Ownership networks and aggregate volatility^{*}

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Abstract

We study how aggregate volatility is influenced by the propagation of idiosyncratic shocks across firms through the network of ownership relations. To this purpose, we use detailed data on cross-holdings as well as relevant balance sheet information for almost the universe of Italian limited liability firms over the period 2005-2013. We first document that the ownership network matters for the correlation across firms' sales. Then, we construct a model where firms are linked through ownership relations and have limited access to credit markets. We characterize key features of the network structure that are relevant for the dynamics of the economy. A calibration to key features of the Italian economy shows that the model-implied volatility can account for a sizable percentage of actual GDP fluctuations. Moreover, we conduct a counterfactual exercise to isolate the role played by the network structure alone in the propagation of idiosyncratic shocks to the aggregate level.

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1 Introduction

In this work we study the relation between the ownership structure across firms and the aggregate volatility of an economy. The influence of the network structure on the aggregate performance has received a growing interest in recent times. The literature typically looks at production networks, trade linkages, or financial liabilities. This paper focuses instead on cross-participations among firms, that is, on firms' interconnections through the internal capital markets of corporate groups. These connections are 1) stable over time, 2) almost acyclic, and 3) potentially important for the propagation of firm-specific shocks, particularly in economies or periods in which firms have limited access to either bank credit or equity markets. We formulate a model where firms participate in both internal capital markets and external debt markets, whose access is possible only subject to collateral constraints à la Kiyotaki and Moore (1997). We calibrate the model to firm-level Italian data, and find that idiosyncratic shocks to firms can generate more than 10% of GDP volatility.

The macro literature is mainly concerned with size effects or Input-Output (I-O) interactions. The transmission mechanism of idiosyncratic shocks to the aggregate level relies on the relaxation of one of the two main hypothesis behind the Law of Large Numbers. Either agents are not homogeneous in size -the so-called Granular mechanism- or the agents' actions are not independent -the so-called Network mechanism-. For example, Gabaix (2011) shows how idiosyncratic shocks to firms can generate non-trivial aggregate fluctuations because of the existence of a fat-tailed size distribution of firms. The aggregate effects of a shock to a large firm are different from the effects of a shock to a small firm, so the more fat-tailed the size distribution of firms, the more likely to generate aggregate volatility the firm-specific shocks. Acemoglu, Carvalho, Ozdaglar, and Tahbaz-Salehi (2012) instead propose the second mechanism, that is, the interdependence in agents' actions.¹ Idiosyncratic shocks to sectors can generate aggregate fluctuations because of the presence of specific sectors that work as hubs in the flow of intermedi-

¹A stream that dates back to Long and Plosser (1983) and Long and Plosser (1987), and passes through Horvath (1998), Dupor (1999), Horvath (2000), Shea (2002), and Carvalho (2007), develops multisectoral RBC models where each sector uses the commodities or capital goods produced in the other sectors as intermediate goods in its own production. Sector-specific shocks can propagate through these production networks and generate aggregate fluctuations. The mathematical and conceptual toolbox for I-O analysis originate from the early work of Leontief (1941) and Hirschman (1958). Foerster, Sarte, and Watson (2011) show how this class of models generates equilibria whose dynamics resemble a dynamic factor model in reduced-form. Carvalho (2014) provides an extensive review of this literature.

ate goods across sectors. Depending on the I-O structure of the economy, sector-specific shocks can generate more or less aggregate volatility. The two mechanisms reflect two different theoretical focuses in the analysis of aggregate volatility. While the granular mechanism looks at the elements on the diagonal of the variance-covariance matrix of agents' outputs in equilibrium, the network mechanism examines the off-diagonal elements of the matrix. In di Giovanni, Levchenko, and Mejean (2014) the authors use French firm-level data to conclude that the network mechanism is twice as important as the granular mechanism in driving aggregate fluctuations.

We focus on ownership linkages. The ownership structure of firms may matter for aggregate volatility because for example the cash flow across firms that belong to the same corporate group may relax the credit constraints of the single firms. If a firm does well, the cash flow it generates permits other firms within the same group to relax their credit constraints and perform well, too. Thus, the propagation of idiosyncratic shocks through the ownership relations can be informative about the length and depth of aggregate business cycles. Moreover, while in normal times firms can access the credit markets on their own, when there is a crisis and credit conditions tighten firms seek alternative ways to finance their activities. We argue that the internal capital markets that arise within corporate groups especially in economic downturns help in explaining the dynamics of the aggregate economy.² The propagation of shocks through ownership relations can take place either vertically or horizontally along the pyramidal structure of a corporate group. In the former case, this can be due to either tunneling, if the propagation happens "upstream" with respect to the line of ownership links, or propping, if the propagation happens "downstream" the ownership lines.³ In the latter case, this can be the product of either a winner-picking or a cross-subsidization attitude by the owner of two subsidiaries.⁴ More in general, internal capital markets can arise within a corporate group depending on the situation of the external capital markets but also of the single member firms. See Gertner, Scharfstein, and Stein (1994) for a theoretical analysis of costs and benefits of the access to internal versus external capital markets by a member firm of a corporate group, and Lamont (1997) for evidence of how such internal

²On the relation between credit availability and investment in Italy, see Gaiotti (2013).

³See Riyanto and Toolsema (2008) for a theoretical explanation of these two phenomena and Dow and McGuire (2009) for empirical evidence on these issues from Japanese *keiretsu*.

⁴See Cestone and Fumagalli (2005) for a stylized analysis of the theoretical underpinnings behind these two alternative strategies, Brusco and Panunzi (2005) for the corporate governance drawbacks in terms of managerial incentives that these strategies can generate, and Bulow, Geanakoplos, and Klemperer (1985) for an early analysis of the macroeconomic consequences of inter-market complementarities in firms' strategies.

capital markets can induce correlation among firms within a corporate group whose economic performance would be otherwise uncorrelated.⁵

The focus of the literature on the I-O network, and more in general on intersectoral linkages, has the advantage of relying on a network structure that seems fairly stable in the extensive margin while changing over the development path in the intensive margin.⁶ This justifies the use of the network structure as a fixed aspect of the technology of the models rather than as a product of an equilibrium interaction.⁷ Other forms of network interdependence across agents such as credit liabilities across firms or trade relations across countries seem less robust on this aspect, and that is the reason why these networks are usually treated as equilibrium constructs.⁸ Nevertheless, ownership linkages among firms are less variable than credit liabilities or trade relations. This is due to both juridical limitations in the transfers and acquisitions of firms' equity and strategic concerns of corporate governance.⁹ Moreover, the ownership network is almost acyclic, as there exists legal restrictions to the extensive presence of cycles in cross-holdings. This permits the interpretation of the diagonal elements of the variance-covariance matrix of firm-level performance as the result of only "granular" volatility.¹⁰ Hence, the ownership network across firms seems suitable as an alternative network mechanism of transmission of idiosyncratic shocks to the aggregate level.

We employ the database *Infocamere* to construct the network structure of ownership relations. This database contains information on almost the

⁵Samphantharak (2006) and Almeida and Kim (2012) provide country-specific empirical evaluations with data from Thailand and South Korea of the rise of internal capital markets within business groups and of the group-specific determinants of these resource reallocations.

⁶See, for example, Jones (2013) for an evolution of I-O tables for countries at different stage of development, and Hausmann and Hidalgo (2011), Atalay, Hortasu, Roberts, and Syverson (2011), Atalay, Hortaçsu, and Syverson (2014) for descriptive characterizations of the network structure of production.

⁷Oberfield (2011) and Carvalho and Voigtländer (2014) move beyond this assumption and endogeneize also the I-O architecture.

⁸See, among many others, Acemoglu, Ozdaglar, and Tahbaz-Salehi (Forthcoming), Elliott, Golub, and Jackson (2014), and Cabrales, Gottardi, and Vega-Redondo (2014) for models of equilibrium financial networks, and Chaney (2014) for micro-founded equilibrium international trade networks.

⁹See Bianchi, Bianco, and Enriques (2001) and Bianco and Nicodano (2006) on the structure of pyramidal groups in Italy and their persistence over time.

¹⁰This reflection problem, to use the terminology of Manski (1993), is instead present in works that use cyclic networks such as the I-O tables, e.g., in di Giovanni, Levchenko, and Mejean (2014). This implies that most likely the share of aggregate volatility assigned to the granular mechanism in those contexts constitutes an upper bound to the actual share, provided that the cyclical iterations of the shocks have the same sign.

universe of Italian limited liability firms, with firm-specific data on the ownership structure and economic performance for the years 2005 to 2013. The information on the ownership structure is compulsory. The registration of the fiscal code of each owner and each firm allows the description of the whole topology of direct and indirect ownership relations. The ownership relations across firms have specific properties from a network theory point of view. They describe a directed, weighed, acyclic, incomplete network, where firms are partitioned into differently connected components, to which we can refer as corporate groups. Moreover, within each corporate group we can distinguish a pyramidal structure with one or more ultimate owners at the top and several subsidiaries at different levels of the hierarchical structure. Hence, any correlation between firms that may arise from the presence of ownership relations is structurally different from the complete cyclic network of I-O relations.¹¹ The inclusion of information about the network structure of ownership relations may thus complement the information of I-O relations and contribute to assign an even larger share of aggregate fluctuations to the propagation of idiosyncratic shocks from interconnected agents. This is especially true for the Italian case, where a large fraction of firms belongs to pyramidal corporate groups and an even larger fraction of total operating revenue is generated within these corporate groups instead of stand-alone firms.¹² Moreover, most of the firms are not quoted, which implies that there is no centralized market for these firms' shares.

We find that in general there is a positive correlation in the firm-level growth rate of operating revenue or operating revenue per worker among owners and participated firms. This is in line with the idea that the ownership network matters for explaining the business cycle. Hence, we construct a model where firms are linked by ownership relations. The set of firms is partitioned into different corporate groups, within each of which there is an ultimate owner firm that owns directly or indirectly all the other firms. Firms face collateral constraints à la Kiyotaki and Moore (1997) in their access to the international credit markets, which induces a different behavior of firms when the collateral requirements or the return rate on debt increase.¹³

¹¹Atalay, Hortaçsu, and Syverson (2014) provide evidence of the relative orthogonality between I-O relations and ownership links in the US, although limited to the case of different establishments of multidivisional firms.

¹²See Bianchi and Bianco (2006) for a descriptive analysis of corporate group structures in Italy.

¹³There are two levels of market incompleteness that we maintain from the original paper of Kiyotaki and Moore (1997). First, there exist only one-period, non-state contingent bonds. Second, each firm's installed capital can be operated only by the firm itself as a factor of production, and each firm cannot commit ex ante to employ the workers it uses in the production of the final good. These assumptions create the threat of debt repudiation

Idiosyncratic shocks to either productivity or collateral requirements affect the accumulation of capital of each firm decided by the ultimate owner of each corporate groups, thus generating a dynamic propagation of shocks across firms. We simulate stochastically a stylized version of the economy to understand how the aggregate volatility depends on different ownership network structures. Moreover, we calibrate the model to key moments of the Italian economy in order to quantify how much of the observed volatility between 2000 and 2013 is due to the propagation of idiosyncratic shocks through ownership relations. We find that idiosyncratic shocks can account for up to 11% of GDP volatility and up to 30% of the volatility of a BVAR's residuals containing both aggregate and idiosyncratic shocks. Lastly, we conduct a counterfactual exercise where we look at what the volatility is if there is no ownership links across firms. This allows us to disentangle the role played by the network of ownership links in the propagation of idiosyncratic shocks to the aggregate level. The exercise reveals that almost one fourth of model-implied volatility is due to the existence of a network structure across firms.

Our paper is close in spirit to Bigio and La'O (2013) as they look at financial frictions in production networks. The network structure of I-O transactions interacts with firm-specific pledgeability constraints, making downstream financial frictions distort upstream input use. Moreover, more vertical I-O structures generate the need for higher liquidity for any allocation and therefore amplify the elasticity of aggregate output to aggregate liquidity. Our model features both a network structure and financial frictions like Bigio and La'O (2013) but does not interact them, as our collateral constraint limits the access to external credit of the single firm independently from the situation of other firms. Thus, the wedges introduced by the financial frictions in firm-specific output depend on the network structure only through the aggregate stochastic discount factor and not through the bilateral firm-to-firm dependence.

The paper is organized as follows. In Section 2 we describe the network structure of ownership relations across Italian firms and its evolution through

that makes the creditors protect themselves through, e.g., the collateralization of debt. In the latter case, the value of the installed capital outside the firm -the liquidation value- is less than how much the firm itself values it, so the firm can systematically bribe the bank and retain the property of the installed capital to its liquidation value. In fact, firms never default on their debt and the market incompleteness takes the form of a collateral constraint that limits the control space in the firms' optimization problems. Dubey, Geanakoplos, and Shubik (2005) present instead a model where defaults occur in equilibrium and the microfoundation of the limits to credit accessibility relies not only on moral hazard but expands to adverse selection and signaling. We leave the exploration of propagation on networks in this context to future research.

time. Moreover, we show how the network of ownership relations influences the correlations between the operating revenues of different firms. In Section 3 we present the model. In Section 4 we simulate the model’s dynamics. Section 5 reports the calibration exercise. Section 6 concludes and proposes lines of future research. The appendix collects all proofs, the data description, and additional figures.

2 Empirical motivation

We use two different datasets in our analysis. The first one reports the ownership relations across all Italian limited liability firms from 2005 to 2013. The second reports the performance of Italian firms from 2005 to 2013. Both datasets are elaborations of the Infocamere database.¹⁴

2.1 The network of ownership relations

This dataset consists of an unbalanced panel that spans 9 waves between 2005 and 2013. Each observation is an ownership relation between a participant firm and a participated firm. Firms are identified by their fiscal code. We also have the detail on the individuals that hold a firm’s shares. Table 1 reports the distribution of the observations across waves and some summary statistics about the network structure. We interpret the ownership relations as links of

Year	Links	Firms	Inter-firm links
2005	2,194,299	768,489	240,651
2006	2,338,620	824,004	258,392
2007	2,369,917	850,535	260,696
2008	2,475,170	895,230	277,586
2009	2,683,885	980,258	309,976
2010	3,445,892	1,294,199	358,281
2011	3,478,461	1,329,161	358,011
2012	3,521,915	1,363,748	359,733
2013	3,530,002	1,375,829	356,204

Table 1: Details for each wave: Number of ownership links, Number of firms, and Inter-firm links.

a network, where firms and owners act as nodes and the share of a firm that each owner has as the strength of the link. If a node “owns” another node, the

¹⁴Details on the elaboration of the original data are provided in the appendix.

opposite is not necessarily true. Hence, our network is directed. There are no reciprocal holdings between firms, and no firm can own a relevant share of itself, that is, there are no self-links. There are isolate circumstances in which these restrictions may not hold, although there are legal limitations to these cases. In fact, there exist legal limitations even to the possibility of a firm owning itself indirectly through a series of ownership relations. Moreover, the ownership relations do not connect all the firm-nodes of the economy. There are disconnected sets of nodes, that is, components. Usually, since the

Figure 1: The firm as a Directed Acyclic Graph with a root.

links are directed and acyclic, we can always identify within each component one or more nodes that have a nil indegree, that is, that are ultimate owners of the firms contained in the component. Hence, the institutional framework dictates that the network of ownership relations is a series of components which are Directed Acyclic Graphs (DAG's), that is, a series of directed subgraphs with no cycles.¹⁵ As all DAG's, the ownership network is a planar

¹⁵Unlike a directed tree, the underlying graph of a DAG is not a tree, in that replacing the directed links with undirected links leads to a graph that may contain cycles. Hence, the rooted tree (a pyramidal corporate structure with an ultimate owner -a root-) is not the best representation for the components of the ownership network. Moreover, owners can be both firms and individuals. This may suggest that we are facing a (directed) bipartite

graph, an aspect which permits an intuitive visualization. Figure 1 provides an example of a DAG. The structure of the ownership links is complex, with the presence of several types of graph across different components.

