Optimal Design of Internal Disclosure

Job Market Paper

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Abstract

This paper studies the joint design of optimal incentive pay and information disclosure in a dynamic moral hazard problem. The principal is more informed about the outcomes of agent's actions and actively manages information available to the agent. Sharing information with the agent increases productivity (for example, allowing a better allocation of resources or effort), but increases the cost of providing incentives. The optimal contract features incomplete information sharing with positive information shared more than negative information and past negative information leading to less information sharing in the future.

1 Introduction

Should a supervisor correct a mistake of an employee? When is it optimal to give feedback to an employee? How does past performance of an employee impact communication between him and his supervisor? To answer these and related questions we must understand how communication in a firm should be structured. I take the perspective of agency theory and model a firm as a dynamic principal-agent relationship. In this classic environment I study the optimal joint design of internal communication and compensation. I show that a less informed agent is cheaper to incentivize due to fewer required incentive constraints. On the other hand information may be

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beneficial for productive reasons. In the presence of performance sensitive pay, an optimal contract has incomplete information sharing. I show how optimal information-sharing rule depends on agent's past performance and demonstrate how under some assumptions it can be characterized using dynamic programming. The model illustrates the optimal resolution to the tension between information sharing and incentive provision. It also shows the dynamic relationship between internal firm transparency and performance.

A contract is a combination of a compensation plan and information disclosure rule. Communication imposed by the optimal contract satisfies three properties. First, the optimal contract typically has incomplete information sharing: good firms are neither fully transparent nor fully opaque with their employees. Second, information attributed to good performance is revealed more frequently than information attributed to bad performance. Third, disclosure of information, which reveals bad performance, leads to less information disclosure in subsequent periods.

Motivated by real-life situations, I assume that the principal obtains private information related to agent's work on the project. In some periods, he can decide to reveal it to the agent and in doing so increase his productivity. Some examples involve correcting an agent's mistake, coaching the agent, or giving a third-party opinion about agent's proposed plans of action. In some situations this information is completely uninformative about agent's performance (for example, some types of training may be equally useful after good and bad performance), but in some cases it can partially or fully reveal principal's objective opinion of the agent's work (for example, shutting down an unpromising project reveals that the manager decided it was unpromising). The principal wishes to make the agent more productive in the future but leaks some information related to the past. I refer to this as information leakage. Information leakage creates a difficult tradeoff between information disclosure and performance pay. On one hand, principal wants to reveal information to get a productivity increase. On the other hand, as it turns out, doing so makes it harder (i.e., more expensive) to motivate the agent in the future.

Consider for example a professional services firm specializing in law or consulting. Typically, the longterm contracts have an up or out structure and provide strong incentives for associates to work hard. Information that makes an associate infer that he is unlikely to make it may sometimes motivate him to work even harder but can also lead him to start looking for a new job and pay less attention to the current employer. Reputational risks to a firm of a not so diligent employee may be significant making such behavior highly undesirable. On the other hand, it is widely acknowledged that junior employees are on a steep learning curve in the first years with the firm and thus coaching and feedback are valuable to the firm. How much information should a junior employee receive in such a firm? An optimal information-sharing rule must balance the costs and

benefits outlined above.

While I cast the model as direct information disclosure, information sharing may also occur indirectly via an observable action undertaken by the principal. Consider, for example, a problem of optimal resource allocation within a firm. Headquarters observes quality of the investment projects generated by individual divisions and decides how many resources to allocate to each division. With decreasing returns to scale, efficient allocation each period equalizes the rates of return on each unit of capital. However, a decision to reallocate resources leads to an information leakage problem since each division observes if it has received additional resources or not. This way each division will know how well it is doing relative to the rest of the firm. If a division perceives itself as weak it may engage in some form of rent seeking behavior instead of focusing on the productive action. The benefits of information sharing in this environment arise from efficient reallocation of resources across divisions, while the costs are coming from the agency conflict coming from separation of ownership and control.

I start with a classic principal-agent model where performance pay is needed to motivate the agent to exert costly private effort (if agent's decisions were verifiable, then the question of optimal information sharing would be trivial). The principal and agent are risk-neutral and the agent has limited liability. Every period the agent decides privately whether to work or shirk. I add to the model the design of information disclosure, as discussed above. Every period some project relevant information is realized and privately observed by the principal. Disclosing information may increase future output and is thus beneficial to the principal. An information-sharing rule describes conditions under which the principal will share the project specific information with the agent. The key tension is the conflict between incentive provision to work and information available to the agent. It turns out that if for a given compensation plan an informed agent finds it incentive compatible to work, an uninformed agent would also find it in his best interest to work. The intuition is best illustrated by comparing contracts in environment where agent observes fully his performance in every period to one in which he observes nothing till the end of the contract. Under full information incentive compatibility constraints have to be satisfied path by path for every realization of the performance measure. Under no information the agent cannot distinguish the paths of the performance measure, and so incentive compatibility constraints have to be satisfied only on average. This logic leads us to conclude that purely from a compensation perspective it would be optimal to reveal no information to the agent.

Performance measure relevant for compensating the agent is correlated with project information and thus when the principal chooses to disclose this information it leads to an information leakage problem that the agent indirectly learns about his performance. Given the cost of information

outlined above this will limit the extent to which information important for production is shared with the agent.

When principal discloses information, agents beliefs over his future compensation change. If information indicating negative performance is disclosed then the agent knows he has done poorly and a pay for performance scheme punishes him. Not only bad feedback can make it hard to motivate the agent in the future, if feedback is asymmetric (for example, more likely to be given when it is good), lack of feedback will have incentive effects as well. When positive information is often disclosed, then lack of disclosure leads the agent to conclude that he has likely done poorly. These features of information disclosure will influence agent's decisions of working in subsequent periods and thus alter the costs of incentive provision.

Characterization of the optimal contract is a difficult problem for the following reason: if the agent is given information in some information nodes and not others, it is in general very hard to determine which multi period incentive compatibility constraints will be binding. In case the agent observes performance measure in every period, we know that it is sufficient to check one-period deviations. However, when he does not, after a deviation the agent has a different belief over in which history node he is, compared to if he followed the recommended action plan. This makes it difficult to compute best deviation strategies (known as the double deviation problem) and the problem is even harder if we try to optimize over information disclosure policies. To gain tractability, I introduce one additional assumption into the model when the agent shirks he privately learns either that his performance was unaffected or that he failed for sure. This structure allows me to characterize optimal thresholds for disclosing bad information via dynamic programming.

The optimal design of the organization is partial and asymmetric transparency. It can be described by history-dependent thresholds: project information is revealed when the benefit of the agent knowing this information is above a certain threshold for disclosure specified by the contract. In every period, there are two thresholds: one for disclosing states that are consistent with recent good performance (i.e., that can be realized with positive probability conditional on good performance) and one that is used when the state fully reveals that recent performance was bad.

The first threshold, for good news, turns out to be zero while the second threshold is typically strictly positive. That is, any project information having positive benefits of disclosure and consistent with good performance is always disclosed but some project signals that would surely reveal bad performance are hidden. This asymmetry in releasing good news and bad news arises naturally in the model even though the manager has no psychological costs of giving bad news. This is a static property of the optimal contract. The second result is how these thresholds change

over time. I show analytically that bad performance leads to less information sharing to follow in subsequent periods. It can also be seen numerically that for long time horizons bad performance leads to a higher disclosure threshold next period.

The problem of optimal information sharing is theoretically interesting in a contracting environment and has so far received surprisingly little attention. In typical dynamic models the agent is informed about the performance measure of his effort at time t before making a $t+1$ effort decision. When there is no persistent effect on production technology, even if the agent shirks at time t it will only have an effect on time t performance measure and performance measures corresponding to subsequent efforts will be unaffected. Another way of saying this is that single deviation incentive compatibility constraints imply global incentive compatibility constraints. This is no longer true in information structures where the agent does not observe time t performance measure before making effort in period $t+1$. In this case if the agent has shirked at time t his subsequent effort profile may include shirking again. This implies that single deviation incentive compatibility constraints may no longer be sufficient for global incentive compatibility. The question that arises is what is the necessary and sufficient incentive compatibility constraint for the optimal contract. There are two difficulties. First, for a given information structure determining the relevant global incentive compatibility constraints is not trivial. Second, for different information structures the relevant constraint are different. I show an intuitive structure on signals observed by the agent under which it is possible to provide an explicit representation of the agents incentive compatibility constraint as well as solve for the optimal information structure via dynamic programming. This allows me to compare the effects of information disclosure over time and show how monetary transfers interact with information sharing.

The contribution of the paper is twofold. I analyze an applied problem of information transparency within the firm. By considering examples of optimal employee management and resource allocation I show that information interactions impact both environments through the channel of optimal incentive provision. This paper thus advances our understanding of information within the firm in a different direction relative to prior literature on dynamic contracts. Theoretically I solve a new class of dynamic contracts where information available to the agent is chosen optimally by the principal. By doing so I explore the limits of classic papers on dynamic contracts to show how to optimally choose information sets for the agent. Under specific assumptions I show the optimal dynamic way to pool agent's information sets to resolve the tension between benefits and costs of information disclosure.

1.1 Related Literature (in progress)

Harris and Raviv (1978) look at a one period incentive provision problem and show that if the agent is more informed about the performance measure before making the effort then he will be more costly to motivate. Holmström and Milgrom (1987) study a problem of repeated incentive provision. They point out that in a long term contract incentives will be cheaper when the agent does not observe intermediate performance signals. Intermediate outcomes make the agent more informed about the final performance measure impacting his incentives to make subsequent effort. These papers come to a conclusion that absent gains from the agent being more informed it is optimal to reveal information only at termination of the agency relationship. Lizzeri et al. (2002) studies a problem very similar to the one analyzed here. The authors study a two period contracting problem when the principal chooses when to reveal agent's performance at time 1. They show that when compensation is designed jointly with the information sharing rule it is optimal to always postpone disclosure to the final period. I step away from the papers above by considering a model where there are benefits of information sharing.

This paper is related to a large literature on dynamic contracting, the closest examples of which are DeMarzo and Fishman (2007), DeMarzo and Sannikov (2006). The primary focus of these papers is to understand optimal labor compensation and firm dynamics in the presence of agency conflicts. The conflict of interest stems from a risk neutral agent with limited liability making private effort decisions over time. In a similar environment I study optimal communication between the principal and the agent. I show that just by limiting communication between the principal and the agent principal's payoff can be largely improved upon.

