The One-Child Policy and Household Savings in China*
PRELIMINARY

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Abstract

This paper analyzes the impact of the 'one child policy' in China on its household saving behavior. First, it develops a life-cycle model with endogenous fertility, intergenerational transfers and human capital accumulation. We show a macroeconomic and a microeconomic channel of a fall in fertility on raising aggregate household saving: at the macroeconomic level, the population composition shifts *initially* towards the middle-aged—the high savers of the economy. At the microeconomic level, (1) expenditures of children fall—despite higher education investment in each child—as quantity substitutes for quality; (2) middle-aged save additionally for retirement in anticipation of reduced transfers from their only child. Second, our quantitative model implies policy-induced changes in aggregate savings and age-saving profiles broadly consistent with estimates from Chinese household-level data. Third, an empirical study using the birth of twins as a source of exogenous increase in fertility is shown to support the micro-economic channels we highlight. Overall, our estimation suggests that the policy is able to account for 30% to 50% of the rise in household savings rate since its implementation in 1980.

1 Introduction

China's household saving rate is staggeringly high in comparison to most other countries, and increasing at a rapid rate. Between 1986 and 2009 it rose from 11.3% to 32.4%.[1] By standard theories, households in a rapid growing economy should borrow against future income to bring forward consumption, and therefore face a declining savings rate rather than a rise. The conundrum has been referred to by both academics and policymakers as a 'Chinese Saving Puzzle' (Modigliani and Cao (2004)), spurring many attempts at explaining it. This paper evaluates the contribution of the 'one-child policy' and its attendant drastic demographic shifts in accounting for this puzzle.

The 'one-child policy', implemented in the late 1970s as part of China's population control program, is a relatively under-studied event— with economic ramifications to a large extent unknown. An immediate question that comes to mind is whether, and to what degree, it has impacted the national saving rate. That concomitant shifts in demographic compositions can directly influence the rate of saving at the aggregate level is well-understood in the classic formulations of the life-cycle motives for saving (Modigliani (1976)): if the young save less than the middle-aged, then the policy should increase aggregate savings with the rising share of the middle-aged during the transition (the *macro-channel*). Yet, a compulsory reduction in fertility can also potentially affect savings *decisions* at the household level (the *micro-channel*). In this paper, we explore these micro and macro-level channels, both theoretically and empirically, and ask how much the advent of the one-child policy can explain the large rise in household saving rate in China. More broadly though, the policy can also be exploited as a natural experiment of an exogenous restriction in fertility to analyze the relationship between household savings and fertility in developing countries.

In the theory we develop, intergenerational transfers—conferred by children to parents in support of old-age consumption—is our primary *micro-channel* through which fertility affects savings: by lowering old-age support from their offspring, a reduction in fertility increases incentives to save before retirement. We believe this is particularly relevant in the Chinese case where such transfers are very common and account for a large share of old-age income. An everyday Chinese adage crystallizes the prominence of this notion: "rear children to provide for old-age" (yang lao fu you). How much parents expect to receive from children in overall transfers depend likely on the number of children (quantity), and also on their individual human capital (quality). Thus, it is reasonable to conjecture that fertility decisions are subject to both quantity and quality decisions—insofar as children are a source of financial means in old age. In particular, a policy aiming at reducing fertility should

[1] Average household saving rate was 4% in OECD economies and 1.7% in the U.S. in 2007.

incentivize parents to invest further in children education (quality). We will investigate such conjecture theoretically and empirically as savings and education decisions should be jointly determined.[2]

In investigating these effects the paper makes three main contributions. First, it develops a multi-period life-cycle model that incorporates intergenerational support, endogenous fertility, and education investment decisions. A simple and tractable four-period model reveals the fundamental structure of the framework and the key mechanisms driving the fertility-saving relationship. The micro-channel through which an exogenous reduction in fertility affects savings behavior is illuminated: parents have greater incentives to save—both because consumption expenditures of children are lower (a 'fewer mouths to feed' effect) and because parents need to save additionally for retirement in anticipation of a reduction in overall transfers from children. The quantity-quality tradeoff induces parents to substitute quantity towards quality in the form of higher investment in children's education. However, higher wages due to greater human capital accumulation of children raise transfers per child, but still do not compensate for the reduction in the number of children. Therefore, the overall amount of transfers falls.

A second contribution is to provide a quantitative multiple periods version of the model that we calibrate using Chinese household-level data. This allows us to simulate the impact of the policy on aggregate savings and to provide finer predictions on the age-savings profile of households in the years following the policy implementation. A distinguishing feature of our paper, and one that sets it apart from the rest of the literature, is our endeavor to bridge the micro-level approach with the macro-level approach in linking demographics to saving. We use household level data to estimate these profiles over the last twenty years and compare it to the model-predicted profiles. The ability of our model to match data on intricate age-saving profiles with a fair degree of accuracy equips our theory with the micro-level evidence requisite for giving credence to our macroeconomic implications. In addition, our model implies a seemingly perverse cross-sectional pattern: that the young cohort's saving rate should be rising faster than the middle-aged cohort's saving rate immediately following the implementation of the one child policy. The reason, according to our framework, is that along the initial stages of the demographic composition, two very different cohorts are coexisting: the younger ones subject to the one child policy—and therefore to the micro-channel of saving that we highlight, and the older cohorts who were not subject to the policy and had multiple children on average. This cross-sectional pattern is confirmed by the data (see

[2]There has been an extensive literature that examines the relationship between fertility and savings, such as Modigliani and Cao (2004), Boldrin and Jones (2002), Chakrabarti (1999), Cisno and Rosati (1992), and Raut and Srinivasan (1994). These studies, however, do not include human capital investment decision made by parents in consideration for their old age transfers.

also Song and Yang (2010) and Chamon and Prasad (2010)), and is shown to be consistent with a modified life-cycle model, for which there has been ample evidence supportive at the aggregate level (Modigliani and Cao (2004), Horioka and Wan (2007); Curtis, Lugauer, and Mark (2011)).

The third main contribution is to check whether the specific micro-level channels that underlie the relationship between fertility and saving in this model hold up in the data. Using the birth of twins as an exogenous deviation of fertility from the one child policy, we find supportive evidence for the three main predictions: that higher fertility leads to (1) lower household savings rate of parents affected by the policy change and (2) lower education investment per child; (3) that transfers from children to parents rise in the quantity and quality of the children. Remarkably, our estimated difference of individual savings rate of parents of twins versus parents of only child is in line with the predictions of our calibrated model. Thus, evidence from twins allow us to partially identify the micro-channel we emphasize. Using these estimates, we then try to evaluate how much of the increase in aggregate savings rate is due to the implementation of the 'one-child policy'. Depending on assumptions made regarding the fertility rate that would have prevailed in the absence of the policy, we find that in 2009, aggregate household savings rate would have been at least 6.5% percentage points lower without the policy and potentially 11% percentage points in a less conservative estimation. The contributions of the micro and macro-channels, moreover, are found to be roughly equal. In other words, we can explain from 30% to 50% of the 21 percentage points increase in household savings rates since the commencement of the one-child policy in 1980. A by-product of our estimates is that our conservative estimates suggests that 20% of the increase in human capital of the new generation of only children can be tied to the policy.

Recent works that bring to the forefront intergenerational transfers in analyzing the relationship between demographics and saving in an overlapping generations structure include Banjeree, Meng and Qian (2010) and Su, Yang and Zhang (2012). These works exclude human capital investment, which is one of the main pillars of our theoretical framework. Thus, fertility decisions in their framework are not subject to a quantity and quality tradeoff that in turn affects saving behavior at various ages of an individual's life cycle. Implications on aggregate saving, as well as the implied individual age saving profiles both in terms of levels and shape, therefore differ. Empirically, this work goes beyond existing studies in providing more direct evidence on the specific channels (education decisions and intergenerational transfers) through which fertility affect saving profiles over time and across generations.

Banerjee et al.(2010) focus on establishing a causal link between fertility and saving without reference to the channel of transfers and education—and find a negative relationship which corroborates with our results albeit using an entirely different identification strategy and data sample. Their work also highlights the importance of children's gender in determining parents' saving behavior–a facet of reality we do not consider in current work. In simultaneous work, Su et al (2012) zeroes in on how one fewer dependent child affects savings behavior at various ages, using yet another altogether-separate identification strategy based on variations across provinces of the implementation of the policy. Yet, their empirical findings are broadly consistent with our theoretical predictions on age-specific saving behavior.

The closest paper to ours is the work by Oliveira (2012). She takes a microeconomic approach in analyzing specifically the relationship between fertility and old-age support in a Becker-Lewis (1973) type of quantity-quality model of fertility. Based on a similar identification strategy, her empirical findings on the link between fertility and intergenerational transfers using Chinese and Indonesian household data, complement and corroborate with one of our main supportive evidence: that transfers from children to parents are increasing in the quantity and quality of children. However, she does not investigate how household saving decisions are, in turn, modified and her stylized two period model does not allow to provide a rigorous quantitative evaluation.

Finally, our paper relates and complements other works aiming at understanding China's perplexingly high national household saving rate in recent years. A few compelling explanations that various past works have explored include: (1) precautionary savings (Blanchard and Giavazzi (2005) and Chamon and Prasad (2010));[3] (2) demographic structural changes (Modigliani and Cao (2004), Ge, Yang and Zhang (2012) and Curtis, Lugauer and Mark (2011));(3) income growth and credit constraints (Coeurdacier, Guibaud and Jin (2012)), potentially in presence of housing expenditures (Bussiere et al. (2013)); (4) gender imbalances and competition in the marriage market (Wei and Zhang (2009)). Yang, Zhang and Zhou (2011) provide a thorough treatment of aggregate facts pertaining to China's saving dynamics, and at the same time present the challenges some of these theories face. The recent availability of household-level data should enable researchers to probe into micro-level patterns and behavior to bear out these macro-level theses—an attempt we make in the current work.

The paper is organized as follows. Section 2 provides certain background information and facts that motivate some key assumptions underlying our framework. Section 3 provides a simple theory that links fertility and savings decisions in an overlapping generations model.

[3]These papers argue that rising unemployment risk and income uncertainty for the workers during a period of economic transition since the 1980s have triggered precautionary saving motives. However, this line of explanation would not fit with the evidence in Yang, Zhang and Zhou (2011) that these uncertainties have in fact decreased over the last ten years in China.

Section 4 develops a calibrated quantitative model to simulate the impact of the policy on aggregate savings as well as age-savings profiles. Macro and micro-level predictions of the model are confronted to their empirical counterpart. Section 5 undertakes an empirical investigation on the main theoretical mechanisms using twin births as source of identification and provides some quantitative guidance on the overall impact of the one child policy on aggregate household savings in China. Section 6 concludes.

2 Motivation and Background

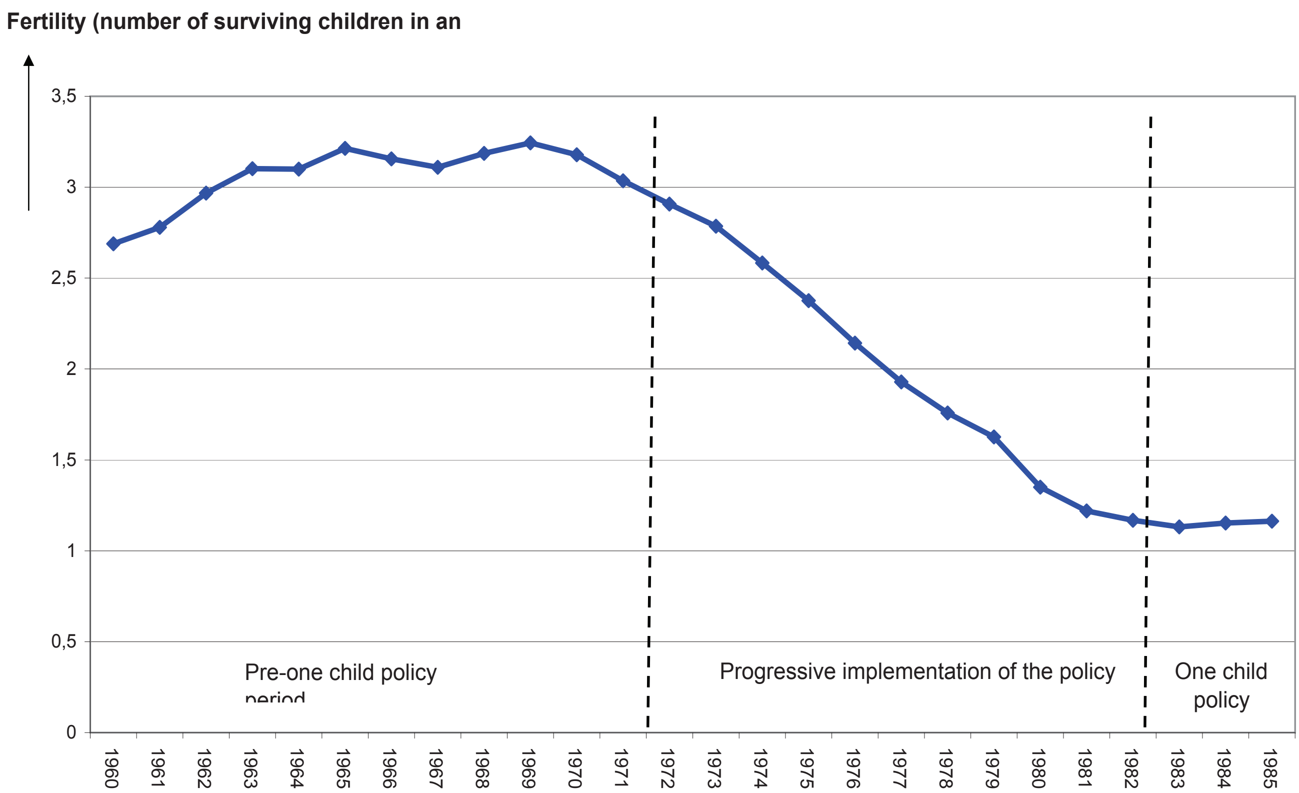
In this section, using various aggregate and household level data sources from China, we provide some stylized facts on (1) the implementation of the 'one-child policy' and its consequences on demographic compositions; (2) the direction and magnitude of intergenerational transfers, as well as (3) education expenditures incurred by households over their lifecycle. The quantitative relevance of the micro and macro-channels as provided by the preliminary evidence below motivates the structure of our theoretical framework. The various micro and macro data sources we use are described in details in Appendix 7.1.

2.1 The One-Child Policy and the Chinese demographic transition

The one-child policy decreed in 1979 was intended to curb the population growth promoted over the course of the Maoist pro-natality agenda. The policy was strictly enforced in urban areas and partially implemented in rural provinces—which, in contrast to urban areas, allowed the birth of two children in the event of a first-born girl. As a consequence of the strict enforcement of the policy, the fertility rate in China dropped sharply from 5.5 children per woman in 1965-1970 to 2.6 between 1980-1985, and very close to one for urban households. Figure 1 shows the evolution of the fertility rate for urban households based on Census data: before 1970, urban households were on average having slightly more than three children. Over the period 1972-1980, the 'one child' policy was progressively put in place and fertility rates started to fall to reach a level very close to one after 1982 (see Banerjee et al.(2010) for further description of the progressive implementation of the policy in the seventies).

It is critical to determine whether the constraint was binding for the Chinese households, and if so, for which households. Our household level data (Urban Household Survey) suggests that the policy was very binding in urban areas, albeit less stringent in rural areas: over the period 2000-2009, conidtionally on having a child, 96% of households of the urban sample had only one child. Urban households are a natural focal point given that our theory pertains

Figure 1: Fertility in Chinese urban areas



Notes: Data source: Census, restricted sample of only urban households

to binding fertility constraints.[4]

The demographic structure has also evolved accordingly, as shown in Table 1. It is characterized by a sharp rise in the median age— from 19.7 years in 1970 to to 34.5 years in 2010, and a rapid decline in the share of young individuals (population of ages 0-20/Total Population) from 51% to 27% over the period, and a corresponding increase of the share of middle aged population (population of ages 30-60/Total Population). While the share of young is expected to drop further until 2050, the share of old (above 60) increases sharply

[4]Some urban households have more than one child. If we abstract for the birth of twins, these households ....

only after 2010, when the generation of only child ages. In other words, the 'one-child policy' leads first to a sharp fall in the share of young individuals relative to middle aged adults, followed by a sharp increase in the share of the elderly only one generation later.

Table 1: Demographic structure in China

	1970	2010	2050
Share of young (age 0-20/Total Population)	51%	27%	18%
Share of middle aged (age 30-60/Total Population)	28%	44%	39%
Share of elderly (age above 60/Total Population)	7%	14%	33%
Median age	19.7	34.5	48.7
Fertility (children per women, urban areas)	3.18 (1965-70)	1.04 (2004-09)	- n/a -

Note: UN World Population Prospects (2011).

2.2 Intergenerational Support

Intergenerational support is prominent and important for Chinese households. Beyond cultural mores, it is stipulated by Constitutional law: "children who have come to age have the duty to support and assist their parents" (article 49). Failure of so doing may even result in law suits. We first document the prominence of intergenerational support in the case of China, using data from the China Health and Retirement Longitudinal Study (CHARLS). The pilot survey was conducted in 2008 for two provinces: Zhejiang (a prosperous coastal province) and Gansu (a poor inland province). The sample includes only households with members above the age of 45 years, but for our purposes we restrict the sample to urban households in which at least one member (respondent or spouse) is older than 60 years of age.[5] Transfers include those within households, i.e. when children and parents are co-residing in the same household. We consider children who are 25 years old and above as adults.

Intergenerational transfers can take on broadly two forms: financial transfers ('direct' transfers) and 'indirect' transfers in the form of co-residence or other in-kind benefits. According to Table 2, 45% of the elderly reside with their children in urban households. Transfers from adult children to parents occur in 65% of households (whereas transfers from the elderly to their children occur in only 4% of the households). They constitute a significant

[5] When including both urban and rural households, our results and descriptive statistics are consistent with those of Lei at al (2012).

share of old-age income: it amounts to an average of 28% of all elderly's pre-transfer income (and up to 41% if one focuses on the sample of transfer receivers).

Table 2: Intergenerational Transfers: Descriptive Statistics

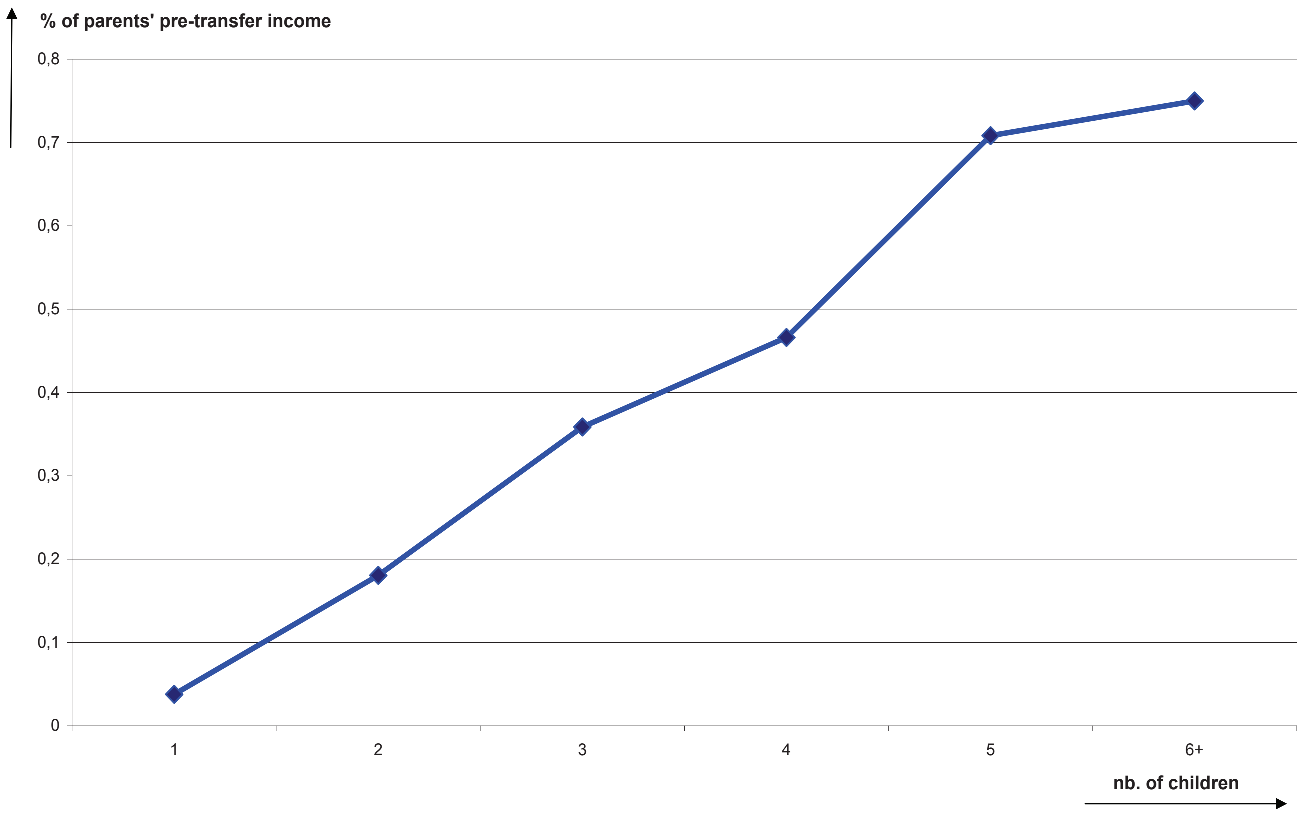
Nbr of hh in restricted sample	321
Av nbr of adult children (25+)	3.4
Share living with adult children	45%
Incidence of transfers	
- from adult children to parents	65%
- from parents to adult children	4%
Net transfers in % of parent's pre-transfer income	
- All parents	28%
- Transfer receivers only	41%

Note: CHARLS(2008) restricted sample of urban households with a respondant/spouse of at least 60 years of age. Transfers is defined as the sum of regular and non-regular financial transfers and the yuan value of in-kind transfers. This includes transfers within households. Net Transfers are transfers from children to parents less the transfers received by children. Gross transfers are defined to be transfers from children to parents

Figure 2 shows that the average transfers (as a % of pre-transfer income) are (i) large in magnitude and (ii) increasing in the number of children. This is very suggestive that the implementation of the 'one-child policy' will lead to a significant fall in transfers towards elderly, which will be a central assumption in our theoretical framework.

Figure 3 provides a snapshot of transfers received and paid at different ages of adulthood (in Chinese yuans). The evolution of net transfers to, and subsequently from, one's children over the course of an individual's life is shown in the left panel of Figure 3. Net transfers are on average negative, and continuously declining before one's child reaches the age of 25, in line with the conjecture of education investment being the main source of transfers to children (see section 2.3 below). After this age, children on average confer increasing amounts of transfers to parents. The pattern is similar when plotting net transfers from children by the average age of the parents (right panels of Fig. 3). Net transfers from children are on average negative for parents below the age of 55. After retirement, at age 60 for male (resp. 55 for female), net transfers become positive. If one considers, co-residence

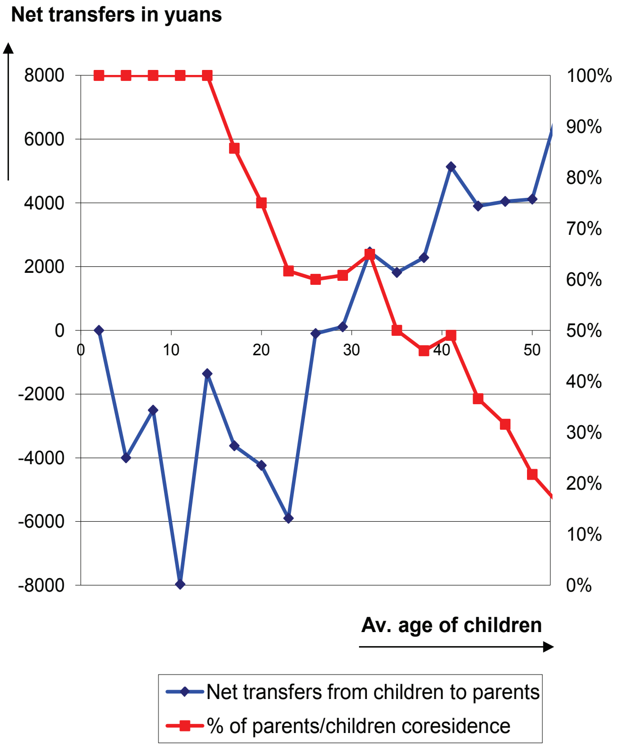
Figure 2: Inter-generational transfers, CHARLS 2008

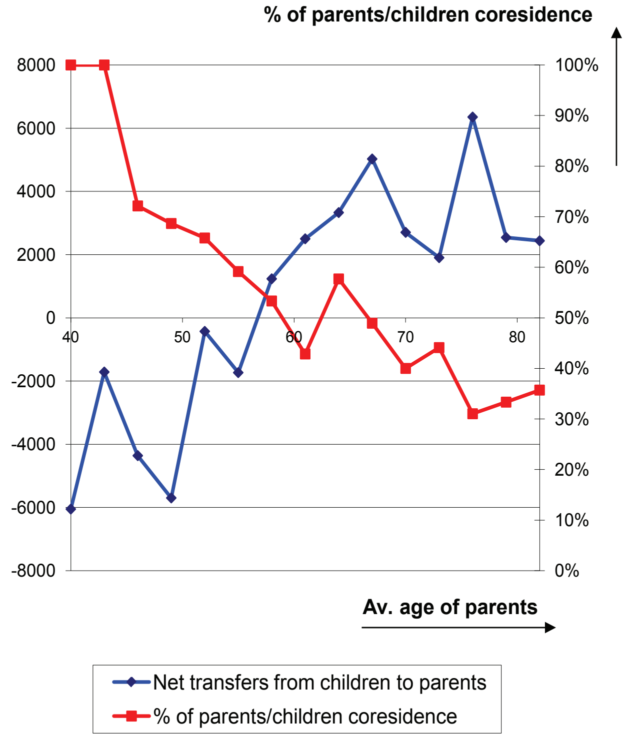


Notes: Data source: CHARLS, 2008, restricted sample of only urban households with respondant/spouse of at least 60 years of age.

as another from of transfers, a similar picture emerges: children leave the parental house as they age; eventually much later on, parents come back to live with their children when they are very old (above 78). Such a timing (and direction) of transfers between children and parents, as well as their magnitude, will motivate our theoretical framework and guide our calibration later on.

