Secondary Market Liquidity
and the Optimal Capital Structure*

[Preliminary and Incomplete]

David M. Arseneau David Rappoport Alexandros Vardoulakis
Federal Reserve Board Federal Reserve Board Federal Reserve Board

February 15, 2015

Abstract

We present a model to study the feedback loop between secondary market liquidity and firm’s financing decisions in primary markets. The model features two key frictions: a costly state verification problem in primary markets, and search frictions in over-the-counter secondary markets. Our liquidity concept depends on the endogenous holdings of assets put for sale relative to the resources available for buying illiquid assets. This creates a feedback loop as issuance in primary markets affects secondary market liquidity, and vice versa through liquidity premia. We show that the privately optimal allocations are inefficient. Both investors and firms can be made better off if firms take on lower leverage and less risk, and investors provide more liquidity. These inefficiencies are established analytically through a set of wedge expressions for key efficiency margins. Our analysis provides a rationale for the effect of quantitative easing on secondary and primary capital markets and the real economy.

Keywords: Market liquidity, secondary markets, capital structure, quantitative easing.

JEL classification: E44, G18, G30.

* We are grateful to Skander Van den heuvel and seminar participants at the Board of Governors of the Federal Reserve for comments. All errors herein are ours. The views expressed in this paper are those of the authors and do not necessarily represent those of Federal Reserve Board of Governors or anyone in the Federal Reserve System.

Emails: david.m.arseneau@frb.gov, david.e.rappoport@frb.gov, alexandros.vardoulakis@frb.gov.
1 Introduction

Secondary market liquidity is an important consideration for investors buying long-term assets. At the same time, the issuance of both short-term and long-term assets affects the liquidity of investment portfolios. This creates a feedback loop between secondary market liquidity and firm’s financing decisions in primary capital markets. How does this feedback loop affect the optimal capital structure of the firm? Are firms’ capital structure decisions efficient? Can large long-term asset purchases have an effect through primary or secondary markets?

This feedback loop between secondary and primary markets is important to understand the real effects of imperfections in financial markets. Herein, we are interested in imperfect secondary trading that gives rise to liquidity risk, as liquidity needs can not be perfectly met selling assets in secondary markets. This is an important friction for the intermediation that takes place outside the traditional banking sector, and potentially through the shadow banking system. Liquidity risk spills over to primary markets where these assets are issued, since investors would require higher premia to invest in them. This direct channel has received considerable attention as it is closely related to the idea of transaction or information costs impeding trading, as well as to the lending channel of monetary policy. Another linkage is the way that asset holdings affect liquidity risk by influencing liquidity in secondary markets. This channel has been less explored in the literature, but it is key to understand how the liability structure of firms matters for the optimal intermediation of liquidity risk and the real economy.

In a seminal paper, Holmström and Tirole (1998) study a similar question to ours, but focus on the liquidity needs of firms to cover operational costs before their investment matures. In contrast, we focus on the liquidity demand of lenders. To this extent, we model the demand for liquidity as in the seminal paper of Diamond and Dybvig (1983), but bring re-trading of long-term assets, aggregate liquidity and the capital structure to the center of our analysis.

We present a three period economy populated by firms that invest in long-term projects and need external financing, and investors who want to transfer funds to consume in all periods. Crucially, they may want to consume before the completion of the firm’s project. There are two key frictions in the model, one in the primary market and one in the secondary market for firm’s financial contracts. The focus of our analysis is on the interaction between these two frictions that arises when investors face liquidity risk.
Ex-ante identical investors supply funds to firms in primary capital markets in the initial period, while firms issue claims against their long-term revenues that materialize in the final period. The contracting problem between the firm and investors is subject to agency frictions, which we model using the Costly State Verification (CVS) framework (Townsend, 1979; Gale and Hellwig, 1985; Bernanke and Gertler, 1989). One of the reasons for our choice is that we are interested in OTC markets for corporate debt and CVS provides a rationale for the use of debt financing. In addition, we are not only interested in the effect of liquidity premia on the level of firms’ borrowing, but also in their effect on credit risk and default premia on corporate bonds, and vice versa. The CSV framework offers a good environment for the joint examination of these issues, since it yields risky debt as the optimal contract between the firm and investors in the primary market.

After the financial contract between the firm and investors has been written and investment decisions have been made, investors receive idiosyncratic (liquidity) shocks to consume before the firms’ investments mature and proceeds are distributed. The shocks are private information and, thus, contingent contracts among patient and impatient investors cannot be written ex-ante. Investors can alternatively self-insure by investing part of their endowment in a storage technology and/or by holding corporate bonds and re-trading them in a secondary market. Corporate bonds are not only a claim on real revenues, but also have a role in facilitating the process of exchange (see also Rocheteau and Wright, 2013). As a result, they are not only valuable for their rates of return, but also for their liquidity services. In a frictionless world, the demand for liquidity would be perfectly satisfied by selling long-term bonds in secondary markets. However, in practice, trading frictions render the trading of long-term assets imperfect and introduce liquidity premia. For assets that are traded in Over-The-Counter (OTC) markets, such as corporate bonds that we have in mind, search frictions are important as suggested empirically by Edwards et al. (2007) and Bao et al. (2011).1

---

1 Bond financing has become one of the most important sources of external financing for U.S. corporations. Figure 3 shows that bond financing is the dominant source of credit liabilities for non-financial corporate firms (Flow of Funds data). This paper focuses on bond financing abstracting from the fact that firms enter into bank loans or other types of borrowing at the same time (see deFiore and Uhlig, 2011, for a model where bank and bond financing coexist). In principle, bank intermediation would be optimal to insure against idiosyncratic liquidity risk in the spirit of Diamond and Dybvig (1983) when bank runs are not very likely (see Cooper and Ross, 1998, and Goldstein and Pauzner, 2005) or bank credit is not sufficiently more expensive than bond financing as in de Fiore and Uhlig. However, Jacklin (1987) shows that the efficiency gains of bank intermediation for investors vanish when secondary capital markets are available and function frictionlessly. This should continue to be true when the associated frictions in secondary markets are not too severe, while bank intermediation would dominate when markets are more imperfect.
In particular, we introduce illiquidity in the secondary market through search frictions between sellers and buyers, or patient and impatient investors in terms of our model. We follow the literature assuming that the terms of trade, after a match is made, are determined by bargaining over the trade surplus. However, the matching function in our model depends on the relative size of buy and sell orders much like the directed search literature.\(^2\) In other words, the probability of one investor type meeting the other type does not only depend on exogenous parameters governing the search frictions or simply their relative size in the population of investors, but also on their holdings of assets put for sale and resources available for buying these assets. This is equivalent to matching by trade, i.e., every buy/sell order is treated in isolation, rather than matching by agent, which would be closer to random search models. As a result, every agent submits multiple orders in the secondary market and there is a probability that each individual offer is matched, but there are no agents who cannot find a counterparty to trade at all. To this extent, we endogenize the aggregate liquidity in the secondary market governing the matching probabilities and, hence, the severity of illiquidity.

Matching by trade has important implications for the interactions of secondary market liquidity and portfolio holdings. Portfolio holdings will respond to the severity of liquidity frictions, which in turn depend on the way that these holdings affect secondary market liquidity. Consequently, private decisions result in externalities operating via the aggregate market liquidity, which private agents do not internalize and render welfare improving policy interventions possible.

Another important feature of our model is that we connect search frictions and secondary market liquidity to the capital and investment decisions of a productive firm, while at the same time pay less attention to asset prices. On one hand, this allows us to endogenize the equilibrium supply of long-term assets. On the other hand, we are interested in studying the effect of liquidity premia on the capital structure and investment, as well as establish the dual link between primary and secondary markets for long-terms claims issued by firms illustrated in Figure 1.

On the positive side, our framework is useful to understand how liquidity premia on long-term assets depend on the decision of firms to issue these claims in the primary market. This is because we do not treat illiquidity as simply an outcome of exogenous

---

\(^2\)See Rogerson et al. (2005) for a detailed discussion and comparison of directed and random search models.
Lenders impose liquidity premia

Primary Market  Secondary Market

Borrowing affects liquidity

Figure 1: Interaction between primary and secondary market for corporate debt

frictions, but rather as an endogenous variable which depends on both the number of assets put for sale and the number of buy orders. Moreover, our mechanism can shed some light on the way that unconventional policies that enhance liquidity in the market affect liquidity premia and most importantly the investment decisions of firm (see Stein, 2014, for a general discussion on the topic).

One of the main results we establish is the effect of illiquidity on the capital structure of the firm. We find that imperfect secondary market liquidity accruing from search frictions leads investors to demand liquidity premia when they purchase corporate bonds in the primary market. In equilibrium, the firm responds by issuing not only fewer, but also less risky bonds. As illiquidity of the liabilities becomes more severe, firms become less leveraged and their liabilities are safer. Consequently, illiquidity implies higher liquidity, but lower default premia. Nevertheless, this implies a lower level of investment reducing aggregate welfare. This happens in equilibrium, despite the fact that lower debt levels enhance liquidity in secondary markets, as this second effect is not enough to offset the rise of liquidity premia.

On the normative side, we identify the externalities arising from the endogenous interaction of private decisions and secondary market liquidity. Agents do not internalize the effect that their decisions have on the aggregate market liquidity in the secondary market, and hence on liquidity premia.³ As a result, the investment and savings decisions are suboptimal; a planner would like the firm to issue even less debt and enhance further secondary market liquidity. In other words, lower risk premia enable firms to enjoy higher profits, even when operating at a lower scale, while investors are not being made worse off.

³This externality is akin to the one identified in Arseneau and Chugh (2012) in a model of directed labour search.
We identify three distorted margins that characterize the inefficiencies in our framework, and describe the three associated instruments that a planner may use to correct these inefficiencies. One key distortion arises because firms do not internalize how primary market issuance affects secondary market liquidity. We show that a tax on leverage induces firms to internalize the impact of their borrowing decisions on secondary market liquidity. This improves the liquidity of their liabilities, reduces the liquidity premia they have to pay and allows them to enjoy higher profits. The second key distortion arises from the fact that investors do not internalize how their portfolio holdings of savings and corporate bonds affects secondary market liquidity. We show that a subsidy on savings encourages investors to hold more liquid assets relative to bonds, which in turn enhances market liquidity. Finally, a third tool that is needed to fully implement the social planner’s solution is imposed directly on the way that secondary trade is conducted conditional on a match being made.

Firms issue inefficiently large quantities of debt and investors hold insufficient liquid assets. The fact that we can clearly identify the distorted margins and the optimal tools to correct them, allows us to make connections with types of policy interventions that have been observed during the Great Recession. Policies that affect the compositions of investors’ portfolio, like quantitative easing, are expected to affect the economy by compressing liquidity premia and affecting firms’ capital choices. Our analysis suggests that these type of policies might ought to be implemented in conjunction with other policies to limit corporate borrowing.

Before turning to the details of our model, we discuss below some of our most important modelling assumptions and connect them to the broader literature.

First, we deviate from the literature introducing search frictions in OTC asset markets by considering that every sell/buy order is matched independently (matching by trade) rather than having investors meeting randomly counterparties to trade their whole portfolio (matching by investor). This assumption distinguishes our framework from Duffie et al. (2005), who first introduced search and bargaining frictions for trade in OTC markets, and studied how illiquidity affects assets prices. In their paper, the holdings of agents participating in the OTC markets do not play an important role for search intensities and equilibrium outcomes, since holdings are restricted to be either 1 or 0. Lagos and Rocheteau (2009) relax the restrictions on asset holdings and utilize the fact that agents can mitigate trading frictions by adjusting their asset positions to reduce their trading needs. Thus, they can study how liquidity premia affect the portfolio holdings of agents, but not
the reverse linkage from portfolios to market liquidity.\footnote{In an extension of their baseline model, Lagos and Rocheteau (2009) also endogenize the matching probabilities by allowing for endogenous entry of deals, which stand on one side on the OTC transaction in their model. Yet, their model exhibits matching by agent making it more difficult to establish the connection between portfolio holdings, aggregate liquidity and matching intensities compared to our matching by trade formulation.} Most of this literature studies the implications of search frictions and illiquidity on asset prices, while we mainly focus on the interaction of primary markets for corporate assets and secondary liquidity in OTC markets.\footnote{He and Milbradt (2014) present a model with search frictions in OTC market for corporate bonds and show how default and liquidity premia, as well as the decision to default, are affected by market liquidity. However, they take the capital structure and investment of the firm as given, which in our model is endogenous, as it plays a crucial role for the interaction of primary and secondary markets.}

Second, the nature of agency frictions in the primary market that we choose is not detrimental for the generality of our results. CSV allows us to jointly study the effect of liquidity premia on the composition (leverage) and the riskiness of the capital structure of the firm. We believe that risk is an important dimension of adjustment for firms’ financing decision through the bond market, but one could instead perceive a situation where firms are credit constrained, and hence the adjustment takes place solely on leverage rather than the optimal combination of risk and leverage. (for example, Holmström and Tirole, 1997; Kiyotaki and Moore, 1997). The general results described above, namely that liquidity premia result in lower, but inefficiently high bond issuance in the primary market, would still hold. The reason is that we are able to disentangle the channel through which market liquidity affects liquidity premia in long-term assets from the choice of the optimal contract/capital structure of the firm, though the two are jointly determined in equilibrium.

