Job Search Behavior over the Business Cycle

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Abstract

In this paper, we study nonemployed workers’ job search behavior. In particular, we analyze how search behavior changes over the business cycle. Theoretically, we show that job search intensity can either be procyclical or countercyclical depending on various factors. Empirically, we first examine how aggregate job search intensity changes over the business cycle. Second, we examine job search behavior at the individual level and analyze, how local labor market conditions affect individuals’ job search behavior.

Keywords: job search, time use, business cycles

JEL Classifications: E24, E32, J22, J64

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1 Introduction

Search effort of firms and workers is one of the most important determinants of aggregate employment. In a frictional labor market, a higher number of matches are formed when both firms and workers make more effort to find a suitable counterpart. Understanding the factors that influence search effort on both the firm and the worker sides, therefore, is essential in analyzing the behavior of aggregate employment.

Firms’ recruiting effort decision over the business cycle has been studied extensively in the past. For example, studies of Beveridge curves find that firms make higher recruiting effort by posting more vacancies during booms.\footnote{See, for example, Blanchard and Diamond (1990).} A recent paper by Davis, Faberman, and Haltiwanger (2010) argues that firms’ recruiting effort in addition to posting vacancies also contributes to the cyclical pattern of how firms match with workers. Relatedly, Davis, Faberman, and Haltiwanger criticize the standard Diamond-Mortensen-Pissarides framework for limiting the firm’s recruiting effort just to the number of vacancies—they emphasize the importance of other inputs in accounting for the microeconomic patterns of hiring.

In this paper, we take a complementary approach and focus on the worker side of the labor market and analyze how workers’ job search effort varies over the business cycle. We show that just counting the number of nonemployed workers (or unemployed workers) is not sufficient in order to measure workers’ job search effort. In that sense, our paper complements Davis, Faberman and Haltiwager (2010)’s argument and extends their critique to the measurement of search effort on the worker side.

Workers’ search effort behavior has not been studied much in the literature both theoretically and empirically. In the basic Diamond-Mortensen-Pissarides model, a firm makes an explicit recruiting effort choice in terms of how many vacancies to post, while a nonemployed worker waits passively until a job offer arrives. In other words, the Diamond-Mortensen-Pissarides type search and matching models abstract from workers’ \textit{intentional} choice of search effort and workers’ search decisions play no role in facilitating match formation. A
recent strand of literature, albeit small, models workers’ search decisions explicitly. In Christiano, Trabandt, and Walentin (2010), search effort by nonemployed households is assumed to move procyclically, and this generates procyclical employment. In Veracierto (2008) and Shimer (2011), a nonemployed worker can choose whether being “unemployed” or “inactive,” and only unemployed workers have a positive probability of finding a job (while they have to incur a cost of search). This discrete choice between “unemployed” or “inactive” can be considered as a discrete choice of search effort.

Despite its theoretical importance, there is even less work done on the empirical front that goes beyond the measurement of unemployment.\(^2\) The main reason for this oversight is the lack of good quality data on job search behavior. We overcome this obstacle by combining two datasets as we describe later.

Beyond the determination of aggregate employment, workers’ search effort also has important implications for policy design. For example, the recent studies of optimal unemployment insurance over the business cycle, such as Kroft and Notowidigdo (2011) and Landais, Michaillat, and Saez (2011), the moral hazard in worker’s search effort (and how it varies over the business cycle) is the central focus in determining the optimal policy.

The paper consists of three parts. We first construct a simple model in order to obtain theoretical predictions on how economic environment influences the worker’s job search effort level. One insight we learn from the model is that there is no ex ante reason to believe that the job search effort is procyclical or countercyclical. There are many effects at work, and it is an empirical question whether some effects dominate others.

Second, we document the macroeconomic time series properties of job search effort. We use the American Time Use Survey (ATUS) and the Current Population Survey (CPS) in order to measure job search effort at the aggregate and individual level. Both datasets have their own shortcomings, and our innovation is to combine the information from both surveys in order to obtain a nationally-representative and monthly time series of job search effort. We

\(^2\)There are a few important exceptions, which we discuss below.
distinguish two different margins of search effort: the “extensive margin” and the “intensive margin,” in an analogy to the labor supply literature. The “extensive margin” refers to “how many nonemployed workers engage in search activities,” while the “intensive margin” refers to “how intensely each searcher is searching.” The aggregate search effort in the economy then can be calculated as the product of the extensive and intensive margins.

Third, we explore the determinants of the job search effort further, by looking at the cross-sectional differences. First, we run regressions with the imputed search time as the independent variable and worker characteristics and economic environment variables as the dependent variables. Of particular interest is how the labor-market tightness and the change in wealth affect the search decision. Second, in order to uncover the nature of the matching technology, we estimate the matching function, incorporating the intensity of search effort.

There are existing papers that utilize the ATUS and the CPS to measure the search effort. Shimer (2004) is an early critic of the way search effort is modeled in typical search-matching models. Shimer uses the CPS measure of job search intensity and shows that the procyclicality of search effort, which is the implication of existing models of job search, is not supported by the data. We extend Shimer’s analysis using an additional data set, the ATUS, and providing a methodology to combine information from two data sets. Krueger and Mueller (2010) and Aguiar, Hurst, and Karabarbounis (2012) are two recent papers which make use of the ATUS to analyze job search behavior. Krueger and Mueller mostly focus on the relationship between search intensity and generosity of unemployment insurance benefits while Aguiar, Hurst, and Karabarbounis analyze the allocation of lost work hours during recessions. DeLoach and Kurt (2012) look at ATUS and analyze the determinants of the search time at the micro level. Their analysis is related to our micro-level regressions in Section 5.1. Our analysis has an advantage of utilize two surveys, the ATUS and the CPS, to have a better measure of job search effort.

This paper is organized as follows. Section 2 presents a simple model, in order to uncover the elements that affect the cyclicality of search behavior in macro and micro level. Section 3
describes the data and explains how we combine the information from two datasets. Section 4 analyzes the cyclicality of search effort at the macro level. Section 5 investigates why we observe the countercyclical search effort from the micro level. Section 6 concludes.

2 Model

In this section, we formulate a simple static model of individual search decision. The model serves two purposes. First, it shows that whether nonemployed workers’ search effort is procyclical or countercyclical is an empirical question—within reasonable settings of the model, search effort can become procyclical or countercyclical. The models of Appendix A confirm this finding in a more elaborate, dynamic setting. Second, it clarifies the intuition regarding the channels that contribute to the cyclicality of search effort. In particular, the model focuses on the effect of labor-market tightness (how many vacancies there are compared to the number of unemployed workers), wages, unemployment compensations, and wealth.

2.1 Setting

Consider a worker who is currently nonemployed. In the beginning of the period, she decides the intensity of job search effort, \( s \in \mathbb{R}_+ \). Search effort is costly, and we assume that the cost (in utility term) is \( c(s) \) where \( c'(\cdot) > 0 \) and \( c''(\cdot) > 0 \). The job-finding probability is assumed to take the form \( f(s, \bar{s}, \theta) \) where \( f \) is increasing and concave in \( s \). The variable \( \bar{s} \) is the search effort of other workers. \( \bar{s} \) affects the job-finding probability of a given worker when there is an externality: for example, it may be the case that when some workers search harder, it reduces the odds of other workers finding a job. \( \theta \) is a parameter that represents the labor market condition surrounding the worker. In addition, we assume that \( f \) is increasing in \( \theta \).

A special case is the standard Diamond-Mortensen-Pissaridis model, which postulates a constant-returns-to-scale matching function \( M(u, v) \)—the number of matches, \( M(u, v) \), is increasing in both the number of unemployed workers \( u \) and the number of vacancies posted \( v \). Given the constant returns, the probability of a worker matching with a job is
\( M(u, v)/u = M(1, \theta) \), where \( \theta \) is defined as \( v/u \). This formulation is a special case where \( f(s, \bar{s}, \theta) \) that does not depend on \( s \) and \( \bar{s} \).

A less trivial special case is the one analyzed by Pissarides (2000, Chapter 5) where job search effort is explicitly modeled. The matching function at the aggregate level (assuming that the individuals are atomistic) is assumed to take the form \( M(\bar{s}u, v) \) where \( M(\bar{s}u, v) \) is constant-returns-to-scale (and increasing) in these two terms. For each individual, the probability of finding a job is \( sM(\bar{s}u, v)/(\bar{s}u) \) which can be rewritten as

\[
    f(s, \bar{s}, \theta) = sM \left( 1, \frac{1}{\bar{s}} \theta \right), \tag{1}
\]

where \( \theta \) is defined as \( v/u \).

