Asset Pricing and Monetary Policy*

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Abstract

This paper examines the role of money in understanding the behavior of asset prices and whether and how monetary policy should react to asset prices such as stock prices and equity premiums. To do so, I introduce money via the form of transaction cost into a production economy with limited stock market participation where agents with lower inter-temporal elasticity of substitution (IES), called non-stockholders, have no access to stock market. In addition to facilitating transactions of consumption goods, money also redistributes wealth by counter-cyclically transferring resources from stockholders to non-stockholders, the main role of non-state contingent bonds. The benchmark model resolves quantitatively the risk premium puzzle and the risk-free return puzzle, matches macroeconomic behavior such as volatilities of output, consumption and investment, and is in line with empirically documented facts about money growth, inflation and asset prices in literature. This model is then used to evaluate alternative policies for money growth rates. I find that monetary policies are welfare improving for both stockholders and non-stockholders if they reduce equity premiums in the economy. These policies include a lower expected money growth, a pro-cyclical money growth rate, and growth rates of money being positively reacting to equity prices or equity premiums, all of which enhance the precautionary saving role of money.

Keywords: recursive preferences, equity premium, monetary policy, business cycles, elastic labor supply

JEL Classification Codes: E32, E37, E42, E52, G12

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1 Introduction

Whether and how monetary policy should respond to asset prices was for many years a deeply debatable subject. Though the central banks' not doing anything with asset prices was a consensus before 2007, the recent crisis has now shifted it.¹ The recent policy of the Fed, buying securities including long-term government debt and privately-issued securities, a policy called "quantitative easing", is definitely an evidence of such a shifting. It is also well-known that even in normal times, it is by buying and selling reserves in the interbank overnight market that the Fed conducts monetary policy, relying on no-arbitrage arguments to link this short-term nominal debt market to the markets for longer-term bonds and equities. Any model which purports to guide and evaluate the wisdom of monetary policies should be consistent with the relative prices of the assets in question. However, the standard models are quantitative failures at reproducing these relative prices (for example, the well-known and persistent equity premium and risk-free return puzzles) and thus not reliable to shed light on monetary policies, especially their welfare implications.

This paper contributes to the debate by first developing a model that is consistent with both macroeconomic and asset market data. The model is an extension of Guvenen (2009), where there are two types of agents, stockholders and non-stockholders, in a production economy. The stockholders, as smaller fraction of total population, have access to both the stock market and bond market, while the non-stockholders are restricted to the latter. Both of them are allowed to borrow only a certain amount based on their wage income. They both have Epstein-Zin-Weil preferences (Epstein and Zin, 1989; Weil, 1990), which disentangle risk aversions from the elasticity of intertemporal substitution (EIS). Consistent with empirical evidence, the non-stockholders, who have relatively lower wealth, are assumed to have a low EIS, whereas the stockholders have a higher elasticity. There is one external firm, whose share is normalized to be 1, that can produce with capital and labor, subject to capital adjustment cost.

Money is introduced via transaction costs on consumption, taking the form of real resources that are consumed during the process of exchange (Brock, 1974, 1990; Schmitt-Grohe and Uribe, 2004). That is, given a volume of consumption goods, higher money balance helps reduce the cost of consumption. It turns out that adding money is important. First, money is one type of assets, meaning that it has to be properly priced as others like bonds and stocks via no-arbitrage. Indeed, since money now competes with bonds in the role of saving intertemporally, it is harder for the model to match asset pricing facts such as a high and countercyclical equity premium. It points out that the model of Guvenen (2009) is not robust if introducing another asset. Second, since money is distributed equally to the agents as a lump-sum transfer and the wealth and preferences of those agents differ, it has real effects on macro fundamentals and hence asset prices through portfolio reallocation. Therefore, money is non-neutral. The Central Banks conduct monetary policy by varying money growth rates.

This model is remarkably good in matching asset price and macroeconomic data with plausible calibrations. It produces higher equity premium and lower risk-free return, 5.3% and 1.6% per annum, respectively. In particular, the match of Sharpe ratio (0.25 vs 0.32 in the data) shows that the risks in the economy are appropriately evaluated. Compared with the literature, the model also performs better in matching volatilities of output growth, the relative volatilities of consumption and investment. Finally, the model is in line with empirically documented relationship between money growth, inflation and asset prices. Specifically, the decrease of expected inflation (money growth rate) explains partially the decline of equity premiums during the past three decades. Unexpected inflations due to real shocks are negatively related with stock prices, real stock returns

¹See Bernanke and Gertler (2001), for example.

and real risk-free returns.

I then use this model to evaluate alternative monetary policies: (1) a change of average money growth rates; (2) a money growth rate reacting to business cycles; (3) a monetary policy responding to equity prices; and (4) a policy reacting to expected risk premiums. All the experiments are compared with a representative agent model with similar setups to shed light on the innovations of having assets appropriately priced. In stark contrast with the conventional wisdom of Friedman rule, my model shows that the monetary policies yield welfare improvements for all types of agents if it drives down the equity premiums. This corresponds to a policy with lower expected inflation, a pro-cyclical monetary policy, and policies that positively responding to stock prices and equity premiums.

This paper relates to a large literature discussing monetary policies and asset prices. Bernanke and Gertler (2001) shows in a small scale macro model that asset prices become relevant only to the extent they may signal potential inflationary or deflationary forces and rules that directly target asset prices appear to have undesirable side effects. Bullard and Schaling (2002) construct a model showing that asset prices targeting can interfere with the minimization of inflation and output variation and under certain conditions, asset price targeting can lead to indeterminacy. On the other hand, Cecchetti (1997), Blanchard (2000), and Mishkin (2000) claimed that considering asset prices improved economic performance. After the crisis, it is now increasingly accepted that, to some degree and extent, mainstreaming reactions to asset price moves in monetary policy is to become a new norm.² Rudebusch et al. (2007) point out that changes in the term premium of bonds has real effects to the economy. Gallmeyer et al. (2007), and Palomino (2010) show that term structure and endogenous inflation are important for understanding monetary policies. Paoli and Zabczyk (2012) shows that the varying precautionary saving due to cyclical risk aversion should lead to large policy errors in turbulent times. However, none of these works or suggestions has been built on a structural model that explains the asset prices data quantitatively. It is therefore likely to misestimate the effects of considering asset prices in monetary policies.

This paper relates to a growing literature that jointly studies asset prices and macroeconomic behavior. Besides Guvenen (2009), many other papers deal with the same problem, including Danthine and Donaldson (2002), Storesletten et al. (2007), Tallarini (2000), etc. However, money is not considered in any of these models. Earlier papers that discuss the relationship between money and asset prices includes Danthine and Donaldson (1986), Marshall (1992), Labadie (1989), Boyle (1990), Balduzzi (1996) and Hodrick et al. (1991), all of which focused on an endowment economy. In particular, Bansal and Coleman (1996) explain equity premium by examining money's role in facilitating transactions. This idea then is inherited by Gust and Lopez-Salido (2010), who consider market segmentation in checking and brokerage account. These two accounts are different in liquidity and infrequent portfolio rebalance leads to the high volatility in consumption for those who are active and then compensated by high risk premium. They show that this model can match some statistics of asset prices and then discuss the welfare gains for different monetary policies. However, they do not explore the performance of the model in matching macroeconomic behavior.