Figure 3: Average Net transfers from Children to Parents, by average age of children (top) and parents (bottom)





Notes: CHARLS 2008, urban households.

2.3 Motives for Savings

Our central thesis that household saving is motivated by education for children in earlier stages of parenthood and for old-age retirement in later stages is born out by basic observations from the data. As reported in Table 3, education and retirement planning are cited to be among the three most important reasons for saving, according to more than half of Chinese households in 2008 (Yao et al (2011)). For 69 percent of rural households, education or retirement are the most important motive for saving. Among these households, the relative importance of these two motives of savings also evolves with age of the respondent: in

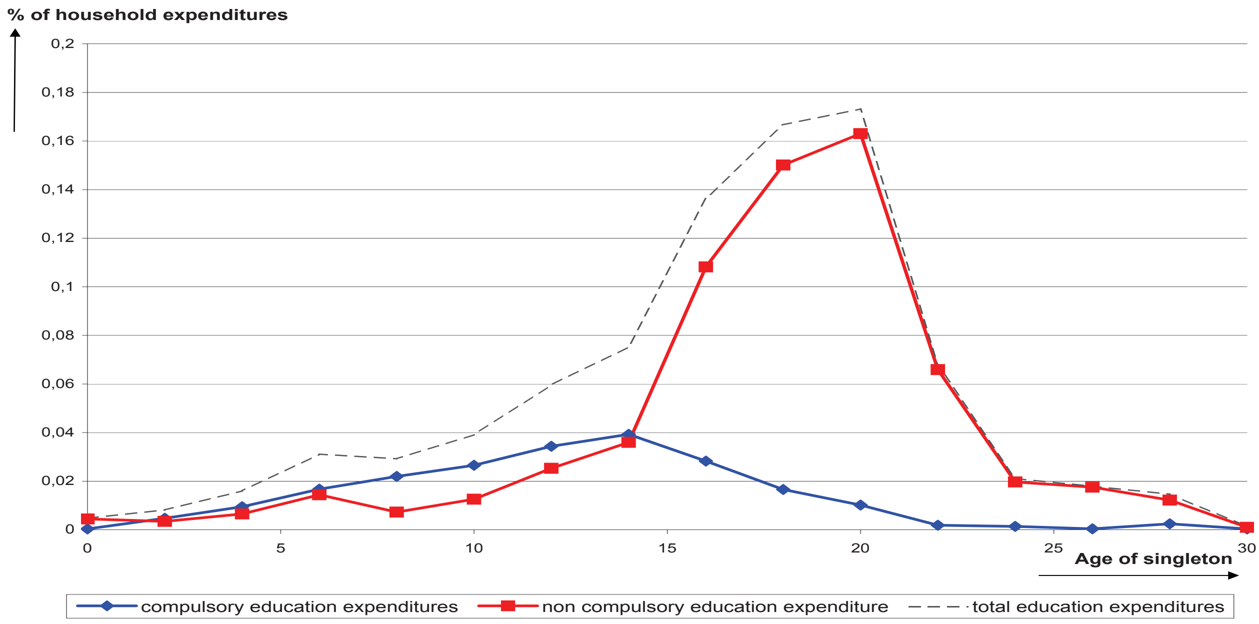
Table 3: Self-Reported Saving Motives. Survey evidence.

Three most-frequently cited motives for saving (Yao et al. (2011))				
Education	Retirement	Housing	Emergency	Wealth preservation
60%	54.4%	28.5%	63.8%	33.4%

First self-reported motive for saving, CHIP, 2002, rural sample				
Education	Retirement	Housing	Sickness	Wedding
43%	26%	7%	5%	12%

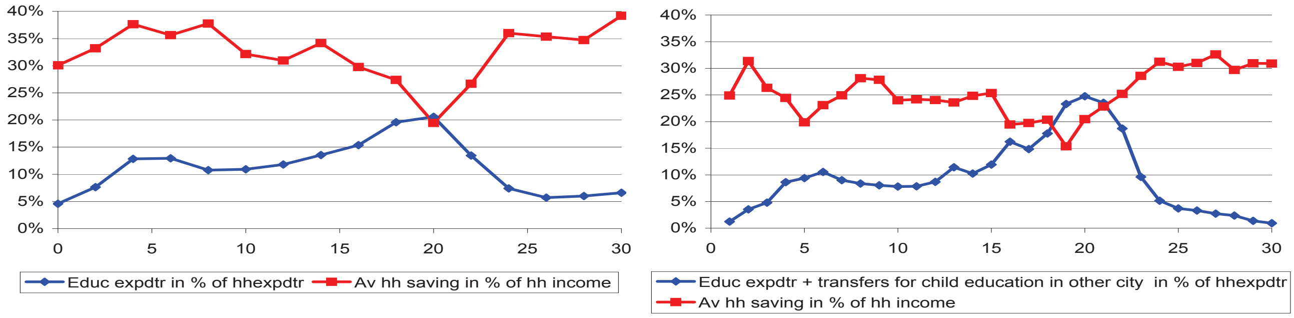
earlier stages of parenthood, respondents report to be saving for their offspring's education, and over time, gradually more for retirement.

Figure 4: Education Expenditures by Age of Only Child



Notes: CHIP (2002), where the sample is restricted to households with only child. This figure graphs compulsory and non-compulsory education expenditures as a share of total household expenditures, by age of the only child.

Figure 5: Household Savings and Education Expenditures



Notes: Sources: Left panel: CHIP (2002). Right panel: RUMICI (2008), where the sample is restricted to households with only child. This graph juxtaposes the average education expenditure (as a share of total expenditures) and household saving (as a share of household income) by the age of the only child.

Why would parents need to save for their children's education expenses in China? To provide some preliminary empirical evidence regarding the importance of education expenditures in the budget of households, we use household data on education expenditures from Chinese Household Income Project (CHIP) in 2002.[6] Figure 4 displays the rapid rise in education expenditures per child (as a share of total household expenditure) in relation to the age of a child, with the sharpest increase occurring between the ages of 15 and 22. The timing of education expenditures will play an important role since it will affect age-saving profiles of optimizing individuals willing to smooth consumption over their lifetime. Education expenditures and household savings will both be endogenous outcomes in our theoretical framework where parents optimally substitute quantity for quality. Yet one can still reasonably expect that the timing of household savings is inversely related to the timing of education expenses— if they constitute a significant fraction of overall expenditures. Figure 5 shows that household savings rate seems to respond broadly to this pattern: it starts to

[6]As a robustness check, we also use an alternative dataset RUMICI (2008), and get similar estimates.

rise when the child is young (before the age of 10)–possibly in preparation for rising future education costs—and starts to decline as these expenses materialize—leading to the sharpest dip in periods where education costs are highest (when the child is around 20); it then subsequently rise as these costs start to subside with the completion of the studies. This basic accumulation and de-cumulation of resources as is related to a child's age is consistent with our view that education expenses are an important driver for household savings behavior, particularly at certain stages of parenthood.

3 Theory

We develop a simple and tractable multi-period overlapping generations model with intergenerational transfers, endogenous fertility and human capital accumulation. Semi-closed form solutions that arise from a parsimonious model reveal the key mechanisms that underlie the relationship between fertility and saving. It is used to analyze the impact of a constriction of fertility on individual and aggregate saving rate, and at the same time form the empirical basis for testable implications. A quantitative version of the model as developed in the later section depicts a more intricate and detailed individual age-saving profile, but the main dynamics are elucidated in a simple four period model.

3.1 Set-up

Consider an overlapping generations economy in which agents live for four periods, characterized by: childhood (k), youth (y), middle-age (m), and old-age (o). The measure of total population N_t at date t comprises the four co-existing generations: $N_t = N_{k,t} + N_{y,t} + N_{m,t} + N_{o,t}$.

An individual born in period $t-1$ does not make decisions on his consumption in childhood, $c_{k,t-1}$, which is assumed to be proportional to parental income. The agent supplies inelastically one unit of labor in youth and in middle-age, and earns a wage rate $w_{y,t}$ and $w_{m,t+1}$, which is used, in each period, for consumption and asset accumulation $a_{y,t}$ and $a_{m,t+1}$. At the end of period t, the young agent then makes the decision on the number of children n_t to bear. In middle-age, in $t+1$, the agent chooses the amount of human capital h_{t+1} to endow to each of his children, and at the same time transfers a combined amount of $T_{m,t+1}$ to his n_t number of children and parents— to augment human capital of the former, and consumption of the latter. In old-age, the agent consumes all available resources, which is financed by gross return on accumulated assets, $Ra_{m,t+1}$ and transfers from children. A consumer thus maximizes the life-time utility including benefits from having n_t children (with

$v > 0$ the parameter governing the preference for children and $0 < \beta < 1$ the discount rate):

$$U_t = \log(c_{y,t}) + v\log(n_t) + \beta\log(c_{m,t+1}) + \beta^2\log(c_{o,t+2})$$

subject to the sequence of budget constraints in each period:

$$\begin{aligned} c_{y,t} + a_{y,t} &= w_{y,t} \\ c_{m,t+1} + a_{m,t+1} &= w_{m,t+1} + Ra_{y,t} + T_{m,t+1} \\ c_{o,t+2} &= Ra_{m,t+1} + T_{o,t+2} \end{aligned} \tag{1}$$

Because of parental investment in education, the individual born in period $t-1$ enters the labor market with an endowment of human capital, h_t. The wage rate depends on this level of human capital, as well as an experience factor $e < 1$, and a deterministic level of economy-wide productivity z_t, so that

$$w_{y,t} = ez_t h_t^{\alpha} \tag{2}$$

$$w_{m,t+1} = z_{t+1} h_t^{\alpha} \tag{3}$$

For analytical convenience, we assume that the gross interest rate R is constant and exogenous. By making this assumption, we sever the link in which saving affects interest rates, and also the potential aggregate feedback of fertility onto interest rates.

Without loss of generality, we also assume that the cost of raising kids are paid by the parents in middle-age, in period $t+1$, for a child born at the end of period t. This assumption corroborates with the empirical fact that the bulk of education costs are born in the period before the child enters the labor market–equivalent to the ages of 15 to 25. The total cost of raising n_t children falls in the mold of a time-cost that is proportional to current wages, $n_t\phi(h_{t+1})w_{m,t+1}$, where $\phi(h) = \phi_0 + \phi_h h_{t+1}$, and $\phi_0 > 0$ and $\phi_h > 0$. The consumption expenditure per child is a fraction ϕ_0 of the parents' wage rate, and the education cost $\phi_h h_{t+1}$ is increasing in the level of human capital—to capture the rising cost of education over a child's course of study.[7]

Transfers made to the middle-aged agent's parents amount to a fraction $\psi n_{t-1}^{\varpi-1}/\varpi$ of current labor income $w_{m,t+1}$, with $\psi > 0$ and $\varpi > 0$. This fraction is decreasing in the number of siblings—to capture the possibility of free-riding among siblings sharing the burden of transfers (see Boldrin and Jones (2002) for a similar outcomes in a 'game of giving'). The combined amount of transfers made by the middle-aged agent in period $t+1$ to his children

[7] This is a key departure from the quantity-quality tradeoff models of Becker and Lewis (1973), later adopted by Oliveira (2012). They assume that costs to quality are independent of the level of quality.

and parents thus satisfy

$$T_{m,t+1} = -\left(n_t\phi(h_{t+1}) + \psi\frac{n_{t-1}^{\varpi-1}}{\varpi}\right) w_{m,t+1},$$

with $\phi(h) = \phi_0 + \phi_h h_{t+1}$.

In old-age, agents become receivers of transfers from a total of n_t number of children, in the amount of

$$T_{o,t+2} = \psi\frac{n_t^{\varpi}}{\varpi} w_{m,t+2}.$$

The life-time resource constraint thus requires that

$$c_{y,t} + \frac{c_{m,t+1}}{R} + \frac{c_{o,t+2}}{R^2} = w_{y,t} + \frac{w_{m,t+1}}{R}\left[1 - n_t\phi(h_{t+1}) - \psi\frac{n_{t-1}^{\varpi-1}}{\varpi}\right] + \frac{\psi n_t^{\varpi}}{\varpi}\frac{w_{m,t+2}}{R^2}$$

Assumption 1 *The young are subject to a credit constraint which is binding in all periods:*

$$a_{y,t+1} = -\theta\frac{w_{m,t+1}}{R}. \tag{4}$$

The young is permitted to borrow up to a constant fraction θ of the present value of future wage income. For a given θ, the constraint more likely to be binding if productivity growth is high enough (compared to R) and the experience parameter e low enough, conditions that we assumed to be met as data suggest. This assumption is necessary to generate realistic savings behaviours of the young. With fast growth and a rising income profile, the model would otherwise imply a sharp borrowing of the young—something that the data does not reveal, as we later on show (see also Coeurdacier, Guibaud and Jin (2012)).

The assumption of log utility implies that the optimal consumption of the middle-age is a constant fraction of the present value of lifetime resources, which consist of disposable income—of what remains after the repayment of debt from the previous period—and the present value of transfers to be received in old-age, less current transfers to children and parents:

$$c_{m,t+1} = \frac{1}{1+\beta}\left[\left(1 - \theta - n_t\phi(h_{t+1}) - \psi\frac{n_{t-1}^{\varpi-1}}{\varpi}\right) w_{m,t+1} + \frac{\psi}{R}\frac{n_t^{\varpi}}{\varpi} w_{m,t+2}\right]$$

It follows from Eq. 1 that the optimal asset holding of a middle-aged individual is

$$a_{m,t+1} = \frac{\beta}{1+\beta}\left[\left(1 - \theta - n_t\phi(h_{t+1}) - \psi\frac{n_{t-1}^{\varpi-1}}{\varpi}\right) w_{m,t+1} - \frac{\psi}{\beta R}\frac{n_t^{\varpi}}{\varpi} w_{m,t+2}\right]. \tag{5}$$

Optimal consumption and asset holdings respond partially to changes in the interest rate R.

The assumption of log utility rules out the substitution and income effect associated with changes in the interest rate, but retains a wealth effect of affecting the present value of future resources. A rise in R thus leads to a fall in middle-age consumption and a corresponding increase in asset holdings.

The old, by consuming all resources and leaving no bequests, enjoy

$$c_{o,t+2} = \frac{\beta}{1+\beta}\left[R\left(1-\theta-n_t\phi_m(h_{t+1})-\psi\frac{n_{t-1}^{\varpi-1}}{\varpi}\right)w_{m,t+1}+\psi\frac{n_t^{\varpi}}{\varpi}w_{m,t+2}\right].$$

Fertility and Human Capital. Fertility decisions hinge on equating the marginal utility of bearing an additional child compared to the net marginal cost of raising the child:

$$\frac{v}{n_t} = \frac{\beta}{c_{m,t+1}}\left(\phi_m(h_{t+1})w_{m,t+1}-\frac{\psi n_t^{\varpi-1}w_{m,t+2}}{R}\right). \tag{6}$$

The right hand side is the net cost, in terms of the consumption good, of having an additional child. The net cost is the current marginal cost of rearing a child, $\partial T_{m,t+1}/\partial n_t$ less the present value of the benefit from receiving transfers next period from an additional child, $\partial T_{o,t+2}/\partial n_t$. Note that the interest rate partly determines the price of having children: an increase in R reduces the relative benefits compared to the costs of having children, and thus raises the price of having children (in terms of middle-age consumption).

The partial effects of changes in interest rate on fertility is the same as its effect on consumption. A rise in R reduces intertemporal wealth by reducing the present value of old-age financial wealth and transfers from children. This wealth effect reduces current consumption and incentives to have children. With our assumption of log utility, the income and substitution effect cancel out, and the wealth effect implies that an increase in R is associated with reduced fertility. This result is the opposite of the positive relation between interest rates and fertility in Barro and Becker (1989): the reason being that in our framework, children should be understood as investment goods.[8] Of course, as we will see, the reduction in the benefits of having children due to the fall in the present value of transfers may also induce parents to invest less in human capital, an act which in turn feeds back onto optimal decisions regarding fertility.

The optimal choice on the children's endowment of human capital h_{t+1} is determined by

$$\frac{\psi}{R}\frac{n_t^{\varpi}}{\varpi}\frac{\partial w_{m,t+2}}{\partial h_{t+1}} = \phi_h n_t w_{m,t+1} \tag{7}$$

[8] In their dynastic model, a rise in interest rate raises per-capita consumption of future generations, and altruism therefore induces people to have more children.

where the marginal gains of having more educated children support oneself in old-age, is equalized to the marginal cost of further educating each child. Using Eq. 3, the above expression yields the optimal choice for h_{t+1}, given n_t and the parent's own human capital h_t, which is predetermined:

$$h_{t+1} = \left[\frac{\alpha\psi}{\phi_h R} \frac{z_{t+2}}{z_{t+1}} \frac{1}{h_t^{\alpha} \varpi n_t^{1-\varpi}} \right]^{\frac{1}{1-\alpha}}. \tag{8}$$

A greater number of children n_t reduces the gains from educating them—a quantity and quality trade-off. This trade-off arises from the fact that the net benefits in terms of transfers are decreasing in the number of children. Indeed, if there were no decreasing returns to transfers, $\varpi = 1$, then there is also no tradeoff. For $\varpi < 1$, the slope of the trade-off depends on $\frac{\alpha\psi}{\phi_h R}\frac{z_{t+2}}{z_{t+1}}$. Given any number of children n_t, incentives to provide further education is increasing in the returns to education (α) and aggregate productivity growth ($\frac{z_{t+2}}{z_{t+1}}$)—which gauges the future benefits relative to the current costs of education. Greater 'altruism' of children for parents (high ψ) increases parental investment in them. Higher marginal cost of education ϕ_h (parents' opportunity cost of h_t) reduces human capital accumulation.

The rate of return R also affects human capital accumulation. As is clear in Eq. 8, a rise in R reduces the marginal benefit of an additional unit of human capital, as the present value of transfers from children falls.

The optimal number of children n_t, combining Eq.6 and 8 satisfies

$$n_t = \left(\frac{v}{\beta(1+\beta)+v} \right) \left(\frac{1-\theta-\psi\frac{n_{t-1}^{\varpi-1}}{\varpi}}{\phi_0 + \phi_h \left(1-\frac{\omega}{\alpha}\right) h_{t+1}} \right). \tag{9}$$

Equations 8 and 9 are two equations that describe the evolution of the two key endogenous variables of in economy $\{n_t; h_{t+1}\}$.

Eq. 9 elucidates the equilibrium relationship between the number of children to bear in relation to the amount of education to provide them. There are two competing effects governing this relationship: the first effect is that higher levels of education per child raises transfers per child, thus motivating parents to have more children. The second effect is that greater education, on the other hand, raises the cost per child, and thus reduces the incentives to having more children. The first effect dominates if diminishing returns of transfers is relatively weak compared to diminishing returns to education, $\omega > \alpha$—in which case the relationship between n_t and h_{t+1} is positive. The second effect dominates when the diminishing returns to education is relatively weak, $\omega < \alpha$, and the relationship between n_t and h_{t+1} is negative. The two effects cancel out when $\omega = \alpha$, and decisions on n_t become independent of human capital decisions.

Definition of Saving Rates. The aggregate saving of the economy in period t, denoted as S_t, is the sum of the aggregate saving of each generation $\gamma = \{y, m, o\}$ coexisting in period t. Thus, $S_t = \sum_\gamma S_{\gamma,t}$, where the overall saving of each generation $S_{\gamma,t}$ is

$$\begin{aligned} S_{y,t} &\equiv N_t^y a_{y,t} \\ S_{m,t} &\equiv N_t^m \left(a_{m,t} - a_{y,t-1}\right) \\ S_{o,t} &\equiv -N_t^o a_{m,t-1}, \end{aligned}$$

where saving is by definition the change in asset holdings over a period, and optimal asset holdings $a_{\gamma,t}$ are given by Eq. 4 and Eq. 5. Let the aggregate saving rate at t be

$$s_t \equiv S_t/Y_t,$$

where Y_t is aggregate labour incomes of the country given by:

$$Y_t \equiv w_{y,t}N_{y,t} + w_{m,t}N_{m,t}$$

Let the individual saving rate of generation γ be denoted as $s_{\gamma,t}$, where the saving rate is defined to be the change in asset holdings over a period divided by the individuals's corresponding labour incomes for the the young and middle-aged and capital incomes for the old:[9]

$$s_{y,t} \equiv \frac{a_{y,t}}{w_{y,t}} \ ; \ s_{m,t} \equiv \frac{a_{m,t} - a_{y,t-1}}{w_{m,t}}; \ s_{o,t} \equiv -\frac{a_{m,t-1}}{(R-1)a_{m,t-1}} = -\left(\frac{1}{R-1}\right)$$

The aggregate saving rate can thus be decomposed into the saving rate of an individual from generation γ and the entire generation's income-contribution to aggregate income:

$$\begin{aligned} s_t &= s_{y,t}\left(\frac{w_{y,t}N_{y,t}}{Y_t}\right) + s_{m,t}\left(\frac{w_{m,t}N_{m,t}}{Y_t}\right) + s_{o,t}\left(\frac{(R-1)\,N_t^o a_{m,t-1}}{Y_t}\right) \\ &= s_{y,t}\left(\frac{n_t w_{y,t}}{y_t}\right) + s_{m,t}\left(\frac{w_{m,t}}{y_t}\right) + s_{o,t}\left(\frac{(R-1)\,a_{m,t-1}}{n_{t-1}y_t}\right) \end{aligned} \quad (10)$$

where it is convenient to introduce aggregate labour income per middle-aged household: $y_t = Y_t/N_{m,t}$.

[9]For analytical convenience, we do not include transfers when defining disposable income of each generations. Results are very similar qualitatively when including transfers but with more cumbersome expressions.

The aggregate saving rate is thus a weighted average of the young and middle-aged's saving rate, less dissavings of the old. The weights depend on both population and relative income weights of the contemporaneous generations. Changes in fertility will thus affect the aggregate saving rate through a 'micro-economic channel'— individual saving behaviors (through the optimal saving rate $s_{m,t}$)—and a 'macroeconomic channel'– changes in the composition of population and income.