Denote the utility that the worker receives from finding a job as \( W \) and the utility from being unemployed as \( U \). Clearly, \( W \) and \( U \) are influenced by the characteristics of the worker and also by the labor market conditions that the worker faces (possibly including \( \theta \), as we will discuss later). We will specify \( W \) and \( U \) in more detail later on. Here, we only assume that \( W > U \). The optimization problem for the worker is

\[
    \max_s f(s, \bar{s}, \theta)W + (1 - f(s, \bar{s}, \theta))U - c(s).
\]

### 2.2 Findings

We next analyze how job search effort responds to various changes in the economic environment.

**Proposition 1** Given \( \bar{s} \) and \( \theta \), job search intensity \( s \) is increasing in \( (W - U) \). It is increasing in \( \theta \) if and only if \( f_{13}(s, \bar{s}, \theta) > 0 \), where \( f_{ij}(s, \bar{s}, \theta) \) represents the cross derivative of \( f(s, \bar{s}, \theta) \) in \( i \)th and \( j \)th terms.

**Proof.** The first order condition is

\[
    c'(s) = f_1(s, \bar{s}, \theta)(W - U), \tag{2}
\]
where \( f_i(s, \bar{s}, \theta) \) is the partial derivative of \( f(s, \bar{s}, \theta) \) with respect to \( i \)th term. From the Implicit Function Theorem,

\[
\frac{ds}{d(W-U)} = \frac{f_1(s, \bar{s}, \theta)}{c''(s) - f_{11}(s, \bar{s}, \theta)(W-U)}
\]

Since \( c''(s) > 0 \), \( f_{11}(s, \bar{s}, \theta) \leq 0 \), and \( f_1(s, \bar{s}, \theta) \geq 0 \), \( ds/d(W-U) \geq 0 \). Similarly,

\[
\frac{ds}{d\theta} = \frac{f_{13}(s, \bar{s}, \theta)(W-U)}{c''(s) - f_{11}(s, \bar{s}, \theta)(W-U)}
\]

and since \( c''(s) > 0 \) and \( f_{11}(s, \bar{s}, \theta) \leq 0 \), and \( W-U > 0 \), \( ds/d\theta \) has the same sign as \( f_{13}(s, \bar{s}, \theta) \).

Note that in the standard Diamond-Mortensen-Pissarides model (such as Pissarides (1985)), \( (W-U) \) also has a direct link to \( \theta \). There, however, both \( \theta \) and \( (W-U) \) are endogenous, and it is difficult to think of an experiment of “moving \( \theta \) exogenously” as we do here. The link between the two in the (extended) Pissarides (1985) model is through general equilibrium: if the outcome of the match is expected to be good, this high surplus is going to be divided between firms and workers through Nash bargaining (therefore a high \( (W-U) \)), while a high surplus for firms leads to more vacancy posting today (therefore a high \( \theta \)).\(^3\) Since here our purpose is to clarify the intuition by abstracting from general equilibrium interactions and dynamic issues, we do not consider this link. Appendix A shows that even in a fully dynamic and general equilibrium model, an equation similar to (2) determines \( s \) and the main message of this section (the cyclicality of \( s \) depends on the setting of the model) carries through.

Proposition 1 provides an analysis of individual search effort decisions. When we look at the data, we need to take into account that \( \bar{s} \) is also determined (by the choice of \( s \) of other people) in the economy and influenced by the economic environment. For simplicity, suppose that the economy consists of homogeneous workers. Then, in equilibrium, \( \bar{s} = s \) has to hold.

\(^3\)A more straightforward and “partial equilibrium” intuition is that \( \theta \) at time \( t \) would affect the unemployment at time \( t + 1 \), and this affects \( (W-U) \) in the next period. This intuition is correct in general, but in Pissarides (1985) these values are not affected by \( u \) at time \( t + 1 \) and thus there is no effect going through this channel.
Under this assumption, we can show prove the “equilibrium version” of Proposition 1. First, we need an additional assumption.

**Assumption 1** \( c''(s) - (f_{11}(s, s, \theta) + f_{12}(s, s, \theta))(W - U) > 0 \) for all \( s \) and \( \theta \).

This is satisfied when \( f_{12}(s, s, \theta) \) is sufficiently small. It is, for example, satisfied under (1) since there \( f_{12}(s, s, \theta) \) is negative. Under Assumption 1, it is straightforward to show the following.

**Proposition 2** Given \( \theta \), job search intensity, \( s \), is increasing in \( (W - U) \). It is increasing in \( \theta \) if and only if \( f_{13}(s, s, \theta) > 0 \).

The proof is omitted since it is similar to the proof of Proposition 1. Thus the result of this “equilibrium outcome” is similar to the outcome of the decision problem in Proposition 1. Note that this result is still not “general equilibrium” in the sense of Pissarides (2000), since variables such as \( \theta \), \( W \), and \( U \) are assumed to be exogenous. This is sufficient for our purpose because we are interested in the workers’ search effort decision in a given environment.

To characterize the determinants of \( W \) and \( U \), we make the following additional assumptions. First, the utility from consumption is represented by a strictly increasing and strictly concave utility function \( u(c) \), where \( c \) is consumption. The worker has an asset level \( a \). If he works, he receives the wage \( w \), and if he is unemployed, he receives the unemployment compensation \( b \). Thus, \( c = w + a \) if he works and \( c = b + a \) if he is unemployed. Then, \( W = u(w + a) \) and \( U = u(b + a) \). (In a dynamic model, \( c \) also depends on the expectations on future income.) Assume that \( w > b \). Then it is straightforward to show the following.

**Corollary 1** The job search intensity, \( s \), is increasing in \( w \), decreasing in \( b \), and decreasing in \( a \) in the context of both Propositions 1 and 2.

**Proof.** We only need to establish how \( W - U \) responds to these parameters since we can then apply Propositions 1 and 2, we Since \( W - U = u(w + a) - u(b + a) \), the first two are straightforward. For \( a \),

\[
\frac{d(W - U)}{da} = u'(w + a) - u'(b + a) < 0
\]
since \( w + a > b + a \). \( \blacksquare \)

During expansions, \( \theta \), \( w \), and \( a \) are typically higher. The first can have positive or negative effect in \( s \), depending on the \( f(s, \bar{s}, \theta) \) function. The second has a positive effect and the third has a negative effect.\(^4\) Therefore, whether \( s \) is procyclical or countercyclical for a nonemployed worker is an empirical issue. This in particular raises two distinct empirical questions: (i) the relative strengths of the effect of \( w \) and \( a \), and (ii) the shape of the \( f(s, \bar{s}, \theta) \) function. At the macroeconomic level, the number of nonemployed worker also fluctuates, adding one more factor in the fluctuations of the aggregate search effort.

### 2.3 Generalizing the matching function

Before discussing our empirical analysis, one issue is worth exploring: When is \( f_{13}(s, s, \theta) \) negative? This is not a trivial question—for example, when we start from the aggregate matching function, a natural formulation such as (1) cannot generate a negative \( f_{13}(s, s, \theta) \): \( f_{13}(s, s, \theta) \) is always positive as long as \( M(su, v) \) is increasing in \( v \). However, intuitively it is quite natural to think of a situation where \( f_{13}(s, s, \theta) \) is negative, that is, the “marginal product” of individuals’ search is smaller when the labor market conditions are better. For example, when there are so many jobs (the firm searches very heavily), that it is almost certain that every worker has a good job offer, additional effort by the worker does not add much to the matching outcome. Based on a similar intuition, Shimer (2004) argues that a concrete micro-founded matching process, which does not necessarily assume complementarity between worker search effort and the labor market conditions, is essential in understanding the job search decisions of workers.

We follow Shimer’s intuition and and consider a “generalized” version of the job finding probability:

\[
f(s, \bar{s}, \theta) = \chi \left( \alpha s^\psi + (1 - \alpha) \left( \frac{s}{\bar{s}} \right)^\xi \theta^\psi \right)^\eta .
\]

\(^4\)If the utility function is linear, the third effect is absent.
When workers are homogeneous, this aggregates up to the matching function

\[ M(\bar{s}, u, v) = \chi \left( \alpha \bar{s}^\psi + (1 - \alpha) \left( \frac{u}{\bar{u}} \right)^\psi \right)^\eta u. \tag{4} \]

We assume that \( \chi > 0 \), \( \alpha \in [0, 1] \), and \( \psi, \xi, \eta \) have the same sign (weakly). This formulation nests some important special cases. For example,

- When \( \xi = \alpha = 0 \) and \( \psi \eta \in (0, 1) \), (3) reduces to the standard Diamond-Mortensen-Pissarides matching function in Cobb-Douglas form.

