Structural Transformation and the U-Shaped Female Labor Supply *

L. Rachel Ngai † Claudia Olivetti‡

February 15, 2015

Abstract

The nature and extent of segmentation of economic activity across genders and its changing roles during the course of economic development has been a central topic of inquiry since Ester Boserup's pioneering work on *Woman's Role in Economic Development.* The evidence, both historically for the U.S. and other developed economies and over large cross-sections of countries, suggests that the relationship between women's labor force participation and economic development is U-shaped.

This paper investigates the link between the U-shaped evolution of womens employment and the process of structural transformation in the course of economic development. Specifically, it shows how this pattern can be rationalized based on a model of structural transformation with home production.

* **VERY Preliminary and VERY Incomplete.**

† LSE

‡ Boston University

1 Introduction

Boserup (1970) and Goldin (1990, 1995) advance the hypothesis that the relationship between womens labor force participation and economic development is U-shaped. In the early stages of economic development, womens participation is high since women tend to be heavily involved as family workers in family farms or businesses, or otherwise working for pay or producing for the market within the household. Womens labor force participation initially falls in the course of economic development, along the declining portion of the U, as the locus of production moves out of household and family enterprises and into factories and offices. This decrease is due both to the negative income effect on womens participation of rising family income and the stigma of womens, particularly married womens, employment as wage-workers in manufacturing. The latter in turn partially reflects the dirty and unpleasant nature of early manufacturing employments, given which, the employment of a wife is taken as an extremely negative reflection on her husbands ability to provide for his family. As economic development progresses, however, womens education and their consequent opportunities for white-collar employment rise. Womens labor force participation then increases along the rising portion of the U both because the higher wages available to women lead to a substitution effect, increasing their labor force participation, and because white-collar employment does not share the same stigma as factory work and wage labor on farms.

The econometric analysis has been broadly supportive of this U-shaped relationship. Goldin (1990, 1986) shows that more inclusive measures of labor supply trace a U-shaped function in the U.S.: after declining for about a century, the female labor force participation rate was as high in 1940 as it was in 1890 and kept rising thereafter. The bottom of the U must have occurred somewhere between 1890 and 1940. Goldin (1995) finds further evidence of a U-shaped female labor supply function with economic development (as measured by GDP per capita) using a large cross-section of countries observed in the first half of the 1980s. Subsequent work by Mammen and Paxson (2000), Lundberg (2010) and Luci (2009) provides additional evidence of a U-shaped labor supply based on larger panels of economies observed in the 1970s and 1980s, 2005, and for the years 1965 to 2005, respectively. As we document in Figure 1 the historical experience of the United States is not unique in comparative perspective. The figure is based on a sample of sixteen developed economies for which data are consistently available for most of the 1890 to 2005 period.[1]

Interestingly, this U-shape seems to be linked to the typical process of sectoral reallocation of employment. This involves redeployment of the labor force, initially, from agriculture to manufacturing and services; which is then followed, as development continues, by a decline in the share of employment in manufacturing but a continued increase in the share in services.[2] Based on the historical cross-country data panel, Figure 2 shows a broad similarity in this experience for both men and women, but with significant gender differences. Women move out of agriculture and into services more rapidly than men do, while mens employment share in manufacturing initially rises more steeply than womens.[3]

The main objective of the paper is to quantitatively establish this link. Specifically, we want to show how a model of structural transformation with home production can generate a U-Shaped female labor supply. The key mechanism in our model is that the gender-specific efficiency parameter is sector-specific. In other words, we assumes that female's comparative advantage is in services, while male's comparative advantage is in agriculture. Labor-productivity growth is uneven across sectors, namely it is fastest in agriculture and slowest in services. The uneven labor productivity itself can be due to either uneven TFP growth or sector-specific capital intensity. Under the assumption that output from the three sectors are poor substitutes, faster labor productivity growth in agriculture implies a decline in the agriculture labor input and a shift of labor inputs in manufacturing and service production. This decline is mainly due to female labor because of the gender-specific comparative advantage. However, because manufacturing jobs are brawn-intensive and dirty, women' labor shifts into home production activities (in services or home manufacturing). This explains the decline in female labor supply. The rising part is similar to Ngai-Petrongolo (2013) and is linked to women's comparative advantage in service sector activities.

The relationship between the process of structural transformation and women's involvement in the labor market has been noted by several authors, especially in relation to the increasing importance of the service sector in the

[1] This figure reproduces Figure 2 in Olivetti (2014). The sample includes: Australia, Belgium, Canada, Denmark, France, Finland, Germany, Ireland, Italy, Netherlands, Norway, Portugal, Spain, Sweden, United Kingdom and the United States.

[2] This process has been extensively documented starting with the work by Kuznets (1966) and Maddison (1980). Recent work by Herrendorf, Rogerson and Valentinyi (2013) provides systematic evidence about the 'facts' of structural transformation for a large cross-section of countries and going back in time as far as possible.

[3] See also table 5.6 in Olivetti (2014).

economy. The idea is that production of goods is relatively intensive in the use of 'brawn' while the production of services is relatively intensive in the use of 'brain'. Since men and women may have different endowments of these factors, with women having a comparative advantage in 'brain' activities, the historical growth in the service sector may impact female participation in the labor market.

Goldin (1995, 2006) notes that service jobs tend to be physically less demanding and cleaner, thus more "respectable" for women entering the labor force, than typical jobs in factories. Thus the expansion of the service sector is well positioned to generate the rising portion of the U. Insofar as the decline in manufacturing and the parallel rise in services are staggered across countries, this development can explain the international variation in women's labor market outcomes. Only a handful of papers in the recent literature have made this connection explicitly (see Blau and Khan, 2003, Rendall, 2010, Akbulut, 2011, Olivetti and Petrongolo, 2011, and Ngai and Petrongolo, 2012). All these papers are concerned with recent trends in female labor force participation in economically advanced economies and suggest that industry structure affect women's work.[4] Other authors have studied the role of home production in explaining the shift towards services but do not explicitly focus on the link with female labor force participation (see Ngai and Pissarides, 2008, Rogerson, 2008, Buera and Kaboski, 2011, 2012).