3.2 The Steady State with Endogenous Fertility

Steady-state fertility and human capital. Assume that in the steady state, $\frac{z_{t+2}}{z_{t+1}} = 1+g_z$; $h_{t+1} = h_t = h_{ss}$, and $n_t = n_{t-1} = n_{ss}$. We consider whether equations (8) and (9) define a unique steady-state solution for $\{n_{ss}; h_{ss}\}$. These equations, in steady-state, are expressed as:

$$\frac{n_{ss}}{1-\theta-\psi\frac{n_{ss}^{\varpi-1}}{\varpi}} = \left(\frac{v}{\beta(1+\beta)+v}\right)\left(\frac{1}{\phi_0+\phi_h\left(1-\frac{\omega}{\alpha}\right)h_{ss}}\right) \qquad (NN)$$

$$h_{ss} = \left(\frac{\alpha\psi\left(1+g_z\right)}{\phi_h R}\right)\frac{n_{ss}^{\varpi-1}}{\varpi} \qquad (QQ).$$

Figure 3.2 depicts graphically the determination of the steady-state values of n_{ss} and h_{ss}. The curve denoted (NN) describes the response of fertility to higher education. This curve has a positive slope so long as $\varpi \geq \alpha$. A higher level of human capital increases transfers and thus induces greater incentives to have children. The curve (QQ) shows the combination of n and h that satisfy the quantity/quality trade-off in children. This curve has a negative slope as a greater number of children raises the cost of educating them and therefore leads to lower human capital investment per child.

The limiting values of n_{NN} and n_{QQ} as $h \rightarrow 0$ is such that $lim_{h\rightarrow 0}(n_{QQ}) > lim_{h\rightarrow 0}(n_{NN})$. This condition ensures that the curves intersect at least once. So long as $\varpi \geq \alpha$, the slopes of these two curves are respectively positive and negative throughout, thus guaranteeing that their intersection is unique. This leads to the following proposition:

Proposition 1 If $\varpi \geq \alpha$, there is a unique steady-state for the number of children $n_{ss} > \left(\frac{v}{\beta(1+\beta)+v}\right)\left(\frac{1}{\phi_0}\right)$ and their education choice $h_{ss} > 0$ to which the dynamic model defined by equations (8) and (9) converges. Also, comparative statics yield

$$\frac{\partial n_{ss}}{\partial g_z} > 0 \text{ and } \frac{\partial h_{ss}}{\partial g_z} > 0\ ;\ \frac{\partial n_{ss}}{\partial R} < 0 \text{ and } \frac{\partial h_{ss}}{\partial R} < 0$$

$$\frac{\partial n_{ss}}{\partial v} > 0 \text{ and } \frac{\partial h_{ss}}{\partial v} < 0;\ \frac{\partial n_{ss}}{\partial \phi_0} < 0 \text{ and } \frac{\partial h_{ss}}{\partial \phi_0} > 0.$$

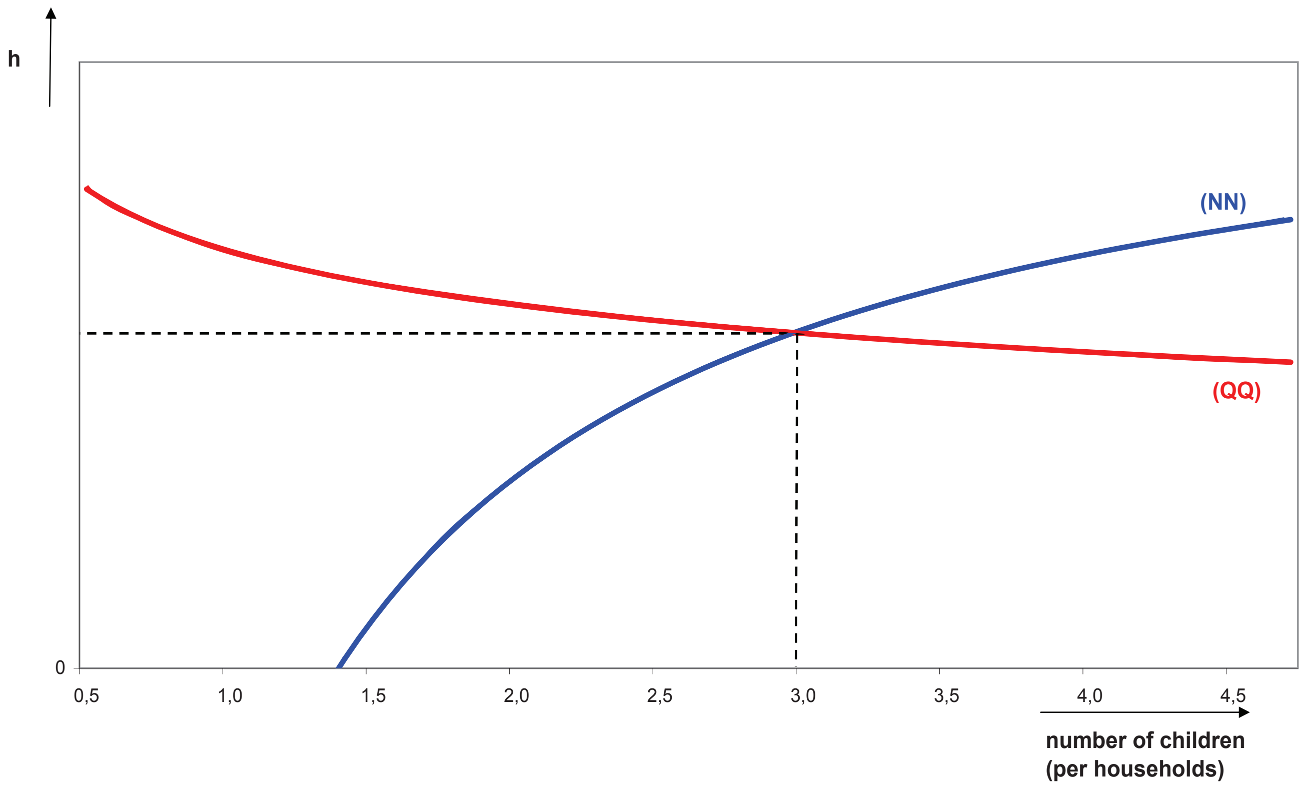


Figure 6: Steady-State Human Capital and Fertility Determination

Proof: See Appendix.

The comparative statics have a quite straightforward interpretation: higher growth (or lower real interest rates) increases the incentives to invest in children, both in terms of quantity and in terms of quality. A stronger preference towards children (or lower costs of raising them) makes parents willing to have more children but less educated (lower 'quality').

Steady-state aggregate savings. We next examine how changes in long-run fertility affect aggregate saving rate in the steady state (s). The saving rate, decomposed into the contribution of contemporaneous generation, from Eq. 10, is in the long run

$$s = \frac{n_{ss}e}{(1+n_{ss}e)}\underbrace{\left(\frac{-\theta(1+g_z)}{R}\right)}_{s_y} + \frac{1}{(1+n_{ss}e)}\underbrace{\left(\kappa(n_{ss})+\frac{\theta}{R}\right)}_{s_m} - \frac{\kappa(n_{ss})(R-1)}{n_{ss}(1+n_{ss}e)(1+g_z)}\underbrace{\left(\frac{1}{R-1}\right)}_{s_o}, \tag{11}$$

where $\kappa(n_{ss}) \equiv a_{m,t}/w_{m,t}$ in the steady-state and is given by the steady-state equivalent of Eq. 5:

$$\kappa(n_{ss}) = \frac{\beta}{1+\beta}\left[(1-\theta) - \underbrace{\left(\phi_0 n_{ss} + \frac{\alpha\psi(1+g_z)}{R}\frac{n_{ss}^{\varpi}}{\varpi}\right)}_{\text{cost of children}} - \underbrace{\psi\frac{n_{ss}^{\varpi-1}}{\varpi}}_{\text{cost of parents}} - \underbrace{\frac{\psi(1+g_z)}{\beta R}\frac{n_{ss}^{\varpi}}{\varpi}}_{\text{benefits from children}}\right],$$

using $h_{ss} = \frac{\alpha\psi(1+g_z)}{R}\frac{n_{ss}^{\varpi}}{\varpi}$ from Eq. 7.

Micro-Economic Channel. The above expression illuminates the three channels through which a reduction in long-run fertility affect optimal asset holdings of a middle-aged individual, and therefore his saving *behavior*. The first channel is to reduce the total cost of children—both because there are 'fewer mouths to feed' ($\phi_0 n_{ss}$ falls) and because total education costs have fallen in spite of the rise in human capital per child ($\alpha\psi(1+g_z)/Rn_{ss}^{\varpi}$ falls).[10] The second effect comes through the impact on the 'cost of parents'—the amount of transfers given to the middle-aged individual's parents ($\psi\frac{n_{ss}^{\varpi-1}}{\varpi}$ rises). As there are fewer siblings among whom the individual can share the burden, total transfers to parents rise, thus reducing the saving rate. The third channel is through the transfers made by children (the term $\frac{\psi(1+g_z)}{R}n_{ss}^{\varpi}$). With a reduction in fertility, the overall amount of transfers from children fall—despite higher human capital per child–thus reducing intertemporal wealth and raising the need to save. The overall micro-economic affect of a reduction in n_{ss} can be summarized by

$$\kappa'(n_{ss}) = \frac{\beta}{1+\beta}\left[-\phi_0 - \frac{(1+\alpha\beta)\psi(1+g_z)}{\beta R}n_{ss}{}^{\varpi-1} + \frac{\psi(1-\varpi)}{\varpi}n_{ss}{}^{\varpi-2}\right].$$

One can see that under the weak assumption that $n_{ss}(1+g_z)(1+\alpha\beta)/(\beta R) > (1-\varpi)/\varpi$, a fall in the steady-state number of children *raises* the steady-state savings rate of the middle-aged. As ϖ approaches 1, the transfers made to the parents are independent of the number of siblings, and a fall in n_{ss} does not reduce saving owing to greater transfers to parents—-

[10]The total cost of education is $n_{ss}h_{ss}$ which is increasing in n_{ss}. In other words, the rise in human capital per child rises by less than the fall in the number of children. This is because the overall reduction in transfers coming from fewer children also reduces incentives to educate heavily in them.

that is, the third term disappears. In this case, $\kappa'(n_{ss})$ is unambiguously negative.

Macro-Economic Channel. The macro-economic channels comprise of changes in the composition of population, and the composition of income attributed to the young and the middle-aged cohorts. This is evident by examining the overall impact of n_{ss} on aggregate saving rate, given by Eq. 11:

$$\begin{aligned}\frac{\partial s}{\partial n_{ss}} &= \underbrace{-\frac{e}{(1+n_{ss}e)}\cdot s - \frac{\kappa'(n_{ss})}{n_{ss}\,(1+n_{ss}e)\,(1+g_z)}}_{\text{income composition effect}} + \underbrace{\frac{1}{1+n_{ss}e}\left[-\frac{e\theta(1+g_z)}{R} + \frac{\kappa(n_{ss})}{n_{ss}^2}\frac{1}{(1+g_z)}\right]}_{\text{population composition effect}} \\ &+ \underbrace{\frac{\kappa'(n_{ss})}{1+n_{ss}e}}_{\text{decision effect (-)}}\ ,\end{aligned}$$

which shows that apart from the micro-economic channel–the 'decision effect' (last term of the equation)—changes to aggregate saving occur through macro-level compositional changes. The first compositional change is an 'income composition effect', which is ambiguous. A reduction in fertility reduces the proportion of the young's contribution to aggregate income, $n_{ss}e$. Thus, more of aggregate income is attributed to the middle-aged savers of the economy and less to the young borrowers—therefore raising aggregate saving rate. On the other hand, under the weak assumption discussed above ($\kappa'(n_{ss}) < 0$), lower fertility increases the interest payments to old dissavers (since aggregate wealth over income in the economy increases) and thus their share in total income.

The second aggregate compositional effect is through the change in the proportion of population. A reduction in n_{ss} reduces the proportion of young borrowers (the first term), thus tending to raise aggregate saving rate, but also increases the proportion of the old dissavers (the second term), thus tending to reduce it. The overall effect of population compositional changes is thus ambiguous. High long-run growth rate g_z tends to raise the weight of the young's borrowing and reduce the weight of the old's dissaving.

However, it is crucial to note that along the transition path towards a steady state with lower fertility, both the income and population composition effects will tend to raise aggregate saving rate. The reason is that the proportion of the middle-age rises immediately but the proportion of dependent elderly will take longer to increase as the population ages. Similarly, the share of income of middle-age raises before the share of income of the old increases.

3.3 Experiment 1: The 'One-child policy'

We first examine the theoretical implications of the one child policy on the aggregate saving rate—comparing to the saving rate under an initial equilibrium of unconstrained fertility. We then theoretically derive conditions from which we can identify the effect of the one-child policy on individual saving behavior (the micro-economic channel)—by examining theoretically the case where some families exogenously deviate from the policy by having twins. We then show conditions under which we can infer a lower bound for the *total* effect on the aggregate saving rate of the policy.

Following the 'one-child policy' in China, suppose that the government enforces a law that compels each agent to have up to a number $n_{\max}$ of children over a certain period $[t_0; t_0 + T]$ with $T > 1$. In the case of the one-child policy, the maximum number of children each individual can have is $n_{\max} = 1/2$). We now examine the transitory dynamics of the key variables following the implementation of the policy, starting from an initial steady-state of unconstrained fertility characterized by $\{n_{t_0-1}; h_{t_0}\}$.

Human capital accumulation. The additional constraint $n \leq n_{\max}$ is now added to the original individual problem. In the interesting scenario in which the constraint is binding, we have the following result:

Lemma 1: Assuming $\alpha < 1/2$. As $T \to \infty$, human capital converges to a new (constrained) steady-state $h_{\max}$ such that:

$$h_{\max} = \left(\frac{\alpha\psi(1+g_z)}{\phi_h R}\right)\frac{n_{\max}^{\varpi-1}}{\varpi} > h_{t_0}$$

This shows that an important implication is that the policy aimed at reducing the population inadvertently *increases* the long-run level of per-capita human capital, thus moving the equilibrium along the (QQ) curve as shown in Figure 3.3.

Proof: From Eq. 8, where $n_{\max}$ substitutes for the choice variable n_t, the dynamics of $\log(h_{t+1})$ is given by

$$\log(h_{t+1}) = \frac{1}{1-\alpha}\log\left(\frac{\alpha\psi}{\phi_h R}\frac{n_{\max}^{\varpi-1}}{\varpi}\right) + \frac{1}{1-\alpha}\log(\frac{z_{t+2}}{z_{t+1}}) - \frac{\alpha}{1-\alpha}\log(h_t),$$

where $\log(h_{t+1})$ is mean-reverting due to $-\frac{\alpha}{1-\alpha} < 1$ for $\alpha < 1/2$. It follows from $n_{t_0-1} > n_{\max}$ that $h_{\max} > h_{t_0}$.

Dynamic of savings following the policy. Assuming constant long-run productivity growth g_z, we next examine the one-period impact of the one-child policy implemented in t_0

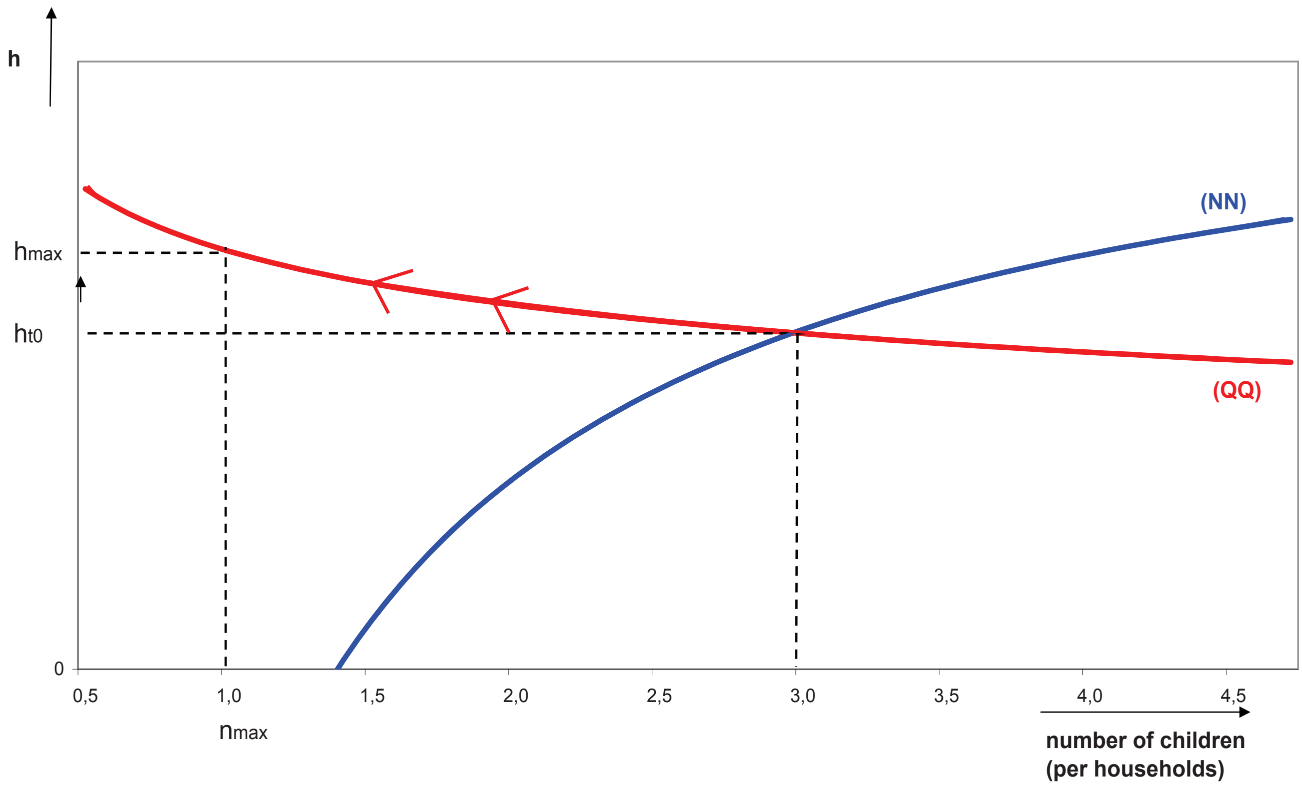


Figure 7: Human Capital and Fertility Determination under the 'one-child policy'

on the dynamics of the aggregate saving rate between t_0 and t_0+1, given by the following lemma:

Lemma 2: For $\omega > 1/2 > \alpha$, imposing the constraint $n_{t_0} \leq n^{max}$ in period t_0 leads to a rise in aggregate saving rate over one period:

$$s_{t_0+1} - s_{t_0} > 0$$

Proof: See Appendix.

The change in aggregate saving rate over the period after the implementation of the policy can be written as:

$$
\begin{aligned}
s_{t_0+1} - s_{t_0} \;=\;& \underbrace{\frac{(n_{t_0-1} - n_{\max})\, e}{1 + n_{\max} e} s_{t_0} + \frac{1}{1 + n_{\max} e} \frac{\theta\,(1+g_z)}{R} \left(n_{t_0-1} - n_{\max} \left(\frac{h_{t_0+1}}{h_{t_0}} \right)^{\alpha} \right)}_{\text{macro-channel (composition effects)}} \qquad (12) \\
&+ \underbrace{\frac{1}{1 + n_{\max} e} \left[\frac{\beta}{1+\beta} \phi_0 \,(n_{t_0-1} - n_{\max}) + \frac{(1+\beta\alpha)}{R(1+\beta)} \frac{\psi\,(1+g_z)}{\varpi} \left(n^{\omega}_{t_0-1} - n^{\omega}_{\max} \left(\frac{h_{t_0+1}}{h_{t_0}} \right)^{\alpha} \right) \right]}_{\text{micro-economic effect}}.
\end{aligned}
$$

$n^{\omega}_{\max} \left(\frac{h_{t_0+1}}{h_{t_0}} \right)^{\alpha}$ is like a quality-adjusted number of children, and is relevant insofar as they contribute to education costs and transfer benefits to parents. The channels through which constrained fertility affects the change in saving rate during the transition are the three channels emphasized before. However, the main difference is that the income and population composition effects only consist of the impact of a proportional reduction in the young cohort. This reduction in fertility has not yet fed into increasing the proportion of the dependent elderly in one generation, and therefore the old's negative impact on saving is absent. All channels exert pressure on the saving rate in the same direction, and it thus rises unambiguous over the period following the implementation of the policy.[11]

3.4 Experiment 2: Identification through 'twins'

Consider the scenario in which all middle-aged individuals exogenously deviate from the 'one-child policy' by having twins. From Eq. 8, the per-capita human capital of the twins (denoted $h^{twin}_{t_0+1}$) must satisfy:

$$
\left(h^{twin}_{t_0+1}\right)^{1-\alpha} h^{\alpha}_{t_0} = \left(\frac{\alpha\psi}{\phi_h R} \frac{z_{t+2}}{z_{t+1}} \right) \frac{(2n_{\max})^{\varpi-1}}{\varpi} < \left(\frac{\alpha\psi}{\phi_h R} \frac{z_{t+2}}{z_{t+1}} \right) \frac{(n_{\max})^{\varpi-1}}{\varpi} = \left(h_{t_0+1}\right)^{1-\alpha} h^{\alpha}_{t_0},
$$

which leads to our first testable implication:

Test 1: Quantity-Quality Tradeoff

$$
\frac{1}{2} < \left(\frac{h^{twin}_{t_0+1}}{h_{t_0+1}} \right) = \left(\frac{1}{2} \right)^{\frac{1-\varpi}{1-\alpha}} < 1 \qquad (\varpi > \alpha). \tag{13}
$$

[11] Along the transition path, we show that $n_{\max} \left(\frac{h_{t_0+1}}{h_{t_0}} \right)^{\alpha} < n_{t_{0-1}}$ and $n^{\omega}_{\max} \left(\frac{h_{t_0+1}}{h_{t_0}} \right)^{\alpha} < n^{\omega}_{t_0-1}$ under the assumption that $\omega > 1/2 > \alpha$.

This illustrates the quantity-quality tradeoff driving human capital accumulation in our framework, and can be identified by comparing twins and only-child. This ratio as measured by the data provides some guidance on the relative strength of ϖ and α. It also shows that the fall in human capital per capita is less than the increase in the number of children, so that total education costs still rise for twins.

Test 2: Microeconomic Effect on Saving.
The second implication is on the micro-economic impact of having twins on the middle-age parent's saving rate decisions. In Appendix, we show that the difference in the saving rate in the case of only-child compared to twins in t_0+1 satisfies, for $\varpi > \alpha$:

$$s_{m,t_0+1} - s^{twin}_{m,t_0+1} = \frac{\beta}{1+\beta}\left(n_{\max}\phi_0 + \frac{(1+\alpha\beta)}{R\beta}\frac{\psi(1+g_z)}{\varpi}n^{\varpi}_{\max}\left(\frac{h_{t_0+1}}{h_{t_0}}\right)^{\alpha}\left(2^{\frac{\varpi-\alpha}{1-\alpha}}-1\right)\right) > 0.$$

Having an only child compared to twins lowers total education and consumption expenditures but also reduces the overall amount of transfers coming from children—-therefore raising the saving rate of an only-child parent above that of a twin parent.

A lower bound of the effect of the policy on savings
Let the micro-economic impact of a policy constraining fertility on the saving rate be $\Delta S(n_{t_0-1})$–the change from the level under unconstrained fertility n_{t_0-1}. From Eq. 12, this change is

$$\Delta S(n_{t_0-1}) = \frac{(1+\beta\alpha)}{R\beta}\frac{\psi\,(1+g_z)}{\varpi}\left(n^{\omega}_{t_0-1} - n^{\omega}_{\max}\left(\frac{h_{t_0+1}}{h_{t_0}}\right)^{\alpha}\right) + \phi_0\,(n_{t_0-1} - n_{\max})\,.$$

Lemma 3: If $n_{t_0-1} = 2n_{\max}$, then

$$\Delta S(n_{t_0-1}) = s_{m,t_0+1} - s^{twin}_{m,t_0+1}$$

Proof: See appendix.
This tells us that under the condition that the initial unconstrained fertility is twice the constrained fertility, we can identify precisely the micro-economic impact of the policy on the aggregate saving rate— by comparing the saving rate of a middle-aged individual with n_{max} kids to that with $2n_{max}$ kids. This also provides us with a lower-bound estimate of the overall impact of the one-child policy on the aggregate saving rate if fertility before the policy was above two children per households. If $n_{t_0-1} > 2n_{\max}$, then

$$\Delta S(n_{t_0-1}) > s_{m,t_0+1} - s^{twin}_{t_0+1}.$$

In other words, if the unconstrained fertility were greater than 2 (as observed in the data for China prior to the policy change), then the 'micro-economic effect' identified on twins is a lower bound of the effect of the policy on the middle-age saving rate. The birth of twins is thus very informative regarding the micro-channel.