There is a subclass of papers on dynamic contracts which explore economic environments where single period incentive compatibility constraints are not sufficient to provide incentives. Such papers include Tchistyi (2006), DeMarzo and Sannikov (2011), He (2012), Varas (2013). In these environments single period incentive compatibility constraints are insufficient for global incentive compatibility as agent's deviations in one period imply that he will wish to deviate again later. Thus even if a contract precludes deviations for one period global incentive compatibility may fail. In this paper I face a similar problem – when the agent does not observe intermediate performance, his off equilibrium beliefs will have an impact on subsequent actions causing the double-deviations problem. This issue is made yet more complicated because as information structures vary so will agent's optimal deviation profile. In short, for some information structures it may be optimal to preclude one period deviations, while others will require different constraints.

The key economic difference between this and a classic dynamic contracting paper is that I assume

the principal observes more information compared to the agent. For instance Baker (1992) notes that it is not a given that the contract will be based on agent's output, but rather some other performance measure, which will lead to incentive distortion. Another related literature is on subjective evaluations Baker et al. (1994), MacLeod (2003) and Fuchs (2007) where the principal privately observes the performance signal before revealing it to the agent. The key difference between my paper and the work on subjective evaluations is that the principal is able to disclose verifiable information to the agent. This commitment to truthful reporting of private information by the principal is similar to classic information disclosure models.

Benefits of information disclosure for technological purposes have been considered in prior contracting literatures. Lizzeri et al. (2002), Ray (2007) consider a model where benefits come from the agent learning his type and thus being put to best use within the firm. In Manso (2011) both principal and agent learn about the type of production technology and information serves as the point of optimal effort allocation.

The link between internal transparency and performance has been suggested in Hornstein and Zhao (2011), who relate transparency of multinational firms to the efficiency of their internal resource allocation. The mechanism of my paper is a different way to look at why poorly performing divisions engage in rent seeking and how optimal information management can be used to prevent this behavior. This way I address the questions raised in Scharfstein and Stein (2000) and Rajan et al. (2000).

2 Model

2.1 Firm and Production Technology

The firm consists of a principal and an agent. It operates a project over time $t = 1, 2, \dots, T$. Every period t the agent decides on effort a_t whether to work ($a_t = 0$) or to shirk ($a_t = 1$). Agent's private cost of working is e . Agent's effort is private and the principal does not observe it. Agent's effort at time t influences the distribution of a binary performance measure $x_t \in \{0, 1\}$ which corresponds to either failure or success of the project at time t . I assume that if agent works the probability of $x_t = 1$ is a constant p . If he shirks then $x_t = 1$ with some, possibly random, probability $\tilde{p}_t \leq p$.¹ The agent knows his effort a_t , but the realization of the performance measure x_t is not directly observed by him, as explained next. If agent shirks, then he privately observes probability of success $\tilde{p}_t$. Agent's decision tree at time t is depicted in Figure 1.

¹ $\tilde{p}_t$ is a random variable independent of past history with the property $\tilde{p}_t \leq p$ for all possible realizations.

Figure 1: Agent's privately observes distribution p_t but not the actual performance measure x_t

2.2 Role of Information Disclosure

Production technology is characterized every period t by the performance measure x_t and state $s_t \in S$.

Figure 2: Performance Measure and Technology State Outcomes

When the performance measure is $x_t = 1$, then the set of possible project states is S_1 while if the performance measure is $x_t = 0$, then the set of possible states is S_0 . Conditional on x_t project state s_t is independent of effort and time. I assume that S_0 and S_1 are subsets of $\mathbb{R}$ of a positive measure. Denote by $G_i(s)$ the distribution of signal $s \in S_i$ conditional on $x = i$. I assume s are distributed continuously with a strictly positive density $g_i(s) > 0$ for all $s \in S_i$.

Profits from the production technology come in two forms. If the agent is uninformed at time t about s_t then the output is perfectly correlated with the performance measure. If the agent is informed at time t about the state of the project s_t , then he will adjust his subsequent actions leading to an additional present value profit of $r(s_t) \geq 0$ which is a function of the current state. I assume that the latter is not related to the agency problem and hence the principal need only take into account the present value of these cash flows. Period t profits are thus given by

$$y_t = \begin{cases} m \cdot x_t + r(s_t) & \text{if agent observes } s_t \text{ at time } t \\ m \cdot x_t & \text{if agent does not observe } s_t \text{ at time } t \end{cases}$$

Information disclosure by the principal can be described by a process $d_t \in \{0, 1\}$ where the principal decides whether or not to give agent access to the state of the project s_t at time t .²

²Since the principal does not benefit from disclosing the actual performance measure x_t it is without loss of generality that I focus on disclosure of s_t

Under this notation

$$y_t = m \cdot x_t + r(s_t) \cdot \mathbb{I}(d_t(s_t) = 1)$$

Timing of events within each period is described in Figure 3.

Figure 3: Timeline of Actions every period

I assume that the principal's decision to share information is binary - he will either reveal the information contained in signal s_t or will not reveal anything.

2.3 Preferences

Principal and agent are risk neutral and discount cash flows at common rate β . Denote by c_t the transfer made from the principal to the agent at time t . If the agent's effort process is a , then his expected utility at $t = 0$ is

$$W_0 = E_{a,d} \left[\sum_{t=1}^T \beta^t \cdot (c_t - a_t \cdot e) \right]$$

where the expectation operator is taken with respect to the probability distribution corresponding to agent's effort a . Principal's utility is given by

$$U_0 = E_{a,d} \left[\sum_{t=1}^T \beta^t \cdot (y_t - c_t) \right] \quad (1)$$

2.4 Contract Design Problem

The firm is a contract between a principal and an agent. The contract specifies disclosure rule $d = \{d_t\}_{t=1}^T$, compensation rule $c = \{c_t\}_{t=1}^T$ and recommended actions $a = \{a_t\}_{t=1}^T$ which satisfy

$$a \in \arg \max_{\hat{a}, d} E_{\hat{a}} \left[\sum_{t=1}^T \beta^t \cdot (c_t - \hat{a}_t \cdot e) \right] \quad (2)$$

A contract is a triplet $\mathcal{C} = (c, d, a)$. I assume that disclosure rule D , compensation C and states of the project S are verifiable by a third party. The novel part of this model is that the principal jointly determines c and d . (in part of the paper I assume parameters are such that $a = 1$ in every

period is optimal). Principal designs contract $\mathcal{C}$ to maximize (1)

$$\max_{\mathcal{C}} E_{a,d} \left[\sum_{t=1}^T \beta^t \cdot (y_t - c_t) \right]$$

subject to (2).

3 Two-Period Example

If agent's effort was contractable, then optimal information sharing would be complete transparency. When there are conflicts of interest however, complete transparency may not be optimal. I show this first in a simple two period model presented in this section. It is very similar to Lizzeri et al. (2002) and is used as an illustration of the incentive cost of disclosing information to the agent in a performance sensitive contract. Incentive provision is cheaper when the agent does not observe his outcomes. However when benefits of information sharing are sufficiently large, they may outweigh the incremental agency costs. Consider an example where $T = 2$, $\beta = 1$, $s_t = x_t$. I assume there are productive gains when the principal corrects the agent if he has failed in the first period $r(0) = r$, $r(1) = 0$.³ I show that full disclosure is preferred to no disclosure only when r is sufficiently large.

3.1 Full Disclosure Contract

In a full information contract the distribution of $\tilde{p}$ does not matter and hence denote $p_L = E[\tilde{p}]$. For a contract revealing s_1 to the agent at $t = 1$ the optimal compensation is given by transfers $\{c_{00}^d, c_{01}^d, c_{10}^d, c_{11}^d\}$. Agent's decision tree under this information structure and given these transfers is:

Figure 4: Full information. Agent distinguishes $x_1 = 0$ and $x_1 = 1$

³This is very much in the spirit of Manso (2011) where the first action corresponds to experimentation.

In order for the agent to find it incentive compatible to implement effort in both periods the following incentive compatibility conditions must be satisfied:

$$\begin{aligned}
pc_{11}^d + (1-p)c_{10}^d - e &\geq p_L c_{11}^d + (1-p_L)c_{10}^d \\
pc_{01}^d + (1-p)c_{00}^d - e &\geq p_L c_{01}^d + (1-p_L)c_{00}^d \\
p(pc_{11}^d + (1-p)c_{10}^d) + (1-p)(pc_{01}^d + (1-p)c_{00}^d) - e &\geq p_L(pc_{11}^d + (1-p)c_{10}^d) + (1-p_L)(pc_{01}^d + (1-p)c_{00}^d)
\end{aligned}$$

Under an optimal contract all three are binding and the transfers can be expressed as

$$c_{00}^d = 0 \quad c_{10}^d = c_{01}^d = \frac{e}{p - p_L} \quad c_{11}^d = \frac{2 \cdot e}{p - p_L}$$

Agent's information rent is

$$W_F = \frac{2p_L}{p - p_L} \cdot e$$

and principal's profits from the contract is

$$U_F = 2pm + 2r - \frac{2pe}{p - p_L} \quad (3)$$

3.2 No Disclosure

A contract not revealing s_1 at $t = 1$ is also given by transfers $\{c_{00}^n, c_{01}^n, c_{10}^n, c_{11}^n\}$. Agent's decision tree given such an information structure and transfers is:

Figure 5: No Information. Agent does not distinguish nodes $x_1 = 0$ and $x_1 = 1$.

It is easy to see that there exists an optimal contract which compensates the agent only in state $x_1 = x_2 = 1$. Agent's incentive compatibility constraint in period one is

$$p^2 \cdot c_{11}^n - 2e \geq E \left[\max \left(\tilde{p}_1 p \cdot c_{11}^n - e, \tilde{p}_1 p_L \cdot c_{11}^n \right) \right] \quad (4)$$

Incentive constraint (4) incorporates the possibility of a one step deviation and is hence always stricter than incentive compatibility at $t = 2$ conditional on working at $t = 1$. Thus (4) is the necessary and sufficient constraint for global incentive compatibility. Consider two illustrative examples.

