Intermediaries as Information Aggregators: 
An Application to U.S. Treasury Auctions*

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Abstract

In most theories of financial intermediation, the intermediaries diversify risk, transform maturity or liquidity, or screen/monitor borrowers. But in U.S. Treasury auctions, none of these rationales apply: Investors can bid directly, assets are highly liquid, dealers do not discipline, screen or diversify fiscal policy risk. Yet, most bids are still intermediated. Motivated by treasury auctions, we explore a new information aggregation theory of intermediaries who observe the order-flow of each client and use that aggregated information to advise all clients. In contrast to underwriting theories where intermediaries extract rents, but reduce revenue variance, information aggregators do the opposite: They increase expected auction revenue, but also make the revenue more sensitive to changes in asset value. We use the model to examine current policy questions, such as the optimal number of intermediaries, the effect of non-intermediated bids and minimum bidding requirements.

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Investors often access markets through intermediaries. Sometimes intermediaries are necessary to access a market: An individual investor cannot enter a trading floor. Sometimes intermediaries lower risk by screening investments or monitoring firm behavior.\footnote{In most macro and asset pricing theories, intermediaries perform a monitoring function. See e.g., ADD REFS.} But in the world’s largest auctions, those for U.S. treasury auctions, investors can choose to bid directly online, the assets are liquid, dealers do not influence U.S. fiscal policy, and dealers cannot screen and select issuances because they are required to participate in every auction. The fact that intermediation is prevalent in markets where none of the typical rationales apply prompts us to examine a new role that intermediaries play and its consequence for asset prices and auction revenue.

We present a new theory of financial intermediaries whose primary function is to collect information from order flow, use it to advise clients, and bid for their in-house demand. In a typical initial public offering (IPO), the underwriter has exclusive access to the primary market and earns a fee for insuring revenues.\footnote{In a “full commitment IPO,” the underwriter also generally earns a large first-day secondary market return, and stabilizes the market value by raising supply elasticity, either offering additional (“greenshoe option”) or buying some of the securities being offered (Ritter and Welch, 2002).} Thus, by assumption, the underwriter lowers expected revenue but also reduces revenue variance. In contrast, our information aggregation intermediaries increase expected revenue and increase revenue variance. By sharing information with their clients, dealers lower the client investors’ risk, which encourages clients to bid more aggressively and raises expected auction revenue. At the same time, more precise information about the future value of the asset makes beliefs and bids more sensitive to changes in that value. Therefore, auction revenue also depends more sensitively on true future value and as a result, is more variable. Thus, information aggregation intermediaries provide value both for investors and for the asset issuer. But their effects on auction revenue are exactly the opposite from those of a traditional underwriter.

We use this theory to study how revenues are affected by policy changes involving the number of intermediaries (hereafter, dealers), the ability of investors to bid directly, and minimum bidding requirements. We learn that additional dealers inhibit information aggregation, even though they reduce monopsony power. The model also highlights the key trade-off investors face when deciding to place direct or intermediated (indirect) bids: by using intermediation, an investor benefits by observing the average demand of other clients of their dealer. On the other hand, they relinquish their own signal to the dealer, lowering the posterior uncertainty faced by other investors, thereby boosting the residual demand for the issue. Finally, minimum bidding requirements have been a controversial tool used to ensure robust dealer participation in every auction. We explain why this requirement makes intermediation more attractive to investors, while also increasing auction revenue.
But despite the fact that the requirement was instituted to stabilize asset prices, we also show how it can, in fact, increase asset price and revenue volatility.

In many markets, intermediaries collect information from order flow and use it to advise clients. We consider Treasury auctions in particular because they are important (being central to funding national debt) and because other common roles for intermediaries (as gate-keepers or monitors) are absent. However, they are also complex and unique in their structure. In modeling these auctions, we attempt to balance a detailed description with a tractable and transparent model which highlights insights that are broadly applicable. The basis for the model is a standard, common-value, uniform-price auction with heterogeneous information, and limit and market orders. We consider a large, finite number of investors who are not strategic bidders, meaning that they take the market-clearing price as given. On top of that foundation, we add five features that distinguish Treasury auctions from other auctions: first, most trades are intermediated by a primary dealer, who bids on behalf of his clients and bids on assets for his own account. Such dealers should act strategically and are expected to consider the effect they have on auction prices. Their primary dealer designation implies a responsibility to serve as a trading counterparty for the Federal Reserve Bank of New York. Second, primary dealers are information aggregators. They use this information to advise clients and the Federal Reserve itself. But primary dealers can also learn from the bidding information they collect.

"[Observing overall patterns of buying in the Treasury market] can be one of the greatest benefits of being a primary dealer, since the service itself often doesn’t pull in big profits directly." (Reuters, 2011)

The first version of the model (Section 2) incorporates these two features and examines the trade-off between the market power that an intermediary enjoys and the information they provide to others.

Using Treasury auction data, we calibrate each version of the model and use it to study the quantitative effect of primary dealers on the mean and variance of auction revenue. The relative magnitude of these effects depends on asset characteristics, like uncertainty, and on investor characteristics, like the quality of private information. We find that dealers contribute most to auction revenue in high-uncertainty times when the auction price does a poor job of aggregating information. By collecting private information from clients and sharing knowledge about the general state of the market, dealers increased expected

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3 As discussed on the New York Fed’s website, primary dealers are “[…] obligated to: (i) participate consistently in open market operations to carry out U.S. monetary policy […], and to provide the New York Fed’s trading desk with market information and analysis helpful in the formulation and implementation of monetary policy. Primary dealers are also required to participate in all auctions of U.S. government debt and to make reasonable markets for the New York Fed when it transacts on behalf of its foreign official account-holders.”
auction revenue at a time when it would otherwise be low. Although this effect is an expected revenue effect and does not reduce the conditional variance of a given auction’s revenue, it does help prevent extreme outcomes such as “failed auctions.” In this way, primary dealers are a hedge against a sudden rise in uncertainty.

The third distinguishing feature of Treasury auctions is that the number of dealers is determined by the New York Fed and fluctuates over time. From around 20 dealers in the 1960s, the number of dealers rose to nearly 50 in the late 80’s and fell back to around 20 by 2007. Policy makers have been debating the merits of a robust primary dealer system for decades (see the Brady, Breeden, and Greenspan (1992) report). Thus, Section 3 adds multiple dealers and examines how the number of dealers affects auction revenue, price stabilization and investor welfare. We find that increasing the number of dealers from 5 to 50 raises expected excess revenue by 15 bps for the baseline calibration and lowers volatility by 4 bps. Thus, for the baseline calibration, the costs of higher monopsony power of dealers outweigh the benefits of information aggregation (which decrease as investors bid through more dealers).

The fourth feature of Treasury auctions is that they are mixed auctions, meaning that investors have the option to bid through a dealer or directly, without any intermediary. While direct bidding has been historically allowed, since 1992, electronic bidding systems and the elimination of deposit requirements for all bidders have facilitated direct bids. Direct bidding allows investors more options, but also suppresses dealers’ ability to aggregate and share information. Section 4 considers a large bidder choosing whether to bid directly or indirectly. The results uncover two valuable lessons. The first lesson is about the optimal number of dealers. When there are few dealers who each aggregate the information of many clients, dealers provide lots of information, making the benefit of indirect bidding high. But the cost of indirect bidding is also high because the dealer reveals the investor’s private signal to many other investors, reducing its value. Our results show that the latter effect dominates. This teaches us that an attempt to reduce the number of dealers to improve information aggregate may be misguided. With too few dealers, demand for dealer-intermediated trade might decline.

The second lesson from the direct bidding model is a new way in which intermediation can amplify negative shocks to asset values. This new financial accelerator arises because bad news gets shared with other investors but good news does not. When an investor sees a good signal, he expects to take a large position in treasuries, which will make his utility quite sensitive to the auction-clearing price. Sharing his good news with others will increase this price and be detrimental to his utility. So, he keeps his information private by bidding directly. But when the news is bad, the investor expects to take a small position
in treasury, which makes his less sensitive to its price. This investor doesn’t mind sharing his signal with others and benefits from the option value that comes with learning new information from other investors. Thus, the low-signal investor bids indirectly through the dealer. When negative signals are shared, they affect the trades of many investors and their price impact is amplified.

Finally, the fifth feature of Treasury auctions are the minimum bidding requirements for primary dealers. Following nearly two decades of less stringent requirements, new operating policies instated in 2010 require primary dealers to bid for a fixed share of Treasury issues, at a price close to the when-issued price. A dealer that consistently fails to bid for a large enough quantity at a high enough price could lose his primary dealer status. Section 5 incorporates such a requirement in the model by including a shadow cost for spending (realized price times quantity) that is too low. By encouraging aggressive bidding, this policy helps to counteract the monopsony power distortion of primary dealers. We show that, while increased minimum bidding raises the expected revenue at auction, it also increases the volatility of auction revenue. Since bidding requirements decrease the monopsony power of primary dealers, dealers act more as price-takers increasing auction volatility. The presence of bidding requirements also impacts the decision of investors to participate directly, with investors more willing to bid through the dealer as bidding requirements increase. Intuitively, the bidding requirements remove one of the costs – the ability of dealers to better exercise monopsony power – associated with bidding through dealers.

Each version of the model adds a new ingredient that reflects a real feature of Treasury auctions. But within each of these models, there is a common tension at the core. The key trade-off is between the dealer’s ability to use market power and their ability to disseminate information in a way that reduces risk and increases auction revenue. This raises the question of whether it would be possible to achieve higher revenue from improved information without the adverse effects of monopsony power. Our model embodies the idea that intermediation and information aggregation are inextricably interlinked: it is the process of intermediating trades that reveals information to dealers. This is the reality that Treasury auction design must acknowledge.

Overview of treasury auction literature. Much of the literature on sovereign auctions studies how the auction format affects revenues. Theoretical work by Chari and Weber (1992), Bikhchandani and Huang (1989), Back and Zender (1993), and Wilson (1979) considers the merits of uniform-price auctions versus other possible alternatives. Empirical work by Nyborg and Sundaresan (1996), Malvey et al. (1995) and Malvey and Archibald (1998) uses aggregate data from 1992-1998 when the U.S. Treasury used both uniform and
discriminatory price auctions to assess the revenue consequences of the auction format. Armantier and Sbaï (2006) and Hortaçsu and McAdams (2010) were able to use micro data on the bids of participants in French and Turkish Treasury auctions to structurally estimate their auction models and quantify the benefits of uniform price auctions. In contrast, our project fixes the auction format to a uniform-price menu auction. Instead, we focus on the effect of intermediation prior to participation in the auction. Specifically, we examine the effect of bidding through primary dealers.