There is also a large literature documenting the relationship between money growth, inflation and asset prices. For example, inflation is negatively correlated with real stock prices if the economy is driven by supply shocks; see Tatom (2011) and Christiano et al. (2010).³ Also the negative relationship between inflation and asset returns is in the spirit of research in finance initiated in the early 1980s. Geromichalos et al. (2007) study the effect of monetary policy on asset prices in an endowment economy with search-based money. When money grows at a higher rate, inflation

²See Canuto (2009), Issing (2009), and Kohn (2007), etc.

³Since inflation is low during stock market booms, so that, Christiano et al. (2010) claim, an interest rate rule that is too narrowly focused on inflation destabilizes asset markets and the broader economy.

is higher and the return on money decreases. In equilibrium, no arbitrage amounts to equating the real return of both objects. Therefore, the price of the asset increases in order to lower its real return.

It is also claimed that the decrease of equity premium during the Great Moderation is due to the decline of inflation; See Siegel (1999), Jagannathan et al. (2000), Claus (2001), and Campbell (2008) for the evidence of decreasing premia and see Beirne and de Bondt (2008) and Kyriacou et al. (2006) for empirical estimation of inflation's role in mitigating premia.⁴ Labadie (1989) explores two ways that inflation affects equity premium in a theoretical model. However, her model is based on an endowment economy. In contrast to her conclusion, my paper shows that it is the expected inflation, not the unexpected one, that determines or closely relates to the equity premium.

The paper is organized as follows. The next section presents the model with solution methods and computational algorithm discussed in section 3. Section 4 explores the role of money in reconciling asset pricing and macroeconomic facts. Section 5 is devoted to policy analysis. Section 6 concludes.

2 The Model

The economy studied is based on the framework of Guvenen (2009). Money is introduced via a transaction cost, taking the form of real resources that are consumed in the process of exchange (Brock, 1974, 1990; Schmitt-Grohe and Uribe, 2004). That is, an increase in the volume of goods exchanged leads to a rise in transaction costs, while higher average real money balances, for a given volume of transactions, lower costs. Compared with other alternative approaches for money in macroeconomics and policy analysis, this setup loosens the tight relationship between money and consumption.⁵

2.1 The Central Bank

In each period t , the central bank issues some new money $(g_t - 1)M_{t-1}^s$ and gives it to the agents as a lump-sum transfer. M_t^s is the per capita money supply in the economy. The money stock follows a law of motion

$$M_t^s = g_t M_{t-1}^s, \quad (1)$$

where g_t is the gross growth rate of money supply. The growth rate of the money supply evolves following:

$$\begin{aligned} \log(g_t) = & (1 - \rho_g) \log(\bar{g}) + \rho_g \log(g_{t-1}) + \\ & \theta_y \log(Y_t/\bar{Y}) + \theta_{EP} \log(\mathbf{E}_t R_{t+1}^{EP}/\bar{R}^{EP}) + \theta_{PE} \log(P_t^s/\bar{P}^s) + \varepsilon_{g,t} \end{aligned} \quad (2)$$

In equation (2), $\varepsilon_g \stackrel{iid}{\sim} N(0, \sigma_g^2)$, $\log(\bar{g})$ is the unconditional mean of the logarithm of the growth rate g_t , $\bar{Y}$, $\bar{R}^{EP}$, and $\bar{P}^s$ are the averages of output, equity premium and stock prices, respectively. This rule allows for a systematic response of money to changes in output, expected equity premium (implied future risks) and equity prices. I evaluate four simple rules: (1) constant money growth

⁴Others claim that it can be due to the declining volatility of technology shocks or other possible shocks/structural changes; see Lettau et al. (2008), for example. .

⁵Wang and Yip (1992) proved the "functional equivalence" between the transactions cost and other approaches like MIU, CIA and shopping time models that are commonly used in macroeconomics and policy analysis. There is another category of money modelling aiming to understand the microfoundation of money, such as search theoretic models by Kiyotaki and Wright (1989) and Nosal and Wright (2010).

when $\rho_g = \theta_y = \theta_{EP} = \theta_{PE} = 0$; (2) monetary rules that are either procyclical ($\theta_y > 0$) or countercyclical ($\theta_y < 0$) when $\rho_g = \theta_{EP} = \theta_{PE} = 0$; (3) rules that respond to equity premiums when $\rho_g = \theta_y = \theta_{PE} = 0$ and $\theta_{EP} \neq 0$; and finally (4) rules that respond to equity prices when $\rho_g = \theta_y = \theta_{EP} = 0$ and $\theta_{PE} \neq 0$.

2.2 Households

There are two types of households who live forever in the economy. The population is constant and normalized to be unity. Let $\mu \in (0, 1)$ denote the measure of stockholders and $1 - \mu$ non-stockholders. Each of them is endowed with one unit of time every period, which he allocates between market work and leisure. They both have Epstein-Zin-Weil preferences (Epstein and Zin, 1989; Weil, 1990) with the following form:

$$U_t^i = \left[(1 - \beta)u_t^i + \beta(E(U_{t+1}^i)^{1-\sigma^i})^{\frac{\rho^i}{1-\sigma^i}} \right]^{\frac{1}{\rho^i}} \quad (3)$$

for $i = h, n$, where throughout the paper the superscripts h and n denote stockholders and non-stockholders, respectively; u_t^i is current utility, a function of consumption c_t^i and labor l_t^i . The conventional interpretation is that $\rho^i < 1$ governs the intertemporal substitution (EIS is $1/(1 - \rho^i)$) and σ^i governs relative risk aversion.

The stockholders choose consumption (c_t^h), bond holdings (b_{t+1}^h), stock shares (s_{t+1}), and nominal money holdings (m_t^h) subject to the following budget constraint:

$$c_t^h + P_t^f b_{t+1}^h + P_t^s s_{t+1} + \frac{m_t^h}{P_t} + \Psi(c_t, \frac{m_t^h}{P_t}) \leq b_t^h + s_t(P_t^s + D_t) + W_t l_t^h + \frac{m_{t-1}^h + (g_t - 1)M_{t-1}^s}{P_t} \quad (4)$$

where the right side is all the real sources that can be spent. It includes values of holding bond and shares, $b_t^h + s_t(P_t^s + D_t)$, wage income, $W_t l_t^h$, and money carried over from previous period, m_{t-1}^h , plus the lump-sum transfer from the central bank $(g_t - 1)M_{t-1}^s$, divided by the price of goods, P_t . Stock prices, P_t^s , and dividend, D_t , are defined in details in the following subsection. Non-stockholders have a similar budget constraint except that they do not own stocks and thus do not choose s_{t+1} .