1. Suppose $\tilde{p}_t \equiv p_L$. Then (4) becomes

$$p^2 \cdot c_{11}^n - 2e \geq p_L^2 \cdot c_{11}^n$$

and optimal transfer is

$$c_{11}^n = \frac{2e}{p^2 - p_L^2}$$

Agent's information rent W_N^1 and principal's profits U_N^1 are

$$\begin{aligned} W_N^1 &= \frac{2p_L^2 e}{p^2 - p_L^2} \\ U_N^1 &= 2pm - \frac{2p^2 e}{p^2 - p_L^2} \end{aligned}$$

Comparing these profits with (3) the principal finds it optimal to give information to the agent as long as

$$r \cdot (1 - p) \geq \frac{2pe}{p - p_L} - \frac{2p^2 e}{p^2 - p_L^2} = \frac{2pe}{(p - p_L)} \cdot \left(1 - \frac{p}{p + p_L}\right) = \frac{2pp_L e}{p^2 - p_L^2}$$

2. Now suppose $\tilde{p}$ conveys more information about agent's subsequent performance:

$$\tilde{p} = \begin{cases} 0 & \text{with probability } q \\ p & \text{with probability } 1 - q \end{cases}$$

Incentive constraint (4) becomes

$$p^2 \cdot c_{11}^n - 2e = (1 - q) \cdot (p^2 c_{11}^n - e)$$

and after some algebra

$$c_{11}^n = \frac{1 + q}{qp^2} \cdot e$$

Because under this information structure the agent observes more about his performance his expected compensation has to be greater than in the previous scenario. Under this

information structure $p_L = E[\tilde{p}] = p(1 - q)$. Agent's information rent is then given by

$$W_N^2 = \frac{p_L}{p} \cdot e$$

and principal's profits are

$$U_N^2 = 2pm - 2e - \frac{p_L e}{p - p_L}$$

Comparing again with (3) principal finds it profitable to disclose information if

$$r \geq \frac{p_L e}{p - p_L}$$

Note that the necessary profitability r is lower here compared to the previous example (keeping the same p_L). This is due to the fact that when $\tilde{p}$ is more informative of the agent's performance measure, the principal does not benefit as much from hiding information.

4 General Properties of the Optimal Contract

Since both principal and agent are risk neutral and equally patient, without loss of generality I can restrict attention to transfers which only occur at time T , meaning that $c_1 = \dots = c_{T-1} = 0$. Three histories are associated with this agency relationship governed by contract $\mathcal{C}$. Public history contains contract terms, disclosure outcomes and information disclosed

$$h_t = \left\{ a_1, \dots, a_t, d_1, \dots, d_t, s_1 \cdot \mathbb{I}(d_1 = 1), \dots, s_t \cdot \mathbb{I}(d_t = 1) \right\}$$

Agent's private history consists of the public history, his private effort decisions and private probabilities of success that he observes

$$h_t^a = h_t \cup \left\{ \hat{a}_1, \dots, \hat{a}_t, \hat{p}_1, \dots, \hat{p}_t \right\}$$

Principal's private history is the public history and all of the project relevant information that has occurred up to time t

$$h_t^p = h_t \cup \left\{ s_1, \dots, s_t, x_1, \dots, x_t \right\}$$

Note that histories do not involve the agent reporting the probability outcomes that he privately observes. The reason is that in contract $\mathcal{C}$ if the agent was meant to work $a_t = 1$ then he will only observe a constant p and hence there is nothing to report. On the other hand as the lemma below illustrates, when the agent is recommended to shirk the contract will not depend on anything that

has happened in that period.

Lemma 1. *Suppose the agent's recommended action profile is $a_{t+1} = 0$ after histories (h_t, h_t^a, h_t^p) . Then there exists an optimal contract in which agent's compensation and subsequent information sharing decisions will not depend on either x_t or $\tilde{p}_t$. Moreover, all project relevant information s will be disclosed to the agent.*

This lemma restricts our attention to contracts where the incentive scheme only depends on the periods when the agent provides effort. Given the lack of recursive structure of the problem this trivial observation considerably simplifies subsequent analysis.

4.1 Public Contracts

For a general contract $\mathcal{C}$ disclosure rule at time t is given by d_t and is measurable with respect to principal's private history h_t^p . This allows general disclosure rules that are not measurable with respect to the public history h_t . This implies that d_t may be informative about agent's performance prior to t . In this case then the agent may have an additional channel through which he can learn about his performance. However the only benefit of information to the principal comes from disclosing current information. I show that it is without loss of generality to avoid this class of contracts altogether and focus on disclosure decisions which are only informative about current performance and do not reveal information about past performance.

Definition. Disclosure decision d_t is public if it is measurable with respect to public history h_{t-1} , project state s_t , agent's performance x_t and a public randomization device.⁴

Definition. Contract $\mathcal{C}$ is public if the corresponding disclosure rule $d = \{d_t\}_{t=1}^T$ consists of public disclosure decisions.

Lemma 2. *Suppose upon disclosure of information at time t the agent places positive probability that $x_t = 1$. Then if disclosure decisions from time t onwards are public then there exists an alternative contract $\tilde{\mathcal{C}} = (\tilde{c}, d, a)$ achieving the same payoffs to both principal and agent for which compensation $\tilde{c}_T$ will be 0 if $x_t = 0$.*

Both participants are risk neutral and hence if the contract prescribes the steepest punishment to the agent at termination it will only improve incentives in prior periods. Lemma 2 simplifies the proof of the next proposition and is used in the subsequent section where incentive compatibility conditions will be explicitly derived for a specific form of $\tilde{p}$.

⁴A public randomization device at time t is some random variable ξ_t independent of the past private and public histories. It may be potentially needed to resolve discreteness problems in this environment similar to Clementi and Hopenhayn (2002) and DeMarzo and Fishman (2007).

Figure 6: Definition of a Public Contract

Assumption 1. *Either $S_0 \subseteq S_1$ or $P(r(s_t) > 0 \mid x_t = 1) < 1$.*

Proposition 1. *For every contract $\mathcal{C}$ there exists a public contract $\mathcal{C}^p$ achieving the same ex-ante payoffs to both principal and agent. If Assumption 1 is satisfied the optimal contract exists.*

From now on I restrict the search for an optimal contract to the space of public contracts. Given Proposition 1 this is without loss of generality.

5 Incentive Provision

Suppose after following the recommended effort profile a in periods $1, \dots, t$ the agent chooses to deviate to a different effort level at time $t+1$. After doing so starting from period $t+2$ onwards the agent will, in general, not follow the recommended effort profile afterwards either – one deviation may cause more subsequent deviations. Denote agent's continuation value in the contract $\mathcal{C}$ given public history h_t if he follows the recommended effort profile a :

$$w_t = E_a \left[\beta^{T-t} \cdot c_T - \sum_{s=t+1}^T \beta^{s-t} \cdot a_s \cdot e \mid h_t \right]$$

This value has to be greater than that if the agent chooses to deviate to an alternative effort profile. Denote:

$$\hat{w}_t = \max_{\hat{a} \in A_t} E_{\hat{a}} \left[\beta^{T-t} \cdot c_T - \sum_{s=t+1}^T \beta^{s-t} \cdot \hat{a}_s \cdot e \mid h_t \right]$$

where

$$A_t = \{\hat{a} : \text{ s.t. } \hat{a}_s = a_s \text{ for } s = 1, \dots, t-1 \text{ and } \hat{a}_{t+1} \neq a_{t+1}\}$$

Effort a is incentive compatible if and only if $w_t \geq \hat{w}_t$ after every public history h_t . This simply states that adhering to the recommended effort level every period is better than deviating and then following some other effort pattern in the future.

For complete information contracts it is typically the case that for an incentive compatible contract if the agent deviates once, he would not find it optimal to deviate again. This is known as checking the contract for one shot deviations by the agent (Rogerson (1985), Sannikov (2008)). As can be seen from the two period example in Section 3 the optimal deviation of the agent will depend on the information structure implemented by the contract.

There is no known general solution to the problem of multiple deviations and it has been addressed differently in the literature.⁵ In this paper by imposing a specific signal structure $\tilde{p}$ it is possible to characterize agent's optimal deviation profile under any optimal contract.

Assumption 2. *Upon shirking the agent observes the realized probability of success/failure*

$$\tilde{p}_t = \begin{cases} 0, & \text{with probability } q \\ p, & \text{with probability } 1 - q \end{cases}$$

When the agent shirks and observes $\tilde{p}_t = 0$ he knows that the outcome of the performance measure is $x_t = 0$. However when $\tilde{p}_t = p$ the agent has the same belief over the distribution of the performance measure as if he had worked in period t . Assumption 2 has little effect in a comparison with a full information contract, but will allow for sufficient tractability to overcome the double deviations problem outlined above while retaining the intuition about pooling agent's incentive constraints.

Assumption 2 has a very realistic interpretation. We can think of the distribution p_t as the non verifiable output produced by the agent. If the agent works and his "output" is high ($p_t = p$) then the performance measure is high with probability p and low otherwise. However when the agent shirks, with some probability he produces output $\tilde{p}_t = p$ and with some probability he doesn't. In the latter case the performance measure x_t of the nonexistence output is assumed to be 0.

⁵See for example Tchisty (2006), DeMarzo and Sannikov (2011), He (2012).

5.1 Optimal Disclosure of Positive News

Proposition 2. *Under an optimal contract every $s \in S_1$ such that $r(s) > 0$ is disclosed at time t . Agent's expected compensation will be the same as under an optimal contract which implements not revealing any outcomes from S_1 .*

When performance is publicly observed and the principal wants the agent to incentivize the agent to work a contract implementing such effort will pay the agent something upon failure (otherwise an agent who knows that he will not be compensated will not work). The expected compensation given observed failure will be less than if success was observed, but it is still greater than 0. When the agent does not observe his failure, however, the principal can pay him 0 conditional on a failure which will be revealed in the final period. The incentives to work in subsequent periods will be coming from compensation of the agent conditional on past success. Thus in expectation even if the agent has failed he still has positive value in the contract, despite the fact that conditional on failure it is 0. This is the quintessential idea behind pooling agent's incentive compatibility constraints. When the agent is uncertain about his performance the optimal contract's incentive compatibility constraint will be slack conditional on good performance and will not be satisfied conditional on bad performance. Thus when the agent does not know he has failed in a period the principal can give him very efficient incentives by only rewarding him for success even when there are subsequent effort decisions to be made by the agent.

Proposition 2 generalizes this intuition to show that not all information sharing leads to an increase in required compensation relative to no information. When information $s_t \in S_1$ is disclosed it changes agent's beliefs about his performance in period t . But since $g_1(s_t) > 0$ information s_t can be attributed with positive probability to $x_t = 1$. To be able to reuse incentives a contract can promise the agent a possibly large transfer in the event $x_t = 1$ and still pay him 0 in the final period if $x_t = 0$. Thus even when disclosed information changes agent's beliefs about past performance it does not have to lead to increased agency costs as long as this information can be attributed to good performance. The changes in agent's beliefs in this case are countered by optimally choosing corresponding transfers c .

The logic above is true for states belonging to S_1 . What happens when s_t is not disclosed at time t ? Under Assumption 1 there exists an optimal contract which will not reveal states $s \in S_1$ for which $r(s) = 0$. This means that when the state is not disclosed at time t it could either mean that $s_t \in S_0 \setminus S_1$ or that $r(s_t) = 0$. Thus nondisclosure could also be attributed to positive performance and the mechanism outlined above will still apply.