Additional studies have considered how economic conditions affect bidding behavior in non-U.S. markets. Nyborg et al. (2002) find that Swedish Treasury bidders vary their demand schedules in response to uncertainty shocks. They further find evidence that, in this multiple price auction, the bidding behavior is consistent with auction models featuring private information, winner’s curse, and champion’s plague. Using bidding data from the Finnish Treasury market, Keloharju et al. (2005) find strong effects of uncertainty, but little evidence of market power. Our analysis builds on these findings by looking at how uncertainty interacts with the presence of primary dealers. Some of the few studies to use data from United States Treasury auctions support our key model assumptions. Coutinho (2013) find that bids forecast the future Treasury price in the when-issued market, and participation in the when-issued market is affected by expectations of the auction outcome. Our model reflects this by modeling Treasury auction participants as setting bids according to their expectations of the future value of the Treasury bill. Other findings support the idea that information acquisition of investors and dealers and quasi-monopsony rent extraction are important elements of auction outcomes. Academics (e.g Lou et al., 2013) and industry analysts find some supporting evidence for these phenomena in “auction concessions”—the fact that Treasury prices decrease into an auction and recover afterwards—implying that primary issuance may be expensive.

1 Treasury Auctions and Primary Dealers

This section describes the workings of Treasury auctions and the role of primary dealers and how we map these features into model assumptions. We then describe the policy changes and debates surrounding the number of dealers, the ability of investors to bid directly, and the requirements placed on dealers to make sufficiently large bids in auctions. These correspond to the three dimensions of policy that we use to model to analyze.

4 “Champion’s plague” is the multi-unit auction version of the winner’s curse, proposed by Ausubel (2004), whereby the more units that a bidder wins, the worse news it is for him.
**Treasury auctions**  In 2013 alone, Treasury issued nearly $8 trillion direct obligations in the form of marketable debt as bills, notes, bonds and inflation protected securities (TIPS), in about 270 separate auctions.\(^5\) The Uniform Offering Circular summarizes all auction rules. An auction begins with an announcement (public notice of sale), one day to one week ahead of the auction. The announcement specifies CUSIPs (an alphanumeric code uniquely identifying an asset), offering amount, issue and maturity dates.

There are two types of bids: Competitive bids specify a quantity and a rate (a discount rate for bills, a nominal yield for notes/bonds, or a real yield for inflation protected securities). Non-competitive bids specify a total amount (value) to purchase at the market-clearing rate. Each bidder can only place a single non-competitive bid with a maximum size of $5 million (with some exceptions).\(^6\) Bids can be direct or indirect. To place a direct bid, investors submit electronic bids to Treasury’s Department of the Public Debt or the Federal Reserve Bank of New York. Indirect bids are placed by a depository institution (banks that accept demand deposits), or dealer on behalf of their clients.

On the auction day, bids are received prior to the auction close, which is typically at 1pm (12pm) for competitive (non-competitive) bids. The auction clears at a uniform price, which is determined by first accepting all non-competitive bids, and then competitive bids in ascending yield or discount rate order.\(^7\) Securities purchased at the auction must be paid on the issue date, when the securities are delivered, generally around 9:15am.

The type of uncertainty faced by Treasury bidders is different from the risks faced by corporate bond investors. Because sovereign secondary markets are deep and liquid, treasury investors can hedge issuer-specific risks by shorting already-issued securities. Newly issued government securities are, however, only imperfect substituted for the outstanding ones because of differential liquidity (Lou et al. (2013), Amihud and Mendelson (1991) and Krishnamurthy (2002)). Investors’ demand for specific issues is the key determinant of this liquidity, and so the key underwriting risks are issue-specific rather than issuer-specific.

Our model captures the idea that main risk comes from the unknown current and future

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\(^5\)Treasury bills are auctioned at a discount from par, do not carry a coupon and have terms of not more than one year. Bonds and notes, instead, pay interest in the form of semi-annual coupons. The maturity of notes range between 1 and 10 years, while the term of bonds is above 10 year. For TIPS, the coupon is applied to an inflation-adjusted principal, which also determines the maturity redeemable principal. TIPS maturities range between 1 and 30 years.

\(^6\)Foreign and international monetary authorities (FIMA) that have accounts a the NY Fed can place bids up to $100 million per account and $1billion in total.

\(^7\)The rate at the auction (or stop-out rate) is then equal to the interest rate that produces the price closest to, but not above, par when evaluated at the highest accepted discount rate or yield at which bids were accepted. The maximum auction award to a single bidder at a given rate is 35 percent of the offering, less the bidder’s reportable net long position in the security. Competitive bidders can place multiple bids at different rates without an overall upper bound. This cap is meant to prevent a single bidder from acquiring a disproportionate amount of securities in the event of a proration, which is when, multiple investors place bids at the stop-out rate.
demands of other investors. Therefore, each investor will know their own issue-specific
demand for this set of treasuries, which is a noisy signal of the aggregate future demand.
Taken together, all those dispersed signals are informative about the future value of the
asset.

Next, we turn to examining the policy changes and policy debates surrounding the three
features of Treasury auctions that we analyze in the model.

The function and number of primary dealers  Primary dealers are the firms, commercial
bank dealer departments or brokers and dealers not associated to banking organiza-
tions, with which the Federal Reserve conducts its open market operations. Figure
1 illustrates that primary dealers, bidding for their own account, are the largest bidder
category at auctions (57 percent of allotted securities). Indirect bidders are the second
largest at 32 percent. They may bid through depository institutions or other brokers or
dealers, but primarily they bid through primary dealers.\footnote{Brokers and dealers include all institutions registered according to Section 15C(a)(1) of the Securities Exchange Act. Indirect bidders also include foreign and international monetary authorities placing competitive bids through the New York Fed. These bids are not parsed out in the auction results and we attempt to estimate their size in the model calibration.} The Federal Reserve Bank of New York (NY Fed) selects primary dealers. Primary dealers are not unique to U.S. but are prevalent in most OECD countries.

In 1960, there were 18 primary dealers. The number of dealers then grew rapidly in the
Figure 2: Number of primary dealers

Note: Data are of the last of the month. Source: Federal Reserve Bank of New York

1970s reaching 37 by the end of that decade (Figure 2). Amid the rapid rise in Federal debt and the sharp increase in interest rate volatility, the number of primary dealers rose further to 46 in the mid-1980s. Regulations and requirements for the primary dealer system remained unchanged from 1997 through 2010, but the number of primary dealers declined reaching its nadir in 2009. The number of primary dealers has since then increased to about 22. The small number of primary dealers makes the U.S. Treasury primary market an imperfectly competitive one Bikhchandani and Huang (1993). The market power of primary dealers is a central feature of our model.

**Direct bidding** Following auction violations between 1990 and 1991, the Joint Report on the Government Securities (Brady et al., 1992) recommended broadening the primary dealer system and increasing auction participation. For the first time, all bidders were permitted to bid in note and bond auctions without deposits, provided that they had an automated payment agreement with a bank. Non-primary registered government broker dealers were also allowed to post bids for their customers.

Direct bidding (through the electronic system TAAPS) has grown from 2 percent of all bids in 2008 to 12 percent in 2014. While auction results do not disclose the number of direct bidders, public remarks of Treasury officials suggest about 1200 direct bidders in 2001, and 825 in 2004. Non-competitive bidders account for only 2 percent of all bids, and include those placed over Treasury Direct by retail investors, as well as indirect non-competitive bids placed by foreign and international monetary authorities (FIMA) that hold securities.
in custody at the NY Fed.

While past policy discussions either considered eliminating indirect bidding (e.g. post-1992, to limit information concentration of primary dealers) or instituting bidding requirements for direct bidders, large bidders in the U.S. currently have the option to bid either way in the amounts they need.

**Minimum bidding requirements** In order to maintain their status, primary dealers have to fulfill a set of obligations, which include: providing the NY Fed trading desk with useful market information, disclosing their treasury trading activity for statistical reporting, maintaining minimum capitalization, and being an active NY Fed counterparty. Prior to 1992, being an active counterparty meant being a “consistent and meaningful participant” in Treasury auctions by submitting bids roughly commensurate with the dealer’s capacity at every auction, and a minimum percentage of the total being offered. In 1997, the NY Fed instituted an explicit counterparty performance scorecard and dealers were evaluated based on the volume of allotted securities. Primary dealers are now expected to bid at all auctions an amount equal to the pro-rata share of the offered amount (in terms of dealer count), with bids that are “reasonable” compared to the market.

2 The Effect of a Single Primary Dealer

Our initial model is designed to illustrate two competing effects: 1) dealers can reduce auction revenue by using market power to keep Treasury prices low; and 2) dealers reduce uncertainty by pooling information from their own and their customers’ demands. Lower uncertainty raises the price that risk-averse investors are willing to bid. Such information pooling increases revenue, particularly in high-uncertainty times when demand might otherwise collapse.

2.1 A Primary Dealer Model

The model has one period, one risky asset (the Treasury security whose issue-specific value is random) and a riskless storage technology with zero net return. The value of the asset is unknown and normally distributed: $f \sim N(\mu, \tau_f^{-1})$. There is a single intermediary. The Treasury auctions off a fixed number of shares (normalized to 1) in a uniform-price auction

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9See (See Appendix E in Brady, Breeden, and Greenspan, 1992) for pre-1992 policies. In 2010 the NY Fed clarified their primary dealer operating policies. (See New York Fed website for the most recent rules.
where only dealers can bid directly (we relax this assumption later). The market-clearing price is \( p \). The intermediary receives orders from investors and trades on their behalf.

There are four types of investors in the economy: small limit-order (price-contingent bids) investors, indexed by \( i = 1, \ldots, N \), potential dealers, indexed by \( d = 1, \ldots, D \), a large limit-order investor with the potential to bid directly, indexed by \( L \), and investors who submit market orders (non-price-contingent bids). In practice, these non-price contingent bids typically come from foreign central banks who need to roll over expiring debt or accumulate liquid assets for exchange rate management. All investors have initial wealth \( W_0 \). The net quantity of market orders is \( x \sim N(\bar{x}, \tau_x^{-1}) \). A limit-order investor (hereafter investor) \( i \) chooses the quantity of the asset to hold, \( q_i \) (which can be negative) at price \( p \) per share, in order to maximize his expected utility, \( E[-\exp(-\rho W_i)] \). The budget constraint dictates that final wealth is

\[
W_i = (W_0 - q_i p) r + q_i f
\]

Before trading, each investor gets a signal about the payoff of the asset. These signals are unbiased, normally distributed and have private noise:

\[
s_i = f + \varepsilon_i,
\]

where \( \varepsilon_i \sim N(0, \tau_{\varepsilon}^{-1}) \).

Each potential dealer has initial wealth \( W_{0,D} \). A dealer \( d \) chooses the quantity of the asset to hold, \( q_d \), at price \( p \) per share in order to maximize his expected utility, \( E[-\exp(-\rho_D W_d)] \), subject to the market clearing. Similar to the small limit-order investors, each dealer gets an unbiased, normally distributed signal about the payoff of the asset:

\[
s_d^D = f + \eta_d,
\]

where \( \eta_d \sim N(0, \tau_d^{-1}) \). The dealer’s utility function and budget constraint are the same as investors’, although he may have a different coefficient of risk aversion. The key difference between dealers and investors is that dealers internalize the effect they have on market prices.

Similarly to the dealers, the large investor also internalizes the effect he has on market prices. The large investor observes an unbiased, normally distributed signal about the payoff of the asset with private noise:

\[
s_L = f + \varepsilon_L,
\]

where \( \varepsilon_L \sim N(0, \tau_L^{-1}) \). After observing the private signal, the large investor chooses the
quantity of the asset to hold, \( q_L \), at price \( p \) per share to maximize his expected utility, 
\[ \mathbb{E} \left[ - \exp \left( - \rho L W_L \right) \right] , \]
subject to the market clearing.