One can also tries to identify a lower bound of the aggregate effect. The first term of the macro-channel (income composition effect) in Eq. 12 can be directly estimated from the data if one can observe the parameter e as well as fertility and savings before the policy implementation (s_{t_0} and n_{t_0-1}). The second term of the macro-channel (population composition effect) is more subtle as it incorporates the endogenous response of human capital accumulation following the policy (term $\left(n_{t_0-1} - n_{\max}\left(\frac{h_{t_0+1}}{h_{t_0}}\right)^{\alpha}\right)$) as well as the degree of credit constraints θ. Data on savings rate of the young workers could be informative regarding the value of θ. Moreover, since data allow us to observe human capital attainment of twins (or equivalently human capital expenditures of families with twins) compared to families with an only child, one can deduce a lower bound for $\left(n_{t_0-1} - n_{\max}\left(\frac{h_{t_0+1}}{h_{t_0}}\right)^{\alpha}\right)$ under the assumption $n_{t_0-1} > 2n_{\max}$. Thus, in principle, from these estimates, one can obtain a lower bound for the *micro-economic* and *macro-economic* channels and thus a lower bound of the overall effect on the aggregate saving rate.

4 A Quantitative OLG Model

We extend our baseline model to more periods in order to capture better savings over the life-cycle. This will allow us to simulate the 'one-child policy' in a calibrated version of the model to obtain the overall impact on savings as well as finer predictions regarding the age-savings profile, predictions that can be confronted to our household-level data.

4.1 Set-up and model dynamics

Timing The life cycle follows a similar structure as before, except that more periods are included to allow for a more elaborate timing of various events and behavior that better capture patterns in the data.

Agents now live for 8 periods, so that in each period, eight generations denoted as $\gamma = \{1; 2...; 8\}$ coexist in the economy. An agent born is a young child/adult for the first two periods, accumulating human capital in the second period, and becomes a young worker in period 3. As a middle-aged parent in between periods 4-6, the individual rears and educates his own child, and make transfers to his now elderly parents. Upon becoming old in period

7 and 8, the individual finances consumption from previous saving and support from his children, and dies with certainty at the end of period 8 without leaving a bequest.

Preferences. Let $c^i_{\gamma,t}$ denote the consumption of cohort γ in period t, with $\gamma \in \{3, 4, ..., 8\}$. The lifetime utility of an agent born at $(t-2)$ and who enters the labour market at date t is

$$U_t = v \log(n_t) + \sum_{\gamma=3}^{8} \beta^\gamma \log(c_{\gamma,t+\gamma-3})$$

where n_t is the number of children the individual chooses to have (children per head), decided at the end of period 3 (β the discount rate and $v > 0$).

Budget Constraints. We still consider an agent entering the labour market at date t. The costs of rearing and educating children, and transfers made to parents are similarly modeled as the simple model, except with a slight modification in the timing. We assume, without loss of generality, that all of these costs and transfers are paid during middle age.[12] The total costs paid for kids during an individual's fourth and fifth periods include: a "mouth to feed expenditure" which is a fraction ϕ_γ of the agent's wage income, for a total of $\phi_\gamma n_t w_{\gamma,t+\gamma-3}$ for a parent with n_t children ($\gamma = \{4, 5\}$); an education cost, born only in period 5, amounting to $\phi_h n_t h_{t+1} w_{5,t+2}$ for all of the children combined. The timing of the education cost is motivated by observations from the data, which reveals that the bulk of 'non-compulsory' education costs over a child's course of study is paid when the child is in between the ages of 15 and 25— right before he enters the labor market. Transfers made to the individuals' parents occur only in period 5 and 6, in the total amount of $-\psi \frac{n_{t-1}^{\varpi-1}}{\varpi} w_{\gamma,t+\gamma-3}$ (with $\psi > 0$ and $\varpi > 0$). For clarity, Figure 4.1 summarizes the timing of income flows and the total costs and transfers, denoted as $T_{\gamma,t+\gamma-3}$ ($\gamma = \{3..., 8\}$) pertaining to the individual in different stages of life. Wages are defined similarly to the simpler version: an individual entering at date t in the labour market with human capital h_t earns $w_{\gamma,t+\gamma-3} = e_{\gamma,t} z_{t+\gamma-3} h_t^\alpha$ for $\gamma = \{3..., 8\}$; $e_{\gamma,t}$ denotes an age effect in the life-income profile (which is potentially time-varying if growth is biased towards certain age-groups) and $z_{t+\gamma-3}$ denotes aggregate productivity at a given date. For simplicity, we will assume productivity growth to be constant, so that $z_{t+1}/z_t = 1 + g_z$.

Optimal Consumption/Saving. As before, we assume that agents can only borrow up to a fraction θ of the present value of their future labor income:

$$a_{\gamma,t+g-3} \geq -\theta \frac{w_{\gamma+1,t+\gamma-2}}{R} \text{ for } \gamma = \{3; ...; 8\}.$$

[12]This is also in line with the average age of first child in the data being 28 years old (average over the period 1975-2005 from UHS).

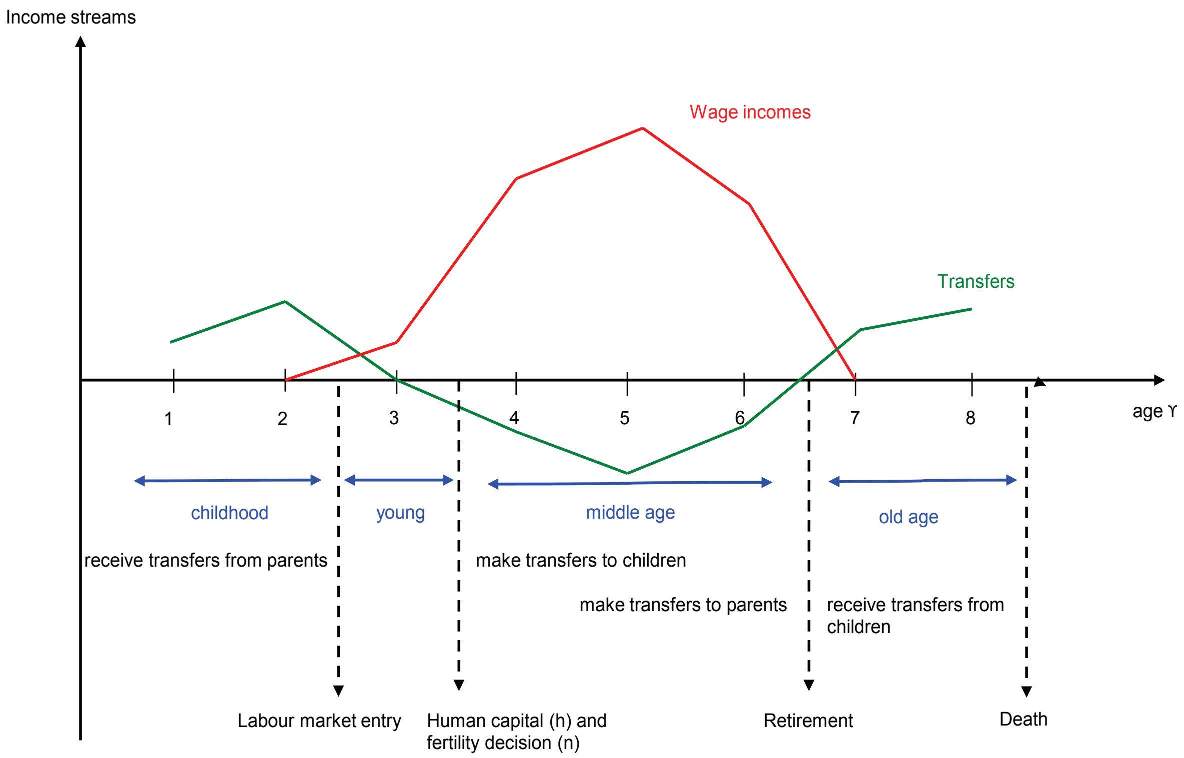


Figure 8: Timing of the quantitative OLG model.

where $a_{\gamma,t}$ denotes total asset accumulation by the end of period t for generation $\gamma = \{3; ...; 8\}$. The gross interest rate R is still taken to be exogenous and constant over time.

We make the assumption that life-income profiles $e_{\gamma,t}$, productivity growth g_z, interest rate and discount factor (β and R) are such that the constraint is only binding in the first period of work ($\gamma = 3$). We check that this is the case along the equilibrium path in our simulations.

The sequence of budget constraints, for an individual born at $t-2$ (and entering labour market at date t), are then:

$$\begin{aligned} c_{3,t} &= w_{3,t} + \theta \frac{w_{4,t+2}}{R} \\ c_{4,t+1} + a_{4,t+1} &= w_{4,t+1}(1-\theta) + T_{4,t+1} \\ c_{\gamma,t+\gamma-3} + a_{\gamma,t+\gamma-3} &= w_{\gamma,t+\gamma-3} + T_{\gamma,t+\gamma-3} + Ra_{\gamma-1,t+\gamma-4} \qquad \text{for } \gamma = \{5;6;7\} \\ c_{8,t+5} &= T_{8,t+5} + Ra_{7,t+4} \end{aligned}$$

The intertemporal budget constaint can be derived when combining the period constraints from $\gamma = 4$ to 8:

$$\sum_{\gamma=4}^{8} \frac{c_{\gamma,t+\gamma-3}}{R^{\gamma-4}} = \sum_{\gamma=4}^{8} \frac{w_{\gamma,t+\gamma-3} + T_{\gamma,t+\gamma-3}}{R^{\gamma-4}} - \theta w_{4,t+1},$$

which, along with the Euler equations, for $\gamma \geq 4$:

$$c_{\gamma+1,t+\gamma-2} = \beta R c_{\gamma,t+\gamma-3} \tag{14}$$

yields

$$\left(\sum_{\gamma=4}^{8} \beta^{\gamma-4} \right) c_{4,t+1} = \sum_{\gamma=4}^{8} \frac{w_{\gamma,t+\gamma-3} + T_{\gamma,t+\gamma-3}}{R^{\gamma-4}} - \theta w_{4,t+1}. \tag{15}$$

Optimal consumption and saving decisions, given $\{n_t; h_{t+1}\}$, are then be pinned down for all periods,

Fertility and human capital. The quantitative model, despite being more complex, yields a similar set of equations capturing the dynamics of fertility and human capital accumulation as in the simple model (see Appendix for detailed derivations):

$$n_t = \left(\frac{1}{1 + \Pi(\beta, v)} \right) \frac{(1-\theta) + G(g) - \psi \frac{n_{t-1}^{\varpi-1}}{\varpi} G(g)}{\phi_0 + (1+g)\phi_h h_{t+1} \left(1 - (1 + (1+g)) \frac{\varpi}{\alpha}\right)} \tag{16}$$

$$h_t^{\alpha} h_{t+1}^{1-\alpha} = \left(\frac{\psi\alpha}{\varpi} \frac{(1+g)^2}{\phi_h} \right) n_t^{\varpi-1} \tag{17}$$

where: $\Pi(\beta, v) = \frac{\beta}{v}(1 + \beta + ...\beta^8)$; $\frac{(1+g_z)}{R} = 1 + g$; $G(g) = (1 + g + (1+g)^2)$ and $\phi_0 = \phi_4 + (1+g)\phi_5$.

Lemma 4 If $(1 + (1+g))\varpi \geq \alpha$, there exists a unique steady-state $\{n_{ss}; h_{ss}\}$— characterized by $n_{ss} > \left(\frac{1}{1+\Pi(\beta,v)}\right)\left(\frac{(1-\theta)+G(g)}{\phi_0}\right)$ and $h_{ss} > 0$— to which the dynamic model defined by Eq. (16) and (17) converge. The modified (NN) and (QQ) curves, describing in the steady-state respectively the choice in fertility given human capital accumulation, and the quantity-quality tradeoff, become:

$$\frac{n_{ss}}{(1-\theta)+G(g)-\psi\frac{n_{ss}^{\varpi-1}}{\varpi}G(g)} = \left(\frac{1/(1+\Pi(\beta,v))}{\phi_0+(1+g)\phi_h h_{t+1}\left(1-(1+(1+g))\frac{\varpi}{\alpha}\right)}\right) \qquad (NN)$$

$$h_{ss} = \left(\frac{\psi\alpha}{\varpi}\frac{(1+g)^2}{\phi_h}\right)n_{ss}^{\varpi-1} \qquad (QQ)$$

Proof: Uniqueness and existence of the steady state is shown in the Appendix.

The intuition for the (NN) and (QQ) curves are analogous to those in the simple case, with comparative statics that equally holds here. An important difference with our simpler version though, is that a finer age-saving profile now emerges from the quantitative model. In order to derive such predictions regarding age-savings profile, we need to calibrate the quantitative model.

4.2 Data and Calibration

Parameters values

Calibrated value are all summarized in Table 4. One period is thought to last 10 years. We make the assumption that before *any* implementation of the 'one child policy' (before 1972), endogenous variables at at their steady-state with optimal steady-state fertility and human capital decisions $\{n_{ss}; h_{ss}\}$. We will use data from various sources described in Appendix 7.1 to calibrate the model.

Life income profile

The life income profile determined by the parameters $\{e_\gamma\}_{3\leq\gamma\leq6}$ are calibrated using individual income data per age-group from UHS. Since we want our estimates of the relative efficiency parameters e_γ to be the least affected by the implementation of the policy, i.e having workers of different age-groups to have similar level of human capital, we use the first year of the UHS sample (1992) for which we have individual wage incomes, even within households.[13] For these years, most of the individuals are constrained by the policy but none of their parents was, which is what matters for the previously made decision regarding their

[13]We also have UHS data for 1986 but they do not provide individual incomes for multigenerational households, which makes it more difficult to back out individual age-income profiles. One can still applying the method developed by Chesher (1998) to estimate individual life income profiles as we do to estimate individual consumption and saving profiles (see below). When doing so, our estimates are very close to the one directly observed from UHS 1992.

Table 4: Calibration of the model parameters. Unless specified otherwise, parameters are equal to their benchmark values in the alternative calibrations (*Low transfers* and *Time-varying income profile*

Parameter	Value	Target (Data source)
β (annual basis)	0.99	/
R (annual basis)	5%	Agg. household savings rate in 1985-1996
g_z (annual basis)	7%	Output growth per worker over 1980-2010 (PWT)
v	0.03	Fertility in 1966-1970 $n_{ss} = 3/2$ (Census)
θ	2%	Av. savings rate of under 25 in 1992-1993 (UHS)
α	0.45	/
ω	0.66	Transfer to elderly wrt the number of siblings (CHARL
$\{\phi_4; \phi_5; \phi_h\}$	$\{12\%; 5\%; 0.45\}$	Education exp. at ages 30-50 in 2008 (CHARLS/UHS)
ψ	12%	Savings rate of age 50-58 in 1986 (UHS)
$(e_\gamma/e_5)_{\gamma=\{3;4;6\}}$	$\{65\%; 90\%; 57\%\}$	Wage income profile in 1992 (UHS)
Alternative calibration		
Low transfers		
ψ	4%	Observed transfers to elderly (CHARLS)
R (annual basis)	3.8%	Agg. household savings rate in 1985-1996
v	0.17	Fertility in 1966-1970 $n_{ss} = 3/2$ (Census)
Time-varying income profile		
$(e_{\gamma,t}/e_5)_{\gamma=\{3;4;6\}}$ for $t \leq 2005$	$\{65\%; 90\%; 57\%\}$	Wage income profile in 1992 (UHS)
$(e_{\gamma,t}/e_5)_{\gamma=\{3;4;6\}}$ for $t > 2006$	$\{65\%; 97\%; 50\%\}$	Wage income profile in 2009 (UHS)

level of human capital. The wage profile across age groups extracted from the UHS as well as our calibrated one are shown in Figure 4.2. In our benchmark calibration, we assume the parameters e_γ to be constant over time. In an extension (*Time-varying income profiles*), we will allow some time variation to reproduce the flattening of the profile for adults below 45 towards the end of the sample period (2006-2009) as shown in Figure 4.2: in this calibration we assume $e_{\gamma,t}$ to be equal to their benchmark for $t \leq 2005$ but slightly different later on in order to match the cross-section of wages for the different age-groups (see Table 4).[14]

Fertility, demographic structure and policy implementation

Before 1972, families were not at all constrained in their fertility decisions. We take the value of the fertility before 1972 as our initial steady-state n_{ss}; to do so we use the fertility

[14]It is important to note that our model does predict a flattening of the curve when observing the cross-section of wages for the different age-groups. Indeed, younger household have higher level of human capital in 2009 than older ones. However, we still fall (slightly) short in explaining fully the flattening of the curve.

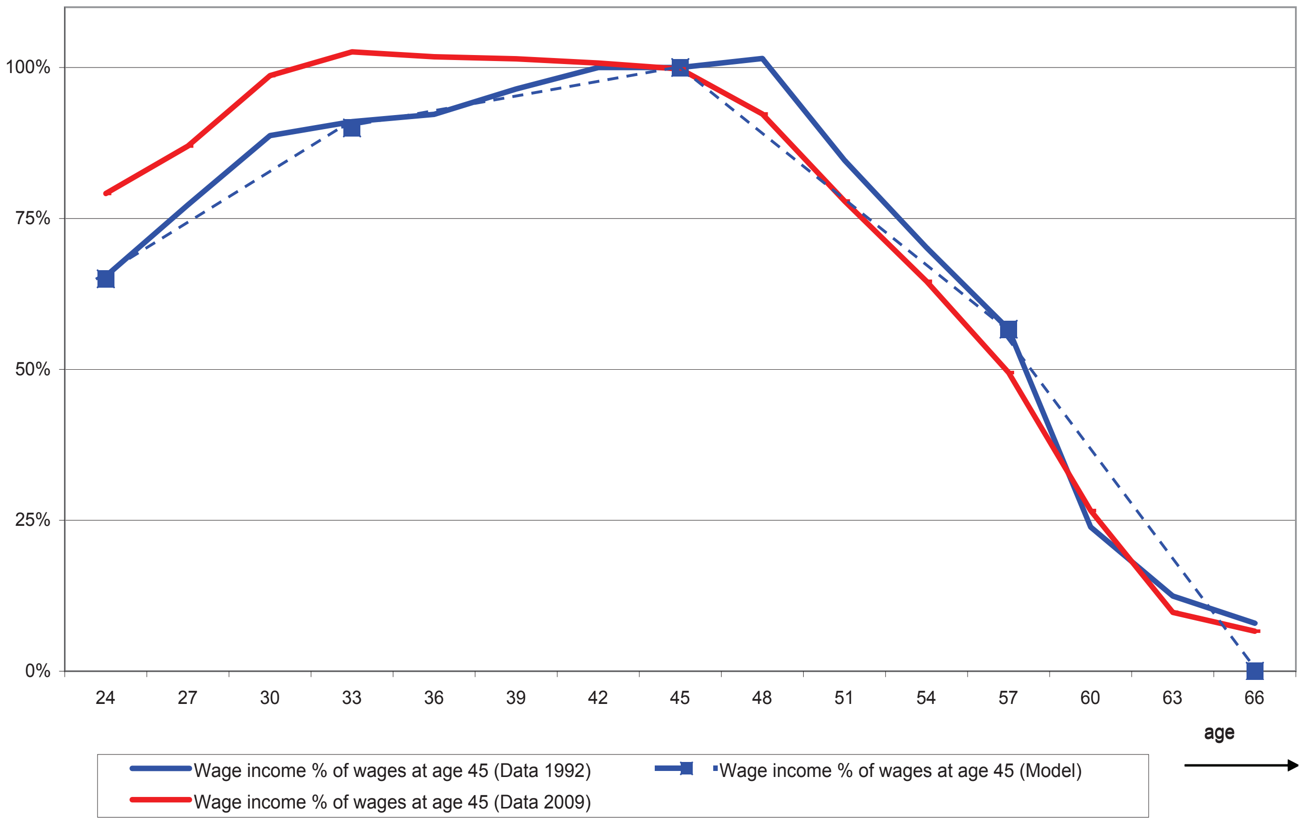


Figure 9: Life income profiles in 1992 and 2009.
Notes: Data source: UHS, 1992 and 2009. Wages includes wages plus self-business incomes.

rate prior to the policy implementation in urban areas based on the number of siblings of adults born between 1966-1971 from the Census; this gives us a benchmark of children (per head) $n_{ss} = n_{t<1972} = \frac{3}{2}$. The initial fertility rate is key to determine the impact of the policy in our simulations. Indeed, one could argue that this is not the appropriate counterfactual since unconstrained fertility nowadays could be different from the one observed before the policy. We initially make this arguably strong assumption but we will perform robustness checks regarding our assumed n_{ss}. The parameter that has to be calibrated to match fertility data is the one governing the preference for children v. Thus, we will set v to make sure that our model delivers a fertility rates n_{ss} of $\frac{3}{2}$ prior to the implementation of the policy. Given initial fertility, one needs to calibrate initial population distribution. To do so we use data from the United Nations in 1965 and calibrate the initial share of each age groups (0-10; 10-20, ..., 60-70 and above 70) to heir empirical counterpart.[15]

While the one-child policy is almost fully binding after 1980 (starting 1982 when looking at Census data), some early attempts to curb population growth were designed between 1972 and 1980, which largely explain the fall in fertility over this period (see Figure 1 in section 2). In our simulations, we will thus assume that the policy is implemented progressively during the seventies, with a first reduction occurring in the mid 70s. We consider cohorts born every four years. In line with fertility data for urban households (see Figure 1), We will assume in particular $n_{1971-1972} = \frac{2.7}{2}$, $n_{1975-1976} = \frac{2.25}{2}$and $n_{1979-1980} = \frac{1.3}{2}$. For any date after 1982, fertility is constrained to $n_{\max} = \frac{1}{2}$.

Transfers

CHARLS (2008) provides exhaustive data on income and transfers from parents to children as well as children to parents for adults aged above 40 in 2008.

We find that for parents aged between 42 and 48, average transfers per child towards children as a % of their wage amount to 25.6%, and based on UHS data education expenditures account for 20%-25% of total expenditures (RUMICI (2008); see Fig 5). This gives $\phi_5 + \phi_h h_{2008-2009} = 20 - 25\%$ (we obtain similar number if one looks at transfers towards children aged between 15 and 21). For children below 15, we find in CHARLS transfers of 10.6% but we have much fewer observations (as CHARLS is restricted to adult above 40). However, very similar estimates stand out for the dataset RUMICI, where the costs of education (as a fraction of total household expenditures) are in between 10% and 15% for a child aged between 5 and 15. We thus set $\phi_4 = 12\%$ to match these targets.Using expenditure data on education from CHIP, we find that (non compulsory) educations costs amount to 2% to4% of total household expenditures for children aged between 15 and 20 but CHIP seems to underestimate total education costs by some margin (lower estimates than in UHS and RUMICI; see Figure 4 and 5); we set $\phi_5 = 5\%$. ϕ_h is set to make sure that at age 45 ($\gamma = 4$), total transfers towards an only child account for about 20% of overall expenditures: this gives $\phi_h = 0.35\%$ (for $\phi_5 = 5\%$).[16] Note however that our estimates based on education expenditures is likely to be a lower bound of the total cost of a child since other transfers (food, co-residence...) take place during childhood.

[15]We would have liked to use data for Chinese urban households instead of the total population but such data are only available from the Census starting 1980. We check though that the future distribution of population by age implied by our imputed fertility rates (see below) is broadly in line with urban population data in the 1980s.

[16]Such values for ϕ_h and ϕ_5 generate a share of non-compulsory education costs at age $15 - 20$ broadly in line with the data (CHIP, 2002).