- If we first set \( \xi = \psi = 1/\eta \) and take a limit of \( 1/\eta \to 0 \), \( f(s, \bar{s}, \theta) \) becomes \( s\chi(\theta/\bar{s})^{1-\alpha} \) and \( M(\bar{s}, u, v) \) becomes \( \chi(\bar{s}u)\alpha v^{1-\alpha} \). Therefore this is a Cobb-Douglas version of (1).

To see that \( f_{13} \) can be either positive or negative, consider a simple special case of (3) where \( \xi = 0 \). Then one can easily show that \( f_{13} < 0 \) if and only if \( \psi(\eta - 1) < 0 \). Since \( \psi \) and \( \eta \) has the same sign, this means that \( \eta \in (0, 1) \).\(^5\)

3 Measuring search effort

This section explains how we measure workers’ job search effort by combining information from the CPS and the ATUS, and creating a measure of job search effort for each worker in the CPS sample at the monthly frequency.

3.1 Data

We use two data sets: the CPS and the ATUS. The CPS is a monthly survey conducted by the U.S. Bureau of Census for the Bureau of Labor Statistics (BLS). It is a primary source of labor force statistics for the population of the United States. The ATUS is conducted by the BLS and individuals are drawn from the exiting samples of the CPS. Respondents are interviewed within 2–5 months of their last CPS interview. The ATUS collects detailed

\(^5\)In this case, additional parametric restrictions may be needed in order to guarantee the properties that are usually assumed in the search-matching literature. Also note that it may be necessary to check the second-order condition for optimal choice of \( s \), since \( f \) function may not be concave. For example, if \( \psi \eta = 1 \) and \( \eta < 1 \), \( f \) is convex in \( s \).
Recently, the ATUS has been used to measure job search effort by some researchers, such as Krueger and Mueller (2010), Aguiar, Hurst, and Karabarbounis (2011), and DeLoach and Kurt (2012). The ATUS has the advantage of having a quantifiable measure of job search effort at the “intensive margin,” that is, how many minutes each nonemployed worker spends for job search. We follow Krueger and Mueller (2010) and identify job search activities as the ones in Table 1. The first category (job search activities) include contacting employer, sending out resumes, and filling out job application, among others.\textsuperscript{6}

The major shortcomings of the ATUS is its small sample size (12,000–21,000 per year) and the short sample period (available only from 2003). In order to overcome these shortcomings, we make use of the available information on job search behavior from the CPS. The advantage of the CPS is a larger sample size (60,000 per month) and a longer sample period (we use the surveys after 1994 redesign). The CPS contains a question about particular search methods that each respondent uses. Conditional on a worker being a nonemployed searcher (that is, an unemployed worker who is not on temporary layoff), the CPS asks what kind of search methods the worker has used.\textsuperscript{7}

There are nine active search methods and three passive search methods. Table 2 lists all

\begin{table}
\centering
\begin{tabular}{|l|}
\hline
Job search activities (050401) \\
Interviewing (050403) \\
Waiting associated with job search interview (050404) \\
Security procedures related to job search/interviewing (050405) \\
Job search activities, not elsewhere specified (050499) \\
\hline
\end{tabular}
\caption{Definitions of job search activities in ATUS}
\end{table}

\textsuperscript{6}See Krueger and Mueller (2011) Table 1 in Appendix A for details.

\textsuperscript{7}There is a small group of nonemployed workers other than searchers who report using a strictly positive number of search methods. Since the CPS documentation states that only searchers are asked about their search methods, we interpret these responses as miscoding and replace them by zeros.
方法和我们使用的缩写。从表中可以看出，许多活动与ATUS中描述的工作搜索活动重叠。我们的想法是， CPS“方法”问题的答案和ATUS“时间”问题的答案之间包含相似的信息，因此我们可以使用“方法”问题来衡量工人的搜索努力。

## 3.2 谁在搜索？

作为分析ATUS和CPS数据的第一步，识别参与搜索活动的工人类型是有用的。表3报告了在ATUS中每个劳动力市场状态的平均时间（以每天分钟计算）用于工作搜索活动。

除了标准分类（就业和非就业；就业、失业和不在劳动力中[NILF]），我们将失业工人分为两类（“临时裁员”和“搜索者”）和不在劳动力中分为两类（“非搜索者”和“其他NILF”）根据Shimer (2004)。搜索者是失业的工人，他们不是临时裁员，非搜索者是不在劳动力中的工人。

In addition to standard categorizations (employed and nonemployed; employed, unemployed, and not in the labor force [NILF]), we divide unemployed workers into two categories ("temporary layoffs" and "searchers") and the not in the labor force into two categories ("non-searchers" and "other NILF") following Shimer (2004). Searchers are the unemployed workers who are not on temporary layoff, and non-searchers are the workers who are not in

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**Table 2: Definitions of job search methods in CPS (the first nine are active, the last three are passive)**

<table>
<thead>
<tr>
<th>Method</th>
<th>Abbreviation</th>
</tr>
</thead>
<tbody>
<tr>
<td>Contacting an employer directly of having a job interview</td>
<td>CE</td>
</tr>
<tr>
<td>Contacting a public employment agency</td>
<td>PU</td>
</tr>
<tr>
<td>Contacting a private employment agency</td>
<td>PR</td>
</tr>
<tr>
<td>Contacting friends or relatives</td>
<td>FR</td>
</tr>
<tr>
<td>Contacting a school or university employment center</td>
<td>EC</td>
</tr>
<tr>
<td>Checking union or professional registers</td>
<td>UN</td>
</tr>
<tr>
<td>Sending out resumes or filling out applications</td>
<td>RA</td>
</tr>
<tr>
<td>Placing or answering advertisements</td>
<td>AD</td>
</tr>
<tr>
<td>Other means of active job search</td>
<td>OA</td>
</tr>
<tr>
<td>Reading about job openings that are posted in newspapers or on the internet</td>
<td>RE</td>
</tr>
<tr>
<td>Attending job training program or course</td>
<td>TR</td>
</tr>
<tr>
<td>Other means of passive job search</td>
<td>OP</td>
</tr>
</tbody>
</table>

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methods and the abbreviations that we will use. From the table, it can be seen that many activities overlap with the job search activities that are described in the ATUS. Our idea is that similar information is contained between the answers to the CPS “methods” question and the ATUS “time” question and therefore we can use the “methods” question in order to measure the search effort by the workers.

**3.2 Who is searching?**

As a first step of in analyzing the ATUS and the CPS data on job search, it is useful to identify the type of workers who engage in search activity. Table 3 reports the average time (in minutes per day) spent for job search activities in the ATUS for workers in each labor market state.

In addition to standard categorizations (employed and nonemployed; employed, unemployed, and not in the labor force [NILF]), we divide unemployed workers into two categories ("temporary layoffs" and "searchers") and the not in the labor force into two categories ("non-searchers" and "other NILF") following Shimer (2004). Searchers are the unemployed workers who are not on temporary layoff, and non-searchers are the workers who are not in
All workers

<table>
<thead>
<tr>
<th>Employed</th>
<th>Nonemployed</th>
</tr>
</thead>
<tbody>
<tr>
<td>0.5</td>
<td>4.6</td>
</tr>
</tbody>
</table>

<table>
<thead>
<tr>
<th>Unemployed</th>
<th>Not in the labor force</th>
</tr>
</thead>
<tbody>
<tr>
<td>27.6</td>
<td>0.5</td>
</tr>
</tbody>
</table>

<table>
<thead>
<tr>
<th>Temp layoff</th>
<th>Searchers</th>
<th>Non-searchers</th>
<th>Other NILF</th>
</tr>
</thead>
<tbody>
<tr>
<td>10.2</td>
<td>30.6</td>
<td>2.3</td>
<td>0.5</td>
</tr>
</tbody>
</table>

<table>
<thead>
<tr>
<th>Attached workers</th>
</tr>
</thead>
<tbody>
<tr>
<td>25.0</td>
</tr>
</tbody>
</table>

Table 3: Average search time (minutes per day) from the ATUS

the labor force but who report that they “want a job.” Also following Shimer’s terminology, searchers and non-searchers are categorized as “attached workers.”