Far less has been written about the transition from agriculture to manufacturing. The declining portion of the U can be linked to the change in the nature of agricultural work as an economy moves away from subsistence agriculture. This change typically involves a shift from very labor-intensive technologies, where women are heavily involved as family workers, to capital-intensive agricultural technologies (such as the plough) where men tend to have a comparative advantage because they require physical strength. De Vries (1994) argues that market production increased (also for women) during the early stages of the industrial revolution but home production gained importance as female labor market participation declined. It is likely that since production in manufacturing was arduous and relatively intensive in the use of 'brawn', especially in the early phases of industrialization in the 19$^{\text{th}}$ century, women, especially married women, were more likely to drop out of the market.

[4] Of course, supply-side factors might be driving the change, although work by Lee and Wolpin (2006) suggests that demand-side factors associated with technical change are likely to be the prevailing force underlying these changes.

The contribution of this paper is to propose a mechanism that can simultaneously generate structural transformation and the full U-shaped pattern for female labor force participation. This is quite novel, if not unique, in this literature. The one notable exception, although the link to female labor supply is not explicit, is Buera and Kaboski (2012). Their theory emphasizes the scale of the productive unit as being important to understand both movements among broad sectors (agriculture, manufacturing, technology) and movements between home and market production. Among other things, scale technologies can generate the movement of services from the market sector to the home sector, and vice versa. To the extent that the division of labor between home and market activities is gendered, this mechanism has the potential to generate a declining female labor supply, associated with the phase of greatest expansion of the manufacturing sector, as well as the increasing portion of the curve, associated with the manufacturing sector decline and the acceleration in the expansion of the service sector.

2 Model

2.1 Background

The setup of the baseline model is a combination of Ngai-Pissarides (2008) and Ngai-Petrongolo (2014):

1. Home production is possible for all types of composite goods c_i, $i = a, m, s$. So there are six "sectors" to allocate labor $\{c_{ih}, c_{im}\}_{i=a,m,s}$. This is consistent with historical account that home production in 19th century produces good substitutes to output from agriculture and manufacturing sectors as well as service sectors. Consumption and time use data can be used to motivate and compare the model's predictions.

2. Focus on sector-specific labor productivity growth $\{A_{ih}, A_{im}\}_{i=a,m,s}$, ignoring the source whether it is TFP or capital intensity. So the model is essentially static and we will drop time subscript throughout. It should not be difficult to obtain labor productivity for market sectors $\{A_{im}\}_{i=a,m,s}$, but not easy to get home productivity growth especially for each sub-categories of home production. Recent attempt by Bridgman (2014) has computed the overall home productivity for 1929-2010 and more recent numbers for Australia, Canada, Japan,

Korea, Netherlands, Norway and South Africa.[5] In practice we can assume all A_{ih} grows at the same rate for $i = a, m, s$.

3. Men and women's labor input are imperfect substitute, combining with a CES function with gender-specific coefficient. This coefficient characterizes comparative advantage of each gender in each sector: $\{\xi_{im}, \xi_{ih}\}_{i=a,m,s}$. Each of these is related to the gender intensity of each sector. There are two special cases worth considering.

   (a) $\xi_{im} = \xi_{ih}$, i.e. the comparative advantage is to do with the nature of the goods rather than where you produce it. The assumption that women has comparative advantage in producing services is also adopted in Ngai and Petrongolo (2014) and Rendall (2014). Renall (2014) provide some evidence based on her earlier work using DOT dataset in Rendall (2010) based on the idea of "brain vs brawn". Ngai and Petrongolo (2014) however focus on arguing the importance of communication skills and empathy required for service production (which needs face-to-face interaction).

   (b) $\xi_{ih} = \xi_h$, i.e. once you stay home, the comparative advantage is the same across all kind of home production. This assumption is used in Buera, Kaboski and Zhao (2013).

   It will be good to have some empirical evidence on which is a more reasonable case.

4. Free mobility across sectors in the market, and across home and market. We might want to add barriers to entering market for women especially for pre-1970 data.[6]

5. Unitary household model, no household bargaining. This should be okay as a model to understand broad macro trends.

6. No externality, so can solve the social planner's problem for allocation of labor inputs across all sectors in the market and at home.

7. Abstract from leisure as both Ramey and Francis (2009) and Aguiar and Hurst (2007) show that the evolution of leisure are similar across genders. This is also related to the "iso-work" hypothesis of Burda,

[5] See Duernecker and Herrendorf (2014) for U.S. and France.

[6] Note that the assumption in case (b) above can generate some observational equivalent results to some forms of barriers to enter the market, especially if one allow these gender-specific parameters to change.

Hamermesh and Weil based on multinational time use data. The model predicts the fraction of total working time that is allocated to market activities – $\left(\mu_f, \mu_m\right)$ in the model.

2.2 Setup

Utility is defined over total consumption by men and women for composite goods C_i where $i = a, m, s$ are agriculture, manufacturing and service "consumption value-added" (see Herrendorf et al 2013 AER).

$$U(.) \equiv \left[\sum_i \omega_i C_i^{\frac{\varepsilon-1}{\varepsilon}}\right]^{\frac{\varepsilon}{\varepsilon-1}}; \quad \sum_i \omega_i = 1 \tag{1}$$

where C_i is a CES of c_{i_h} (home-produced) and c_{i_m} (market-produced) consumption goods.

$$C_i = \left[\psi_i c_{i_h}^{\frac{\sigma-1}{\sigma}} + (1-\psi_i)\, c_{i_m}^{\frac{\sigma-1}{\sigma}}\right]^{\frac{\sigma}{\sigma-1}}. \tag{2}$$

The production function for each good $j = \{c_{i_m}, c_{i_h}\}_{i=a,m,s}$ follows

$$c_j = A_j N_j, \quad N_j = \left[\xi_j l_{fj}^{\frac{\eta-1}{\eta}} + \left(1-\xi_j\right) l_{mj}^{\frac{\eta-1}{\eta}}\right]^{\frac{\eta}{\eta-1}}, \tag{3}$$

where productivity growth γ_j is sector-specific.