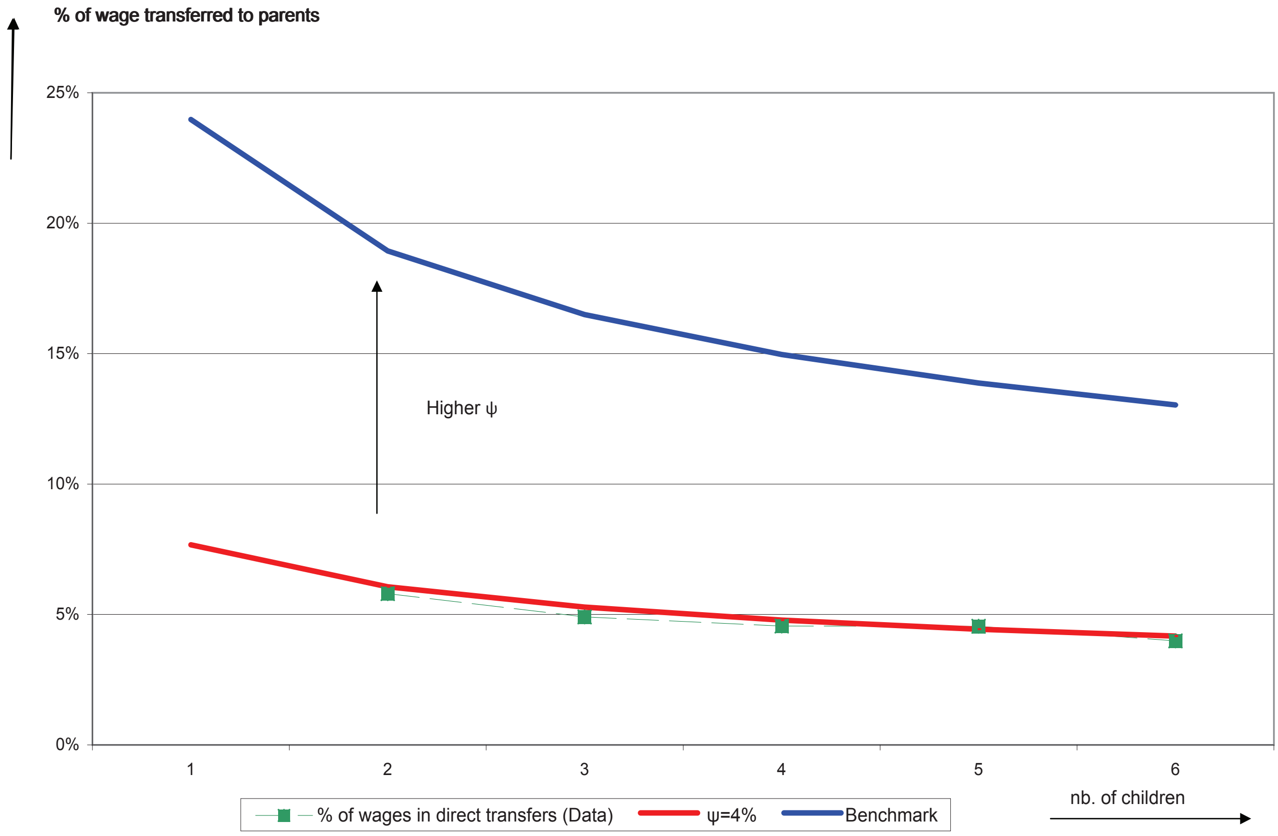


Figure 10: Transfers to old-age (% of wage income) as a function of the number of siblings. *Notes*: Data source: CHARLS (2008) and UHS (2008). We drop of only-child individuals due to lack of observations.

The calibration of transfers towards parents, which are the most important ones for our purpose, is less straightforward. First, two parameters need to be identified (ψ and ϖ), the first one governs the overall amount of transfers, while the latter governs the incentives of children to free-ride on the transfers provided by their siblings. In principle, one needs simply to observe total transfers, as well as how they vary with the number of siblings. From CHARLS (2008)[17], we get the average share of direct transfers towards parents as % of the child wage (as a function of the child age). We find it to be in between 4% and 7% of

[17]**[mixed with UHS (2008); to explain.**

wage incomes between age 42 and 54 (note that due high growth rates, this still lead to a quantitatively relevant fraction of incomes of the old). Figure 4.2 provides evidence on how the share transferred to parents is decreasing with the number of siblings individual have. This evidence would be enough to calibrate ψ and ϖ (see Figure 4.2) where we can match the transfer function $\psi\frac{n^{\varpi-1}}{\varpi}$ implied by the data with $\psi = 3.8\%$ and $\varpi = 0.66$. However, this is likely to be a *lower-bound* for ψ since this is equivalent to disregard any 'indirect' transfers through in-kind benefits (such as co-residence or healthcare) and/or social security system. Compared to children, we believe it is essential to take into account 'indirect' transfers towards elderly since the insurance they provide should have a strong impact on savings decisions for adults in their forties/fifties. As stated in section 2.3, co-residence and various other in-kind transfers are important 'indirect' transfers that could lead to higher estimates of ψ and potentially lower estimates of ϖ (it is likely to be financed by only one of the siblings). Social security is another form of 'indirect' intergenerational transfers. Any contribution to social security (as a % of wages) would shift upwards the transfer function shown in Figure 4.2, leading to higher estimate of ψ as well as ϖ (free-riding issues due to the presence to siblings are much less relevant in this case).[18] As explained in Song and Yang (2010), the retirement system in China is quite complex and has evolved significantly over the last twenty years. Basically, it is evolving since the 90s from a system of social security centrally-planned but with little coverage across the population (less than 50%) to a pay-as-you-go system. The former is the one we are interested if we want to estimate properly the amount of intergenerational transfers, however data do not allow us to distinguish between the two.[19]

While we keep $\varpi = 0.66$ estimated from 'direct' transfers throughout our calibrations, we remain agnostic regarding the values of ψ. In our preferred benchmark specification, we set ψ in order to obtain savings rates of adults in their fifties that match the UHS data in 1986: this yields $\psi = 12\%$ but we perform robustness checks with a lower value of ψ, i.e 3.8%, the latter matching our observations on 'direct' transfers.

Credit constraints

Motivated by the lack of credit card and mortgage markets and the very low levels of

[18]We could have consider a different transfer function of the form: $\psi_r + \psi\frac{n^{\varpi-1}}{\varpi}$ where ψ_r stands for the rate of contribution to social security while $\psi\frac{n^{\varpi-1}}{\varpi}$ stands for 'direct' transfers and in-kind benefits (other 'indirect direct' transfers). Results are however qualitatively and quantitatively very similar, the reason why we restrict ourselves to the identification of two parameters.

[19]Data on pension contributions from UHS (2008) shows that this amounts $xx\%$ of wages for individuals aged between xx and xx. However, contributions to pensions include both the old system (social security) and the new one (pay-as-you-go) [here we need to make some efforts to provide a reasonable number; one major issue is that the old system is disappearing]

household debt in China (less than 10% of GDP in 2008), we set a very low ability of young households to borrow against future income $\theta = 2\%$. Such a value provides reasonable estimates of the dissavings of the young, consistent with age-savings profiles observed throughout the last twenty years.

Production parameters, real interest rates and discount rates

Output growth rates per capita in China are very high throughout the thirty years period 1980-2010. Real growth rate of output per worker averages over the period to 8.2% (from Penn World Tables). In our model, part of this rate of growth occurs endogenously through human capital accumulation and thus 8.2% is an upper-bound of g_z: we set $g_z = 7\%$ which will lead to a real output growth per worker in the other of magnitude of 8%. We set the decreasing marginal return to education α to 0.45, in line with the empirical growth literature (see Mankiw, Romer and Weil (1992)).

We lack of empirical observations for Chinese average real return on assets R faced by households over the last thirty years. One could use an average rate of interest on long-term foreign bonds (this would give $R-1 \approx 4\%$ based on 10 years US T-bills over the period 1986-2006) but, while consistent with the model, this could be a poor proxy of the real return on assets faced by Chinese households. As it turns out, the calibration of R is quite irrelevant for our results since R and the discount rate β are calibrated simultaneously to provide an aggregate savings rate in the mid-eighties in line with the data ($s_{1985-1986} \approx 10\%$). In our simulations, we set $\beta = 0.99$ (on an annual basis) in all calibrations adjust $R-1$ accordingly (we find reasonable values for the real interest rates with an order of magnitude of $4-5\%$ in our calibrations).

Steady-state aggregate savings and age savings profiles before the implementation of the 'one child policy'

Figure 4.2 displays the steady-state saving rate at different stages of life before the policy implementation, denoted as $s_{\gamma,t}$ where $\gamma = \{3; ...8\}$ for our benchmark calibration as well as the ones with alternative values of intergenerational transfers parameter ψ.[20] Note that since young adult are credit constrained, the age-savings profile is unchanged in 1976 and only starts to change once the first generation of adults concerned by the policy reach age $\gamma = 4$, i.e their mid-thirties which occurs in the beginning of the eighties. We do not have household data for this period but for comparison purposes, we also plot the data implied age-savings

[20]We do not show the savings rate of the very old (which are very large dissavers) reaching age $\gamma = 8$ in the case $\psi = 4\%$ since they are of small interest for our purpose and would massively affect the scale of the graph.

profile in 1986 (first year of the sample of UHS data). Note that, in principle, only households under 40 are affected by the policy in 1986 (and mostly through its partial implementation in the seventies), so this remains a valid point of comparison for most middle-aged adults.[21]

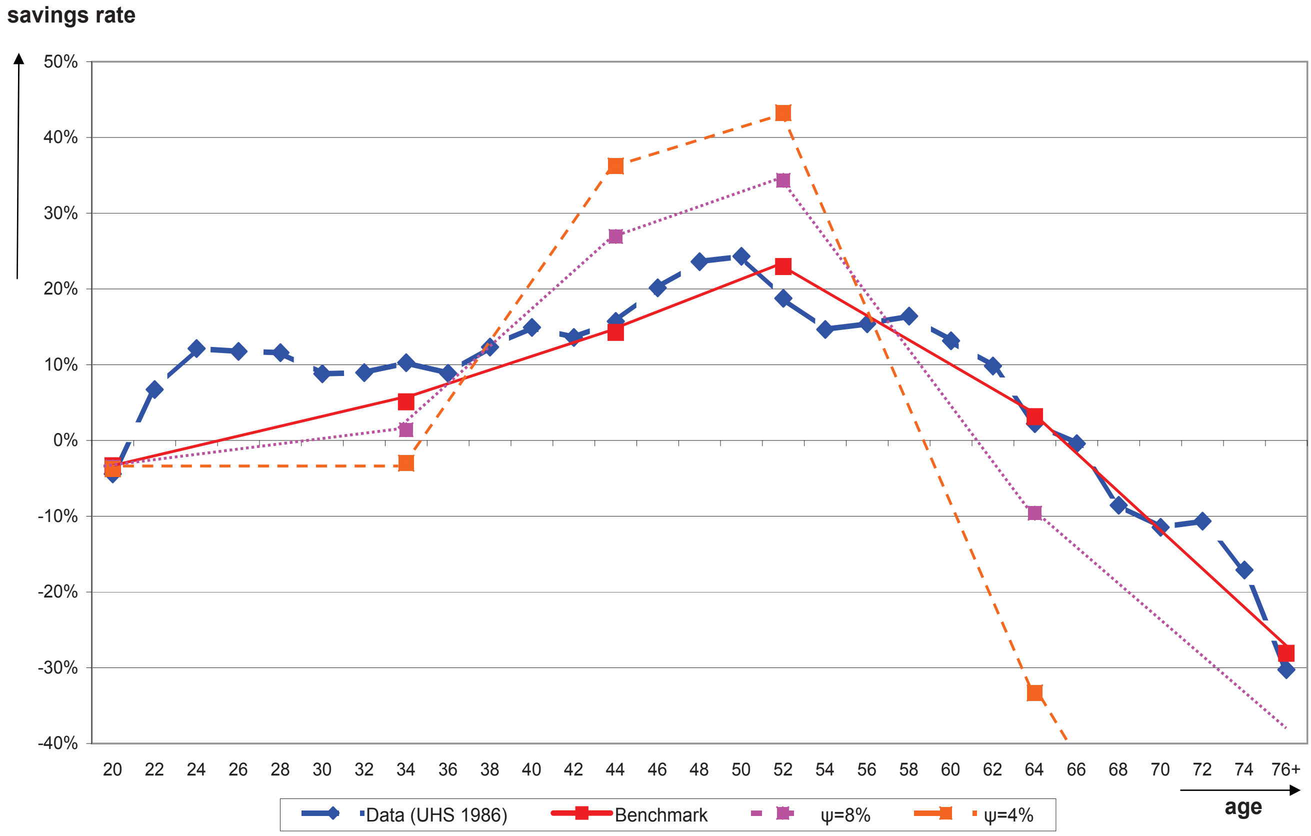


Figure 11: Initial age-savings profiles. Model with different values of ψ versus data

Notes: Data source: UHS, 1992. Individual profiles are constructed using Chesher (1998). See Technical Appendix and Coeurdacier, Guibaud and Jin (2012). Model implied profile are steady-state age-saving profiles with $n_{ss} = \frac{3}{2}$. For the different values of ψ, the real interest rate R is adjusted to keep aggregate savings constant to their 1985 value.

As shown in Coeurdacier, Jin and Guibaud (2012), estimating individual age-savings

[21] Young individuals are also not affected by the policy in our theoretical framework due to time-invariant credit constraints. Remind also that the average of the first child is in the eighties is 28.

profile in presence of multigenerational households (more than 50% of our observations) is a complex task and standard estimates based on the age of the household head are not accurate. We provide a detailed technical appendix to show how we can recover individual age-savings from household level data in presence of multigenerational household following a method initially developed by Chesher (1998). This relies on estimating individual consumption from household level consumption data using variations in the family composition as identification strategy. Individual savings are then estimated using these estimates of individual consumption as well as observed individual incomes (see Technical Appendix and Coeurdacier, Guibaud and Jin (2012) for more details).

Figure 4.2 illustrates how the amount of old-age support is key to generate age-savings profile in line with the data. Our benchmark calibration with high level of intergenerational transfers matches fairly well the level of savings of young and middle-aged individuals as well as the dissavings of the old. We underestimate slightly savings of young adults (between 26 and 30) but we see that as a good sign for our theory since these are the first cohorts affected by the policy change. Calibrations with lower values of ψ tend to underestimate savings of young middle-aged (in their 30s), overestimate savings of individuals in their 50s (with lower transfers, individuals have much more incentives to save for retirement). Due to much larger wealth accumulation, dissavings of the old also become much larger than in the data.

4.3 One child policy simulations

We aim at studying the transitory dynamics of the model following a 'one-child policy' starting from our unconstrained steady-state denoted $\{n_{ss}; h_{ss}\} = \{n_{t_0-1}; h_{t_0+1}\}$ and a given (steady-state) age-savings profile $\{s_{\gamma,t_0}\}_{\gamma=\{3;...8\}}$. The policy is implemented at date $t_0 = 1982$ and is assumed to be binding (with the exception of twin births).[22]

Analytical solutions are very cumbersome so we proceed through numerical simulations to describe the model dynamics following the implementation of the policy, using the calibration shown above (see Table 4).

Transitory dynamics of the calibrated model

As before, the maximization program remains the same with the binding constraint $n \leq n_{\max}$. After t_0, equation (17) still holds but the number of children is now $n_{\max}$:

$$h_{t+1}^{1-\alpha} h_t^{\alpha} = \left(\frac{\psi\alpha}{\varpi} \frac{(1+g)^2}{\phi_h} \right) \frac{n_{\max}^{\varpi-1}}{\varpi} \tag{18}$$

[22]Note that $h_{t_0+1} = h_{t_0}$ since the people holding a human capital of h_{t_0+1} are born in $t_0 - 1$ before the policy is implemented.

This describes the dynamics of human capital accumulation with initial condition h_{t_0+1}. Given the dynamics of human capital h_t for $t \geq t_0 + 1$, one can easily back out consumption/savings decisions for each age group or $t \geq t_0 + 1$ using the appropriate intertemporal budget constraint (equation (15)) and the Euler equation (14). This allows to simulate aggregate savings over income as well as age-savings profile.

Aggregate savings

Figure **??** aggregate savings over labour income in the years following the policy for in our benchmark and in the data. Model estimates are linearly interpolated at the various dates (1979, 1989, ..., 2009). Our model generates roughly 50% of the total increase in aggregate savings over the last thirty years. Interestingly, calibrations for different values of intergenerational transfers ψ provide a reasonable match of aggregate savings and a very similar pattern (the increase of savings rates over the period 1981-2009 is 11.5% in our *Benchmark*, and 9% for $\psi = 3.8\%$ (*Low transfers*), compared to 21% in the data).[23]

Aggregate results are robust across calibrations because aggregate savings raise for two reasons: the 'micro-economic channel' (change in savings decisions for a given age group) and the 'macro-economic channel' (changes in income and population composition). It turns out that for a high value of ψ such as in our benchmark calibration, the 'micro-channel' becomes larger as the loss in transfers following the policy becomes more important, but the 'macro-economic channel' is dampened since age-savings profiles are flatter as shown in Figure 4.2. To the opposite, lower values of ψ imply a stronger 'macro-economic channel' and a weaker 'micro-economic channel'. As both channels are, roughly speaking, substitutes, the increase in aggregate savings is similar across the different calibrations. Finally, a flattening of the life income-profile generates stronger incentives to save for households in the thirties, which are an important part of the Chinese population in 2009, generating a further increase in aggregate savings in the calibration with *Time-varying income profiles* (see Figure **??**).

Age-saving profiles

Figure 4.3 (upper panel) represents the predicted age-savings profile $\{s_{\gamma,t}\}_{\gamma=\{3;...8}$ for $t = 1998$ (upper panel) and 2010 (lower panel) as well as their empirical counterpart for 1998 and 2009, thus for our *Benchmark* calibration (and for *Time varying income profiles*.[24] In order to build Figure 4.3, we identify properly the individuals born after the policy

[23] Household savings rates are a bit noisy at the beginning of the sample so we average the first 4 years (1981-1984) to compute the overall increase.

[24] We do not present profiles in the case of 'Low transfers' since we know they will not be comparable with the data.

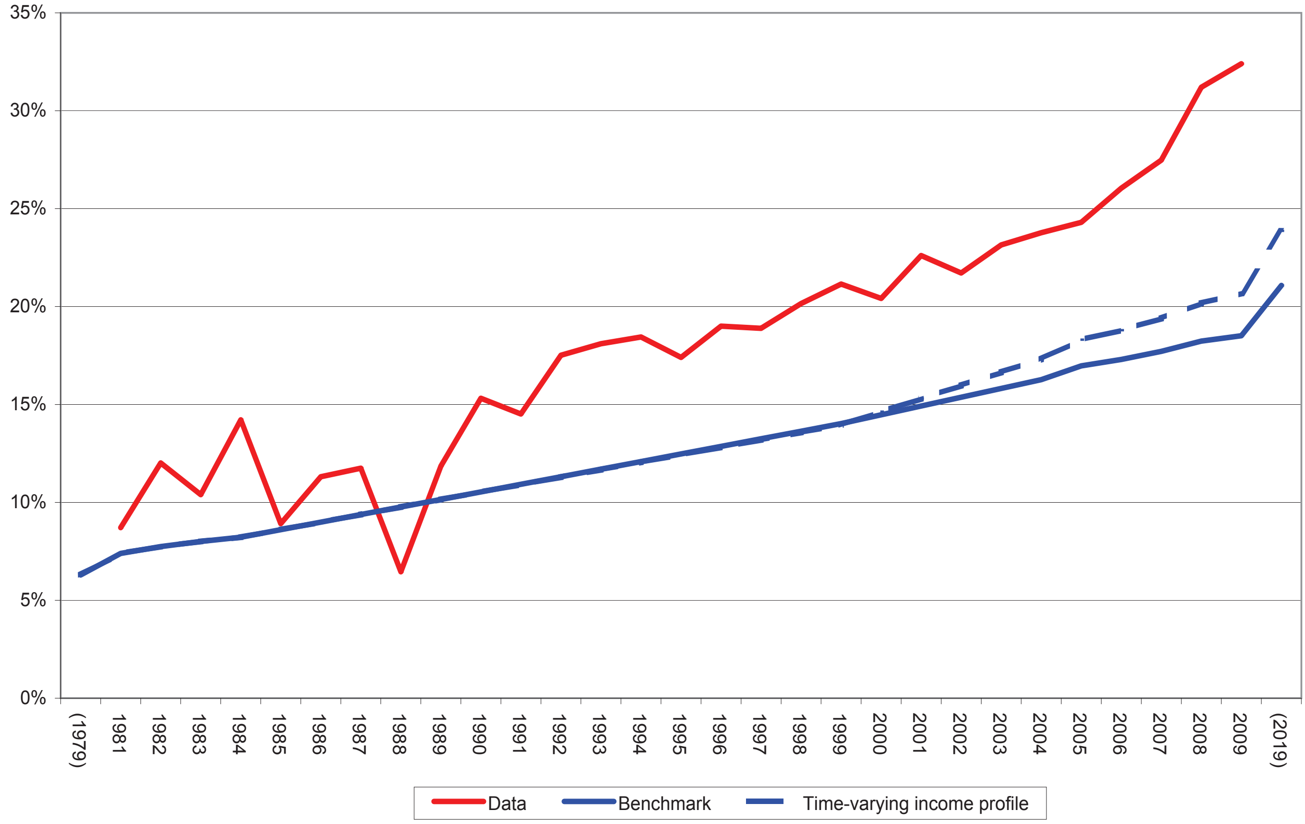
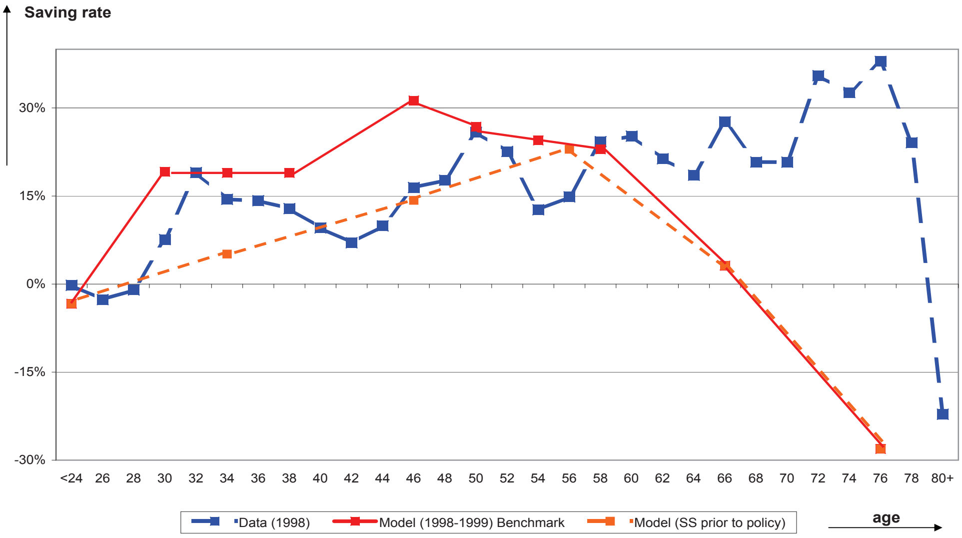


Figure 12: Aggregate household saving rate. Model versus data
Notes: Data source: To be done

implementation and the ones born when the policy was only partially implemented (between 1972 and 1980) or not affected at all (the oldest). To do so, we assume that a given cohort has a flat savings profile at a given date over the following age brackets [22-26], [30-38], [42-50] and [54-60] (for $\gamma = 3, .., 5$) but cohorts born at different dates (and thus affected differently by the policy) coexist at a given date: cohorts are born every four years from 1940 onwards and have their first child at the age of 28 (average age of first child over the last 30 years).[25]

[25]For instance, in 2009, adults aged 30 have an only child and are also born at the time were the policy was almost fully implemented (in 1979). They differ from adults aged 38 which also have an only child but



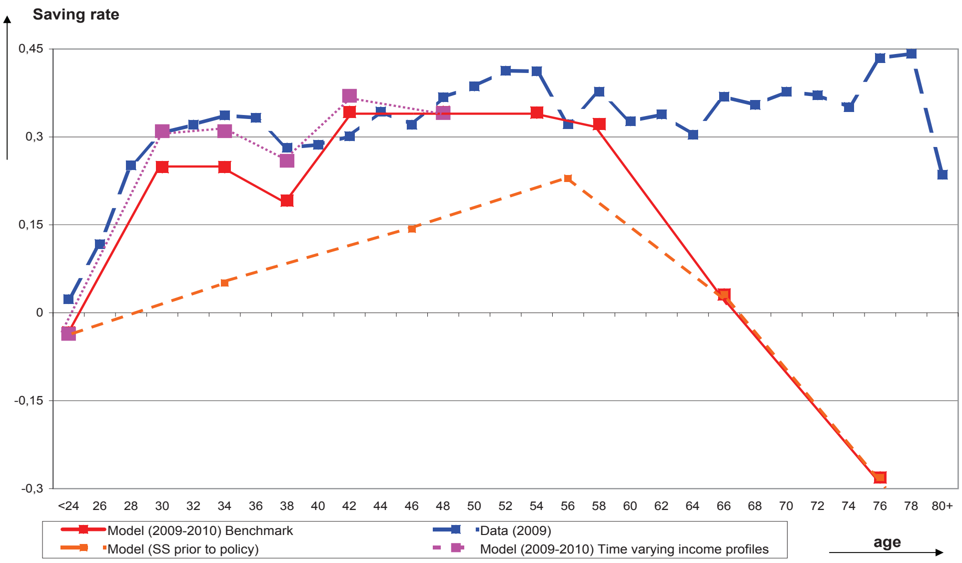


Figure 13: Age-savings profiles in 1998 and 2009. Model versus data

Notes: Data source: UHS, 1998 and 2009. Individual profiles are constructed using Chesher (1998). See Technical Appendix and Coeurdacier, Guibaud and Jin (2012).

have more siblings (since born in 1971 before the policy implementation).