Nevertheless, there is a fundamental difference between models featuring collateral constraints and our framework with respect to the concept of liquidity. The former emphasize funding liquidity, i.e., how much funds can be raised by pledging the firm’s assets, while our theory highlights the importance of market liquidity, i.e., the ease with which assets can be traded (Brunnermeier and Pedersen, 2009). Yet, this distinction for firm’s assets is immaterial in our framework, since the firm does not face any liquidity risk, such that the pledgability of its assets would matter as in Holmström and Tirole (1998), and only the liquidity of financial contracts matters. Thus our conclusions would carry to a framework where firms are collateral constrained.

Third, we have abstracted from issues related to adverse selection arising from asym-
metrically informed agents participating in the secondary market. In a seminal paper, Gorton and Pennacchi (1990) show how the information sensitivity of financial contracts affects their liquidity in secondary markets, and study the capital structure of the firm and efficient intermediation. Although similar in spirit, our approaches differ with respect to the frictions resulting in illiquid liabilities of the firm. Gorton and Pennacchi consider informational asymmetries between buyers and sellers in the secondary market for corporate liabilities and show that investors respond by demanding informationally insensitive assets, notably riskless debt. In contrast, we consider search frictions that limits trade in secondary markets. Although the informational sensitivity of firms’ liability is important, especially to understand investors’ participation decisions in these markets, we have chosen to abstract from such considerations and focus on the illiquidity of liabilities stemming from imperfect trading to understand the feedback between secondary and primary markets. In other words, we consider trade frictions to focus on the effect of changes in holdings of illiquid assets, i.e., the intensive margin of the portfolio problem, abstracting from information frictions that are important to understand the extensive margin of this problem.

Finally, we have abstracted from aggregate liquidity risk. Arguably the channel we examine is important in determining financing decisions and liquidity premia even when there is no aggregate liquidity risk in the economy. When investors face aggregate liquidity risk which cannot be hedged due to market incompleteness, liquidity provision in the form of aggregate savings/reserves may be suboptimally low (Bhattacharya and Gale, 1987; Allen and Gale, 2004). In our paper, liquidity underprovision stems from trading frictions rather than aggregate shocks, which yields important implications for the liquidity premia of corporate bonds during periods that aggregate liquidity shocks are expected to occur rather infrequently. Consequently, our mechanism could potentially explain the fluctuations in liquidity and default risk premia, as well as firms’ leverage.

---

6 There is an important literature following this tradition, such as Dang et al. (2011) and Gorton and Ordoñez (2014). Guerrieri and Shimer (2014) examine how adverse selection about the quality of assets affects their liquidity premia. They differ from the search microfoundations of illiquidity because the difficulty of finding a buyer depends primarily on the extent of private information rather than the availability of trading opportunities. Like us, but for different reasons, they suggest that unconventional policy interventions, such as asset purchase, can enhance the liquidity of assets not included in the purchase programs. Nevertheless, they do not study how illiquidity and policy interventions affect the equilibrium supply of assets, i.e. they abstract from corporate finance issues.

7 Liquidity underprovision may also stems from fire-sales externalities (Lorenzoni, 2008) or hidden trades undoing the efficient sharing of liquidity risk across impatient and patient agents as in Farhi et al. (2009).
even when aggregate liquidity shortages are unlikely or excluded due to the presence of unconventional policies, such as quantitative easing.

The rest of the paper proceeds as follows. Section 2 presents the model and derives the equilibrium conditions. Section 3 shows how secondary market liquidity interacts with the optimal financing decisions of the firm. Section 4 presents the social planner’s problem, and identifies the externalities inherent in the private economy as well as the optimal policy mix. Section 5 analyses the effect of quantitative easing on secondary market liquidity and financing decisions. Finally, section 6 concludes.

2 Model

2.1 Physical Environment

There are three time periods \( t = 0, 1, 2 \), a single consumption good, and two type of agents: entrepreneurs and investors. Entrepreneurs have long-term investment projects and may fund these projects with internal funds and borrowing from investors. Ex ante identical investors loan funds to entrepreneurs, but once that lending has taken place and while production is underway, investors are subject to a preference (liquidity) shock which reveals whether they are impatient, and hence prefer to consume earlier rather than later, or patient. These two types of investors trade their assets in secondary asset markets with search frictions (see Figure 2).

There is a mass one of ex ante identical entrepreneurs, who are endowed with \( n_0 \) units of consumption at \( t = 0 \). Entrepreneurs invest to maximize the return on their equity, i.e., to maximize profits per unit of endowment. The technology is linear and delivers \( R_k \omega \) at \( t = 2 \), per unit invested at \( t = 0 \). The random variable \( \omega \) is an idiosyncratic productivity shock that hits after the project starts, and is privately observed by the entrepreneur. It is distributed according to the cumulative distribution function \( F \), with unit mean. The (expected) gross return \( R^k \) is assumed to be known at \( t = 0 \), as there is no aggregate uncertainty in the model. In order to produce, the firm must finance investment, denoted \( k_0 \), either through its own funds or by issuing financial contracts to investors. So profits equal total revenue in period 2, \( R^k \omega k_0 \), minus payment obligations from financial contracts. Entrepreneurs represent the corporate sector in our model, so we will talk about entrepreneurs’ projects and firms interchangeably.

There is a mass one of ex ante identical investors, who are endowed with \( e_0 \) units
of consumption at $t = 0$. Investors have unknown preferences at $t = 0$, and learn their preferences at $t = 1$. At $t = 1$ investors realize if they are patient or impatient consumers, a fraction $1 - \delta$ will turn out to be patient and a fraction $\delta$ impatient. Patient consumers have preferences only for consumption in $t = 2$, $u^P(c_1, c_2) = c_2$, whereas impatient consumers have preferences for both consumption in $t = 1$ and $2$, but discount period 2 consumption at rate $\beta$, $u^I(c_1, c_2) = c_1 + \beta c_2$.

Investors in both period 0 and 1 have access to a storage technology with yield $r > 0$, i.e., every unit stored yields $1 + r$ units of consumption in the next period. The amount stored in period $t$ is denoted $s_t$. In addition, at $t = 0$, they can invest in financial contracts issued by entrepreneurs in primary markets; and, at $t = 1$, they can buy and sell assets in secondary markets with search frictions (see Figure 2). Both the primary and secondary markets are described in detail below.\(^8\)

In what follows we make the following assumptions.

---

\(^8\) Note that since $r > 0$ and since investors preferences have been assumed time separable and risk neutral, there was no loss of generality in abstracting away from consumption at $t = 0$ for investors, and consumption at $t = 1$ for patient investors.
Assumption 1 (Technology) The long-term return of the productive technology is larger than the cumulative two-period storage return, i.e., $R^k > (1 + r)^2 > R^k(1 - \mu)$.

Assumption 2 (Impatience) The rate of preference of impatient investors is such that $\beta \leq 1/(1 + r)$.

Assumption 3 (Productivity distribution) Let $h(\omega) = dF(\omega)/(1 - F(\omega))$, the hazard rate of the productivity distribution. Then it is assumed that $\omega h(\omega)$ is increasing.

Assumption 4 (Investors Deep Pockets) It is assumed that investors’ (total) endowment $e_0$ is significantly higher than entrepreneurs’ (total) endowment $n_0$, i.e., $e_0 >> n_0$.

Assumption 1 is necessary for there to be a role for the entrepreneurial sector, $R^k > (1 + r)^2$, and to rule out equilibria where the entrepreneurs are always monitored, $(1 + r)^2 > R^k(1 - \mu)$. Assumption 2 will make impatient investors to behave as impatient agents when the interest rate is $r$. Assumption 3 will ensure that there is no credit rationing in equilibrium, as we discuss below. Finally, Assumption 4 ensures that investors can meet the credit demand of entrepreneurs.

2.2 The Financial Contract

Entrepreneurs finance the investment project using either internal funds, $n_0$, or by selling long-term financial contracts to investors in the primary corporate debt market. The contract specifies an amount, $b_0$, borrowed from investors at $t = 0$ and a promised gross interest rate, $Z$, made upon completion of the project at $t = 2$. If entrepreneurs cannot make the promised interest payments, investors can take all firm’s proceeds paying a monitoring cost, equal to a fraction $\mu$ of the value of assets.

The $t = 0$ budget constraint for the entrepreneur is given by

$$k_0 \leq n_0 + b_0.$$  \hspace{1cm} (1)

For what follows it will be useful to define the entrepreneur’s leverage, $l_0$, as the ratio of assets to (internal) equity $k_0/n_0$.

The entrepreneur is protected by limited liability, so its profits are always non-negative. Thus, the firm’s expected profit in period $t = 2$ is given by

$$E_0 \max \{0, R^k \omega_0 k_0 - Zb_0\}$$
Limited liability implies that the entrepreneur will default on the contract if the realization of \( \omega \) is sufficiently low such that the payoff of the long-term project falls below the promised payout; that is, when \( R^k \omega k_0 < Z_0 \). This condition defines a threshold productivity level, \( \bar{\omega} \), such that the entrepreneur defaults when
\[
\omega < \bar{\omega} = \frac{Z_0 - 1}{R^k}.
\]
The productivity threshold is increasing in the spread between the promised return and the expected return on the entrepreneur investment, and increasing in firm’s leverage. Alternatively, this relationship gives the debt contract’s optimal gross interest rate \( Z \) as a function of leverage and the productivity threshold.

For notational convenience, we define
\[
G(\bar{\omega}) \equiv \int_{\bar{\omega}}^{\infty} \omega dF(\omega) \quad \text{and} \quad \Gamma(\bar{\omega}) \equiv \bar{\omega}(1-F(\bar{\omega}))+G(\bar{\omega}).
\]
The function \( G(\bar{\omega}) \) equals the truncated expectation of entrepreneurs’ productivity given default. The function \( \Gamma(\bar{\omega}) \) equals the expected value of a random variable equal to \( \omega \) if there is default (\( \omega < \bar{\omega} \)) and equal to \( \bar{\omega} \) when there is not (\( \omega \geq \bar{\omega} \)). It follows that \( R^k k_0 \Gamma(\bar{\omega}) \) corresponds to the expected transfers from entrepreneurs to investors.

Then, firms’ objective, expected profits per unit of endowment, or return on equity, can be expressed using the previous notation as
\[
\frac{1}{n_0} E_0 \max \{0, R^k \omega k_0 - Z_0 \} = [1 - \Gamma(\bar{\omega})] R^k l_0 = \frac{l_0 - 1}{l_0} R^k \Gamma(\bar{\omega}) - \mu G(\bar{\omega}),
\]
(2)
The objective of the firm in equation (2) is written in terms of return to equity rather than total profits. However, both formulation would yield the same equilibrium results as \( n_0 \) is positive and given.

Similarly, the expected payoffs of bond contracts can be expressed as
\[
\int_{\bar{\omega}}^{\infty} Z_0 dF(\omega) + (1-\mu) \int_{0}^{\bar{\omega}} R^k \omega k_0 dF(\omega) = k_0 R^k [\Gamma(\bar{\omega}) - \mu G(\bar{\omega})].
\]
Therefore, the expected gross return of holding a bond to maturity \( R^b \) is given by
\[
R^b = \frac{l_0}{l_0 - 1} R^k [\Gamma(\bar{\omega}) - \mu G(\bar{\omega})],
\]
(3)
which is a function of only leverage and the productivity threshold.

For the subsequent analysis it will be useful to note that the previously introduced
functions $G$ and $\Gamma$ have the following properties (see Bernanke et al. 1999): (i) $0 \leq \Gamma(\bar{\omega}) \leq 1$; (ii) $\Gamma'(\bar{\omega}) = 1 - F(\bar{\omega})$; (iii) $\Gamma''(\bar{\omega}) < 0$; (iv) $\lim_{\bar{\omega} \to 0} \Gamma(\bar{\omega}) = 0$; (v) $\lim_{\bar{\omega} \to \infty} \Gamma(\bar{\omega}) = 1$; (vi) $\lim_{\bar{\omega} \to 0} G(\bar{\omega}) = 0$; and (vii) $\lim_{\bar{\omega} \to \infty} G(\bar{\omega}) = 1$.