The intermediary observes the demand of all limit-order investors and potential dealers, and bids on their behalf.

Describing Information Sets and Updating Beliefs with Correlated Signals  Investors observe three types of information. They observe their private signals \( s_i \). In addition, primary dealers (intermediaries) observe the bids of all investors who bid with them and disclose the average order to their investors. Since bids will turn out to be linear functions of beliefs, revealing average bids and revealing average signals is equivalent. Thus, investment through a primary dealer allows investors to observe an average of other investors’ signals \( \bar{s} \). The final piece of information is the auction-clearing price \( p \). Of course, the agent does not know this price at the time he bids. However, the agent conditions his bid \( q(p) \) on the realized auction price \( p \). Thus, each quantity \( q \) demanded at each price \( p \) conditions on the information that would be conveyed if \( p \) were the realized price. Since \( p \) contains information about the signals that other investors received, the investor uses a signal derived from \( p \) to form his posterior beliefs about the asset payoff. We conjecture, and later verify, that the price is a linear function of the average investors’ signal \( \bar{s} \) and \( x \):

\[ p = A + B \bar{s} + C x. \]  

It follows that \( \frac{p-A-C\bar{s}}{B} \) provides an unbiased signal about \( f \): 
\[ \frac{p-A-C\bar{s}}{B} \sim N(f, 1/N \tau^{-1}_e + \tau^{-1}_p) \]
where \( \tau_p = (B/C)^2 \tau_x \).

For every agent, we use Bayes’ law to update beliefs about \( f \). But because the signals agents observe have correlated signal errors, we need to use a procedure that adjusts for this correlation. The following general information structure is one we can use to solve all of the versions of the model that follow in the paper.

Let \( S \) be the vector of all signals (including price signals) available to any agent. Let \( Z \) be the vector of shocks. For this model, the signals are the individual signals of each investor, an average of all investors’ signals, revealed by the dealer, and the information gleaned from the market-clearing price that the investor conditions on when forming his bid \( q(p) \) for each price realization \( p \).

\[
S = \begin{bmatrix} s_1 & \ldots & s_N & s_L & s^D_1 & \ldots & s^D_d & \bar{s} & \frac{p-A-C\bar{s}}{B} \end{bmatrix}'
\]
\[
Z = \begin{bmatrix} \varepsilon_1 & \ldots & \varepsilon_N & \varepsilon_L & \eta_1 & \ldots & \eta_D & x \end{bmatrix}'.
\]
where
\[ \bar{s} \equiv (\tau_L + N\tau_e + D\tau_d)^{-1} \left( \tau_L s_L + \tau_e \sum_{i=1}^{N} s_i + \tau_d \sum_{d=1}^{D} s_d^D \right) = \omega_L \tau_L + \omega_e \sum_{i=1}^{N} s_i + \omega_d \sum_{d=1}^{D} s_d^D. \]

Then, we can represent the vector of signals as
\[ S = 1_{N+D+3} f + \Pi Z, \]
where \( 1_{N+D+3} \) is an \((N + D + 3) \times 1\) vector of ones and the \((N + D + 3) \times (N + D + 2)\) matrix \( \Pi \) is given by
\[
\Pi = \begin{bmatrix}
I_N & 0 & 0 & 0 \\
0 & 1 & 0 & 0 \\
0 & 0 & I_D & 0 \\
\omega_e \cdot 1_N' & \omega_L & \omega_d \cdot 1_D' & 0 \\
\omega_e \cdot 1_N' & \omega_L & \omega_d \cdot 1_D' & C_B \\
\end{bmatrix}.
\]
where \( I_N \) denotes the \(N \times N\) identity matrix. The matrix \( \Pi \) tells us how signals weight the orthogonal shocks. Each private signal \( s_i \) has a weight of one on its own signal noise. The price signal \((p - A)/B\) is the average of all the \(N + D + 1\) signals, plus supply noise \(x\). Thus, its weight on each individual signal noise is \( \omega_e \), on large investor’s noise \( \omega_L \), and \( \omega_d \) on each dealer’s signal noise. Of course, some of these signals are redundant. But our filtering algorithm will put zero weight on signals that provide no additional information.

Each agent in the economy observes a subset of these signals. Let \( X_j \) be an operator that selects the subset of all signals observed by agent \( j \). (See appendix A for definition of this operator.) In particular, this first version of the model has the following information structure.

1. Individual investors observe their own private signal. In addition, the primary dealer who intermediates their trade tells them the average of all clients’ orders. This reveals the average of all investors signals. It is equivalent to knowing the signal of every other investor.\(^{10}\) Thus, investor \( i \) observes \( s_i, \bar{s}, (p - A - C\bar{x})/B \) is the vector of \( i \)'s signals.

2. Each dealer observes their own private signal, as well as the average signal, and conditions on the price information. Thus, \( X_d S = [s_d, \bar{s}, (p - A - C\bar{x})/B] \) is a vector

\(^{10}\)Of course, specifically revealing trades or information acquired from any client to another client would be illegal. But dealers often discuss general buying trends with clients. In this model, revealing the trend is informationally equivalent to revealing each signal because the investor would just average the individual signals himself. In a model where some agents know more than others, this might no longer be true.
of all the signals known to dealer $d$ at the time when he invests.

3. The large investor observes his own private signal and the average signal, and conditions on the price information. Thus, $X_L S = [s_L, \bar{s}, (p - A - C\bar{x})/B]$ is a vector of all the signals known to the investor at the time when he invests.

We can now solve for the beliefs of any agent $j$ (investor or dealer) who observes the vector of signals $X_j S$. Since all the uncertain quantities are normally distributed, we have the following optimal linear projections

$$
E[f|X_j S] = (1 - \beta'1_m)\mu + \beta'X_j S \quad \text{where}
$$

$$
\beta \equiv \mathbb{V}(X_j S)^{-1} \mathbf{Cov}(f, X_j S) \quad (3)
$$

$$
\mathbb{V}[f|X_j S] = \mathbb{V}(f) - \mathbf{Cov}(f, X_j S)' \mathbb{V}(X_j S)^{-1} \mathbf{Cov}(f, X_j S) \equiv \tau_j^{-1}, \quad (4)
$$

where the covariance matrix is

$$
\mathbf{Cov}(f, X_j S) = \mathbf{Cov}(f, X_j (1_{N+2} f + \Pi Z))
$$

$$
= X_j (\mathbb{E}[1_{N+2} f^2 + \Pi Z f] - \mathbb{E}[f] \mathbb{E}[1_{N+2} f + \Pi Z]) = 1_m \tau_j^{-1} \quad (5)
$$

$$
\mathbb{V}(X_j S) = X_j \mathbb{V}(1_{N+2} f + \Pi Z) X_j' = X_j (\tau_j^{-1} 1_{N+2} 1_{N+2}' + \Pi \mathbb{V}[Z] \Pi') X_j', \quad (6)
$$

and the variance matrix of the shocks is $\mathbb{V}[Z] = \text{diag}([\tau^{-1}_e, \tau^{-1}_L, \tau^{-1}_D, \tau^{-1}_x])$.

Although this matrix information structure may seem cumbersome for the simple problem at hand, it allows us to examine various forms of the model with minimal changes to the setup. The different versions of the model we explore in the following section will change two things about the model: The set of signals, summarized by $\Pi$, and the information sets of each agent, summarized by a set of $X_j$’s.

**Investors’ problem.** We consider a problem where there is a large finite number of investors. The key feature that distinguishes investors from dealers here is that dealers have market power and use it, while investors behave competitively, as if they had no

---

11Note that the following formulas are just like the OLS formulas in a context where all the means, variances and covariances of variables are known. The OLS additive constant $\alpha$ is $(1_N - \beta')1_N \mu$. $\beta$ is the infinite sample version of $(X'X)^{-1}X'g$. The conditional mean here is analogous to the optimal linear estimate in the OLS problem. This equivalence holds because in linear systems, both OLS and Bayesian estimators are consistent. It also has the Bayesian interpretation as a weighted average of normal priors and signals, where each is weighted by their relative precision. Here that precision is adjusted to account for correlation.
impact on the equilibrium price. The problem of investor \( i \) is

\[
\max_{q_i(p)} \mathbb{E}[\exp(-\rho W_i)|X_iS] \\
\text{s.t. } W_i = q_i(f - p).
\]

Since the investor’s posterior beliefs about \( f \) are normally distributed, we can use the properties of a log-normal random variable to evaluate the expectation of the investor’s objective function. It then follows that the FOC of the problem is to bid the following set of price-quantity pairs:

\[
q_i(p) = \frac{1}{\rho} \nabla [\mathbb{E}[f|X_iS]^{-1} (\mathbb{E}[f|X_iS] - p)].
\] (7)

The optimal bid function is the inverse function: \( p(q_i) \).

This is a standard portfolio expression in an exponential-normal portfolio problem. The fact that it is an auction rather than a competitive market doesn’t change choices. The novelty of the model is in modeling the large, strategic players (dealers) and in thinking carefully about how primary dealers affect the information sets that determine the conditional mean and variance of the asset payoff.

**Dealer’s problem.** The dealer’s problem is different because the dealer is a large investor who accounts for the fact that he has price impact and market power.\(^\text{12}\)

\[
\max_{q_d,p} \mathbb{E}[\exp(-\rho_D W_d)|X_dS] \\
\text{s.t. } W_d = W_{0,D} + q_d(f - p), \\
x + \sum_{i=1}^{N} q_i + \sum_{d=1}^{D} q_d + q_L = 1.
\]

The second constraint is the market clearing condition and reflects that the dealer must choose his quantity and the price so that the market clears.

Applying the formula for mean and variance of the dealer’s beliefs about \( f \), we find that the dealer’s mean and variance of beliefs are the same as investors’.

By substituting the equation for \( W_d \) in the dealer’s problem into the objective function,\(^\text{12}\) we use exponential (CARA) utility here for both investors and dealers to keep the problem tractable. Of course, this rules out wealth effects on portfolio choices. At the same time, we want to capture the idea that dealers, with larger balance sheets, hold larger positions of risky assets. Therefore, we assign dealers a larger absolute risk aversion. In other words, we are capturing wealth effects with differences in risk aversion.
evaluating the expectation and taking the log, we can simplify the dealer’s problem to be \( \max_{q_d,p} q_d (E[f|X_dS] - p) - \frac{1}{2} \rho_D q_d^2 \sqrt{V[f|X_dS]} \) s.t. the market clearing condition (10). Taking the first order condition with respect to \( q_d \), we obtain

\[
q_d (p) = \frac{E[f|X_dS] - p}{\rho_D \sqrt{V[f|X_dS]} + dp/dq_d} \equiv M_D (E[f|X_dS] - p).
\]

(11)

The term \( dp/dq_d \) measures the price impact of a dealer’s bid. As the price impact increases, the dealer’s demand becomes less sensitive to his beliefs about the value of the security.