5.2 Incentive Compatibility and Promise Keeping Constraints

When $s_t \in S_0 \setminus S_1$ is disclosed, then it automatically reveals $x_{t+1} = 0$ with probability 1. As noted earlier signal s_{t+1} is independent of agent's performance conditional on $x_{t+1} = 0$. Hence without loss of generality I can restrict attention to contracts which do not depend on the specific signal realization from $S_0 \setminus S_1$, but only on whether or not this signal has been disclosed. Denote agent's continuation value in the contract $\mathcal{C}$ given public history h_t

$$w_t = E_a \left[\beta^{T-t} \cdot c_T - \sum_{s=t+1}^T \beta^{s-t} \cdot a_s \cdot e \mid h_t \right]$$

Similarly define period $t + 1$ continuation values as functions of the states revealed to the agent at $t + 1$

$$\begin{aligned} w_{t+1}^s &= E_a \left[\beta^{T-t-1} \cdot c_T - \sum_{k=t+2}^T \beta^{k-t-1} \cdot a_k \cdot e \mid h_t, d_{t+1} = 1, s \right] \\ w_{t+1}^N &= E_a \left[\beta^{T-t-1} \cdot c_T - \sum_{k=t+2}^T \beta^{k-t-1} \cdot a_k \cdot e \mid h_t, d_{t+1} = 0 \right] \end{aligned}$$

From the Law of Iterated expectation it follows that

$$w_t = \beta \cdot \left[\int_{d_{t+1}(s)=1} w_{t+1}^s \cdot P_t(s \mid d_{t+1}(s) = 1, a_{t+1} = 1) ds + P_t(d_{t+1} = 0) \cdot w_{t+1}^N - e \right]$$

which is called the Promise Keeping constraint.

Corollary 1. *From Proposition 2 it follows that there exists an optimal contract for which*

$$w_{t+1}^s = w_{t+1}^N, \quad s \in S_1$$

This corollary follows from the proof of Proposition 2. Promise Keeping constraint can now be simplified to

$$w_t = \beta \cdot \left((1-p) \cdot \int_{d_{t+1}(s)=1} w_{t+1}^s \cdot g_0(s) ds + (1 - P_t(d_{t+1}(s) = 1, s \in S_0 \setminus S_1)) \cdot w_{t+1}^N - e \right)$$

I denote w_{t+1}^0 to be the single continuation value of the agent conditional on bad performance

being revealed. Formally:

$$w_{t+1}^0 = w_{t+1}^s \quad \text{for } s_{t+1} \in S_0 \setminus S_1 \quad \text{and} \quad d_{t+1} = 1$$

Probability of receiving negative feedback in period $t + 1$ given public history h_t is

$$f_t^0 = \mathbb{P}(d_{t+1} = 1, s_{t+1} \in S_0 \setminus S_1 \mid h_t)$$

This further simplifies the promise keeping constraint to

$$w_t = \beta \cdot \left[f_t^0 \cdot w_{t+1}^0 + (1 - f_t^0) \cdot w_{t+1}^N - a_{t+1} \cdot e \right]$$

Proposition 3 presents the necessary and sufficient conditions for the dynamics of agent's continuation value under which he finds it optimal to work. This condition uses both the results obtained for public contracts and the specific signal structure of $\tilde{p}$ given by Assumption 2.

Proposition 3. *Public contract $\mathcal{C}$ implements effort $a_{t+1} = 1$ at time t if and only if*

$$f_t^0 \cdot w_{t+1}^0 \leq (1 - p) \cdot \left(\frac{w_t}{\beta} - \frac{1 - q}{q} \cdot e \right) \quad (5)$$

Proof. Denote by $\hat{E}$ the expectation operator under the assumption that the agent takes the optimal subsequent effort profile provided the information in the expectation operator. Then

$$\begin{aligned} \hat{E}_t[w_{t+1} \mid p_{t+1} = \tilde{p}_{t+1}] &\leq \mathbb{E}_t[w_{t+1} \mid p_{t+1} = p] - e \\ (1 - q) \cdot \mathbb{E}_t[w_{t+1} \mid \tilde{p}_{t+1} = p] + q \cdot \hat{E}_t[w_{t+1} \mid \tilde{p}_{t+1} = 0] &\leq \mathbb{E}_t[w_{t+1} \mid p_{t+1} = p] - e \\ \hat{E}_t[w_{t+1} \mid \tilde{p}_{t+1} = 0] &\leq \mathbb{E}_t[w_{t+1} \mid \tilde{p}_{t+1} = p] - \frac{e}{q} \end{aligned}$$

Under the optimal public contract

$$\begin{aligned} \hat{E}_t[w_{t+1} \mid \tilde{p}_{t+1} = 0] &= \hat{E}_t[w_{t+1} \mid x_{t+1} = 0] \\ &= \mathbb{E}_t[w_{t+1} \mid x_{t+1} = 0, d_{t+1} = 1, s_{t+1} \in S_0 \setminus S_1] \cdot \mathbb{P}_t(d_{t+1} = 1, s_{t+1} \in S_0 \setminus S_1 \mid x_{t+1} = 0) \\ &\quad + \hat{E}_t[w_{t+1} \mid x_{t+1} = 0, d_{t+1} = 1, s_{t+1} \in S_1] \cdot \mathbb{P}_t(d_{t+1} = 1, s_{t+1} \in S_1 \mid x_{t+1} = 0) \\ &\quad + \hat{E}_t[w_{t+1} \mid x_{t+1} = 0, d_{t+1} = 0] \cdot \mathbb{P}(d_{t+1} = 0 \mid x_{t+1} = 0) \end{aligned}$$

By definition of f_t^0 we derive

$$\begin{aligned} P_t(d_{t+1} = 1, s_{t+1} \in S_0 \setminus S_1 | x_{t+1} = 0) &= \frac{P_t(d_{t+1} = 1, s_{t+1} \in S_0 \setminus S_1, x_{t+1} = 0)}{P_t(x_{t+1} = 0)} \\ &= \frac{P_t(d_{t+1} = 1, s_{t+1} \in S_0 \setminus S_1,)}{P_t(x_{t+1} = 0)} \\ &= \frac{f_t^0}{1-p} \end{aligned}$$

Also note that if the agent has been unsuccessful but was not informed of this, then he will get 0 compensation at time T which he knows because he has privately observed $\tilde{p}_{t+1} = 0$. In this case it is optimal for him to exert no effort which means that

$$\hat{E}_t[w_{t+1} | x_{t+1} = 0, d_{t+1} = 1, s_{t+1} \in S_1] = \hat{E}_t[w_{t+1} | x_{t+1} = 0, d_{t+1} = 0] = 0$$

Hence

$$\hat{E}_t[w_{t+1} | \tilde{p}_{t+1} = 0] = \frac{f_t^0}{1-p} w_{t+1}^0$$

Using the promise keeping constraint

$$w_t = \beta \cdot \left[E_t[w_{t+1} | p_{t+1} = p] - e \right]$$

the I.C. constraint becomes

$$\frac{f_t^0}{1-p} \cdot w_{t+1}^0 \leq \frac{w_t}{\beta} - \frac{1-q}{q} \cdot e$$

□

When the agent is incentivized to provide effort $a_{t+1} = 1$ the promise keeping and incentive compatibility conditions can be jointly written as

$$\begin{cases} \frac{f_t^0}{1-p} \cdot w_{t+1}^0 \leq \frac{w_t}{\beta} - \frac{1-q}{q} \cdot e \\ w_t = \beta \cdot \left[f_t^0 \cdot w_{t+1}^0 + (1-f_t^0) \cdot w_{t+1}^N - e \right] \end{cases}$$

Constraint (5) imposes limitations on the ability of the principal to motivate the agent to exert effort. Agent's limited liability implies $w_{t+1}^0 \geq 0$ and hence to incentivize $a_{t+1} = 1$ agent's time t continuation value must satisfy

$$w_t \geq \beta \cdot \frac{1-q}{q} \cdot e$$

6 Optimal Information Disclosure

Assumption 3. *Value m is sufficiently large so that it is optimal to incentivize the agent to provide effort $a_t = 1$ after all histories.*

It is possible to choose m sufficiently large so that the benefit of information disclosure given by $r(\cdot)$ does not justify not providing the agent with incentives for any amount of time. The simple idea behind Assumption 3 is that no information is sufficiently valuable to outweigh premature termination of the agent.⁶

Denote by π_t the agent's on equilibrium belief that $x_k = 1$ for all $k \leq t$ for which outcome $s_k \in S_0 \setminus S_1$ was not disclosed. The process follows a Bayesian updating rule:

$$\pi_{t+1} = \begin{cases} \pi_t \cdot \text{P}(x_{t+1} = 1 \mid d_{t+1} = 0), & \text{if } d_{t+1} = 0 \\ \pi_t \cdot \text{P}(x_{t+1} = 1 \mid s_{t+1} \in S_1), & \text{if } d_{t+1} = 1, s_{t+1} \in S_1 \\ \pi_t, & \text{if } d_{t+1} = 1, s_{t+1} \in S_0 \setminus S_1 \end{cases}$$

Lemma 3. *Under an optimal public contract period T expected transfers after history h_T are given by $\frac{w_T}{\pi_T}$. Agent's expected compensation is w_T .*

History h_t can be summarized by a pair (w_t, π_t) . Without loss, in the search for an optimal contract I restrict attention to contracts which are Markov in agent's promised utility and time. Final period transfers will depend on belief π_T at time T , however due to risk neutrality this does not have an effect on expected compensation

While we began with searching for an optimal contract with a single time horizon T it will be useful to characterize contracts for all maturities $t \leq T$. I will show that the payoffs between these contracts over time will be linked via dynamic programming. Define

$$b_t(w) = \max_{\hat{c}_t} \text{E} \left[\sum_{k=1}^t \beta^k \cdot y_k - \beta^t \cdot \hat{c}_t \right]$$

subject to

$$\begin{cases} \bar{a}^t \in \arg \max_{\hat{a}} \text{E}_{\hat{a}} \left[\beta^t \cdot \hat{c}_t - \sum_{k=1}^t \beta^k \cdot \hat{a}_k \cdot e \right] \\ w = \text{E} \left[\beta^t \cdot \hat{c}_t - \sum_{k=1}^t \beta^k \cdot e \right] \end{cases}$$

⁶If information is such that the principal benefits on it only if the agent works in the next period then complete effort contract will always be optimal as long as $pm \geq e^{\frac{1-q}{q}}$.

and given Proposition 2 functions $b_t(w)$ are well defined (an optimal contracts exist). For $t = 0$ there are no effort decisions to be made and hence principal's value function reflects the principal owing w to the agent. Thus $b_0(w) = -w$.