For each gender $g = f, m$, the following resource constraint holds:

$$\sum_i \left(l_{g_m} + l_{g_h}\right) = l_g. \tag{4}$$

Allowing the representative female and representative male to have different endowment of total working time allows for flexibility of calibration at later stage. (Time use data reveals l_f/l_m are fairly stable for the last 50 years, with l_f slightly larger than l_m).

This complete the setup of the model. Given sector-specific labor productivity growth, the model solves time allocation by sector and by gender: $\{l_{fj}, l_{mj}\}_{j=\{im,ih\}_{i=a,m,s}}$. The model also delivers gender wage ratio, relative prices, employment shares and market participation of each gender (measured as $\left(\mu_f, \mu_m\right)$ the fraction of time they allocated to the market sectors). Our key interest is to see whether the model can predicts a N-shaped or U-shape for μ_f and a monotonically declining μ_m. Of course, other objects such as gender wage ratio is interesting if we have historical series on it.

The quantitative exercise is to calibrate the model to match a particular period and let sector-specific labor productivity growth (interacted with different degree of substitutability across goods, and across market and home) predicts time allocation over time. The driving forces $\left\{\gamma_{i_m}, \gamma_{i_h}\right\}_{i=a,m,s}$ and $\{\varepsilon, \sigma, \eta\}$ are set directly to the estimates in the data. The rest of the relevant parameters are set to match data targets. More specifically, there are 13 data targets including wage ratio and the time allocation of each gender. The parameters to be matched are

- 6 gender-specific parameters $\left\{\xi_{i_h}, \xi_{i_m}\right\}_{i=a,m,s}$,
- 6 parameters that are product of ω_i and A_{i_m}, and product of ψ_i and A_{i_h}, $i = a, m, s$
- the ratio of l_f/l_m

Ngai-Petrongolo chooses to match the model to the end of the sample based on the argument that this perfectly competitive model should fit the data best when culture and gender discrimination concerns are at their minimal in the 21st century. Then the predictions of the model can be interpreted as how much can sector-specific labor productivity growth, through the process of structural transformation and marketization, can explain the dynamic path of time allocation for each gender. For Ngai-Petrongolo there is also a huge advantage of doing this for practical reason. Data required to pin the parameters are more likely to be available for the end of the sample than the beginning, e.g. gender's time allocation in all three sectors (goods, services and home) of Ngai-Petrongolo. For this paper, it is slightly tricky because home agriculture and home manufacturing have both disappeared by 21st century. So one may have to focus on one of the special case mentioned in point 3 above.

3 Equilibrium Allocation

Given there is no externality, we focus on the planner's solution for the allocation and the corresponding gender wage ratio is equal to the ratio of value of marginal product of labor.

The model can be solved backward in two steps: first optimize across home (i_h) and market (i_m), then optimize across different types of composite goods $i = a, m, s$.

Suppose at optimal, L_{fi} and L_{mi} are female and male labor input allocated into producing composite good $i = a, m, s$, the optimal allocation across market and home is to choose $(l_{fi_h}, l_{fi_m}, l_{mi_h}, l_{mi_m})$ to maximize C_i in (2) subject to the production function (3) and the following constraint for each gender:

$$l_{gi_h} + l_{gi_m} = L_{gi}; \quad g = f, m \tag{5}$$

Free mobility of labor across home and market for each gender $g = f, m$ implies

$$\frac{\partial C_i}{\partial c_{i_h}} A_{i_h} \left(\frac{\partial N_{i_h}}{\partial l_{gi_h}} \right) = \frac{\partial C_i}{\partial c_{i_m}} A_{i_m} \left(\frac{\partial N_{i_m}}{\partial l_{gi_m}} \right); \quad g = f, m \tag{6}$$

The set of equations (5)-(6) are four equations solving for the four unknown $(l_{fi_h}, l_{fi_m}, l_{mi_h}, l_{mi_m})$ as functions of (L_{fi}, L_{mi}). More specifically, using (6) across gender, their ratio of marginal productivity (which is the gender wage ratio in decentralized economy) can be solved as[7]

$$w = \frac{\xi_{i_h}}{1 - \xi_{i_h}} \left(\frac{l_{mi_h}}{l_{fi_h}} \right)^{1/\eta} = \frac{\xi_{i_m}}{1 - \xi_{i_m}} \left(\frac{l_{mi_m}}{l_{fi_m}} \right)^{1/\eta}, \tag{7}$$

which relates the gender intensities across market and home according to the gender-specific parameter ξ_j.

Once $(l_{fi_h}, l_{fi_m}, l_{mi_h}, l_{mi_m})$ as functions of (L_{fi}, L_{mi}), the second step is to choose (L_{fi}, L_{mi}) to maximize the utility function (1) subject to the production functions (3) and time constraint (4).

3.1 Special Case

To derive full analytical solution, we focus on the special case 1:

$$\xi_{i_h} = \xi_{i_m} = \xi_i \quad, i = a, m, s. \tag{8}$$

In other words the gender comparative advantage is associated with the type of goods but not where the goods is produced.

[7] It is worth pointing out that this is a model that studies between-industry component in accounting for changes in gender ratios. Thus without any within-industry change over time, the model cannot simultaneously account for a rise in gender wage ratio and an increase in female intensity in all sectors. This is a result of adopting a CES labor composite function. However, within-in industry shifts are allowed, e.g. a rise in ξ_j then both rise in wage ratio and female intensity can co-exist. This has been shown for data post 1968 in Ngai-Petrongolo (2014).

