bar{c}_b} - \\ &\quad \theta \left(\frac{A_t}{A_C}\right)^{-\gamma} \frac{1}{r} \frac{1 - \left(1 - \left(\frac{A_t}{\bar{A}_B}\right)^{-\gamma}\right) - (\hat{c}_b - \bar{c}_b) \gamma \left(\frac{A_t}{\bar{A}_B}\right)^{-\gamma} \frac{1}{\bar{A}_B} \frac{\partial \bar{A}_B}{\partial \bar{c}_b}}{1 - (1-\lambda) \left(\frac{A_t}{A_C}\right)^{-\gamma}} \end{aligned}$$

where $\frac{\partial \bar{A}_B}{\partial \bar{c}_b} = \beta(1-\theta)$. Based on the above

$$\begin{aligned} \left. \frac{\partial \Delta\hat{G}}{\partial \bar{c}_b} \right|_{\bar{c}_b = \hat{c}_b} &= -\left(\frac{A_t}{\hat{A}_B}\right)^{-\gamma} \left(\frac{\gamma}{\hat{A}_B} \frac{\bar{c}_b \theta}{r} + \alpha(1+\gamma) \right) \beta(1-\theta) + \\ &\quad \frac{\theta}{r} \left(\frac{A_t}{A_C}\right)^{-\gamma} \frac{1 - \left(\frac{A_t}{\bar{A}_B}\right)^{-\gamma}}{1 - (1-\lambda) \left(\frac{A_t}{A_C}\right)^{-\gamma}} \\ &= -\left(\frac{A_t}{\hat{A}_B}\right)^{-\gamma} \left(\frac{\gamma \theta}{r} + \alpha(1+\gamma) \beta(1-\theta) \right) + \\ &\quad \frac{\theta}{r} \left(\frac{A_t}{A_C}\right)^{-\gamma} \frac{1 - \left(\frac{A_t}{\bar{A}_B}\right)^{-\gamma}}{1 - (1-\lambda) \left(\frac{A_t}{A_C}\right)^{-\gamma}} \\ &= -\left(\frac{A_t}{\hat{A}_B}\right)^{-\gamma} \left(\frac{\gamma \theta}{r} + \alpha(1+\gamma) \beta(1-\theta) \right) + \\ &\quad \frac{\theta}{r} \frac{1 - \left(\frac{A_t}{\bar{A}_B}\right)^{-\gamma}}{\left(\frac{A_t}{A_C}\right)^{-\gamma} - (1-\lambda)}. \end{aligned}$$

For $\left(\frac{A_t}{A_c}\right)^\gamma - (1 - \lambda) \geq 1$ or $\lambda \geq 2 - \left(\frac{A_t}{A_c}\right)^\gamma$

$$\begin{aligned} \frac{\partial \Delta \hat{G}}{\partial \bar{c}_b} \Big|_{\bar{c}_b = \hat{c}_b} &\leq - \left(\frac{A_t}{\hat{A}_B} \right)^{-\gamma} \left(\frac{\gamma \theta}{r} + \alpha(1 + \gamma)\beta(1 - \theta) \right) + \frac{\theta}{r} \left(1 - \left(\frac{A_t}{\hat{A}_B} \right)^{-\gamma} \right) \\ &= - \left(\frac{A_t}{\hat{A}_B} \right)^{-\gamma} (1 + \gamma) \left(\frac{\theta}{r} + \alpha\beta(1 - \theta) \right) + \frac{\theta}{r}. \end{aligned} \quad (\text{A.10})$$

Next, assume that $\hat{c}_b \geq c_b^*$. Then, based on (4)

$$\begin{aligned} \hat{c}_b &\geq \frac{A_t}{\beta(1 - \theta)} \left(\frac{\theta}{r} \right)^{\frac{1}{\gamma}} \left[(\gamma + 1) \left(\frac{\theta}{r} + \alpha\beta(1 - \theta) \right) \right]^{-\frac{1}{\gamma}} \\ \frac{\beta(1 - \theta)\hat{c}_b}{A_t} &\geq \left(\frac{\theta}{r} \right)^{\frac{1}{\gamma}} \left[(\gamma + 1) \left(\frac{\theta}{r} + \alpha\beta(1 - \theta) \right) \right]^{-\frac{1}{\gamma}}. \end{aligned}$$