Table 3 reveals large differences in search time among different categories. Unemployed workers spend much more time searching for a job compared to employed workers or the NILF workers. There is also some difference in search time between workers on temporary layoff and searchers, but workers on temporary layoff do spend significant time searching. Therefore, it is clear from Table 3 that the answer to “who is searching?” is “unemployed workers.” Given that the main criteria needed to be classified as unemployed is job search, this observation is not surprising. However, we find these calculations useful since they assure us that job search time in the ATUS is a reasonable measure of job search effort.

### 3.3 Linking the ATUS and the CPS

As we have discussed, the ATUS provides an empirically plausible measure of job search effort. However, it has the disadvantage of having a small sample size and spanning a short sample period. In order to overcome these shortcomings of the ATUS, we link the information of the job search methods in the CPS to the job search time in the ATUS.

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8This is a larger category than “marginally attached workers”—a marginally attached worker has to be available for working and have searched during the past 12 months (but not past four weeks), in addition to reporting that she wants a job.
Since the CPS question on job search methods is also asked to the ATUS sample, we have information on job search methods (and relevant worker characteristics) as well as time spent on job search for these respondents. To see how these two measures are related, we categorize workers (limited to searchers) based on the number of methods they report to use, and plot average minutes per day for job search activities in Figure 1.

Figure 1 indicates that search time and number of methods used exhibit a strong positive correlation. At the same time, the number of methods is not an ideal measure of the search effort, since it does not convey any information on the relative importance of each method in workers’ job search activities. It is likely that workers allocate their search time differently across different methods, considering the effectiveness of each method in finding a job. In order to estimate the “weights” on each method, we run a regression of the form

$$s_{it} = \alpha + \sum_k \beta_k M_{it}^k + \mathbf{x}_{it}' \gamma + \varepsilon_{it},$$

where $s_{it}$ is the ATUS search time for worker $i$ in period $t$ (minutes per day), $\alpha$ is a constant, $M_{it}^k$ is a dummy which is 1 if the worker $i$ answers that she used the method $k$ at time $t$, and $\beta_k$ is the associated coefficient. There are twelve $M_{it}^k$ dummies for each of the methods in Table 2. $\mathbf{x}_{it}$ is the vector of controls and $\gamma$ is the associated coefficient vector. $\varepsilon_{it}$ is the
error term.

There are two sets of controls. The first is a set of worker characteristics which may affect job search decision. We mostly follow Shimer (2004) in the choice of these controls. They are the quartic of age \((\text{age}_{it})\) and dummies for education levels \((\text{educ}_{1it} \text { for high school diploma, } \text{educ}_{2it} \text { for some college, and } \text{educ}_{3it} \text { for college plus)}\), black \((\text{black}_{it})\), female \((\text{female}_{it})\), and married \((\text{married}_{it})\). We also add the interaction term of \(\text{female}_{it} \times \text{married}_{it}\) since being married is likely to affect men and women differentially in terms of job search behavior. The second is a set of dummies for labor market states, such as being NILF but not categorized as a nonsearcher \((\text{otherNILF}_{it})\), being on temporary layoff \(\text{layoff}_{it}\), and being a nonsearcher \((\text{NS}_{it})\). These controls are intended to capture the heterogeneity of the search time for the respondents who do not answer the CPS question on job search methods. Note that only searchers report information on methods, while job search time is reported by everyone in the ATUS sample.

The regression coefficients are repotted below. The abbreviations of the CPS methods are reported in Table CPSmethods). * indicates the coefficient being significant at 5% level, ** indicates significant at 1% level, and *** indicates significant at 0.1% level.

\[
\hat{s}_{it} = 14.6***CE_{it} + 20.0***PU_{it} + 22.5***PR_{it} + 11.5***FR_{it} + 13.1***EC_{it} + 8.3*UN_{it} \\
+ 23.7***RA_{it} + 11.4***AD_{it} + 15.3***OA_{it} + 18.0***RE_{it} + 38.9***TR_{it} + 4.6OP_{it} \\
- 7.4age_{it} + 0.3age_{it}^2 - 0.004*age_{it}^3 + 0.00002*age_{it}^4 \\
+ 0.6educ_{1it} + 2.4***educ_{2it} + 4.1***educ_{3it} \\
- 0.5black_{it} - 1.8**female_{it} + 0.6married_{it} - 2.5**female_{it} \times \text{married}_{it} \\
+ 11.9***\text{otherNILF}_{it} + 20.2***\text{layoff}_{it} + 13.4***\text{NS}_{it} + 63.2\text{const} \\
\]

Most of the coefficients on the CPS methods are statistically different from zero at the 0.1% significance level. Equation 6 allows us to calculate the “estimated search time” for each worker in the CPS, even for the ones who were not selected to be interviewed by the ATUS.\(^9\)

\(^9\)Note that each coefficient can be interpreted as the average number of minutes spent for that particular
Figure 2: Average search minutes per day for all nonemployed workers and unemployed workers, actual and imputed. ATUS sample.

Figure 2 provides a comparison of the time series of the actual minutes and the imputed minutes within the ATUS sample. The imputed minutes track the actual minutes closely with the exception of 2004 and 2005.

In the remainder of the paper, we use $\hat{s}_{it}$, our imputed minutes, as the search effort measure for the CPS sample. This measure is a nontrivial extension of Shimer’s measure since it exploits information on job search from the ATUS. Note that Shimer’s measure is a special case of the regression (5)—it corresponds to the case where $\beta_k$ is restricted to be the same for all $k$ and $\gamma$ is zero. In this special case, $\hat{s}_{it}$ reduces to the number of search methods in the CPS, the measure used by Shimer (2004). Our measure is superior to Shimer’s, as long as the ATUS search time is a good quantitative representation of the actual search effort.\(^\text{10}\)

method.

\(^{10}\)We have repeated all exercises in Section 4 using Shimer’s (2004) measures. All results remain the same qualitatively.
Figure 3: The time series of extensive margins: $U/(U + N)$ and $U/(U + NS)$

4 Cyclicality of search effort

In this section, we describe how nonemployed workers’ search behavior has changed over time. We measure both the “extensive margin” and the “intensive margin” of the worker’s search. The extensive margin refers to the number of people who are actively searching, and the intensive margin refers to the intensity of each searcher’s search effort.

4.1 The extensive margin

As we discussed in the previous section, the most reasonable answer to “who is searching?” is “the unemployed workers.” Figure 3 plots the ratio of unemployed worker to all nonemployed workers and the ratio of unemployed workers to the sum of the number of unemployed workers and the number of nonsearchers. where $U$ is the unemployed, $N$ is not in the labor force,
Figure 4: The time series of the time dummies $\mu_s$. The sample population is $U + N$ for the dotted line and $U + NS$ for the solid line.

and $NS$ is nonsearchers.\textsuperscript{11} The choice of the denominator is dictated by the choice of the appropriate set of “entire population of workers” in the analysis, and in the most general context, it corresponds to the “entire population who are older than 16.”\textsuperscript{12} In that case, the appropriate denominator is all nonemployed workers. Another reasonable choice is to exclude workers who do not want to work. In that case the sum of unemployed workers and the nonsearchers is the appropriate denominator.

As can be seen from the Figure 3, the extensive margin of the search effort is clearly countercyclical. This is not a surprising observation given that the strong countercyclicality of unemployment has been widely documented. This property holds even after we control for the demographic composition of the $U + N$ (or $U + NS$) pool—this is shown in Figure

\textsuperscript{11} This sum corresponds to “generalized unemployment” in the language of Krusell et al. (2010).

\textsuperscript{12} In this paper, the age range is from 25 to 70.
4. In order to control for the changes in composition, we estimate a linear probability model that is similar to Shimer’s (2004). We run the regression (to the $U + N$ and $U + NS$ sample)

$$y_{it} = x_{it} \delta + \sum_s \mu_s m_s + \varepsilon_{it}, \quad (7)$$

$y_{it}$ is 1 if $i \in U$ and 0 if $i \notin U$, $x_{it}$ is the same set of controls as in (6) with the coefficient vector $\delta$, and $m_s$ is the month dummy that takes 1 if $s = t$ and 0 otherwise. Figure 3 plots the coefficients on the month dummy, $\mu_s$, for each $s$. These coefficients provide an estimate of how much being in a particular month raises the probability of being in $U$. Similar to Figure 3, this time series is also countercyclical.