Clearly $R^b$ decreasing in $l_0$ as leverage dilutes lenders claim on the collateral. Moreover, is increasing in $\bar{\omega}$ when the latter is chosen optimally, i.e. $\Gamma'(\bar{\omega}) - \mu G'(\bar{\omega}) > 0$ in equilibrium. This derives from the fact that we are interested in non-rationing equilibria, where the firm can compensate investors by offering them a higher expected return. On the contrary, rationing solutions would require $\Gamma'(\bar{\omega}) - \mu G'(\bar{\omega}) \leq 0$. Finally, note that the expected return is known is period 0 and 1, since there is no aggregate uncentainty or new information arriving after investors and the firm have agreed on the terms of lending. This means that idiosyncratic liquidity shocks in period 1 do not affect $R^b$ and investors would trade bonds in a secondary market promising this expected return.

### 2.3 The Secondary OTC Market

The ex post heterogeneity introduced by the preference shock generates potential gains from trading corporate debt in a secondary market. Impatient investors want to exchange long-term, imperfectly liquid, bonds for consumption, as they would rather consume at the end of period 1 rather than holding the bond to maturity until period 2 (Assumption 2). Patient investors want to exchange lower yielding storage for higher yielding corporate debt.

In order for such a trade to take place, buy and sell orders must be paired up according to a matching technology which aligns them. Impatient investors submit sale orders, one for each bond they are ready to sell at a given price $q_1$. Patient investors submit buy orders, one for each package of $q_1$ units of storage they are ready to exchange for a bond.

We model the OTC market such that matching is by trade, as opposed to by investor.\footnote{This can be though of as money chasing bonds, instead of investors chasing investors.} Suppose, in aggregate, there are $A$ sell (or ask) orders and $B$ buy orders. The matching function is assumed to be constant returns to scale and is given by

$$m(A, B) = \nu A^\alpha B^{1-\alpha},$$

with $1 > \nu > 0$ a scaling constant.

We define a concept of market liquidity as the ratio of buy orders to sell orders, or

---

\[9\]
\[ \theta = B/A. \] When \( \theta \) is large (small), the secondary market is relatively (ill)liquid in the sense that the number of buy orders is large (small) relative to sell orders. This makes it easier (harder) for impatient investors to transform their corporate bonds into storage, which can then be consumed.

Indeed, the probability that a sell order is executed is expressed as

\[
f(A, B) = \min \left[ 1, \frac{m(A, B)}{A} \right] \quad \text{or} \quad f(\theta) = \min [1, m(1, \theta)],
\]

and the probability that a buy order is executed is expressed as

\[
p(A, B) = \min \left[ 1, \frac{m(A, B)}{B} \right] \quad \text{or} \quad p(\theta) = \min [1, m(\theta^{-1}, 1)].
\]

Once a buy order and a sell order are matched, the terms of trade are determined via Nash bargaining over the total surplus. The surplus that accrues to an impatient investor is given by

\[ S^I(q_1) = q_1 - \beta R^b. \]

From the seller’s perspective, a trading match yields additional liquid wealth from unloading the incremental bond sold at price \( q_1 \). If the seller walks away from the match she holds the bond, which matures in the final period, delivering an expected payout of \( R^b \) in \( t = 2 \), which is discounted at rate \( \beta \).

On the other side of the market, the surplus that accrues to a patient investor from matching funds in storage with a unit of bonds for sale is given by

\[ S^P(q_1) = \frac{R^b}{1 + r} - q_1. \]

The value of a trading match to a buyer is the present value of the (expected) return on the bond, net of the price that needs to be paid for each bond in the secondary market.\(^{10}\)

The price of the debt contract on the secondary market is determined via Nash Bargaining. That is the price is that which maximizes the Nash product of the respective surpluses,

\[
\max_{q_1} \left( S^I(q_1) \right)^\psi \left( S^P(q_1) \right)^{1-\psi},
\]

\(^{10}\) The present value for patient investors is calculated using \( 1 + r \) and not the rate of time preferences, equal to 1. This is the case since patient investors at the margin will save at rate \( 1 + r \) rather than substitute future for present consumption.
where \( \psi \in (0, 1) \) is the bargaining power of sellers.

The solution of the Nash bargaining problem yields a price

\[
q_1 = R^b \left( \frac{\psi}{1+r} + (1 - \psi)\beta \right).
\]

Note that giving full bargaining power to sellers \( \psi = 1 \) drives the price of the bond to the "bid" price, or the price that extracts full rent from the buyer, \( q_1 = \frac{R^b}{1+r} \). By the same token, giving full bargaining power to buyers (\( \psi = 0 \)) drives the price of the bond to the "ask" price, or the price that extracts full rent from the seller, \( q_1 = \beta R^b \). Thus, the spread for the possible equilibrium prices in the secondary market is determined by the preferences of impatient households. The spread equals

\[
\left( \frac{1}{1+r} - \beta \right) R^b
\]

where \( R^b \) is a function of \((l_0, \bar{\omega})\) (and primitives: \( R^k, F, \ldots \)).

### 2.4 Investors

As described above, investors are ex ante identical and are endowed with \( e_0 \) units of consumption. At \( t = 0 \) they can allocate their wealth across two assets: the storage technology and debt contracts. Thus, the budget constraint is given by

\[
s_0 + b_0 = e_0,
\]

where \( s_0, b_0 \geq 0 \), i.e. borrowing at the storage rate or short-selling corporate debt are not allowed.

The storage technology, denoted \( s_0 \), pays a fixed rate of return \( 1 + r \) at \( t = 1 \) in units of consumption. The proceeds of this investment, if not consumed, can be reinvested to earn an additional return of \( 1 + r \) between period 1 and 2, again paid in units of consumption. In this sense, storage is a liquid investment, as at any point in time it can be costlessly transformed into consumption. Alternatively, the corporate bond has an expected payoff of \( R^b \), but only at the beginning of \( t = 2 \). Moreover, for an investor to turn her bond into consumption at \( t = 1 \), she will have to post an order in a secondary market characterized

\[\text{\footnotesize\footnote{Since the mass of both entrepreneurs and investors equals one, and we focus on the symmetric equilibrium, we abuse notation and denote the individual supply and demand of debt by } b_0.}\]
by search frictions. So the bond is illiquid, as it does not allow investors to transform their investment costlessly into consumption in period 1.

The relative illiquidity of corporate debt comes into play because at the beginning of \( t = 1 \), a fraction \( \delta \) of investors receive a preference shock that makes them discount future consumption by rate \( \beta \). Moreover, Assumption 2 implies that impatient investors strictly prefer to consume in period 1 relative to period 2. In contrast, the remaining fraction \( 1 - \delta \) are patient investors, who only enjoy consumption in \( t = 2 \).

Thus, impatient investors find themselves holding corporate debt contracts which cannot easily be transformed into period \( t = 1 \) consumption. Ideally, they would like to sell this asset to patient investors who are willing to give up units of liquid storage in exchange for the higher yielding corporate debt. This trading activity takes place in an OTC secondary market characterized by search frictions. As described above, impatient investors looking to unload corporate debt contracts in the secondary market only will get their orders executed with endogenous probability \( f(\theta) \). Similarly, patient investors looking to purchase corporate debt in the secondary market only will get their orders executed with endogenous probability \( p(\theta) \). If a buy and a sell order are lucky enough to be matched in the OTC market a bilateral trade takes place and units of bonds are exchanged for units of storage at the agreed upon price, \( q_1 \).

To describe the portfolio choice problem of investors, it is useful to first consider the optimal behavior of impatient and patient investors in \( t = 1 \) when they arrive to that period with a generic portfolio of storage and bonds \((s_0, b_0)\).

### 2.4.1 Impatient Investors

By Assumption 2 at \( t = 1 \) impatient investors want to consume in the current period. They can consume the payout from investing in storage, \( s_0(1 + r) \), plus the additional proceeds from placing \( b_0 \) sell orders in the OTC market. These orders are executed with probability \( f \) and each executed order yields \( q_1 \) units of consumption. Thus, the expected consumption of impatient investors in period 1 is given by

\[
c_1^I = s_0(1 + r) + fq_1b_0.
\]  

(9)

On the other hand, with probability \( 1 - f \) orders are not matched and impatient investors are forced to carry debt contracts into period 2. Therefore, expected consumption
in the final period is given by

\[ c_2^I = (1 - f)R^b b_0 \]  

(10)

and the utility derived from \( c_2^I \) is discounted by \( \beta \).

### 2.4.2 Patient Investors

Patient investors only value consumption in the final period and will be willing to place buy orders in the OTC market if there is a surplus to be made, i.e., if \( q_1 \leq R^b / (1 + r) \). The price determination in the OTC market guarantees that this is always the case, thus patient investor would ideally like to exchange all of the lower yielding units of storage for higher yielding corporate debt. But their buy orders will be executed only with probability \( p \).

Therefore, expected storage holdings at the end of \( t = 1 \), \( s_1^P \), are equal to a fraction \( 1 - p \) of the available liquid funds \( s_0(1 + r) \), i.e.,

\[ s_1^P = (1 - p)s_0(1 + r). \]

On the other hand, patient investors place \( s_0(1 + r)/q_1 \) buy orders, of which a fraction \( p \) is expected to be executed. So patient investors expect to increase their bond holding by \( ps_0(1 + r)/q_1 \) units. It follows that expected consumption in the final period equals

\[ c_2^P = (1 - p)s_0(1 + r)^2 + \left[ b_0 + p\frac{s_0(1 + r)}{q_1} \right] R^b. \]  

(11)

That is, the payout from units of storage that were not traded away in the secondary market plus the expected payout from corporate debt holdings.

### 2.4.3 Optimal Portfolio Allocation

In the initial period investors solve a portfolio allocation problem, choosing between storage and bonds to maximize discounted expected lifetime utility

\[ U = \delta(c_1^I + \beta c_2^I) + (1 - \delta)c_2^P \]

subject to the period 0 budget constraint (8), and the expressions for expected consumption of impatient and patient investors (9)-(11).

Given the optimal behavior of patient and impatient investors described above, we
can define the discounted expected utility from investing in storage and bonds in period 0: $U_s$ and $U_b$, respectively. It follows from above that

$$U_s = \delta(1 + r) + (1 - \delta) \left( (1 - p)(1 + r)^2 + p(1 + r) \frac{R^b}{q_1} \right),$$

(12)

$$U_b = \delta \left( f q_1 + \beta (1 - f) R^b \right) + (1 - \delta) R^b.$$  

(13)

Note that both of these expressions depend on the characteristics of the financial contract, $(l_0, \bar{\omega})$, through the expected return on holding the bond to maturity $R^b$; and on the characteristics of the secondary market, $(q_1, \theta)$, through the secondary market price $q_1$ and matching probabilities $f(\theta)$ and $p(\theta)$.

Using these definitions, we can express the asset demand correspondence that maximizes the investors portfolio problem as

$$\begin{cases} 
    s_0 = 0, & b_0 = e_0 \text{ if } U_s < U_b \\
    s_0 \in [0, e_0], & b_0 = e_0 - s_0 \text{ if } U_s = U_b \\
    s_0 = e_0, & b_0 = 0 \text{ if } U_s > U_b
\end{cases}$$

That is, when the expected benefit of holding storage in period 0 is dominated by the benefit of holding the bond, then investors will demand only bond in period 0. On the contrary, if the expected benefit of holding storage is greater than then expected utility of buying the bond in period 0, then investors will only hold storage in the initial period. Finally, if both expected utilities are equal, investors will be indifferent between investing in storage and bonds initially and their demands will be an element of the set of feasible portfolio allocations: $s_0, b_0 \in [0, e_0]$, such that the total value of assets equal the initial endowment (8). Given assumptions 1 and 4, interior equilibria will obtain (i.e. $U_s = U_b$ and $s_0, b_0 > 0$), thus we focus our analysis on them.

All told, in equilibrium it must be that the two assets in period 0 yield the same expected discounted utility, so the return to storage equals the return to lending to entrepreneurs,

$$U_s(l_0, \bar{\omega}, q_1, \theta) = U_b(l_0, \bar{\omega}, q_1, \theta).$$

(14)

For future reference we label equation (14) as the investors’ break-even condition. Note that the discounted expected utility from investing in storage, $U_s$, is not smaller than the
discounted expected utility in financial autarky: \( U_a = \delta(1 + r) + (1 - \delta)(1 + r)^2 \), since the return of buying a bond in the secondary market is at least, \( 1 + r \) (equation 7). So investors are always willing to provide liquidity in secondary markets.