In equation (4), I introduce real money demand by having the transaction cost Ψ , which is a function of consumption c and real money balances m/P . Feenstra (1986) demonstrates that the transaction costs satisfy the following condition for all $c, m/P \geq 0$: Ψ is twice continuously differentiable and $\Psi \geq 0$; $\Psi(0, m/P) = 0$; $\Psi_c \geq 0$; $\Psi_{m/P} \leq 0$; $\Psi_{cc}, \Psi_{m/P, m/P} \geq 0$; $\Psi_{c, m/P} \leq 0$; and $c + \Psi(c, m/P)$ is quasi-convex, with expansion paths having a nonnegative slope.⁶⁷

2.3 The Firm

There is an aggregate firm producing a single good that can be used for either consumption or investment by using capital (K_t) and labor (L_t) inputs according to a Cobb-Douglas technology:

⁶If no firm leverage (detailed below), then a portfolio adjustment cost for bonds has to be added. Whenever the agents change their bond positions, they incur a cost of $\frac{\phi}{2}(b_{t+1}^h/b_t^h - 1)^2$, where ϕ is the semi-elasticity of adjustment cost over changes. The necessity for adding this cost is that money essentially has a similar feature as bonds, which is more risk-free when no leverage. By facilitating goods transactions, it gives the agents an implicitly determined return which is nearly risk-free. Hence, it makes no difference for a household to choose between bonds and money. By adding such a cost, assets in the economy have a nonsingular return matrix and hence the agents can distinguish bonds from money when making a decision.

⁷Tough with different forms of the functional form, money has been introduced in this approach widely in the literature, see Sims (2003), Schmitt-Grohe and Uribe (2004), etc.

$Y_t = Z_t K_t^\theta L_t^{1-\theta}$, where $\theta \in (0, 1)$ is the factor share parameter. The productivity level evolves according to:

$$\log(Z_{t+1}) = \rho_z \log(Z_t) + \varepsilon_{z,t+1}, \varepsilon_z \stackrel{iid}{\sim} N(0, \sigma_z^2) \quad (5)$$

The objective of the firm is to maximize the value of the firm, which equals the value of future dividend stream generated by the firm, $\{D_{t+j}\}_{j=1}^\infty$, discounted by the marginal rate of substitution process of stockholders, $\{\Lambda_{t,t+j}\}_{j=1}^\infty$. Specifically, the firm's problem is to solve:

$$P_t^s = \max_{\{I_{t+j}, L_{t+j}\}} E_t \left[\sum_{j=1}^\infty \Lambda_{t,t+j} D_{t+j} \right] \quad (6)$$

subject to the law of motion for capital, which features adjustment costs in investment:

$$K_{t+1} = (1 - \delta)K_t + \Phi\left(\frac{I_t}{K_t}\right)K_t \quad (7)$$

where P_t^s is the ex-dividend value of the firm. The number of shares outstanding is normalized to unity for convenience and hence P_t^s is the stock price. Strict convexity of $\Phi(\cdot)$ captures the difficulty of quickly changing the level of capital installed in the firm, which is necessary if the model is to generate realistic asset prices, particularly equity prices; see Cochrane (1991). An equity share is the right to own the entire stream of dividends, defined by the profits net of wages and investments: $D_t = Z_t K_t^\theta (L_t)^{1-\theta} - W_t L_t - I_t - \chi \bar{K}$.⁸

2.4 Financial Markets

There are two types of assets traded in this economy: a one-period bond and an equity share (stock). The non-stockholders can freely trade the risk-free bond while restricted from participating in the stock market. In contrast, the stockholders have access to both markets and hence are the sole capital owners in the economy. As in Guvenen (2009), portfolio constraints are imposed to avoid Ponzi schemes.

2.5 Individuals' Dynamic Problem and Equilibrium

A change in variables is introduced so that the problem solved by the households is stationary. That is, let $\hat{m}_t^i = m_t^i / M_t^s$ and $\hat{P}_t = P_t / M_t^s$. In addition, I let $\hat{m}^i = \hat{m}_{t-1}^i$ be the money balances of agent i at the beginning of period t . In each given period, the portfolio of each group is a function of the *beginning-of-period* capital stock, K , the aggregate bond holdings of non-stockholders after production, B , the *beginning-of-period* aggregate money holdings of non-stockholders, $\bar{M}$, the gross rate of money supply, g , and the technology level, Z . Let Υ denote the aggregate state vector $(K, B, \bar{M}, Z, g)$. The dynamic programming of a stockholder can be expressed as follows (primes indicate next period values):

$$V^h(\omega^h; \Upsilon) = \max_{c^h, l^h, b^{h'}, s^{h'}, \hat{m}^{h'}} \left[(1 - \beta) u(c^h, l^h)^{\rho^h} + \beta (E(V^h(\omega^{h'}; \Upsilon') | Z, g)^{1-\sigma^h})^{\frac{\rho^h}{1-\sigma^h}} \right]^{\frac{1}{\rho^h}} \quad (8)$$

⁸Guvenen (2009) claims that (in his note 4) the existence of leverage has an effect on quantity allocations. However, this is not true when money, a competing asset, is introduced (?).

s.t.

$$\omega^h + W(\Upsilon)l^h \geq c^h + P^f(\Upsilon)b^{h'} + P^s(\Upsilon)s' + \frac{\hat{m}^{h'}}{\hat{P}(\Upsilon)} + \Psi(c, \frac{\hat{m}^{h'}}{\hat{P}(\Upsilon)}) \quad (9)$$

$$\omega^{h'} = b' + s'(P^s(\Upsilon') + D(\Upsilon')) + \frac{\hat{m}^{h'} + g' - 1}{\hat{P}(\Upsilon')g'} \quad (10)$$

$$K' = \Gamma_K(\Upsilon), B' = \Gamma_B(\Upsilon), \widehat{M}' = \Gamma_{\widehat{M}}(\Upsilon) \quad (11)$$

$$b^{h'} \geq \underline{B} \quad (12)$$

where ω^h denotes financial wealth; b^h and s are individual bond and stock holdings, respectively; Γ_K , Γ_B and $\Gamma_{\widehat{M}}$ denote the laws of motion for the wealth distribution which are determined in equilibrium; and P^f is the equilibrium bond pricing function. The problem of a non-stockholder can be written as above with $s' \equiv 0$, and the superscript h replaced with n .

A *stationary recursive competitive equilibrium* for this economy is given by a pair of value functions, $V^i(\omega^i; \Upsilon)$; consumption, labor supply, bond holding decision rules and money holding decision rules for each type of agent, $c^i(\omega^i; \Upsilon)$, $l^i(\omega^i; \Upsilon)$, $b^{i'}(\omega^i; \Upsilon)$, and $\hat{m}^{i'}(\omega^i; \Upsilon)$; a stockholding decision rule for stockholders, $s'(\omega^i; \Upsilon)$; stock, bond and consumption goods pricing functions, $P^s(\Upsilon)$, $P^f(\Upsilon)$, and $\hat{P}(\Upsilon)$; a competitive wage function, $W(\Upsilon)$; an investment function for the firm, $I(\Upsilon)$; laws of motion for aggregate capital, aggregate bond holdings of non-stockholders, and aggregate money holdings of non-stockholders, $\Gamma_K(\Upsilon)$, $\Gamma_B(\Upsilon)$, and $\Gamma_{\widehat{M}}(\Upsilon)$; and a marginal utility process, $\Lambda(\Upsilon)$, for the firm such that:

(1) Given the pricing functions and laws of motion, the value function and decision rules of each agent solve that agent's dynamic problem.