3.1.1 Allocation across Market and Home

Under the special case, (7) implies equal gender intensity across market and home, and together with (5), they imply:

$$\frac{l_{mi_h}}{l_{fi_h}} = \frac{l_{mi_m}}{l_{fi_m}} = \frac{L_{mi}}{L_{fi}}. \tag{9}$$

Using (9), the composite labor in (3) for market and home production can be written as

$$N_{i_k} = L_{fi_k}\left[\xi_i + (1-\xi_i)\left(\frac{L_{mi}}{L_{fi}}\right)^{\frac{\eta-1}{\eta}}\right]^{\frac{\eta}{\eta-1}}; \quad k = m, h, \tag{10}$$

thus

$$\frac{N_{i_m}}{N_{i_h}} = \frac{l_{fi_m}}{l_{fi_h}} = \frac{l_{mi_m}}{l_{mi_h}}, \tag{11}$$

which implies the "physical" marginal product of labor (for both genders) are proportional across home and market given

$$\frac{\partial N_{i_k}}{\partial l_{gi_k}} = \xi_{i_k}\left(\frac{N_{i_k}}{l_{gi_k}}\right)^{1/\eta}, \quad k = m, h; \quad g = f, l. \tag{12}$$

Using this result, and by substituting (2) in (6), we derive the optimal consumption allocation across home and market:

$$\frac{c_{i_m}}{c_{i_h}} = \left(\frac{A_{i_m}}{A_{i_h}}\right)^{\sigma}\left(\frac{1-\psi_i}{\psi_i}\right)^{\sigma}. \tag{13}$$

Condition (13) can be interpreted as the "marketization" condition for outsourcing home-produced good i to be market-produced. It says if market productivity rises relative to home productivity in producing good i, then more of good i will be marketized. It could also be driven by change in preference for consuming home-produced good through parameter ψ_i.

The time allocation across market and home can now be derived from substituting production function (3) into (13) and making use of the condition (11)

$$\frac{l_{fi_m}}{l_{fi_h}} = \frac{l_{mi_m}}{l_{mi_h}} = \alpha_i \equiv \left(\frac{A_{i_m}}{A_{i_h}}\right)^{\sigma-1}\left(\frac{1-\psi_i}{\psi_i}\right)^{\sigma}. \tag{14}$$

Condition (14) is the "marketization" of time for producing each good i for each gender. This is the key condition for marketization participation for each gender. The ratio α_i changes over time due to change in technology or preferences. It is important to note that this implies that marketization forces are similar across genders for given (L_{fi}, L_{mi}). However, the implied dynamics for market participation are different across genders because the equilibrium L_{f_i} behaves differently than L_{mi} due to different comparative advantage across genders.

Finally, substituting (14) into the time constraint (5), we derive explicitly for each gender g

$$l_{gi_h} = \frac{L_{gi}}{1+\alpha_i}; \quad l_{g_{i_m}} = \frac{\alpha_i L_{g_i}}{1+\alpha_i} \quad g = f, m. \tag{15}$$

Using the production function (3), at optimal,

$$C_{i_h} = \frac{A_{i_h}}{1+\alpha_i} \left[\xi_j L_{fj}^{\frac{\eta-1}{\eta}} + \left(1-\xi_j\right) L_{mj}^{\frac{\eta-1}{\eta}} \right]^{\frac{\eta}{\eta-1}} \tag{16}$$

We next derive a hypothetical production function for the composite good i and proceed to solving time allocation across different type of good i. This is equivalent to deriving C_i as a function of L_{fi} and L_{mi}. Using (13) and (16), composite consumption (2) can be expressed as output of a hypothetical production function

$$C_i = B_i H_i \tag{17}$$

where the equilibrium productivity index is (see Appendix)

$$B_i \equiv \left[\psi_i^{\sigma} A_{i_h}^{\sigma-1} + (1-\psi_i)^{\sigma} A_{i_m}^{\sigma-1} \right]^{\frac{1}{\sigma-1}} \tag{18}$$

and the equilibrium composite of labor is

$$H_i \equiv \left[\xi_i L_{fi}^{\frac{\eta-1}{\eta}} + (1-\xi_i) L_{mi}^{\frac{\eta-1}{\eta}} \right]^{\frac{\eta}{\eta-1}} \tag{19}$$

3.1.2 Allocation across Sectors

Our final step is to maximize utility (1) by choosing $(L_{fi}, L_{mi})_{i=a,m,s}$ subject to (17)-(19) and resource constraints (4) rewriting as

$$\sum_{i=a,m,s} L_{gi} = l_g; \quad g = f, m. \tag{20}$$

Free mobility of labor across sectors implies equal marginal rate of substitution across sectors, using the result (9) to obtain:

$$w = \frac{\xi_i}{1-\xi_i}\left(\frac{L_{mi}}{L_{fi}}\right)^{1/\eta}; \quad i = a, m, s, \tag{21}$$

which states that gender intensity are different across sector i because of the sector-specific gender parameter ξ_i (which is the same reason why gender intensities are equal across market and home within the same i). Substitute this into (19) to obtain an useful formula

$$\frac{H_i}{L_{fi}} = \xi_i^{\frac{\eta}{\eta-1}}\left[1+\left(\frac{1-\xi_i}{\xi_i}\right)^{\eta} w^{\eta-1}\right]^{\frac{\eta}{\eta-1}} \tag{22}$$

Equalizing the value of marginal product of L_{fi} across sectors i and j using utility function (1), (19) and the hypothetical production function (17) to obtain (See Appendix)

$$\frac{L_{fi}}{L_{fj}} = \frac{F_i(w)}{F_j(w)}; \quad F_i(w) = \omega_i^{\varepsilon}\left(\xi_i^{\frac{\eta}{\eta-1}} B_i\right)^{\varepsilon-1}\left[1+\left(\frac{1-\xi_i}{\xi_i}\right)^{\eta-1} w^{\eta-1}\right]^{\frac{\varepsilon-\eta}{\eta-1}}. \tag{23}$$

As in Ngai-Petrongolo (2014), under plausible case of $\eta > 1 > \varepsilon$, higher w will lead to an increase in L_{fi}/L_{fj} if and only if $\xi_i > \xi_j$. There are two substitution margins at work: input and output substituability. The input substituability operates through parameter η : higher female relative wage induces substitution away from female labor input especially in sector with lower ξ_i, thus female labor shifts away from sector with low ξ_i. The output substituability operates through parameter ε : higher female relative wage induces substitution away from sector that uses female more intensively, thus female labor shifts away from sector with high ξ_i.When $\eta > \varepsilon$, the input substitution dominates, thus higher w induce a net shifts of female labor away from sector with low ξ_i.