6.1 Dynamic Program

For notational convenience denote the principal's expected profits in period t that stem from agent's direct effort and disclosure of information in S_1

$$\hat{m} = m \cdot p + \int_{S_1} r(s) \cdot (pg_1(s) + (1-p)g_0(s)) ds$$

Without loss of generality set $S_0 \setminus S_1 = [0, 1]$ and $r(s)$ is weakly increasing in s on $[0, 1]$ and $g_0(s) \equiv g_0$ on $[0, 1]$. The disclosure of outcomes in $S_0 \setminus S_1$ follows a threshold rule each period

$$d_{t+1}(s_{t+1}) = \begin{cases} 1, & \text{if } r(s_{t+1}) \geq r_t^* \\ 0, & \text{if } r(s_{t+1}) < r_t^* \end{cases}$$

For notational convenience it is helpful to denote $l_t^0 = r^{-1}(r_t^*)$. Then given that $r(\cdot)$ is weakly increasing outcome $s \in [0, 1]$ will be disclosed only if $s \geq l_t^0$. Expected probability at time t of receiving feedback at time $t + 1$ is:

$$f_t^0 = (1-p) \cdot g_0 \cdot (1-l_t^0)$$

Given that I'm looking for an optimal contract which is Markov in time and agent's promised utility w_t the disclosure threshold will be a function of time and agent's continuation value $l_t^0 = l_t^0(w_t)$. The following lemma characterizes principal's value functions in a dynamic program and shows how to compute the disclosure thresholds numerically. Define functions $\{\hat{b}_t\}_{t=0}^\infty$ by induction:

$$\hat{b}_{t+1}(w) = \max_{l^0} \left[\hat{m} + (1-p) \cdot \int_{l^0}^1 r(s) \cdot g_0 ds + (1-p)g_0(1-l^0) \cdot \beta \hat{b}_t(w^0) + (1-(1-p)g_0(1-l^0)) \cdot \beta \hat{b}_t(w^N) \right] \quad (6)$$

subject to

$$\left\{ \begin{array}{ll} g_0(1 - l^0) \cdot w^0 = \frac{w}{\beta} - \frac{1 - q}{q} \cdot e & \text{I.C.} \\ \frac{w}{\beta} = (1 - p)g_0(1 - l) \cdot w^0 + (1 - (1 - p)g_0(1 - l)) \cdot w^N - e & \text{I.R.} \\ \frac{w^0}{\beta} \geq \frac{1 - q}{q} \cdot e, \quad \frac{w^N}{\beta} \geq \frac{1 - q}{q} \cdot e & \text{I.C. + Limited Liability} \end{array} \right.$$

The initial condition is given by $\hat{b}_0(w) = -w$.

Lemma 4. *If functions $\hat{b}_k$ are concave for $k = 1, \dots, T$, then $\hat{b}_T = b_t$.*

The proof is based on a standard backward induction argument and is omitted. Lemma 4 summarizes the previous steps of the analysis to arrive to the fact that the optimal contract can be solved for as a dynamic program. By computing functions $\hat{b}_t$ recursively it is possible to check numerically if they are concave so the concavity is testable. This lemma characterizes principal's objective function over time and allows us to compute the optimal information rule in terms of disclosure thresholds l_t^0 . The latter maximize the following expression:

$$\max_{l_t^0} \left[(1 - p) \cdot \int_{l_t^0}^1 r(s) \cdot g_0 ds + (1 - p)g_0(1 - l^0)\beta b_t(w^0) + (1 - (1 - p)g_0(1 - l^0)) \cdot \beta b_t(w^N) \right]$$

where

$$w^0 = \frac{\frac{w}{\beta} - \frac{1 - q}{q} \cdot e}{g_0(1 - l^0)}, \quad w^N = \frac{p \cdot \left(\frac{w}{\beta} - \frac{1 - q}{q} \cdot e \right) + \frac{e}{q}}{1 - (1 - p)g_0(1 - l^0)}$$

The unconstrained first order condition is given by

$$-\frac{r(l)}{\beta} + b_t(w^N) - b_t(w^0) - w^N \cdot b'_t(w^N) + w^0 \cdot b'_t(w^0) = 0 \quad (7)$$

Proposition 4. *For each w the disclosure threshold $l(t, w)$ satisfies $l(t, w) \geq \underline{l}(w)$ where the latter solves*

$$\frac{w}{e \cdot \beta} \cdot \frac{q}{1 - q} - 1 = \beta \cdot g_0 \cdot (1 - \underline{l}(w))$$

subject to $\underline{l}(w) \in [0, 1]$. Moreover $l(t, w) = \underline{l}(w)$ for w sufficiently close to $\beta \cdot e \cdot \frac{1 - q}{q}$.

Proposition above states the precise tension outlined in the introduction. When the agent needs to be incentivized to work and his promised utility is low then the principal has no choice but to

limit the amount of news disclosure. When w is small, then agency considerations are the only important factor in providing feedback – the principal will give as much information as possible as long as incentives next period can still be given.

6.2 Numerical Example 1

Now I can revisit the example of Section 3. Consider parameters $\hat{m} = 1, \beta = 8, e = 0.2, p = 0.5, q = 0.3$. Even though in Section 3 it was assumed that $s_t = x_t$ it is easy to map this into the environment of Section 6 by saying that $S_0 \cap S_1 = \emptyset$ and $r(s) = r \cdot \mathbb{I}\{s \in [0, 1]\}$.

Figure 7: Benefit of revealing bad information $r(0) = 1$ and $T = 2$.

Principal's value function in the two period contract turns out to be linear and the kink corresponds to the point of full disclosure. The intuition why is clear – after the kink the value function is decreasing with the slope of one which means that any increase in promised utility does not have any productive benefits. Figure 7 corresponds to $r = 1$ which turns out to be sufficiently large that the principal finds it optimal to implement full information.

When r is small, as illustrated in by Figure 8 the principal would find it optimal to not disclose intermediate signals to the agent. In this two period environment with constant returns on information disclosure it turns out to be optimal to either implement full opacity or full transparency

6.3 Numerical Example 2

Now consider a different example where signal profitability varies over time. Suppose $S_0 \cap S_1 = \emptyset$, $r(s) = s$ for $s \in [0, 1]$, $g_0 = 1$, and $\hat{m} = 1, \beta = 0.8, e = 0.2$. Concavity of functions $\hat{b}$ is checked

Figure 8: Benefit of revealing bad information $r(0) = 0.1$ and $T = 2$.

Figure 9: Benefit of revealing bad information $r(0) = 1$ and $T = 10$.

numerically and if it holds, then principal's value function solves the Bellman equation

$$b_{t+1}(w) = \max_l \left[\hat{m} + (1-p) \cdot \frac{1-l^2}{2} + (1-p)(1-l) \cdot \beta b_t(w^0) + (1 - (1-p)(1-l)) \cdot \beta b_t(w^N) \right]$$

Suppose $T = 2$. The value function is computed numerically and depicted on Figure 10.

In a two period model optimal contract will always implement have $l_2^0(w) = \underline{l}(w)$. To plot thresholds for disclosure I will plot $r_t^* = r(l_t^0)$ for disclosure of news from $S_0 \setminus S_1$. For expositional purposes I define $r(l_t^1)$ to be the threshold for disclosure of information from S_1 which is 0 according to Proposition 2. The optimal information sharing rule as a function of promised utility of the

Figure 10: Principal's value function, $T = 2$.

Figure 11: Optimal disclosure thresholds, $T = 2$. Left: threshold for disclosure of negative information. Right: threshold for disclosure of positive information.

agent is depicted in Figure 11. As w increases the threshold for news disclosure decreases because the agency problem becomes less binding.

Given that the problem is of finite horizon dynamic programming, the solution will not be stationary in time. For example for $T = 10$ principal's value function and optimal disclosure thresholds are given by Figures 12 and 13.

While there is nothing special about the plot of the value function it is interesting to contrast Figures 11 and 13. While both are decreasing as w increases, when there are many periods remaining the principal chooses to disclose less negative information in the early periods for intermediate ranges of w . This is due to the fact that the principal does not wish to give the agent negative

Figure 12: Value Function for $T = 10$

Figure 13: Optimal disclosure thresholds, $T = 10$. Left: threshold for disclosure of negative information. Right: threshold for disclosure of positive information.

feedback when the profit of it is not sufficiently large. In other words when there are many periods the optimal level of feedback may sometimes be the interior solution given by (7). This leads to the question – what happens when the time horizon becomes long and the “end” effects are no longer present?

6.4 Long Time Horizon

In this section I note that when T becomes large there exists a “stationary” solution to the agency problem in the sense of Proposition 5.

Proposition 5. *There exists a unique function $b(\cdot)$ such that $b(w) + w$ is bounded, which solves*

$$b(w) = \max_{l^0} \left[\hat{m} + (1-p) \cdot \int_{l^0}^1 r(s) \cdot g_0 ds + (1-p)g_0(1-l^0) \cdot \beta b(w^0) + (1-(1-p)g_0(1-l^0)) \cdot \beta b(w^N) \right]$$

for $t = 0, \dots$, subject to

$$\left\{ \begin{array}{ll} g_0(1-l^0) \cdot w^0 = \frac{w}{\beta} - \frac{1-q}{q} \cdot e & I.C. \\ \frac{w}{\beta} = (1-p)g_0(1-l) \cdot w^0 + (1-(1-p)g_0(1-l)) \cdot w^N - e & I.R. \\ \frac{w^0}{\beta} \geq \frac{1-q}{q} \cdot e, \quad \frac{w^N}{\beta} \geq \frac{1-q}{q} \cdot e & I.C. + \text{Limited Liability} \end{array} \right.$$

If condition in Lemma 4 is satisfied for all T then $b_t \rightarrow b$ as $t \rightarrow \infty$ uniformly in w .

For the same parameters as the numerical example from Section 6.3 I compute the stationary solution $b(\cdot)$. This value is illustrated in Figure 14.

Figure 14: $T = \infty$, value function

Optimal stationary disclosure threshold in this case solves the stationary problem

$$\max_{l^0} \left[(1-p) \cdot \int_{l^0}^1 r(s) \cdot g_0(s) ds + (1-p)g_0(1-l^0)\beta b(w^0) + (1-(1-p)g_0(1-l^0)) \cdot \beta b(w^N) \right]$$

subject to $w^0 \geq \beta \cdot e \cdot \frac{1-q}{q}$ and $w^N \geq \beta \cdot e \cdot \frac{1-q}{q}$, where

$$w^0 = \frac{\frac{w}{\beta} - \frac{1-q}{q} \cdot e}{g_0(1-l^0)}$$

$$w^N = \frac{p \left(\frac{w}{\beta} - \frac{1-q}{q} \cdot e \right) + \frac{e}{q}}{1 - (1-p)g_0(1-l^0)}$$

I solve this numerically for $l^0 = l^0(w)$ and plot the optimal threshold $r(l^0)$ in 15.