(2) Given $W(\Upsilon)$ and the equilibrium discount rate process obtained from $\Lambda(\Upsilon)$, the investment function $I(\Upsilon)$ and the labor choice of the firm, $L(\Upsilon)$, are optimal.

(3) All markets clear: (a) $\mu b^{h'}(\bar{\omega}^h; \Upsilon) + (1 - \mu)b^{n'}(\bar{\omega}^n; \Upsilon) = 0$ (bond market); (b) $\mu s'(\bar{\omega}^h; \Upsilon) = 1$ (stock market); (c) $\mu l^h(\bar{\omega}^h; \Upsilon) + (1 - \mu)l^n(\bar{\omega}^n; \Upsilon) = L(\Upsilon)$ (labor market); (d) $\mu \hat{m}^{h'}(\bar{\omega}^h; \Upsilon) + (1 - \mu)\hat{m}^{n'}(\bar{\omega}^n; \Upsilon) = 1$ (money market); and (e) $\mu[c^h(\bar{\omega}^h; \Upsilon) + \Psi(c^h(\bar{\omega}^h; \Upsilon), \hat{m}^{h'}(\bar{\omega}^h; \Upsilon)/\hat{P}(\Upsilon))] + (1 - \mu)[c^n(\bar{\omega}^n; \Upsilon) + \Psi(c^n(\bar{\omega}^n; \Upsilon), \hat{m}^{n'}(\bar{\omega}^n; \Upsilon)/\hat{P}(\Upsilon))] + I(\Upsilon) = Y(\Upsilon)$ (goods market), where $\bar{\omega}^i$ denotes the wealth of each type of agent in state Υ in equilibrium.

(4) Aggregate laws of motion are consistent with individual behavior: (a) $K' = (1 - \delta)K + \Phi(I(\Upsilon)/K)K$; (b) $B' = (1 - \mu)b^n(\bar{\omega}^n, \Upsilon)$; and (c) $\widehat{M}' = (1 - \mu)\hat{m}^n(\bar{\omega}^n, \Upsilon)$.

(5) There exists an invariant probability measure $\mathbf{P}$ defined over the ergodic set of equilibrium distributions.⁹

3 Quantitative Analysis

3.1 Solution Methods

The solution method used is a direct application of policy function iteration proposed by Coleman (1989, 1990). This gives a global solution over the entire state space. There are two other methods that are popular for solving this class of models. The first one is value function iteration method, which is used by Krusell and Smith (1997), Storesletten et al. (2007) and Guvenen (2009), among others. The application of this method to incomplete asset pricing models is computationally inefficient. The second one can be roughly categorized as approximation methods, including

⁹Details for solving the model are in *Appendix B*.

loglinear approximation (e.g. Backus et al. 2007, 2010), affine method (e.g. Shamloo and Malkhozov (2010)), perturbation methods (e.g. Malkhozov and Shamloo, 2009b; Kim et al. (2005)). The problem with this method is that it gives local solutions around where you approximate (usually, the steady state). When the problem of interest is actually not in steady state, the local solution is useless. Second, the far away from the steady state, the bigger the approximation errors are. This may give you misleading policy functions and particularly underestimate the fluctuations in asset prices, which is important for explaining asset pricing facts.

To solve the model, I substitute individual state variables out and keep only aggregate state variables. The new system of equations then is used as an input for the solver. The computational algorithm is detailed in Appendix B. It turns out my algorithm is much faster than that of Guvenen (2009). A test of running the model without money takes about 250 hours on a 3-GHz Intel dual-core Xeon cpu by his algorithm while mine only several minutes.

3.2 Baseline Parameterization

A model period corresponds to one month of calendar time. Table 1 summarizes the baseline parameter choices. I start with the parameterization of productivity and money growth. As for the technology shock, the AR(1) coefficient ρ_z is set to be 0.976 at monthly frequency in order to match the 0.95 autocorrelation of Solow residuals at quarterly frequencies. The standard deviation σ_z is set to be 0.015 to match the standard deviation of H-P filtered output in quarterly data. Similarly, ρ_g is set to be 0.17 at monthly frequency to match the 0.005 autocorrelation of $M0$ growth rate in US data. For the benchmark model, $\bar{g}$ is set to be 1.0025 to match a 3% annual inflation. The variance of money growth, σ_g , is set to be either 0 for only expected inflation or 0.0045 for unexpected inflation.

EIS parameters for stockholders and non-stockholders are set to be 0.3 and 0.1, respectively.¹⁰ I set discount factor β to be 0.93 to get 1.69% risk-free return in the benchmark model. I then set depreciation rate $\delta = 0.02$, capital share $\theta = 0.36$ to roughly match the US capital-output ratio of 8 in quarterly data. Capital adjustment cost coefficient is set to be 0.99 to match relative volatilities of consumption and investment over output.¹¹ Participation rate and borrowing constraints are the same to those in Guvenen (2009).

Table 1 here!

3.2.1 Portfolio Adjustment Cost

It deserves special discussion on choosing the value of ϕ , which governs how costly households adjust their bond positions. Theoretically, ϕ can be calibrated by the ratio of value added by financial department over GDP in national income account, which is approximately 5% in US. However, changing the value of ϕ gives no difference in the model economy and the simulated results show that the ratio of cost incurred over output is always negligible and close to 0.¹² Nevertheless, such a cost must exist to make bonds perform different from money in the sense of getting risk-free return. By experimenting with different values, I find that ϕ should be bigger than 0.1. Here I set it to be 1 for simplicity.

¹⁰However, debates about the right values for these parameters persist. See Guvenen (2006), Blundell et al. (1994), among others.

¹¹Capital adjustment cost function takes the form of $\Phi(I_t/K_t) = a_1(I_t/K_t)^{1-1/\xi} + a_2$, where $a_1 = \frac{\delta^{1/\xi}}{1-1/\xi}$, and $a_2 = \frac{\delta}{1-\xi}$. The parameter ξ governs how easily investment can be transformed into capital.

¹²Some others also find that this cost typically is very small, though they have a different modeling setup. For instance, Barber and Odean (2000) calculate a similar cost varying from 0.01% to 0.1% of the portfolio value.

3.2.2 Utility Functions

I consider two different specifications for the current utility function. First, I begin with the case where labor supply is elastic as a benchmark. The current utility therefore takes the form of Greenwood et al. (1988):

$$u(c_t^i, l_t^i) = \left[c_t^i - \psi \frac{(l_t^i)^\xi}{\xi} \right]^{\rho^i} \quad (13)$$

where l_t^i is the labor supply of each agent at period t . There is no uniform agreement about the correct value of the Frisch elasticity $\frac{1}{\xi-1}$. So I set it to be 1/3 for the benchmark model and try different values, including an estimate of 1 from Kimball and Shapiro (2003), to see the model performance. Finally, ψ is chosen to match a target value of $\bar{L} = 0.33$. In order to provide a simple comparison, I also consider the case with inelastic labor supply and assume that current utility function is of the standard power form: $u(c_t^i, l_t^i) = c_t^{\rho^i}$.