The standard relative price effects in the structural transformation literature also present here: higher relative productivity B_i/B_j, reduces price of good i relative to j, shifts labor away from producing good i into producing good j because goods i and j are poor substitutes $(\varepsilon < 1)$.

Substitute (23) into resource constraint to derive labor allocation as a function of gender wage ratio w :

$$L_{fi} = \frac{l_f F_i(w)}{\sum\limits_{j=a,m,s} F_j(w)}; \quad i = m, s. \tag{24}$$

Using (21) male's labor allocation is

$$L_{mi} = \left(\frac{1-\xi_i}{\xi_i} w\right)^\eta L_{fi}; \quad i = m, s \tag{25}$$

Finally, substitute (25) into the resource constraint for male to obtain an implicit function that solves for w :

$$\frac{l_m}{l_f} = \sum_{i=a,m,s} \left(\frac{1-\xi_i}{\xi_i} w\right)^\eta \frac{F_i(w)}{\sum\limits_{j=a,m,s} F_j(w)}. \tag{26}$$

Once equilibrium gender wage ratio w is derived from (26), equilibrium variables $(L_{fi}, L_{mi})_{i=a,m,s}$ are derived in (24)-(25), time allocation across all six sectors of the economy are then obtained from (15), which complete the derivation of the model.

3.1.3 Market Participation

We now turn to the main variable of interesting: market participation. Let μ_g be the fraction of time that each gender allocated to market production. For female, by definition

$$\mu_f = \sum_{i=a,m,s} \frac{l_{fi_m}}{l_f} = \sum_{i=a,m,s} \frac{l_{fi_m}}{L_{fi}} \left(\frac{L_{fi}}{l_f}\right)$$

substituting (15) and (24),

$$\mu_f = \sum_{i=a,m,s} \left(\frac{\alpha_i}{1+\alpha_i}\right) \frac{F_i(w)}{\sum\limits_{j=a,m,s} F_j(w)}. \tag{27}$$

Note that if the productivity growth $\gamma_{i_m} > \gamma_{i_h}$, then definition (14) implies $\alpha_i \longrightarrow \infty$, implying eventually all home production are marketized, thus $\mu_f \longrightarrow 1$. This is not to say that female labor market participation converges to 1. It is stating that female's market work relative to total work converges to 1. In other words, if leisure is added to the utility and its functional form gives rise to some balanced growth path where leisure is a constant fraction of time in the long run, then female's labor market participation is simply one minus such fraction.

For male, by definition,

$$\mu_m = \sum_{i=a,m,s} \frac{l_{mi_m}}{l_m} = \frac{l_f}{l_m} \sum_{i=a,m,s} \frac{l_{mi_m}}{L_{mi}} \left(\frac{L_{mi}}{L_{fi}}\right) \left(\frac{L_{fi}}{l_f}\right),$$

substituting (15), (21) and (25),

$$\mu_m = \frac{l_f}{l_m} w^\eta \sum_{i=a,m,s} \left(\frac{1-\xi_i}{\xi_i}\right)^\eta \left(\frac{\alpha_i}{1+\alpha_i}\right) \frac{F_i(w)}{\sum\limits_{j=a,m,s} F_j(w)}.$$

Finally using the equilibrium wage in (26) to obtain

$$\mu_m = \frac{\sum\limits_{i=a,m,s} \left(\frac{1-\xi_i}{\xi_i}\right)^\eta \left(\frac{\alpha_i}{1+\alpha_i}\right) \frac{F_i(w)}{\sum\limits_{j=a,m,s} F_j(w)}}{\sum\limits_{i=a,m,s} \left(\frac{1-\xi_i}{\xi_i}\right)^\eta \frac{F_i(w)}{\sum\limits_{j=a,m,s} F_j(w)}}. \tag{28}$$

There are two remarks about comparing market participation of female and male in (27) and (28). First, rising gender wage ratio tends to increase μ_m relative to μ_f. This is the substitution effect from the CES labor composite. This is discussed in footnote above. There is nothing wrong with it since the model is not about within sector shifts. Quantitatively, since w is endogenous in this model, so this equilibrium channel of w on market participation is likely to be small. Second, a gender-specific shifts such as a general increase in ξ_i for all i will put a downward pressure in μ_m relative to μ_f. So this within-sector force can helps to generate a monotonic decline in male's market participation.

Now let's ignore the equilibrium effect through w and also fix ξ_i to be constants. The important forces that drive the dynamics of market participations are the relative productivity growth rate and their interaction with the ranking of ξ_i. Consider the plausible case that

$$\xi_s > \max\{\xi_a, \xi_m\}, \tag{29}$$

$$\gamma_{ia_m} > \gamma_{im_m} > \gamma_{is_m}, \tag{30}$$

$$\gamma_{i_m} > \gamma_{i_h} = \gamma_h, \tag{31}$$

where the last equality is for simplicity. The key is that $\left(\gamma_{i_m} - \gamma_{i_h}\right)$ is largest for agriculture, then manufacturing and least in services.

Under (30) and (31), α_i are increasing, but at the fastest speed in agriculture, follow by manufacturing, then services. From (18), the growth rate of B_i is

$$\gamma_{B_i} = \frac{\psi_i^\sigma A_{i_h}^{\sigma-1}\gamma_{i_h} + (1-\psi_i)^\sigma A_{i_m}^{\sigma-1}\gamma_{i_m}}{\psi_i^\sigma A_{i_h}^{\sigma-1} + (1-\psi_i)^\sigma A_{i_m}^{\sigma-1}}, \tag{32}$$

which is a weight average of productivity growth for market and home technologies. Under (31), γ_{B_i} is increasing over time but at the fastest speed in agriculture, then manufacturing, then services. So the definition of $F_i(w)$ in (23) implies that $F_i(w)$ is falling fastest in agriculture, then manufacturing, finally services. Thus the dynamics of μ_f in (27) are driven by interaction of rising α_i and falling $F_i(w)$; while such interaction is weighted in addition by $\left(\frac{1-\xi_i}{\xi_i}\right)^\eta$ for μ_m in (28). Here is a conjecture of different stages for the dynamics of μ_f and μ_m:

1. When home agriculture is substantially large (pre-1870), the fast marketization of home agriculture dominates structural transformation, so major shift from home to market. This tend to increase market participation for both genders.