Figure 15: Optimal disclosure thresholds, $T = \infty$. Left: threshold for disclosure of negative information. Right: threshold for disclosure of positive information.

Figure 15 illustrates that when w is small only the most profitable information will be disclosed. As w is increasing this threshold is declining as the agency costs of information sharing are decreasing. This intuition is summarized in Lemma 5.

Lemma 5. *For long time horizons optimal disclosure threshold l leads to optimal promised utility of the agent to satisfy $w^0 \leq w \leq w^N$ implying that*

$$b(w^0) + w^0 \leq b(w) + w < b(w^N) + w^N$$

Moreover this inequality is strict whenever $w > \beta \cdot e \cdot \frac{1-q}{q}$.

Lemma 5 states that after good performance the value of future information sharing will be greater than when bad performance is disclosed. In Figure 16 I plot the changes in agent's promised utility upon receiving negative news vs not receiving news or receiving positive news.

Figure 16: Evolution of agent's promised utility

6.5 Commitment to the Communication Rule

In many settings where communication plays a role, the principal privately observes agent's performance measure. The assumption of commitment to a disclosure rule is thus important.

Proposition 6. *Suppose the principal can commit to a compensation plan as a function of processes $\{x_t, s_t\}_{t=1}^T$, but cannot commit to when to release information. The optimal contract remains the same.*

This implies that compensation, or “career path”, of the agent serves as the ultimate commitment device.

7 Interpreting the Results

7.1 Disclosure of Positive Information

Information that can be attributed to good performance will be shared as long as it is profitable. This challenges common intuition about information disclosure – when you disclose positive information, then upon non-disclosure the agent will believe that he has likely done poorly. Proposition 2 shows that it is possible to design transfers in a way to mitigate this information friction. Thus disclosure of positive information does not introduce additional costs coming from the agency problem. This means that good performance is disclosed more frequently than bad performance. This is a different mechanism to understand the leniency bias outlined in Murphy and Cleveland (1995) and is similar to one proposed by Prendergast (1999).

Proposition 2 also shows that, absent information disclosure benefits, opacity is not uniquely optimal. When there is a small benefit of communication, information that can be attributed to good performance will be shared. In this way a contract involving communication is more robust to the value of information compared to a contract which simply implements opacity.

7.2 Disclosure of Negative Information

The optimal contract also specifies how negative performance will be communicated to the agent. As shown by Proposition 4 when agent's continuation value in the firm drops sufficiently low there is a conflict between disclosing bad performance and providing incentives to work in subsequent periods. As shown by Lemma 5 for low promised value the contract will disclose maximum amount of information possible as to not violate the lower bound on agent's continuation value required for him to work in the next period. Optimal information disclosure specifies thresholds on profitability

of negative information as a function of agent's promised utility w in that period. When w is small only the most profitable information related to bad performance will be disclosed.

The effect and importance of internal communication through feedback is noted in Lazear (1998) and Murphy and Cleveland (1995). Typically feedback is thought to lead to increases in employee productivity and value. In this paper I show that there is the opposite channel as well - after good performance the agent will receive more information about his performance which reinforces his productivity in subsequent periods.

7.3 Model Applications

I consider two applications of my stylized model: employee management and internal resource allocation. The first is an example where information disclosure is a result of direct communication via performance appraisals and feedback. The second is a case where project management actions taken by the principal leak information to the agent about his performance.

When thinking of managing employees at the workplace my model sheds light on what information should be revealed to the agent. I show that praise should be given when it leads to increases in agent's productivity. Thus performance evaluations and communication will suffer from leniency as pointed out in Murphy and Cleveland (1995) and Prendergast (1999). On the other hand criticism should only be given to agents who have been performing sufficiently well in the past. Thus even though feedback will improve agent's performance in the future, the required compensation to motivate the agent will outweigh this potential benefit.

Consider the example of resource reallocation in a firm. Headquarters observes project quality of individual projects and then decides whether or not to allocate funds to them. Suppose at the beginning of each period a given division starts of with K units of capital. Then for one period the headquarters can decide whether to scale the division up or down depending on the profitability of the project generated. For this application it is useful to think that $S_0 \cap S_1 = \emptyset$ and the benefits from scaling up come from S_1 and scaling down in S_0 . Moreover Assumption 1 dictates that for some $s \in S_1$ we have $r(s) = 0$. This interpretation is that some good projects would not benefit of added capital. The model predicts that divisions with good performance will be allocated capital more efficiently, while divisions with bad past performance will not have their capital removed. This problem is largely similar to Scharfstein and Stein (2000), Rajan et al. (2000).

7.4 Relation to Full Information Dynamic Contracts

When the principal is able to hide information from the agent, compensating the agent for effort becomes cheaper. This is especially pronounced when agent's continuation value in the contract becomes low and the firm liquidation becomes a potential threat. To see this I compute the optimal contract which rewards the agent

Figure 17: Pareto Frontier, $e = 0.2$

Figure 18: Pareto Frontier, $e = 0.4$

The difference in payoffs from the contract becomes more significant as agent's effort becomes more costly. In the same example as above for $e = 0.4$ the value functions are plotted on Figure 18. When effort cost increases, the discrepancy in profits between a full information contract and the optimal contract decreases.

8 Conclusion

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Appendix

Proof of Lemma 1

Suppose after history h_t the agent was not incentivized to work meaning that the recommended $a_{t+1} = 1$. This means that the principal observed x_{t+1} privately and the agent observed $\tilde{p}_{t+1}$. The subsequent contract will depend on x_{t+1} and the agent reporting $\tilde{p}_{t+1}$ to the principal. Revelation principle establishes that it is sufficient to restrict attention to contracts under which the agent reports $\tilde{p}_{t+1}$ truthfully.

Consider an alternative contract where after history h_t the principal publicly randomizes $\hat{x}_{t+1}$ and $\hat{p}_{t+1}$ such that $(\hat{x}_{t+1}, \hat{p}_{t+1}) \stackrel{law}{=} (x_{t+1}, \tilde{p}_{t+1})$ when the agent didn't work. The distribution of the subsequent contract is unaffected but there will be less truthful reporting constraints going forward.

Proof of Lemma 2

By definition of effort profile a in the triplet (C, D, a) the following global incentive compatibility condition must be satisfied

$$\mathbb{E}_{a,t-1} \left[\beta^{T-t} \cdot c_T - \sum_{k=t}^T \beta^{k-t} \cdot a_k \right] \geq \max_{\hat{a}, t-1} \mathbb{E}_{\hat{a}, t} \left[\beta^{T-t} \cdot c_T - \sum_{k=t}^T \beta^{k-t} \cdot \hat{a}_k \right]$$

Provided that we restrict attention to $a_t = 1$ the deviation we want to preclude is $\hat{a}_t = 0$. Then

$$\begin{aligned} & \mathbb{E}_{a,t-1} \left[p \cdot \mathbb{E}_{a,t} \left[\beta^{T-t} \cdot c_T - \sum_{k=t}^T \beta^{k-t} \cdot a_k \mid x_t = 1 \right] + (1-p) \cdot \mathbb{E}_{a,t} \left[\beta^{T-t} \cdot c_T - \sum_{k=t}^T \beta^{k-t} \cdot a_k \mid x_t = 0 \right] \right] \geq \\ & \mathbb{E}_{\hat{a}, t-1} \left[\tilde{p}_1 \cdot \mathbb{E}_{\hat{a}, t} \left[\beta^{T-t} \cdot c_T - \sum_{k=t}^T \beta^{k-t} \cdot \hat{a}_k \mid x_t = 1 \right] + (1-\tilde{p}_1) \cdot \mathbb{E}_{\hat{a}, t} \left[\beta^{T-t} \cdot c_T - \sum_{k=t}^T \beta^{k-t} \cdot \hat{a}_k \mid x_t = 0 \right] \right] = \end{aligned}$$

Now move compensation from states $d = 0, x = 0$ to states $d = 0, x = 1$ outcome by outcome in the following way

$$\hat{c}_T = c_T|_{d_t=0, x_t=1} + \frac{1-p}{p} \cdot c_T|_{d_t=0, x_t=0}$$

Taking into account

$$\frac{1-p}{p} \cdot \tilde{p}_1 \leq 1 - \tilde{p}_1$$

Follow the inequalities

$$\begin{aligned}
& \mathbb{E}_{t-1} \left[\tilde{p}_1 \cdot \mathbb{E}_{\hat{a},t} \left[\beta^{T-t} \hat{c}_T - \sum_{k=t}^T \beta^{k-t} \hat{a}_k \mid x_t = 1 \right] + (1 - \tilde{p}_1) \cdot \mathbb{E}_{\hat{a},t} \left[\beta^{T-t} \hat{c}_T - \sum_{k=t}^T \beta^{k-t} \hat{a}_k \mid x_t = 0 \right] \right] = \\
& \quad \mathbb{E}_{\hat{a},t-1} \left[\tilde{p}_1 \cdot \mathbb{E}_{\hat{a},t+1} \left[\beta^{T-t} \cdot \left(c_T + \frac{1-p}{p} \cdot c_T|_{d_t=0, x_t=0} \right) - \sum_{k=t}^T \beta^{k-t} \hat{a}_k \mid x_t = 1 \right] \right] = \\
& \mathbb{E}_{t-1} \left[\tilde{p}_1 \cdot \mathbb{E}_{\hat{a},t} \left[\beta^{T-t} \hat{c}_T - \sum_{k=t}^T \beta^{k-t} \hat{a}_k \mid x_t = 1 \right] + \frac{1-p}{p} \cdot \tilde{p}_1 \cdot \mathbb{E}_{\hat{a},t} \left[\beta^{T-t} \hat{c}_T - \sum_{k=t}^T \beta^{k-t} \hat{a}_k \mid x_t = 1 \right] \right] \leq \\
& \mathbb{E}_{t-1} \left[\tilde{p}_1 \cdot \mathbb{E}_{\hat{a},t} \left[\beta^{T-t} c_T - \sum_{k=t}^T \beta^{k-t} \hat{a}_k \mid x_t = 1 \right] + (1 - \tilde{p}_1) \cdot \mathbb{E}_{\hat{a},t} \left[\beta^{T-t} c_T - \sum_{k=t}^T \beta^{k-t} \hat{a}_k \mid x_t = 0 \right] \right] \leq \\
& \mathbb{E}_{t-1} \left[\tilde{p}_1 \cdot \mathbb{E}_{\hat{a},t} \left[\beta^{T-t} c_T - \sum_{k=t}^T \beta^{k-t} \hat{a}_k \mid x_t = 1 \right] + (1 - \tilde{p}_1) \cdot \mathbb{E}_{\hat{a},t} \left[\beta^{T-t} c_T - \sum_{k=t}^T \beta^{k-t} \hat{a}_k \mid x_t = 0 \right] \right]
\end{aligned}$$

Thus for the contract defined by $(\hat{C}, D, a)$ the incentive compatibility constraints can be weakly improved and hence it is sufficient to look for the optimal contract under such restrictions.