Although the results on the extensive margin provide some useful information about search effort decision, this is an imperfect measure of each worker’s search effort. In addition, in the context of many search models, the extensive margin is effectively irrelevant, since in these models the “entire population of workers” is considered as employment plus unemployment. Thus we examine the “intensive margin” in the next subsection.

4.2 The intensive margin

We use the imputed minutes, $\hat{s}_{it}$, calculated using Equation 6 as our measure of the intensive margin. To obtain a longer time series for search effort we backcast average job search time starting from 1994 even though the ATUS sample starts in 2003. Figure 5 plots the time series of the average $\hat{s}_{it}$ in minutes per day. The solid line is the average minutes per day that an unemployed worker spends for search activities. We also plot (in dotted line) the corresponding number for searchers. They clearly exhibit a countercyclical pattern. [Still need to add the ATUS data]

In order to see if the same pattern holds after controlling for the composition of unemployed workers, we run a similar regression as (7) with $\hat{s}_{it}$ for the unemployed worker on the left-hand side. Figure 6 plots the coefficients on month dummies. The countercyclical pattern remains even after controlling for composition.

The takeaway from the previous section and this section is that at both extensive and
Figure 5: The time series of the estimated average search effort (minutes per day) for unemployed workers and searchers
intensive margins, nonemployed workers search harder during recessions.

4.3 Total

Total search effort by nonemployed workers in the economy can be calculated as [the fraction of unemployed in population] \times [intensive margin]. This is the counterpart of $s$ (and $\bar{s}$) in the nonemployed worker’s matching probability $f(s, \bar{s}, \theta)$ in Section 2. Figure 7 shows the total search effort. There are three series, depending on the group of workers included in the calculations. The smallest is the minutes of searchers only (that is, the minutes of all other nonemployed workers is counted as zero), and the next is the minutes of unemployed workers, and the largest counts all minutes of nonemployed workers. We count unemployed workers as the extensive margin, so the second one is the most natural measure, but we plot the other two because searchers’ $\hat{s}_{it}$ is the most accurately measured (since they are the ones who report about their search methods) and some not in the labor force workers also have nonzero $\hat{s}_{it}$s (although they are small). As one can infer from the previous two sections’
Figure 7: The total search effort (sum of all minutes divided by the total nonemployed population), using the minutes of only searchers, only unemployed workers, and all nonemployed results, total search effort in Figure 7 (all three series) also exhibits a countercyclical pattern.

If we multiply the series in Figure 7 by \((U + N)/(U + N + E)\), we obtain the total search effort in the economy. This is the natural counterpart of the input \(su\) in the matching function \(M(su, v)\) in Section 2. These series are plotted in Figure 8. Again, we observe a countercyclical pattern.

The fact that there are cyclical fluctuations in the intensive margin has important implications for accounting for the cyclical changes in the job-finding process. If we consider the number of unemployed workers as the input to the matching function (that is, \(M(u, v)\) instead of \(M(su, v)\), where \(u = U/(E + U + N)\)), we measure the search effort only using the extensive margin. Figure 9 plots these two series (for the total search effort, we use only the unemployed workers’ minutes), normalizing the initial level to one. There are significant discrepancies, especially in recent years, indicating the importance of appropriately taking
Figure 8: The total search effort (sum of all minutes divided by the total population), using the minutes of only searchers, only unemployed workers, and all nonemployed workers.
Figure 9: The total search effort (sum of all minutes divided by the total population), using the minutes of unemployed workers ($sU/(E+U+N)$), compared to the number of unemployed workers $U/(E+U+N)$. Both are normalized to 1 at the beginning of 1994.

the intensive margin into account. In the context of the recent outward shift of Beveridge curve documented by many researchers, our observation deepens the “puzzle” of why it has shifted outwards, since the increase in intensive margin of the search effort should move the Beveridge curve inwards, other things being equal.

5 Why countercyclical?

All our results indicate that the search effort is countercyclical, both at the intensive and the extensive margins. This section makes several attempts to account for this countercyclicality, especially in the intensive margin.

Unobserved heterogeneity is an important issue in the measurement of the cyclicality of the intensive margin, since the pool of searchers change over time. We have shown that
aggregate search effort is countercyclical, and this holds for individual intensive margin as well. This observation is true even after controlling for observed heterogeneity. However, in addition to changes in observed characteristics of searchers, there is a countercyclical bias stemming from unobserved heterogeneity when we consider individual-level effort. Suppose that searchers are heterogeneous in their desire to work, and this heterogeneity is unobservable. In addition, suppose that a worker with a strong preference to work tends to make higher search effort and tends to transit to employment more quickly than other workers. In booms, workers tend to transit to employment more quickly, and “high search effort” workers may disappear from the unemployment pool faster. As a result, the unemployment pool would be dominated by workers with less desire to work during booms. This may lead to a countercyclical bias in the observed search effort. We deal with this issue of unobserved heterogeneity by exploiting the panel aspect of the CPS and find that unobserved heterogeneity indeed has an impact in analyzing the response of individual search effort to external environment.

Before we go into the analysis, it is useful to note what our observation so far would imply for our original motivation. Our goal was to provide insights on the determinant of cyclicality of employment, in particular whether it is supply-driven or demand-driven. Our observation so far already provides some answer—even with the existence of unobserved heterogeneity, it is very unlikely that an economy with an entirely supply-driven economy (as in Veracierto (2008) and Christiano, Trabandt, and Walentin (2012)) produces a countercyclical total search effort. In other words, if the aggregate employment and unemployment are entirely supply-driven, the total search effort has to be procyclical. Appendix B shows this formally in the context of a Pissarides (1985)-style model.

5.1 Estimating the individual decision rules

In the context of our simple model above, job search behavior is strongly influenced by either labor market conditions, $\theta$, and present value of wealth, $a$.

We run individual-level regressions to uncover the factors that influence individuals’ de-
cision for \( s \). The regression equation is
\[
s_{it} = \delta_{\theta_{it}} + x_{it}'\delta_x + w_{it}'\delta_w + \varepsilon_{it},
\]

[To be completed]

5.2 Estimating the matching function

We run the regression
\[
\ln(M_{kt}) = \delta_0 + \delta_s \ln(\bar{s}_{kt}) + \delta_u \ln(u_{kt}) + \delta_v \ln(v_{kt}) + \varepsilon_{kt}.
\]

Here, \( k \) denotes submarket \( k \). Compare this to the log-linearized version of the generalized matching function (4) (and suppose that we normalize the data so that \( \tilde{s} = \tilde{v} = \tilde{u} = 1 \)). Then
\[
\delta_s = \frac{\tilde{M}_s}{M} \bar{s} = \alpha \psi \eta \bar{s}^\psi \left( \alpha \bar{s}^\psi + (1 - \alpha) \left( \frac{\bar{v}}{\bar{u}} \right)^\psi \right)^{-1} = \alpha \psi \eta,
\]
\[
\delta_u = \frac{\tilde{M}_u}{M} \bar{u} = 1 - (1 - \alpha) \psi \eta \left( \frac{\bar{v}}{\bar{u}} \right)^\psi \left( \alpha \bar{s}^\psi + (1 - \alpha) \left( \frac{\bar{v}}{\bar{u}} \right)^\psi \right)^{-1} = 1 - (1 - \alpha) \psi \eta,
\]
and
\[
\delta_v = \frac{\tilde{M}_v}{M} \bar{v} = 1 - \delta_u
\]
hold, where \( \tilde{M}_i \) is the partial derivative of \( M \) function with respect to \( i \) (\( \tilde{\cdot} \) denotes evaluating at the steady-state). Here, \( \delta_v + \delta_u = 1 \) because the matching function is constant returns in \( v \) and \( u \).

Now we try to infer the structural parameter from the regression. We have two equations with three unknowns. Clearly, \( \psi \) and \( \eta \) cannot be distinguished in this log-linear approximation—in order to distinguish them we have to expand the function further. It is easy to obtain the estimates of \( \alpha \) and \( \psi \eta \), since \( \delta_s / \delta_v = \alpha / (1 - \alpha) \) and \( \delta_s + \delta_v = \psi \eta \).