Based on (2)

$$\begin{aligned} \frac{\hat{A}_B}{A_t} &\geq \left(\frac{\theta}{r} \right)^{\frac{1}{\gamma}} \left[(\gamma + 1) \left(\frac{\theta}{r} + \alpha\beta(1 - \theta) \right) \right]^{-\frac{1}{\gamma}} \\ \left(\frac{A_t}{\hat{A}_B} \right)^{-\gamma} &\geq \frac{\theta}{r} \left[(\gamma + 1) \left(\frac{\theta}{r} + \alpha\beta(1 - \theta) \right) \right]^{-1} \\ - \left(\frac{A_t}{\hat{A}_B} \right)^{-\gamma} &\leq - \frac{\theta}{r} \left[(\gamma + 1) \left(\frac{\theta}{r} + \alpha\beta(1 - \theta) \right) \right]^{-1}. \end{aligned} \quad (\text{A.11})$$

By using (A.11) in (A.10), we get

$$\frac{\partial \Delta \hat{G}}{\partial \bar{c}_b} \Big|_{\bar{c}_b = \hat{c}_b} \leq - \frac{\theta}{r} + \frac{\theta}{r} = 0.$$

This means that there exists $\bar{c}_1$ such that, for any $c_c \in (0, \bar{c}_1)$, $\Delta \hat{G} \leq 0$. Given that $G(A_0, A_B^*; c_b^*, 0)$ is fixed, for any $c_c \in (0, \bar{c}_1)$, $G(A_0, \bar{A}_B; \bar{c}_b, c_c) \geq G(A_0, A_B^*; c_b^*, 0)$.

Finally, we prove item (iii) of the proposition. Since $\hat{c}_b > \bar{c}_b$, based on (A.2), the optimal default-triggering boundary drops from $\hat{A}_B = \beta(1 - \theta)\hat{c}_b$ to $\bar{A}_B = \beta(1 - \theta)\bar{c}_b$. Given this and the closed-form solution for the cost of bankruptcy from Proposition 1

$$BC(A_t, \bar{A}_B; \bar{c}_b) - BC(A_t, \hat{A}_B; \hat{c}_b) = \alpha \bar{A}_B \left(\frac{A_t}{\bar{A}_B} \right)^{-\gamma} - \alpha \hat{A}_B \left(\frac{A_t}{\hat{A}_B} \right)^{-\gamma}$$

$$\begin{aligned}
&= \alpha \bar{c}_b \beta (1 - \theta) \left(\frac{\bar{c}_b \beta (1 - \theta)}{A_t} \right)^\gamma - \alpha \hat{c}_b \beta (1 - \theta) \left(\frac{\hat{c}_b \beta (1 - \theta)}{A_t} \right)^\gamma \\
&= (\bar{c}_b^{\gamma+1} - (\hat{c}_b)^{\gamma+1}) \alpha \frac{(\beta(1 - \theta))^{\gamma+1}}{A_t^\gamma}.
\end{aligned}$$

Since $\bar{c}_b < \hat{c}_b$, the last term above is strictly negative. Therefore, $BC(A_t, \bar{A}_B; \bar{c}_b) < BC(A_t, \hat{A}_B; \hat{c}_b)$. $\square$

Proof of Proposition 5. We can use (2) to re-write (8) as

$$S(A_t; c_b, c_c) = \left(\frac{c_b}{r} - c_b(1 - \theta)\beta \right) \left(\frac{A_t}{c_b(1 - \theta)\beta} \right)^{-\gamma} = c_b \left(\frac{1}{r} - (1 - \theta)\beta \right) \left(\frac{c_b(1 - \theta)\beta}{A_t} \right)^\gamma.$$