3.2.3 Transaction Cost of Consumption

The transaction cost function takes the following form:

$$\Psi(c, \frac{m^i}{P}) = \zeta c \exp(-\alpha \frac{m^i}{Pc})$$

where ζ and α are positive constants. One can prove that this form satisfies all the conditions claimed by Feenstra (1986).¹³ Here we treat it as the cost of maintaining the ATM/payment system. To calibrate ζ and α , we use the data from the FRED database of Federal Reserve Bank of St. Louis: Consumption is real monthly expenditures on nondurables (PCENDC96) and services (PCESC96); the money supply is $M0$ (CURRSL); real balances are the money supply divided by GDP deflator (GDPDEF). The average monthly expenses for one ATM are roughly \$1450 in 2006.¹⁴ In the same year, the total number of ATM used in US is 395,000.¹⁵ All quantity variables are divided by the resident U.S. population (CNP16OV). Our first goal is to match the average transaction cost, which is 1.2% of consumption goods.¹⁶¹⁷

Following Robert E. Lucas (2000) and Ireland (2009), the second goal is to match the money demand elasticity on interest rate with the following form:

$$\ln(m) = \ln(B) - \xi r,$$

where $\hat{\xi} = -1.88$ is estimated from the data. We vary the values of ζ and α to match these two targets and get $\zeta = 0.05$ and $\alpha = 0.6$, respectively.

¹³Note that we are using a different form from those used in Bansal and Coleman (1996) and Marshall (1992), where a power function is used. The reason that we use the exponential form rather than the power form is that it avoids the possibility of such a solution that one agent holds negative money balance in the latter case. Also the definition of transaction cost is different from Schmitt-Grohe and Uribe (2004).

¹⁴Data source: 2006 ATM deployer study.

¹⁵See <http://www.creditcards.com/credit-card-news/atm-use-statistics-3372.php>.

¹⁶Humphrey et al. (2003) have a different estimation of the cost of payment system and the benefit from using more ATMs.

¹⁷Marshall (1992) estimates a 0.8% cost of output. Barber et al. (2009) use a complete trading history of all investors in Taiwan and find that individual investor losses are equivalent to 2.2 percent of Taiwan's GDP or 2.8 percent of total personal income.

4 The Role of Money in Asset Pricing

Table 2 shows the performance of the benchmark model. Compared with the literature, I have got a better matching to the basic asset pricing and business cycle facts. Equity premium is 5.29% percent a year while risk-free return is 1.69%. It also delivers a Sharpe ratio (0.25) close to that in the data (0.32). While the variances of output growth, the relative volatility of investment are close to those in the data, volatilities of consumption and labor are not so well matched, which have been found hard to match in the literature.

Table 2 here!

The model is also consistent with what are empirically documented about the relations between inflation and asset prices, given the source of inflation is technology shocks. For example, Tatom (2011) documents that Inflation and real stock prices are negatively correlated, depending on the sources of inflation. This relation is mostly apparent during Great Inflation, 1965-84. Giovannini and Labadie (1991) documents that when inflation is high, realized real stock returns and interest rates are low, and vice versa. The main channel here is via no-arbitrage between money and other real assets. Table 3 reports that a negative relationship between inflation and real stock prices, real stock returns and real interest rates. The comovement of inflation and nominal interest rate is no surprise. Finally, the model gives a very high negative correlation between inflation and equity premiums, which might be debatable. The logic of this high relation is: Suppose these a positive shock, output is high and hence the equity price, which leads to high stock prices; at the same time, non-stockholders require a lower risk-free return to save because their marginal utility of consumption is lower; given that money supply is constant, a lower price level (inflation) followed.

Table 3 here!

5 The Determinants of Declining Equity Premium

It is widely documented that the equity premium has been going down during the last three decades, the so-called Great Moderation; see Siegal (1999), Jagannathan et al. (2000), Clause and Thomas (2001), and Campbell (2007). However, the reason for such a trend is still on debate. Besides a declining volatility of technology shocks and improvement of market imperfection, one competing explanation is that this is due to the decrease of inflation since sustained low inflation implies less uncertainty about the future.¹⁸ Beirne and de Bondt (2008) claims that these two are closely related. Kyriacou et al. (2006) shows that inflation can exaggerate equity premium. Labadie (1989) established a endowment economy model to explore the two ways of inflation to affect equity premium, namely by the assessment of an inflation tax and the presence of an inflation premium.

Here I explore the role of decreasing inflation by experimenting with different money growth rates. Table 4 shows that the decrease of inflation is one source of the declining equity premium. However, the equity premium goes down only 0.02 – 0.1 percent by one percent decrease in inflation.¹⁹ Given the the inflation decreased from an average of 8% to 2%, the decline that can be

¹⁸See Campbell and Vuolteenaho (2004), Ritter and Warr (2002), Lettau et al. (2008), for example.

¹⁹It needs mention that it is the expected inflation, not the unexpected one, that determines or closely relates to the equity premium, in contrast with Labadie (1989).

explained by inflation is only approximately 0.22 percent.²⁰ Also note that as inflation goes higher, its role in driving equity premium is deteriorating. This shows that hyperinflations do not lead to infinitely higher equity premia.

Table 4 here!

The reason is that money has two roles, transaction and intertemporal saving. As money growth (inflation) becomes higher, money's role for saving will be dominated by bonds, which means that the demand curve for money has a kink as money growth rate is higher. The role of money in driving down the equity premium can also be seen by the Euler equations that price bonds and stocks. The pricing kernel of the stockholders is

$$M_{t+1}^h = \beta \left(\frac{V_{t+1}^h}{[E_t V_{t+1}^{h1-\sigma^h}]^{\frac{1}{1-\sigma^h}}} \right)^{1-\rho^h-\sigma^h} \left(\frac{c_{t+1}^h}{c_t^h} \right)^{\rho^h-1} \frac{1 + \Phi'(\hat{m}_t^h / \hat{P}_t c_t^h)}{1 + \Phi'(\hat{m}_{t+1}^h / \hat{P}_{t+1} c_{t+1}^h)} \quad (14)$$

and

$$E_t M_{t+1}^h R_{t+1}^{EP} = 0 \quad (15)$$

where $R_{t+1}^{EP} = \frac{P_{t+1}^s + D_{t+1}}{P_t^s} - \frac{1}{\bar{P}}$ is the equity premium. From equation (15) we get $E_t M_{t+1} E_t R_{t+1}^{EP} + Cov(M_{t+1}^h, R_{t+1}^{EP}) = 0$,²¹ where we conclude that money matters because M_{t+1}^h depends on money growth via portfolio reallocation and so does $Cov(M_{t+1}^h, R_{t+1}^{EP})$.²² Since the real value of money is only a small fraction of agents' wealth, its role in resolving equity premium puzzle is limited. As money growth goes up, the real value of money carried from previous period, $\frac{\hat{m}^i}{\hat{P}_g}$, is decreasing with a slower rate, and thus inflation drives up less and less equity premium. That is, as money growth rate goes up, the uncertainty induced by inflation is decreasingly declining.