2.5 Equilibrium

The equilibrium of the model is defined as follows.

**Definition 1 (Competitive Equilibrium)** We say that \((l_0, \bar{\omega}, q_1, \theta)\) is a competitive equilibrium if and only if:

1. Given the characteristics of the secondary market \((q_1, \theta)\), the debt contract is described by \((l_0, \bar{\omega})\) maximizes entrepreneurs’ return on equity subject to investors’ break even condition (14).

2. Market liquidity corresponds to \(\theta = (1 - \delta)(1 + r)s_0/q_1/(\delta b_0)\).

3. \(q_1\) is determined via Nash Bargaining.

4. All agents have rational expectations about \(q_1\) and \(\theta\).

The equilibrium of the model is described by the entrepreneur’s choice of \(l_0\) and \(\bar{\omega}\) to maximize the payoff of the risky investment project. Entrepreneurs’ profits are higher when leverage \(l_0\) is higher and when the promised payout is lower, that is, when the threshold productivity level \(\bar{\omega}\) is lower. But entrepreneurs are constrained in their choices of \(l_0\) and \(\bar{\omega}\) as they need to offer terms that make financial contracts attractive to investors: investors’ break-even condition (14).

Entrepreneurs are aware that when selling in the secondary market, investors obtain a price that depends on the contract characteristics. In fact, the price is determined via Nash bargaining (equation 7). Substituting the secondary market price in the investors’ break-even condition, the entrepreneur’s problem can be written as

\[
\max_{l_0, \bar{\omega}} [1 - \Gamma(\bar{\omega})] R^k l_0
\]

\[
s.t. \quad \frac{\delta(1 + r) + (1 - \delta)\left[ (1 - p)(1 + r)^2 + p(1 + r)\left(\frac{\psi}{1+r} + (1 - \psi)\beta\right)^{-1}\right]}{\delta\left( f\left(\frac{\psi}{1+r} + (1 - \psi)\beta\right) + \beta(1 - f)\right) + (1 - \delta)} = R^k(l_0, \bar{\omega}).
\]
Let $\lambda$ be the multiplier on the entrepreneur’s constraint, then the entrepreneur’s privately optimal choice of leverage is given by

$$[1 - \Gamma(\Bar{\omega})] = \lambda \frac{\Gamma(\Bar{\omega}) - \mu G(\Bar{\omega})}{(l_0 - 1)^2}$$

That is, the marginal increase in profits for entrepreneurs need to be proportional to the marginal reduction in discounted expected utility of financial contracts for investors.

Similarly, the privately optimal choice for the risk profile of corporate debt is given by

$$\Gamma'(\Bar{\omega}) = \lambda \frac{\Gamma'(\Bar{\omega}) - \mu G'(\Bar{\omega})}{l_0 - 1}.$$  

That is, the marginal increase in profits for entrepreneurs need to be proportional to the marginal reduction in discounted expected utility of financial contracts for investors.

Taking a ratio of the equations (15) and (16) gives

$$l_0 = 1 + \frac{\Gamma(\Bar{\omega}) - \mu G(\Bar{\omega})}{1 - \Gamma(\Bar{\omega})} \frac{\Gamma'(\Bar{\omega})}{\Gamma'(\Bar{\omega}) - \mu G'(\Bar{\omega})}.$$  

Together, this equation, which describes the privately optimal debt contract, the investors’ break-even condition given by equation (14), and the expressions that characterize the secondary market ($q_1$, $\theta$) provide a complete description of the equilibrium of the model.

Finally, note that both the price in the secondary market $q_1$ and the market liquidity in this market $\theta$ can be expressed as a function of the characteristics of the optimal financial contract $(l_0, \Bar{\omega})$. In fact, the price is a function of the expected return on holding the bond to maturity $R^b$, which depends on $(l_0, \Bar{\omega})$; whereas we can write market liquidity as

$$\theta = \frac{(1 - \delta)(1 + r)}{\delta b_0 q_1} = \frac{(1 - \delta)(1 + r)\left(\frac{\Bar{\omega}}{\Bar{r}_0} - (l_0 - 1)\right)}{\delta q_1(l_0 - 1)}.$$  

### 3 Frictions and the (Ir)relevance of OTC Trade

The demand for liquidity by impatient investors in the interim period, and the inability to perfectly insure against liquidity shocks due to the trading frictions in the secondary OTC market introduce a liquidity premium for holding corporate debt. As a result, there
is a direct link from the secondary market to the primary market for corporate debt, since entrepreneurs need to offer borrowing rates that compensate investors not only for credit, but also for liquidity risk, as shown in Figure 1.

However, our model features another link between the two markets, which results from the fact that the matching technology in the secondary market depends on the number of buy and sell orders rather than simply on the exogenously given relative percentage of impatient to patient investors. This differentiates our analysis from models of random matching, and introduces an important channel through which financing decisions affect secondary market liquidity. Moreover, this will have important implications for optimal regulatory policy and the efficacy of quantitative easing, which we analyze in sections 4 and 5.

The purpose of this section is to analyze the role of trading frictions in the secondary OTC market on the optimal capital structure of entrepreneurs and to establish the dual link between the primary and secondary markets. Section 3.1 derives the conditions on exogenous parameters under which liquidity frictions do not matter for the capital structure, i.e. the model collapses to the benchmark CSV environment. When these conditions are not satisfied, liquidity premia will be positive and will affect the capital structure of entrepreneurs. We show that when liquidity premia increase for a variety of reasons, then entrepreneurs will borrow less and credit risk will go down despite the fact that a lower amount of corporate debt enhances secondary market liquidity (section 3.2). Our main theoretical findings are followed by a simple numerical exercise in section 3.3 to make the results more concrete. All proofs are relegated to the Appendix.

We begin by defining a benchmark interest rate that is the return on a two-period bond that could hypothetically be traded in a perfectly liquid secondary market, if such a market were available to investors. Naturally, such a hypothetical contract needs to deliver in expectation the same return as a strategy of investing only in storage both in the initial period and in period $t = 1$. This gives rise to the following definition.

**Definition 2 (Liquid Two-period Rate)** The liquid two-period rate is defined as the gross interest rate on a perfectly liquid two-period bond.\(^{12}\)

\[
R^f \equiv (1 + r)^2.
\]

\(^{12}\)No arbitrage under perfectly liquid markets implies that trading a two-period bond should yield the same expected return for investors to rolling over one period safe investments, i.e. $\delta \cdot R^f / (1 + r) + (1 - \delta) \cdot R^f = \delta \cdot (1 + r) + (1 - \delta) \cdot (1 + r)^2$. 

21
To be clear, trade in the secondary market is subject to search and matching frictions, so investors do not have access to such a financial contract. Instead, the benchmark rate is useful because it allows us to decompose the total gross return on the financial contract written by the firm into a default and a liquidity premium.

In order to do this, express the gross return of the firm’s contract relative to the benchmark rate, $Z/R^\ell$. Then, this relative return is decomposed into a component owing to default risk, $Z/R^b$, and a component owing to liquidity risk, $R^b/R^\ell$. With this decomposition, we have the following definitions for the default and liquidity premia, respectively.

**Definition 3 (Default and Liquidity Premia)** The default premium on the firm’s debt contract is given by

$$\Phi^d \equiv \frac{Z}{R^b}$$

and the the liquidity premium is given by

$$\Phi^\ell \equiv \frac{R^b}{R^\ell}$$

**Remark 1 (Investors Break-even Condition and Liquidity Premium)** If investors correctly expect the period 1 bond price to be determined via Nash Bargaining, then the investors’ break-even condition (14) can be expressed as

$$(1 + r)^2 \Phi^\ell = R^b$$

with the liquidity premium being only a function of secondary market liquidity and being given by

$$\Phi'(\theta) = \frac{1}{1 + r} \frac{\delta + (1 - \delta) \left[ (1 - p(\theta))(1 + r) + p(\theta) \left( \frac{\psi}{1 + \gamma} + (1 - \psi)\beta \right)^{-1} \right]}{\delta \left[ f(\theta) \left( \frac{\psi}{1 + \gamma} + (1 - \psi)\beta \right) + \beta(1 - f(\theta)) \right] + (1 - \delta)}$$

We now turn to proving our main results.

### 3.1 A Frictionless Benchmark

Our first result, stated in Proposition 1, establishes the conditions under which trade in the secondary market is irrelevant, so that OTC liquidity has no bearing on the firm’s optimal capital structure.

**Proposition 1 (Irrelevance of OTC Trade)** Under the following conditions, there is no liquidity
premium, i.e., \( \Phi^t = 1 \), implying that the model collapses to the benchmark costly state verification model:

1. All investors are patient, so that \( \delta = 0 \);

2. Impatient investors discount at rate \( \beta = \frac{1}{1+r} \); and

3. Impatient investors extract their full value from frictionless sales in the secondary market, which is true for \( \psi = 1 \) and \( \{ e_0 \geq \bar{e}_0 : f(\theta) = 1 \} \).

The case in which \( \delta = 0 \) is straightforward. When all investors are patient, there are no gains from trade and hence no exchange in the secondary market. The price of liquidity trivially goes to zero and the model collapses to the standard costly state verification setup presented in, for example, Townsend (1979) and Bernanke and Gertler (1989).

The same result obtains for the second and third cases though for different reasons. In both cases, there are indeed potential gains from trade in the secondary market but investors are indifferent to whether these gains are realized relative to an alternative investment strategy of holding only the storage asset for two consecutive periods. The indifference reflects the fact that in both special cases the returns are equated across the two assets.

When impatient investors discount future consumption at exactly the rate of return that comes from holding a unit of storage, so that \( \beta = \frac{1}{1+r} \), the two returns are equated due to an assumption about preferences.

The final case makes a particular assumption about the nature of secondary market trade. When corporate debt can be sold off frictionlessly in the secondary market, \( f(\theta) = 1 \), and impatient investors have full bargaining power in setting the terms of trade, so that \( \psi = 1 \), the resulting price of corporate debt in the secondary market ensures that the expected rate of return on primary issued corporate debt and storage are equated. In turn, \( f(\theta) = 1 \) requires that there is enough storage at \( t = 1 \) that all buy orders can be satisfied. Given that in this frictionless environment, entrepreneurs can only operate below a certain upper bound for leverage as an outcome of costly state verification, all the remaining endowment of investors not used for lending will be invested in the storage technology, which drives the probability that impatient investors find a match to 1. We derive this threshold for investors endowment in the proof of Proposition 1 in the Appendix.
3.2 OTC Trade in the Secondary Market

More generally, trading frictions in the secondary market imply that investors require additional compensation for bearing liquidity risk. Lemma 1 summarizes how liquidity in the secondary market affects investor’s demand for corporate debt in the primary market, i.e. they establish the direct link in Figure 1. In other words, shows that investors demand higher compensation in term of the expected return $R^b$ for a given level of borrowing when liquidity in the secondary market, $\theta$, decreases. Proposition 2 performs comparative statics on the exogenous parameters listed in Proposition 1 and shows how they affect secondary market liquidity and the overall demand for corporate debt.

Finally, in Proposition 3 we study how the optimal debt structure changes in equilibrium in response to changes in secondary market liquidity, i.e. how the dual link between primary and secondary markets determines the equilibrium levels of leverage and risk.

**Lemma 1 (Secondary Market Liquidity and Liquidity Premia)** Investor require a higher liquidity premium $\Phi^l$ when the secondary market liquidity $\theta$ is lower. Moreover, the semielasticity of the liquidity premium $\Phi^l$ with respect to secondary market liquidity $\theta$ is lower than $\Phi^f$ in absolute terms, assuming that matching probabilities are strictly smaller than 1.

**Proposition 2 (Investors’ Bond Demand)** Investors require a higher hold-to-maturity return on the bond, $R^b$, and a higher liquidity premium $\Phi^f$ when

1. (Liquidity shock) The probability of becoming impatient is higher, i.e., $\delta$ is higher;
2. (Preferences) Impatient investors discount the future more heavily, i.e., $\beta$ is lower; and
3. (Endowments) Investors have fewer funds to invest in storage, i.e., $e_0$ is lower.

The two results above quantify the effect of liquidity premia on borrowing rates in the primary market. Higher $\Phi^f(\theta)$ induces investors to require a higher expected return $R^b$ to invest in corporate bonds, which is given by equation (3): $R^b = l_0/(l_0 - 1)R^k [\Gamma(\omega) - \mu G(\omega)]$. As a result, investors will require a higher compensation for risk, $\omega$, for a given level of borrowing (or leverage, $l_0$), or equivalently they will reduce their demand for corporate debt for any given level of compensation and leverage will go down.