Proof of Proposition 1

Sufficiency of Public Contracts. Proof by backward induction. Suppose disclosure decisions $d_{t+1}, \dots, d_T$ are public. I will show that for such a contract $\mathcal{C}$ there exists a different contract $\hat{\mathcal{C}} = (\hat{C}, \hat{D}, a)$ with the same ex-ante payoff to the principal. The disclosure decisions will be such that $\hat{d}_k = d_k$ for $k = 1, \dots, t-1$ while the subsequent decisions $\hat{d}_k$ for $k = t, \dots, T$ are public. Denote

$$f_t(s) = \mathbb{P}(d_t(s) = 1 \mid h_{t-1})$$

Principal's total expected at time t conditional on public history h_{t-1}

$$\int_S r(s) f_t(s) dG(s) = p \cdot \int_S r(s) f_t(s) dG_1(s) + (1-p) \cdot \int_S r(s) f_t(s) dG_0(s)$$

Define disclosure decision $\hat{d}_t$

$$\hat{d}_t(s) = \begin{cases} 1 & \text{with probability } f_t(s) \\ 0 & \text{with probability } 1 - f_t(s) \end{cases}$$

This leads to a public history $\hat{h}_t$ which is different from a public history h_t . Note that $\hat{d}_t$ is distributed in the same way as d_t . Disclosure decisions d_k for $k \geq t+1, \dots, T$ are public, but with respect to history

h_t . This means that they are dependent on d_t . Similarly define $\hat{d}_k$ to be dependent on $\hat{h}_t$ to be a function of $\hat{d}_t$ in the same way that d_k is dependent on d_t .

Denote by π_t^s the beliefs of the agent regarding his past outcomes after he was disclosed signal s at time t . Similarly π_t^N are his beliefs when he was not disclosed anything. Given a public history h_{t-1} there is also the space of agent's beliefs π_{t-1} before any period t disclosure or no disclosure.

$$\begin{aligned}\hat{\pi}_t^s &= \pi_{t-1} \times P(x_1 | s_1) \\ \hat{\pi}_t^N &= \pi_{t-1} \times P(x_1 | N)\end{aligned}$$

Similar to proof of Lemma 2 before the only relevant statistic over the beliefs going forward is whether or not all past outcomes were 1 or not. This would strictly improve incentives beforehand. Denote by $t_1, \dots, t_l$ such that $t_k < t$ and for which the agent is not fully certain about the outcome being $x_k = 0$. Denote the corresponding beliefs by

$$\begin{aligned}p_t^s &= \pi_t^s(x_{t_1} = \dots = x_{t_k} = 1) \\ \hat{p}_t^s &= \hat{\pi}_t^s(x_{t_1} = \dots = x_{t_k} = 1)\end{aligned}$$

The transfers for contract $\hat{C}$ conditional on history t will be

$$\begin{aligned}\hat{c}_t(s) &= \frac{p_t^s}{\hat{p}_t^s} \cdot c_t(s) \\ \hat{c}_t(N) &= \frac{p_t^N}{\hat{p}_t^N} \cdot c_t(N)\end{aligned}$$

Contract $\hat{C} = (\hat{C}, \hat{D}, a)$ implements a public disclosure decision from period t onwards. Repeating this procedure we will arrive to a contract which is public and results in the same ex-ante utility for the agent and principal.

Existence of an Optimal Contract. I will only need to show the existence of an optimal contract when $r(s) = 0$ for all $s \in S_1$. If this holds then the construction in Part 2 of this proof will show how to obtain optimal contracts for a general $r(\cdot)$. Proposition 1 allows me to restrict attention to public contracts.

Any contract specifies a decision tree for the agent. From every node there are four edges. They are characterized by a pair: whether information was disclosed or not and what is the recommended effort profile next period. In terms of notation in period t the agent receives information d_t and next period's effort a_{t+1} . where γ_t^i and κ_t^i are probabilities and κ_t^i will be different conditional on $d_t = 0$ or $d_t = 1$ but I suppress it for notational convenience.

After public history h_t at time $t + 1$ the agent observes a message which is given by a disclosure decision. Denote by α_t agent's belief that $x_t = 1$ at time t

$$\alpha_t = P(x_t = 1 \mid d_t)$$

The effort profile itself may serve as a signal of performance, but similar to the proof of Proposition 3 it is sufficient to consider contracts where the decision about a_{t+1} is driven by disclosure decision d_t . Beyond that decision of $a_{t+1} = 0$ or $a_{t+1} = 1$ is driven by public randomization.

For contract $\mathcal{C}$ denote $U(\mathcal{C})$ to be principal's expected utility from this contract as given by (1). Given that principal's payoff from any contract is bounded from above there must exist a sequence of contracts $\mathcal{C}^n$ for which

$$\lim_{n \rightarrow \infty} U(\mathcal{C}^n) = \sup_{\mathcal{C}} U(\mathcal{C})$$

where the supremum is taken over all contracts implementing effort profile a . For every contract $\mathcal{C}^n$ agent's decision tree is the same and has at most 4^T nodes. Thus every contract $\mathcal{C}^n$ specifies a finite number of probabilities α . Denote the set of all of these probabilities for a contract $\mathcal{C}^n$ by $\bar{\alpha}^n$. Given that they belong to a bounded finite dimensional set there exists a converging subsequence

$$\lim_{k \rightarrow \infty} \bar{\alpha}^{n_k} = \bar{\alpha}^*$$

Hence without loss assume that the original sequence is converging to some $\bar{\alpha}^*$. Thus agent's beliefs in this contract converge to some well specified limit $\bar{\alpha}^*$.

If transfers c^n for the set of contracts were to be bounded, then an optimal contract exists since in that case we could have obtained a converging subsequence of transfers. If transfers were diverging then there would exist a subsequence in the original sequence for which $c_T^{n_k}$ was diverging to $+\infty$ for $k \rightarrow \infty$ for one of the finite histories of the contract. Fix the rest of the history and show that within this subsequence there exists an alternative contract whose transfer after this given history is bounded above.

If $c_T^{n_k}$ diverges to $+\infty$ this means that the probability of reaching this transfer is converging to 0. This means that as long as the contract follows a threshold disclosure rule, then the probability of this history

is given by

$$P_0(h_T^*) = \prod_{k=1}^T \gamma_t^{d_t} \cdot \kappa_t^{a_t}$$

where the public history corresponding to h_T^* is given by $(d_1, a_1, \dots, d_T, a_T)$. Because the outcomes of $s \in S_0 \setminus S_1$ are independent from the past and effort decisions $a_{t+1} = 1$ vs $a_{t+1} = 0$ are based on public randomization the probability of history h_T^* is the product of these probabilities over time.

Given that $P_0(h_T^*) \cdot c_T$ has to be finite in the limit the probability of history h_T^* has to converge to 0. Because of the fact that the contract is public this means that some of the probabilities in the expression (??) have to converge to 0. Denote by t' the last such element for which the probability is converging to 0. Thus either $\gamma_{t'}^{n, d_{t'}}$ converges to 0 or $\kappa_{t'}^{n, a_{t'}}$ is converging to 0.

1. Suppose $\gamma_{t'}^{n, d_{t'}}$ converges to 0. Then this must imply that $d_{t'} = 1$. This must mean that conditional on $d_{t'} = 1$ agent's information rent is infinitely increasing as $n \rightarrow \infty$. However it is clear that the biggest amount of information rent required to compensate the agent is the rent of a full information contract for any subtree. The contract would be well defined but imply that the required continuation value to the agent from this contract is smaller. Incentives could be strictly improved by taking the remainder of the continuation value of the agent and making it contingent on high performance in all periods starting from period t and onwards. The proof will be similar to Lemma 2.
2. Suppose that $\kappa_{t'}^{n, a_{t'}}$ converges to 0. This means that the principal randomizes the effort profile and with some probability making the agent very rich. Similar to the logic above there will exist a subsequent full information contract which achieves incentive compatibility in the rest of the history while the extra transfer can be moved to $\kappa_{t'}^{n, \hat{a}_{t'}}$ where $\hat{a}_{t'} \neq a_{t'}$.

Proof of Proposition 2

Previous section has established that there exists an optimal contract for environments where information in S_1 is never disclosed. Now I will show that for a general value $r(s)$ there exists a contract delivering the agent the same expected compensation, but disclosing all project states $s_t \in S_1$ for which $r(s_t) > 0$ at time t .

Suppose there exists an contract that achieves the result above for all histories of length $t - 1$. In other words all $s_k \in S_1$ for which $r(s_k) > 0$ are disclosed for $k = 1, \dots, t - 1$ and subsequent disclosure decisions $d_t, d_{t+1}, \dots$ are public and do not disclose outcomes in S_1 . Denote

$$\Delta = \{s \in S_1 : r(s) > 0\}$$

Define disclosure rule $\hat{d}_t$ the following way

$$\hat{d}_t(s) = \begin{cases} 1 & \text{if } s \in \Delta \\ d_t(s) & \text{if } s \notin \Delta \end{cases}$$

Subsequent disclosure rules will treat disclosure of $s \in S_1$ as no disclosure and thus the subsequent contract will by construction remain public. I will the transfers associated with this history in the following way

$$d_t = 0 \begin{cases} \xrightarrow{s \in \Delta} \hat{d}_t = 1, s_t \in \Delta \\ \xrightarrow{s \notin \Delta} \hat{d}_t = 0 \end{cases}$$

Given that the subsequent disclosure decisions will be identical I will adjust the transfers paid out to the agent in the final period as a function of the disclosure decision $\hat{d}$. Consider also the alternative compensation profile:

$$\hat{c}_T = \begin{cases} \frac{P_t(x_t = 1 | d_t = 0)}{P_t(x_t = 1 | s_t)} \cdot c_T, & \text{if } \hat{d}_t = 1 \text{ and } s_t \in \Delta \\ \frac{P_t(x_t = 1 | d_t = 0)}{P_t(x_t = 1 | \hat{d}_t = 0)} \cdot c_T & \text{if } \hat{d}_t = 0 \\ c_T, & \text{if } \hat{d}_t = 1 \text{ and } s_t \in S_0 \setminus S_1 \end{cases}$$

If the agent follows the same effort profile as before then compensation $\hat{c}_T$ has the same expected value as compensation c_T . Now all we need is to check for incentive compatibility to hold at time t . Suppose the agent worked before time t and is considering a deviation to no effort at t . Upon agent's deviation there are two possibilities. Either with probability $(1 - q)$ the distribution of his continuation value will be exactly the same as on equilibrium value, or with probability q he will get 0. If he gets 0 and is disclosed his performance then his promised utility is the same as of the original contract $\mathcal{C}$. If he is not disclosed his performance then he gets 0 which is also the same as under the original contract.