We can analyze several characteristics of this network. First, we look at the strength of the ownership links. Figure 2 presents the frequency distribution of the link strengths, with the majority of the links concentrated around 50% -the level of ownership sufficient to exercise full control of the firm by the same owner- and the rest divided between an interval of weaker relations - from right above 0% to 33%- and the full ownership -100%-. Note that these thresholds, and especially 50% and 100%, reflect well-known facts about firms' corporate governance.

Figure 2: Distribution of inter-firm shares over the years. Each subplot reports the frequency distribution of ownership links in a given year.

Second, we consider the degree distributions. Given that we deal with a directed network, we have to distinguish between the indegree and the outdegree of each node. The indegree of a node in our framework is the

graph, with one group of agents owning the other group. Nevertheless, also this would be a partial representation of the ownership network, since members of the firm group may own each other.

number of owners that have participations in a node. By construction, the indegree is zero for all nodes that are individuals. The indegree distribution represents the frequency of nodes with a certain indegree. The outdegree of a node instead is the number of firms in which a node has participations. The outdegree distribution represents the frequency of nodes with a certain outdegree.¹⁶ We document the evolution of the network structure over time in Table 2. There does not seem to be relevant changes in the overall net-

Year	Average share	Average indegree	Average outdegree
2005	0.35	1.74	1.98
2006	0.36	1.74	1.98
2007	0.37	1.71	1.95
2008	0.37	1.70	1.98
2009	0.39	1.66	2.08
2010	0.38	1.66	2.00
2011	0.39	1.64	1.97
2012	0.40	1.63	1.95
2013	0.40	1.62	1.93

Table 2: Network characteristics for each wave. Average ownership share, Average participants for each firm (indegree), Average participations for each firm (outdegree).

work structure over the 9 years we consider. There seems to be a marginal increase in the concentration of ownership over time, which is certified by both the increase in the average shares from 35% in 2005 to 40% in 2013 and the decrease in the average indegree. In other words, ownership of limited liability firms seems to become more concentrated both in the intensive and in the extensive margin. The evolution of the average outdegree, that is, the number of participations, seems to show a decrease only after the peak in 2009.

2.2 The performance of Italian firms

We have an unbalanced panel of firm-level data that spans 8 years from 2005 to 2012. There are around 6.8M observations evenly distributed over time, from which we count around 1.5M firms identified by their fiscal codes. We have information about the operating revenue and the number of employees,

¹⁶In the appendix, Figure 16 reports the (log) indegree distribution in 2006 and Figure 17 reports the (log) outdegree distribution in 2006.

among other characteristics. We report some summary statistics about the sample in Table 3.

Year	Obs	Total sales	Avg sales	Total empl	Avg empl
2005	674,648	1,939,775,540,197	2,875,241	10,122,618	11
2006	695,919	2,355,155,242,712	3,384,238	9,978,636	11
2007	812,014	2,374,191,541,275	2,923,831	10,724,489	11
2008	896,167	2,168,598,597,352	2,419,860	11,480,218	11
2009	940,413	2,209,487,626,884	2,349,486	12,248,393	10
2010	956,191	2,393,628,411,359	2,503,295	-	-
2011	950,883	2,445,287,168,211	2,571,596	12,889,671	10
2012	905,992	2,448,882,818,251	2,702,985	9,178,686	9

Table 3: Summary statistics for each wave. Number of observations, Total sales, Average sales, Total employment, Average employment. Employment in 2010 is not available in the original database, as only a limited sample of selected firms reported a non-missing employment level in that year.

As of 2007, the total operating revenue of our sample is around €2.4 trillion, while Italian gross output is around €3.1 trillion (source: National accounts). Total employment of our sample amounts to almost 11M, while Italian total employment is 25.3M (source: National accounts). Hence, our sample covers around 2/3 of aggregate gross production and more than 1/3 of aggregate employment.

2.3 The propagation of shocks through ownership relations

We merge the two datasets by matching the fiscal code of each owner in the first database with the performance information contained in the second database for that fiscal code. We repeat this procedure for all the years from 2006 to 2012. In this way, we can check whether firms that share an ownership relation are likely to have similar economic performance. We take into account the growth rate of sales as a measure of firms' performance. In order to make estimates comparable, we standardize each firm's growth rate at the industry-period level. Hence, our measure u_{it} is such that

$$u_{it} = \frac{s_{it} - \mu_{st}}{\sigma_{st}},$$

where s_{it} is the growth rate of sales of Firm i at time t , μ_{st} is the average growth rate of sales in sector s at time t , and σ_{st} is the standard deviation

of the growth rate of sales in sector s at time t . Otherwise, correlations in different time periods or different sectors may appear different just as a result of either a different mean performance or a different standard deviation of such performance. In Figure 3, we find that there seems to exist a linear dependence in sales' growth between two firms that share an ownership link.

Figure 3: Linear dependence of growth rate of sales (standardized at the industry-period level).

Firms that belong to the same business group may perform similarly for reasons other than cross-participations. To control for this, we recalculate the correlation replacing u_{it} with a variable that measures each firm's performance relative to its group. We construct business groups using the ownership relations. We consider a threshold of more than 50% share to consider a participated firm as a member of the same business group of the participant firm. This approach has several implications that we discuss in Section 5. We obtain therefore a partition of the set of firms into different business groups, and control for the average performance of each group before comparing the sales' growth rates of two adjacent firms. In order to control for groups' performance, we run an OLS regression such that

$$s_{it} = \beta_0 + \beta_1 \underbrace{\mu_t}_{\text{Aggregate}} + \beta_2 \underbrace{\mu_{st}}_{\text{Sector}} + \beta_3 \underbrace{\mu_{ct}}_{\text{Group}} + u_{it},$$

where μ_t is the average growth rate of sales across all firms and μ_{ct} is the average growth rate of sales across firms that belong to the same business

group as Firm i . Hence, our alternative measure u_{it} is the residual of this OLS regression.¹⁷ Figure 4 illustrates the correlation in this case.¹⁸

Figure 4: Linear dependence of growth rate of sales, controlling for aggregate, sector, and group effects among firms.

In light of these results, we conclude that there is reduced-form evidence that there exist a dependence in the performance across firms that share an ownership relation. Hence, there is scope for the construction of a model that may provide a structural interpretation to this dependence.

3 The model

The model consists of a small open economy composed of firms and households. There are two international markets, the credit and the product market, and two national markets, the equity and the labor market. The credit market is characterized by an infinite supply of credit at the exogenous return

¹⁷For this exercise we must consider only business groups with at least 5 firms. By construction, there exists a negative correlation equal to -1 among firms that belong to groups with 2 member firms, once we subtract the group average. The same applies to firms that belong to groups of small size.

¹⁸We conduct also three robustness exercises, reported in the appendix. Figure 18 looks at potential nonlinearities in the dependence across adjacent firms' residual growth rates of sales. In Figure 19 we consider the growth rate of sales per worker instead of the growth rate of sales. In Figure 20 we consider only the correlation among firms that are linked by majority shares.

rate R_t , subject to collateral constraints. The product market is characterized by an infinite demand of goods at an exogenous price that we normalize to 1.

The households are identical and distributed on a continuum of mass 1. There is a countable and finite set $\mathcal{N}$ of firms. Each firm can own and be owned by other firms. We denote with θ_{ij} the direct share that firm i has in firm j . The direct and indirect ownership relations describe a partition of the set $\mathcal{N}$ of firms into different corporate groups $\mathcal{N}_u$, that is, $\coprod \mathcal{N}_u = \mathcal{N}$. Firms are structured within each corporate group in a pyramid defined by their ownership relations, with an ultimate owner firm u at the top of each corporate group and the rest of the firms j below it. The ultimate owner controls all the firms within its corporate group. Firms cannot own their own shares, and by construction ultimate owners are firms whose shares are not owned by any other firm. In the case of ultimate owners, only the households own their shares. Figure 5 illustrates with an example the structure of the economy.

Figure 5: The structure of the economy.

The equity market refers to the market for shares in the ultimate owners. There is no market for the shares of firms that are controlled by other firms, so

that the partition of firms into different corporate groups and the pyramidal structures within each group are fixed and part of the technology of the model. The labor market clears when the quantity of labor demanded by firms is equal to the quantity of labor supplied by households. We first describe more in detail the preference side and the technology side, and then move to the definition and description of the general equilibrium.

3.1 Households

There is a continuum of mass 1 of identical households. They cannot access the credit market and are the owners of all shares of firms that are not owned by other firms. Each household works, trades in equities, and consumes. It maximizes its utility that depends on consumption and leisure, that is,

$$\max \mathbb{E} \sum_{\tau=0}^{+\infty} \beta^\tau U(C_\tau - G(L_\tau)),$$

where C_t is consumption, L_t is labor supply, β is the discount factor, and $U(\cdot)$ is a constant-relative-risk-aversion (CRRA) utility function that is additively separable in consumption and leisure. In this way, there is no wealth effect on the labor supply. Differently from Greenwood, Hercowitz, and Huffman (1988) we assume that the function $G(\cdot)$ is linear in labor. Given the Gorman form of the utility, there exists a representative household whose problem is

$$\max \mathbb{E} \sum_{\tau=0}^{+\infty} \beta^\tau \frac{(C_\tau - \psi L_\tau)^{1-\sigma} - 1}{1-\sigma}, \quad (1)$$

where $\sigma > 0$ and $\psi > 0$. The household owns shares $\theta_{uu\tau}$ of ultimate owners u . It receives dividends $D_{u\tau}$ and trades only in the shares of the ultimate owners, as there does not exist a market for firms that are not ultimate owners. Hence, the household can buy and sell shares of ultimate owner u at the price $P_{u\tau}$. Moreover, the household works and earns the wage W_τ . Thus, the household's budget constraint is

$$\sum_{u \in \mathcal{U}} \theta_{uu\tau+1} P_{u\tau} + C_\tau \leq W_\tau L_\tau + \sum_{u \in \mathcal{U}} \theta_{uu\tau} (D_{u\tau} + P_{u\tau}), \quad (2)$$

where $\mathcal{U}$ is the set of ultimate owners. The first order conditions (FOC) of the household's problem yield

$$W_\tau = \psi, \quad (3)$$

and

$$P_{u\tau} (C_\tau - \psi L_\tau)^{-\sigma} = \beta \mathbb{E}_\tau [(C_{\tau+1} - \psi L_{\tau+1})^{-\sigma} (D_{u\tau} + P_{u\tau+1})], \quad (4)$$

for every u in $\mathcal{U}$. If we iterate (4) forward and impose a no-bubble condition we obtain that the price of u 's share at time τ is

$$P_{u\tau} = \mathbb{E}_\tau \left[\sum_{t=\tau}^{+\infty} \beta^{t-\tau} \frac{(C_t - \psi L_t)^{-\sigma}}{(C_\tau - \psi L_\tau)^{-\sigma}} D_{ut} \right], \quad (5)$$

for every u . Moreover, given the structure of the utility function, the budget constraint always binds and

$$C_\tau = W_\tau L_\tau + \sum_{u \in \mathcal{U}} \theta_{uu\tau} D_{u\tau} - \sum_{u \in \mathcal{U}} (\theta_{uu\tau+1} - \theta_{uu\tau}) P_{u\tau}. \quad (6)$$

Conditions (3), (5), and (6) describe the necessary optimality conditions for the preference side of the economy.

3.2 Firms

There is a countable and finite set $\mathcal{N}$ of firms, and a partition $\{\mathcal{N}_u\}_{u \in \mathcal{U}}$ of this set which represents the corporate groups. We assume that ownership linkages occur only across firms that belong to the same corporate group and not across firms of different corporate groups. In other words, the set of components of the ownership network coincide with the set $\{\mathcal{N}_u\}_{u \in \mathcal{U}}$ of corporate groups. We call ultimate owner a firm u in $\mathcal{N}$ such that $\theta_{iu} = 0$ for every $i \in \mathcal{N}$ and $i \neq u$. By construction, there exists at most one ultimate owner in each corporate group. We assume that an ultimate owner u controls the other firms that belong to the same corporate group $\mathcal{N}_u$.¹⁹

Let us first look at the single firms. Each firm j combines technology A_{jt} , capital K_{jt} , and labor L_{jt} to produce a unique homogeneous final good Y_{jt} according to

$$Y_{jt} = A_{jt}^{1-\epsilon} \left(K_{jt}^{\alpha_j} L_{jt}^{1-\alpha_j} \right)^\epsilon, \quad (7)$$

where $\epsilon \in (0, 1)$ the share of technology A_{jt} in output, $\alpha_j \in (0, 1)$ is the relative weight of capital, $1 - \alpha_j$ is the relative weight of labor. The firm-specific technical level A_{jt} is exogenous and follows an iid process that we

¹⁹In the data there may be various exceptions to these assumptions. Control may not coincide with direct or indirect ownership, a corporate group may not have a unique ultimate owner firm, the set of corporate groups is not necessarily a partition of the set of firms but rather a set of overlapping subsets of firms, and firms may share ownership links with firms of other corporate groups. We leave the discussion of all these issues to Section 5 about calibration.

specify below. By convention, we denote with K_{jt} the capital accumulated up to time t whose level is decided at time $t - 1$, while firms adjust the demand for labor L_{jt} at time t . Firm j distributes dividends D_{jt} in each period to its owners, and receives dividends from all other firms where it holds a share θ_{ji} . An ultimate owner distributes its dividends only to the households, while firms that do not have a share in any other firm do not receive any dividends. Firms access external financing through the emission of one-period non-state contingent bonds. Again, by convention we denote with B_{jt+1} the amount of bonds issued at time t to be repaid at time $t + 1$. The return rate R_t on bonds is exogenous and follows a process we specify below. Lastly, the firm has to pay the total wage bill $W_t L_{jt}$. Each firm is subject to a flow-of-funds constraint that balances outflows and inflows from and to the firm. The inflows of firm j are the sales Y_{jt} , the revenue $\sum_{i \in \mathcal{N}_j^{in}} \theta_{ji} D_{it}$ that comes in the form of dividends D_{it} distributed by any other firm i of which firm j holds a share θ_{ji} , and newly contracted debt B_{jt+1} . The outflows of firm j are instead the dividends D_{jt} it distributes, the total labor bill $W_t L_{jt}$ it pays, and the repayment $R_t B_{jt}$ of debt it had previously contracted, where R_t is the exogenous return rate in the international credit markets. The difference between inflows and outflows is devoted to internal investment I_{jt} . Hence, the flow-of-funds constraint of firm j is

$$D_{jt} + W_t L_{jt} + R_t B_{jt} + I_{jt} = Y_{jt} + \sum_{i \in \mathcal{N}_j^{in}} \theta_{ji} D_{it} + B_{jt+1}. \quad (8)$$

The capital K_{jt} represents the value of the assets in the active side of firm j 's balance sheet. Its law of motion is

$$K_{jt+1} = I_{jt} + (1 - \delta) K_{jt}, \quad (9)$$

where I_{jt} is the investment of firm j at time t and $\delta \in (0, 1)$ is the depreciation rate of capital.