We, now, turn to how entrepreneurs adjust the contract they offer to investors when the latter demand higher compensation for all level of leverage. As already discussed, this will influence further the liquidity in the secondary market giving rise to a round of
second order effects. Proposition 3 shows how the equilibrium leverage and riskiness of debt, i.e. the privately optimal contract, adjust to a change in market liquidity, which can be induced by any shock in the variables in Proposition 2.

**Proposition 3 (Equilibrium Comparative Statics)** In the private equilibrium, the firm’s optimal leverage, $l_0$, and the riskiness of the contracts it offers up in the primary market, $\bar{\omega}$, both decrease when

1. (Liquidity shock) The probability of becoming impatient is higher, i.e., $\delta$ is higher;
2. (Preferences) Impatient investors discount the future more heavily, i.e., $\beta$ is lower; and
3. (Endowments) Investors have fewer funds to invest in storage, i.e., $e_0$ is lower.

As investors demand greater compensation for liquidity risk, the cost of funding goes up. According to Proposition 3, entrepreneurs adjusts on two margins (recall that the debt contract is two-dimensional); they offer fewer contacts in the primary market and the contracts offered are less risky relative to an equilibrium in which the firm’s debt can be traded frictionlessly. The intuition that entrepreneurs do not respond by increasing their leverage and riskiness of debt in an attempt to compensate investors for bearing liquidity risk is that less risky debt contracts characterized by lower levels of leverage mitigate the drop in market liquidity and are thus more favourable. This would not be the case if the capital structure did not affect the liquidity in the secondary market.

### 3.3 A Numerical Illustration

In order to make the theoretical results presented thus far a bit more concrete, we conduct a simple numeric exercise. For our numerical example, we set: the initial endowments of firms’ and investors to $n_0 = 0.2$ and $e_0 = 1$; the returns to the investment and storage technologies to $R^k = 1.2$ and $r = 0.01$; the preference parameter for early consumption by impatient investors to $\beta = 0.85$; the idiosyncratic shock on the long-term investment, $\omega$, follows a log-normal distribution with variance set to 0.25 and $E\omega = 1$; the monitoring cost to $\mu = 0.2$; the parameter in the matching function to $\nu = 0.5$ and $\alpha = 0.5$; and the bargaining power of impatient investors to $\psi = 1$.

Figure 4 shows the equilibrium in the frictionless benchmark (under any of the conditions outlined in Proposition 1). Given that the optimal contract is two dimensional, $(\bar{\omega}, l_0)$ and all over variables, such that market liquidity and the secondary price depend on these
two contract terms, we illustrate the equilibrium in a diagram of risk and leverage. The household’s breakeven condition is shown by the red line and the firm’s isoprofit lines are shown by the green lines. The firm’s profits are increasing as isoprofit lines move toward the lower right hand quadrant of the figure. The private equilibrium in the economy without frictions is given by the tangency between the break-even condition and the isoprofit line shown by the solid black dot.

Figure 5 graphically illustrates Case 1 of Proposition 2. As the share of impatient investors increases the liquidity premium rises, which drives down the supply of debt in the primary market. The resulting equilibrium has a lower level of leverage and a less risky debt contract (as we proved in Proposition 3).

Finally, Figure 6 presents a decomposition of the total return paid on the primary debt contract into the portion attributed to default risk and the portion attributed to liquidity risk. The figure shows that lower levels of investment and leverage due to increased liquidity demand result in lower corporate bond premia. Naturally, the liquidity premia go up, but default premia decrease since the firm is offering a lower \( \bar{\omega} \) (Proposition 3).

4 The Efficient Structure of Corporate Debt

We analyze the efficient structure of corporate debt by considering a social planner that is constrained by the presence of matching frictions in the secondary market.\(^{13}\)

Define \( w^F \) and \( w^I = 1 - w^F \) as the arbitrary weights that the planner places on the welfare of firms, \( U^F \), and investors, \( U^I \), respectively. Using these weights, we can write the planner’s problem as

\[
\max_{\bar{\omega}, l_0, q_1, \theta} w^F \left( l_0 R^{k} [1 - \Gamma(\bar{\omega})] \right) \\
+ (1 - w^F) \left( \left( \frac{e_0}{n_0} - l_0 + 1 \right) U_s(l_0, \bar{\omega}, q_1, \theta) + (l_0 - 1) U_b(l_0, \bar{\omega}, q_1, \theta) \right)
\]

subject to:

\[
\theta q_1(l_0 - 1) = \frac{\delta}{1 - \delta} (1 + r) \left( \frac{e_0}{n_0} - l_0 + 1 \right),
\]

where: \( U_s(l_0, \bar{\omega}, q_1, \theta) \) and \( U_b(l_0, \bar{\omega}, q_1, \theta) \) denote the expected return to savings and lending, respectively. Also, note that for notational convenience, we are imposing the definition of

\(^{13}\)Our concept of the social planning equilibrium is one of the “second best” or a “constrained socially efficient” equilibrium.
tightness in the OTC market as an explicit constraint. The social planning problem differs from the competitive equilibrium in three main respects: (1.) the planner maximizes directly over the expected utility of both firms and investors; (2.) the planner need not respect the investor’s break even condition; and (3.) the planner can directly influence the thickness of trading in the OTC market through its choice of $\theta$ and $q_1$.

The efficient level of risk, $\tilde{\omega}$, equates the Pareto-weighted marginal gain that accrues to both firms and investors from an incrementally riskier contract:

$$w^R l_0 R^b \Gamma' (\tilde{\omega}) = (1 - w^R) \left[ \left( \frac{e_0}{n_0} - l_0 + 1 \right) \frac{\partial U_s}{\partial \tilde{\omega}} + (l_0 - 1) \frac{\partial U_b}{\partial \tilde{\omega}} \right]$$ (22)

Let $\Psi$ denote the multiplier on the definition of $\theta$. The efficient level of leverage, $l_0$, is given by

$$w^R R^b (1 - \Gamma (\tilde{\omega})) = (1 - w^R) \left( U_s - U_b - \left( \frac{e_0}{n_0} - l_0 + 1 \right) \frac{\partial U_s}{\partial l_0} - (l_0 - 1) \frac{\partial U_b}{\partial l_0} \right) - F_{l_0}(\Psi)$$ (23)

where $F_{l_0}(\Psi) \equiv \Psi \left( \theta q_1 + \frac{\delta}{1 - \delta} (1 + r) \right)$ and $\Psi$ the multiplier on (21). As with the optimal choice of risk, the optimal amount of leverage, $l_0$, equates the marginal gain that accrues to the firm from increasing leverage to the marginal cost stemming from the fact that investors will need additional compensation for that leverage. Moreover, the planner takes into account how leverage influences liquidity in the secondary market.

Taking the ratio of equations (22) and (23) above and then substituting in the associated derivatives gives the following expression:

$$\frac{1 - \Gamma (\tilde{\omega})}{l_0 \Gamma' (\tilde{\omega})} = \frac{1}{l_0 - 1} \frac{\Gamma (\tilde{\omega}) - \mu G (\tilde{\omega})}{\Gamma' (\tilde{\omega}) - \mu G' (\tilde{\omega})} + \frac{l_0 (U_s - U_b - \frac{1}{1 - \delta} F_{l_0}(\Psi))}{(1 - \delta) \left( \left( \frac{e_0}{n_0} - l_0 + 1 \right) p_{\frac{1}{q_1}} (1 + r) + l_0 (\Gamma (\tilde{\omega}) - \mu G' (\tilde{\omega})) \right) + \delta (l_0 - 1) \beta (1 - f)}$$ (24)

Finally, using the first order condition on $\theta$ to solve for the value of $\Psi$ and substituting the resulting expression into the first order condition on $q_1$ gives the efficient price of corporate debt traded in the secondary market. We have that

$$(l_0 - 1) (1 - \delta) (1 + r) \left( p(\theta) \frac{R^b}{q_1} + \theta p'(\theta) \right) = \left( \frac{e_0}{n_0} - l_0 + 1 \right) \delta \left( q_1 f(\theta) - \theta f'(\theta)(q_1 - \beta R^b) \right)$$ (25)
The planner can use the fire sale price, $q_1$, to affect liquidity, $\theta$, both directly (as can be seen from the definition of $\theta$) as well as indirectly. The indirect effect comes from the fact that—as is standard in search models—the terms of trade governs the distribution of the match surplus that is formed in OTC trade.

With this in mind, the planner chooses the optimal fire sale price such that the expected marginal utility from an incremental change in $q_1$ is equated across patient and impatient investors. Intuitively, as the fire sale price declines, for example, the patient investor captures more of the surplus for a given trade. But, at the same time trading opportunities are more difficult to come by because the lower fire sale price shifts market liquidity in a way that is detrimental to patient investors. On the other side of the market, patient investors give up match surplus as the fire sale price declines, but they gain from the increase in liquidity—it is easier to unload the unwanted asset, albeit at a lower price. Taken together, the optimal fire sale price strikes a balance between redistribution of the match surplus between patient and impatient investors and the effects of a change in market liquidity.

4.1 Distortions in the Structure of Corporate Debt

All told, there are three distortions operating in the private equilibrium of the model. The first distortion comes from the fact that investors do not internalize how the liquidity premium they demand for holding corporate debt influences the firm’s capital structure. This distortion is evident from the fact that, despite the investor’s break even condition being a necessary condition for the private equilibrium, the planner need not respect it in implementing the socially efficient allocations. The second distortion comes from the fact that the firm does not internalize how debt issuance in the primary market influences liquidity in the secondary market. Finally, the third distortion comes from the fact that a simple surplus sharing rule—such as one that comes from Nash bargaining—does not take into account the fact that the fire sale price has a direct effect on secondary market liquidity.

4.1.1 A Special Case

This section present a special case for the social planner’s problem, which is useful to isolate the externalities arising from the supply side of corporate debt, namely the decision of the firm to borrow in the primary market while taking secondary market liquidity as
given. In particular we are interested in policy interventions that can make the firm better-off without reducing the welfare of investors, thus replicating the constrained efficient allocations. As a result, we set $w^F = 1$ in the social welfare function and force that investors’ welfare is still as high as in the competitive equilibrium.

We make the following two assumption to neutralize the externalities stemming from the portfolio allocations of investors and the price in the secondary market. The first assumption is that $\psi = 1$, i.e. impatient investors have all the bargaining power after a match is made in secondary trade. The second assumption is that the planner needs to respect the pricing function arising from Nash bargaining and given by equation (7), i.e. the planner can only intervene in period 0 and lets markets operate without intervention thereafter.

These two assumptions together imply that $q_1 = R^k/(1 + r)$ and that the return from savings to investors given by equation (12) is equal to the outside option or the return from autarky, i.e. $U_s = U_a = \delta(1 + r) + (1 - \delta)(1 + r)^2$. Using the break-even condition (14) for investors in the private equilibrium we obtain their welfare is equal to the autarky return on their endowment and thus constant, i.e. $\mathbb{U}^l = e_0U_a$.

Given that the planner sets $w^F = 1$ her general problem outlined in section 4 can be re-written as

$$\max_{\omega, l_0, q_1} \left( l_0 R^k \left[ 1 - \Gamma(\omega) \right] \right)$$

subject to:

$$\theta q_1(l_0 - 1) \geq \frac{\delta}{1 - \delta} (1 + r) \left( \frac{e_0}{n_0} - l_0 + 1 \right), \quad (26)$$

$$(U_b - U_s) \geq 0. \quad (27)$$

Assume that condition (27) is not binding. Then, for $w^F = 1$, the optimality condition (22) becomes $R^k(1 - \Gamma(\omega)) = -F_b(\Psi)$, which is a contradiction since the participation constraint of the firm requires $1 - \Gamma(\omega) > 0$ and $F_b(\Psi) > 0$. Thus, (27) is binding and is the same to the break-even condition of investors in the competitive economy.

However, the planner takes into consideration how the contract it offers to investors affects secondary market liquidity (through condition (26)). Thus, the optimality condition

---

14Condition 27 is the participation constraint for investors that the planner needs to respect. It says that the expected utility for investors cannot be lower than their outside option, i.e. $s_0U_s + b_0U_n \geq U_s \Rightarrow (U_b - U_s)(l_0 - 1)\eta_0 \geq U_a - U_s = 0$ given that $U_s = U_a$ under the aforementioned special assumptions. We also consider equilibria with active borrowing, i.e. $l_0 > 1$. 