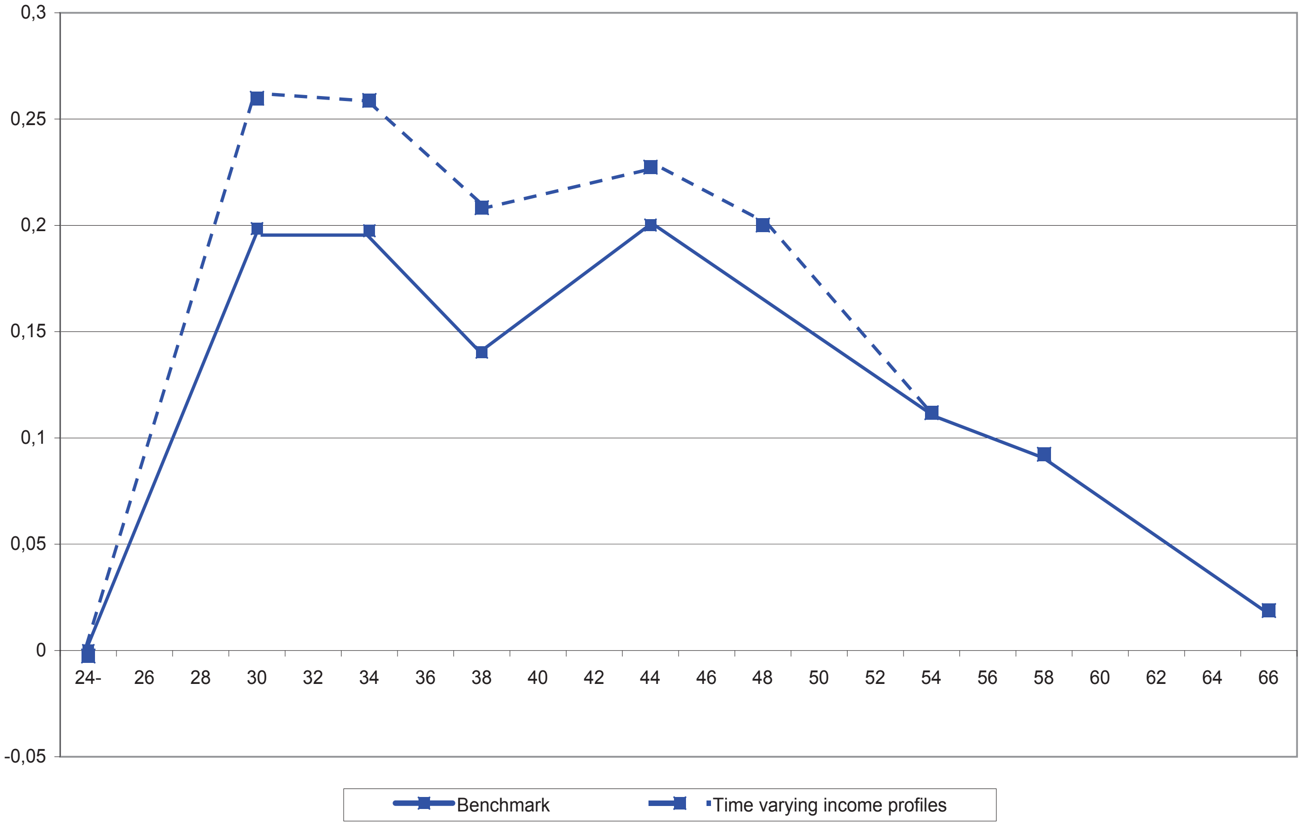


Figure 14: Change in savings rates across age between the initial steady-state and 2009. Model Predictions.
Notes: Cohorts are born every four years since 1940. Calibrations of the parameters are shown in Table 4.

When inspecting Figure 4.3, results are quite striking if we abstract from the old retirees. At both dates, an particularly so in 2009, we match fairly well the saving profiles of the middle-aged group. In particular, if we compare to our initial distribution of savings rates, two movements in the profile deserve further comments: one is simply an overall shift upwards of the curve that we capture quantitatively pretty well. This overall shift has to do with the fall in in current expenditures to raise children and the fall future transfers that provides incentives to everyone to save more. The other one is a significant flattening of the curve for the middle-age group (30 to 60). While in 1986 (and in our model), the top of the curve was clearly attained by individuals in the fifties, in the years following the policy, the curve flattens. This implies that savings rates have been increasing *faster* for individuals in their thirties than for older age groups, a feature of the data well noticed by Chamon and Prasad (2010) and Song and Yang (2010).[26] Two factors contribute to this finding: first, individuals in their thirties are the *first* affected by the policy along the transition. Individuals in their fifties had their children in the mid-seventies and are only partially affected by the policy change. Second, they are the ones that are the *most* affected by the policy: they can free up more resources for saving due to lower expenditure to raise children. Adults in their forties also see their overall expenditures falling but this is dampened as they have to spend more resources per kid to provide better education (quantity-quality trade-off). This result is confirmed and reinforced by Figure 4.3 where we investigate the variations over a longer time-span (1979-2009) of the saving rates of the different age groups as predicted by the model. One should notice, that our benchmark model fails however in explaining the sharp increase in savings rate of retirees over the period and falls slightly short in explaining the overall increase in savings of younger adults (in their thirties). Allowing for *time-varying life income profiles* solves the latter issue as shown in Figure 4.3. As the life-income profile flattened in the recent years, younger adults in their thirties had more incentives to save (see Song and Yang (2010) and Guo and Perri (2013) for a similar point).

Human capital accumulation

[to be written]

Identification through 'twins'

We simulate the income/consumption profile of an individual giving birth to twins at a date $t > t_0$. The savings rate at different age (compared to the savings rate at the same age of an individual having one child as prescribed by the policy) is shown in Table 5 for our benchmark calibration. We find that at age $\gamma = 4$ and $\gamma = 5$, the saving rate is resp. 7.2% and 9.5% lower than for households with an only child in 2009. This is so because they spend less per child as well as save less for retirement since household with two children will have higher old-age support than households with an only child. At age $\gamma = 6$, households were only subject to the partial implementation of the policy (their children are born before 1982). Thus differential savings rate when having twins depend on how the birth of twins

[26] An important difference emerges from our data compared to Chamon and Prasad (2010) and Song and Yang (2010). Individual savings rates did not increase for young adults childless (both in the model and the data). Our individual savings data differs because we correct for biases due to multigenerational households in the estimation of the profiles (see Coeurdacier, Guibaud and Jin (2012)).

affect their overall fertility (compared to households not having twins) an cannot be inferred directly form the model. One can however safely argues that for these households, the overall impact on savings of having twins should be much weaker compared to younger households. Similar arguments hold for older households (age $\gamma = 7$ and $\gamma = 8$). If having twins does not affect the optimal number of kids, one should observe no difference between their rate of savings and the the one of parents not having twins.[27] Finally, we estimate the differences in terms of human capital of twins versus only child and the differences in terms of education expenditures *per child* across the two types of households (for children aged 15-25, as a % of total expenditures). We find that our calibrated model suggest a 20% difference in human capital achievement between twins and only child, which in terms of observable accounts for a 8% difference in human capital expenditures at the age of $40 - 45$ ($\gamma = 5$) for the parents (as a % of overall expenditures).

Table 5: Twin experiment. Comparison of outcomes for families with twin births and families with an only child in 2009 (*Benchmark calibration*)

Savings rate			
	Only child	Twins	Difference
$s_{\gamma=4}$ (30–40)	24.7%	17.5%	7%
$s_{\gamma=5}$ (40–50)	34%	24.5%	9%
Human capital			
	Only child	Twins	% Difference
$(h_{2009} - h_{ss})/h_{ss}$	45%	15%	$\left(\frac{h_{only} - h_{twin}}{h_{only}}\right) = 20\%$
$(\phi_4 + \phi_h h_{2009}) / (c_{4,2009} + \phi_4 + \phi_h h_{2009})$	20.6%	12.6%	7.3%

5 Empirical Evidence

In what follows, we test three main implications of our theoretical model: higher fertility leads to (1) lower household savings rate; (2) lower education investment per child; (3) transfers from children to parents rise in the quantity and quality of the children. To obtain an exogenous variation in fertility, we adopt the incidence of twins as an instrument for variations in household size. Twinning becomes an ideal natural experiment in the case of China after the implementation of the one child policy, as all families are allowed only one

[27]Another possibility could be that having twins is less costly than having two separate births due to economies of scale. In this case, one could expect parents of twins not affected by the policy to save more.

birth–and thus differences in the number of children do reflect differences in preferences. Where exogenous restrictions on fertility do not apply to most circumstances, Rosenzweig and Wolpin (1980) show that the incidence of twinning at first birth can also serve as an appropriate instrument. Indeed, to the extent that the probability of twinning increases with the number of births, comparing outcomes of households with birth of twins with those who had singletons in all pregnancies may also capture differences in outcomes that are attributable to variations in preferences for the number of children. It is in this instance that Rosenzweig and Wolpin (1980) and Oliveira (2012) use twin births that occurred in the first pregnancy as a source of exogenous variation in the supply of children, on the presumption that these women who gave birth first to twins are likely to have preferred the same number of children to those who had singletons during first birth.

Moreover, insofar as we are interested only in extrapolating the saving behavior of an individual from household-level saving, which potentially combines differential saving behavior of members at various stages of the lifecycle—we do not want to include households with multiple generations of adults. For this reason, only 'nuclear' households are considered—that is, households in which their members consist of only parents, their only child, or twins—here taken to be the two children of the household head with identical age.[28]

5.1 Household Savings Rate

Table 6 provides descriptive statistics of the average household savings rate for households with one child and those with twins. Indeed, the saving rate is on average lower in households with twins for every income bracket. To examine whether fertility has a causal effect on household savings rate, we perform an OLS regression analysis using the incidence of twinning as an instrument for variations in the number of children across households.

The results displayed in Table 7 shows that the birth of twins has a highly significant and adverse effect on household saving rate, reducing it by an average of 6 to 7 percentage points. In Column (1), household income is excluded lest it should be an outcome variable—household members may decide to work more to meet higher expenditures of a greater number of children, or, conversely, lower the labor supply of mothers. Columns (2)-(4), however, show that the results are robust to controlling for household income. The effect becomes even stronger when excluding the top 10% of the poorest and richest households, further increasing the fall in savings rate by an additional percentage point. It is remarkable that

[28]The data provides information only for children co-residing with the household head. One potential problem is that twins could be separated in a way that only one of the two stays with the parents. In order to limit this possible source of bias, we restrict our sample to children aged 18 years old and below. These would be the children born after the implementation of the one child policy.

the order of magnitude of the estimates from the data are of very similar order of magnitude than the ones from our calibrated model (and not statistically different at standard confidence intervals; see Table 5).

Although the indirect effect of twinning on household savings rate is unobservable, the direct effect on increasing household expenditures can be estimated. Table 8 shows that twinning mostly raises education and food expenditures (Columns 2 and 3), while having no significant impact on other types of expenditures or on household income (Columns (4) and (5)).

Table 6: Descriptive statistics: Saving Rates in Households with Only Child vs Twins

	Singleton household	Twins households
Nbr of obs.	51,572	903
Average hh saving rate	22.0%	16.8%
−lowest 20% income	6.7%	0.2%
−second lowest	18.4%	18.0%
−middle income group	24.1%	21.1%
−second highest	29.4%	22.3%
−top 20% income	35.8%	31.5%

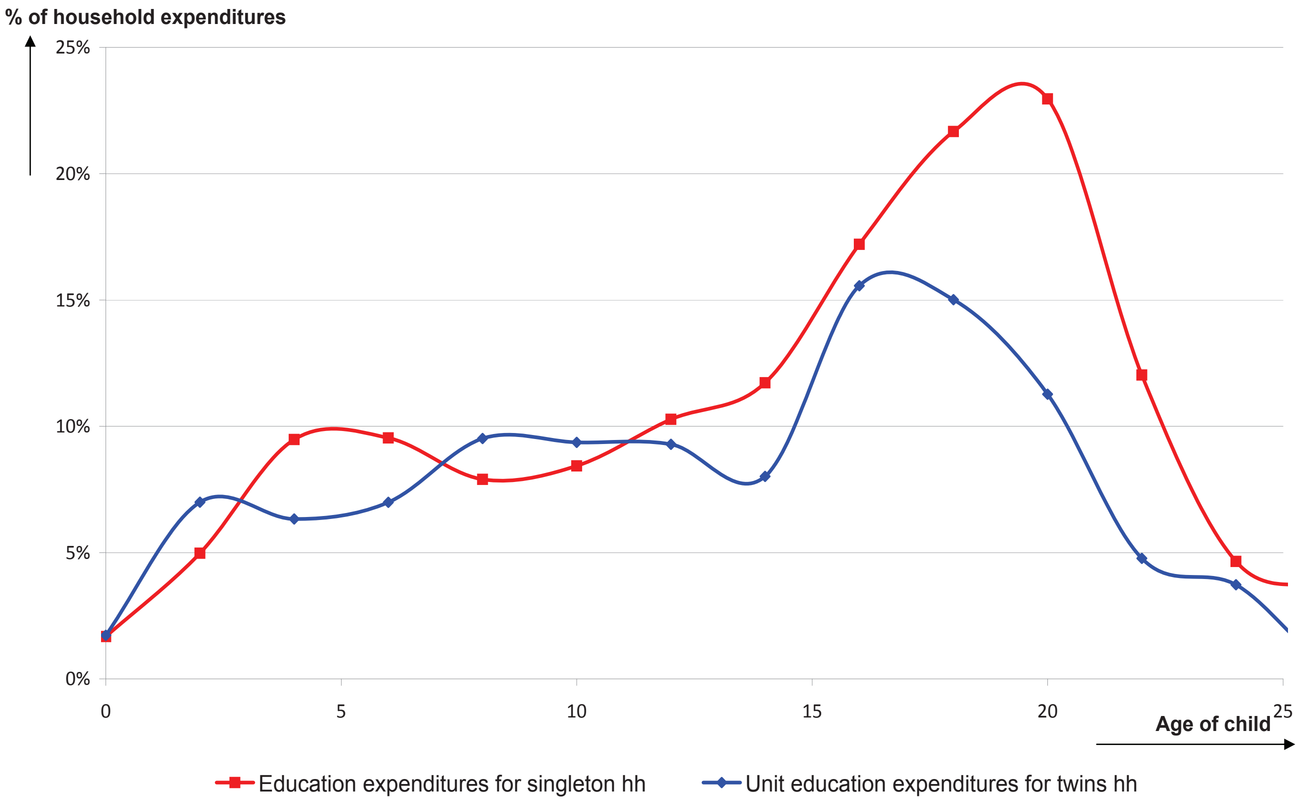
Note: UHS, 2002-2009, restricted sample of nuclear households—those in which the members consist of the parents and the only child or twins.

5.2 The Education Channel

Our model predicts that restricting fertility leads to substitution towards higher human capital investment per child. Indeed, the data reveals a significant increase in overall education attainment in recent years –with the gross enrollment rate in senior secondary education rising from 43% to 79% between 2000 and 2009, and that of higher education rising from 11.5% to 27% between 2000 and 2011.

The flip side of this argument is that that education investments per child decreases with additional children. Figure 15 reveals an interesting pattern: while the average education expenditure (% of total household expenditure) per child is similar for only child and for twins during compulsory education (of ages up to 15 years old), it is significantly higher for only child during non-compulsory education (of ages 16-22 years old)—as much as the average education expenditure put together for twins—at age 20. The difference of 7 to 10 % in terms of % of consumption expenditure across the two groups is also of an order of magnitude very close to the model predictions. No overt selection bias seems to be at play as the distribution for the proportions of observations of households with twins relative to only child by age is conserved across ages (Figure 16).[29]

Figure 15: Education Expenditures: Only Child vs. Twins

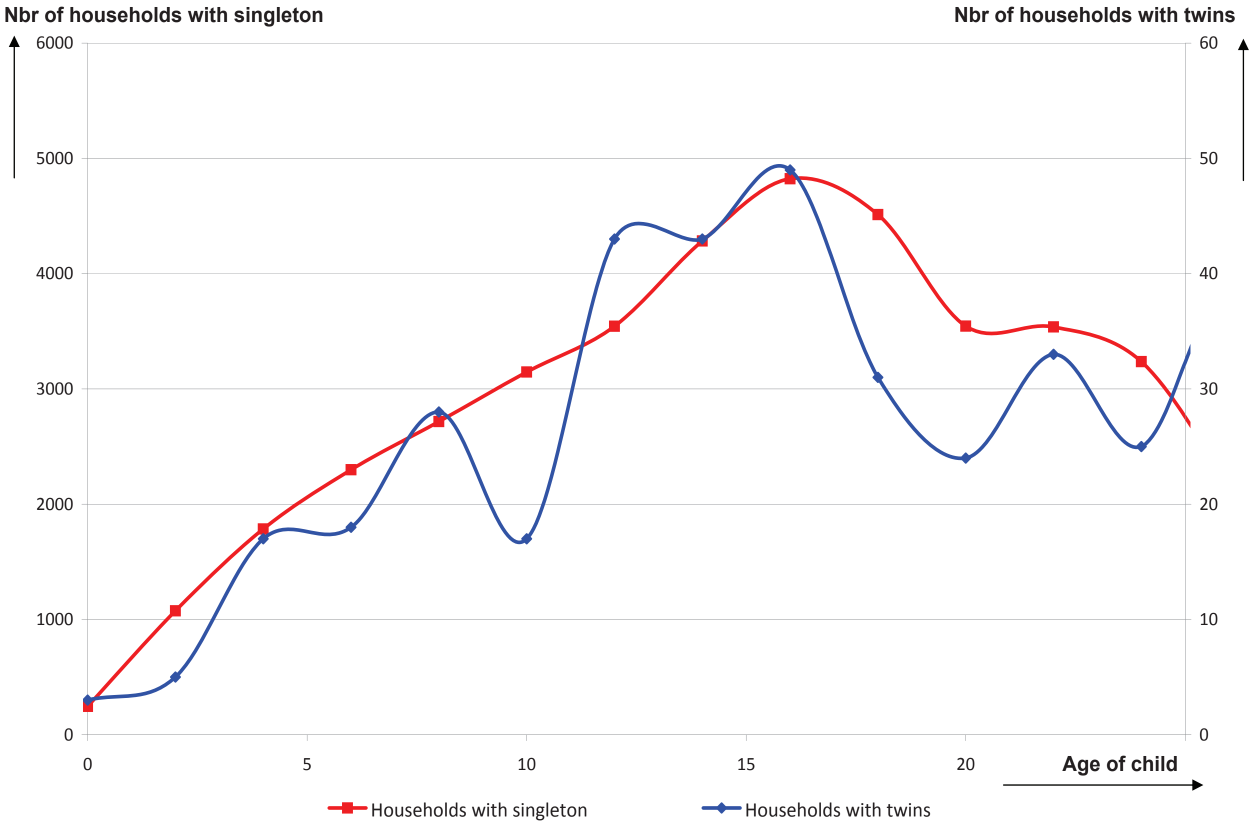


UHS (2002-2006), restricted sample of nuclear households. This figure displays the average education expenditure per child (as a share of of total household expenditure) by age of the child, over the period 2002-2006.

These descriptive results are affirmed by regression analysis that yields estimates of the

[29]We do not observe children when they leave the household in the UHS data and one could have been worried that twins and only child would leave the household at earlier age.

Figure 16: Number of observations for twins/singleton nuclear households (2002-2006 average).



Notes: UHS (2002-2006).

effect of twinning on education investment for each child. As Column (1) of Table 9 indicates, there is no significant difference between education expenditure per child during compulsory education years for households with twins compared to those with only child, but a vast and significant difference during the years of higher education (Column 2): parents with twins of ages 16-22 invest on average 6 percentage points (of total household expenditure) less than parents with only children on each child.

This quantity-quality tradeoff argument is also reflected by measures of education attainment: twins are about on average twice as less likely to pursue either higher or secondary education than their only-child peers. Table 9 shows that the odds ratio is 0.461 for secondary education and 0.557 for higher education— with the incidence of twins.[30] It is possible that twins are of potential lower quality compared to singletons–for example, by having lower weights at birth (net of family-size effects)—and parents may be in turn invest less on their education and substitue investment in favor of their singleton offspring— a concern raised by (Rosenzweig and Zhang (2009)). The problem is less serious, however, when households can only have one births in the case of recent periods in China. Oliveira (2012) also finds no systematic differences between singletons and twins.

5.3 Transfers

The third implication is that transfers from children to parents are increasing in the quantity and quality of children. Data on transfers come from CHARLS, for the year 2008. We estimate the effect of both the number of children and their education level on net transfers received by the parents, in a sample that includes only urban households (Table 11) and in a larger sample that combines both rurual and urban households (Table 12). The sample is restricted to parents above the age of 60—for whom the one child policy was more or less not yet binding.

As before, the definition for transfers is: the sum of regular and non regular financial transfers and the yuan value of in-kind transfers. Net transfers is taken to be those from children to parents less transfers received by children. Gross transfers are transfers from children to parents.

Table 11 indicates that an additional child has a positive and significant effect on net tranfers to parents— on average conferring an additional ... yuan (dollars)–equivalent to ... of household income (Column 2). It is clear that higher education of children leads to a significant increase in net transfers received by the parents (Columns 2 and 4), in both samples. For those households with children under the age of 25, net transfers are on average falling in children's education level, most likely reflecting the transfers used to finance education expenditures. For households with adult children (of ages above 25), net transfers are rising in children's level of education (Column (4)). Overall, there is a significant and positive effect of both the quantity and quality of children on net transfers received by parents.[31]

[30] Results are similar when controlling for household income and therefore hereby omitted for convenience.

[31] Similar results hold when replacing net transfers by gross transfers, results of which are hereby omitted for convenience.

5.4 Back-of-Envelope Calculations on Household Savings Rate

[very preliminary]

One can back out form empirical estimates some counterfactual savings rate for a different policy. First, one ask how moving from restricting households to having one child to households being able to have two children affect household savings (assuming that having twins in this regard is not too different from having two children). For households with children less than 18 years of age, the effect of having an additional child is to reduce the savings rate of these households by 6.16 percentage points (estimates taken from Table 7).We label this as the 'decision effect' (micro-economic channel). This brings down the aggregate household savings rate from 32.41 percent to 29.83 percent (a reduction of 2.58 percentage points). The second impact comes from changes in the demographic composition, labeled as the 'composition effect' (macro-economic channel), which is estimated by doubling the observations of individuals between the ages of 18-29—those who are now adults but who were subject to the one child policy (born after 1980). We assume that consumption is equally shared between these individuals and their hypothetical siblings. This composition effect further reduces the aggregate savings rate by 3.50 percentage points. The combined effect is therefore a fall of 6.09 percent, which amounts to roughly 30 percent of the total 21 percentage points rise in household savings rate between 1982-2009. Note that such estimates are purely driven by empirical estimates. Note surprisingly, they are not very far away from the model predictions since our simulations generate similar micro-economic channel and our age-savings profiles for young and middle-aged households are fairly close to the data. Simulating a two child policy generates an household savings rate 8% lower in our model. This such a number can also be interpreted as a lower bound of the aggregate saving effect of the one child policy. Our baseline model estimates of section 4 were imputing roughly half of the increase in aggregate savings to the one child policy. This is based on a counterfactual where families have on average 3 children. With a more conservative counterfactual of 2 children per households, one gets that the one child policy contributed to a third of the increase in household savings.[32]

6 Conclusion

We show in this paper that exogenous fertility restrictions in China may have lead to a rise in household savings rate—by altering savings decisions at the household level, and

[32]Discuss if the estimated effect is a lower/upper bound (economies of scale associated with raising twins/average natural fertility rate is above 2 per woman....

demographic compositions at the aggregate level. We explore the quantitative implications of these channels in a simple model linking fertility and saving through intergenerational transfers that depend on the quantity and quality of offspring. Predictions on the age saving profile become richer and more subtle than that of the standard lifecycle hypothesis, where both human capital investment and intergenerational transfers are absent. These patterns are shown to be born out by profiles estimated from the data.