Given that $\beta = \frac{\gamma}{r(1 + \gamma)}$,

$$\frac{dS(A_t; c_b, 0)}{dc_b} = \left(\frac{1 + \gamma}{r} - (1 + \gamma)(1 - \theta)\beta \right) \left(\frac{A_t}{A_B} \right)^{-\gamma} = \frac{1}{r}(1 + \gamma\theta) \left(\frac{A_t}{A_B} \right)^{-\gamma} > 0.$$

Since $BC(A_t; \hat{c}_b, 0) = 0$, the total value of the firm in the presence of government subsidy when it does not issue CCB is

$$G(A_t; c_b, 0) = A_t + TB(A_t; c_b, 0) + S(A_t; c_b, 0).$$

Based on the closed-form solution for $TB(A_t; c_b, 0)$ and equation (8)

$$G(A_t; c_b, 0) = A_t + \frac{\theta c_b}{r} \left(1 - \left(\frac{A_t}{A_B} \right)^{-\gamma} \right) + \left(\frac{c_b}{r} - A_B \right) \left(\frac{A_t}{A_B} \right)^{-\gamma}.$$

Given budget equation, it is easy to see that $G(A_t; c_b, 0)$ from above is higher than the total value of the firm before the guarantee was issued by

$$S(A_t; c_b, 0) + BC(A_t; c_b) = \left(\frac{c_b}{r} - A_B \right) \left(\frac{A_t}{A_B} \right)^{-\gamma} + \alpha A_B \left(\frac{A_t}{A_B} \right)^{-\gamma} > 0.$$

$\square$

Proof of Proposition 6. The only statement that remains to be proved is that $G(A_t; c_b^g, 0) - G(A_t; \bar{c}_b, c_c) > 0$.

Denote $G(A_t; c_b^g, 0) - G(A_t; \bar{c}_b, c_c)$ by $\Delta\tilde{G}$. When the capital structure of the firm includes only equity and straight debt the closed-form solution for the value of tax benefits is

$$TB(A_t; c_b^g, 0) = \frac{\theta c_b^g}{r} \left(1 - \left(\frac{A_t}{A_B^g} \right)^{-\gamma} \right). \quad (\text{A.12})$$

Given (14), (A.12), the closed-form solution for the value of tax benefits from Proposition 1, and equation (8) for the value of government subsidy

$$\begin{aligned} \Delta\tilde{G} = & \frac{\theta c_b^g}{r} \left(1 - \left(\frac{A_t}{A_B^g} \right)^{-\gamma} \right) - \frac{\theta \bar{c}_b}{r} \left(1 - \left(\frac{A_t}{\bar{A}_B} \right)^{-\gamma} \right) - \frac{\theta c_c}{r} \left(1 - \left(\frac{A_t}{A_C} \right)^{-\gamma} \right) + \\ & \left(\frac{c_b^g}{r} - A_B^g \right) \left(\frac{A_t}{A_B^g} \right)^{-\gamma} - \left(\frac{\bar{c}_b}{r} - \bar{A}_B \right) \left(\frac{A_t}{\bar{A}_B} \right)^{-\gamma}. \end{aligned} \quad (\text{A.13})$$

By multiplying both side of equation (11) by θ and using the closed-form solutions for the values of straight and CCB, we get

$$\begin{aligned} \frac{c_b^g \theta}{r} \left(1 - \left(\frac{A_t}{A_B^g} \right)^{-\gamma} \right) + \left(\frac{A_t}{A_B^g} \right)^{-\gamma} (1 - \alpha) A_B^g \theta = & \frac{\bar{c}_b \theta}{r} \left(1 - \left(\frac{A_t}{\bar{A}_B} \right)^{-\gamma} \right) + \left(\frac{A_t}{\bar{A}_B} \right)^{-\gamma} (1 - \alpha) \bar{A}_B \theta + \\ & \frac{c_c \theta}{r} \left(1 - \left(\frac{A_t}{A_C} \right)^{-\gamma} \right) + \theta \left(\lambda \frac{c_c}{r} \right) \left(\frac{A_t}{A_C} \right)^{-\gamma}. \end{aligned}$$