The higher-order (log) approximation of (4) can be conducted by further Taylor expansion. In particular, denoting \( \hat{s} \) as the log-deviation from the steady-state, we can expand
\[
\ln(M) = \ln(M(\tilde{s}e^\hat{s}, \tilde{u}e^{\tilde{\hat{s}}}, \tilde{v}e^{\tilde{\hat{v}}}))
\]
with respect to \( \hat{s}, \hat{u}, \) and \( \hat{v} \). The second-order approximation yields

\[
\ln(M) \approx \sum_{i=s,u,v} \hat{M}_i \hat{M}^2 + \sum_{i=s,u,v} \frac{1}{2} \left( \frac{\hat{M}_i \hat{M}^2 - \hat{M}^2_i}{M^2} \right) \hat{M}^2 + \sum_{i,j=s,u,v} \frac{\hat{M}_i \hat{M}_j - \hat{M}_i \hat{M}_j}{M^2} \hat{M}^2\hat{M}^2,
\]

where \( M_{ij} \) is the cross derivative of \( M \) function with respect to \( i \) and \( j \). We can run the regression corresponding to this:

\[
\ln(M_{kt}) = \delta_0 + \delta_s \ln(\bar{s}_{kt}) + \delta_u \ln(u_{kt}) + \delta_v \ln(v_{kt})
\]

\[
+ \delta_{ss} \ln(\bar{s}_{kt})^2 + \delta_{uu} \ln(u_{kt})^2 + \delta_{vv} \ln(v_{kt})^2
\]

\[
+ \delta_{su} \ln(\bar{s}_{kt}) \ln(u_{kt}) + \delta_{sv} \ln(\bar{s}_{kt}) \ln(v_{kt}) + \delta_{uv} \ln(u_{kt}) \ln(v_{kt}) + \epsilon_{kt}.
\]

\( \delta_s, \delta_u, \) and \( \delta_v \) are the same as above. The rest are (after applying for the normalization):

\[
\delta_{ss} = \frac{1}{2} \alpha(1 - \alpha) \psi^2 \eta,
\]

\[
\delta_{uu} = \frac{1}{2} \left[ (1 - \alpha) \psi \eta (\alpha \psi + 1) - 1 \right],
\]

\[
\delta_{vv} = -\frac{1}{2} (1 - \alpha) \psi \eta (\alpha \psi - 1),
\]

\[
\delta_{sv} = -\alpha(1 - \alpha) \psi \eta,
\]

\[
\delta_{su} = \alpha(1 - \alpha) \psi \eta,
\]

and

\[
\delta_{uv} = -\alpha(1 - \alpha) \psi \eta.
\]

The important coefficients are \( \delta_{sv} \) and \( \delta_{su} \). The signs of these are directly linked to the sign of \( \eta \). In order to have “substitutability” in the job finding rate, we expect \( \eta \) to be positive.

[To be completed]

6 Conclusion

[To be completed]
References


Appendix

A A general equilibrium search-matching model

This section presents an infinite-horizon general equilibrium model.

A.1 General setup

The aggregate number of matches at each period is dictated by the matching function $M(u_t, v_t; \bar{s}_t)$, where $\bar{s}_t$ is the average search effort in the economy, $v_t$ is the aggregate vacancy, and $u_t$ is the number of unemployed workers at time $t$. At the individual level, matching is stochastic, and the probability of worker $i$ finding a job is $f(s_{it}, \bar{s}_t, \theta_t)$, where $s_{it}$ is his search effort and $\theta_t \equiv v_t/u_t$. The probability of a firm finding a worker is $q(\bar{s}_t, \theta_t)$. The separation probability of a matched job-worker pair is $\sigma$. The job-worker match produces $z_t$ unit of consumption goods, and $z_t$ follows a Markov process.

A.1.1 Unemployment dynamics

The total population is 1, and therefore the number of employed workers is $1 - u_t$. The dynamics of the unemployment is dictated by

$$u_{t+1} = \sigma(1 - u_t) + (1 - f(s_{it}, \bar{s}_t, \theta_t))u_t.$$  \hfill (8)

A.1.2 Value functions

Let the (aggregate) state variable at time $t$ be $S_t \equiv (u_t, z_t)$. From a firm’s perspective, the value of being matched with a worker, $J(S_t)$, is:

$$J(S_t) = z_t - w(S_t) + \beta E[(1 - \sigma)J(S_{t+1}) + \sigma V(S_{t+1})],$$  \hfill (9)

where $V(S_t)$ is the value of vacancy and $w(S_t)$ is the wage paid to the worker. The expectation $E[\cdot]$ is taken with the information of $S_t$. The value of vacancy is

$$V(S_t) = -\kappa + \beta E[q(\bar{s}_t, \theta_t)J(S_{t+1}) + (1 - q(\bar{s}_t, \theta_t))V(S_{t+1})].$$  \hfill (10)
For the worker’s side, the value of being employed, \( W(S_t) \), is

\[
W(S_t) = w(S_t) + \beta E[(1 - \sigma)W(S_{t+1}) + \sigma U(S_{t+1})],
\]

and the value of being unemployed, \( U(S_t) \), is

\[
U(S_t) = \max_{s_{it}} \{ b - c(s_{it}) + \beta E[f(s_{it}, \bar{s}_t, \theta_t)W(S_{t+1}) + (1 - f(s_{it}, \bar{s}_t, \theta_t))U(S_{t+1})] \}. \tag{12}
\]

The first-order condition for the right-hand side is:

\[
c'(s_{it}) = \beta f_1(s_{it}, \bar{s}_t, \theta_t)E[W(S_{t+1}) - U(S_{t+1})]. \tag{13}
\]

Denote \( s_{it} \) that satisfies (13) by \( s^*_{it} \).

A.1.3 Wage determination

Let

\[
\bar{J}(w; S_t) = z_t - w + \beta E[(1 - \sigma)J(S_{t+1}) + \sigma V(S_{t+1})|S_t]
\]

and

\[
\bar{W}(w; S_t) = w + \beta E[(1 - \sigma)W(S_{t+1}) + \sigma U(S_{t+1})].
\]

The wage is determined by the generalized Nash bargaining with the worker’s bargaining power \( \gamma \in (0, 1) \). Then \( w \) solves

\[
(1 - \gamma)(\bar{W}(w; S_t) - U(S_t)) = \gamma(\bar{J}(w; S_t) - V(S_t)). \tag{14}
\]

A.1.4 Free entry and equilibrium

We assume free entry to vacancy posting, \( V(S_t) = 0 \). From (10),

\[
\kappa = \beta q(s_t, \theta_t)E[J(S_{t+1})]
\]

holds, and (9) can be rewritten as

\[
J(S_t) = z_t - w(S_t) + \beta(1 - \sigma)E[J(S_{t+1})].
\]
Therefore,

\[ J(S_t) = z_t - w(S_t) + \frac{(1 - \sigma)\kappa}{q(s_t, \theta_t)}. \]

Using this to the right-hand side of (15) yields

\[ \kappa = \beta q(\bar{s}_t, \theta_t) E \left[ z_{t+1} - w(S_{t+1}) + \frac{(1 - \sigma)\kappa}{q(\bar{s}_{t+1}, \theta_{t+1})} \right]. \tag{16} \]

From (11) and (12),

\[ W(S_t) - U(S_t) = w(S_t) - b + c(s^*_t) + \beta E[(1 - \sigma - f(s^*_t, \bar{s}_t, \theta_t))(W(S_{t+1}) - U(S_{t+1}))]. \]

This can be rewritten as

\[ w(S_t) = W(S_t) - U(S_t) + b - c(s^*_t) - \beta E[(1 - \sigma - f(s^*_t, \bar{s}_t, \theta_t))(W(S_{t+1}) - U(S_{t+1}))]. \]

From (14),

\[ W(S_t) - U(S_t) = \frac{\gamma}{1 - \gamma} J(S_t). \]

Thus

\[ w(S_t) = \frac{\gamma}{1 - \gamma} J(S_t) + b - c(s^*_t) - \beta(1 - \sigma - f(s^*_t, \bar{s}_t, \theta_t)) \frac{\gamma}{1 - \gamma} E[J(S_{t+1})]. \]

Once again, from (15),

\[ w(S_t) = \frac{\gamma}{1 - \gamma} J(S_t) + b - c(s^*_t) - \frac{\gamma}{1 - \gamma} (1 - \sigma - f(s^*_t, \bar{s}_t, \theta_t)) \frac{\kappa}{q(s_t, \theta_t)}. \]