2. When home agriculture disappear and market agriculture sector is substantially large compared to other market sectors (around 1900), structural transformation dominates, so major shift into producing services (both at home and in the market). The marketization of service is weak compared to structural transformation because of the differences in productivity growth rates. This tends to decrease market participation for both genders, especially for female because they have comparative advantage in producing services.

3. When market agriculture disappears and manufacturing becomes small as well (around 1950s), structural transformation force is weak, the main shift is from home services to market services. This tends to increase market participation for both genders but especially for female because of comparative advantage.

Stage 2 and 3 are very likely to produce the U-shape female market participation. As for male, it is more likely to be flat or more like L-shape given the comparative advantage argument given.

Starting from stage 1, e.g. from 1800, it is likely to get a small hump-shape for male (is this true in the data?). The N-shape for female is plausible if the initial allocation is such that lots of women is performing home agriculture (this could be obtained by having $\xi_a > \xi_m$).

3.2 Comparison to Ngai-Petrongolo 2014

The model presented in Ngai and Petrongolo (2014) is a special case of our model where agriculture is absent ($\omega_a \to 0$) and manufacturing can only be produced in the market ($\psi_m \to 0$). Their Proposition 3 and 4 are especially of interest to this paper.

Their Proposition 3 states that for both genders, the share of market hours falls with structural transformation but rise with marketization. In other words, both μ_f and μ_m are non-montonic. In theory, they could generate a U-shaped female labor supply as well. The intuition is that structural transformation through the decline in goods sector dominates marketization initially while the reverse is true when the size of goods sector becomes small. However, quantitatively, they show that the implied μ_f is monotonically increasing. One important reason for this is that the goods sector is already quite small (42% of employment) for the beginning of their sample period 1968-72, thus the structural transformation force is weak. The beginning of our sample period is set to be 1890, when the goods sector (agriculture and manufacturing) was substantially larger with 70% of employment. This is a clear example of strong structural transformation force.

Their Proposition 4 further establish that given women's comparative advantage in services, both structural transformation and= marketization lead to a rise μ_f/μ_m. It will be interesting to check whether our more general setup implies a monotonic increase in μ_f/μ_m as in the simpler setup or there is a U-shape as well.

We first express μ_f and μ_m in a form that is comparable to Ngai-Petrongolo. Rewrite (27) as

$$\mu_f = [I(w) - 1]\left[1 - \frac{F_s(w)}{\sum\limits_{j=a,m,s} F_j(w)}\right] + 1 - \left(\frac{F_s(w)}{\sum\limits_{j=a,m,s} F_j(w)}\right)\left(\frac{1}{1+\alpha_s}\right). \tag{33}$$

where

$$I(w) \equiv \left(\frac{\alpha_a}{1+\alpha_a}\right)\frac{F_a(w)}{F_a(w)+F_m(w)} + \left(\frac{\alpha_m}{1+\alpha_m}\right)\frac{F_m(w)}{F_a(w)+F_m(w)}. \tag{34}$$

Under the special case where $\omega_a \to 0$ and $\psi_m \to 0$, equation (14) implies $\alpha_m \to \infty$ and equation (23) implies $F_a(w) \to 0$, $I(w) \to 1$ and (33) becomes

$$\mu_f = 1 - \left(\frac{1}{1+\frac{F_m(w)}{F_s(w)}}\right)\left(\frac{1}{1+\alpha_s}\right) \tag{35}$$

which is identical as in Ngai-Petrongolo. It shows structural transformation (falling $\frac{F_m(w)}{F_s(w)}$) reduces μ_f while marketization (rising α_s) increases μ_f. Comparing (33) with (35) provides insight into whether distinguishing goods into agriculture and manufacturing and allowing home production in agriculture and manufacturing are important for understanding the U-shaped female labor supply.

The term $I(w)$ which is a weighted average of the marketization forces in agriculture and manufacturing. Using (14) and (23), and the definition of L_{gi} in (5),

$$I(w) = \left(\frac{l_{f_{am}} + L_{f_{mm}}}{L_{f_a} + L_{f_m}}\right)$$

which is the share of market hours to total working hours used in the production of goods (agriculture and manufacturing). This is increasing over time regardless of the overall size of the goods sector, but it is converging to one (from below). Its effect on μ_f, however, is through the term

$$[I(w) - 1]\left[1 - \frac{F_s(w)}{\sum\limits_{j=a,m,s} F_j(w)}\right] = -\left(\frac{l_{f_{ah}} + L_{f_{mh}}}{L_{f_a} + L_{f_m}}\right)\left(\frac{L_{f_a} + L_{f_m}}{L_{f_a} + L_{f_m} + L_{f_s}}\right),$$

which is a combination of the marketization and structural transformation happening across sectors. Clearly, this term tends to lower the level of μ_f. The first bracket is falling (due to marketization) and the second bracket is also falling due to structural transformation. So over time this term tends to increase μ_f. Therefore the declining part of μ_f in this general model compared to Ngai-Petrongolo hinges crucially on how quickly $\frac{F_s(w)}{\sum\limits_{j=a,m,s} F_j(w)}$ or $\frac{L_{f_s}}{L_{f_a}+L_{f_m}+L_{f_s}}$ rises in the two models. In the general model, both marketization and structural transformation are lowering $(L_{f_a} + L_{f_m})$ relative to L_{f_s}. In Ngai-Petrongolo model, only structural transformation is lowering $(L_{f_a} + L_{f_m})$, so it is reasonable to expect a faster decline in μ_f due to this second term. However, it is not clear how this compared with the rising force from the first term.