---
defining the optimal combination of $\bar{\omega}$ and $l_0$ is written as

$$l_0 = 1 + \frac{\Gamma(\bar{\omega}) - \mu G(\bar{\omega}) - \hat{\zeta} \frac{\partial f}{\partial 0}}{1 - \Gamma(\bar{\omega})} \frac{\Gamma'(\bar{\omega})}{\Gamma'(\bar{\omega}) - \mu G'(\bar{\omega}) + \hat{\zeta} \frac{\partial f}{\partial 0}}.$$  \hspace{1cm} (28)

where $\hat{\zeta} = \zeta(l_0 - 1)^2$, $\check{\zeta} = \zeta(l_0 - 1)$, and

$$\zeta = \frac{\delta(1 + r) + (1 - \delta)(1 + r)^2}{\delta((1 + r)^{-1} - \beta) + \beta} + (1 - \delta) \delta((1 + r)^{-1} - \beta) > 0.$$

Comparing conditions (17), which describes the optimal contract curve in the competitive equilibrium, and (28), we see that the planner takes into consideration the impact of the contract terms on secondary market liquidity through the effect of probability $f$. Note that $\partial f / \partial l_0, \partial f / \partial \bar{\omega} < 0$. Hence, in the planner solution the firm should be compensated with higher leverage for a given level of $\bar{\omega}$, or equivalently should offer a lower $\bar{\omega}$ for a given leverage compared to the competitive solution. Any of these two options would violate the break-even condition of investors who require higher compensation, i.e. $\bar{\omega}$, to lend more.

The planner can respond by either increasing both $l_0$ and $\bar{\omega}$ compared to the competitive equilibrium, or choose lower levels for both (following the steps in the proof of Proposition 3 it can be shown that the two contract terms move in the same direction at the optimal solution in the social planner’s problem as well). Applying the envelope theorem at the competitive equilibrium we get that

$$d U^F(l_0, \bar{\omega}) = \frac{\partial U^F}{\partial l_0} dl_0 + \frac{\partial U^F}{\partial \bar{\omega}} d\bar{\omega} = -\lambda \left[ \frac{\partial B^F}{\partial l_0} dl_0 + \frac{\partial B^F}{\partial \bar{\omega}} d\bar{\omega} \right] = \lambda \frac{\partial B^F}{\partial f} df,$$

where $B^F$ is the budget set over which the firm optimizes or equivalently the break-even condition of investors, and $\lambda$ the associated Lagrange multiplier. A Pareto improvement is possible if the planner chooses $df > 0$ since $\partial B^F / \partial f > 0$. In other words, the planner needs to adjust $k_0$ and $\bar{\omega}$ such that $(\partial f / \partial l_0) dl_0 + (\partial f / \partial \bar{\omega}) d\bar{\omega} > 0$. Given that $\partial f / \partial l_0, \partial f / \partial \bar{\omega} < 0$, the planner chooses lower $(l_0, \bar{\omega})$. This would result in a (weak) Pareto improvement, since investors earn their outside option as in the competitive equilibrium.

Private firms do not internalize the effect of the leverage and risk decision on secondary market liquidity. As a result, they over-borrow and, hence, investors demand a higher compensation for risk. This externality signals that there is an under-supply of liquidity.
If firms internalized the importance of secondary market liquidity, they would take less leverage to allow their debt being traded in a more liquid market. Hence, they would improve the terms they borrow.

Finally, to decentralize the planner’s solution, recall that only one margin is distorted (condition (28)), while the remaining equilibrium conditions are the same between the planner and the competitive economy. Also, the planner would like to implement a solution characterized by lower leverage. We show that a distortionary tax on leverage, with the tax revenue returned to the firm lump-sum, can implement the constrained efficient allocations. The objective of the firm can be written as $l_0 \cdot R^k [1 - \Gamma(\bar{\omega})] - \tau l_0 + T$, where $T = \tau l_0$ is the lump-sum transfer.

The tax that decentralizes the planner’s solution is:

$$\tau^l = -\zeta \cdot \left( \frac{\partial f}{\partial l_0} + \frac{1 - \Gamma(\bar{\omega})}{k_0 \cdot \Gamma'(\bar{\omega})} \frac{\partial f}{\partial \omega} \right) > 0.$$  

(29)

**Numerical Illustration:** We use the numerical example in section 3.3 to illustrate how the planner’s solution differ from the one that private agents choose. Under the aforementioned assumption the planner has the same objective as the competitive firm. Thus, the isoprofits lines are the same in both problems. Figure 7 shows the social planner’s and private equilibrium solution for two cases; $\delta = 0$ and $\delta > 0$. In a frictionless environment, the planner’s solution coincides with the private equilibrium (as we proved in Proposition 1). However, when there is a positive demand for liquidity, $\delta > 0$ and $\beta < (1 + r)^{-1}$, and secondary market liquidity is not sufficiently high to guarantee $f(\theta) = 1$, as it the case in our example, the planner would choose lower leverage and a less risky capital structure, i.e. lower $l_0$ and $\bar{\omega}$. As explained above, the reason is the the planner internalizes how leverage decision affect secondary market liquidity. The Figure shows that this induces the planner to consider a steeper budget set (or break-even condition) compared to the one considered by competitive firms, which take market liquidity as given (this can be seen analytically by comparing conditions (17) and (28)). As a result, the planner understand how lower leverage improves borrowing terms on the margin.

5 Quantitative Easing as part of the Optimal Policy Mix

In this section, we study the effect of unconventional policies like the ones implemented by central banks following the Great Recession of 2007-09. We model these policies as
direct purchases of long-term illiquid assets, financed by short-term liquid liabilities of the central bank. This seems a reasonable first order approximation for the policies of the Federal Reserve during this period. Carpenter et al. (2013) document that lending facilities and asset purchases during this period were financed primarily with deposits from depository institutions, or reserves. This instruments are redeemable liabilities issued by the central bank, lining up well with our modeling assumption.

In our model, short-term central bank liabilities need to offer the same return as storage, and will be referred to as reserves; whereas long-term illiquid assets will be represented with financial contracts, i.e., firms’ debt that is retraded in OTC markets (much like Treasuries and Agency MBS). Specifically, we consider quantitative easing as follows. In period $t = 0$, and before markets open, the central bank announces an amount of bonds that it will purchase in period 0 and will hold to maturity, $\bar{b}$. This bonds are financed by issuing $\bar{s}$ units of reserves to balance the budget in period 0. Thus, the central bank budget constraint in period 0 is simply $\bar{b} = \bar{s}$. In period 1 the central bank will keep its bond holdings, so it needs to roll over its reserves $\bar{s}$ and pay interest on them. Let $\bar{s}'$ equal central bank borrowing through reserves in period 1, then the budget constraint of the central bank requires that $\bar{s}' = (1 + r)\bar{s}$.\(^{15}\)

The effect of quantitative easing can be summarized by the following proposition.

**Proposition 4 (Effect of Quantitative Easing)** Quantitative easing increases secondary market liquidity $\theta$, compress liquidity premia $\Phi^\ell$ and increase issuance and the riskiness of firms’ debt.

### 6 Conclusion

We presented a model to study the feedback loop between secondary market liquidity and firm’s financing decisions in primary capital markets. We showed that imperfect secondary market liquidity accruing from search frictions results in positive liquidity premia, lower levels of leverage—or equivalently lower debt issuance,—and less credit risk in primary markets. Lower issuance in primary markets enhances liquidity in secondary markets, but this effect is not enough to offset the rise of liquidity premia.

Furthermore, this feedback loop creates externalities operating via secondary market

---

\(^{15}\) In practice, the long-term assets held by central banks pay interest in the interim period, and in an environment of low interest rate will generate a positive net-interest income for the central bank, but for simplicity we abstract from these considerations. See, for instance, Carpenter et al (2013) for estimates of net-interest income for the Federal Reserve.
liquidity, as private agents do not internalize how their borrowing and liquidity provision decisions affect secondary market liquidity. We showed that a tax on leverage makes a firm to internalize the impact of its borrowing decisions on secondary market liquidity. Less borrowing enhances the liquidity in the secondary market for its debt claims, reducing the liquidity premia on these debt claims and allowing the firm to earn higher expected profits. In addition, we showed that a subsidy on savings can incentivize investors to provide the socially efficient level of liquidity in secondary markets. A third distortion we characterize regards the way that secondary trade is conducted conditional on a match being made. Finally, we showed how unconventional policies like quantitative easing are expected to affect both secondary market liquidity and debt issuance in primary capital markets. By substituting illiquid assets for liquid short-term securities, these policies increase secondary market liquidity and compress liquidity premia. Our analysis suggests that these type of policies ought to be implemented in conjunction with other policies to limit corporate borrowing.

In our model firms borrow more, and investor provide less liquidity, than what is socially optimal, as both type of agents fail to internalize how they affect secondary market liquidity. This result is similar to other results in the literature of overborrowing and liquidity under supply. However, our result has different policy prescriptions as two policy tools tools are needed to restore efficiency. This contrasts with previous results, which has just focussed on one of these inefficiencies (Fostel and Geanakoplos, 2008; Farhi, Golosov and Tsyvinski, 2009); or where borrowers are also liquidity providers and one policy instrument is enough to restore efficiency (Holmstro¨m and Tirole, 1998; Caballero and Krishnamurthy, 2001; Lorenzoni, 2008; Jeanne and Korinek, 2010; Bianchi, 2011).

Our model suggest a set of testable predictions for the relationship between the availability of short-term liquid assets and liquidity premia. In our model there is only one set of investors who participate in OTC markets, but in practice there are many, potentially segmented OTC markets. In this context, the intuition of our model will predict that liquidity premia for a given asset, should be inversely related to the liquidity of the portfolio of the participants in the OTC market for that asset. Along these lines, our model predicts that quantitative easing financed with bank reserves should have an effect on the liquidity premia of all the securities traded in OTC markets where banks are relevant participants, not only affect the liquidity premia of the illiquid assets purchased by central banks.

Finally, this paper leaves open questions that we are taking on future work. First, we would like to explore the quantitative relevance of the mechanisms described herein.
For that we have deliberately stayed very close to the quantitative model of Bernanke et al (1999), and we are planning to explore the quantitative prescription of our model. Second, in practice many different assets are traded in OTC markets, a dimension that we have abstracted from in our analysis but seems important in practice. Future work should explore the relationship between market segmentation in OTC trade and secondary market liquidity. Two important considerations that we abstracted from will have to be accounted for in this work: what are the strategic incentives in such an environment?, and, how is liquidity allocated across these markets?

References


Appendix

A Proofs

Proof of Proposition 1: When there is no need to compensate investors for liquidity risk, the expected return from lending to entrepreneurs is equal to the outside option of holding storage. In other words, the liquidity premium is zero, i.e. \( \Phi^1(\theta) = 1 \) and \( R_b = (1 + r)^2 \) or \( k_0R^k[\Gamma(\tilde{\omega}) - \mu G(\tilde{\omega})] = (k_0 - n_0)(1 + r)^2 \). This is equivalent to the benchmark costly state verification model where investors are only compensated for credit risk. Note that entrepreneurs’ profits do not depend directly on secondary market liquidity. We proceed by showing that \( \Phi^1(\theta) = 1 \) under the three alternative condition stated in Proposition 1.

Condition 1: \( \delta = 0 \). This implies that secondary market liquidity \( \theta \rightarrow \infty \), hence \( p(\theta) = 1 \). Setting \( \delta = 0 \) and \( p(\theta) = 1 \) yields \( \Phi^1(\theta) = 1 \).

Condition 2: \( \beta = (1 + r)^{-1} \). Simple substitution yields \( \Phi^1(\theta) = 1 \).

Condition 3: \( \psi = 1 \) and \( f(\theta) = 1 \). Simple substitution yields \( \Phi^1(\theta) = 1 \).

For any given distribution for the idiosyncratic productivity shock \( \omega \), the upper threshold \( \tilde{\omega} \) that entrepreneurs can promise to investors under perfect secondary markets is given by \( \Gamma'(\tilde{\omega}) - \mu G'(\tilde{\omega}) = 0 \). After this point there will be credit rationing (reference??) and leverage will start falling (see Proposition 3 why \( \tilde{\omega} \) is lower when secondary markets are imperfect). Denote this solution by \( \tilde{\omega}_0 \). This implies a maximum amount of borrowing, \( \bar{b}_0 \), or leverage, \( \tilde{l}_0 = (\bar{b}_0 + \eta_0)/\eta_0 \), which is given by the break-even condition \( (\bar{b}_0 + n_0)R^k[\Gamma(\tilde{\omega}) - \mu G(\tilde{\omega})] = \bar{b}_0(1 + r)^2 \). In turn this implies a lower bound for investors’ endowment, \( \bar{e}_0 \), such that \( m(1, \theta) > 1 \) for \( e_0 > \bar{e}_0 \) given that \( \theta = (1 + \delta)(1 + r)\left(\frac{\bar{e}_0}{\bar{e}_0} + 1 - \bar{l}_0\right) / \left(\tilde{\omega}_1(\bar{l}_0 - 1)\right) \) is increasing in \( e_0 \) and the highest value for \( q_1 \) is \( \bar{l}_0/(\bar{l}_0 - 1)R^k[\Gamma(\bar{\omega}) - \mu G(\bar{\omega})] \).