Firms that issue bonds access the infinite supply of credit at rate R_t provided by the international credit markets. We suppose that this supply of credit is subject to a collateral constraint à la Kiyotaki and Moore (1997), that is,

$$B_{jt+1} \leq \kappa_{jt} K_{jt+1}. \quad (10)$$

The collateral consists of the fraction κ_{jt} of capital K_{jt+1} that creditors can recover out of the book-value liquidation of the firm's assets.²⁰ We assume

²⁰As discussed in Bianchi (2012), a market-value collateral introduces a fire-sale externality. However, since there are no adjustment costs on investment in our model, Tobin's Q is equal to 1 so the book value coincides with the market value. See Bianchi and Mendoza (2010) for an analysis of the fire-sale channel.

that κ_{jt} is the realization of an iid process on the support set $[0, 1]$. We assume that the firms in our model have a limited liability, so that the collateral provided in the credit contracts cannot exceed a portion of the installed capital of the single firm.²¹

In every period t a manager in each ultimate owner firm u maximizes the value P_{ut} of the ultimate owner firm. In order to do so, the manager drives the decisions of both the ultimate owner firm and all firms that are controlled by the ultimate owner, subject to all the constraints of the firms that belong to the corporate group $\mathcal{N}_u$. Let us call N_u the number of firms in the corporate group $\mathcal{N}_u$ whose ultimate owner is firm u . The ultimate owner faces N_u flow-of-funds constraints, N_u laws of motion for capital, and N_u collateral constraints. We can simplify the problem of the ultimate owner by nesting the flow-of-funds constraints into one. Let us represent the system of N_u flow-of-funds constraints in matrix form,

$$\mathbf{D}_t = \mathbf{Y}_t + \mathbf{B}_{t+1} - R_t \mathbf{B}_t - W_t \mathbf{L}_t - I_t + \mathbf{\Theta} \mathbf{D}_t,$$

where $\mathbf{D}_t$, $\mathbf{Y}_t$, $\mathbf{B}_{t+1}$, $\mathbf{B}_t$, $\mathbf{L}_t$, and $\mathbf{I}_t$ are all $N_u \times 1$ vectors with typical elements D_{jt} , Y_{jt} , B_{jt+1} , B_{jt} , L_{jt} , and I_{jt} , while $\mathbf{\Theta}$ is a $N_u \times N_u$ matrix with typical element θ_{ij} , where i and j belong to corporate group $\mathcal{N}_u$. The matrix $\mathbf{\Theta}$ represents the direct ownership relations. Then, the matrix

$$\mathbf{M} \equiv [\mathbf{I} - \mathbf{\Theta}]^{-1} = \sum_{k=0}^{+\infty} \mathbf{\Theta}^k, \quad (11)$$

where $\mathbf{\Theta}^k$ is the k -th power of $\mathbf{\Theta}$, represents the indirect ownership relations of any length k .²² The typical element of $\mathbf{M}$ is the indirect share (plus 1) m_{ij} of firm i in firm j , that is,

$$m_{ij} = \sum_{k=0}^{+\infty} \theta_{ij}^{[k]} = 1 + \theta_{ij} + \sum_{i'} \theta_{ii'} \theta_{i'j} + \sum_{i''} \sum_{i'} \theta_{ii''} \theta_{i''i'} \theta_{i'j} + \dots$$

²¹If the owners were partially or fully responsible for the debt contracted by the controlled firms, then the specification of the collateral constraint itself would not be the same. A creditor may add in the liquidation value of a firm's collateral the potentially "deep pockets" of the controlling owners. Moreover, this "unlimited liability" would open up the possibility for a unit of installed capital in a controlling firm to be used as "indirect" collateral in more than one controlled firm's credit contract. Given the nature of our data and the focus of the paper, we leave the exploration of these potential deviations for future research.

²²This geometric sum is finite because by construction there does not exist cycles of direct or indirect ownership relations. Hence, the powers of $\mathbf{\Theta}$ have nonnil entries only up to $\bar{k}$, where $\bar{k}$ is the maximal distance between the ultimate owner and the farthest subsidiary along the ownership lines. Since $\lim_{k \rightarrow \infty} \mathbf{\Theta}^k = \mathbf{0}$, the matrix $[\mathbf{I} - \mathbf{\Theta}]$ is nonsingular and its inverse can be expressed by Neumann series, that is, $\sum_{k=0}^{+\infty} \mathbf{\Theta}^k$.

Thus, we can rewrite the system as

$$\mathbf{D}_t = \mathbf{M} [\mathbf{Y}_t + \mathbf{B}_{t+1} - R_t \mathbf{B}_t - W_t \mathbf{L}_t - \mathbf{I}_t],$$

where $\mathbf{M}$ is defined in (11). Thus, the dividends of the ultimate owner are

$$D_{ut} = \sum_{j \in \mathcal{N}_u} m_{uj} [Y_{jt} + B_{jt+1} - R_t B_{jt} - W_t L_{jt} - I_{jt}], \quad (12)$$

where m_{uj} is (u, j) -th element of $\mathbf{M}$.²³ Since the ownership relations describe a directed weighted acyclic graph, $m_{uu} = 1$ for the ultimate owner at the top of the pyramid, $m_{uj} = \theta_{uj}$ for all firms that belong to the first layer of the pyramidal structure of the corporate group, $m_{uj} = \theta_{uj} + \sum_i \theta_{ui} \theta_{ij}$ for all firms that belong to the second layer of the pyramid, and so on.²⁴

²³This accounting procedure of the value of an organization in presence of cross-holdings is reminiscent of Brioschi, Buzzacchi, and Colombo (1989). See Elliott, Golub, and Jackson (2014) for further reference.

²⁴To understand the intuition behind this, let us look at the flow-of-funds constraint of the ultimate owner,

$$D_{ut} = Y_{ut} + B_{ut+1} - R_t B_{ut} - W_t L_{ut} - I_{ut} + \sum_{j \neq u} \theta_{uj} D_{jt}.$$

Since the dividends D_{jt} paid by firm j respect (8) as well, we can substitute it inside the constraint of the ultimate owner,

$$\begin{aligned} D_{ut} = & Y_{ut} + B_{ut+1} - R_t B_{ut} - W_t L_{ut} - I_{ut} \\ & + \sum_{j \neq u} \theta_{uj} \left[Y_{jt} + B_{jt+1} - R_t B_{jt} - W_t L_{jt} - I_{jt} + \sum_{i \neq j} \theta_{ji} D_{it} \right], \end{aligned}$$

where D_{it} is the dividends paid by firm i that is owned any firm j that belong to the ultimate owner u . Let us rewrite this constraint as

$$\begin{aligned} D_{ut} = & Y_{ut} + B_{ut+1} - R_t B_{ut} - W_t L_{ut} - I_{ut} \\ & + \sum_{j \neq u} \theta_{uj} [Y_{jt} + B_{jt+1} - R_t B_{jt} - W_t L_{jt} - I_{jt}] + \sum_{j \neq u} \sum_{i \neq j} \theta_{uj} \theta_{ji} D_{it}. \end{aligned}$$

We can repeat the substitution infinite times in order to account for indirect connections of any length across firms through a series of direct ownership relations. If we repeat this process for infinite times we obtain exactly (12), since after a series of ownership relations that start with the ultimate owner we reach the end of the pyramid and no further links can be found between the firms at the bottom of the pyramid and any other firm of the corporate group.

The problem of the ultimate owner u at time τ is then

$$\max_{\left\{ \begin{smallmatrix} L_{jt}, K_{jt+1}, \\ B_{jt+1}, X_{ijt} \end{smallmatrix} \right\}_{t \geq \tau, j \in \mathcal{N}_u}} \mathbb{E}_\tau \left[\sum_{t=\tau}^{+\infty} \beta^{t-\tau} \frac{(C_t - \psi L_t)^{-\sigma}}{(C_\tau - \psi L_\tau)^{-\sigma}} \right. \\ \left. \sum_{j \in \mathcal{N}_u} m_{uj} [Y_{jt} + B_{jt+1} - R_t B_{jt} - W_t L_{jt} - I_{jt}] \right], \quad (13)$$

subject to (9) and (10) for every $j \in \mathcal{N}_u$ and every $t \geq \tau$. The first necessary condition for the optimal solution of (13) is the FOC with respect to L_{jt} , that is,

$$L_{jt} = \frac{1}{W_t} \epsilon (1 - \alpha_j) Y_{jt}, \quad (14)$$

for every $j \in \mathcal{N}_u$ and for every $t \geq \tau$. This static condition expresses the optimal choice of labor demand as proportional to output Y_{jt} and the labor share $\epsilon(1 - \alpha_j)$, and inversely proportional to the wage W_t . The second condition is the FOC with respect to K_{jt+1} ,

$$m_{uj} = m_{uj} \mathbb{E}_t \left[\beta_{t+1} \left(\epsilon \alpha_j \frac{Y_{jt+1}}{K_{jt+1}} + 1 - \delta \right) \right] + \kappa_{jt} \xi_{jt}, \quad (15)$$

for every j and for every t , where ξ_{jt} is the Lagrange multiplier associated to the collateral constraint (10), and

$$\beta_{t+1} \equiv \beta \left(\frac{C_{t+1} - \psi L_{t+1}}{C_t - \psi L_t} \right)^{-\sigma} \quad (16)$$

is the stochastic discount factor. The multiplier ξ_{jt} is a measure of how binding the collateral constraint is. The condition (15) means that a unit of capital is worth m_{uj} to the ultimate owner in terms of distributed dividends, so in the decision about how much capital to allocate to firm j to alter its marginal productivity the ultimate owner takes into account how much the collateral constraint of firm j is relaxed by the potential increase in the collateral. An increase in the looseness κ_{jt} of the collateral constraint increases how much an additional unit of capital relaxes the constraint. The capital chosen today K_{jt+1} acts as collateral in (10), so its shadow price today depends on how binding the constraint is (ξ_{jt}) and how much increasing the collateral relaxes the constraint (κ_{jt}). The third condition is the FOC with respect to B_{jt+1} ,

$$\xi_{jt} = m_{uj} [1 - \mathbb{E}_t [\beta_{t+1} R_{t+1}]], \quad (17)$$

for every j and for every t . This condition casts a relation between the price of the collateral constraint ξ_{jt} and the exogenous return rate R_{t+1} . The higher the return on borrowed funds, the lower the amount of bonds that the ultimate owner finds optimal firm j to issue, the less binding the collateral constraint. Nevertheless, if firm j is important for ultimate owner u in the sense of a higher m_{uj} , then it is optimal to borrow more money to realize cash flow in firm j , thus making the collateral constraint more binding. The last condition is the complementary slackness of the collateral constraint,

$$\xi_{jt} (\kappa_{jt} K_{jt+1} - B_{jt+1}) = 0, \quad (18)$$

for every j and for every t , where the constraint can occasionally bind depending on the shocks.

3.3 Intertemporal competitive general equilibrium

We need only the market clearing conditions to close the model and be able to define an intertemporal general equilibrium for our economy. First, since the Gorman form of households' preferences allows for a representative household, the equity market for shares in the ultimate owners clears at time τ simply as

$$\theta_{uu\tau} = 1, \quad (19)$$

for every $u \in \mathcal{U}$, where $\theta_{uu\tau}$ is the total demand of shares in the ultimate owner u and 100% is its total exogenous supply. Second, the labor market clears at time τ if

$$\sum_{j \in \mathcal{N}} L_{j\tau} = L_{\tau}, \quad (20)$$

where the left-hand side is the total demand of labor expressed by all the firms that populate the economy, and the right-hand side is the supply of labor by the representative household. Since we are in a small open economy, the credit market consists of a perfectly elastic credit supply coming from abroad and a demand expressed by all the firms in the economy.

Definition 1. An intertemporal competitive general equilibrium is a sequence

$$\{C_{\tau}, L_{\tau}, W_{\tau}, \{\theta_{uu\tau+1}, P_{u\tau}, \{\{K_{jt+1}, L_{jt}, B_{jt+1}\}_{j \in \mathcal{N}}\}_{t \geq \tau}\}_{u \in \mathcal{U}}\}_{\tau \geq 0}$$

such that

- (i) $\{C_{\tau}, L_{\tau}, \{\theta_{uu\tau+1}\}_{u \in \mathcal{U}}\}_{\tau \geq 0}$ solves the representative household problem (1) given $\{W_{\tau}, \{P_{u\tau}\}_{u \in \mathcal{U}}\}_{\tau \geq 0}$,

- (ii) $\{\{K_{jt+1}, L_{jt}, B_{jt+1}\}_{j \in \mathcal{N}}\}_{t \geq \tau}$ solves ultimate owner u 's problem (13) at time τ given $\{C_t, L_t, W_t, R_t, \{A_{jt}, \kappa_{jt}\}_{j \in \mathcal{N}}\}_{t \geq \tau}$ for every $u \in \mathcal{U}$ and for every $\tau \geq 0$,
- (iii) the market clearing conditions (19) and (20) hold for every $\tau \geq 0$, and
- (iv) $\{R_\tau, \{A_{j\tau}, \kappa_{j\tau}\}_{j \in \mathcal{N}}\}_{\tau \geq 0}$ follow their stochastic processes.

The problems (1) and (13) of the representative household and of each ultimate owner are convex problems, so the necessary conditions for optimality are also sufficient. Thus, the sequence

$$\{C_\tau, L_\tau, W_\tau, \{\theta_{uu\tau+1}, P_{u\tau}, \{\{K_{jt+1}, L_{jt}, B_{jt+1}\}_{i,j \in \mathcal{N}}\}_{t \geq \tau}\}_{u \in \mathcal{U}}\}_{\tau \geq 0}$$

is an equilibrium if and only if it is the solution of the system of equations (3), (4), (6), (7), (8), (12), (14), (15), (17), (18), (19), and (20) given the stochastic processes for κ_{jt} , A_{jt} , and R_t for every j and for every t .

The collateral constraint in the access to the credit market implies two types of equilibria. On the one hand, the collateral constraint may not bind, that is, $B_{jt+1} < \kappa_{jt}K_{jt+1}$ for some firm j at some point in time t . On the other hand, the collateral constraint may be binding, that is, $B_{jt+1} = \kappa_{jt}K_{jt+1}$.²⁵ The dynamics of the economy are different depending on the two types of equilibria. Thus, we first define and characterize the steady state equilibrium in which we are interested and then move to the analysis of the economy's dynamics around that steady state.