6 The Welfare Effects of Money: Quantitative Estimations

Compared with a standard classic model, this section answers the following two questions:²³ First, what is the optimal money growth rate? Second, should monetary policy be countercyclical or respond to asset prices, such as equity prices or expected equity premium? We answer these two questions by comparing the welfare under different policy regimes. Specifically, suppose $V^B(\Upsilon)$ and $V^A(\Upsilon)$ are the value functions for the benchmark and under the alternative monetary policies. The welfare change is measured as percentage change of consumption in the benchmark, that is, to find τ such that

$$\begin{aligned} V^A(\Upsilon) &= V^B(\Upsilon)(1 + \tau) \\ &\equiv \left[(1 - \beta)u(c(1 + \tau), l)^\rho + \beta(E(V(\omega'; \Upsilon')|Z, g)^{1-\sigma})^{\frac{\rho}{1-\sigma}} \right]^{\frac{1}{\rho}} \end{aligned} \quad (16)$$

²⁰Tough it is not the theme of this paper to estimate contributing role of the declining variance of technology shocks, a rough estimation shows that it explains most of the decrease in equity premium. It can be shown that inflation depends approximately on the ratio of money growth rate to technology growth rate, i.e., $\pi \propto \frac{g^M}{g^c}$ where $g^c \approx 1/3g^z$; and hence, $\pi \propto 3\frac{g^M}{g^z}$, which shows that change of g^z plays a more important role.

²¹Equity premium is thus $E_t R_{t+1}^{EP} = -Cov(M_{t+1}^h, R_{t+1}^{EP})/E_t M_{t+1}$.

²²In the representative agent model as I present in Appendix A, the pricing kernel becomes $M_{t+1} = \beta \left(\frac{V_{t+1}}{[E_t V_{t+1}^{1-\sigma}]^{\frac{1}{1-\sigma}}} \right)^{1-\rho-\sigma} \left(\frac{c_{t+1}}{c_t} \right)^{\rho-1} \frac{1+\Phi'(1/\bar{P}c)}{1+\Phi'(1/\bar{P}'c)}$, which is not affected by the injection of money.

²³See Appendix A for the counterpart of the model presented in this paper, which is a representative agent model.

In doing so, we can get the welfare change for each state vector Υ . I then simulate a long time-series ($T = 50,000$) under the benchmark and then calculate the average welfare change $\bar{\tau}$. If $\bar{\tau}$ is positive (negative), then we say there is a welfare gain (loss).

6.1 The Optimal Money Growth Rate

In the last section, we have concluded that risk premia are of different size since different money growth rates deliver different risks and, in particular, different risk-sharing allocations in the economy. Table 5 reports the welfare implications of different expected money growth rates, comparing with the zero-inflation case.

Table 5 here!

As shown in the table, the total welfare, the welfare of stockholders and non-stockholders are decreasing with inflation rates. Therefore, zero money growth rate is not optimal. In contrast, a deflation is welfare improving. The logic is that now there is less uncertainty about inflation, and thus less precautionary saving motivation from non-stockholders, which then means that they consume more and buy less bonds. Stockholders now have to pay higher return to borrow, but they also borrow less. The equilibrium is that both of them better off.

6.2 Implications of Alternative Monetary Policies

This subsection considers three alternative monetary policies, compared with the benchmark model. The first case is whether monetary policies should be procyclical or countercyclical, where we set $\rho_g = \theta_{EP} = \theta_{PE} = 0$ and $\theta_y = -0.05, -0.025, -0.01, 0.01, 0.025, 0.05$, respectively. Figure 1 shows the equity premia and risk-free returns with each parameter values. It turns out that procyclical policy tends to drive up risk-free return and down the equity premium, where equity returns are of almost no changes. The welfare change is shown in Panel A of Table 6. In contrast with the popular "leaning against the wind" advice, I find that procyclical monetary policy is welfare-improving. The logic is similar as in the last subsection. Procyclical money injection makes the saving role of money stronger and holding money now is less risky, which amounts to less uncertainty about inflation. Higher return of money transmits to higher return on bonds and lower equity premium via no-arbitrage.

Figure 1 here!

The second is to explore whether monetary policy should respond to asset prices, say real equity prices. In this case, $\rho_g = \theta_y = \theta_{EP} = 0$ and $\theta_{PE} = -0.05, -0.025, -0.01, 0.01, 0.025, 0.05$, respectively. Figure 2 shows again the equity premia and risk-free return with different coefficients of monetary policy responding to equity prices. Welfare implications are similar to the rules reacting to business cycles, as shown in Panel B of Table 6. The reason for this result is that asset prices are highly correlated with output ($\text{corr}(P_t^s, y_t) \approx 0.999$).

Figure 2 here!

Table 6 here!

Finally, I consider the case where monetary policy responds to (expected) equity premiums. If equity premium is high, policy makers view that there is higher risk in the economy and thus mop it down by setting θ_{EP} positive, which is a countercyclical policy. Without surprise, such a policy drives down equity premium and thus improves the welfare of the whole economy, as shown in Figure 3 and Panel C of Table 6.

Figure 3 here!

7 Conclusion

This paper first builds a monetary model with production and limited stock participation to reconcile with asset prices and macroeconomic data. Specifically, it not only resolves the equity premium and risk-free return puzzles, matches volatilities of macro fundamentals, but also is in line with empirical findings about the relations among inflation, money growth rate and asset prices.

This model is then used to estimate alternative monetary policies, compared with a standard classic model where there is only one representative agent model. In contrast with conventional wisdom of Friedman rule, saying optimal inflation should be the negative of real returns on other assets, this paper shows that monetary policies are welfare improving if they drive down the equity premium and raise risk-free returns. This manifests a procyclical monetary policy, a positive response of monetary policy to stock prices and risk premiums.

Appendix A: The Representative Model

There is only one stand-in household who live forever in the economy. The population is normalized to be unity, who is endowed with one unit of time every period, which he allocates between market work and leisure. The agent has similar Epstein-Zin-Weil preference as in the heterogeneous model:

$$U_t = \left[(1 - \beta)u_t + \beta(E(U_{t+1})^{1-\sigma})^{\frac{\rho}{1-\sigma}} \right]^{\frac{1}{\rho}} \quad (\text{A1})$$

where u_t is again current utility, a function of consumption c_t and labor l_t . The household has a similar budget constraint as stockholders:

$$c_t + P_t^f b_{t+1} + P_t^s s_{t+1} + \frac{m_t}{P_t} + \Psi(c_t, \frac{m_t}{P_t}) \leq b_t + s_t(P_t^s + D_t) + W_t l_t + \frac{m_{t-1} + (g_t - 1)M_{t-1}^s}{P_t} \quad (\text{A2})$$

where the form of transaction cost Ψ is kept the same.