Forwarding one period, taking expectation, and using (15) once again,

\[ E[w(S_{t+1})] = \frac{\gamma}{1 - \gamma} \frac{\kappa}{\beta q(s_{t+1}, \theta_{t+1})} + b - E[c(s^*_{t+1})] - \frac{\gamma}{1 - \gamma} \frac{(1 - \sigma - f(s^*_{t+1}, \bar{s}_{t+1}, \theta_{t+1}))\kappa}{q(s_{t+1}, \theta_{t+1})}. \tag{17} \]

Let us impose the equilibrium condition and denote \( s_t = s^*_t = \bar{s}_t \). Then combining (16) and (17) we obtain

\[ \frac{\kappa}{1 - \gamma} = \beta q(s_t, \theta_t) E \left[ z_{t+1} - b + c(s_{t+1}) + \frac{1 - \sigma - \gamma f(s_{t+1}, s_{t+1}, \theta_{t+1})}{1 - \gamma} \frac{\kappa}{q(s_{t+1}, \theta_{t+1})} \right]. \tag{18} \]
The equation (13) can be rewritten as
\[ c'(s_t) = f_1(s_t, s_{t'}, \theta_t) \frac{\gamma \kappa}{1 - \gamma q(s_t, \theta_t)}. \] (19)

Equations (18) and (19) determine the dynamics of \( \theta_t \) and \( s_t \). Note that the variable \( u \) do not appear in both (18) and (19). This implies that the dynamics of \( \theta_t \) and \( s_t \) (both jump variables) are not influenced by \( u \) (only influenced by \( z \)). Once we know the dynamics of \( \theta_t \) and \( s_t \) from (18) and (19), we can determine the dynamics of unemployment by (8) and \( u_0 \).

A.2 Pissarides (1985) model (no effort choice)

A special case is when \( s_t \) is constant, which boils down to the standard Pissarides (1985) model. This case is easy to analyze. Assume that \( f(\theta) = \chi \theta^{1-\eta} \) and \( q(\theta) = \chi \theta^{-\eta} \), where \( \chi > 0 \) and \( \eta \in (0, 1) \). Then, log-linearizing (18) around the steady-state yields (the “tilde” (\( \tilde{\cdot} \)) denotes the value at the steady state and the “hat” (\( \hat{\cdot} \)) denotes the log deviation from the steady state)

\[ A\hat{\theta}_t = E[\tilde{\varepsilon}_{t+1} + B\hat{\theta}_{t+1}], \]

where \( A \equiv \kappa \eta \tilde{\theta}^{\eta}/(1 - \gamma) \beta \chi \) and \( B \equiv [(1 - \sigma)\kappa \eta \tilde{\theta}^{\eta}/(1 - \gamma) \chi] - [\gamma \kappa \tilde{\theta}/(1 - \gamma)]. \)

Assume that \( \tilde{\varepsilon}_{t+1} = \rho \tilde{\varepsilon}_t + \varepsilon_{t+1} \), where \( \rho \in (0, 1) \) and \( \varepsilon_{t+1} \) is a mean zero random variable (thus \( \tilde{\varepsilon} = 1 \)). Since the equilibrium \( \hat{\theta} \) has to take the form

\[ \hat{\theta}_t = C\tilde{\varepsilon}_t, \]

using the method of undetermined coefficients,

\[ C = \frac{\rho}{\underline{A} - \rho B} = \frac{1 - \gamma}{\kappa \tilde{\theta}^{\eta} \left( \left[ \frac{1}{\rho \beta} - (1 - \sigma) \right] \frac{\eta}{\chi} + \gamma \tilde{\theta}^{1-\eta} \right)}. \] (20)

This makes it clear that, for example, for given \( \tilde{\theta} \) the amplification (\( C \)) is large when \( \kappa \) is small. This is the background of Hagedorn and Manovskii’s (2008) main result. (In order to keep \( \tilde{\theta} \) and other parameters constant, a small \( \kappa \) requires a large value of \( b \).)
A.3 Pissarides (2000, Ch 5) model

Now, let’s go back to the original model, with (18) and (19). Assume that 
\[ c(s) = \phi s^{\omega} / \omega, \]
where \( \omega > 1 \). As in Pissarides (2000, Ch 5), assume that the matching function takes the form of \( M(\tilde{su}, v) \) and the worker’s job finding rate is 
\[ f(s, \tilde{s}, \theta) = s M(1, \theta / \tilde{s}). \]
In particular, assume a Cobb-Douglas function for the matching function:
\[ M(\tilde{su}, v) = \chi(\tilde{su})^{\eta} v^{1-\eta}, \]
where \( \chi > 0 \) and \( \eta \in (0, 1) \). The worker’s job finding probability is
\[ f(s, \tilde{s}, \theta) = \chi s \left( \frac{\theta}{\tilde{s}} \right)^{1-\eta}. \]
The probability of a vacancy finding a worker is
\[ q(\tilde{s}, \theta) = \chi \tilde{s}^{\eta} \theta^{-\eta}. \]

The equation (18) is now
\[ \frac{\kappa}{1-\gamma} = \beta \chi s_t^{\eta} \theta_t^{-\eta} E \left[ z_{t+1} - b + \frac{\phi}{\omega} s_{t+1}^{\omega} + \frac{1-\sigma - \gamma \chi s_{t+1}^{\eta} \theta_{t+1}^{1-\eta}}{1-\gamma} \frac{\kappa}{\chi s_{t+1}^{\eta} \theta_{t+1}^{-\eta}} \right]. \]
Rearranging, this can be rewritten as
\[ \frac{\kappa}{(1-\gamma)\beta \chi} s_t^{\eta} \theta_t^{\eta} E = E \left[ z_{t+1} - b + \frac{\phi}{\omega} s_{t+1}^{\omega} + \frac{(1-\sigma)\kappa}{(1-\gamma)\chi} s_{t+1}^{\eta} \theta_{t+1}^{\eta} - \frac{\gamma \kappa}{1-\gamma} \theta_{t+1}^{1-\eta} \right]. \]

Log-linearizing this yields
\[ \frac{\kappa \tilde{s}^{-\eta} \tilde{\theta}^{\eta}}{(1-\gamma)\beta \chi} (\theta_t - \tilde{s}_t) = E \left[ \tilde{z}_{t+1} + \phi \tilde{s}_{t+1}^{\omega} \tilde{s}_{t+1}^{\omega} + \frac{(1-\sigma)\kappa \tilde{s}^{-\eta} \tilde{\theta}^{\eta}}{(1-\gamma)\chi} (\tilde{\theta}_{t+1} - \tilde{s}_{t+1}) - \frac{\gamma \kappa \tilde{\theta}}{1-\gamma} \tilde{\theta}_{t+1} \right]. \]

The equation (19) can be rewritten as
\[ \frac{\phi s_{t+1}^{\omega-1}}{1-\gamma} = \gamma \kappa \frac{\theta_t}{s_t}. \]
This can be solved as
\[ s_t = \left( \frac{\gamma \kappa}{(1-\gamma)\phi} \frac{\theta_t}{s_t} \right)^{\frac{1}{\omega}}. \]
Log-linearizing,
\[ \dot{s}_t = \frac{1}{\omega} \theta_t. \]  
(23)
This makes it clear that \( s_t \) responds positively to \( \theta_t \). This is no surprise given our results in the simple model. There is no effect of wealth given the linear utility and \( b \) stays constant. The job finding probability is complementary between \( s \) and \( \theta \), which tends to make \( s \) move in the same direction as \( \theta \). The effect of wage enhances it, since the wage also tends to be procyclical. It responds less when the curvature of the effort cost function (\( \omega \)) is larger.

Using (23), (21) can be rewritten as
\[ A \dot{\theta}_t = E[\ddot{z}_{t+1} + B \dot{\theta}_{t+1}], \]
using the same assumption on \( \ddot{z} \) as before and following the same steps (we used the fact that (22) also holds in the steady state), we obtain
\[ \dot{\theta}_t = C \ddot{z}_t, \]
where
\[ C = \frac{\omega}{(\omega - 1) \kappa \tilde{\theta}^\eta} \left( \frac{1 - \gamma}{\rho^\beta} - (1 - \sigma) \right) \frac{\eta}{\chi \tilde{s}^\eta + \gamma \tilde{\theta}^{1-\eta}}. \]
This is remarkably similar to (20). The only differences are (i) \( \chi \) is now replaced by \( \chi \tilde{s}^\eta \), since now this is the “effective” match efficiency on average, and (ii) the term \( \omega/(\omega - 1) \) is multiplied in front, since the movement of \( s \) influences the cyclical movement of the probability of a vacancy finding a worker, changing the incentive for vacancy posting. There is a “magnification” (when \( \chi \tilde{s}^\eta \) is replaced by \( \chi \)), since \( \omega/(\omega - 1) > 1 \). This was observed by Merz (1995) and Gomme and Lkhagvasuren (2011) in related, numerically-solved models.