Note: it seems like new analytical results require further restriction on parameters. For example, we might have to think about the ranking of ξ_a and ξ_m (under the assumption that $\xi_s > \max\{\xi_a, \xi_m\}$) :

1. if $\xi_a = \xi_m$, i.e. women's comparative advantage is only to do with services, nothing to do with distinction across manufacturing and agriculture.

2. if $\xi_a > \xi_m$, this might add some additional decline in women's μ_f when agriculture is shrinking
3. if $\xi_a < \xi_m$, this might add some additional rise in women's μ_f when manufacturing is expanding

To proceed, it will be helpful to look at the male/female hours ratio in agriculture and manfucturing sectors; both their levels and their dynamics over time (this is fact D4 above).

4 References

References

[1] Aguiar, Mark and Erik Hurst. 2007. "Measuring Trends in Leisure: The Allocation of Time Over Five Decades ". Quarterly Journal of Economics 122: 969-1006.

[2] Bridgman, Benjamin. 2013. "Home Productivity", Working Paper 0091, Bureau of Economic Analysis.

[3] Burda, Michael, Daniel Hamermesh and Phillipe Weil. 2013. "Total Work and Gender: Facts and Possible Explanations," Journal of Population Economics

[4] Buera, Francisco J., Joseph P. Kaboski, and Min Qiang Zhao. 2013. "The Rise of Services: the Role of Skills, Scale, and Female Labor Supply."NBER working paper No. 19372.

[5] Duernecker, George and Berthod Herrendorf 2014. "On the Allocation of Time –A Quantitative Analysis of U.S. and France." Unpublished manuscript, Arizona State University.

[6] Claudia Goldin 1995. "The U-shaped Female Labor Force Function in Economic Development and Economic History. In T.P.Schultz, ed., *Investment in Women's Human Capital and Economic Development*, pp.61-90.

[7] Ngai and Petrongolo 2014 "Gender gaps and the rise of the service economy"

[8] Ngai, L. Rachel and Christopher A. Pissarides. 2008. " Trends in Hours and Economic Growth". Review of Economic Dynamics 11: 239-256.

[9] Olivetti, Claudia. 2014. “The Female Labor Force and Long-run Development: The American Experience in Comparative Perspective”. In L. Platt Boustan, C. Frydman and R.A. Margo (eds.), Human Capital in History: The American Record. Chicago: Chicago University Press.

[10] Ramey, Valery A. and Neville Francis, “A Century ofWork and Leisure,” American Economic Journal: Macroeconomics, 2009, 1, 189–224.

[11] Rendall, Michelle. 2010. “Brain versus Brawn: The Realization ofWomen’s Comparative Advantage.”Mimeo, University of Zurich.

[12] Rendall, Michelle. 2014. “The Service Sector and Female Market Work.”Mimeo, University of Zurich.

5 Appendix

5.1 Hypothetical production function

From (2),

$$C_i = \left[\psi_i + (1-\psi_i)\left(\frac{c_{i_m}}{c_{i_h}}\right)^{\frac{\sigma-1}{\sigma}}\right]^{\frac{\sigma}{\sigma-1}} c_{i_h}$$

Using (3) and (13),

$$C_i = \left[\psi_i + (1-\psi_i)\left(\left(\frac{A_{i_m}}{A_{i_h}}\right)\left(\frac{1-\psi_i}{\psi_i}\right)\right)^{(\sigma-1)}\right]^{\frac{\sigma}{\sigma-1}} A_{i_h} N_{i_h}.$$

Using (10) and (15),

$$\begin{aligned} & C_i \\ = \quad & \left[\psi_i + (1-\psi_i)\left(\left(\frac{A_{i_m}}{A_{i_h}}\right)\left(\frac{1-\psi_i}{\psi_i}\right)\right)^{(\sigma-1)}\right]^{\frac{\sigma}{\sigma-1}} \bullet \\ & \left(\frac{A_{i_h} L_{fi}}{1+\alpha_i}\right)\left[\xi_i + (1-\xi_i)\left(\frac{L_{mi}}{L_{fi}}\right)^{\frac{\eta-1}{\eta}}\right]^{\frac{\eta}{\eta-1}}. \end{aligned}$$

So the H_i is direct result and the productivity index is

$$B_i \quad = \quad \left[\psi_i + (1-\psi_i)\left(\left(\frac{A_{i_m}}{A_{i_h}}\right)\left(\frac{1-\psi_i}{\psi_i}\right)\right)^{(\sigma-1)}\right]^{\frac{\sigma}{\sigma-1}} \frac{A_{i_h}}{1+\alpha_i}$$

$$= \quad \psi_i^{\frac{\sigma}{\sigma-1}} A_{i_h} \frac{\left[1+\left(\frac{A_{i_m}}{A_{i_h}}\right)^{\sigma-1}\left(\frac{1-\psi_i}{\psi_i}\right)^{\sigma}\right]^{\frac{\sigma}{\sigma-1}}}{1+\alpha_i}$$

using definition of α_i in (14)

$$B_i = (1+\alpha_i)^{\frac{1}{\sigma-1}}\, \psi_i^{\frac{\sigma}{\sigma-1}} A_{i_h}$$

or

$$\begin{aligned} B_i &= \psi_i^{\frac{\sigma}{\sigma-1}} A_{i_h} \left[1+\left(\frac{A_{i_m}}{A_{i_h}}\right)^{\sigma-1}\left(\frac{1-\psi_i}{\psi_i}\right)^{\sigma}\right]^{\frac{1}{\sigma-1}} \\ &= \left[\psi_i^{\sigma} A_{i_h}^{\sigma-1} + (1-\psi_i)^{\sigma} A_{i_m}^{\sigma-1}\right]^{\frac{1}{\sigma-1}}. \end{aligned}$$