The problem of the firms is unchanged except now the value of the firm is discounted by the MRS of the representative agent, $\{\Lambda_{t,t+j}\}_{j=1}^{\infty}$. Since there is only one type of agents, they can buy both stocks and bonds. As I did in the paper, a change in variables is introduced so that the problem solved by the households is stationary. That is, let $\hat{m}_t = m_t/M_t^s$ and $\hat{P}_t = P_t/M_t^s$. The state vector becomes $\Upsilon = (K, Z, g)$. The dynamic programming of the household can be expressed as follows:

$$V(\omega; \Upsilon) = \max_{c, l, b', s', \hat{m}'} \left[(1 - \beta)u(c, l)^{\rho} + \beta(E(V(\omega'; \Upsilon')|Z, g)^{1-\sigma})^{\frac{\rho}{1-\sigma}} \right]^{\frac{1}{\rho}} \quad (\text{A3})$$

s.t.

$$\omega + W(\Upsilon)l \geq c + P^f(\Upsilon)b' + P^s(\Upsilon)s' + \frac{\hat{m}'}{\hat{P}(\Upsilon)} + \Psi(c, \frac{\hat{m}'}{\hat{P}(\Upsilon)}) \quad (\text{A4})$$

$$\omega' = b' + s'(P^s(\Upsilon') + D(\Upsilon')) + \frac{\hat{m}' + g' - 1}{\hat{P}(\Upsilon')g'} \quad (\text{A5})$$

$$K' = \Gamma_K(\Upsilon) \quad (\text{A6})$$

In equilibrium, $\hat{m}_t = 1, b_t = 0$ and $s_t = 1$ for all t , and the budget constraint (A4) becomes $c_t + \Psi(c_t, \frac{1}{\hat{P}_t}) = Y_t - I_t$.

Appendix B: Model Solution and Computational Algorithm

Let λ^i and μ^i be the Lagrange multipliers of budget constraint and bond borrowing constraint, respectively. Then solved Euler equations are as follows:

$$\begin{aligned}
 (A1) : V^i(\omega^i; \Upsilon) &= \left[(1 - \beta)u(c^i, 1 - l^i)^{\rho^i} + \beta \left[E(V^i(\omega^{i'}; \Upsilon') | Z, g)^{1 - \sigma^i} \right]^{\frac{\rho^i}{1 - \sigma^i}} \right]^{\frac{1}{\rho^i}} \\
 (A2) : c^i : V^{i1 - \rho^i} (1 - \beta) u^{i\rho^i - 1} u_{c^i}^i &= \lambda^i [1 + \Psi(c, \frac{\widehat{m}^{h'}}{\widehat{P}})] \\
 (A3) : l^i : V^{i1 - \rho^i} (1 - \beta) u^{i\rho^i - 1} u_{l^i}^i &= \lambda^i W \\
 (A4) : b^i : V^{i1 - \rho^i} \beta (E(V^{i'}(.))^{1 - \sigma^i})^{\frac{\rho^i}{1 - \sigma^i} - 1} E \left[V^{i'}(.)^{-\sigma^i} \lambda^{i'} (1 + \phi(b^{i''} - b^{i'})) \right] &= \lambda^i (P^f + \phi(b^{i'} - b^i)) - \mu^i \\
 (A5) : s^i : V^{h1 - \rho^h} \beta (E(V^{h'}(.))^{1 - \sigma^h})^{\frac{\rho^h}{1 - \sigma^h} - 1} E \left[V^{h'}(.)^{-\sigma^h} \lambda^{h'} (P^{s'} + D') \right] &= \lambda^h P^s \\
 (A6) : K^i : \beta (E(V^{h'}(.))^{1 - \sigma^h})^{\frac{\rho^h}{1 - \sigma^h} - 1} E \left\{ V^{h'}(.)^{1 - \rho^h - \sigma^h} u^{h\rho^h - 1} u_{c^h}^{h'} \times \right. \\
 \left. \left[f_1(K', L') + \Phi'(\frac{I'}{K'})^{-1} [1 - \delta + \Phi(\frac{I'}{K'}) - \Phi'(\frac{I'}{K'}) \frac{I'}{K'}] \right] \right\} &= u^h \rho^h - 1 u_{c^h}^h \Phi'(\frac{I}{K})^{-1}.
 \end{aligned}$$

In the above, we redefine $\mu_+^i = \max\{\mu, 0\}^2$ the multipliers on the borrowing constraints and $\mu_-^i = \max\{-\mu, 0\}^2$, following Garcia and Zangwill (1981). This allows us to turn a system of Kuhn-Tucker conditions into a standard system of nonlinear equations, which can then be solved using standard methods. Borrowing constraints for these two agents are:

$$\begin{aligned}
 (A7) \quad \mu_-^h &= b^{h'} - \underline{B}, \text{ and} \\
 (A8) \quad \mu_-^n &= b^{n'} - \underline{B}, \text{ respectively.}
 \end{aligned}$$

Define $B' = (1 - \mu)b^{n'}$ and get $b^{n'} = B'/(1 - \mu)$. With bond market clearing condition, I have $b^{h'} = -B'/\mu$. Similarly, define $\widehat{M}' = (1 - \mu)\widehat{m}^{n'}$ and get $\widehat{m}^{n'} = \widehat{M}'/(1 - \mu)$. With money market clearing condition, I have $\widehat{m}^{h'} = (1 - \widehat{M}')/\mu$. By plugging these formulas into the system of equations above, I substituted out individual state variables in the system. And the new equation system is the input for the solver. The algorithm is then to solve $K'(\Upsilon)$, $I(\Upsilon)$, $B'(\Upsilon)$, $\widehat{M}'(\Upsilon)$, $C^h(\Upsilon)$, $C^n(\Upsilon)$, $\widehat{P}(\Upsilon)$, $P^s(\Upsilon)$, $P^f(\Upsilon)$, $\lambda^h(\Upsilon)$, $\lambda^n(\Upsilon)$, $\mu^h(\Upsilon)$, $\mu^n(\Upsilon)$, $T(\Upsilon)$, $V^h(\Upsilon)$, $V^n(\Upsilon)$, $l^h(\Upsilon)$, $l^n(\Upsilon)$ following these steps:

Step 1. Generate a discrete grid for the economy's capital, bond and money positions: $G_K = \{K_1, K_2, \dots, K_{NK}\}$, $G_B = \{K_1, K_2, \dots, K_{NB}\}$, $G_{\widehat{M}} = \{K_1, K_2, \dots, K_{N\widehat{M}}\}$ and the shock state spaces $G_Z = \{K_1, K_2, \dots, K_{NZ}\}$, $G_g = \{K_1, K_2, \dots, K_{Ng}\}$. Choose an interpolation scheme for evaluating the functions outside the grid of capital, bonds and money. I use 7, 12, and 10 points in the grid for capital, bonds and money, respectively. Functions are interpolated using a piecewise linear approximation.

Step 2. Conjecture $K'^j(\Upsilon)$, $I^j(\Upsilon)$, $B'^j(\Upsilon)$, $\widehat{M}'^j(\Upsilon)$, $C^{hj}(\Upsilon)$, $C^{nj}(\Upsilon)$, $\widehat{P}^j(\Upsilon)$, $P^{sj}(\Upsilon)$, $P^{fj}(\Upsilon)$, $\lambda^{hj}(\Upsilon)$, $\lambda^{nj}(\Upsilon)$, $\mu^{hj}(\Upsilon)$, $\mu^{nj}(\Upsilon)$, $T^j(\Upsilon)$, $V^{hj}(\Upsilon)$, $V^{nj}(\Upsilon)$, $l^{hj}(\Upsilon)$, $l^{nj}(\Upsilon) \forall K \in G_K, \forall B \in G_B, \forall \widehat{M} \in G_{\widehat{M}}, \forall Z \in G_Z, \forall g \in G_g$, where superscript j indexes the iteration number. Set $j = 1$.