From our empirical estimates of the impact of an additional child on household saving, we find that the 'one-child policy' can account for at least 30 percent of the rise in the aggregate household saving rate since the enforcement of the policy in 1980. Moreoever, the necessary components that constitute the micro-level channel through which fertility impinges on household saving are broadly consistent with the data.

We believe that shifts in demographics as understood through the lens of a life-cycle model remains to be a powerful factor in accounting for the high and rising national saving rate in China—particularly when augmented with important features capturing the realities of its households, and particularly when buttressed by compatible micro-level evidence which we show. The tacit implication—on a broader scale—is that the one-child policy provides a natural experiment for understanding the link between fertility and savings behavior in any economy— in which intergenerational transfers serve as a reliable and consistent means to old-age living—be it a matter of under-developed pension systems, or be it merely a matter of culture and tradition.

References

[1] Attan, I. (2011), Au pays des enfant rares: la Chine vers une crise dmographique, Fayard, 275p.

[2] Banerjee, Meng, Qian (2010), "The life cycle model and household savings: micro evidence from urban China", Mimeo Yale.

[3] Barro, and Becker (1989), "Fertility Choice in a Model of Economic Growth", Econometrica, vol. 57(2), pp. 481-501.

[4] Becker, G. and Lewis, H.G., 1973. On the Interaction between the Quantity and Quality of Children. The Journal of Political Economy (81), 279-288.

[5] Bernanke, B (2005), "The Global Saving Glut and the U.S. Current Account Deficit" ,Homer Jones Lecture, St. Louis, Missouri, April 14, 2005.

[6] Boldrin, M. and Jones, L.E, 2002. Mortality, Fertility, and Saving in a Malthusian Economy. Review of Economic Dynamics, Evol. 5(4), 775-814

[7] Bourguignon, Browing, Chiappori & Lechene (1993), "Intra-household allocation of consumption", Revue d'Economie et de statistique, N23.

[8] Bussiere, M., Kalantzis, Y., Lafarguette, R. and T. Sicular, 2013. Understanding Household Savings in China: the Role of the Housing Market and Borrowing Constraints. mimeo Banque de France.

[9] Carroll, Weil (1994), "Saving and growth: A Reinterpretation," Carnegie-Rochester Conference Series on Public Policy .

[10] Chamon, Prasad (2010), "Why Are Saving Rates of Urban Households in China Rising?", American Economic Journal: Macroeconomics, pp. 93-130, January.

[11] Chan, Kwok Ho (2011), "Private Old-Age Support in China: New Evidence from the Rich and the Poor", mimeo.

[12] Chesher, Andrew (1997), "Diet Revealed? Semiparametric Estimation of Nutrient Intake-Age Relationships", Journal of the Royal Statistical Society, 389-420.

[13] Chesher, Andrew (1998), "Individual Demands from Household Aggregates: Time and Age Variation in the Composition of Diet", Journal of Applied Econometrics, 505-524.

[14] Clark, T. (2008), "Plight of the Little Emperors", Psychology Today, July 1st. Coeurdacier, Guibaud, Jin (2012), "Credit Constraints and Growth in a Global Economy", mimeo.

[15] Curtis, Lugauer & Mark (2011), "Demographic Patterns and Household Saving in China", mimeo March 2011.

[16] Deaton, Angus and Christina H. Paxson (2000) "Growth and Saving Among Individuals and Households," Review of Economics and Statistics, 82(2), 212-25.

[17] Duflo, Esther (2003), Grandmothers and Granddaughters: Old Age Pensions and Intrahousehold Allocation in South Africa", The World Bank Economic Review.

[18] Edlund, Li, Yi, Zhang (2007), "Sex Ratios and Crime: Evidence from China's One-Child Policy", IZA working paper.

[19] Fong, V. (2002), "China's One-Child Policy and the Empowerment of Urban Daughters", American Anthropologist 104, no. 4 2011, pp.1098-1109.

[20] Ge, S., Yang, D., and J. Zhang. (2012), " Population Policies, Demographic Structural Changes, and the Chinese Household Saving Puzzle ", IZA Discussion paper 7026.

[21] Gibson, Huang, and Rozelle (2003), "Improving Estimates of Inequality and Poverty from Urban China's Household Income and Expenditure Survey", Review of Income and Wealth.

[22] Greenspan, A (2009), "The Fed Didn't Cause the Housing Bubble", Wall Street Journal, March 11, 2009.

[23] Gruber, N. (2012), "Keeping with the Zhangs: household saving behavior as a function of relative wealth", mimeo 2012.

[24] Guonan, Yi (2010), "China's High Saving Rate: Myth and Reality", BIS Working Paper, Number 312.

[25] Guo, S. and Perri, F., 2013. The allocation puzzle is not as bad as you think, mimeo University of Minnesota, 2013.

[26] Horioka, Wan (2007), "The Determinants of Household Saving in China: A Dynamic Panel Analysis of Provincial Data," Journal of Money, Credit and Banking 39(8) pp.2077-2096.

[27] Lee, Jinhyun (2009), "Estimating Individual Non-linear Health Expenditure and Income Relationships using Household Data : the Indonesian SUSENAS Case", mimeo UCL.

[28] Li, Zhang and Zhu (2008), "The Quantity-Quality Trade-Off of Children in a Developing Country: Identification Using Chinese Twins", Demography, February; 45(1): 223–243.

[29] Mankiw, N., D. Romer and D.N. Weil, 1992. A Contribution to the Empirics of Economic Growth. The Quarterly Journal of Economics , 107 (2), pp. 407-437

[30] Meng, Xin (2006), "Demographic change, pension reform, and household saving in urban China", slides for presentation at the International Symposium on Contemporary Labor Economics, December 18th, 2006.

[31] Modigliani, F., 1976. Life-cycle, individual thrift, and the wealth of nations. American Economic Review, 76(3), 297–313.

[32] Modigliani, Cao, (2004), "The Chinese Saving Puzzle and the Life-Cycle Hypothesis", Journal of Economic Literature, 42, pp. 145-170.

[33] Naughton, B. (2007), The Chinese Economy: Transitions and Growth, MIT Press.

[34] Oliveira, J. (2012), The Value of Children: Intergenerational Transfers, Fertility, and Human Capital, mimeo, Yale University.

[35] Rosenzweig, and Wolpin (1980), Testing the Quantity-Quality Fertility Model: The Use of Twins as a Natural Experiment, Econometrica, vol. 48(1), pages 227-40, January.

[36] Salditt, Whiteford, Adema (2007),"Pension Reform in China: Progress and Prospects", OECD Social, Employment and Migration Working Papers No. 53

[37] Song, and Yang (2010), "Life cycle earnings and the household saving puzzle in a fast growing economy", mimeo May 2010.

[38] Xie, Yu and Zhu (2009), Do Sons or Daughters Give More Money to Parents in Urban China?, Journal of Marriage and Family , pp.174-186.

[39] Yang, Zhang and Zhou (2011), "Why are savings rates so high in China ?", IZA discussion paper No. 5465

[40] Yao, Wang, Weagley and Liao (2011), "Household Saving Motives: Comparing American and Chinese Consumers", Family and Consumer Sciences Research Journal Volume 40, Issue 1, pages 28–44, September.

7 Appendix

7.1 Data

RUMICI 2008 (where we can see all children alive)

CHARLS
UHS
...

7.2 Theory

[to be cleaned]

Proof of Proposition 1
[Convergence and stability to be done. Proof to be put in the appendix]
Uniqueness.

If $\{n_{ss}; h_{ss}\}$ exists, then it must satisfy by the definition of SS the set of equations:

$$\begin{aligned} \frac{n_{ss}}{1-\theta-\psi\frac{n_{ss}^{\varpi-1}}{\varpi}} &= \left(\frac{v}{\beta(1+\beta)+v}\right)\left(\frac{1}{\phi_0+\phi_h\left(1-\frac{\omega}{\alpha}\right)h_{ss}}\right) \\ h_{ss} &= \left(\frac{\alpha\psi\left(1+g_z\right)}{\phi_h R}\right)\frac{n_{ss}^{\varpi-1}}{\varpi} \end{aligned}$$

Plugging the second equation into the first one gives:

$$\frac{n_{ss}}{1-\theta-\psi\frac{n_{ss}^{\varpi-1}}{\varpi}} = \left(\frac{v}{\beta(1+\beta)+v}\right)\left(\frac{1}{\phi_0+\left(\frac{\alpha}{\omega}-1\right)\left(\frac{\psi(1+g_z)}{R}\right)n_{ss}^{\varpi-1}}\right)$$

Introduce $N_{ss} = n_{ss}^{\varpi-1}$. This can be rewritten as:

$$N_{ss}^{-1/(1-\omega)} - \left(\frac{v}{\beta(1+\beta)+v}\right)\left(\frac{1-\theta-\frac{\psi}{\omega}N_{ss}}{\phi_0+(\alpha-\omega)\frac{(1+g_z)}{R}\frac{\psi}{\omega}N_{ss}}\right) = 0$$

Consider $G(x) = x^{-1/(1-\omega)} - \left(\frac{v}{\beta(1+\beta)+v}\right)\left(\frac{1-\theta-\frac{\psi}{\omega}x}{\phi_0+(\alpha-\omega)\frac{(1+g_z)}{R}\frac{\psi}{\omega}x}\right)$ for $x > 0$.
$\lim_{x\to+\infty} G(x) = \left(\frac{v}{\beta(1+\beta)+v}\right)\frac{\psi/\omega}{\left(\frac{\alpha}{\omega}-1\right)\left(\frac{\psi(1+g_z)}{R}\right)} < 0$ if $\varpi > \alpha$
$\lim_{x\to 0^+} G(x) = +\infty$

$$G'(x) = -\frac{x^{-\omega/(1-\omega)}}{1-\omega} + \frac{v\psi/\omega}{\beta(1+\beta)+v}\frac{\phi_0-(\omega-\alpha)\frac{(1+g_z)}{R}}{\left(\phi_0+\left(\frac{\alpha}{\omega}-1\right)\frac{(1+g_z)}{R}\psi x\right)^2}$$

Two cases: if $\phi_0 - \left(1-\frac{\alpha}{\omega}\right)\frac{(1+g_z)}{R} \leq 0$ then $G(x)$ is monotonic (decreasing) on $]0;+\infty[$.

In the other case, $G(x)$ is decreasing first to a minimum value $x_{\min} > 0$ such that $\frac{x_{\min}^{-\omega/(1-\omega)}}{1-\omega} = \frac{(\omega-\alpha)\frac{(1+g_z)}{R}-\phi_0}{\left(\phi_0+\left(\frac{\alpha}{\omega}-1\right)\frac{(1+g_z)}{R}\psi x_{\min}\right)^2}$ and $G(x_{\min}) < 0$ and then increasing.for $x > x_{\min}$.

In both case, from the intermediate value theorem, there is a unique $N_{ss} > 0$ such that $G(N_{ss}) = 0$. this pins down the uniqueness of $\{n_{ss}; h_{ss}\}$, which are both > 0.

Moreover, if we define implicitly the unique n_0 as follows:

$$\frac{n_0}{1-\theta-\psi\frac{n_0^{\varpi-1}}{\varpi}} = \left(\frac{v}{\beta(1+\beta)+v}\right)\left(\frac{1}{\phi_0}\right)$$

We get immediately since $\varpi \geq \alpha$ that $n \geq n_0$.

Proof of Proposition 2

Define aggregate labour income in the economy as:

$$Y_{t+1} = (1+n_t e)\, N^m_{i,t+1} w_{t+1,m}$$

Population evolution
$N^m_{t+1} = N_{y,t} = n_{t-1} N^o_{t+1}$
$N^y_{t+1} = n_t N_{y,t} = n_t N^m_{t+1}$
Savings of the young: $S_{y,t+1} = N^y_{t+1} s_{y,t+1} = -\theta n_t N^m_{t+1} \frac{w_{m,t+2}}{R}$
Savings of the middle aged:
$S_{m,t+1} = N^m_{t+1}\left(s_{m,t+1} - s_{y,t}\right) = N^m_{t+1}\left(\frac{\beta}{1+\beta}\left(1-\theta-n_t\phi(h_{t+1}) - \psi\frac{n_{t-1}^{\varpi-1}}{\varpi}\right) w_{t+1,m} - \frac{1}{R(1+\beta)}\psi\frac{n_t^{\varpi}}{\varpi} w_{t+2,m} + \theta\frac{w_m}{}\right.$
Dissavings of the old:
$S_{o,t+1} = -N^o_{t+1} a_{m,t-1} = -\frac{N^m_{t+1}}{n_{t-1}}\left(\frac{\beta}{1+\beta}\left(1-\theta-n_{t-1}\phi(h_t) - \psi\frac{n_{t-2}^{\varpi-1}}{\varpi}\right) w_{t,m} - \frac{1}{R(1+\beta)}\psi\frac{n_{t-1}^{\varpi}}{\varpi} w_{t+1,m}\right)$
Aggregate savings at date $t+1$
$S_{t+1} = N^y_{i,t+1} s_{y,t+1} + N^m_{i,t+1}\left(s_{m,t+1} - s_{y,t}\right) - N^o_{i,t+1} s_{m,t-1}$
$S_{t+1} = N^m_{i,t+1}\left[n_t s_{y,t+1} + \left(s_{m,t} - s_{y,t}\right) - \frac{s_{m,t-1}}{n_{t-1}}\right]$

$$\frac{S_{t+1}}{N^m_{i,t+1} w_{t+1,m}} = \left[\begin{array}{c} \frac{\theta}{R}\left(1-n\left(1+g_z\right)\right) + \frac{\beta}{1+\beta}\left(1-\theta-n\phi(h)-\psi\frac{n^{\varpi-1}}{\varpi}\right)\left(1-\frac{1}{n(1+g_z)}\right) \\ -\frac{1}{R(1+\beta)}\psi\frac{n^{\varpi}}{\varpi}\left[(1+g_z)-\frac{1}{n}\right] \end{array}\right]$$

So steady-state aggregate savings over labour income at date $t+1$:

$$\begin{aligned}
\left(\frac{S}{Y}\right)_{t+1} &= \frac{1}{1+ne}\left[\begin{array}{c} \frac{\theta}{R}\left(1-n\left(1+g_z\right)\right) + \frac{\beta}{1+\beta}\left(1-\theta-n\phi_m(h)-\psi\frac{n^{\varpi-1}}{\varpi}\right)\left(1-\frac{1}{n(1+g_z)}\right) \\ +\frac{1}{R(1+\beta)}\psi\frac{n^{\varpi-1}}{\varpi}\left(1-n\left(1+g_z\right)\right) \end{array}\right] \\
&= \frac{1}{1+ne}\left(\begin{array}{c} \left(\frac{\theta}{R}+\frac{1}{R(1+\beta)}\psi\frac{n^{\varpi-1}}{\varpi}\right)\left(1-n\left(1+g_z\right)\right) \\ +\frac{\beta}{1+\beta}\left(1-\theta-n\phi(h)-\psi\frac{n^{\varpi-1}}{\varpi}\right)\left(1-\frac{1}{n(1+g_z)}\right) \end{array}\right) \\
&= \frac{1}{1+ne}\left(\begin{array}{c} \left(\frac{\theta}{R}+\frac{1}{R(1+\beta)}\psi\frac{n^{\varpi-1}}{\varpi}\right)\left(1-n\left(1+g_z\right)\right) \\ +\frac{\beta}{1+\beta}\left(1-\theta-n\phi_0-\phi_h nh-\psi\frac{n^{\varpi-1}}{\varpi}\right)\left(1-\frac{1}{n(1+g_z)}\right) \end{array}\right) \\
&= \frac{1}{1+ne}\left(\begin{array}{c} \left(\frac{\theta}{R}+\frac{1}{R(1+\beta)}\psi\frac{n^{\varpi-1}}{\varpi}\right)\left(1-n\left(1+g_z\right)\right) + \\ \frac{\beta}{1+\beta}\left(1-\theta-n\phi_0-\phi_h nh-\psi\frac{n^{\varpi-1}}{\varpi}\right)\left(1-\frac{1}{n(1+g_z)}\right) \end{array}\right)
\end{aligned}$$

Steady state decompostition of savings:

$$\begin{aligned}
\left(\frac{S_y}{Y}\right)_{t+1} &= -\frac{n_{ss}}{1+n_{ss}e}\left(1+g_z\right)\frac{\theta}{R} \\
\left(\frac{S_m}{Y}\right)_{t+1} &= \frac{1}{(1+n_{ss}e)}\left(\frac{\beta}{1+\beta}\left(1-\theta-n\phi(h_{ss})-\psi\frac{n_{ss}^{\varpi-1}}{\varpi}\right) - \frac{1+g_z}{R(1+\beta)}\psi\frac{n_{ss}^{\varpi}}{\varpi}+\frac{\theta}{R}\right)
\end{aligned}$$

$\psi\frac{\alpha n_{ss}^{\varpi}}{R\varpi}(1+g_z)=\phi_h n_{ss}h_{ss}$

$$\begin{aligned}
\left(\frac{S_m}{Y}\right)_{t+1} &= \frac{1}{(1+n_{ss}e)}\left(\frac{\beta}{1+\beta}\left(1-\theta-n\phi_0-\psi\frac{\alpha n_{ss}^{\varpi}}{R\varpi}(1+g_z)-\psi\frac{n_{ss}^{\varpi-1}}{\varpi}\right)-\frac{1+g_z}{R(1+\beta)}\psi\frac{n_{ss}^{\varpi}}{\varpi}+\frac{\theta}{R}\right)\\
&= \frac{1}{(1+n_{ss}e)}\left(\frac{\beta}{1+\beta}\left(1-\theta-n\phi_0-\psi\frac{n_{ss}^{\varpi-1}}{\varpi}\right)-\frac{(1+g_z)}{R(1+\beta)}\frac{\psi}{\varpi}(1+\beta\alpha)n_{ss}^{\varpi}+\frac{\theta}{R}\right)\\
&= \frac{1}{(1+n_{ss}e)}\left(\frac{\beta}{1+\beta}(1-\theta)+\frac{\theta}{R}-\frac{\beta}{1+\beta}n_{ss}\phi_0-\frac{(1+g_z)}{R(1+\beta)}\frac{\psi}{\varpi}(1+\beta\alpha)n_{ss}^{\varpi}-\frac{\beta}{1+\beta}\frac{\psi}{\varpi}n_{ss}^{\varpi-1}\right)\\
&= \frac{1}{(1+n_{ss}e)}\left(\frac{\beta}{1+\beta}(1-\theta)+\frac{\theta}{R}-\frac{\beta}{1+\beta}n_{ss}\phi_0-\frac{\beta}{1+\beta}\frac{\psi}{\varpi}n_{ss}^{\varpi-1}\left(1+\frac{n_{ss}(1+g_z)(1+\beta\alpha)}{R\beta}\right)\right)
\end{aligned}$$

$$\begin{aligned}
\left(\frac{S_o}{Y}\right)_{t+1} &= -\frac{1}{n_{ss}(1+n_{ss}e)}\left(\frac{\beta}{1+\beta}\left(1-\theta-n_{ss}\phi_0-\psi\frac{\alpha n_{ss}^{\varpi}}{R\varpi}(1+g_z)-\psi\frac{n_{ss}^{\varpi-1}}{\varpi}\right)\frac{1}{1+g_z}-\frac{1}{R(1+\beta)}\psi\frac{n_{ss}^{\varpi}}{\varpi}\right)\\
&= -\frac{1}{n_{ss}(1+n_{ss}e)}\left(\frac{\beta}{1+\beta}\frac{1}{1+g_z}(1-\theta-n_{ss}\phi_0)-\frac{1}{1+g_z}\frac{\beta}{1+\beta}\psi\frac{n_{ss}^{\varpi-1}}{\varpi}-\frac{1}{R(1+\beta)}\frac{\psi}{\varpi}n_{ss}^{\varpi}(1+\beta\alpha)\right.\\
&= -\frac{1}{n_{ss}(1+n_{ss}e)}\frac{\beta}{1+\beta}\left(\frac{1}{1+g_z}(1-\theta)-\frac{1}{1+g_z}n_{ss}\phi_0-\frac{1}{1+g_z}\frac{\psi}{\varpi}n_{ss}^{\varpi-1}\left(1+\frac{n_{ss}(1+g_z)(1+\beta\alpha)}{R\beta}\right.\right.
\end{aligned}$$

$$\left(\frac{S}{Y}\right)_{t+1}=\frac{1}{(1+n_{ss}e)}\left(\begin{array}{c}\frac{\theta}{R}\left(1-n_{ss}(1+g_z)\right)+\frac{\beta}{1+\beta}(1-\theta)\left(1-\frac{1}{n_{ss}(1+g_z)}\right)\\ -\frac{\beta}{1+\beta}\left(n_{ss}\phi_0+\frac{\psi}{\varpi}n_{ss}^{\varpi-1}\left(1+\frac{n_{ss}(1+g_z)(1+\beta\alpha)}{R\beta}\right)\right)\left(1-\frac{1}{n_{ss}(1+g_z)}\right)\end{array}\right)$$

Proof of Lemma 2

$$\begin{aligned}
\left(\frac{S}{Y}\right)_{t_0} &= \frac{1}{(1+n_{t_0-1}e)}\left(\frac{\theta}{R}\left(1-n_{t_0-1}(1+g_z)\right)+\frac{\beta}{1+\beta}(1-\theta)\left(1-\frac{1}{n_{t_0-1}(1+g_z)}\right)\right)\\
&\quad -\frac{\beta}{1+\beta}\frac{1}{(1+n_{t_0-1}e)}\left(n_{t_0-1}\phi_0+\frac{\psi}{\varpi}n_{t_0-1}^{\varpi-1}\left(1+\frac{n_{t_0-1}(1+g_z)(1+\beta\alpha)}{R\beta}\right)\right)\left(1-\frac{1}{n_{t_0-1}(1+g_z)}\right)
\end{aligned}$$

$$\begin{aligned}
\left(\frac{S}{Y}\right)_{t_0+1} &= \frac{1}{(1+n_{\max}e)}\left[\begin{array}{r}-\frac{\theta}{R}n_{\max}\left(\frac{w_{m,t+2}}{w_{t+1,m}}\right)+\left(\frac{\beta}{1+\beta}\left(1-\theta-n_{\max}\phi(h_{t_0+1})-\psi\frac{n_{t_0-1}^{\varpi-1}}{\varpi}\right)-\frac{1}{R(1+\beta)}\psi\frac{n_{\max}^{\varpi}}{\varpi}\left(\frac{w}{w}\right.\right.\\ -\frac{1}{n_{t_0-1}}\frac{\beta}{1+\beta}\left(1-\theta-n_{t_0-1}\phi(h_{t_0})-\psi\frac{n_{t_0-1}^{\varpi-1}}{\varpi}\right)\left(\frac{z_{t_0}}{z_{t_0+1}}\right)-\frac{1}{n_{t_0-1}}\frac{1}{R(1+\beta)}\psi\frac{n_{t_0}^{\varpi}}{\varpi}\end{array}\right.\\
&= \frac{1}{(1+n_{\max}e)}\left[\begin{array}{r}-\frac{\theta}{R}n_{\max}\left(\frac{w_{m,t+2}}{w_{t+1,m}}\right)+\frac{\theta}{R}+\left(\frac{\beta}{1+\beta}\left(1-\theta-n_{\max}\phi(h_{t_0+1})-\psi\frac{n_{t_0-1}^{\varpi-1}}{\varpi}\right)-\frac{1}{R(1+\beta)}\psi\frac{n_{\max}^{\varpi}}{\varpi}\right.\\ -\frac{1}{n_{t_0-1}}\left(\frac{1}{1+g_z}\right)\frac{\beta}{1+\beta}\left(1-\theta-n_{t_0-1}\phi(h_{t_0})-\psi\frac{n_{t_0-1}^{\varpi-1}}{\varpi}\right)-\frac{1}{n_{t_0-1}}\frac{1}{R(1+\beta)}\psi\frac{n_{t_0}^{\varpi}}{\varpi}\end{array}\right.
\end{aligned}$$