By rearranging terms

$$\begin{aligned} \frac{c_b^g \theta}{r} \left(1 - \left(\frac{A_t}{A_B^g} \right)^{-\gamma} \right) - \frac{\bar{c}_b \theta}{r} \left(1 - \left(\frac{A_t}{\bar{A}_B} \right)^{-\gamma} \right) - \frac{c_c \theta}{r} \left(1 - \left(\frac{A_t}{A_C} \right)^{-\gamma} \right) = & \left(\frac{A_t}{\bar{A}_B} \right)^{-\gamma} (1 - \alpha) \bar{A}_B \theta + \\ \theta \left(\lambda \frac{c_c}{r} \right) \left(\frac{A_t}{A_C} \right)^{-\gamma} - \left(\frac{A_t}{A_B^g} \right)^{-\gamma} (1 - \alpha) A_B^g \theta. \end{aligned} \quad (\text{A.14})$$

Now we can use (A.14) in (A.13) to get

$$\begin{aligned}
\Delta \tilde{G} &= \left(\frac{A_t}{\bar{A}_B} \right)^{-\gamma} (1 - \alpha) \bar{A}_B \theta + \theta \left(\lambda \frac{c_c}{r} \right) \left(\frac{A_t}{A_C} \right)^{-\gamma} - \left(\frac{A_t}{A_B^g} \right)^{-\gamma} (1 - \alpha) A_B^g \theta + \\
&\quad \left(\frac{c_b^g}{r} - A_B^g \right) \left(\frac{A_t}{A_B^g} \right)^{-\gamma} - \left(\frac{\bar{c}_b}{r} - \bar{A}_B \right) \left(\frac{A_t}{\bar{A}_B} \right)^{-\gamma} \\
&= \theta \left(\lambda \frac{c_c}{r} \right) \left(\frac{A_t}{A_C} \right)^{-\gamma} + \left(\frac{c_b^g}{r} \left(\frac{A_t}{A_B^g} \right)^{-\gamma} - \frac{\bar{c}_b}{r} \left(\frac{A_t}{\bar{A}_B} \right)^{-\gamma} \right) + ((1 - \alpha)\theta + 1) \bar{A}_B \left(\frac{A_t}{\bar{A}_B} \right)^{-\gamma} - \\
&\quad ((1 - \alpha)\theta + 1) A_B^g \left(\frac{A_t}{A_B^g} \right)^{-\gamma}.
\end{aligned}$$

Given (2) and $\beta = \frac{\gamma}{r(1+\gamma)}$,

$$\begin{aligned}
\Delta \tilde{G} &= \theta \left(\lambda \frac{c_c}{r} \right) \left(\frac{A_t}{A_C} \right)^{-\gamma} + \left(\frac{c_b^g}{r} \left(\frac{A_t}{A_B^g} \right)^{-\gamma} - \frac{\bar{c}_b}{r} \left(\frac{A_t}{\bar{A}_B} \right)^{-\gamma} \right) - \\
&\quad ((1 - \alpha)\theta + 1) (1 - \theta) \frac{\gamma}{(1 + \gamma)} \left(\frac{c_b^g}{r} \left(\frac{A_t}{A_B^g} \right)^{-\gamma} - \frac{\bar{c}_b}{r} \left(\frac{A_t}{\bar{A}_B} \right)^{-\gamma} \right) \\
&= \theta \left(\lambda \frac{c_c}{r} \right) \left(\frac{A_t}{A_C} \right)^{-\gamma} + \\
&\quad \left(1 - ((1 - \alpha)\theta + 1) (1 - \theta) \frac{\gamma}{(1 + \gamma)} \right) \left(\frac{c_b^g}{r} \left(\frac{A_t}{A_B^g} \right)^{-\gamma} - \frac{\bar{c}_b}{r} \left(\frac{A_t}{\bar{A}_B} \right)^{-\gamma} \right). \tag{A.15}
\end{aligned}$$