Step 3. Solve for the values of $K'^{j+1}(\Upsilon)$, $I^{j+1}(\Upsilon)$, $B'^{j+1}(\Upsilon)$, $\widehat{M}'^{j+1}(\Upsilon)$, $C^{hj+1}(\Upsilon)$, $C^{nj+1}(\Upsilon)$, $\widehat{P}^{j+1}(\Upsilon)$, $P^{sj+1}(\Upsilon)$, $P^{fj+1}(\Upsilon)$, $\lambda^{hj+1}(\Upsilon)$, $\lambda^{nj+1}(\Upsilon)$, $\mu^{hj+1}(\Upsilon)$, $\mu^{nj+1}(\Upsilon)$, $T^{j+1}(\Upsilon)$, $V^{hj+1}(\Upsilon)$, $V^{nj+1}(\Upsilon)$, $l^{hj+1}(\Upsilon)$, $l^{nj+1}(\Upsilon) \forall K \in G_K, \forall B \in G_B, \forall \widehat{M} \in G_{\widehat{M}}, \forall Z \in G_Z, \forall g \in G_g$.

Step 4. Iterate Step 3 until convergence. I require maximum discrepancy (across all points in the state space) between consecutive iterations to be less than 10^{-7} for aggregate capital, bonds, money, and value functions of each agent.

Appendix C: Tables and Figures

Table 1: Baseline Parameterization

parameter		Value
β^*	Time discount	0.93
$1/(1 - \rho^h)$	EIS for stockholders	0.3
$1/(1 - \rho^n)$	EIS for non-stockholders	0.1
μ	Participation rate	0.2
ρ_z^*	Persistence of aggregate shocks	0.95
ρ_g^*	Persistence of aggregate money supply	0.005
θ	Capital share	0.36
ξ	Capital adjustment cost coefficient	0.99
δ^*	Depreciation rate	0.02
$\underline{B}$	Borrowing limit	$6\bar{W}$
$\bar{g}^{**}$	Average growth rate of money supply	1.03
σ_ε	Standard deviation of technology shocks	0.015
σ_g	Standard deviation of monetary shocks	0.0045
$\sigma^h = \sigma^n$	Relative risk aversion	10/6/2
χ	Firm leverage	0.15

”*” indicates that the reported value refers to the implied quarterly value for a parameter that is calibrated to monthly frequency, while ”**” indicates the implied annual value. $\bar{W}$ is the average monthly wage rate in the economy.

Table 2: Comparison of Statistics

	Asset Pricing Facts	Model
R^E	8.11% (19.30%)	6.98% (21.14%)
R^f	1.94% (5.44%)	1.69% (8.02%)
R^{EP}	6.17% (19.40%)	5.29% (21.55%)
Sharpe Ratio	0.32	0.25
	Business Cycle Facts	Model
$\sigma(Y)$	1.89	2.71
$\sigma(C)/\sigma(Y)$	0.40	0.79
$\sigma(C^h)/\sigma(Y)$	—	0.93
$\sigma(C^m)/\sigma(Y)$	—	0.72
$\sigma(I)/\sigma(Y)$	2.39	2.31
$\sigma(L)/\sigma(Y)$	0.80	0.26

Table 3: Correlations

	$Corr(\pi, R^f)$	$Corr(\pi, i^f)$	$Corr(\pi, R^s)$	$Corr(\pi, R^{EP})$	$Corr(\pi, P^s)$
Model Statistics	-0.08926	0.98911	-0.99418	-0.97979	-0.12590

Table 4: Different Money Growth Rates and Equity Premiums

Inflation (π)	-1%	0%	3%	6%	9%	12%
Equity Premiums	4.95%	5.05%	5.29%	5.46%	5.54%	5.59%

Table 5: Optimal Money Growth Rates

Inflation (π)	-1%	0%	3%	6%	9%	12%
Total	0.0068	0.0047	0	-0.0044	-0.0093	-0.0142
Stockholders	0.0080	0.0052	0	-0.0046	-0.0099	-0.0149
Non-Stockholders	0.0062	0.0045	0	-0.0043	-0.0091	-0.0138

Figure 1: Equity premiums and Risk-free Returns under the Alternative Monetary Policy: Responding to Business Cycles

Figure 2: Equity premiums and Risk-free Returns under the Alternative Monetary Policy: Responding to Equity Prices

Figure 3: Equity premiums and Risk-free Returns under the Alternative Monetary Policy: Responding to Equity Premiums

Table 6: Welfare Implications of Alternative Monetary Policies

	Parameters	S	N	Total Welfare
Benchmark	$\rho_g = \theta_{EP} = \theta_{PE} = \theta_y = 0$	0	0	0
Panel A: To business cycles	$\rho_g = \theta_{EP} = \theta_{PE} = 0$			
Procyclical	$\theta_y = 0.05$	0.0040	0.0042	0.0041
	$\theta_y = 0.025$	0.0011	0.0017	0.0015
	$\theta_y = 0.01$	0.0003	0.0006	0.0005
Countercyclical	$\theta_y = -0.01$	0.0000	-0.0004	-0.0003
	$\theta_y = -0.025$	-0.0010	-0.0016	-0.0014
	$\theta_y = -0.05$	-0.0021	-0.0033	-0.0029
Panel B: To equity prices	$\rho_g = \theta_{EP} = \theta_y = 0$			
	$\theta_{PE} = 0.05$	0.0113	0.0094	0.0101
	$\theta_{PE} = 0.025$	0.0034	0.0039	0.0038
	$\theta_{PE} = 0.01$	0.0008	0.0013	0.0012
	$\theta_{PE} = -0.01$	-0.0006	-0.0011	-0.0010
	$\theta_{PE} = -0.025$	-0.0021	-0.0030	-0.0027
	$\theta_{PE} = -0.05$	-0.0039	-0.0054	-0.0049
Panel C: To equity premium	$\rho_g = \theta_{PE} = \theta_y = 0$			
	$\theta_{EP} = 0.10$	-0.00005	0.00008	0.00004
	$\theta_{EP} = 0.075$	0.00001	0.00007	0.00005
	$\theta_{EP} = 0.05$	0.00008	0.00011	0.00010
	$\theta_{EP} = 0.025$	0.00018	0.00019	0.00019
	$\theta_{EP} = 0.01$	0.00028	0.00030	0.00029
	$\theta_{EP} = 0.00$	0.00015	0.00079	0.00058
	$\theta_{EP} = -0.01$	0.00019	0.00020	0.00020
	$\theta_{EP} = -0.025$	0.00013	0.00012	0.00012
	$\theta_{EP} = -0.05$	0.00005	0.00003	0.00004
	$\theta_{EP} = -0.075$	0.00000	-0.00003	-0.00002

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