5.2 Time allocation across sectors

Equalizing the value of marginal product of L_{fi} across sectors i and j

$$\frac{\partial U}{\partial C_i} B_i \frac{\partial H_i}{\partial L_{fi}} = \frac{\partial U}{\partial C_j} B_j \frac{\partial H_j}{\partial L_{fj}}$$

substituting utility function (1) and (19)

$$\frac{\omega_i}{\omega_j}\left(\frac{C_j}{C_i}\right)^{1/\varepsilon} B_i = B_j \frac{\xi_j \left(\frac{H_j}{L_{fj}}\right)^{1/\eta}}{\xi_i \left(\frac{H_i}{L_{fi}}\right)^{1/\eta}}$$

substitute the hypothetical production function (17) to obtain,

$$\frac{\omega_i}{\omega_j}\left(\frac{B_j H_j}{B_i H_i}\right)^{1/\varepsilon} B_i = B_j \frac{\xi_j \left(\frac{H_j}{L_{fj}}\right)^{1/\eta}}{\xi_i \left(\frac{H_i}{L_{fi}}\right)^{1/\eta}}$$

simply to

$$\frac{L_{fi}}{L_{fj}} = \left(\frac{\omega_i}{\omega_j}\right)^{\varepsilon}\left(\frac{B_j}{B_i}\right)^{1-\varepsilon}\left(\frac{\xi_i}{\xi_j}\right)^{\varepsilon}\left(\frac{H_j/L_{fj}}{H_i/L_{fi}}\right)^{1-\frac{\varepsilon}{\eta}}$$

Finally substitute (22) to obtain (23).

Figure 1: Female labor force participation and economic development: 1890-2005

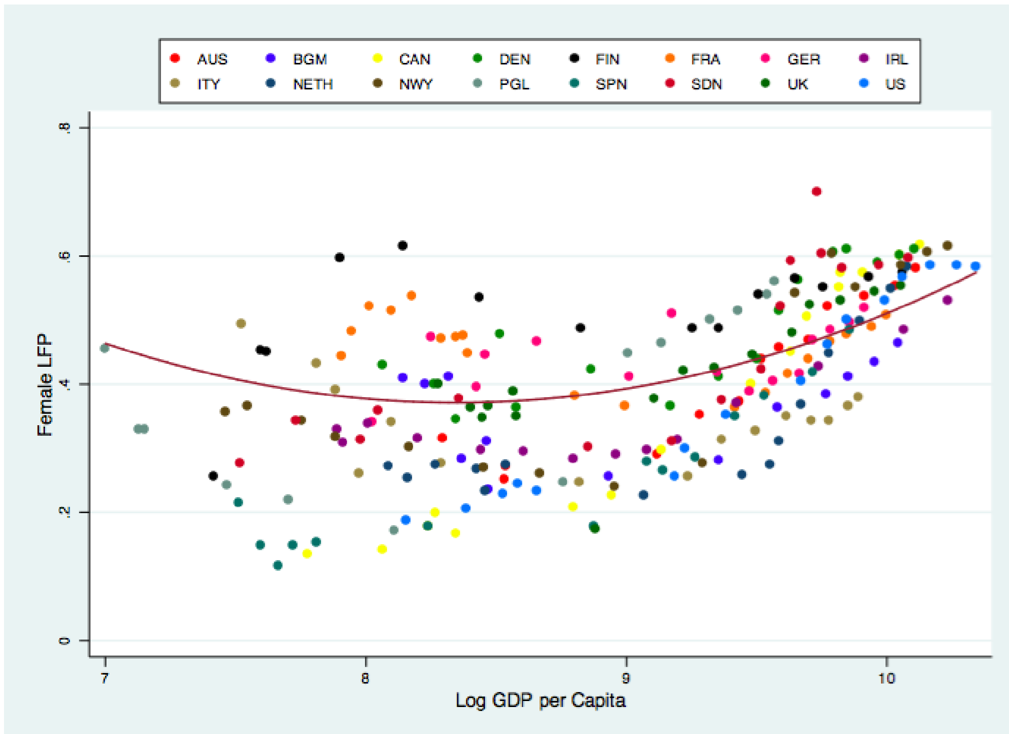
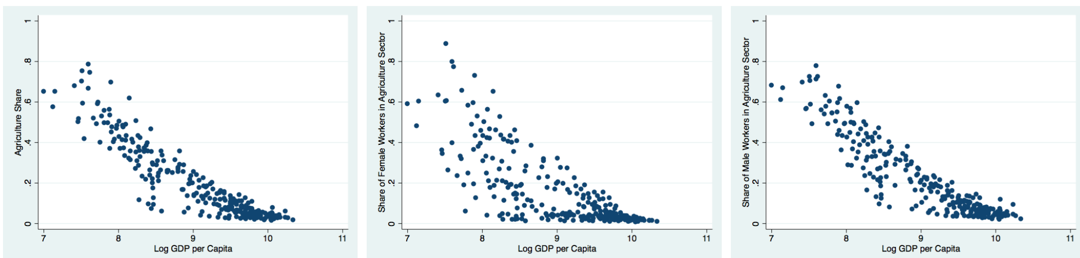


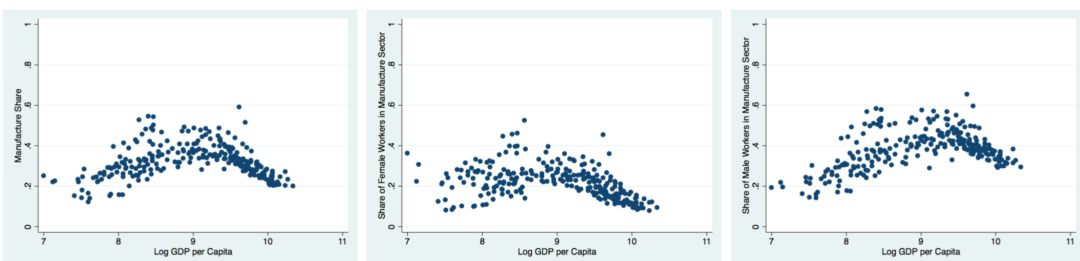
Figure 2: Sectoral employment shares by gender: Developed economies, 1890-2005

All | Females | Males

Panel A: Agricultural Sector



Panel B: Manufacturing Sector



Panel C: Service Sector