In (A.15) the first term on the right-hand side is positive, $\theta \left(\lambda \frac{c_c}{r} \right) \left(\frac{A_t}{A_C} \right)^{-\gamma} > 0$. Also, for $c_b^g > \bar{c}_b$, $\left(\frac{c_b^g}{r} \left(\frac{A_t}{A_B^g} \right)^{-\gamma} - \frac{\bar{c}_b}{r} \left(\frac{A_t}{\bar{A}_B} \right)^{-\gamma} \right) >$

0. Finally,

$$1 - ((1 - \alpha)\theta + 1) (1 - \theta) \frac{\gamma}{(1 + \gamma)} > 1 - (\theta + 1) (1 - \theta) \frac{\gamma}{(1 + \gamma)} = 1 - (1 - \theta^2) \frac{\gamma}{(1 + \gamma)} > 0.$$

All the terms on the right-hand side of equation (A.15) are positive. Therefore, $\Delta \tilde{G} > 0$. $\square$

Proof of Proposition 7. Consider a TBTF firm with a capital structure that includes equity and straight debt paying coupon c_b . We try to analyze the effect of replacing a portion of straight debt with CCB on the values of government subsidy and equity.

In the presence of government subsidy at any time t the following budget equation holds

$$A_t + TB(A_t; \hat{c}_b, 0) + S(A_t; \hat{c}_b, 0) = W(A_t; \hat{c}_b, 0) + U^B(A_t; \hat{c}_b, 0). \quad (\text{A.16})$$

There are no bankruptcy costs, $BC(A_t; \hat{c}_b, 0) = 0$, and debt is risk-free, $U^B(A_t; \hat{c}_b, 0) = \frac{\hat{c}_b}{r}$.

The firm is to replace a portion of its current straight debt with CCB paying c_c . The remaining straight debt will be paying coupon $\bar{c}_b$, such that $\bar{c}_b < \hat{c}_b$. The government guarantee remains in place, so straight debt will still be risk-free, $U^B(A_t; \bar{c}_b, c_c) = \frac{\bar{c}_b}{r}$. As before, straight debt holders should be indifferent between exchanging their holdings for CCB and continuing to hold straight debt

$$U^B(A_t; \hat{c}_b, 0) = U^C(A_t; \bar{c}_b, c_c) + U^B(A_t; \bar{c}_b, c_c). \quad (\text{A.17})$$

Equation (A.17) is equivalent to equation (6) in Section 4. The key difference, though, is that after a TBTF firm announces its decision to replace straight debt with CCB the value of existing straight debt does not change. Debt is risk-free and, therefore, contrary to what we had before, the announcement does not affect its default boundary.

After the firm replaces a portion of its straight debt with CCB for any time t the following budget equation will hold

$$A_t + TB(A_t; \bar{c}_b, c_c) + S(A_t; \bar{c}_b, c_c) = W(A_t; \bar{c}_b, c_c) + U^B(A_t; \bar{c}_b, c_c) + U^C(A_t; \bar{c}_b, c_c). \quad (\text{A.18})$$

Given (A.16), (A.17) and (A.18),

$$\begin{aligned} W(A_t; \hat{c}_b, 0) - W(A_t; \bar{c}_b, c_c) &= TB(A_t; \hat{c}_b, 0) - TB(A_t; \bar{c}_b, c_c) + \\ &S(A_t; \hat{c}_b, 0) - S(A_t; \bar{c}_b, c_c). \end{aligned} \quad (\text{A.19})$$

Since $\hat{c}_b > \bar{c}_b$, based on Proposition 5 (part 1), $S(A_t; \hat{c}_b, 0) - S(A_t; \bar{c}_b, c_c) > 0$. We proved the first part of the proposition.