Proof of Lemma 1: We want to show that the derivative of the liquidity premium wrt liquidity is negative. Denote by \( C \) and \( D \) the numerator and denominator in \( \Phi^1(\theta) \) given by 20. Then,

\[
C = \delta + (1 - \delta)\left[(1 - p)(1 + r) + p\left(\frac{\psi}{1 + \psi}\right)^{-1}\right] > 0 \quad (A.1)
\]

\[
D = \delta\left[f\left(\frac{\psi}{1 + \psi}(1 - \psi)\beta + \beta(1 - f)\right) + (1 - \delta)\right] > 0
\]

where the inequalities follow from the fact that probabilities and returns are non-negative. In
addition, denote $C_\theta$ and $D_\theta$ the derivatives of $C$ and $D$, respectively, wrt $\theta$. Then

$$C_\theta = \frac{\partial C}{\partial \theta} = (1 - \delta) \left[ \left( \frac{\psi}{1+r} + (1 - \psi) \beta \right)^{-1} - (1 + r) \right] \frac{df(\theta)}{d\theta} \leq 0$$

$$D_\theta = \frac{\partial D}{\partial \theta} = \delta \psi \left( \frac{1}{1+r} - \beta \right) \frac{df(\theta)}{d\theta} \geq 0$$

where the inequalities follow from Assumption 2, equations (5) and (6), and that the matching function $m(A, B)$ is increasing in both total sales and total buy orders. From equation (20) we have that

$$\frac{d\Phi^\ell}{d\theta} = \frac{1}{1+r} \left[ \frac{C_\theta D - CD_\theta}{D^2} \right] \leq 0 \quad (A.2)$$

where the inequality follows from the previously established inequalities: $D, C, D_\theta \geq 0$ and $C_\theta \leq 0$.

Regarding the second part of the Lemma, the semielasticity of the liquidity premium, $\Phi^\ell$, with respect to the secondary market liquidity, $\theta$, is written as:

$$z_{\Phi^\ell, \theta} = \theta \frac{d\Phi^\ell}{d\theta} = \frac{\theta}{1+r} \frac{C_\theta D - CD_\theta}{D^2}, \quad (A.3)$$

using A.2. We assume that the matching probabilities are strictly smaller than 1. In this case, $f(\theta) = \nu \theta^{1-a}$ and $p(\theta) = \nu \theta^{-a}$. As a result, we have the following relationship between the matching probabilities and their derivatives: $\theta \frac{d f(\theta)}{d\theta} = (1 - a) f(\theta)$ and $\theta \frac{df(\theta)}{d\theta} = -ap(\theta)$. Then,

$$C_\theta \theta = -\alpha C + \alpha [\delta + (1 - \delta)(1 + r)] \leq 0$$

$$D_\theta \theta = (1 - \alpha) D - (1 - \alpha) [\beta \delta + (1 - \delta)] \geq 0$$

where the inequalities follow from Assumption 2. In fact, it follows from Assumption 2 that $C \geq \delta + (1 - \delta)(1 + r)$, and $D \geq \delta \beta + (1 - \delta)$. Then $|z_{\Phi^\ell, \theta}| < \Phi^\ell$ requires:

$$-\frac{\theta}{1+r} \frac{C_\theta D - CD_\theta}{D^2} < \frac{1}{1+r} \frac{C}{D} \quad \Leftrightarrow \quad CD + \theta C_\theta D - \theta CD_\theta > 0$$

But the last inequality follows from above. In fact,

$$CD + \theta C_\theta D - \theta CD_\theta = CD + D (-\alpha C + \alpha [\delta + (1 - \delta)(1 + r)] - C [(1 - \alpha)D - (1 - \alpha) [\beta \delta + (1 - \delta)]])$$

$$= \alpha D [\delta + (1 - \delta)(1 + r)] + C(1 - \alpha) [\beta \delta + (1 - \delta)] > 0$$

Proof of Proposition 2: From the investors’ break-even condition (19), we see that an increase in the liquidity premium, $\Phi^\ell$, induces investors to require a higher expected return $R^b$ to invest in corporate bonds. Hence, the liquidity premium $\Phi^\ell$ and the hold-to-maturity bond return $R^b$ are proportional to one another. In fact,

$$(1 + r)^2 \Phi^\ell = R^b \quad \Rightarrow \quad \frac{dR^b}{d\Phi^\ell} = \frac{R^b}{\Phi^\ell}$$
For this proof we consider the liquidity premium a function of both secondary market liquidity, \( \theta \), and model parameters \( \delta \) and \( \beta \). That is, we can write the liquidity premium as \( \Phi^\ell(\theta, \delta, \beta) \).

**Case 1: Effect of \( \delta \).** Want to show that
\[
\frac{d\Phi^\ell}{d\delta} = \frac{\partial \Phi^\ell}{\partial \delta} + \frac{\partial \Phi^\ell}{\partial \theta} \frac{\partial \theta}{\partial \delta} > 0
\]

From the definition of secondary market liquidity, given in equation (18), and considering the dependence of secondary market pricing on liquidity premia, we have that
\[
\frac{\partial \theta}{\partial \delta} = -\frac{\theta \delta (1 - \delta)}{1 - \theta q_1 dR^b \Phi^\ell} = -\frac{\theta}{\delta (1 - \delta)} - \frac{\theta}{\Phi^\ell} \frac{d\Phi^\ell}{d\delta}
\]

Using this expression we get
\[
\frac{d\Phi^\ell}{d\delta} = \frac{\partial \Phi^\ell}{\partial \delta} - \frac{1}{\Phi^\ell} \frac{d\Phi^\ell}{d\delta} = \frac{\partial \Phi^\ell}{\partial \delta} - \frac{\partial \Phi^\ell}{\partial \theta} \frac{\partial \theta}{\partial \delta}
\]

where \( z_{\Phi^\ell, \theta} \) is the semielasticity of the liquidity premium with respect to secondary market liquidity, which is negative and strictly greater than \( -\Phi^\ell \) (Lemma 1). Therefore, \( 1 + z_{\Phi^\ell, \theta}/\Phi^\ell > 0 \).

It is left to show that \( \frac{\partial \Phi^\ell}{\partial \delta} > 0 \). For that we use the notation introduced in equation (A.1). In addition, denote \( C_\delta \) and \( D_\delta \) the derivatives of \( C \) and \( D \), respectively, wrt \( \delta \). Then
\[
C_\delta = \frac{\partial C}{\partial \delta} = 1 - \left[(1 - p)(1 + r) + p \left(\frac{\psi}{1 + \psi} + (1 - \psi)\beta\right)^{-1}\right]
\]
\[
D_\delta = \frac{\partial D}{\partial \delta} = \left[f\left(\frac{\psi}{1 + \psi} + (1 - \psi)\beta\right) + \beta(1 - f)\right] - 1
\]

Then, from equation (20) we have that
\[
\frac{\partial \Phi^\ell}{\partial \delta} = \frac{1}{1 + r} \left[\frac{C}{D} - \frac{C D}{D^2}\right]
\]

which is strictly greater than zero if and only if
\[
C_\delta D > C D_\delta
\]
\[
C_\delta [\delta D_\delta + 1] > [\delta C_\delta + 1 - C_\delta] D_\delta
\]
\[
C_\delta > [1 - C_\delta] D_\delta
\]
or
\[
1 - \left[(1 - p)(1 + r) + p \left(\frac{\psi}{1 + \psi} + (1 - \psi)\beta\right)^{-1}\right]
\]
\[
> \left[(1 - p)(1 + r) + p \left(\frac{\psi}{1 + \psi} + (1 - \psi)\beta\right)^{-1}\right] \left[f\left(\frac{\psi}{1 + \psi} + (1 - \psi)\beta\right) + \beta(1 - f)\right] - 1
\]
\[
\Rightarrow 1 > \left[(1 - p)(1 + r) + p \left(\frac{\psi}{1 + \psi} + (1 - \psi)\beta\right)^{-1}\right] \left[f\left(\frac{\psi}{1 + \psi} + (1 - \psi)\beta\right) + \beta(1 - f)\right]
\]
This inequality follows from Assumption 2. It is easy to check that after distributing terms in the previous expression the four remaining terms are not greater than 1. In fact, $\beta \leq 1/(1 + r)$ imply that

$$
\beta(1 + r) \leq 1, \quad \left( \frac{\psi}{1+r} + (1 - \psi)\beta \right)(1 + r) \leq 1, \quad \text{and} \quad \left( \frac{\psi}{1+r} + (1 - \psi)\beta \right)^{-1} \beta \leq 1
$$

Case 2: Effect of $\beta$. Want to show that

$$
d\Phi^\ell \frac{d}{d\beta} = \frac{\partial \Phi^\ell}{\partial \beta} + \frac{\partial \Phi^\ell}{\partial \theta} \frac{\partial \theta}{\partial \beta} < 0
$$

For that we use the notation introduced in equation (A.1). In addition, denote $C_\beta$ and $D_\beta$ the derivatives of $C$ and $D$, respectively, with respect to $\beta$. Then

$$
C_\beta = \frac{\partial C}{\partial \beta} = -(1 - \delta) p \left( \frac{\psi}{1+r} + (1 - \psi)\beta \right)^2 (1 - \psi) \leq 0
$$

$$
D_\beta = \frac{\partial D}{\partial \beta} = \delta f(1 - \psi) + 1 - f = \delta(1 - \psi) f \geq 0
$$

Then,

$$
\frac{\partial \Phi^\ell}{\partial \beta} = \frac{1}{1 + r} \left[ \frac{C_\beta}{D} - \frac{C D_\beta}{D^2} \right] \leq 0
$$

as $C_\beta < 0, D_\beta \geq 0$ and $C, D > 0$.

From the definition of secondary market liquidity, given in equation (18), and considering the dependence of the secondary market price on liquidity premia, we have that

$$
\frac{\partial \theta}{\partial \beta} = -\frac{\theta}{q_1} \left[ \frac{\partial q_1}{\partial \beta} + \frac{\partial q_1}{\partial R} \frac{\partial R}{\partial \beta} \frac{d \Phi^\ell}{d \beta} \right] = -\theta \left[ (1 - \psi) \left( \frac{\psi}{1+r} + (1 - \psi)\beta \right)^{-1} + \frac{1}{\Phi^\ell} \frac{d \Phi^\ell}{d \beta} \right]
$$

Thus,

$$
d\Phi^\ell \frac{d}{d\beta} = \frac{\partial \Phi^\ell}{\partial \beta} - (1 - \psi) \left( \frac{\psi}{1+r} + (1 - \psi)\beta \right)^{-1} \theta \frac{\partial \Phi^\ell}{\partial \theta} \frac{1}{1 + z_{q_1,\theta}/\Phi^\ell}
$$

where $z_{q_1,\theta}$ is the semielasticity of the liquidity premium with respect to secondary market liquidity. From Lemma 1 the denominator, $1 + z_{q_1,\theta}/\Phi^\ell$, is strictly positive. But the sign of the numerator is ambiguous. The reason is that a higher $\beta$ on one hand reduces the preference for liquidity by impatient households, i.e., $\partial \Phi^\ell / \partial \beta < 0$. But on the other hand it increases the secondary market price, $q_1$, which pushes market liquidity $\theta$ down and liquidity premia up. This second force, represented by the second term in the numerator, depends crucially on the bargaining power of impatient investors: the lower their bargaining power the more important the effect of their valuation, i.e., $\beta$, will be on the price.