Definition 2. A deterministic steady state is an intertemporal competitive general equilibrium such that $C_t = C$, $L_t = L$, $W_t = W$, $\theta_{uut+1} = \theta_{uu}$, $P_{ut} = P_u$, $K_{jt+1} = K_j$, $L_{jt} = L_j$, and $B_{jt+1} = B_j$ as long as $\kappa_{jt} = \kappa_j$, $A_{jt} = A_j$, and $R_t = R$, for every t , every $u \in \mathcal{U}$, and every $j \in \mathcal{N}$.

The deterministic steady state is the equilibrium that features constant values of all the endogenous variables in the absence of shocks. By (16) we

²⁵In the first case, the slackness condition (18) implies that $\xi_{jt} = 0$. This makes it indifferent for firm j to raise funds internally through investment I_{jt} or externally through new debt B_{jt+1} , making thus indeterminate the composition of the balance sheet of firm j . In the second case, the structure at time t of the balance of firm j is determined by the looseness κ_{jt} . From (17) we derive the expression for ξ_{jt} as a function of the expectation at t over β_{t+1} and R_{t+1} . Hence, the only situation in which ξ_{jt} can be nil is when $\mathbb{E}_t[\beta_{t+1}R_{t+1}] = 1$, since as long as firm j belongs to corporate group $\mathcal{N}_u$, by construction $m_{uj} > 0$. The condition $\mathbb{E}_t[\beta_{t+1}R_{t+1}] = 1$ may occasionally be true and affects all the firms in the economy. Hence, it is possible that there may be periods where the balance sheet of all firms is undetermined.

can trivially derive the steady state value of the stochastic discount factor, that is, $\beta_{t+1} \mid_{\text{at steady state}} = \beta$. Hence, we can distinguish at least two distinct deterministic steady state, the first when $\beta R \geq 1$ and the second when $\beta R < 1$. Since we are interested in analyzing the dynamics of an economy when firms are financially constrained, we state the following assumption.

Assumption 1. *The steady state value of the interest rate, R , is such that*

$$\beta R < 1,$$

that is, is less than the inverse of the discount factor.

Proposition 1. *If Assumption 1 holds, then there exists a unique deterministic steady state characterized by*

$$\begin{aligned} Y_j &= A_j C_{Kj}^{\frac{\epsilon}{1-\epsilon} \alpha_j} C_{Lj}^{\frac{\epsilon}{1-\epsilon} (1-\alpha_j)}, \\ K_j &= \frac{\beta}{1 - \beta(1 - \delta) - \kappa_j(1 - \beta R)} \epsilon \alpha_j Y_j, \quad L_j = C_{Lj} Y_j, \\ B_j &= k_j K_j, \quad \xi_j = m_{uj} (1 - \beta R) > 0, \\ L &= \sum_{j \in \mathcal{N}} L_j, \quad W = \psi, \end{aligned}$$

and

$$C = \psi L + \sum_{u \in \mathcal{U}} \sum_{j \in \mathcal{N}_u} m_{uj} [1 - ((R - 1)\kappa_j + \delta) C_{Kj} - \psi C_{Lj}] Y_{jt},$$

where

$$C_{Kj} \equiv \frac{\beta}{1 - \beta(1 - \delta) - \kappa_j(1 - \beta R)} \epsilon \alpha_j \quad \text{and} \quad C_{Lj} \equiv \frac{1}{\psi} \epsilon (1 - \alpha_j).$$

The heterogeneity across firms is entirely driven by the idiosyncratic characteristics A_j , α_j , and κ_j . These differences are amplified or smoothed out by the degree ϵ of returns to scale to capital and labor, that is, without considering the contribution of productivity A_j to the formation of value added at the firm- j level. By changing the value of ϵ we can change how skewed the size distribution of firms is. If ϵ is close to 1, then small differences in the capital intensity α_j or the ease in accessing the credit market κ_j can yield large differences in value added Y_j across firms. If ϵ is close to 0, then most of

the differences across firms can be explained by differences in the productivity A_j .²⁶ The steady state values of firm-specific variables do not depend on how the firms are distributed among different corporate groups, nor of their position within those corporate groups. The network structure of ownership relations affects only the aggregate variables, in that it influences how the dividends distributed by all firms are aggregated within each corporate group and distributed to the household. Different network structures change the m_{uj} 's, that is, the indirect ownership relations between the ultimate owners and the firms they control.²⁷

3.4 Dynamics of the constrained economy

Since we can characterize the deterministic steady state, we are able to log-linearize explicitly the system of equations that describes the equilibrium around that steady state. In this way we can look at the dynamics of the economy from a situation of financial distress of the firms. We only look at the dynamics around the unique deterministic steady state under the condition $\beta R < 1$.

Let us denote with a hat the log-deviations of all variables from their steady state values. Hence, we denote with $\hat{Y}_{jt}$ the log-deviation of Y_{jt} from its steady state value, that is,

$$\hat{Y}_{jt} \equiv \ln Y_{jt} - \ln Y_j.$$

The log-linearization of the system of equations that describe the equilibrium yields the log-deviation of firm-specific and aggregate variables as a function of the future stream and past values of exogenous shocks $\hat{A}_{jt}$, $\hat{\kappa}_{jt}$, and $\hat{R}_t$.

Proposition 2. *Suppose Assumption 1 holds. Then, the log-linearized equilibrium around the deterministic steady state is such that*

$$\hat{Y}_{jt} = \hat{A}_{jt} + C_{Yj}\kappa_j(1 - \beta R)\hat{\kappa}_{jt-1} - C_{Yj}\kappa_j\beta R\hat{R}_t + C_{Yj}(1 - \kappa_j)\hat{\beta}_t \quad (21)$$

and

$$\hat{\beta}_t = \pi_R(\mathbf{L})\hat{R}_t - \pi_A(\mathbf{L})\hat{\mathbf{A}}_t - \pi_\kappa(\mathbf{L})\hat{\kappa}_{t-1}, \quad (22)$$

where

$$C_{Yj} = \frac{\epsilon\alpha_j}{1 - \epsilon} \frac{1}{1 - \beta(1 - \delta) - \kappa_j(1 - \beta R)},$$

²⁶If $\epsilon = 1$, then firms' size is indeterminate and so are the rest of firm-specific variables. If $\epsilon = 0$, then $Y_{jt} = A_{jt}$ for every j with no role for capital or labor.

²⁷This affects the steady state level of consumption and may thus even permit welfare comparisons among different network structures.

$\pi_R(\mathbf{L})$ is a polynomial of the lead operator $\mathbf{L}$, $\pi_A(\mathbf{L})$ and $\pi_\kappa(\mathbf{L})$ are $1 \times N$ vectors of polynomials of the lead operator $\mathbf{L}$, and $\hat{\mathbf{A}}_t$ and $\hat{\kappa}_{t-1}$ are $N \times 1$ vectors of firm-specific shocks. Both $\pi_R(\mathbf{L})$, $\pi_A(\mathbf{L})$, and $\pi_\kappa(\mathbf{L})$ depend on the ownership network.

The production of firm j at time t is a function of firm j -specific shocks, that is, $\hat{A}_{jt}$, $\hat{\kappa}_{jt-1}$, and $\hat{R}_t$, and of shocks that hit all the other firms in the economy, that is, $\hat{A}_{it}$, $\hat{\kappa}_{it-1}$, and again $\hat{R}_t$, for every $i \in \mathcal{N}$. Let us look at the contemporaneous effect of the shocks, that is, the responses on impact. The effects of the different shocks depends on the role played by the stochastic discount factor. Without the last component of the response of output in (21), the effect of a productivity shock $\hat{A}_{jt}$ is positive since it trivially increases the productivity of the factors of production. The effect of a favorable shock $\hat{\kappa}_{jt}$ to the collateral constraint, that is, an increase on how loose the constraint is, does not have contemporaneous effects since it simply alters the ability to access the credit markets and the path of capital accumulation. A looser collateral constraint in $t - 1$ permits a faster capital accumulation and increases therefore the production in period t . The effect of a shock $\hat{R}_t$ to the return rate on bonds is negative, since it makes access to credit costlier. Nevertheless, the overall effect of any of these three shocks on production at time t depends also on their indirect effect through the fluctuations of the stochastic discount factor $\hat{\beta}_t$. Even if we consider only the contemporaneous effects, the response can be quantitatively and even qualitatively different, since productivity and collateral shocks decrease the stochastic discount factor while return shocks increase it. Hence, they partially compensate and potentially even offset the direct effect of the shocks on production. All these considerations are limited to the response of production on impact. The dynamic propagation of shocks depends exclusively on their indirect effect through the stochastic discount factor, which depends itself on the whole law of motion (22) for the stochastic discount factor. The same applies to the effect of the idiosyncratic shocks to the other firms in the economy. Their effect on the production of firm j is limited to their effect on the stochastic discount factor, both on impact and in the following periods. The impulse response functions of firms' output and aggregate consumption are discussed in Section 4.

Suppose that the collateral constraint is as tight as possible at steady state, that is, κ_j is close to 0. In this case firm j basically cannot access the credit markets and $B_{jt+1} \approx 0$. Then, it does not care about the return rate on bonds, except to the extent that an increase in the return rate causes a higher stochastic discount factor. If instead the collateral constraint is loose, that is, if κ_j is close to 1, then firm j can access the credit markets

easily and a shock to the return rate on bonds has a negative effect on firm j 's output, independently of its effect on the stochastic discount factor. Intermediate levels of the looseness of the collateral constraint imply that the overall effect of an increase in the return on bonds on firm j 's production depends on whether the effect on firm j 's bond emission dominates on the effect on the stochastic discount factor.

As equations (21) shows, the network structure plays a role in firm j 's response to both idiosyncratic and aggregate shocks, but only through the dynamics (22) of the stochastic discount factor. The network structure affects how the dividends distributed by all firms are aggregated within each corporate group and then distributed eventually to the household. Hence, the ownership relations do not enter the optimal decisions of the ultimate owners, but they do affect the image of the ultimate owners' problems, that is, the equilibrium values of the corporate groups. Hence, they affect the resources distributed to the household by the ultimate owners and therefore its consumption patterns, which in turn determines the stochastic discount factor and the propagation mechanism of any shock. This is also the reason why the shocks that hit other firms affect firm j 's production only through the stochastic discount factor.

4 Simulation

We perform a stochastic simulation of a stylized economy.²⁸ In general, since some of the constraints may stop to bind as a result of a sufficiently large shock, we should use either a global solution method like dynamic programming on a fine grid for the state variables or a piecewise linear perturbation solution such as in Guerrieri and Iacoviello (2015). However, our model has firm-specific collateral constraints. Hence, we have a different equilibrium depending on whether any of the N constraints binds. Moreover, the position of the firm with the non-binding constraint within the network structure influences the equilibrium, too. For each firm, its collateral constraint can bind or not, and so can the collateral constraints of the firms connected to it. For example, the equilibrium will differ if two firms within the same corporate group have binding constraints, or only one has it. Since the computational complexity rises exponentially with the number of firms, we focus on the equilibrium where all collateral constraints are binding. We need $\mathbb{E}_t[\beta_{t+1}R_{t+1}] < 1$ to hold so that the multiplier ξ_{jt} associated to the collateral constraint of any firm j is strictly positive for every t . This can

²⁸We employ IRIS for the simulation and estimation exercises. See Benes, Johnston, and Plotnikov (2014) for documentation.

be achieved either with a sufficiently low discount factor β or by excluding from the support set of the stochastic processes realizations of conditional shocks that would be sufficiently large to drive the stochastic discount factor, $\beta_{t+1} = \beta (C_t - \psi L_t) / (C_{t+1} - \psi L_{t+1})$, below the expected future realization of the interest rate R_{t+1} . Under these numerical conditions, solving the model with standard linear rational expectations tool kits like Blanchard and Kahn (1980) provides a reasonable approximation.

We want to keep a simple structure of the economy so as to distinguish explicitly the effects of the different shocks and the implied aggregate volatilities. The economy is composed by 4 firms, among which Firm 1 is the ultimate owner. All other firms are either directly or indirectly owned by Firm 1. We compare 5 different network structures, which we illustrate in Figure 6.

Figure 6: The 5 cases of network structures.

The first distinction is with respect to the number of links. Cases 1 through 4 report network structures with only three links, while Case 5 introduces a fourth link. Case 1 refers to a star network, that is, where the ultimate owner is the direct owner of all other firms. Case 2 represents a first prolongation of the ownership lines. In this case, the ultimate owner owns only two firms, while one of the two subsidiaries own a firm as well. Case 3 takes one step forward and presents a structure where the ultimate owner owns stock only in another firm, which itself owns the other two firms. Case 4 presents the other opposite in the spectrum of possible network topologies with only 3 links. The ultimate owner owns directly only one firm, and so do Firm 2 and Firm 3, while Firm 4 is simply a subsidiary with no participation in any other firm. This structure presents thus a line of intermediate ownerships between the ultimate owner and the last subsidiary. Case 5 introduces an additional link, so it is fundamentally different from the other topologies. It consists of a star network as in Case 1, but with an additional link between two of the subsidiaries of the ultimate owner. We assume that the participations are homogeneous across structures and consist of a 50% ownership of the other firms' equities. In Case 5, we assume that the subsidiary that is

participated by both the ultimate owner and the other subsidiary is owned 50%-50%.

We fix the values of the parameters of the model in a way that makes firms identical apart from their position within the network. In particular, we assign the same share $\alpha_j = \alpha$ of capital in the production functions of the different firms, as well as the same steady state level of the collateral constraint's looseness $\kappa_j = \kappa$. Although the relative values of α , ϵ , β , σ , ψ , and R are not important at the moment, they become so for the calibration exercise in Section 5. Hence, we fix them at the levels described in Table 5, and leave their justification for later on. We define the properties of the stochastic processes R_t , A_{jt} , and κ_{jt} in the following way.

Assumption 2. *The economy is subject to the following stochastic processes.*

- i) *The return rate R_t on bonds follows a first-order Markov process such that*

$$R_t = (1 - \rho_R)R_{ss} + \rho_R R_{t-1} + \varepsilon_t^R,$$

where $\rho_R \in (0, 1)$, ε_t^R is normally distributed with mean 0 and variance σ_R^2 , and $R \in (0, 1/\beta)$.

- ii) *The productivity A_{jt} of firm j is composed of an idiosyncratic component a_{jt} and a common component a_t , that is,*

$$A_{jt} = a_{jt}a_t.$$

Both components follow first-order Markov processes in logs such that

$$\log(a_{jt}) = (1 - \rho_{a_j}) \log(a_j^{ss}) + \rho_{a_j} \log(a_{jt-1}) + \varepsilon_{jt}^a$$

and

$$\log(a_t) = (1 - \rho_a) \log(a^{ss}) + \rho_a \log(a_{t-1}) + \varepsilon_t^a,$$

where ε_{jt}^a and ε_t^a are normally distributed with mean 0 and variance $\sigma_{a_j}^2$ and σ_a^2 , for every j .