Denote $W(A_t; \hat{c}_b, 0) - W(A_t; \bar{c}_b, c_c)$ by $\Delta \tilde{W}$. When the capital structure of the firm includes only equity and straight debt the closed-form solution for the value of tax benefits is

$$TB(A_t; \hat{c}_b, 0) = \frac{\theta \hat{c}_b}{r} \left(1 - \left(\frac{A_t}{\hat{A}_B} \right)^{-\gamma} \right). \quad (\text{A.20})$$

Given (A.19), (A.20), the closed-form solution for the value of tax benefits from Proposition 1, and equation (8) for the value of government subsidy

$$\begin{aligned} \Delta \tilde{W} = & \frac{\theta \hat{c}_b}{r} \left(1 - \left(\frac{A_t}{\hat{A}_B} \right)^{-\gamma} \right) - \frac{\theta \bar{c}_b}{r} \left(1 - \left(\frac{A_t}{\bar{A}_B} \right)^{-\gamma} \right) - \frac{\theta c_c}{r} \left(1 - \left(\frac{A_t}{\bar{A}_C} \right)^{-\gamma} \right) + \\ & \left(\frac{\hat{c}_b}{r} - \hat{A}_B \right) \left(\frac{A_t}{\hat{A}_B} \right)^{-\gamma} - \left(\frac{\bar{c}_b}{r} - \bar{A}_B \right) \left(\frac{A_t}{\bar{A}_B} \right)^{-\gamma}. \end{aligned} \quad (\text{A.21})$$

By multiplying both side of equation (A.17) by θ and using the closed-form solutions for the values of straight and CCB, we get

$$\begin{aligned} \frac{\hat{c}_b \theta}{r} \left(1 - \left(\frac{A_t}{\hat{A}_B} \right)^{-\gamma} \right) + \left(\frac{A_t}{\hat{A}_B} \right)^{-\gamma} (1 - \alpha) \hat{A}_B \theta = & \frac{\bar{c}_b \theta}{r} \left(1 - \left(\frac{A_t}{\bar{A}_B} \right)^{-\gamma} \right) + \left(\frac{A_t}{\bar{A}_B} \right)^{-\gamma} (1 - \alpha) \bar{A}_B \theta + \\ & \frac{c_c \theta}{r} \left(1 - \left(\frac{A_t}{\bar{A}_C} \right)^{-\gamma} \right) + \theta \left(\lambda \frac{c_c}{r} \right) \left(\frac{A_t}{\bar{A}_C} \right)^{-\gamma}. \end{aligned}$$

By rearranging terms

$$\begin{aligned} \frac{\hat{c}_b \theta}{r} \left(1 - \left(\frac{A_t}{\hat{A}_B} \right)^{-\gamma} \right) - \frac{\bar{c}_b \theta}{r} \left(1 - \left(\frac{A_t}{\bar{A}_B} \right)^{-\gamma} \right) - \frac{c_c \theta}{r} \left(1 - \left(\frac{A_t}{\bar{A}_C} \right)^{-\gamma} \right) = & \left(\frac{A_t}{\bar{A}_B} \right)^{-\gamma} (1 - \alpha) \bar{A}_B \theta + \\ \theta \left(\lambda \frac{c_c}{r} \right) \left(\frac{A_t}{\bar{A}_C} \right)^{-\gamma} - \left(\frac{A_t}{\hat{A}_B} \right)^{-\gamma} (1 - \alpha) \hat{A}_B \theta. \end{aligned} \quad (\text{A.22})$$