A.4 A model with “substitutive” matching function

Now assume that
\[ M(\tilde{s}, u, v) = \chi \left( \alpha \tilde{s}^\psi + (1 - \alpha) \left( \frac{v}{u} \right)^\psi \right)^\eta u. \]
and
\[ f(s, \bar{s}, \theta) = \chi(\alpha s^\psi + (1 - \alpha)\theta^\psi)^\eta, \]
where \( \chi > 0, \alpha \in (0, 1), \eta \in (0, 1), \psi > 0, \) and \( \psi \eta < 1. \) It follows that
\[ q(\bar{s}, \theta) = \chi(\alpha \bar{s}^\psi + (1 - \alpha)^\theta^\psi)^\eta \theta^{-1}. \]

Note that \( f_{13} < 0 \) is satisfied in this formulation.

The equation (18) can be rearranged to
\[ \frac{\kappa}{(1 - \gamma)\beta^\chi} (\alpha s_t^\psi + (1 - \alpha)\theta_t^\psi)^{-\eta} \theta_t \]
\[ = E \left[ z_{t+1} - b + \phi \omega s_{t+1}^\omega - \frac{(1 - \sigma)\kappa}{(1 - \gamma)^\chi} (\alpha s_{t+1}^\psi + (1 - \alpha)\theta_{t+1}^\psi)^{-\eta} \theta_{t+1} - \frac{\gamma \kappa}{1 - \gamma} \theta_{t+1} \right] \tag{24} \]
and the equation (19) is\(^\text{13}\)
\[ \phi s_t^\omega - \frac{\alpha \eta \gamma \kappa s_t}{(1 - \gamma)\alpha s_t^\psi + (1 - \alpha)\theta_t^\psi}. \]

Rearranging and log-linearizing, we obtain
\[ \hat{s}_t = \frac{\alpha \hat{s}^\psi - (\psi - 1)(1 - \alpha)\hat{\theta}^\psi}{\omega \alpha \hat{s}^\psi + (\omega - \psi)(1 - \alpha)\hat{\theta}^\psi} \hat{\theta}_t. \tag{25} \]

Denote the right-hand side as \( \Xi \hat{\theta}_t \). Similarly to (23), the absolute value of \( \Xi \) is small when \( \omega \) is large. In contrast to (23), here \( \hat{s} \) can react \textit{negatively} to \( \hat{\theta} \) (i.e. \( \Xi > 0 \)) if \( \psi \) is sufficiently large (or \( \alpha \) is sufficiently small). Note that \( f_{13} < 0 \) is not sufficient for this because the effect from wage still exists.

Log-linearizing (24) and using (25), we obtain the equation
\[ A\hat{\theta}_t = E[\hat{z}_{t+1} + B\hat{\theta}_{t+1}], \]
\(^\text{13}\)We also have to check the second-order condition in this case, because the concavity of \( f \) in \( s \) is not necessarily guaranteed. It is
\[ \phi(\omega - 1)s_t^\omega - \frac{\gamma \kappa \eta s_t^\psi - \psi - 1}{(1 - \gamma)(\alpha s_t^\psi + (1 - \alpha)\theta_t^\psi)} \left( \psi - 1 + \frac{\psi(\eta - 1)\alpha s_t^\psi}{\alpha s_t^\psi + (1 - \alpha)\theta_t^\psi} \right) > 0. \]
where
\[
A = \frac{\kappa}{(1-\gamma)\beta\chi} (\alpha \tilde{s}^\psi + (1-\alpha)\tilde{\theta}^\psi)^{-\eta \tilde{\theta}} \left[ 1 - \eta \frac{\alpha \psi \tilde{s}^\psi \Xi + (1-\alpha)\psi \tilde{\theta}^\psi}{\alpha \tilde{s}^\psi + (1-\alpha)\tilde{\theta}^\psi} \right]
\]
and
\[
B = \frac{\kappa}{1-\gamma} \left( \frac{1-\sigma}{\chi} (\alpha \tilde{s}^\psi + (1-\alpha)\tilde{\theta}^\psi)^{-\eta \tilde{\theta}} \left[ 1 - \eta \frac{\alpha \psi \tilde{s}^\psi \Xi + (1-\alpha)\psi \tilde{\theta}^\psi}{\alpha \tilde{s}^\psi + (1-\alpha)\tilde{\theta}^\psi} \right] - \gamma \tilde{\theta} \right) + \phi \tilde{s}^\omega \Xi.
\]

As before, this can be solved as
\[
\hat{\theta}_t = \frac{\rho}{A - \rho B} \hat{z}_t.
\]

Below we solve this numerically. Several parameter values are given upfront: set $\omega = 4$, $\alpha = 0.1$, $\psi = 2$, and $\eta = 0.3$. One period is assumed to be one month: thus set $\beta = 0.987^{\frac{1}{12}}$ and $\rho = 0.95^{\frac{1}{12}}$ (both from Cooley and Prescott (1995)). Following Shimer (2005), set $\gamma = 0.72$ and $\sigma = 0.034$. Let $\tilde{\theta} = 1$ and $\tilde{s} = 1$ in the steady state (later these will pin down $\kappa$ and $\phi$). Set $\chi$ so that the steady-state job finding rate $\chi (\alpha \tilde{s}^\psi + (1-\alpha)\tilde{\theta}^\psi)^\eta = \chi (0.1 + 0.9)^{0.3} = 0.45.$ We set the value of nonemployment, $b - \phi \tilde{s}^\omega / \omega = b - \phi / 4 = 0.7$, along the line of Hall and Milgrom (2008). $\kappa$ and $\phi$ are set from the steady-state version of (18) and (19).

The resulting parameter values are $\chi = 0.45$, $\phi = 0.0161$, $b = 0.708$, and $\kappa = 0.104$. The movement of endogenous variables are governed by
\[
\hat{s}_t = -0.3636 \hat{\theta}_t
\]
and
\[
\hat{\theta}_t = 3.3750 \hat{z}_t.
\]

Thus in this case $s_t$ reacts negatively to the change in $\theta_t$ (and therefore $z_t$).

**B Implications of countercyclical search effort**

This section shows that our evidence goes against a model where the cyclical movement of employment and unemployment are entirely supply-driven. We show that the observed total search effort of such an economy is procyclical.
Consider of an infinite-horizon model with continuum of workers, who are either employed or nonemployed. Workers are heterogeneous and each selects a search effort \( s \) when nonemployed. Suppose that the job-finding probability is an increasing function of \( s \): \( f(s) \). Employment is purely driven by this search effort, so that \( s \) is the only factor that affects the job-finding probability. The separation probability is the same across workers and constant over time (as in Pissarides (1985)) at \( \delta \).

Below we compare the steady states of a “good state” and “bad state”. In the U.S. economy, the job-finding probability is sufficiently large so that the cyclical movement is well-approximated by steady state comparisons (see, for example, Shimer (2005)). Below, we assume that \( f'(s)s/f(s) \leq 1 \) (if we consider \( f(s) = s^\zeta \), this means that \( \zeta \leq 1 \)).

From the above assumptions, the steady-state probability of a worker with search effort \( s \) being nonemployed is \( \delta/(f(s) + \delta) \). Suppose that \( s \) is distributed across the entire population with the distribution function \( G(s) \). Then the steady-state nonemployment rate in the entire economy is

\[
\int \frac{\delta}{f(s) + \delta} dG(s) \tag{26}
\]

and the observed total search effort by nonemployed workers is

\[
\int s \frac{\delta}{f(s) + \delta} dG(s). \tag{27}
\]

Suppose that in a boom, \( G(s) \) increases in the sense of the first-order stochastic dominance. Then the nonemployment rate (26) falls in the boom and the observed total effort (27) rises. The first is straightforward from the fact that \( \delta/(f(s) + \delta) \) is decreasing in \( s \). The second is because \( s\delta/(f(s) + \delta) \) is increasing in \( s \) when \( f'(s)s/f(s) \leq 1 \). The observed effort per nonemployed worker is

\[
\int s \frac{\delta}{f(s) + \delta} dG(s) / \int \frac{\delta}{f(s) + \delta} dG(s)
\]

and this is even more procyclical.