- iii) *The looseness κ_{jt} of the collateral constraint of firm j is composed of an idiosyncratic component $\tilde{\kappa}_{jt}$ and a common component κ_t , that is,*

$$\kappa_{jt} = \tilde{\kappa}_{jt}\kappa_t.$$

Both components follow first-order Markov processes in logs such that

$$\log(\tilde{\kappa}_{jt}) = (1 - \rho_{\kappa_j}) \log(\tilde{\kappa}_j^{ss}) + \rho_{\kappa_j} \log(\tilde{\kappa}_{jt-1}) + \varepsilon_{jt}^{\tilde{\kappa}}$$

and

$$\log(\kappa_t) = (1 - \rho_\kappa) \log(\kappa^{ss}) + \rho_\kappa \log(\kappa_{t-1}) + \varepsilon_t^\kappa,$$

where $\varepsilon_{jt}^{\tilde{\kappa}}$ and ε_t^κ are normally distributed with mean 0 and variance $\sigma_{\tilde{\kappa}_j}^2$ and σ_κ^2 , for every j .

We set ρ_a , ρ_{a_j} , ρ_κ , and ρ_{κ_j} to 0, and a_j^{ss} , a^{ss} , $\tilde{\kappa}_j^{ss}$, and κ^{ss} to 1. We simulate stochastically the economy around the deterministic steady state for 2000 periods and compute both the impulse response functions of firms' output and aggregate consumption to the different shocks and the implied aggregate volatility, that is, the coefficient of variation of aggregate consumption through the 2000 periods. We perform this simulation for the 5 network structures described above.

4.1 Impulse response functions

We compare the impulse response functions of output and consumption between the two polar cases of network structures with 3 links, that is, Case 1 (the star network) and Case 4 (the line network). Figure 7 reports the impulse response functions of the output of each firm to the idiosyncratic shocks to the productivity of each firm.

We report the dynamic propagation of the shocks for 20 quarters. The diagonal graphs represent the effect of a shock to a certain firm on the production of that firm. The response on impact is high as described by (22). The dynamic propagation of the shocks is the product of its effect on the stochastic discount factor. The two curves represent the two cases, Case 1 (in blue) and Case 4 (in red). The off-diagonal graphs represent instead the response of a firm's output to a productivity shock that hits another firm in the economy. Column 1 represents the response of each firm's output to a positive shock to Firm 1, that is, to the ultimate owner. The shock increases production of Firm 1 on impact while it does not affect the production of the other firms. This shock affects the stochastic discount factor so it alters the capital accumulation decided by the ultimate owner for both itself and the rest of the firms. Hence, the shape of the dynamic propagation of the shock is similar across firms, since it simply reflects the effect on the stochastic discount factor. The graph shows how the effect of the shocks is more persistent in Case 4 than in Case 1. The difference between these cases is due to the change in the law of motion for the stochastic discount factor described in (22). Different network structures imply different responses of the stochastic discount factor to the shocks. The intuition behind is that different network

Figure 7: Impulse response functions of firm-specific output to idiosyncratic productivity shocks. Comparison between a star network (in blue) and a line network (in red). Each line reports the response of the same firm to shocks occurring to the same firm and to all other firms.

structures imply different intertemporal reallocations of resources by the ultimate owner across its controlled firms. In Case 1, the owner can fully pass the consequences of the shock to its controlled parties, making the effect of the shock more short lived. In Case 4, the owner does pass the shock to its controlled firms, but the transmission is limited by its cash-flow rights in these firms. The indirect ownership between Firm 1 and Firm 4 is smaller in Case 4 than in Case 1, hence the shock reverberates in the economy for a longer period. The second column shows the response of the firms to a shock to Firm 2. Similarly to what happens when the shock hits Firm 1, the response at impact is limited to the firm that is hit, while the dynamic propagation is common across firms. The magnitude of the response, compared to when the shock hits the ultimate owner, is smaller for all firms, since the effect on the ultimate owner's flow-of-funds constraint is smaller the smaller

is the indirect ownership relation. This smaller effect is evident from the analysis of the third column. A shock to Firm 3 has the same effect as a shock to Firm 2 in the case of a star network (Case 1), since both firms are directly owned by the ultimate owner. In the case of a line network (Case 4), the response is smaller as a consequence of a shock to Firm 3 than to a shock to Firm 2. This is due to the fact in this case the ultimate owner owns a smaller cash-flow right on Firm 3 than on Firm 2, so its response to a shock to the former is smaller than to a shock to the latter. This also results in a magnitude of the response that is smaller for Case 4 than for Case 1, contrary to what happens in the event of shock to the ultimate owner or Firm 2. An even smaller response is caused by a shock to Firm 4 for Case 4, as the separation in terms of ownership links between the ultimate owner and the firm hit by the shock is maximal. Figure 8 shows that the comparison of responses to shocks to the collateral constraint is similar to shocks to firm-specific productivities.

As Figure 8 shows, output responds to collateral shocks with a lag. An increase in κ_{jt} increases the value of capital as a collateral device for firm j , thus increasing the accumulation of capital in period t , but this turns into a higher output only in the following period. The rest of the dynamic propagation of the shock is due to its effect on the stochastic discount factor.

Figure 9 displays the response of aggregate consumption to either idiosyncratic or aggregate shocks. For simplicity we report only the case of a shock to the ultimate owner. For idiosyncratic shocks, aggregate consumption's response is less persistent in the case of a star network than in the case of a line network. Aggregate shocks instead seem to generate larger responses in the case of a star network. The greater impact is due to the impossibility for the ultimate owner to smooth out the shock across its subsidiaries. By hitting all firms contemporaneously, the impact of a shock to the rate R_t or the common components of the collateral constraints (κ_t) and the productivity (a_t) depends simply on the degree of separation between the household and the firms. In the case of the line network, the effect on Firm 2, 3, and 4 is attenuated by the indirect connection. Part of the effect of the shock is lost since the ultimate owner owns the subsidiaries only partially. The lower cash flow generated by a shock to the return rate affects the ultimate owner only limited to its share into the different firms. In the case of the star network, the ultimate owner receives the full consequence of a shock to all its controlled firms.

Figure 8: Impulse response functions of firm-specific output to idiosyncratic collateral shocks. Comparison between a star network (in blue) and a line network (in red). Each line reports the response of the same firm to shocks occurring to the same firm and to all other firms.

Figure 9: Impulse response functions of aggregate consumption to idiosyncratic and aggregate shocks. Comparison between a star network (in blue) and a line network (in red). The subplots report the response of aggregate consumption to either a productivity shock to Firm 1, a collateral shock to Firm 1, an aggregate collateral shock, or a shock to the interest rate.

4.2 Comparison of implied aggregate volatilities

We compare the implied volatilities of the different network structures. We take the coefficient of variation, that is, the standard deviation divided by the mean, of the path of aggregate consumption through the 2000 periods considered in our stochastic simulation. We use only idiosyncratic shocks on both productivities and collateral constraints. In this way, we have a sense of how volatile our stylized economies are as a result of the idiosyncratic shocks. Table 4 reports the comparison across the five cases.

	Case 1	Case 2	Case 3	Case 4	Case 5
Moment	Star	Tree 1	Tree 2	Line	DAG
σ_C	0.0890	0.0877	0.0872	0.0869	0.0896
μ_C	3.3070	3.1988	3.0924	3.0387	3.4109
σ_C/μ_C	0.0269	0.0274	0.0282	0.0286	0.0263

Table 4: Standard deviation, mean, and coefficient of variation implied by different network structures.

The volatility of the economy consistently decreases from Case 1 through Case 4. The driver of this decrease is the ultimate owner’s ability to diversify the idiosyncratic shocks across its subsidiaries. In the case of a star network, the propagation of the shock is short-lived, while in the case of the line network the ultimate owner is less able to smooth out the shocks through time. The two intermediate cases, Case 2 and Case 3, yield intermediate levels of aggregate volatility. Note that the standard deviation decreases from Case 1 to Case 4, but this is simply due to the decrease in the steady state levels of aggregate consumption. If we conduct the same simulation using only aggregate shocks, we obtain the opposite results. More diversified economies are more volatile as a result of aggregate shocks.

The internal structure of business groups has an impact on the aggregate volatility in our model. The size and sign of this impact depends on the nature of the shocks. For idiosyncratic shocks, more diversified economies are less volatile. Moreover, diversification is more effective in smoothing out idiosyncratic shocks the closer to the household side the agent that carries it out is. The most effective diversification occurs when the households carry it out. The ultimate owners are the second most effective agents. The firms that are directly participated by the ultimate owners are in third position, and so on. For aggregate shocks, more diversified economies are more volatile, and the closer the diversification is to the ultimate owner and the household side, the higher is the aggregate volatility.

5 Calibration and aggregate volatility

In this section we calibrate the model to aggregate moments of the Italian data and quantify which percentage of the Italian actual GDP volatility can be explained by idiosyncratic productivity shocks.

First, we calibrate the structural parameters of the model to standard values. We set the elasticity ψ of labor supply and the relative risk aversion σ to 1. The product $\epsilon(1 - \alpha)$ of returns to scale and (homogeneous) labor importance must be equal to the labor share in a firm's production. We follow Orsi and Turino (2014) and assume a stationary labor share for Italy equal to 0.58. Thus, once we choose ϵ , we can derive $1 - \alpha$. We follow Bhattacharya, Guner, and Ventura (2013) and set the returns to scale parameter at 0.765. The depreciation rate δ matches the stationary annual depreciation rate derived in Orsi and Turino (2014), which is 4.64% for Italy. The discount factor is equal to 0.946. The steady state real interest rate is set to 4.69%, which corresponds to the average short-term real interest rate charged on bank credit to nonfinancial firms over the period 2000 – 2013. We set to zero the standard deviations of all shocks except the idiosyncratic

Figure 10: Representativeness of the sample. Median growth rate of sales (our sample) versus gross output growth (from National Accounts).

productivity shocks, and impose a persistence ρ_{a_j} of the productivity shocks equal to 0.99. We set the idiosyncratic shocks' volatility σ_{a_j} to the median observed coefficient of variation of annual firm sales growth in our sample, which covers the period 2006 – 2013. The annual coefficient of variation of 7.47% corresponds to a quarterly coefficient of variation of 1.82%. Lastly, we set the steady state levels a_j^{ss} and a^{ss} of the productivity shocks to 1, and the steady state levels $\tilde{\kappa}_j^{ss}$ and κ^{ss} of the collateral constraint shocks to 0.5.

Figure 10 illustrates why the calibration of the model to the median firms is justified. The median growth rate of sales in our sample comoves closely with the gross output growth derived from National Accounts. Moreover, our sample of firms is representative of the Italian economy because total sales amount to around 75% of Italian gross output (e.g., in 2007 total sales are €2.37 trillion, whereas Italian gross output in the same year is €3.15 trillion). We report all the details of the calibration in Table 5.

Description	Parameter	Value
El. labor supply	ψ	1
RRA	σ	1
Returns to scale	ϵ	0.765
Capital share	α	0.549
Depreciation rate	δ	0.012
Discount factor	β	0.972
Interest rate (ss)	R_{ss}	$(1 + 4.69/100)^{1/4} = 1.01$
Idiosyncratic volatility	σ_{a_j}	$(1 + 7.47/100)^{1/4} - 1 = 1.82\%$

Table 5: Parameter values for calibration.

Second, we calibrate three key aspects of the network structure of ownership relations. We set the strength of the linkages uniformly to 50%, which is the mode of the shares' distribution in our sample and covers almost 1/3 of all observations (see Figure 2). Moreover, we can reconstruct the business groups from the ownership data by considering only the shares that are strictly above 50%. This has two consequences. On the one hand, it reduces the complexity of intra-group cross-holdings and the average group size. On the other hand, each firm belongs unambiguously to at most one group and each group has at most one ultimate owner by construction. Moreover, any threshold equal or below 50% implies the emergence of a supercomponent in the ownership network structure, that is, a group of firms directly or indirectly connected to each other that covers a relevant portion of the economy. If we consider all the shares, the size of the supercomponent covers more than 16% of the firms in the economy (around 100,000 firms). Since the existence

of this supercomponent is difficult to interpret economically and no single ultimate owner could be identified within it, we assume that a firm belongs to another firm within the same business group if the participation of one into the other is higher than 50%.²⁹ Figure 11 reports the size distribution of business groups that emerge if we consider only strict majority shares.

Figure 11: The size distribution of business groups when we consider only shares strictly higher than 50%. Both axes are in log scale. We report the distribution for 2005, 2009, and 2013, and we add a linear fit across the data points for each year.

The distribution does not seem to change significantly over time. If we parametrize it with a Pareto distribution of the form

$$\log f(N_u) = \alpha + \beta \log N_u,$$

where $f(N_u)$ is the frequency at which a business group of size N_u appears, an OLS regression would yield $\hat{\alpha} = 11.02918$ and $\hat{\beta} = -2.901756$. This means that the minimum number N of firms that we should consider in the calibration in order to generate a comparable distribution is higher than what is computationally feasible. Hence, we can adopt only qualitatively the fat

²⁹While a common feature of random graphs, the emergence of the supercomponent and

Figure 12: The internal network structure of the largest business group in 2005 (ignoring shares below or equal to 50%).

tail of business groups' distribution. We set the number N of firms to 100 and distribute them into 83 business groups. The biggest group consists of 16 firms, the second and third consist of 2 firms each, and the rest of the groups consist of stand-alone firms. The last aspect we take into account is the internal organization of the groups. In our calibrated model, only the

Figure 13: The calibrated network structure.

biggest group contains enough firms to show a structure, as the rest of the

the evolution of its size over time may be informative about the nature of the ownership network. We leave its analysis to future research.

groups are either stand-alone firms or two-firm groups. Figure 12 shows the structure of the biggest group in 2005 (considering only shares above 50%). The structure seems to be dominantly star-like. Hence, we assume that the largest group in our calibrated model consists of an ultimate owner with 15 direct participations. The resulting network structure is reported in Figure 13.

5.1 The relevance of idiosyncratic shocks

Given our calibration, we simulate the model at a quarterly frequency for the years 2000 to 2013, and compare the model-implied standard deviation of GDP and consumption growth rates over the period with the data. We consider the residual of a BVAR estimation featuring GDP, private and public consumption, investment, exports, imports, unemployment rate, employment growth, short and long run interest rates, effective nominal Euro Area exchange rate, the oil price expressed in US dollars, and the debt over GDP ratio. In Table 6 we report the standard deviation over the period of the actual data, of the residual once we control for the BVAR variables, and of the calibrated model. Figure 14 shows the same point graphically.

Variable	Data	Data residual	Model
GDP growth	0.7670	0.2813	0.0855
Consumption growth	0.5102	0.1684	0.0469

Table 6: Volatilities of GDP and consumption growth in the data, in the residual of the BVAR estimation, and in the model.

The idiosyncratic productivity shocks generate around 30% of actual GDP volatility that cannot be explained through the BVAR, and more than 11% of overall GDP volatility. In the case of consumption volatility, these proportions are 28% and 9%, respectively.