Now we can use (A.22) in (A.21) to get

$$\begin{aligned}
\Delta \tilde{W} &= \left(\frac{A_t}{\bar{A}_B} \right)^{-\gamma} (1 - \alpha) \bar{A}_B \theta + \theta \left(\lambda \frac{c_c}{r} \right) \left(\frac{A_t}{A_C} \right)^{-\gamma} - \left(\frac{A_t}{\hat{A}_B} \right)^{-\gamma} (1 - \alpha) \hat{A}_B \theta + \\
&\quad \left(\frac{\hat{c}_b}{r} - \hat{A}_B \right) \left(\frac{A_t}{\hat{A}_B} \right)^{-\gamma} - \left(\frac{\bar{c}_b}{r} - \bar{A}_B \right) \left(\frac{A_t}{\bar{A}_B} \right)^{-\gamma} \\
&= \theta \left(\lambda \frac{c_c}{r} \right) \left(\frac{A_t}{A_C} \right)^{-\gamma} + \left(\frac{\hat{c}_b}{r} \left(\frac{A_t}{\hat{A}_B} \right)^{-\gamma} - \frac{\bar{c}_b}{r} \left(\frac{A_t}{\bar{A}_B} \right)^{-\gamma} \right) + ((1 - \alpha)\theta + 1) \bar{A}_B \left(\frac{A_t}{\bar{A}_B} \right)^{-\gamma} - \\
&\quad ((1 - \alpha)\theta + 1) \hat{A}_B \left(\frac{A_t}{\hat{A}_B} \right)^{-\gamma}.
\end{aligned}$$

Given (2) and $\beta = \frac{\gamma}{r(1+\gamma)}$,

$$\begin{aligned}
\Delta \tilde{W} &= \theta \left(\lambda \frac{c_c}{r} \right) \left(\frac{A_t}{A_C} \right)^{-\gamma} + \left(\frac{\hat{c}_b}{r} \left(\frac{A_t}{\hat{A}_B} \right)^{-\gamma} - \frac{\bar{c}_b}{r} \left(\frac{A_t}{\bar{A}_B} \right)^{-\gamma} \right) - \\
&\quad ((1 - \alpha)\theta + 1) (1 - \theta) \frac{\gamma}{(1 + \gamma)} \left(\frac{\hat{c}_b}{r} \left(\frac{A_t}{\hat{A}_B} \right)^{-\gamma} - \frac{\bar{c}_b}{r} \left(\frac{A_t}{\bar{A}_B} \right)^{-\gamma} \right) \\
&= \theta \left(\lambda \frac{c_c}{r} \right) \left(\frac{A_t}{A_C} \right)^{-\gamma} + \\
&\quad \left(1 - ((1 - \alpha)\theta + 1) (1 - \theta) \frac{\gamma}{(1 + \gamma)} \right) \left(\frac{\hat{c}_b}{r} \left(\frac{A_t}{\hat{A}_B} \right)^{-\gamma} - \frac{\bar{c}_b}{r} \left(\frac{A_t}{\bar{A}_B} \right)^{-\gamma} \right). \tag{A.23}
\end{aligned}$$

In (A.23) the first term on the right-hand side is positive, $\theta \left(\lambda \frac{c_c}{r} \right) \left(\frac{A_t}{A_C} \right)^{-\gamma} > 0$. Also, for $\hat{c}_b > \bar{c}_b$, $\left(\frac{\hat{c}_b}{r} \left(\frac{A_t}{\hat{A}_B} \right)^{-\gamma} - \frac{\bar{c}_b}{r} \left(\frac{A_t}{\bar{A}_B} \right)^{-\gamma} \right) >$

0. Finally,

$$1 - ((1 - \alpha)\theta + 1) (1 - \theta) \frac{\gamma}{(1 + \gamma)} > 1 - (\theta + 1) (1 - \theta) \frac{\gamma}{(1 + \gamma)} = 1 - (1 - \theta^2) \frac{\gamma}{(1 + \gamma)} > 0.$$

All the terms on the right-hand side of equation (A.23) are positive. Therefore, $\Delta \tilde{W} > 0$. This proves the second part of the proposition. $\square$