**Large investor’s problem.** The large investor’s problem is similar to that of a dealer

\[
\max_{q_L,p} \quad E[-\exp(-\rho_L W_L)|X_LS] \quad \text{s.t.} \quad W_L = W_{0,L} + q_L (f - p),
\]

(12)

and s.t. the market clearing condition (10). Applying the formula for mean and variance of the large investor’s beliefs about \( f \), we find that the large investor’s mean and variance of beliefs are the same as investors’.

By substituting the equation for \( W_L \) in the investor’s problem into the objective function, evaluating the expectation and taking the log, we can simplify the investor’s problem to be \( \max_{q_L,p} q_L (E[f|X_LS] - p) - \frac{1}{2} \rho_L q_L^2 \sqrt{V[f|X_LS]} \), subject to market clearing (10). Taking the first order condition with respect to \( q_L \), we obtain

\[
q_L (p) = \frac{E[f|X_LS] - p}{\rho_D \sqrt{V[f|X_LS]} + dp/dq_L} \equiv M_L (E[f|X_LS] - p).
\]

(14)

The term \( dp/dq_L \) measures the price impact of the large investor’s bid. As the price impact increases, the investor’s demand becomes less sensitive to his beliefs about the value of the security.

**Model Solution** The belief updating problem in this model becomes very simple because the signal from the primary dealer \( \bar{s} \) contains all the relevant information from the investor’s signal \( s_i \) and the price signal \( (p - A)/B \). The less-informative redundant signals get zero weight in the updating rule. Thus, the conditional expectation collapses to

\[
E[f|X_iS] = \frac{\tau_f \mu + (\tau_L + N \tau_s + D \tau_d) \bar{s}}{\tau_f + \tau_L + N \tau_s + D \tau_d}
\]
and the conditional variance becomes $\mathbb{V}[f|X_iS] = (\tau_f + \tau_L + N\tau_\varepsilon + D\tau_d)^{-1} \equiv \hat{\tau}^{-1}$. These beliefs are the same for all agents. In the model with only one intermediary, there is no information asymmetry.

Thus, the small investors have a demand function (inverse bid function) of $q(p) = 1/\rho(\tau_f \mu + N\tau_\varepsilon \bar{s} - p(\tau_f + N\tau_\varepsilon))$. To solve for a dealer’s demand, we need to solve for the market-clearing price and then differentiate it to get $dp/dq_d$. Substituting investors’ aggregate demand $q(p)$ into the market-clearing condition (10), we get

$$
p = \frac{(-1 + x + q_d)}{(D - 1) M_D + M_L + N\rho^{-1}\hat{\tau}} + \frac{\tau_f \mu + (\tau_L + N\tau_\varepsilon + D\tau_d)\bar{s}}{\hat{\tau}}. \quad (15)
$$

Finally, we differentiate the price function (15) to get $dp/dq_d = \left((D - 1) M_D + M_L + N\rho^{-1}\hat{\tau}\right)^{-1}$.

Substituting this in the first order condition (11) gives us the implicit solution for $M_D$:

$$M_D^{-1} = \rho_D \hat{\tau}^{-1} + \left((D - 1) M_D + M_L + N\rho^{-1}\hat{\tau}\right)^{-1}.
$$

Similarly, differentiating the market clearing price with respect to $q_L$, and substituting into the first order condition (14) gives us the implicit solution for $M_L$:

$$M_L^{-1} = \rho_L \hat{\tau}^{-1} + (DM_D + N\rho^{-1}\hat{\tau})^{-1}.
$$

Next, we substitute the demand of each type of agent into the price function to determine the revenue of the auction. Recall that the quantity auctioned was normalized to 1. So price and revenue are equivalent and equal to:

$$p = \frac{-1 + x}{DM_D + M_L + N\rho^{-1}\hat{\tau}} + \frac{\tau_f \mu + (\tau_L + N\tau_\varepsilon + D\tau_d)\bar{s}}{\hat{\tau}}. \quad (16)
$$

Note that this is a linear function of $\bar{s}$ and $x$. This verifies the linear price conjecture ($p = A + B\bar{s} + Cx$). From this solution, we get the following coefficients in the price function:

$$A = -\frac{1}{DM_D + M_L + N\rho^{-1}\hat{\tau}} + \frac{\tau_f \mu}{\tau_f + \tau_L + D\tau_d + N\tau_\varepsilon}, \quad (17)
$$

$$B = \frac{\tau_f + D\tau_d + N\tau_\varepsilon}{\tau_f + \tau_L + D\tau_d + N\tau_\varepsilon}, \quad (18)
$$

$$C = \frac{1}{DM_D + M_L + N\rho^{-1}\hat{\tau}}. \quad (19)
$$

**Auction Revenue** Since we normalized the supply of the Treasuries to one, price and auction revenue are the same. What we want the model to tell us is what the expected
revenue is, what the variance of that revenue is, and how that mean and variance compare to other primary dealer arrangements. In every auction, the unconditional expected revenue will be $A + B\mu + C\bar{x}$, which is

$$E[p] = \mu - \frac{1 - \bar{x}}{DM_D + ML + N\rho^{-1}r}$$

(20) and unconditional revenue variance will be $B^2\psi[\bar{s}] + C^2\tau^{-1}r_x$.

2.2 A Benchmark: An Auction Without Primary Dealers

To understand the costs and benefits of having primary dealers, we need a benchmark model to compare our primary dealer model to. A natural benchmark is a model with no intermediaries. Instead, each type of limit order investor submits bids on their own behalf. The investors’ information set now contains only their private signal $s_i$ and the price information $(p - A - C\bar{x})/\tilde{B}$, but not the average signal that was previously provided by the intermediary. Conjecture that the equilibrium settle price will have the form

$$p = A_C + \frac{BC}{N} \sum_{i=1}^{N} s_i + F_C s_L + \frac{EC}{D} \sum_{d=1}^{D} s_d + C C x,$$

(21) with $\tilde{B} = B_C + F_C + E_C$. The set of possible signals, as described by the matrix $\Pi$ will remain the same, with $\omega_e = B_C / \left(N \cdot \tilde{B}\right)$, $\omega_L = F_C / \tilde{B}$, $\omega_d = E_C / \left(D \cdot \tilde{B}\right)$, but the vector of signals observed by investor $i$ is now $X^C_i S = [s_i, (p - A - C\bar{x})/B]$. This is the same as the signal selection matrix in the previous problem, except without the middle row which put the average of all investors’ signals in the information set. This matrix gives the investor only his own private signal and the price information. Similarly, the signals observed by a dealer $d$ and the large investor $L$ are $X^C_d S = [s_d, (p - A - C\bar{x})/B]$ and $X^C_L S = [s_L, (p - A - C\bar{x})/B]$. All investors and dealers observe the price level. But in contrast to the previous case, investors will now learn any new information from the price level.

Result 1. Without primary dealers, the auction revenue is $p = A_C + \frac{BC}{N} \sum_{i=1}^{N} s_i + F_C s_L +$
\[
\frac{E^C}{D} \sum_{d=1}^{D} s_d^D + C^C x, \text{ where}
\]
\[
A^C = -C^C \left(1 + \left( N_\hat{\tau} \beta_I (2) + M^C_L \beta_L (2) + DM^C_D \beta_D (2) \right) \frac{A + C\bar{x}}{B}\right)
\]
\[
+ C^C \left( N_\hat{\tau} \left(1 - \beta_I (1) - \beta_I (2)\right) + M^C_L \left(1 - \beta_L (1) - \beta_L (2)\right) + DM^C_D \left(1 - \beta_D (1) - \beta_D (2)\right) \right) \mu
\]
\[
B^C = C^C N_\rho^{-1} \hat{\tau} \beta_I (1),
\]
\[
F^C = C^C M^C_L \beta_L (1)
\]
\[
E^C = C^C DM^C_D \beta_D (1)
\]
\[
C^C = -\hat{B} \left[ N_\rho^{-1} \hat{\tau} \left( \beta_I (2) - \hat{B} \right) + M_L \left( \beta_L (2) - \hat{B} \right) + DM^C_D \left( \beta_D (2) - \hat{B} \right) \right]^{-1},
\]

and the elasticity of the dealer’s demand and the large investor’s demand to information are given, respectively, by
\[
(M^C_D)^{-1} = \rho_D \hat{\tau}^{-1}_D - \hat{B} \left[ N_\rho^{-1} \hat{\tau} \left( \beta_I (2) - \hat{B} \right) + M_L \left( \beta_L (2) - \hat{B} \right) + (D - 1) M^C_D \left( \beta_D (2) - \hat{B} \right) \right]^{-1}
\]
\[
(M^C_L)^{-1} = \rho_L \hat{\tau}^{-1}_L - \hat{B} \left[ N_\rho^{-1} \hat{\tau} \left( \beta_I (2) - \hat{B} \right) + DM^C_D \left( \beta_D (2) - \hat{B} \right) \right]^{-1}.
\]

The result yields an implicit solutions for the price coefficients. Normally, a simple competitive market model like this has simple closed form solutions for prices. But the complication in this model is that the average of investors’ signals \( \bar{s} \) is not equal to the true payoff. The presence of public and private signal noise adds the extra layer of complexity to the solution. But that extra complexity is easy to resolve numerically and it is essential in the following model to understand how the number of dealers affects information aggregation and the risk of failed auctions.

### 2.3 Calibration and Data

We calibrate model parameters using data from two main data sources: Treasury auction results and market prices. The target moments are shown in Table 1. Our sample starts in June 2009 and ends in June 2014.\(^{13}\) To study a comparable sample and estimate yield curves, we restrict attention to 2-, 3-, 5-, 7- and 10-year notes and exclude bills, bonds and TIPS.

For each maturity we compute the mean share of securities allotted to primary dealers, direct and indirect bidders. As discussed above, the definition of indirect bidders from

\(^{13}\)We begin the sample in June 2009 because of a shift in the definition of indirect bids at that time. A rule change reclassified some bids, “guaranteed bids,” previously reported as direct by primary dealers, as indirect bids. Note that we exclude amounts allotted to the Fed’s own portfolio, the System Open Market Account or “SOMA,” which are an add-on to the auction.
official auction results includes FIMA competitive bids placed through the NY Fed. Because these bids do not provide information to primary dealers, we attempt to abstract from them using a simple imputation. We reclassify these bids as part of the noise trader group, as they do not reflect the short-term issue-specific value (more on this in the next paragraph) because foreign official investors have long investment horizons investing reflecting foreign exchange strategies.