In order to elicit the role played by idiosyncratic shocks in the realized GDP path of the Italian economy, we calibrate also the volatilities of the other shocks. We set the aggregate productivity shocks' volatility σ_a to the coefficient of variation of the mean sales growth over the sample period in our data. The annual coefficient of variation is 0.57%, so the corresponding value at the quarterly frequency is 0.14%. We do not have information about the looseness of the collateral constraints in our sample, so we maintain the order of magnitude of the productivity shocks and we set the standard deviations $\sigma_{\kappa_j}^2$ and σ_{κ}^2 of the corresponding shocks to 1% and 0.1%. We keep the persistences ρ_a , ρ_{κ_j} , and ρ_{κ} to the same level of ρ_{a_j} equal to 0.99. We use

Figure 14: Italian GDP growth (percentage points) according to the data, the residual of the BVAR estimation, and the model simulation with only idiosyncratic productivity shocks.

as observables GDP (GDP_t), consumption ($Cons_t$), investment (Inv_t), and short term interest rates ($Short_t$).³⁰ We express all variables in quarter-on-quarter growth rates except for the short term interest rate which we use in levels. We allow for measurement errors in all variables except the interest rates. Our measurement equations are

$$\begin{aligned}
 GDP_t &= 100 \left(\frac{\sum_{j \in \mathcal{N}} Y_{jt} - \sum_{j \in \mathcal{N}} Y_{jt-1}}{\sum_{j \in \mathcal{N}} Y_{jt-1}} + \eta^{GDP_t} \right), \\
 Cons_t &= 100 \left(\frac{C_t - C_{t-1}}{C_{t-1}} + \eta^{Cons_t} \right), \\
 Inv_t &= 100 \left(\frac{\sum_{j \in \mathcal{N}} I_{jt} - \sum_{j \in \mathcal{N}} I_{jt-1}}{\sum_{j \in \mathcal{N}} I_{jt-1}} + \eta^{Inv_t} \right), \\
 Short_t &= 100 (R_t^4 - 1),
 \end{aligned}$$

³⁰All data come from Eurostat except for the short term interest rates (Bank of Italy).

where η^{GDP_t} , η^{Const_t} , and η^{Inv_t} are measurement errors with standard deviation equal to 0.01. We can then extract the shocks from the observables using a Kalman filter and analyze which role the model assigns to each type of shock given our calibration. We report the filtered series of shocks and measurement errors as the benchmark case in Figure 21, Figure 22, and Figure 23 of the appendix.

Figure 15: Contributions of different shocks to Italian GDP growth (in percentage points). We extract the shocks through a Kalman filter and obtain the alternative series by simulating the model with all the shocks (without measurement errors), with only collateral shocks (both aggregate κ_t and idiosyncratic $\tilde{\kappa}_{jt}$), with only aggregate productivity shocks a_t , and with only idiosyncratic productivity shocks a_{jt} .

We use the filtered shocks to simulate again the model separately for each shock type, so as to gauge the role played by the idiosyncratic shocks. The results for the GDP are illustrated in Figure 15. By construction, all the shocks combined fit quite well the GDP, net of the measurement errors. The combined shocks are able to generate around 90% of actual GDP volatility. Feeding to the model only with the collateral shocks generates almost countercyclical responses, while aggregate productivity shocks alone follow qualitatively the actual GDP series. Their implied volatility represent around 40% and 35% of actual GDP volatility. The simulated GDP series are the

closest to the actual data both qualitatively and quantitatively in the case of idiosyncratic productivity shocks alone, as these shocks are able to generate up to almost 70% of observed volatility. Hence, we conclude that given our calibration the model assigns a dominant role to idiosyncratic shocks rather than other shock types.

5.2 Counterfactual exercise

We conduct a counterfactual exercise where we try to elicit the role that the ownership network structure plays in the propagation of idiosyncratic shocks. We compare the simulations of the benchmark model described above with the simulations of a model where firms are not connected through ownership links and are directly owned by the household, holding all other aspects constant. The differences in the results between the benchmark model and the counterfactual model inform us about how the network structure alters the dynamics of the economy and the weight of idiosyncratic shocks in observed aggregate fluctuations.

First, we simulate a model with only idiosyncratic productivity shocks with the same calibration as in Table 5, so as to obtain a counterfactual version of Table 6. The implied volatility of GDP growth in this case is 0.0654, while the implied volatility of consumption growth is 0.0333. Hence, the network structure accounts for around 23.5% of the model-implied GDP volatility, 7.2% of the volatility not explained by the BVAR, and 2.6% of overall GDP volatility. The proportions in the case of the consumption volatility are 29.0%, 8.1%, and 2.7%, respectively. The impact of the same shocks is higher when the network is in place, as the business groups channel the idiosyncratic shocks of the participated firms to the ultimate owners' flow-of-funds constraints and therefore to the households. Households' budget constraints are less affected by firm shocks the lower is the transmission of these shocks through the network. If all firms are stand-alone and therefore owned 100% by the households, their budget constraint is just a sum of the dividends of each firm. Thus, the law of large numbers applies fully, as long as dividends across firms are not correlated. If instead there is a network of ownership relations and the subsidiaries' dividends impact only indirectly on the budget constraint of the household, then the latter appears as a weighed sum of dividends, where the weights correspond to the indirect cash flow rights of each ultimate owner into the firms that belong to its business group.³¹

³¹If the strategic interactions and the externalities across firms within the same group go beyond the mechanisms we model here, then not only there is a lack of homogeneity in size across firms that impacts the usual averaging-out across firm-specific shock, but also

Variable	All shocks	Idiosyncratic productivity shocks
Benchmark model	0.6953	0.5343
Counterfactual model	0.6899	0.4301

Table 7: Model-implied volatility of GDP growth for the benchmark calibrated model and the counterfactual model with no ownership links, for the cases with all shocks and with only idiosyncratic productivity shocks. The volatility of GDP growth is 0.7670 in the data.

Second, we use the counterfactual model with no ownership links to extract the shocks through the Kalman filter, thus obtaining counterfactual series of the shocks. We report the evolution of these shocks as the counterfactual case in Figure 21, Figure 22, and Figure 23 of the appendix. We then simulate again the model with either all the shocks or with only the idiosyncratic productivity shocks. In the first simulation, the shocks are able to generate exactly the same amount of volatility that they generate in the benchmark case, that is, around 90% of actual GDP volatility. In the second simulation, the idiosyncratic shocks alone generate a higher share of aggregate fluctuations in the benchmark case, that is, around 70% of actual GDP volatility, than in the counterfactual case, that is, only 56% of actual GDP volatility. Table 7 reports the results of these simulations.³²

6 Conclusion

The macroeconomic dynamics of an economy may not depend only on the aggregate shocks that hit all the agents at the same time. There is a nontrivial portion of aggregate volatility that may just be the result of the propagation of idiosyncratic shocks to the aggregate level. Part of this propagation can simply reflect a granular effect, where agents that represent relevant portions of aggregate production have an impact on the overall economy. The residual aggregate volatility that we cannot explain through either aggregate shocks or “granular” shocks is far from being simply a measure of our ignorance.

a lack of independence of those shocks.

³²The differences in the extracted shocks are small between the benchmark case and the counterfactual case, as reported in Figures 21 to 23 of the appendix. If we use the same shocks extracted with the benchmark model for both the benchmark case and the counterfactual case, thus controlling for differences in the extracted shock series, the results are similar. The benchmark shocks with the counterfactual model generate 96% of actual GDP volatility if we simulate the model with all shocks and 57% of the same volatility if we simulate the model with only the idiosyncratic productivity shocks.

In fact, having information about the underlying network structure of the economy that regulates the structural interactions across the agents can help to explain a further share of this residual volatility.

We document a reduced-form dependence in economic performance across limited liability firms that share an ownership link, and we construct a model that features an explicit role for the ownership network structure. One example of underlying network structure consists of the input-output relations across firms, as pinned down at the industry-by-industry level by the I-O tables (the so-called production network). Another example are the firm-by-firm ownership relations, that this paper explores. Suppose that the firms are the finest grains into which we can disaggregate an economy. Then, we can think of at least three limitations in the determination and use of an underlying network structure across firms in macroeconomic analysis. First, the measurement of the interactions is subject to error, as observation of firm-by-firm relations relies mostly on self-reporting. Ownership relations instead exist and can produce effects only if they are recorded. Hence, the measurement error is minimized by the compulsory nature of the record. Second, the identification of the network structure is not trivial, as most of the observations are the result of an equilibrium rather than the actual fundamental connection across firms. In particular, the shocks that propagate across the network relations should be orthogonal to the determinants of the observed network structure. The legal limitations to alienate an ownership share especially when this grants the control of the participated firm help to consider the propagation of shocks at the quarterly frequency less dependent on the effect of the shock on the ownership link itself. Third, certain network structures do not allow to disentangle granular and propagation effects of idiosyncratic shocks from the observed performance of firms. The acyclicity of the ownership network permits this distinction, net of other potential general equilibrium feedback effects.

We simulate our model and derive theoretical predictions about the response of the economy to different shocks. Other things equal, with idiosyncratic shocks the volatility is higher the lower the diversification and the farther the diversification is from the ultimate owner and the preference side. The opposite holds with aggregate shocks. We calibrate the model to mimic key aspects of the Italian economy. We can thus quantify how much of the aggregate volatility experienced in Italy can be traced back to the impact of idiosyncratic shocks. Moreover, we can invert the model and use it to extract the series of idiosyncratic and aggregate shocks that are necessary to generate the observed behavior of aggregate variables. Thus, we can conduct a counterfactual exercise where we compare the volatility implied by a model with the calibrated network structure with the volatility implied by a

model with no ownership links. This helps up to understand which portion of the model-implied volatility can be traced back to the propagation of shocks through ownership relations.

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A Appendix: Proofs

Notation: We distinguish matrices with boldface upper-case letters and vectors with boldface lower-case letters. We use the script font for sets.

Proof of Proposition 1. The equilibrium solution can be summarized by the following system of equations,

$$\begin{aligned}
W_\tau &= \psi, \\
\theta_{uu\tau} &= 1, \\
\sum_{j \in \mathcal{N}} L_{jt} &= L_\tau, \\
C_\tau &= \psi L_\tau + \sum_{u \in \mathcal{U}} D_{ut}, \\
D_{ut} &= \sum_{j \in \mathcal{N}_u} m_{uj} [Y_{jt} + B_{jt+1} - R_t B_{jt} - W_t L_{jt} - I_{jt}], \\
Y_{jt} &= A_{jt}^{1-\epsilon} \left(K_{jt}^{\alpha_j} L_{jt}^{1-\alpha_j} \right)^\epsilon, \\
I_{jt} &= K_{jt+1} - (1 - \delta) K_{jt}, \\
L_{jt} &= \frac{1}{W_t} \epsilon (1 - \alpha_j) Y_{jt}, \\
K_{jt+1} &= \mathbb{E}_t \left[\frac{m_{uj} \beta_{t+1}}{m_{uj} \mathbb{E}_t [1 - \beta_{t+1} (1 - \delta)] - \kappa_{jt} \xi_{jt}} \epsilon \alpha_j Y_{jt+1} \right], \\
\xi_{jt} &= m_{uj} [1 - \mathbb{E}_t [\beta_{t+1} R_{t+1}]], \\
B_{jt+1} &\leq \kappa_{jt} K_{jt+1}, \\
\xi_{jt} (\kappa_{jt} K_{jt+1} - B_{jt+1}) &= 0, \\
\beta_{t+1} &= \beta \left(\frac{C_t - \psi L_t}{C_{t+1} - \psi L_{t+1}} \right)^\sigma.
\end{aligned}$$

We focus on the deterministic steady state equilibrium and we suppose that Assumption 1 holds. Since $\beta R < 1$, the steady state value ξ_j of the multiplier ξ_{jt} on the collateral constraint is such that

$$\xi_j = m_{uj}(1 - \beta R) > 0,$$

for every j . Hence, the collateral constraint at the deterministic steady state is always binding and by the complementary slackness condition we have that

$$B_j = \kappa_j K_j.$$

From the law of motion for capital we obtain that the investment is

$$I_j = \delta K_j.$$

The stochastic discount factor β_{t+1} becomes $\beta_{t+1} = \beta$ at steady state (for simplicity, $\beta_{t+1} |_{ss}$), since by construction the household must be indifferent between consumption and leisure in t and in $t + 1$. We can substitute the expressions for ξ_j and $\beta_{t+1} |_{ss}$ in the FOC on capital of firm j . At steady state this becomes

$$K_j = \frac{\beta}{1 - \beta(1 - \delta) - \kappa_j(1 - \beta R)} \epsilon \alpha_j Y_j.$$

The steady state level of labor demand by firm j is

$$L_j = \frac{1}{\psi} \epsilon (1 - \alpha_j) Y_j.$$

Since we have the expressions for K_j and L_j , we can derive the steady state level of firm j 's output,

$$Y_j = A_j \left[\frac{\beta}{1 - \beta(1 - \delta) - \kappa_j(1 - \beta R)} \epsilon \alpha_j \right]^{\frac{\epsilon}{1-\epsilon} \alpha_j} \left[\frac{1}{\psi} \epsilon (1 - \alpha_j) \right]^{\frac{\epsilon}{1-\epsilon} (1 - \alpha_j)},$$

which we can substitute back in the expressions for K_j and L_j to obtain the steady state levels of capital and labor in firm j given the parameter values and the absence of shocks. The consumption of the household is then

$$C = \psi L + \sum_{u \in \mathcal{U}} \sum_{j \in \mathcal{N}_u} m_{uj} F_j,$$

where

$$F_j \equiv \left[1 - \frac{[(R - 1)\kappa_j + \delta] \beta}{1 - \beta(1 - \delta) - \kappa_j(1 - \beta R)} \epsilon \alpha_j - \epsilon (1 - \alpha_j) \right] Y_j$$

is the flow-of-funds generated by firm j at steady state, and the aggregate labor supply at steady state is $L = \sum_{j \in \mathcal{N}} L_{jt}$ by the market clearing condition. We define the vector $\mathbf{u}$ as an $N \times 1$ vector that identifies those firms that are ultimate owners, that is, where $u_j = 1$ if $j \in \mathcal{U}$ and $u_j = 0$ if $j \notin \mathcal{U}$, for every $j \in \mathcal{N}$. Thus, we can rewrite the consumption as

$$C = \psi L + \mathbf{u}^\top \mathbf{M} \mathbf{f},$$

where $\mathbf{M}$ is the $N \times N$ matrix with typical element m_{ij} and $\mathbf{f}$ is the $N \times 1$ vector with typical element F_j . □

Proof of Proposition 2. The wage W_t and the equity θ_{ut} in the ultimate owner are constant in equilibrium, so their log-deviations from their steady state values are nil. In other words,

$$\hat{W}_t = 0,$$

$$\hat{\theta}_{ut} = 0.$$

Moreover, the market clearing condition for the labor market yields

$$\hat{L}_t = \sum_{j \in \mathcal{N}} \frac{L_j}{L} \hat{L}_{jt},$$

and the budget constraint of the household can be written as

$$\hat{C}_t = \frac{\psi L}{C} \hat{L}_t + \sum_{u \in \mathcal{U}} \frac{D_u}{C} \hat{D}_{ut},$$

where

$$\hat{D}_{ut} = \sum_{j \in \mathcal{N}_u} \frac{m_{uj} F_j}{D_u} \hat{F}_{jt},$$

$$\hat{F}_{jt} = \frac{1}{F_j} \tilde{F}_{jt},$$

and

$$\tilde{F}_{jt} \equiv Y_j \hat{Y}_{jt} - B_j \hat{B}_{jt+1} - R B_j (\hat{R}_t + \hat{B}_{jt}) - \psi L \hat{L}_{jt} - I_j \hat{I}_{jt}.$$