References

- [1] Adrian, Tobias and Markus K. Brunnermeier (2009). "*Covar*", Version dated August 25, 2009, available at: <http://www.princeton.edu/markus/research/papers/CoVaR.pdf>
- [2] Bernanke, Ben S. (2009). "*Regulatory Reform*", Testimony before the Committee on Financial Services, U.S. House of Representatives, Washington, D.C. October 1, 2009.
- [3] Bhanot, K., Mello, A.S., (2006). "*Should corporate debt include a rating trigger?*", Journal of Financial Economics 79: 69-98.
- [4] Dudley, W. C. "*Some Lessons from the Crisis*," Remarks at the Institute of International Bankers Membership Luncheon, New York City, October 13, 2009.
- [5] Duffie, D. "*Dynamic Asset Pricing Theory*", 3-rd ed., Princeton University Press, 2001.
- [6] Duffie, D. "*Contractual Methods for Out-of-Court Restructuring of Systemically Important Financial Institutions*", Submission Requested by the U.S. Treasury Working Group on Bank Capital, draft of November 9, 2009.
- [7] Eisenbeis, Robert A. and Larry D. Wall (2002), "*Reforming Deposit Insurance and FDICIA*", Economic Letter, Federal Reserve Bank of Atlanta, First Quarter 2002.
- [8] Flaherty, Jim (2010), "*Speech by the Minister of Finance to the Canada Forum Euromoney Conference*", April 21, 2010, available: <http://www.fin.gc.ca/n10/10-0291-eng.asp>
- [9] Flannery, Mark J. (2005), "*No Pain, No Gain: Effecting Market Discipline via Reverse Convertible Debentures*", in Hal. S. Scott (ed.), Capital Adequacy Beyond Basel: Banking Securities and Insurance, Oxford University Press 2005.
- [10] Flannery, Mark J. (2009a), "*Stabilizing Large Financial Institutions with Contingent Capital Certificates*", paper presented at the "Symposium on banking and Financial Reform", Federal Reserve Bank of San Francisco, September 11, 2009.

- [11] Flannery, Mark J. (2009b), "*Market-Valued Triggers Will Work for Contingent Capital Instruments*", Solicited Submission to U.S. Treasury Working Group on Bank Capital, November 6, 2009.
- [12] Hart, Oliver and Luigi Zingales (2009), "*A New Capital Regulation for Large Financial Institutions*", Nota Di Lavoro, 124.009, available at: <http://www.feem.it/userfiles/attach/200912111646414124-09.pdf>
- [13] Goldstein, R., Ju, N., Leland, H., (2001). "*An EBIT-based model of dynamic capital structure*", Journal of Business 74:483-512.
- [14] Kashyap, Rajan, and Stein (2008), "*Rethinking Capital Regulation, Federal Reserve Bank of Kansas City symposium on Maintaining Stability in a Changing Financial System*", available at: <http://www.kc.frb.org/publicat/sympos/2008/KashyapRajanStein.03.12.09.pdf>
- [15] Manso, Gustavo, Bruno Strulovici, and Alexei Tchistyi (2009), "*Performance-Sensitive Debt*", forthcoming, Review of Financial Studies.
- [16] Leland, H.E., (1994). "*Corporate debt value, bond covenants, and optimal capital structure*", Journal of Finance 49:1283-1252.
- [17] McDonald, Robert. "*Contingent Capital with a Dual Price Trigger*", SSRN-id1553430.pdf.
- [18] Modigliani, F., Miller, (1958). "*The cost of capital, corporation finance and the theory of investment*", American Economic Review 48:267-297.
- [19] Raviv, Alon, (2004). "*Bank Stability and Market Discipline: Debt-for-Equity Swap versus Subordinated Notes*", available at SSRN:<http://ssrn.com/abstract=575862>
- [20] Squam Lake Working Group on Financial Regulation (2009), "*Can Expedited Resolution Mechanism for Distressed Financial Firms: Regulatory Hybrid Securities*", Center for Geoeconomic Studies, Council in Foreign Relations, Working Paper, April 2009, available at: http://www.cfr.org/content/publications/attachments/Squam_Lake_Working_Paper3.pdf
- [21] Strabulaev, I., (2007). "*Do Tests of Capital Structure Theory Mean What They Say?*", The Journal of Finance, 62.

- [22] Wall, Larry D. (2009), "*Prudential Discipline for Financial Firms: Micro, Macro, and Market Structures*", ADBI Working Paper 176. Tokyo: Asian Development Bank Institute. Available at: <http://www.adbi.org/working-paper/2009/12/10/3401.prudential.discipline.financial.firms/>