The numerator is negative if and only if

$$
(1 - \psi) \left( \frac{\psi}{1+r} + (1 - \psi)\beta \right)^{-1} \theta [C_\theta D - C D_\theta] - C_\beta D + CD_\beta > 0
$$

40
From Assumption 2 we have

\[
\beta \leq \frac{1}{1+r} \quad \Leftrightarrow \quad \beta \leq \frac{\psi}{1+r} + (1-\psi)\beta \leq \frac{1}{1+r} \quad \Leftrightarrow \quad 1 + r \leq \left( \frac{\psi}{1+r} + (1-\psi)\beta \right)^{-1} \leq \beta^{-1}
\]

Using these inequalities, that \(\theta[C_\theta D - CD_\theta] < 0\) (Lemma 1), and the expressions derived above for \(C, D, C_\theta, D_\theta, C_\beta,\) and \(D_\beta,\) we have

\[
\frac{1 - \psi}{1+r} \left( \frac{\psi}{1+r} + (1-\psi)\beta \right)^{-1} \theta[C_\theta D - CD_\theta] - C_\beta D + CD_\beta \\
\geq \frac{1 - \psi}{\beta(1+r)} \theta[C_\theta D - CD_\theta] - C_\beta D + CD_\beta \\
\geq \frac{1 - \psi}{\beta(1+r)} (-CD + \alpha[\delta + (1-\delta)(1+r)]D + (1-\alpha)[\beta\delta + 1 - \delta]C) \\
+ (1-\delta)p \left( \frac{\psi}{1+r} + (1-\psi)\beta \right)^{-2} (1-\psi)D + \delta(1-\psi f)C \\
\geq \frac{1 - \psi}{\beta(1+r)} (-CD + \alpha[\delta + (1-\delta)(1+r)]D + (1-\alpha)[\beta\delta + 1 - \delta]C) \\
+ (1-\psi)(1-\delta)p(1+r)^2D + \delta(1-\psi f)C
\]

Taking the limit \(\psi \to 1\) we obtain:

\[
\lim_{\psi \to 1} \frac{d\Phi^f}{d\beta} \to \frac{\partial \Phi^f}{\partial \beta} \left( 1 + \zeta_{\phi,\theta} \right)^{-1} < 0.
\]

Similarly, taking the limit \(\psi \to 0\) we obtain:

\[
\lim_{\psi \to 0} \frac{d\Phi^f}{d\beta} \to \frac{-(1-\delta)p\beta^{-2}(1 + \zeta_{\phi,\theta})D + (1-\delta)p\beta^{-1}(1+r)\zeta_{\phi,\theta} - \delta C}{D^2(1 + \zeta_{\phi,\theta})} < 0,
\]

since \(\zeta_{\phi,\theta} = -\alpha > -1\) assuming a constant returns to scale matching function.

**Case 3: Effect of \(\epsilon_0\).** Show that

\[
\frac{d\Phi^f}{d\epsilon_0} = \frac{\partial \Phi^f}{\partial \epsilon_0} + \frac{\partial \Phi^f}{\partial \theta} \frac{\partial \theta}{\partial \epsilon_0} < 0. \quad (A.4)
\]

It is easy to see that \(\frac{\partial \Phi^f}{\partial \epsilon_0} = 0\). Changing \(\epsilon_0\) will only affect the liquidity premium through the secondary market liquidity and in particular through its effect on \(s_0 = \epsilon_0 - b_0\) given that we have fixed leverage in this exercise. Thus,

\[
\frac{d\Phi^f}{d\epsilon_0} = \frac{\partial \Phi^f}{\partial \theta} \frac{\partial \theta}{\partial \epsilon_0} s_0 = \frac{\partial \Phi^f}{\partial \theta} \frac{\partial \theta}{\partial \epsilon_0} s_0 < 0. \quad (A.5)
\]
Proof of Proposition 3:
Case 1: Comparative Statics on $\delta$. From equation 17 we had that

$$l_0(\bar{\omega}) = 1 + \frac{\Gamma(\bar{\omega}) - \mu G(\bar{\omega})}{1 - \Gamma(\bar{\omega})} \frac{\Gamma'(\bar{\omega})}{\Gamma'(\bar{\omega}) - \mu G'(\bar{\omega})}$$

On the other hand, from equation (18)

$$\theta(l_0, \bar{\omega}, \delta) = \frac{(1 - \delta)(1 + r)(\frac{\omega}{\omega_0} - (l_0 - 1))}{\delta(\frac{\psi}{1 + \psi} + (1 - \psi)\beta) R^b(l_0, \bar{\omega})(l_0 - 1)}$$

Using these expressions we can express the investors’ break-even condition as

$$(1 + r)^2 \Phi^f(\theta(l_0(\bar{\omega}), \bar{\omega}, \delta), \delta) = R^b(l_0(\bar{\omega}), \bar{\omega})$$

By the Implicit Function Theorem if the derivative of the previous expression wrt $\bar{\omega}$ is different than 0 then we can define $\bar{\omega}(\delta)$ and calculate its derivative from the previous expression. We want to show that $\frac{d\bar{\omega}}{d\delta} \leq 0$.

Fully differentiating wrt to $\bar{\omega}$ we obtain

$$(1 + r)^2 \left\{ \frac{\partial \Phi^f}{\partial \theta} \left[ \frac{\partial \theta}{\partial l_0} \frac{dl_0}{d\bar{\omega}} + \frac{\partial \theta}{\partial \bar{\omega}} \frac{d\bar{\omega}}{d\delta} + \frac{\partial \theta}{\partial \delta} \right] + \frac{\partial \Phi^f}{\partial \delta} \right\} = \frac{\partial R^b}{\partial l_0} \frac{dl_0}{d\bar{\omega}} \frac{d\bar{\omega}}{d\delta} + \frac{\partial R^b}{\partial \bar{\omega}} \frac{d\bar{\omega}}{d\delta}$$

Thus,

$$\frac{d\bar{\omega}}{d\delta} = \frac{H}{J}$$

with

$$H = -(1 + r)^2 \left\{ \frac{\partial \Phi^f}{\partial \theta} \frac{\partial \theta}{\partial \delta} + \frac{\partial \Phi^f}{\partial \delta} \right\}$$

$$J = (1 + r)^2 \left\{ \frac{\partial \Phi^f}{\partial \theta} \left[ \frac{\partial \theta}{\partial l_0} \frac{dl_0}{d\bar{\omega}} + \frac{\partial \theta}{\partial \bar{\omega}} \right] - \frac{\partial R^b}{\partial l_0} \frac{dl_0}{d\bar{\omega}} - \frac{\partial R^b}{\partial \bar{\omega}} \right\}$$

From Lemma 1 $\frac{\partial \Phi^f}{\partial \bar{\omega}} \leq 0$ and from Proposition 2 $\frac{\partial \Phi^f}{\partial \delta} \geq 0$, with strict inequality if $\beta < 1/(1 + r)$. In addition,

$$\frac{\partial \theta}{\partial \delta} = -\frac{\theta}{\delta(1 - \delta)} < 0$$

Thus, $H \leq 0$, with strict inequality if $\beta < 1/(1 + r)$.

Next we want to show that $J > 0$. For that first note that from Assumption 3 in a non-credit rationing equilibrium $\frac{dl_0}{d\bar{\omega}} \geq 0$. In fact, differentiating we obtain

$$\frac{dl_0}{d\bar{\omega}} = \frac{\Gamma'(\bar{\omega})}{1 - \Gamma(\bar{\omega})} + \frac{\Gamma(\bar{\omega}) - \mu G(\bar{\omega})}{1 - \Gamma(\bar{\omega})} \left\{ \frac{\Gamma'(\bar{\omega})}{\Gamma'(\bar{\omega}) - \mu G'(\bar{\omega})} \right\} \geq 0$$

where the inequality follows from $\Gamma'(\bar{\omega}) - \mu G'(\bar{\omega}) > 0$ in a non-rationing equilibrium; $\Gamma'(\bar{\omega})G''(\bar{\omega}) - \mu G'(\bar{\omega}) < 0$.
\( \Gamma''(\hat{\omega})G'(\hat{\omega}) = \frac{d^2(\hat{\omega}\hat{\omega})}{d\tilde{\omega}^2}(1 - F(\hat{\omega}))^2 > 0; \Gamma'(\hat{\omega}) > 0; \) and \( \Gamma(\hat{\omega}) < 1. \)

Second, note that from equation (3) we have that

\[
\frac{\partial R^b}{\partial l_0} = -\frac{R^b}{l_0(l_0 - 1)} < 0 \quad \text{and} \quad \frac{\partial R^b}{\partial \hat{\omega}} = \frac{R^b[\Gamma'(\hat{\omega}) - \mu G'(\hat{\omega})]}{\Gamma(\hat{\omega}) - \mu G(\hat{\omega})} > 0
\]

where the last inequality follows from \( \Gamma'(\hat{\omega}) - \mu G'(\hat{\omega}) > 0 \), which is the case in a non-rationing equilibrium.

Third, using the previous expressions we get that

\[
\frac{\partial \theta}{\partial l_0} = -\frac{\theta}{l_0}\left(\frac{n_0 + 1}{n_0} + (l_0 - 1)\right) < 0 \quad \text{and} \quad \frac{\partial \theta}{\partial \hat{\omega}} = -\frac{\theta [\Gamma'(\hat{\omega}) - \mu G'(\hat{\omega})]}{\Gamma(\hat{\omega}) - \mu G(\hat{\omega})} < 0
\]

where the first inequality follows from Assumption 4, whereas the second inequality follows from \( \Gamma'(\hat{\omega}) - \mu G'(\hat{\omega}) > 0. \)

It follows from above that

\[
\frac{\partial \Phi_f}{\partial \theta} \left[ \frac{\partial \theta}{\partial l_0} \frac{dl_0}{d\hat{\omega}} + \frac{\partial \theta}{\partial \hat{\omega}} \right] \geq 0
\]

It is just left to show that

\[
\frac{\partial R^b}{\partial l_0} \frac{dl_0}{d\hat{\omega}} + \frac{\partial R^b}{\partial \hat{\omega}} < 0
\]

which is the case iff

\[
\frac{1}{l_0} \frac{dl_0}{d\hat{\omega}} > (l_0 - 1) \frac{\Gamma'(\hat{\omega}) - \mu G'(\hat{\omega})}{\Gamma(\hat{\omega}) - \mu G(\hat{\omega})} = \frac{\Gamma'(\hat{\omega})}{1 - \Gamma(\hat{\omega})}
\]

\[
\Leftrightarrow \frac{1 - \Gamma(\hat{\omega})}{\Gamma'(\hat{\omega})} \frac{dl_0}{d\hat{\omega}} - l_0 > 0
\]

Substituting in the expressions for \( \frac{dl_0}{d\hat{\omega}} \) and \( l_0(\hat{\omega}) \) above we get

\[
1 + \frac{\Gamma(\hat{\omega}) - \mu G(\hat{\omega})}{\Gamma'(\hat{\omega})[\Gamma'(\hat{\omega}) - \mu G'(\hat{\omega})]} \left\{ \frac{[\Gamma'(\hat{\omega})]^2}{1 - \Gamma(\hat{\omega})} + \frac{\mu [\Gamma'(\hat{\omega})G''(\hat{\omega}) - \Gamma''(\hat{\omega})G'(\hat{\omega})]}{\Gamma'(\hat{\omega}) - \mu G'(\hat{\omega})} \right\} - 1
\]

\[
- \frac{\Gamma(\hat{\omega}) - \mu G(\hat{\omega})}{1 - \Gamma(\hat{\omega})} \frac{\Gamma'(\hat{\omega})}{\Gamma'(\hat{\omega}) - \mu G'(\hat{\omega})} = \frac{[\Gamma(\hat{\omega}) - \mu G(\hat{\omega})][\mu [\Gamma'(\hat{\omega})G''(\hat{\omega}) - \Gamma''(\hat{\omega})G'(\hat{\omega})] - 1}{\Gamma'(\hat{\omega})[\Gamma'(\hat{\omega}) - \mu G'(\hat{\omega})]} > 0
\]

Therefore, we conclude that \( J > 0 \) and \( \frac{d\theta}{d\hat{\omega}} < 0 \). It follows from \( \frac{dl_0}{d\hat{\omega}} > 0 \) that \( \frac{dl_0}{d\hat{\omega}} < 0 \)

**Proof of Proposition 4:** In the presence of quantitative easing we need to distinguish between firms’ borrowing and investors lending. Let \( l_0 \) be investors lending, then firms’ borrowing equals
\( k_0 - n_0 = b_0 + \bar{b} \). Then, secondary market liquidity, in equilibrium, is given by

\[
\theta = \frac{(1 - \delta)(1 + r)s_0}{\delta b_0} = \frac{(1 - \delta)(1 + r)\left(\frac{e_0 + \bar{b}}{n_0} - (l_0 - 1)\right)}{\delta \left(\frac{\psi}{1+r} + (1 - \psi)\bar{b}\right) R^b(l_0, \bar{\omega}) \left(l_0 - 1 - \frac{\bar{b}}{n_0}\right)}
\]

Then, \( \partial \theta / \partial \bar{b} > 0 \). From Lemma 1 this imply that liquidity premia compress and from Proposition 3 this imply that both leverage and the riskiness of debt \( \bar{\omega} \) increase in equilibrium.
Tables and Figures

Figure 3
Figure 4

Figure 5