Thus,

$$\hat{C}_t = \frac{\psi L}{C} \hat{L}_t + \sum_{u \in \mathcal{U}} \sum_{j \in \mathcal{N}_u} \frac{m_{uj}}{C} \tilde{F}_{jt}.$$

Moreover, the log-deviation of the stochastic discount factor is

$$\hat{\beta}_{t+1} = \sigma \left(\widehat{C_t - \psi L_t} - \widehat{C_{t+1} - \psi L_{t+1}} \right),$$

that is,

$$\hat{\beta}_{t+1} = \sigma \sum_{u \in \mathcal{U}} \sum_{j \in \mathcal{N}_u} \frac{m_{uj}}{\sum_{u \in \mathcal{U}} D_u} \left[\tilde{F}_{jt} - \tilde{F}_{jt+1} \right], \quad (23)$$

From this expression, we deduce that, in order to characterize the dynamics of the aggregate variables, we first have to derive the dynamics of the firm-specific variables. First,

$$\hat{Y}_{jt+1} = (1 - \epsilon) \hat{A}_{jt+1} + \epsilon \left[\alpha_j \hat{K}_{jt+1} + (1 - \alpha_j) \hat{L}_{jt+1} \right].$$

Second,

$$\begin{aligned} \hat{K}_{jt+1} = & \hat{Y}_{jt+1} \\ & + \frac{1}{1 - \beta(1 - \delta) - \kappa_j(1 - \beta R)} (1 - \kappa_j) \hat{\beta}_{t+1} \\ & + \frac{1}{1 - \beta(1 - \delta) - \kappa_j(1 - \beta R)} \kappa_j (1 - \beta R) \hat{\kappa}_{jt} \\ & - \frac{1}{1 - \beta(1 - \delta) - \kappa_j(1 - \beta R)} \kappa_j \beta R \hat{R}_{t+1}, \end{aligned}$$

Third,

$$\hat{L}_{jt+1} = \hat{Y}_{jt+1}.$$

So, we can derive an expression of $\hat{Y}_{jt}$ as simply a function of shocks and the log-deviation of the stochastic discount factor, that is,

$$\begin{aligned} \hat{Y}_{jt+1} = & \frac{\epsilon}{1 - \epsilon} \alpha_j \frac{1}{1 - \beta(1 - \delta) - \kappa_j(1 - \beta R)} (1 - \kappa_j) \hat{\beta}_{t+1} \\ & + \hat{A}_{jt+1} \\ & + \frac{\epsilon}{1 - \epsilon} \alpha_j \frac{1}{1 - \beta(1 - \delta) - \kappa_j(1 - \beta R)} \kappa_j (1 - \beta R) \hat{\kappa}_{jt} \\ & - \frac{\epsilon}{1 - \epsilon} \alpha_j \frac{1}{1 - \beta(1 - \delta) - \kappa_j(1 - \beta R)} \kappa_j \beta R \hat{R}_{t+1}. \end{aligned} \quad (24)$$

Thus,

$$\begin{aligned}\hat{K}_{jt+1} = & \frac{1 - \epsilon(1 - \alpha_j)}{1 - \epsilon} \frac{1}{1 - \beta(1 - \delta) - \kappa_j(1 - \beta R)} (1 - \kappa_j) \hat{\beta}_{t+1} \\ & + \hat{A}_{jt+1} \\ & + \frac{1 - \epsilon(1 - \alpha_j)}{1 - \epsilon} \frac{1}{1 - \beta(1 - \delta) - \kappa_j(1 - \beta R)} \kappa_j(1 - \beta R) \hat{\kappa}_{jt} \\ & - \frac{1 - \epsilon(1 - \alpha_j)}{1 - \epsilon} \frac{1}{1 - \beta(1 - \delta) - \kappa_j(1 - \beta R)} \kappa_j \beta R \hat{R}_{t+1}\end{aligned}$$

Since we log-linearize the system of equation around the deterministic steady state, then we look at the dynamics in a situation where the collateral constraint binds for all firms. Hence, in the system of equations that we have to log-linearize, $B_{jt+1} = \kappa_{jt} K_{jt+1}$. This implies that

$$\hat{B}_{jt+1} = \hat{\kappa}_{jt} + \hat{K}_{jt+1},$$

that is,

$$\begin{aligned}\hat{B}_{jt+1} = & \frac{1 - \epsilon(1 - \alpha_j)}{1 - \epsilon} \frac{1}{1 - \beta(1 - \delta) - \kappa_j(1 - \beta R)} (1 - \kappa_j) \hat{\beta}_{t+1} \\ & + \hat{A}_{jt+1} \\ & + \left(1 + \frac{1 - \epsilon(1 - \alpha_j)}{1 - \epsilon} \frac{1}{1 - \beta(1 - \delta) - \kappa_j(1 - \beta R)} \right) \hat{\kappa}_{jt} \\ & - \frac{1 - \epsilon(1 - \alpha_j)}{1 - \epsilon} \frac{1}{1 - \beta(1 - \delta) - \kappa_j(1 - \beta R)} \kappa_j \beta R \hat{R}_{t+1}\end{aligned}$$

Moreover, the log-deviation of the investment is

$$\hat{I}_{jt} = \frac{K_j}{I_j} \hat{K}_{jt+1} - (1 - \delta) \frac{K_j}{I_j} \hat{K}_{jt},$$

that is,

$$\begin{aligned}
I_{jt}\hat{I}_{jt} = & K_j \frac{1 - \epsilon(1 - \alpha_j)}{1 - \epsilon} \frac{1}{1 - \beta(1 - \delta) - \kappa_j(1 - \beta R)} (1 - \kappa_j) \hat{\beta}_{t+1} \\
& + K_j \hat{A}_{jt+1} \\
& + K_j \frac{1 - \epsilon(1 - \alpha_j)}{1 - \epsilon} \frac{1}{1 - \beta(1 - \delta) - \kappa_j(1 - \beta R)} \kappa_j (1 - \beta R) \hat{\kappa}_{jt} \\
& - K_j \frac{1 - \epsilon(1 - \alpha_j)}{1 - \epsilon} \frac{1}{1 - \beta(1 - \delta) - \kappa_j(1 - \beta R)} \kappa_j \beta R \hat{R}_{t+1} \\
& - (1 - \delta) K_j \frac{1 - \epsilon(1 - \alpha_j)}{1 - \epsilon} \frac{1}{1 - \beta(1 - \delta) - \kappa_j(1 - \beta R)} (1 - \kappa_j) \hat{\beta}_t \\
& - (1 - \delta) K_j \hat{A}_{jt} \\
& - (1 - \delta) K_j \frac{1 - \epsilon(1 - \alpha_j)}{1 - \epsilon} \frac{1}{1 - \beta(1 - \delta) - \kappa_j(1 - \beta R)} \\
& \quad \kappa_j (1 - \beta R) \hat{\kappa}_{jt-1} \\
& + (1 - \delta) K_j \frac{1 - \epsilon(1 - \alpha_j)}{1 - \epsilon} \frac{1}{1 - \beta(1 - \delta) - \kappa_j(1 - \beta R)} \kappa_j \beta R \hat{R}_t.
\end{aligned}$$

Hence, we can substitute all the log-deviations of the endogenous variables into (23) to obtain an expression for the dynamics of the stochastic discount

factor. In order to do that, let us look at $(\tilde{F}_{jt} - \tilde{F}_{jt-1})$, that is,

$$\begin{aligned}
\tilde{F}_{jt} - \tilde{F}_{jt+1} &= \\
&= \left[(Y_j - \psi L_j) \frac{\epsilon \alpha_j}{1 - \epsilon(1 - \alpha_j)} + (1 - \delta)K_j - RB_j \right] C_{\beta j}(1 - \kappa_j) \hat{\beta}_t \\
&- \left[(Y_j - \psi L_j) \frac{\epsilon \alpha_j}{1 - \epsilon(1 - \alpha_j)} + (1 - \delta)K_j - RB_j - B_j + K_j \right] \\
&\quad C_{\beta j}(1 - \kappa_j) \hat{\beta}_{t+1} \\
&+ [K_j - B_j] C_{\beta j}(1 - \kappa_j) \hat{\beta}_{t+2} \\
&+ [(Y_j - \psi L_j) + (1 - \delta)K_j - RB_j] \hat{A}_{jt} \\
&- [(Y_j - \psi L_j) + (1 - \delta)K_j - RB_j + K_j - B_j] \hat{A}_{jt+1} \\
&+ [K_j - B_j] \hat{A}_{jt+2} \\
&+ \left[\left[(Y_j - \psi L_j) \frac{\epsilon \alpha_j}{1 - \epsilon(1 - \alpha_j)} + (1 - \delta)K_j - RB_j \right] C_{\beta j} - RB_j \right] \hat{\kappa}_{jt-1} \\
&- \left[\left[(Y_j - \psi L_j) \frac{\epsilon \alpha_j}{1 - \epsilon(1 - \alpha_j)} - RB_j + (1 - \delta)K_j - B_j + K_j \right] \right. \\
&\quad \left. C_{\beta j} \kappa_j (1 - \beta R) - RB_j - B_j \right] \hat{\kappa}_{jt} \\
&+ [[K_j - B_j] C_{\beta j} \kappa_j (1 - \beta R) - B_j] \hat{\kappa}_{jt+1} \\
&- \left[\left[(Y_j - \psi L_j) \frac{\epsilon \alpha_j}{1 - \epsilon(1 - \alpha_j)} - (1 - \delta)K_j + RB_j \right] \right. \\
&\quad \left. C_{\beta j} \kappa_j \beta R + RB_j \right] \hat{R}_t \\
&+ \left[\left[(Y_j - \psi L_j) \frac{\epsilon \alpha_j}{1 - \epsilon(1 - \alpha_j)} - RB_j + (1 - \delta)K_j + K_j - B_j \right] \right. \\
&\quad \left. C_{\beta j} \kappa_j \beta R + RB_j \right] \hat{R}_{t+1} \\
&+ [B_j - K_j] C_{\beta j} \kappa_j \beta R \hat{R}_{t+2},
\end{aligned}$$

where

$$C_{\beta j} \equiv \frac{1 - \epsilon(1 - \alpha_j)}{1 - \epsilon} \frac{1}{1 - \beta(1 - \delta) - \kappa_j(1 - \beta R)}.$$

We can substitute $\mathbb{E}_{t-1} [\tilde{F}_{jt-1}] - \mathbb{E}_t [\tilde{F}_{jt}]$ into (23) and obtain

$$\begin{aligned}
\mathbf{u}^\top \mathbf{M} \left[\mathbf{w}_1 \hat{\beta}_t - \left(\mathbf{w}_2 + \frac{\mathbf{f}}{\sigma} \right) \hat{\beta}_{t+1} + \mathbf{w}_3 \hat{\beta}_{t+2} \right] = \\
\mathbf{u}^\top \mathbf{M} \left[\mathbf{w}_4 \hat{R}_t - \mathbf{w}_5 \hat{R}_{t+1} - \mathbf{w}_6 \hat{R}_{t+2} \right. \\
- \Delta(\mathbf{w}_7) \hat{\mathbf{A}}_t + \Delta(\mathbf{w}_8) \hat{\mathbf{A}}_{t+1} - \Delta(\mathbf{w}_9) \hat{\mathbf{A}}_{t+2} \\
\left. - \Delta(\mathbf{w}_{10}) \hat{\kappa}_{t-1} + \Delta(\mathbf{w}_{11}) \hat{\kappa}_t - \Delta(\mathbf{w}_{12}) \hat{\kappa}_{t+1} \right], \tag{25}
\end{aligned}$$

where

$$\mathbf{u}^\top \mathbf{M} \mathbf{x} = \sum_{u \in \mathcal{U}} \sum_{j \in \mathcal{N}_u} m_{uj} x_j$$

for any vector $\mathbf{x}$ whose typical element is x_j . The vector $\mathbf{x}$ can be $\mathbf{1}$, $\mathbf{f}$, or $\mathbf{w}_l$ for every $l \in \{1, \dots, 14\}$, where the typical element of $\mathbf{w}_l$ is W_j^l for every $l \in \{1, \dots, 14\}$, where

$$\begin{aligned}
W_j^1 &\equiv \left[(Y_j - \psi L_j) \frac{\epsilon \alpha_j}{1 - \epsilon(1 - \alpha_j)} + (1 - \delta) K_j - R B_j \right] C_{\beta_j} (1 - \kappa_j), \\
W_j^2 &\equiv \left[(Y_j - \psi L_j) \frac{\epsilon \alpha_j}{1 - \epsilon(1 - \alpha_j)} + (1 - \delta) K_j - R B_j - B_j + K_j \right] C_{\beta_j} (1 - \kappa_j), \\
W_j^3 &\equiv [K_j - B_j] C_{\beta_j} (1 - \kappa_j), \\
W_j^4 &\equiv \left[(Y_j - \psi L_j) \frac{\epsilon \alpha_j}{1 - \epsilon(1 - \alpha_j)} - (1 - \delta) K_j + R B_j \right] C_{\beta_j} \kappa_j \beta R + R B_j, \\
W_j^5 &\equiv \left[(Y_j - \psi L_j) \frac{\epsilon \alpha_j}{1 - \epsilon(1 - \alpha_j)} - R B_j + (1 - \delta) K_j + K_j - B_j \right] \\
&\quad C_{\beta_j} \kappa_j \beta R + R B_j, \\
W_j^6 &\equiv [B_j - K_j] C_{\beta_j} \kappa_j \beta R, \\
W_j^7 &\equiv [(Y_j - \psi L_j) + (1 - \delta) K_j - R B_j], \\
W_j^8 &\equiv [(Y_j - \psi L_j) + (1 - \delta) K_j - R B_j + K_j - B_j], \\
W_j^9 &\equiv [K_j - B_j], \\
W_j^{10} &\equiv \left[\left[(Y_j - \psi L_j) \frac{\epsilon \alpha_j}{1 - \epsilon(1 - \alpha_j)} + (1 - \delta) K_j - R B_j \right] C_{\beta_j} - R B_j \right],
\end{aligned}$$

$$W_j^{11} \equiv \left[\left[(Y_j - \psi L_j) \frac{\epsilon \alpha_j}{1 - \epsilon(1 - \alpha_j)} - RB_j + (1 - \delta)K_j - B_j + K_j \right] \right. \\ \left. C_{\beta j} \kappa_j (1 - \beta R) - RB_j - B_j \right],$$