To calibrate the auction price and fundamental value we first note that, up to rounding, the auction price clears at par. The stop-out coupon rate is, instead, a function of issue-specific value as well as the term structure of interest rates at the time of the auction, which depends on factors unrelated to the auction, such as monetary policy and inflation expectations. We focus on issue-specific fundamentals, or the “on-the-run” value of the issue, for two reasons. First, an investor can easily hedge interest rate risk into the auction by shorting a portfolio of currently outstanding securities. Second, from the issuer perspective, the stop-out rate could be very low because of low interest rates, but an issue could still be “expensive” relative to the rate environment due to auction features, which is what we are after. To strip out the aggregate interest-rate effects, we assume that the bidder enters the auction with an interest-rate-neutral portfolio, which holds one unit of the auctioned security and shorts a replicating portfolio of bonds trading in the secondary market. This portfolio is equivalent to the excess revenue on the current issue, relative to outstanding securities. Thus, price $p$ in our model corresponds to the auction price, minus the present value of the security’s cash-flows, where future cash flows are discounted using a yield curve. The fundamental value $f$ in the model corresponds to the value of the interest-rate neutral portfolio on the date when the security is delivered to the winning bidders (close of issue date). The issue date in our sample lags the auction date by an average of 5.5 days with a standard deviation of about 2.3 days. For example, in 1, the average revenue from selling a new 10-year note is 37.18 basis points higher than the replicating portfolio formed using outstanding securities. Thus, we calibrate the model to have this average asset payoff. This excess revenue is positive across all maturities. This is the well-known “on-the-run” premium.

We fit the parameters of the full model (the model of Section 5) separately for each maturity to account for potential differences in duration and/or liquidity of the different issues. The

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14 From Treasury International Capital (TIC), as of August 2014, about $6 trillion of securities are held by foreign investors, while from the Fed Board’s H.4.1 release, foreign and international monetary accounts (FIMA) at the NY Fed are about $3.4 trillion as of that time. Assuming that the portfolio composition and bidding strategy of FIMA and non-FIMA are similar then an estimate of FIMA’s share of competitive bids reported as indirect ones is: 3.3/6 X all foreign bids (from investment allotment) less FIMA non-competitive bids that are reported separately.

15 We estimate a Svensson yield curve following the implementation details of Gürkaynak, Sack, and Wright (2007) but using intraday price data as of 1pm, which is when the auction closes (data from Thomson Reuters TickHistory).
Table 1: Calibration targets and model-implied values

Note: Prices and excess revenues are all expressed in basis points, while shares are in fractions.

<table>
<thead>
<tr>
<th></th>
<th>Data</th>
<th>Model</th>
</tr>
</thead>
<tbody>
<tr>
<td>A</td>
<td>-17.01</td>
<td>-19.23</td>
</tr>
<tr>
<td>B + F</td>
<td>0.97</td>
<td>0.89</td>
</tr>
<tr>
<td>C</td>
<td>124.38</td>
<td>86.68</td>
</tr>
<tr>
<td>Error Std. Dev.</td>
<td>29.72</td>
<td>53.90</td>
</tr>
<tr>
<td>Expected excess revenue</td>
<td>37.18</td>
<td>27.92</td>
</tr>
<tr>
<td>Volatility of excess revenue</td>
<td>72.64</td>
<td>85.09</td>
</tr>
<tr>
<td>Indirect share</td>
<td>0.25</td>
<td>0.15</td>
</tr>
<tr>
<td>Dealer share</td>
<td>0.53</td>
<td>0.44</td>
</tr>
<tr>
<td>Direct share</td>
<td>0.10</td>
<td>0.08</td>
</tr>
</tbody>
</table>

The objective function matches a few moments from the model to their empirical counterparts: the pricing coefficients $A$, $B$ and $C$ in equation (1), the mean and variance of the price of the interest-rate-risk-neutral portfolio $p$ at auction, the mean and variance of the price of the portfolio $f$ at issuance, the mean allotted share and variance of non-competitive bids $x$ (including the FIMA trades, or market orders), the mean allotted share to primary dealers, $\sum_{d=1}^{D} q_d$, the mean allotted share to indirect bidders ($\sum_{i=1}^{N} q_i$), and the mean allotted share to direct bidders, $q_L$. We obtain sample estimates of $A$, $B$ and $C$ by regressing the stop-out-price at each auction on a constant, the end-of-day secondary price on the issue date of the auction security (data from Bloomberg LP) and the non-competitive bids. As shown in Table 1, consistent with the model, excess revenues are positively correlated to the fundamental value on issue date (positive $B + F$), and it also increases with the share of securities allocated to market orders (positive $C$).

Table 2: Calibrated parameters

<table>
<thead>
<tr>
<th></th>
<th>$\mu$</th>
<th>$\tau_{f}^{-\frac{1}{2}}$</th>
<th>$\tau_{e}^{-\frac{1}{2}}$</th>
<th>$\tau_{d}^{-\frac{1}{2}}$</th>
<th>$\tau_{L}^{-\frac{1}{2}}$</th>
<th>$\bar{x}$</th>
<th>$\rho$</th>
<th>$\rho_D$</th>
<th>$\rho_L$</th>
<th>$\chi$</th>
</tr>
</thead>
<tbody>
<tr>
<td></td>
<td>40.77</td>
<td>73.51</td>
<td>540.41</td>
<td>169.77</td>
<td>29.08</td>
<td>0.06</td>
<td>0.12</td>
<td>45421.00</td>
<td>3416.10</td>
<td>1401.70</td>
</tr>
</tbody>
</table>

2.4 Results: Dealers and Auction Revenue

The left panel of Figure 3 plots the auction revenue with a dealer $E[p]$ and without a dealer $E[p^C]$ (line labelled competitive). We see that the expected revenue gain from the presence of a single dealer is $E[p] - E[p^C]$. This revenue differential (distance between the solid and dashed lines) varies from about 12 basis points to nearly 100 basis points. The revenue gain comes from dealer information aggregation and is larger when the fundamental uncertainty about the future asset value ($\tau_f^{-1}$) is large. The reason the dealer increases auction revenue is that he is providing his clients with information. This information makes the Treasury less risky to them. They are willing to bid more for a less risky asset, which increases the auction revenue. This effect is strongest when their uncertainty is the absence of dealer
Figure 3: Dealers increase revenue in times of high uncertainty

(a) Expected auction revenue

(b) Standard deviation of revenue

Information is greatest. That no-dealer level of uncertainty is governed by $\tau_f^{-1}$.

The right panel of Figure 3 plots the conditional variance of auction revenue with a dealer $\text{Var}[p]$ and without a dealer $\text{Var}[p^C]$ (line labelled competitive). We see that dealers increase the variance of auction revenue and that this effect is largest when investors are most uncertain about the future value of Treasuries. The size of the increase in variance is $\text{Var}[p] - \text{Var}[p^C]$, which ranges from 12 - 35 basis points in the figure. This higher variance arises because dealers make investors better informed about the future value of the Treasury. If they had no information about the future value, bidders would always bid the same amount and revenue would be constant. With precise information, bidders condition their bids on the information. When the true future value of the Treasury fluctuates, investors learn this with a high degree of accuracy, their bids adjust to reflect the information and as a result, auction revenue varies.

We also examined how the effect of dealers varies as we change the variance of the non-competitive bids. When these bids are less predictable, auction clearing prices are less clear signals about the true value of the asset. This makes the value of information aggregation greater, which makes dealers more valuable in expected auction revenue terms—but the magnitude of this effect is tiny.

2.5 Dealers as Insurance Against Uncertainty Shocks

In typical models of underwriting, dealers stabilize auctions by reducing revenue variance. In our model, dealers increase the conditional variance of revenue, but they do provide revenue insurance in another way, by providing a hedge against uncertainty shocks.

Figure 3 shows that an auction with a dealer delivers lots more revenue than one with-
out when fundamental uncertainty ($\tau_f^{-1}$) is high. The result comes from the feedback of uncertainty through price information. In the absence of a dealer, when beliefs are uncertain, investors make less aggressive bids, meaning that their demand is less sensitive to the private information they have. However, when bids are less sensitive to private information, the market-clearing price contains less information, relative to the noise of the non-competitive bids. Thus, investors are all the more uncertain because the price does not reveal much information to them either.

The presence of a dealer avoids this problem. By providing investors signals about the average trade, without noise, the investors do not have to rely on price to aggregate information. So, when price become a more noisy signal, it does not affect the investors who trade through dealers because they already observe that average signal from other agents clearly, getting their information directly from the dealer. Since the difference between the price information and the dealer information is greatest when prices have the most noise, and prices have the most noise when uncertainty is high, auction revenue diverges in high-uncertainty times. This ability of dealers to raise revenue in times when it would otherwise be low by improving the information environment and reducing uncertainty is valuable to Treasury auctioneers.

Conditional on high uncertainty, revenue is still less predictable with the dealer than without. What we learn is that dealers make revenue more sensitive to the issue-specific value of the asset, because they aggregate and provide information about that value. But they make the auction revenue less sensitive to changes in uncertainty.

3 Multiple Dealers and Information Aggregation

Increasing the number of primary dealers, who bid on behalf of their clients, disperses information across a larger number of participants. In other words, dealer competition inhibits information-aggregation. On the other hand, a larger number of dealers increases competitive pressures and limits rent extraction. This section studies these trade-offs.

When we move from one to two dealers, there is more than just a quantitative change in the solution. It introduces the effect of learning from prices. When there was one dealer, the dealer aggregated all the signals in the market and there was no more information for the price to reveal. But with more than one dealer, no single dealer knows or reveals all the signals that make up the price. Therefore, there is information to learn from the realized auction-clearing price and agents condition their bids on the information that would be revealed about what others know if that price were realized.
3.1 A multiple dealer model

We now expand the set of intermediaries that can place bids at the auction on behalf of their clients. There are \( \Delta \leq \bar{\Delta} \) intermediaries that participate in the auction. Each intermediary \( k, k = 1, \ldots, \Delta \), receives orders from an equal number of investors and bids on their behalf. Thus, each intermediary observes the orders of \( N/\Delta \equiv \nu \) investors and places bids on their behalf. When we change the number of intermediaries, we want to isolate the information and competition aspects on this change. We do not want to change the total demand for the asset by changing the number or size of market participants. Therefore, we include \( \bar{\Delta} \) strategic investors, with the same preferences and risk aversion as dealers. When we change the number of dealers from \( \Delta \) to \( \Delta + 1 \), we reduce the number of equivalent non-dealer investors from \( \bar{\Delta} - \Delta \) to \( \bar{\Delta} - \Delta - 1 \). This holds fixed the total non-information demand for Treasuries. Similarly to the one dealer case described above, each intermediary \( k \) discloses the average signal \( \bar{s}_k \) of his clients to all his clients.