Proof of Lemma 5

Standard result from dynamic contracts. Proof can be provided upon request.

Proof of Lemma 6

Lemma 4 + Blackwell's contraction mapping condition.

9 Appendix 2. Notes and Further Analysis

Binding Agency Problem

Proposition 7. *For $r(s) = r$ there exist parameters for which the agency problem is always binding, meaning that $l_t(w) = \underline{l}_t(w)$. The value functions defined in Lemma 3 are concave.*

Consider an example where $r(s) = 1$ and $\hat{m} = 0$ for simplicity. In this section I will show that for a subset of model parameters the solution to the agency problem involves a constantly binding agency problem and the resulting value function is concave.

Consider function $g(w)$ defined by

$$b_t(w) = g_t(w) + \sum_{k=t}^T \beta^{k-t} \cdot \hat{m} - w$$

and $g_t(w)$ satisfies

$$g_t(w) = \max_f \left[R(f) + (1-f) \cdot \beta g_{t+1} \left(\frac{p \left(w - \frac{1-q}{q} \right) + \frac{1}{q}}{\beta(1-f)} \right) + f \cdot \beta g_{t+1} \left(\frac{(1-p)(w - \frac{1-q}{q})}{\beta f} \right) \right]$$

where $R(f)$ is increasing and concave. Derivative w.r.t. f^0 is given by

$$\begin{aligned} \Delta(f, w) = & R'(f) - \beta g \left(\frac{p(w - \frac{1-q}{q}) + \frac{1}{q}}{\beta(1-f)} \right) + \frac{p(w - \frac{1-q}{q}) + \frac{1}{q}}{\beta(1-f)} \cdot \beta \frac{dg_{t+1}}{dw} \left(\frac{p(w - \frac{1-q}{q}) + \frac{1}{q}}{\beta(1-f)} \right) \\ & + \beta g \left(\frac{(1-p)(w - \frac{1-q}{q})}{\beta f} \right) - \frac{(1-p)(w - \frac{1-q}{q})}{\beta f} \cdot \beta \frac{dg}{dw} \left(\frac{(1-p)(w - \frac{1-q}{q})}{\beta f} \right) \end{aligned}$$

Then either f is determined by $\Delta(f) = 0$ or this is a corner solution leading to $\Delta(f) > 0$. As long as $g(\cdot)$ is sufficiently continuous the optimal f as a function of continuation value w will be continuous. For $w = \frac{1-q}{q}$ the optimal amount of feedback to give is 0 and so the incentive constraint is clearly binding. Denote by w_1 the first point where the corresponding $f(w_1)$ is given by the binding incentive compatibility constraint

$$f(w_1) = \frac{(1-p) \cdot (w_1 - \frac{1-q}{q})}{\beta \cdot \frac{1-q}{q}}$$

Note that

$$\Delta(f, w) \geq R'(f) - \left(\beta \cdot \frac{1-q}{q} + \frac{1}{pq} \right) \cdot \frac{dg}{dw} \left(\frac{1-q}{q} \right)$$

And a sufficient condition would be

$$\frac{dg}{dw} \left(\frac{1-q}{q} \right) \leq \frac{R'(1)}{\beta \frac{1-q}{q} + \frac{1}{pq}}$$

$$\begin{aligned}
\left(\beta \cdot \frac{1-q}{q} + \frac{1}{pq} \right) \cdot \frac{q(1-p)}{\beta(1-q)} &\leq 1 \\
1-p + \frac{(1-p)}{\beta p(1-q)} &\leq 1 \\
\frac{1-p}{\beta p(1-q)} &\leq p \\
1-p &\leq \beta p^2(1-q)
\end{aligned}$$

Then

$$\begin{aligned}
\frac{dg_t}{dw} \left(\frac{1-q}{q} \right) &= \frac{q(1-p)}{\beta(1-q)} \cdot R'(0) + p \frac{dg_{t+1}}{dw} \left(\frac{1}{\beta q} \right) + (1-p) \frac{dg_{t+1}}{dw} \left(\frac{1-q}{q} \right) + \frac{q(1-p)}{\beta(1-q)} \beta \left(g_{t+1} \left(\frac{1-q}{q} \right) - g_{t+1} \left(\frac{1}{\beta q} \right) \right) \\
\frac{dg_t}{dw} \left(\frac{1-q}{q} \right) &\leq \frac{q(1-p)}{\beta(1-q)} \cdot R'(0) + p \frac{dg_{t+1}}{dw} \left(\frac{1}{\beta q} \right) + (1-p) \frac{dg_{t+1}}{dw} \left(\frac{1-q}{q} \right) - \frac{(1-\beta+\beta q)(1-p)}{\beta(1-q)} \cdot \frac{dg_{t+1}}{dw} \left(\frac{1}{\beta q} \right) \\
\frac{dg_t}{dw} \left(\frac{1-q}{q} \right) &\leq \frac{q(1-p)}{\beta(1-q)} \cdot R'(0) + \left(1 - \frac{1-p}{\beta(1-q)} \right) \frac{dg_{t+1}}{dw} \left(\frac{1}{\beta q} \right) + (1-p) \frac{dg_{t+1}}{dw} \left(\frac{1-q}{q} \right)
\end{aligned}$$

Suppose $\beta \leq \frac{1}{2-q}$. Then $\frac{dg_{t+1}}{dw} \left(\frac{1}{\beta q} \right) = 0$ since the optimal contract just shares all of the information with the agent. Then this is valid for negative Poisson shocks.

$$\frac{dg_t}{dw} \left(\frac{1-q}{q} \right) \leq \frac{q(1-p)}{\beta(1-q)} \cdot R'(0) + (1-p) \frac{dg_{t+1}}{dw} \left(\frac{1-q}{q} \right)$$

and hence

$$\begin{aligned}
\frac{dg_t}{dw} \left(\frac{1-q}{q} \right) &\leq \frac{q(1-p)}{\beta(1-q)} \cdot R'(0) + (1-p) \frac{dg_{t+1}}{dw} \left(\frac{1-q}{q} \right) \\
\frac{dg_t}{dw} \left(\frac{1-q}{q} \right) &\leq \frac{R'(0)q(1-p)}{\beta p(1-q)}
\end{aligned}$$

The optimality condition then translates to

$$\frac{R'(1)}{\beta \frac{1-q}{q} + \frac{1}{pq}} \geq \frac{R'(0)q(1-p)}{\beta p(1-q)}$$

Hence for p satisfying the above condition (sufficiently close to 1) the optimal contract implements maximum possible disclosure.

Concavity of the value function. Suppose for some w the constraint on $\underline{s}$ is binding. This means the principal reveals everything to the agent and by envelope theorem:

$$\frac{db_t}{dw}(w) = p \cdot \frac{db_{t+1}}{dw} \left(\frac{w - \frac{1-q}{q} + \frac{1}{pq}}{\beta} \right) + (1-p) \cdot \frac{db_{t+1}}{dw} \left(\frac{w - \frac{1-q}{q}}{\beta} \right)$$

There is another constraint which could possibly be binding and that is $w^i \geq \frac{1-q}{q}$. and concavity of b_t follows by induction in t when the optimal f is interior or $f = 1$ (since the upper bound on f is independent of w). Now suppose condition $f = \frac{w+1-\frac{1}{q}}{\beta \cdot \frac{1-q}{q}}$ is binding. Then

$$\begin{aligned} w^0 &= \frac{1-q}{q} \\ w^1 &= \frac{w+1-f(1-p) \cdot \beta \frac{1-q}{q}}{\beta(fp+1-f)} \\ \frac{\partial f}{\partial w} &= \frac{q}{\beta(1-q)} \end{aligned}$$

and

$$b_t(w) = mp + \bar{f}p \cdot \delta_1 + f^0(1-p) \cdot \delta_0 + (1-f^0(1-p)) \cdot \beta b_{t+1}(w^1) + f^0(1-p) \cdot \beta b_{t+1}\left(\frac{1-q}{q}\right)$$

Then at the points where the incentive problem is indeed binding we have

$$\begin{aligned} \frac{db_t}{dw}(w) &= \delta_0 \cdot \frac{q(1-p)}{\beta(1-q)} - \frac{q(1-p)}{\beta(1-q)} \cdot \beta b_{t+1}(w^1) + \frac{q(1-p)}{\beta(1-q)} \cdot \beta b_{t+1}\left(\frac{1-q}{q}\right) + \beta(fp+1-f) \frac{db_{t+1}}{dw}(w^1) \cdot \frac{dw^1}{dw} \\ &= \delta_0 \cdot \frac{q(1-p)}{\beta(1-q)} - \frac{q(1-p)}{1-q} \cdot b_{t+1}(w^1) + \frac{q(1-p)}{1-q} \cdot b_{t+1}\left(\frac{1-q}{q}\right) + (fp+1-f) \cdot \frac{db_{t+1}}{dw}(w^1) \cdot \beta \frac{dw^1}{dw} \end{aligned}$$

When the constraint seizes to bind, the concavity condition is given by

$$\begin{aligned} \delta_0 \frac{q(1-p)}{\beta(1-q)} - \frac{q(1-p)}{1-q} b_{t+1}(w_1) + \frac{q(1-p)}{1-q} b_{t+1}\left(\frac{1-q}{q}\right) &\geq (1-p) \frac{db_{t+1}}{dw}\left(\frac{1-q}{q}\right) - \frac{q(1-p)}{\beta(1-q)} \frac{\partial b_{t+1}}{\partial w}(w^1) w^1 \\ \delta_0 \frac{q}{\beta(1-q)} - \frac{q}{1-q} b_{t+1}(w_1) + \frac{q}{1-q} b_{t+1}\left(\frac{1-q}{q}\right) &\geq \frac{db_{t+1}}{dw}\left(\frac{1-q}{q}\right) - \frac{q}{\beta(1-q)} \frac{\partial b_{t+1}}{\partial w}(w^1) w^1 \\ \delta_0 - b_{t+1}(w_1) + b_{t+1}\left(\frac{1-q}{q}\right) &\geq \frac{1-q}{q} \cdot \frac{db_{t+1}}{dw}\left(\frac{1-q}{q}\right) - w^1 \cdot \frac{\partial b_{t+1}}{\partial w}(w^1) \end{aligned}$$

which is the first order condition for the given feedback level to be indeed optimal.