$$W_j^{12} \equiv [[K_j - B_j] C_{\beta j} \kappa_j (1 - \beta R) - B_j].$$

The notation $\Delta(\mathbf{x})$ indicates the $N \times N$ diagonal matrix whose j -th diagonal element is j -th element of $\mathbf{x}$, for any vector $\mathbf{x}$. We denote now with future value of any variable with the lead operator $\mathbf{L}$, that is, $\mathbf{L}x_t = x_{t+1}$, for any x_t . Hence, we can rewrite (25) as

$$\mathbf{u}^\top \mathbf{M} \left[\mathbf{w}_1 - \left(\mathbf{w}_2 + \frac{\mathbf{f}}{\sigma} \right) \mathbf{L} + \mathbf{w}_3 \mathbf{L}^2 \right] \hat{\beta}_t = \mathbf{u}^\top \mathbf{M} \left[[\mathbf{w}_4 - \mathbf{w}_5 \mathbf{L} - \mathbf{w}_6 \mathbf{L}^2] \hat{R}_t \right. \\ \left. - [\Delta(\mathbf{w}_7) - \Delta(\mathbf{w}_8) \mathbf{L} + \Delta(\mathbf{w}_9) \mathbf{L}^2] \hat{\mathbf{A}}_t \right. \\ \left. - [\Delta(\mathbf{w}_{10}) - \Delta(\mathbf{w}_{11}) \mathbf{L} + \Delta(\mathbf{w}_{12}) \mathbf{L}^2] \hat{\kappa}_{t-1} \right],$$

that is,

$$\mathbf{u}^\top \mathbf{M} \mathbf{W}_\beta(\mathbf{L}) \hat{\beta}_t = \mathbf{u}^\top \mathbf{M} \left[\mathbf{W}_R(\mathbf{L}) \hat{R}_t - \mathbf{W}_A(\mathbf{L}) \hat{\mathbf{A}}_t - \mathbf{W}_\kappa(\mathbf{L}) \hat{\kappa}_{t-1} \right], \quad (26)$$

where

$$\mathbf{w}_\beta(\mathbf{L}) \equiv \left[\mathbf{w}_1 - \left(\mathbf{w}_2 + \frac{\mathbf{f}}{\sigma} \right) \mathbf{L} + \mathbf{w}_3 \mathbf{L}^2 \right],$$

$$\mathbf{w}_R(\mathbf{L}) \equiv [\mathbf{w}_4 - \mathbf{w}_5 \mathbf{L} - \mathbf{w}_6 \mathbf{L}^2],$$

$$\mathbf{W}_A(\mathbf{L}) \equiv [\Delta(\mathbf{w}_7) - \Delta(\mathbf{w}_8) \mathbf{L} + \Delta(\mathbf{w}_9) \mathbf{L}^2],$$

$$\mathbf{W}_\kappa(\mathbf{L}) \equiv [\Delta(\mathbf{w}_{10}) - \Delta(\mathbf{w}_{11}) \mathbf{L} + \Delta(\mathbf{w}_{12}) \mathbf{L}^2].$$

As long as the process in (26) admits an ARMA representation and that representation is invertible, we can obtain an expression of $\hat{\beta}_t$ as a function of the shocks to the return rate $\hat{R}_t$, the idiosyncratic productivity $\hat{\mathbf{A}}_t$, and the collateral constraints $\hat{\kappa}_t$.³³ Suppose that the roots of $\mathbf{u}^\top \mathbf{M} \mathbf{w}_\beta(\mathbf{L})$ lie outside the unit circle. Then, we can iterate forward the process and write the log-deviation of the stochastic discount factor from its steady state value at time t as

$$\hat{\beta}_t = \pi_R(\mathbf{L}) \hat{R}_t - \pi_A(\mathbf{L}) \hat{\mathbf{A}}_t - \pi_\kappa(\mathbf{L}) \hat{\kappa}_{t-1}, \quad (27)$$

³³See Evans and Honkapohja (1986) for a complete characterization of the ARMA representation.

where

$$\begin{aligned}\pi_R(\mathbf{L}) &\equiv [\mathbf{u}^\top \mathbf{M} \mathbf{w}_\beta(\mathbf{L})]^{-1} [\mathbf{u}^\top \mathbf{M} \mathbf{w}_R(\mathbf{L})], \\ \boldsymbol{\pi}_A(\mathbf{L}) &\equiv [\mathbf{u}^\top \mathbf{M} \mathbf{W}_\beta(\mathbf{L})]^{-1} [\mathbf{u}^\top \mathbf{M} \mathbf{W}_A(\mathbf{L})], \\ \boldsymbol{\pi}_\kappa(\mathbf{L}) &\equiv [\mathbf{u}^\top \mathbf{M} \mathbf{W}_\beta(\mathbf{L})]^{-1} [\mathbf{u}^\top \mathbf{M} \mathbf{W}_\kappa(\mathbf{L})].\end{aligned}$$

Note that while $\pi_R(\mathbf{L})$ is a polynomial in $\mathbf{L}$ that describes the effect of $\hat{R}_t$ on $\hat{\beta}_t$, and $\boldsymbol{\pi}_A(\mathbf{L})$ and $\boldsymbol{\pi}_\kappa(\mathbf{L})$ are $1 \times N$ vectors whose j -th entries are the polynomials in $\mathbf{L}$ that describe the effect of $\hat{A}_{jt}$ and $\hat{\kappa}_{jt-1}$ on $\hat{\beta}_t$. We can now substitute (27) into (24) and obtain

$$\begin{aligned}\hat{Y}_{jt} &= \hat{A}_{jt} + C_{Yj} \kappa_j (1 - \beta R) \hat{\kappa}_{jt-1} - C_{Yj} \kappa_j \beta R \hat{R}_t + C_{Yj} (1 - \kappa_j) \hat{\beta}_t \\ &= \hat{A}_{jt} + C_{Yj} \kappa_j (1 - \beta R) \hat{\kappa}_{jt-1} - C_{Yj} [\boldsymbol{\pi}_A(\mathbf{L}) \hat{\mathbf{A}}_t + \boldsymbol{\pi}_\kappa(\mathbf{L}) \hat{\boldsymbol{\kappa}}_{t-1}] \\ &\quad - C_{Yj} [\kappa_j \beta R - (1 - \kappa_j) \pi_R(\mathbf{L})] \hat{R}_t,\end{aligned}\tag{28}$$

where

$$C_{Yj} \equiv \frac{\epsilon \alpha_j}{1 - \epsilon} \frac{1}{1 - \beta(1 - \delta) - \kappa_j(1 - \beta R)}.$$

□

B Appendix: Data

The Infocamere raw data reports a fine detail about firms. Table 8 lists the possible juridical forms and their frequency distribution for the 2005 wave. In light of the concentration of the distribution of the juridical forms, we focus only firms that show limited liability of the owners, that is, SPA's ("SP" and "AU") and SRL's ("SR" and "SU"). This restriction comprises around 97% of the distribution of the juridical forms. We impose this restriction in the data because depending on the juridical form it is quite natural to think that the institutional framefork that the agents face may be considerably different, thus altering their incentives and interactions.

Infocamere provides also the type of right that each individual, be it a physical or a juridical subject, has on the equity of each firm. Unfortunately, this type of information is available only until 2009. Table 9 lists the types of right and their frequency distribution for the 2005 wave. The (full) ownership relations ("01 - PROPRIETA'") cover around 96% of the distribution of the types of right. Hence, we decide to ignore this detail in order not to lose all years from 2010 to 2013.

We correct also for those firms whose share sum up to more than 100% and for firms that report owning themselves. In this way, we drop less than 2% of the links.³⁴

³⁴All codes are available upon request.

Code	Description	Frequency	Percent
AA	SOCIETA' IN ACCOMANDITA PER AZIONI	4,715	0.04
AC	ASSOCIAZIONE	226	0.00
AE	SOCIETA' CONSORTILE IN ACCOMANDITA SEMPLICE		
AF	ALTRE FORME	87	0.00
AI	ASSOCIAZIONE IMPRESA	2	0.00
AL	AZIENDA SPECIALE DI ENTE LOCALE		
AM	AZIENDA MUNICIPALE	3	0.00
AN	SOCIETA' CONSORTILE IN NOME COLLETTIVO		
AP-AR	AZIENDA PROVINCIALE / AZIENDA REGIONALE		
AS	SOCIETA' IN ACCOMANDITA SEMPLICE	13,353	0.11
AT	AZIENDA AUTONOMA STATALE		
AU	SOCIETA' PER AZIONI CON UNICO SOCIO	37,134	0.29
AZ	AZIENDA SPECIALE	115	0.00
CC	CONSORZIO CON ATTIVITA' ESTERNA	14,540	0.11
CE	COMUNIONE EREDITARIA	13	0.00
CI	SOCIETA' COOPERATIVA A RESPONSABILITA ILLIMITATA	7	0.00
CL	SOCIETA' COOPERATIVA A RESPONSABILITA LIMITATA	360	0.00
CM	CONSORZIO MUNICIPALE		
CN	SOCIETA' CONSORTILE	1,701	0.01
CO	CONSORZIO	21,201	0.17
CR	CONSORZIO INTERCOMUNALE		
CS	CONSORZIO SENZA ATTIVITA' ESTERNA		
CZ	CONSORZIO DI CUI AL DLGS 267/2000	99	0.00
DI	IMPRESA INDIVIDUALE		
EC-ED	ENTE PUBBLICO COMMERCIALE / ENTE DIRITTO PUBBLICO		
EE-EI	ENTE ECCLESIASTICO / ENTE IMPRESA		
EM	ENTE MORALE		
EN	ENTE	123	0.00
EP	ENTE PUBBLICO ECONOMICO	7	0.00
ER	ENTE ECCLESIASTICO CIVILMENTE RICONOSCIUTO		
FI	FONDAZIONE IMPRESA	2	0.00
FO	FONDAZIONE	480	0.00
GE	GRUPPO EUROPEO DI INTERESSE ECONOMICO	235	0.00
IC-ID	ISTITUTO DI CREDITO / ISTIT DI CREDITO DI DIRITTO PUBBLICO		
IF-IR	IMPRESA FAMILIARE / ISTITUTO RELIGIOSO		
LL-MA	AZIENDA SPECIALE DI CUI AL DLGS 267/2000 / MUTUA ASSICURAZ		
OC	SOCIETA' COOPERATIVA CONSORTILE	75	0.00
OO	COOPERATIVA SOCIALE	71	0.00
OS	SOCIETA' CONSORTILE COOPERATIVA A RESPONSABILITA' LIMITATA		
PA-PC	ASSOCIAZIONE IN PARTECIPAZIONE / PICCOLA SOCIETA' COOP		
PS	PICCOLA SOCIETA' COOPERATIVA A RESPONSABILITA' LIMITATA	12	0.00
SA	SOCIETA' ANONIMA		
SC	SOCIETA' COOPERATIVA	820	0.01
SE	SOCIETA' SEMPLICE	611	0.00
SF	SOCIETA' DI FATTO	2,159	0.02
SI	SOCIETA' IRREGOLARE		
SL	SOCIETA' CONSORTILE A RESPONSABILITA' LIMITATA	250,201	1.97
SM	SOCIETA' DI MUTUO SOCCORSO		
SN	SOCIETA' IN NOME COLLETTIVO	14,372	0.11
SO	SOCIETA' CONSORTILE PER AZIONI	46,800	0.37
SP	SOCIETA' PER AZIONI	1,358,555	10.70
SR	SOCIETA' A RESPONSABILITA' LIMITATA	10,439,388	82.22
SS	SOCIETA' COSTITUITA IN BASE A LEGGI DI ALTRO STATO	132	0.00
ST	SOGGETTO ESTERO	9	0.00
SU	SOCIETA' A RESPONSABILITA' LIMITATA CON UNICO SOCIO	489,678	3.86
SV	SOCIETA' TRA AVVOCATI		
SZ	SOCIETA' NON PREVISTA DALLA LEGISLAZIONE ITALIANA	1	0.00
XX	NON PRECISATA		
Total		12,697,287	100.00

Table 8: Distribution of juridical forms, 2005

Code	Description	Frequency	Percent
00	PROPRIETA' NON DICHIARATA	4	0.00
01	PROPRIETA'	12,697,287	96.36
02	USUFRUTTO	161,870	1.23
03	PEGNO	27,603	0.21
04	CONFERIMENTO		
05	NUDA PROPRIETA'	193,147	1.47
06	PIGNORAMENTO	951	0.01
07	CONFISCA	61	0.00
08	PEGNO (ESTINZIONE)	204	0.00
09	INTESTAZIONE FIDUCIARIA	24,277	0.18
10	SEQUESTRO GIUDIZIARIO	1,178	0.01
11	REINTESTAZIONE	28	0.00
12	PEGNO (MODIFICAZIONE)	2,355	0.02
13	USUFRUTTO (ESTINZIONE)	159	0.00
14	USUFRUTTO (MODIFICAZIONE)	10,835	0.08
15	INTESTAZIONE FIDUCIARIA (ESTINZIONE)	16	0.00
16	PIGNORAMENTO (MODIFICAZIONE)	60	0.00
17	PIGNORAMENTO (ESTINZIONE)	4	0.00
18	SEQUESTRO GIUDIZIARIO (MODIFICAZIONE)	69	0.00
19	SEQUESTRO GIUDIZIARIO (ESTINZIONE)	1	0.00
20	INTESTAZIONE FIDUCIARIA (MODIFICAZIONE)	34	0.00
99	ALTRO	95	0.00
		56,131	0.43
Total		13,176,369	100.00

Table 9: Distribution of types of right, 2005

C Appendix: Additional figures and robustness exercises

Figure 16: Indegree distribution. Frequency distribution of firms with a given number of participants. Both axes are in log scale.

Figure 17: Outdegree distribution. Frequency distribution of firms with a given number of participations. Both axes are in log scale.

Figure 18: Nonlinear dependence of growth rate of sales, controlling for aggregate, sector, and group effects among firms.

Figure 19: Linear dependence of growth rate of sales per worker (standardized at the industry-period level).

Figure 20: Linear dependence of growth rate of sales per worker considering only majority shares (standardized at the industry-period level).

D Appendix: Filtered shocks

Figure 21: Idiosyncratic shocks obtained through the Kalman filter using the benchmark model with the calibrated ownership network and the counterfactual model with no ownership links. We order the shocks by type (productivity shocks on the left column, collateral shocks on the right column) and by firm type (participant firm in the first line, participated firm in the second line, and stand-alone firm in the third line). The participant firm is the ultimate owner and the participated firm is one of the subsidiaries of the biggest corporate group in the benchmark case. The stand-alone firm is a firm that does not belong to any corporate group and is directly owned by the household. In the counterfactual case, all firms are stand-alones (the counterfactual shock series are the same across firm types).

Figure 22: Aggregate shocks obtained through the Kalman filter using the benchmark model with the calibrated ownership network and the counterfactual model with no ownership links. We order the shocks by type (productivity shocks in the first line, collateral shocks in the second line, interest rate shocks in the third line).

Figure 23: Realizations of the measurement errors obtained through the Kalman filter using the benchmark model with the calibrated ownership network and the counterfactual model with no ownership links. We order the measurement errors by which observable they are associated to (GDP in the first line, consumption in the second line, and investment in the third line).