**Information Sets**  
Clients \( I_d \) of dealer \( d \) observe three types of signals: their own private signal \( s_i \), the average signal received by their dealer \( \bar{s}_d \) and the settle price \( p \). We guess (and later verify) that the market-clearing (settle) price is a linear function of the average signals \( \{\bar{s}_k\}_{k=1}^{\Delta} \) and of the market orders \( x \)

\[
p = A + B\bar{s}_1 + \frac{F}{\bar{\Delta} - 1} \sum_{k=2}^{\bar{\Delta}} \bar{s}_k + Cx, \tag{22}
\]

where

\[
\bar{s}_1 = \frac{\tau_L s_L + \tau_e \sum_{i \in I_1} s_i + \tau_d \sum_{d \in I_1} s_d^D}{\tau_L + \nu \tau_e + \delta \tau_d} \equiv \omega_L s_L + \omega_1 \sum_{i \in I_1} s_i + \omega_d \sum_{d \in I_1} s_d^D
\]

\[
\bar{s}_k = \frac{\tau_e \sum_{i \in I_k} s_i + \tau_d \sum_{d \in I_k} s_d^D}{\nu \tau_e + \delta \tau_d} \equiv \omega_2 \sum_{i \in I_k} s_i + \omega_d \sum_{d \in I_k} s_d^D
\]

Let \( S \) be the vector of all signals available to agents in this economy. The vector of shocks in this case is the same as in the one intermediary case but the signal vector is now expanded to include the dealer-level average signals

\[
S = \begin{bmatrix}
s_1 & \ldots & s_N & \bar{s}_1 & \ldots & \bar{s}_\Delta & \frac{p-A-Cx}{\bar{B}}
\end{bmatrix}',
\]

where \( \bar{B} = B + F \).
This version of the model has the following information structure: Individual investors observe their own private signal $s_i$. In addition, the primary dealer $k$ who intermediates their trade tells them the average of their clients’ orders, revealing $\bar{s}_k$. This reveals the average of their clients’ signals. It is equivalent to knowing the signal of every other investor who trades with that dealer.\(^\text{16}\) Finally, for each bid in his menu of price-quantity pairs, this investor conditions on the information that would be revealed if $p$ were the realized auction price. Thus, investor $i$ who is assigned to intermediary $k$ conditions his beliefs on the signal vector $X_iS = [s_i, \bar{s}_k, (p - A - C\bar{x})/B]$.

Each dealer observes his own private signal $s_d^D$, the average signal of clients of their intermediary $k$ and conditions on the information in the realized auction price. His signal vector is $X_dS = [s_d^D, \bar{s}_k, (p - A - C\bar{x})/B]$.

The large investor observes his own private signal $s_L$, the average signal of the clients of intermediary 1 and conditions on the information in the realized auction price. His signal vector is $X_LS = [s_L, \bar{s}_1, (p - A - C\bar{x})/B]$.

As in the previous Section, $X_jS$ is a vector of all the signals known to agent $j$ at the time when he invests. Of course, some of these signals are redundant. But our filtering algorithm will put zero weight on signals that provide no additional information.

**Bidding schedules.** Conditional on their information set, the bidding problem of an indirect bidder $i$ is given by (7), the optimal bidding schedule of a dealer by (11), and the bidding schedule of the large investor by (11). The effect of the number of dealers on bids and revenue comes through the expectation and variance terms. Conditional on those beliefs, investors bid in the same way regardless of the number of dealers.

**Model solution.** With more than one dealer, the average signal $\bar{s}_k$ observed by intermediary $k$ no longer summarizes the information contained in the equilibrium settle price. Since prices now contain some information that is independent from the dealer’s signal, the belief updating problem in this model is slightly more involved than in the one dealer case. Each investor uses the price to form a second unbiased signal about $f$.

\(^{16}\)Of course, specifically revealing trades or information acquired from any client to another client would be illegal. But dealers often discuss general buying trends with clients. In this model, revealing the trend is informationally equivalent to revealing each signal because the investor would just average the individual signals himself. In a model where some agents know more than others, this might no longer be true.
Figure 4: More dealers reduce the mean and standard deviation of revenue. This effect is larger with high uncertainty (black bars) than with low uncertainty (white bars).

(a) Expected auction revenue
(b) Standard deviation of revenue

Result 2. With $\Delta$ primary dealers, auction revenue is given by (22) where

$$A = C \left[ -1 + \left( \nu \rho^{-1} \hat{\tau}_1 + \delta M_{D,1} + M_L \right) \left( 1 - \beta_1 (1) - \beta_1 (2) \right) \mu - \frac{\beta_1 (2)}{B} (A + C \bar{x}) \right]$$

$$+ C (\Delta - 1) \left( \nu \rho^{-1} \hat{\tau}_2 + \delta M_{D,2} \right) \left( 1 - \beta_2 (1) - \beta_2 (2) \right) \mu - \frac{\beta_2 (2)}{B} (A + C \bar{x})$$

$$B_1 = C \left( \nu \rho^{-1} \hat{\tau}_1 + \delta M_{D,1} + M_L \right) \beta_1 (1)$$

$$B_2 = (\Delta - 1) C \left( \nu \rho^{-1} \hat{\tau}_2 + \delta M_{D,2} \right) \beta_2 (1)$$

$$C = -\hat{B} \left[ \left( \nu \rho^{-1} \hat{\tau}_1 + \delta M_{D,1} + M_L \right) \left( \beta_1 (2) - \hat{B} \right) + (\Delta - 1) \left( \nu \rho^{-1} \hat{\tau}_2 + \delta M_{D,2} \right) \left( \beta_2 (2) - \hat{B} \right) \right]^{-1}.$$ 

3.2 Results: The Effect of Multiple Dealers

The moments of interest are the mean and variance of auction revenue. Recall that since the quantity of Treasury securities sold is normalized to 1, the price and revenue are identical. Therefore, in the plots that follow, we report the expected price and the variance of that price, varying one exogenous parameter at a time. In each exercise, all parameters other than the one being varied are held at their calibrated values.

Adding dealers reduces the auction revenue (Figure 4). An auction with 20 dealers earns about 10% less on-the-run premium than as an auction with 1 dealer in the baseline calibration. When prior uncertainty about the future value of the asset is high (precision $\tau_f$ is low), the reduction in excess revenue is closer to 30%.
There are two effects working in opposite directions here. First, introducing more dealers increases competition and reduces the market power of each dealer. That increases revenue. Second, more dealers disperse information. With two dealers, each dealer sees half of all the signals. With three dealers, each observes a third. Each time a new dealer is added, all dealers have less information. Since dealers disclose their information to their clients, all investors have less precise information sets, and are therefore more uncertain about the asset value, when the number of dealers rises. Because they are risk averse, more uncertain investors reduce their bids and reduce average auction revenue.

The standard deviation of auction revenue declines in the number of dealers (Figure 4 right panel) because dealers are segmenting information sets. When information about the true value of the asset is less precise, investors’ bids respond less to changes in that true value. Since investors’ bids are less sensitive to a random variable, they are also less variable themselves and create less variance in revenue. That effect shows up as a lower standard deviation of auction revenue from increasing the number of dealers.

4 Mixed Auctions: Choosing Direct or Indirect Bidding

Another distinguishing feature of U.S. Treasury auctions is that they are mixed, meaning that despite the presence of primary dealers, direct bidding by non-dealers is also allowed. This choice to trade with or without an intermediary affects auction revenue and lowers the revenue-maximizing number of dealers and amplifies the effect of low signals on auction revenue. To explore these effects in a simple setting, we now allow one investor to choose between bidding directly or indirectly through an intermediary.

Information structure and the intermediation choice The large investor’s choice to bid directly or indirectly (through an intermediary) only affects the information structure of the large investor, the dealer and all the investors who bid with the same dealer. By affecting the information sets of all these investors, the choice to bid directly or indirectly affects the information content of the price $s_p$ as well.

When the large investor chooses to bid directly on his own behalf, the first dealer’s signal is the average of the first $\nu$ investors’ and the first $\delta$ dealers’ signals: $\bar{s}_1 = \omega_{\nu,2} \sum_{i=1}^{\nu} s_i + \omega_{\delta,2} \sum_{d=1}^{\delta} s^D_d$. As in the previous model, investor $i$ who bids through intermediary $k$ observes signals $X_i S = [s_i, \bar{s}_k, s_p]$ and dealer $d$ observes $X_d S = [s^D_d, \bar{s}_k, s_p]$. The large investor observes only his own signal and the price information: $X_L S = [s_L, s_p]$. 

27
**Solution: Auction Outcomes** If the large investor bids indirectly, the problem and the solution are the same as in the previous section. Therefore, we solve now for the case with direct bidding. The following result shows that the auction price is a linear function of the dealer-level average individual investor signals, \( \bar{s}_d \), the signal of the large investor, \( s_L \), and of market orders \( x \). Since the supply of the asset is 1, the price and the auction revenue are the same.

**Result 3.** With \( \Delta \) dealers and 1 large investor who bids directly, the auction revenue is

\[
p = A + \frac{B}{D} \sum_{d=1}^{D} \bar{s}_d + Cx + Fs_L \quad \text{where} \quad (27)
\]

\[
A = -C \left( 1 - (N \rho^{-1} \hat{\tau}_D + DM_D) \left[ (1 - \beta_D (1) - \beta_D (2)) \mu - \beta_D (2) \frac{A + Cx}{B + F} \right] \right)
\]

\[
+ CM_L \left[ (1 - \beta_L (1) - \beta_L (2)) \mu - \beta_L (2) \frac{A + Cx}{B + F} \right] \quad (28)
\]

\[
B = C \left( \frac{N}{D} \rho^{-1} \hat{\tau}_D + M_D \right) D \beta_D (1) \quad (29)
\]

\[
C = - \left[ \left( \frac{\beta_D (2)}{B + F} - 1 \right) (N \rho^{-1} \hat{\tau}_D + DM_D) + M_L \left( \frac{\beta_L (2)}{B + F} - 1 \right) \right]^{-1} \quad (30)
\]

\[
F = CM_L \beta_L (1). \quad (31)
\]

**Intermediation choice** Given the solution to the auction outcomes, we can now consider the large investor’s choice of whether to bid directly or indirectly. The large investor chooses \( \lambda \in \{ID, D\} \) to maximize expected utility.

\[
\max_{\lambda \in \{ID, D\}} M_{L, \lambda} E_{L, \lambda} [f - p] - \frac{\rho_L}{2} M_{L, \lambda}^2 E_{L, \lambda} [f - p]^2 \hat{\tau}_{L, \lambda}. \]

The numerical results that follow teach us about the properties of this choice.

**4.1 Results**

In figure 5, we see that a large investor with a medium or high signal always bids directly, while an investor with a low signal may or may not choose to bid through a dealer. If we interpret this large investor as a foreign central bank, this fits with the fact that most of the largest traders do not bid through primary dealers most of the time. They cite the fact that they do not want to alert dealers to their trades. Given the obvious cost of revealing one’s information to an intermediary, why trade through a dealer when the signal is low?
Figure 5: Mixed auctions.

Note: If a bar is missing from the bidding decision plot, bidding indirectly is optimal.
“Low signal”: $s_L = \mu/2$; “average signal”: $s_L = \mu$; “high signal”: $s_L = 1.5\mu$.

(a) Direct bidding decision

(b) Auction Revenue Mean and Stdev

The reason is that investors with low signals, in this example, hold fewer treasuries, in the absence of additional information. (See low large investor allocation in Table 3.) If they do not expect to hold any treasury bonds, they are not helped or hurt (in expectation) if the price of those bonds incorporates their private information. Thus, the cost of bidding through a dealer is very low. But if the dealer provides them with new information that makes them subsequently take a higher position in treasuries, they can benefit from the dealer’s information. Since the benefits outweigh the costs, the large investor with a low signal sometimes bids through the dealer.

Figure 5 also illustrates how the number of dealers affects the intermediation decision. On the left, where the number of dealers is low, the bars are one, meaning that the large investor chooses to bid directly (not through a dealer), after observing low, average and high signals. But with more dealers, the large investor is more likely to bid indirectly - specifically in cases where his signal is far away from the prior mean.