Using $\phi_h n_{\max}h_{t_0+1}=\left(\frac{\alpha\psi}{R}(1+g_z)\left(\frac{h_{t_0+1}}{h_{t_0}}\right)^{\alpha}\right)\frac{n_{\max}^{\varpi}}{\varpi}=\left(\frac{\alpha\psi}{R}\right)\frac{n_{\max}^{\varpi}}{\varpi}\left(\frac{w_{m,t+2}}{w_{m,t+1}}\right)$

$$\left(\frac{S}{Y}\right)_{t_0+1} = \frac{1}{(1+n_{\max}e)} \left[\begin{array}{l} -\frac{\theta}{R} n_{\max}\left(\frac{w_{m,t+2}}{w_{m,t+1}}\right) + \frac{\theta}{R} + \frac{\beta}{1+\beta}(1-\theta)\left(1 - \frac{1}{n_{t_0-1}(1+g_z)}\right) - \frac{1}{R(1+\beta)}\psi\frac{n_{\max}^{\varpi}}{\varpi}\left(\frac{w_{m,t+2}}{w_{m,t+1}}\right)(1+ \\ -\frac{\beta}{1+\beta}\phi_0\left(n_{\max} - \frac{1}{1+g_z}\right) - \frac{1}{n_{t_0-1}}\frac{1}{R(1+\beta)}\psi\frac{n_{t_0-1}^{\varpi}}{\varpi}(1+\beta\alpha) - \frac{\beta}{1+\beta}\psi\frac{n_{t_0-1}^{\varpi-1}}{\varpi}\left(1 - \frac{1}{n_{t_0-1}(1+g_z} \end{array}\right.$$

$$\left(\frac{S}{Y}\right)_{t_0+1} - \left(1 + \frac{(n_{t_0-1}-n_{\max})e}{1+n_{\max}e}\right)\left(\frac{S}{Y}\right)_{t_0} = \frac{1}{1+n_{\max}e}\left[\begin{array}{c} -\frac{\theta}{R}\left(n_{\max}\left(\frac{w_{m,t+2}}{w_{m,t+1}}\right) - n_{t_0-1}(1+g_z)\right) \\ -\frac{(1+\beta\alpha)}{R(1+\beta)}\frac{\psi}{\varpi}\left(n_{\max}^{\varpi}\left(\frac{w_{m,t+2}}{w_{m,t+1}}\right) - (1+g_z)n_{t_0-1}^{\varpi}\right) - \frac{\beta}{1+\beta}\phi_0\,(n_{\max} \end{array}\right.$$

$$= \frac{1}{1+n_{\max}e}\left[\begin{array}{c} -\frac{\theta}{R}(1+g_z)\left(n_{\max}\left(\frac{h_{t_0+1}}{h_{t_0}}\right)^{\alpha} - n_{t_0-1}\right) \\ -\frac{(1+\beta\alpha)}{R(1+\beta)}\frac{\psi}{\varpi}(1+g_z)\left(n_{\max}^{\varpi}\left(\frac{h_{t_0+1}}{h_{t_0}}\right)^{\alpha} - n_{t_0-1}^{\varpi}\right) - \frac{\beta}{1+\beta}\phi_0\,(n_{\max} - n_{t_0-1}) \end{array}\right]$$

Proof of the the inequalities:

$$\begin{aligned} h_{t_0} &= \left(\frac{\alpha\psi}{\phi_h R}(1+g_z)\right)\frac{(n_{t_0-1})^{\varpi-1}}{\varpi}(h_{t_0+1})^{1-\alpha} h_{t_0}^{\alpha} = \left(\frac{\alpha\psi}{\phi_h R}(1+g_z)\right)\frac{(n_{\max})^{\varpi-1}}{\varpi} \\ &\Rightarrow \left(\frac{h_{t_0+1}}{h_{t_0}}\right) = \left(\frac{n_{t_0-1}}{n_{\max}}\right)^{\frac{1-\omega}{1-\alpha}} \\ &\Rightarrow n_{\max}^{\omega}\left(\frac{h_{t_0+1}}{h_{t_0}}\right)^{\alpha} = n_{\max}^{\omega}\left(\frac{n_{t_0-1}}{n_{\max}}\right)^{\alpha\frac{1-\omega}{1-\alpha}} = n_{t_0-1}^{\alpha\frac{1-\omega}{1-\alpha}}\, n_{\max}^{\frac{\omega-\alpha}{1-\alpha}} \end{aligned}$$

Proof of Lemma 3

$$\Delta S(2n_{\max}) = \frac{\beta}{1+\beta}\phi_0 n_{\max} + \frac{(1+\beta\alpha)}{R(1+\beta)}\frac{\psi}{\varpi}(1+g_z)\, n_{\max}^{\omega}\left(\frac{h_{t_0+1}}{h_{t_0}}\right)^{\alpha}\left(2^{\omega}\left(\frac{h_{t_0+1}}{h_{t_0}}\right)^{-\alpha} - 1\right)$$

$$\begin{aligned} h_{t_0} &= \left(\frac{\alpha\psi}{\phi_h R}(1+g_z)\right)\frac{(2n_{\max})^{\varpi-1}}{\varpi}\ ;\ (h_{t_0+1})^{1-\alpha} h_{t_0}^{\alpha} = \left(\frac{\alpha\psi}{\phi_h R}(1+g_z)\right)\frac{(n_{\max})^{\varpi-1}}{\varpi} \\ &\Rightarrow \left(\frac{h_{t_0+1}}{h_{t_0}}\right) = 2^{\frac{1-\omega}{1-\alpha}} \Rightarrow 2^{\omega}\left(\frac{h_{t_0+1}}{h_{t_0}}\right)^{-\alpha} = 2^{\frac{\varpi-\alpha}{1-\alpha}} \end{aligned}$$

Thus:

$$\begin{aligned} \Delta S(2n_{\max}) &= \frac{\beta}{1+\beta}\phi_0 n_{\max} + \frac{(1+\beta\alpha)}{R(1+\beta)}\frac{\psi}{\varpi}(1+g_z)\, n_{\max}^{\omega}\left(\frac{h_{t_0+1}}{h_{t_0}}\right)^{\alpha}\left(2^{\frac{\varpi-\alpha}{1-\alpha}} - 1\right) \\ &= \left(\frac{S_m}{Y_m}\right)_{t_0+1} - \left(\frac{S_m^{twin}}{Y_m}\right)_{t_0+1} \end{aligned}$$

Proof of proposition 3

The first-order condition with respect to n_t is:

$$\frac{v}{n_t} + \beta^5\frac{\psi n_t^{\varpi-1} w_{5,t+4}}{c_{7,t+4}} + \beta^6\frac{\psi n_t^{\varpi-1} w_{6,t+5}}{c_{8,t+5}} = \beta\frac{\phi_4 w_{4,t+1}}{c_{4,t+1}} + \beta^2\frac{(\phi_5 + \phi_h h_{t+1})\, w_{5,t+1}}{c_{5,t+1}}$$

Which can be rewritten using Eq. (14):

$$\left(\frac{c_{8,t+5}}{w_{4,t+1}}\right)\frac{v}{\beta^5 n_t} = R^3\left[R\phi_4 + (\phi_5 + \phi_h h_{t+1})(1+g_z)\right] - \left[R + (1+g_z)\right]\psi n_t^{\varpi-1}(1+g_z)^3\left(\frac{h_{t+1}}{h_t}\right)^{\alpha} \tag{19}$$

The first-order condition with respect to h_t is:

$$\begin{aligned}
\beta^2\psi\frac{n_t^{\varpi}}{\varpi}\left[\frac{\beta}{c_{8,t+5}}\frac{\partial w_{6,t+5}}{\partial h_{t+1}} + \frac{1}{c_{7,t+4}}\frac{\partial w_{5,t+4}}{\partial h_{t+1}}\right] &= n_t\phi_h\left(\frac{w_{5,t+2}}{c_{5,t+2}}\right) \\
\beta^2\psi\frac{n_t^{\varpi}}{\varpi}\alpha h_{t+1}^{\alpha-1}z_{t+4}\left[\beta^{-2}R^{-3}(1+g_z) + (\beta R)^{-2}\right] &= n_t\phi_h z_{t+2}h_t^{\alpha}
\end{aligned}$$

$$h_t^{\alpha}h_{t+1}^{1-\alpha} = \left(\frac{\psi\alpha}{\varpi}\frac{(1+g_z)^2}{R^2\phi_h}\right)n_t^{\varpi-1}$$

or

$$\psi(1+g_z)^2 n_t^{\varpi-1}\left(\frac{h_{t+1}}{h_t}\right)^{\alpha} = \frac{\varpi}{\alpha}R^2\phi_h h_{t+1}$$

So Eq. (19) becomes:

$$\begin{aligned}
\left(\frac{c_{8,t+5}}{w_{4,t+1}}\right)\frac{v}{R^3\beta^5 n_t} &= R\left[R\phi_4 + (\phi_5 + \phi_h h_{t+1})(1+g_z)\right] - \left[R + (1+g_z)\right](1+g_z)\frac{\varpi}{\alpha}\phi_h h_{t+1} \\
\left(\frac{c_{4,t+1}}{w_{4,t+1}}\right)\frac{Rv}{(1+g_z)\beta n_t} &= R\phi_4/(1+g_z) + \phi_5 + \phi_h h_{t+1}\left(1 - \beta\left(1 + (1+g_z)/R\right)\frac{\varpi}{\alpha}\right) \\
\left(\frac{c_{4,t+1}}{w_{4,t+1}}\right) &= \frac{(1+g_z)\beta}{Rv}\left[\left(R\phi_4/(1+g_z) + \phi_5\right)n_t + \phi_h n_t h_{t+1}\left(1 - \left(1 + (1+g_z)/R\right)\frac{\varpi}{\alpha}\right)\right]
\end{aligned}$$

Using Eq. (15):

$$\begin{aligned}
\left({}^{8}_{\gamma=4}\beta^{\gamma-4}\right)\left(\frac{c_{4,t+1}}{w_{4,t+1}}\right) &= {}^{8}_{\gamma=4}\frac{w_{\gamma,t+\gamma-3}/w_{4,t+1} + T_{\gamma,t+\gamma-3}/w_{4,t+1}}{R^{\gamma-4}} - \theta \\
&= (1-\theta-\phi_4 n_t) - (\phi_5(1+g_z) + \phi_h(1+g_z))\frac{n_t}{R} - \psi\frac{n_{t-1}^{\varpi-1}}{\varpi}\frac{(1+g_z)}{R}\left(1 + \frac{(1+g_z)}{R}\right) \\
&\quad +{}^{2}_{\gamma=1}\frac{(1+g_z)^{\gamma}}{R^{\gamma}} +{}^{8}_{\gamma=7}\frac{\psi\frac{n_t^{\varpi}}{\varpi}w_{\gamma-2,t+\gamma-3}/w_{4,t+1}}{R^{\gamma-4}} \\
&= (1-\theta-\phi_4 n_t) - (\phi_5(1+g_z) + \phi_h(1+g_z))\frac{n_t}{R} - \psi\frac{n_{t-1}^{\varpi-1}}{\varpi}\frac{(1+g_z)}{R}\left(1 + \frac{(1+g_z)}{R}\right) \\
&\quad +{}^{2}_{\gamma=1}\frac{(1+g_z)^{\gamma}}{R^{\gamma}} + \psi\frac{n_t^{\varpi}}{\varpi}\left(\frac{h_{t+1}}{h_t}\right)^{\alpha}\left(\frac{1+g_z}{R}\right)^3\left(1 + \frac{(1+g_z)}{R}\right) \\
&= 1-\theta-\phi_4 n_t - (\phi_5 + \phi_h h_{t+1}(1+g_z))n_t - \psi\frac{n_{t-1}^{\varpi-1}}{\varpi}\frac{(1+g_z)}{R}\left(1 + \frac{(1+g_z)}{R}\right) \\
&\quad +{}^{2}_{\gamma=1}\frac{(1+g_z)^{\gamma}}{R^{\gamma}} + \frac{\varpi}{\alpha}\phi_h n_t h_{t+1}\left(\frac{1+g_z}{R}\right)\left(1 + \frac{(1+g_z)}{R}\right)
\end{aligned}$$

Introduce $\Pi(\beta, v) = \beta \left(^{8}_{\gamma=4}\beta^{\gamma-4}\right)/v$

$$\begin{aligned}
& \frac{R\,(1-\theta)}{1+g_z} + \left(1 + \frac{(1+g_z)}{R}\right) - (R\phi_4/(1+g_z) + \phi_5)_5\, n_t - \psi\frac{n_{t-1}^{\varpi-1}}{\varpi}\left(1 + \frac{(1+g_z)}{R}\right) + \phi_h n_t h_{t+1}\left[\frac{\varpi}{\alpha}\left(1 + \right.\right. \\
= \;& \Pi(\beta, v)\left[(R\phi_4/(1+g_z) + \phi_5)\, n_t + \phi_h n_t h_{t+1}\left(1 - (1 + (1+g_z)/R)\,\frac{\varpi}{\alpha}\right)\right] \\
& \frac{R\,(1-\theta)}{1+g_z} + \left(1 + \frac{(1+g_z)}{R}\right) - \psi\frac{n_{t-1}^{\varpi-1}}{\varpi}\left(1 + \frac{(1+g_z)}{R}\right) \\
= \;& (1 + \Pi(\beta, v))\, n_t\left[(R\phi_4/(1+g_z) + \phi_5) + \phi_h h_{t+1}\left(1 - (1 + (1+g_z)/R)\,\frac{\varpi}{\alpha}\right)\right] \\
n_t = \;& \frac{(1-\theta) + \left(\frac{(1+g_z)}{R} + \left(\frac{(1+g_z)}{R}\right)^2\right) - \psi\frac{n_{t-1}^{\varpi-1}}{\varpi}\left(\frac{(1+g_z)}{R} + \left(\frac{(1+g_z)}{R}\right)^2\right)}{(1+\Pi(\beta, v))\left[\left(\phi_4 + \phi_5\frac{(1+g_z)}{R}\right) + \frac{(1+g_z)}{R}\phi_h h_{t+1}\left(1 - (1 + (1+g_z)/R)\,\frac{\varpi}{\alpha}\right)\right]}
\end{aligned}$$

Introduce $\phi_0 = \left(\phi_4 + \phi_5\frac{(1+g_z)}{R}\right) = \phi_4 + (1+\gamma)\phi_5$ and $\frac{(1+g_z)}{R} = 1 + g$

Two equations very similar to the simpler model:

$$\begin{aligned}
n_t &= \left(\frac{1}{1+\Pi(\beta, v)}\right)\frac{(1-\theta) + \left(1 + g + (1+g)^2\right) - \psi\frac{n_{t-1}^{\varpi-1}}{\varpi}\left(1 + g + (1+g)^2\right)}{\phi_0 + (1+g)\phi_h h_{t+1}\left(1 - (1 + (1+g))\,\frac{\varpi}{\alpha}\right)} \\
h_t^{\alpha} h_{t+1}^{1-\alpha} &= \left(\frac{\psi\alpha}{\varpi}\frac{(1+g)^2}{\phi_h}\right) n_t^{\varpi-1}
\end{aligned}$$

Table 7: Household saving rate: Twin Identification

VARIABLES	(1) All households	(2) All households	(3) w/o highest and lowest hh income deciles	(4) w/o highest and lowest hh expenditures deciles
Twins hh dummy	-0.0612***	-0.0616***	-0.0713***	-0.0574***
	(0.0178)	(0.0174)	(0.0196)	(0.0149)
Av parents age	0.0114*	0.00928	0.00625*	0.00824***
	(0.00663)	(0.00653)	(0.00321)	(0.00265)
Squared parents age	-0.000122*	-9.58e-05	-6.95e-05*	-9.60e-05***
	(6.96e-05)	(6.84e-05)	(3.71e-05)	(3.07e-05)
Child age	0.00940**	0.00975**	0.00400**	0.00618***
	(0.00389)	(0.00388)	(0.00192)	(0.00179)
Squared child age	-0.000634***	-0.000687***	-0.000368***	-0.000437***
	(0.000179)	(0.000181)	(8.22e-05)	(8.90e-05)
Av parents educ	0.0148	0.0204	-0.0408***	-0.0211**
	(0.0246)	(0.0243)	(0.0129)	(0.0106)
Squared parents educ	0.000940	-0.00233	0.00421***	0.00523***
	(0.00182)	(0.00192)	(0.00112)	(0.000885)
Hh income		5.47e-06***		
		(4.86e-07)		
Squared hh income		-0***		
		(0)		
Observations	42,669	42,669	33,921	34,050
R-squared	0.009	0.023	0.013	0.073
Years dummies	YES	YES	YES	YES
Province dummies	YES	YES	YES	YES

Robust standard errors in parentheses
*** p<0.01, ** p<0.05, * p<0.1

Notes: Data source: UHS (2002-2009) restricted sample of nuclear households: those with either an only child or twins.

Table 8: Expenditures: Twin Identification

VARIABLES	(1) Savings (yuans)	(2) Food expdt	(3) Educ. expdt	(4) Other expdt	(5) Hh income
Twin hh dummy	-2,213**	1,044***	1,290***	942.8	1,065
	(937.9)	(335.6)	(275.8)	(955.5)	(1,383)
Av. parents age	581.2***	135.1***	126.2***	-205.4	637.2***
	(159.8)	(40.90)	(24.49)	(147.9)	(187.6)
Squared parents age	-5.895***	-1.339***	-1.262***	1.091	-7.405***
	(1.746)	(0.453)	(0.284)	(1.597)	(2.018)
Child age	-163.1	-23.05	-67.04***	169.3	-83.81
	(119.0)	(29.53)	(17.67)	(106.1)	(140.4)
Squared child age	-0.0486	2.270*	8.221***	-2.952	7.490
	(5.098)	(1.234)	(0.791)	(4.636)	(5.836)
Av parents educ	-3,031***	513.6***	425.4***	-2,042***	-4,133***
	(838.8)	(185.1)	(143.3)	(712.1)	(951.5)
Squared parents educ	473.8***	20.75	3.994	463.8***	962.3***
	(75.64)	(16.24)	(12.85)	(64.43)	(85.12)
Observations	26,465	26,465	26,465	26,465	26,465
R-squared	0.100	0.326	0.122	0.160	0.401
Years dummies	YES	YES	YES	YES	YES
Province dummies	YES	YES	YES	YES	YES

Robust standard errors in parentheses

*** p<0.01, ** p<0.05, * p<0.1

Notes: UHS (2002-2006) restricted sample of nuclear households. Households saving rate is defined as household income less household consumption divided by household income.

Table 9: Education Expenditures: Twin Identification

	(1)	(2)	(3)	(4)
VARIABLES	Unit educ expdtr (in yuans) child below 15yrs	Unit educ expdtr (in yuans) 15 to 22yrs	Unit educ expdtr (% of hh income) child below 15yrs	Unit educ expdtr (% of hh income) 15 to 22yrs
Twins hh dummy	-277.4	-1,051***	-0.00429	-0.0506***
	(179.2)	(284.2)	(0.00698)	(0.0100)
Child age	98.16***	4,697***	0.000792	0.183***
	(25.63)	(328.1)	(0.000763)	(0.0133)
Squared child age	-4.570***	-124.9***	-4.92e-06	-0.00497***
	(1.503)	(8.922)	(4.49e-05)	(0.000359)
Av parents age	142.3***	776.6***	0.00566***	0.0325***
	(26.80)	(150.4)	(0.000878)	(0.00557)
Squared parents age	-1.195***	-7.077***	-4.67e-05***	-0.000297***
	(0.316)	(1.583)	(9.84e-06)	(5.70e-05)
Av parents educ	222.3	983.4***	0.00920**	0.0407***
	(159.6)	(316.7)	(0.00460)	(0.0132)
Squared parents educ	19.30	-17.21	-0.000842**	-0.00411***
	(14.21)	(28.37)	(0.000387)	(0.00111)
Observations	16,993	16,946	16,993	16,946
R-squared	0.121	0.075	0.041	0.029
Years dummies	YES	YES	YES	YES
Province dummies	YES	YES	YES	YES

Robust standard errors in parentheses
*** p<0.01, ** p<0.05, * p<0.1

Notes: UHS (2002-2006) restricted sample of nuclear households: those with either an only child or twins.

Table 10: Education Attainment: Twin Identification (LOGIT)

VARIABLES	(1) Secondary educ dummy	(2) odds ratio	(3) Higher educ dummy	(4) odds ratio
Twins hh dummy	-0.775**	0.461**	-0.586**	0.557**
	(0.301)	(0.139)	(0.228)	(0.127)
Child age	5.615***	274.5***	10.03***	22,783***
	(0.804)	(220.8)	(0.571)	(13,020)
Squared child age	-0.135***	0.874***	-0.228***	0.796***
	(0.0202)	(0.0177)	(0.0141)	(0.0112)
Av parents age	0.645***	1.905***	0.787***	2.198***
	(0.0946)	(0.180)	(0.127)	(0.278)
Squared parents age	-0.00615***	0.994***	-0.00741***	0.993***
	(0.000950)	(0.000944)	(0.00128)	(0.00127)
Av parents educ	1.460***	4.305***	1.156***	3.177***
	(0.201)	(0.867)	(0.151)	(0.479)
Squared parents educ	-0.0972***	0.907***	-0.0690***	0.933***
	(0.0188)	(0.0171)	(0.0134)	(0.0125)
Observations	15,226	15,226	15,226	15,226
Years FE	YES	YES	YES	YES
Province FE	YES	YES	YES	YES

Robust standard errors in parentheses

*** $p < 0.01$, ** $p < 0.05$, * $p < 0.1$

Notes: UHS (2002-2006) restricted sample of nuclear households: those with either an only child or twins.

Table 11: Urban Household-level Net Transfers from Children to Parents (of ages 45 and above)

NET TRANSFERS	(1) All households Urban only	(2) Parents (60+) Urban only	(3) Children (up to 25) Urban only	(4) Children (25+) Urban only
Av children educ	-196.5	812.5***	-2,106***	860.1**
	(408.0)	(254.2)	(587.6)	(349.0)
Nbr children	649.6*	581.2*	2,851	472.2
	(344.4)	(311.5)	(2,141)	(296.7)
Av children age	280.6***	173.4*	-460.8	292.3***
	(107.6)	(97.85)	(777.7)	(110.2)
Av parents age	-112.4	-156.2	243.0	-180.9**
	(98.71)	(109.6)	(252.2)	(87.57)
Hh pretransfer inc	-0.0343*	-0.0111	-0.0132	-0.0502*
	(0.0203)	(0.0131)	(0.0105)	(0.0273)
Constant	-2,053	1,749	-183.5	-1,216
	(2,979)	(4,904)	(8,456)	(2,944)
Observations	664	308	185	479
R-squared	0.064	0.024	0.043	0.102

Robust standard errors in parentheses
*** p<0.01, ** p<0.05, * p<0.1

Notes: CHARLS (2008). The baseline sample in (1) includes all urban households with children and respondent or spouse of ages 45 and above; (2) includes adds all households with parents of an average age greater or equal to 60 years of age to the baseline sample; (3) includes the baseline sample in addition to all households with children of an average age less than 26 years old; (4) includes the baseline sample in addition to all households with children of an average age greater than or equal to 26 years old.

Table 12: Urban and Rural Household-level Net transfers from Children to Parents (of ages 45 and above)

VARIABLES	(1) All households Urban and rural	(2) Parents (60+) Urban and rural	(3) Children (up to 25) Urban and rural	(4) Children (25+) Urban and rural
Av children educ	-125.9	736.7***	-2,025**	826.1***
	(347.5)	(148.7)	(838.7)	(220.3)
Nbr children	439.9***	409.9***	1,035	340.8***
	(150.5)	(131.8)	(672.4)	(127.2)
Av child age	153.4***	87.13**	-192.9	131.6***
	(46.24)	(35.69)	(338.8)	(44.19)
Av parents age	-58.10	-87.18**	154.1	-89.62**
	(41.87)	(41.14)	(137.6)	(35.48)
Hh pretransfer inc	-0.0360*	-0.0134	-0.00925	-0.0516**
	(0.0189)	(0.0110)	(0.00947)	(0.0258)
Urban dummy	667.7	1,495***	-990.0	981.0**
	(693.7)	(547.3)	(2,152)	(485.6)
Constant	-1,506	-17.47	2,149	-1,448
	(1,381)	(1,898)	(7,037)	(1,396)
Observations	1,511	713	425	1,086
R-squared	0.044	0.054	0.048	0.097

Robust standard errors in parentheses
*** $p<0.01$, ** $p<0.05$, * $p<0.1$

Notes: CHARLS (2008). The baseline sample in (1) includes all urban and rural households with children and respondent or spouse of ages 45 and above; (2) includes adds all households with parents of an average age greater or equal to 60 years of age to the baseline sample; (3) includes the baseline sample in addition to all households with children of an average age less than 26 years old; (4) includes the baseline sample in addition to all households with children of an average age greater than or equal to 26 years old.