There are two competing effects underlying the relationship between the number of dealers and intermediation choice. When the number of dealers is larger, each dealer aggregates fewer signals and provides less precise information to their clients. Thus the benefit of intermediation is less. On the other hand, sharing the private signal is less costly with more dealers because the dealer will reveal the signal to fewer other clients. Also, the dealer himself can profit less from the investors’ signals when more dealers reduce the competitive power of any one dealer. Our results teach us that for the calibrated parameter values, the second effect dominates. Thus, for around 20 dealers (the current level), intermediation is more valuable when the number of dealers is large.
This result offers an important caveat to the discussion in the previous section about the optimal number of dealers. If policy-makers reduce the number of dealers to improve information aggregation, the results tell us to keep in mind that with a smaller set of dealers, intermediation may become less desirable. The net effect of limiting dealer entry could be to reduce intermediated trade, and with it, auction revenue.

As in the previous models, expected auction revenue and its standard deviation are higher when agents bid through dealers. When the number of dealers increases, revenue falls in cases where the bidders eschew intermediaries and bid directly.

4.2 A New Financial Accelerator

This result reveals a new way in which financial intermediaries amplify shocks. When signals about the value of a financial asset are negative, this information is more likely to be shared with a dealer and his clients. Positive signals are less likely to be shared because an investor who receives a positive signal then expects to take a large portfolio position in the asset and faces a high expected cost from sharing his information. But sharing a bad signal places that signal in the information set of many more investors and causes a large number of investors to demand less of the asset. Thus, bad signals may affect the demand of more investors than good signals do and have a larger effect on asset prices.

This effect shows up as a distribution of auction revenues that has unconditional negative skewness. The first panel of Table 3 reveals that the unconditional skewness is −1.05. The fourth panel reveals that most of this skewness comes from states where the large investor gets a low signal. When shocks are good, they have a moderate effect on the asset price and the auction revenue. But when nature draws a bad realization of the asset’s value, large investors observe negative signals. These investors choose to share their low signals with primary dealers, which in turn lowers the demands of other investors and has a significantly negative effect on the auction revenue.

5 Minimum Bidding Requirements

The final feature of Treasury markets that we examine is the requirements that primary dealers be consistent, active participants in Treasury auctions. The model clarifies how such a bidding requirement affects the costs and benefits of primary dealers. Does this lessen the market power distortion or impede information aggregation? Does it substitute for or complement dealer entry? How does a minimum bidding requirement affect the decision of investors to bid through dealers?
In any given auction, there is no strict bidding requirement. But if on average, a dealer is not buying a sufficient quantity of Treasuries, his primary dealer status can be revoked. To capture the essence of this dynamic requirement in a static model, we model the bidding requirement as a cost levied on a dealer who purchases too little. Conversely, a dealer who purchases a large dollar amount of Treasuries faces a relaxed bidding constraint in the future. We model this benefit as a current transfer. Thus, for a dealer who purchases a dollar amount $qp$ of Treasuries through the auction, there is a low-bid penalty of $\chi_0 - \chi qp$, or equivalently a benefit of $\chi qp - \chi_0$.

We introduce this minimum bidding policy in the model from the previous section. The investors’ objectives are the same as before. But the dealers’ problem becomes

$$\max_{qD,p} E[-\exp(-\rho_D W_D)|X_DS]$$

s.t. $W_D = W_{0,D} + qD(f - p) - \chi_0 + \chi qDp$, \hspace{1cm} (33)

and subject to the market-clearing condition (10). By substituting the equation for $W_D$ in the dealer’s problem into the objective function, evaluating the expectation and taking the log, and dropping the constant terms that do not affect optimization, we can simplify the dealer’s bidding problem to be: max$_{qD,p}$ $qD(E[f|X_DS] - p(1 - \chi)) - \frac{1}{2}\rho_D qD^2 V[f|X_DS]$, s.t. the market clearing constraint (10). Taking the first order condition with respect to $qD$, we obtain

$$qD(p) = \frac{E[f|X_DS] - p(1 - \chi)}{\rho_D V[f|X_DS] + (1 - \chi)dp/dqD},$$

(34)

where $\tau_D \equiv \tau_f + \tau_\epsilon$ is the precision of the dealer’s beliefs about the payoff.

Note that the bidding requirement shows up like a dealer price subsidy, encouraging the dealer to purchase more of the asset. It also mitigates the effect of dealer market power by multiplying the $dp/dqD$ term by a number less than one. It does not change expectations or the investors’ problem, except through the change in equilibrium price coefficients. The following results show how the penalty $\chi$ affects the pricing coefficients. As before, there are two cases: one where the large bidder bids directly and another where they bid through dealer 1. We explore each in turn.

**Result 4.** In an auction with $\Delta$ dealers and 1 large investor who bids directly, the auction revenue is

$$p = A + B \sum_{d=1}^D \bar{s}_d + Cx + Fs_L,$$

(35)
where $A$, $B$ and $F$ are given by (28), (29) and (31) and

$$C = -\left[ \left( \frac{\beta_D(2)}{B + F} - 1 \right) (N \rho^{-1} \tau_D + DM_D) + DM_D \chi + M_L \left( \frac{\beta_L(2)}{B + F} - 1 \right) \right]^{-1}$$

Note that it appears as though the cost to bidding too little $\chi$ only enters through the coefficient $C$. However, the penalty $\chi$ also changes dealers’ demand coefficient $M_D$ and by making dealers less responsive to the price, it changes the large investor’s demand coefficient $M_L$ as well.

By examining the price impact of a dealer, we see that one clear effect of minimum bidding requirements is to help counteract the tendency of dealers to use their market power to hold price low. The proof of Result 4 shows that

$$\frac{dp}{dq_d} = -\left[ (N \rho^{-1} \tau_D^{-1} + M_D(D - 1)) \left( \frac{\beta_D(2)}{B + F} - 1 \right) + M_D(D - 1) \chi + M_L \left( \frac{\beta_L(2)}{B + F} - 1 \right) \right]^{-1}.$$  

Notice that the bidding cost raises the term in brackets, which is raised to the power $-1$ and multiplied by $-1$, to make it positive. Thus, an increase in the penalty $\chi$ make the effect of dealer demand on the price smaller. This reduces the strategic power of dealers.

Next, we turn to the case where the large investor bids indirectly, through dealer 1.

**Result 5.** In an auction with $N$ dealers and 1 large investor who bids indirectly through dealer 1, the auction revenue is

$$p = A + B_1 \bar{s}_1 + \frac{B_2}{D - 1} \sum_{d=2}^{D} \bar{s}_d + Cx.$$  

(36)

where $A$, $B_1$ and $B_2$ are given by (23), (24) and (25) and

$$C = -\hat{B} \left[ (\nu \rho^{-1} \hat{\tau}_1 + \delta M_{D,1} + M_L) \left( \beta_1(2) - \hat{B} \right) + (\Delta - 1) (\nu \rho^{-1} \hat{\tau}_2 + \delta M_{D,2}) \left( \beta_2(2) - \hat{B} \right) + \chi \hat{B} \delta (M_{D,1} + (\Delta - 1)M_{D,2}) \right]^{-1}.$$

As before, the minimum bidding penalty $\chi$ affects the pricing coefficient $C$. It also affects $A$, $B_1$ and $B_2$ through the coefficients $M_{D,1}$, $M_{D,2}$ and $M_L$ that multiply expected profit to produce dealers’ and the large investor’s demands.
Figure 6: Auction with a Low-Bid Penalty.

Note: If a bar is missing from the bidding decision plot, bidding indirectly is optimal. “Low signal”: $s_L = \mu/2$; “average signal”: $s_L = \mu$; “high signal”: $s_L = 1.5\mu$.

(a) Direct bidding decision  (b) Auction Revenue Mean  (c) Auction Revenue St. Dev.

Results  A stringent bidding requirement (high $\chi$) discourages the use of primary dealers. Investors who observe medium and high signals bid directly all the time. But low-signal bidders choose to use dealers only when $\chi$ is low.

An increase in the penalty $\chi$ for low bidding does raise the expected revenues by incentivizing dealers to make larger bids. That effect is intuitive. However, contrary to policy wisdom, low-bid penalties do not reduce auction revenue risk. Figure 6 (panel b) shows that more stringent bidding requirements increase the volatility of auction revenue.

6 Conclusions

The vast majority of Treasury auction bids are placed by a small set of primary dealers for either their own or their customers’ account. Standard monopsony arguments would suggest that such a concentrated bidding structure may be a costly system for issuing Treasury securities. In fact, primary dealers could use not only market power, but also information to their advantage, lowering auction revenue and their customers’ returns.

Using data from U.S. Treasury auctions, we quantified the costs and benefits of the primary dealer system and evaluated policies such as increasing dealer entry, allowing investors to bid directly, and minimum bidding requirements.

The common theme throughout the results is a reversal in the common wisdom about dealers as underwriters. The prevailing thinking about dealers is that they may reduce auction revenue, but with the benefit that they reduce the revenue risk as well. We find the opposite. In every model, the one result that consistently reappears is: When investors bid through dealers, the mean and variance of auction revenue rises.

By making investors better-informed, dealers reduce their risk, which encourages them to
bid more and raises revenue. But when investors are better informed, their demand is more sensitive to the true payoff (secondary market value) of the asset. This higher sensitivity of better-informed traders makes auction revenue also more sensitive to shocks, increasing revenue variance. This difference highlights why it matters which theory of intermediation we use in making policy. This type of information aggregation is surely not the only role that intermediaries play in asset markets. But since other roles for intermediaries do not seem to be applicable in the context of treasury auctions, this setting provided a useful laboratory for isolating and investigating this facet of intermediation.
References


A Technical Details

The $X$ operators that select the relevant signals that each agent observes are as follows. In each case, the operator $X$ is a linear matrix operation. It is described by a matrix of zeroes and ones that pre-multiplies the vector of all signals.

$$X_i = \begin{bmatrix} 1_i' & 0 & 0'_{D} & 0 & 0 \\ 0'_N & 0 & 0'_{D} & 1 & 0 \\ 0'_N & 0 & 0'_{D} & 0 & 1 \end{bmatrix}$$

where $1_i$ is an $(N \times 1)$ vector of $n$ zeros with a 1 in the $i$th position and $0_N$ is an $N \times 1$ vector of zeros. The matrix $X_i$ is constructed so that it picks off from the vector of all signals $S$ only the signals that agent $i$ observes.

Similarly, for the dealer,

$$X_d = \begin{bmatrix} 0'_{N} & 1_d' & 0 & 0 \\ 0'_N & 0 & 0'_{D} & 1 & 0 \\ 0'_N & 0 & 0'_{D} & 0 & 1 \end{bmatrix}$$

and for the large investor, the $X$ matrix is

$$X_L = \begin{bmatrix} 0'_{N} & 1 & 0'_{D} & 0 & 0 \\ 0'_N & 0 & 0'_{D} & 1 & 0 \\ 0'_N & 0 & 0'_{D} & 0 & 1 \end{bmatrix}.$$

**Competitive model without dealers** In the model without intermediation, the $X$ operators that select the relevant signals that each agent observes are as follows.

$$X_i^C = \begin{bmatrix} 1_i' & 0 & 0'_{D} & 0 & 0 \\ 0'_N & 0 & 0'_{D} & 0 & 1 \end{bmatrix}.$$

$$X_d^C = \begin{bmatrix} 0'_{N} & 1_d' & 0 & 0 \\ 0'_N & 0 & 0'_{D} & 0 & 1 \end{bmatrix},$$

and by the large investor

$$X_L^C = \begin{bmatrix} 0'_{N} & 1 & 0'_{D} & 0 & 0 \\ 0'_N & 0 & 0'_{D} & 0 & 1 \end{bmatrix}.$$

B Proofs

**Proof of Result 1: Price in the no dealer model.** Let the price conjecture in this model be

$$p = A^C + B^C S + C^C x.$$
Then, an investor’s unbiased price signal is \((p - AC)/BC = \bar{s} + CC/BCx\). That is an unbiased signal about \(f\). However, because investor \(i\)’s signal \(s_i\) is contained in the average signal \(\bar{s}\), the price signal and the private signal have correlated signal errors. Using equation (2), we find that the weights on the two signals are

\[
\beta^C = \frac{\psi}{\tau_f} \left[ \frac{N\tau_f\tau C^2}{\tau_x(N\tau_x + \tau_f)} \right] \\frac{(N - 1)\tau_f}{N\tau_x + \tau_f - 1},
\]

where \(\psi \equiv [(N - 1)/N\tau_x^{-1} + C^2/B^2\tau_x^{-1}(\tau_x + \tau_f)/(\tau_x + \tau_f/N)]^{-1}\). Thus,

\[
E[f|X^C_iS_i] = (1 - \beta^C(1) - \beta^C(2))\mu + \beta^C(1)s_i + \beta^C(2)p - AC/BC. \quad (37)
\]

That implies an average investors’ expectation is

\[
\bar{E}[f|X^C,S] = (1 - \beta^C(1) - \beta^C(2))\mu + \beta^C(1)\bar{s} + \beta^C(2)p - AC/BC. \quad (38)
\]

Using (4), we find that each investor’s conditional variance is

\[
V[f|X^C_iS_i] = \tau_f^{-1} - \psi\tau_f^{-1} \left[ \frac{N\tau_x(C^2/BC)^2\tau_x^{-1} + N - 1}{N\tau_x + \tau_f - 2\tau_f^{-1}} \right]. \quad (39)
\]

The investors’ demand functions (bids) in the competitive market model are given by equation (7). Substituting these bids into the market clearing condition \(x + \sum_{i=1}^N q_i = 1\) yields \(x + N\rho^{-1}V[f|X^C,S]^{-1}(\bar{E}[f|X^C,S] - p^C) = 1\). Solving for \(p\) yields a competitive auction price and revenue of

\[
p^C = \frac{BC}{BC - \beta^C(2)} \left[ (1 - \beta^C(1) - \beta^C(2))\mu + \beta^C(1)\bar{s} - \frac{\beta^C(2)AC}{BC} - N^{-1}\rho V[f|X^C_iS_i] (1 - x) \right]. \quad (40)
\]

Matching coefficients yields the implicit solution to the price equation in the result. The existence of a set of coefficients verifies the price conjecture.

Since the supply of the asset is one, auction revenue is the price of the asset.

**Proof of Result 2: Price in the N dealer model with all investors bidding through a dealer.** The vector of signals \(S\) is given by

\[
S = \begin{bmatrix}
  s_1 & \ldots & s_N & s_L & \bar{s}_1 & \ldots & \bar{s}_D & \frac{p - A}{B_1 + B_2}
\end{bmatrix}',
\]

or, equivalently,

\[
S = 1_{N+D+2}f + \Pi Z,
\]
where the \((N + D + 2) \times (N + 2)\) matrix \(\Pi\) is given by

\[
\Pi = \begin{bmatrix}
I_N & 0_N & 0_N \\
\omega_e \cdot 1'_{N/D} & \cdots & 0'_{N/D} & \omega_L \\
0'_{N/D} & \cdots & \omega_e - \omega_L & 0 \\
\frac{B_1}{B_1 + B_2} \omega_e \cdot 1'_{N/D} & \cdots & \frac{1}{N} B_2 & \frac{D}{N} \cdot 1'_{N/D} & B_2 \\
\frac{B_1}{B_1 + B_2} & \cdots & \frac{D}{N} \cdot 1'_{N/D} & 0 & \frac{C}{B_1 + B_2} \\
\end{bmatrix}.
\]

Investors bidding through dealer 1 use the average signal \(\bar{s}_1\) and the price to form expectations. Using equation (2), we have

\[
\beta_1 = \tau_f^{-1} |\nabla (X_1 S)|^{-1} \left[ -\frac{B_2 B_1}{(B_1 + B_2)^2} \left( \frac{\omega_e^2 N}{D} \tau_x^{-1} + \frac{\omega_e^2 \tau_L - \omega_L^2}{L} \right) + \left( \frac{B_2}{B_1 + B_2} \right)^2 \left( \frac{\omega_e^2 N}{D} \right) \tau_x^{-1} + \left( \frac{B_2}{B_1 + B_2} \right)^2 \left( \frac{1}{N D - 1} \right) \tau_x^{-1} \right],
\]

so that

\[
\mathbb{E} [f | X_1 S] = (1 - \beta_1 (1 - \beta_1 (2))) \mu + \beta_1 (1) \bar{s}_1 + \beta_1 (2) \left( \frac{p - A}{B_1 + B_2} \right),
\]

and the variance of the beliefs of investors bidding through dealer 1 is

\[
\nabla [f | X_1 S] = \tau_f^{-1} - \tau_f^{-2} |\nabla (X_1 S)|^{-1} \left( \left( \frac{C}{B_1 + B_2} \right)^2 \tau_x^{-1} + \left( \frac{B_2}{B_1 + B_2} \right)^2 \left( \frac{\omega_e^2 N}{D} + \frac{1}{N D - 1} \right) \tau_x^{-1} \right)
\]

\[
\equiv \tau_1^{-1}.
\]

Similarly, for the inference of \(f\) for investors bidding through any other dealer is given by:

\[
\mathbb{E} [f | X_d S] = (1 - \beta_2 (1 - \beta_2 (2))) \mu + \beta_2 (1) \bar{s}_d + \beta_2 (2) \left( \frac{p - A}{B_1 + B_2} \right),
\]

where

\[
\beta_2 = \tau_f^{-1} |\nabla (X_1 S)|^{-1} \left[ \left( \frac{B_1}{B_1 + B_2} \right)^2 \left( \frac{\omega_e^2 N}{D} \tau_x^{-1} + \frac{\omega_e^2 \tau_L - \omega_L^2}{L} \right) + \left( \frac{C}{B_1 + B_2} \right)^2 \tau_x^{-1} + \left( \frac{B_1 B_2}{(B_1 + B_2)^2} \right) \left( \frac{1}{N D - 1} \right) \tau_x^{-1} \right],
\]

and the variance of their beliefs by

\[
\nabla [f | X_d S] = \tau_f^{-1} - \tau_f^{-2} |\nabla (X_1 S)|^{-1} \left( \left( \frac{B_1}{B_1 + B_2} \right)^2 \left( \frac{\omega_e^2 N}{D} \tau_x^{-1} + \frac{\omega_e^2 \tau_L - \omega_L^2}{L} \right) + \left( \frac{C}{B_1 + B_2} \right)^2 \tau_x^{-1} + \left( \frac{1}{N D - 1} \right) \tau_x^{-1} \right),
\]

\[
\equiv \tau_2^{-1}.
\]

When the large investor bids indirectly through a dealer, dealer 1 has a different market impact than the other dealers. In particular, from the market clearing constraint, we
have:

\[ 1 = x + \left( \frac{N}{D} \rho^{-1} \hat{\tau}_1 + M_{D,1} + M_L \right) \left( (1 - \beta_1 (1) - \beta_1 (2)) \mu + \beta_1 (1) \tilde{s}_1 + \beta_1 (2) \left( \frac{p - A}{B_1 + B_2} \right) - p \right) \]

+ \sum_{d=2}^{D} \left( \frac{N}{D} \rho^{-1} \hat{\tau}_2 + M_{D,2} \right) \left( (1 - \beta_2 (1) - \beta_2 (2)) \mu + \beta_2 (1) \tilde{s}_d + \beta_2 (2) \left( \frac{p - A}{B_1 + B_2} \right) - p \right).

Taking the derivative with respect to \( q_1 \), we obtain the price impact of dealer 1:

\[
\frac{dp}{dq_1} = - \left[ \left( \frac{N}{D} \rho^{-1} \hat{\tau}_1 + M_L \right) \left( \frac{\beta_1 (2)}{B_1 + B_2} - 1 \right) + (D - 1) \left( \frac{N}{D} \rho^{-1} \hat{\tau}_2 + M_{D,2} \right) \left( \frac{\beta_2 (2)}{B_1 + B_2} - 1 \right) \right]^{-1},
\]

so that \( M_{D,1} \) solves

\[
M_{D,1}^{-1} = \rho_{D} \hat{\tau}_1^{-1} - \left[ \left( \frac{N}{D} \rho^{-1} \hat{\tau}_1 + M_L \right) \left( \frac{\beta_1 (2)}{B_1 + B_2} - 1 \right) + (D - 1) \left( \frac{N}{D} \rho^{-1} \hat{\tau}_2 + M_{D,2} \right) \left( \frac{\beta_2 (2)}{B_1 + B_2} - 1 \right) \right]^{-1}.
\]

Similarly, the price impact of any other dealer is:

\[
\frac{dp}{dq_d} = - \left[ \left( \frac{N}{D} \rho^{-1} \hat{\tau}_1 + M_L + M_{D,1} \right) \left( \frac{\beta_1 (2)}{B_1 + B_2} - 1 \right) + (D - 1) \left( \frac{N}{D} \rho^{-1} \hat{\tau}_2 + M_{D,2} \right) \left( \frac{\beta_2 (2)}{B_1 + B_2} - 1 \right) \right]^{-1},
\]

and the price impact of the large investor is:

\[
\frac{dp}{dq_L} = - \left[ \left( \frac{N}{D} \rho^{-1} \hat{\tau}_1 + M_{D,1} \right) \left( \frac{\beta_1 (2)}{B_1 + B_2} - 1 \right) + (D - 1) \left( \frac{N}{D} \rho^{-1} \hat{\tau}_2 + M_{D,2} \right) \left( \frac{\beta_2 (2)}{B_1 + B_2} - 1 \right) \right]^{-1}.
\]

Thus, \( M_{D,2} \) and \( M_L \) are given implicitly as solutions to

\[
M_{D,2}^{-1} = \rho_{D} \hat{\tau}_2^{-1} - \left[ \left( \frac{N}{D} \rho^{-1} \hat{\tau}_1 + M_L + M_{D,1} \right) \left( \frac{\beta_1 (2)}{B_1 + B_2} - 1 \right) + (D - 1) \left( \frac{N}{D} \rho^{-1} \hat{\tau}_2 + M_{D,2} \right) \left( \frac{\beta_2 (2)}{B_1 + B_2} - 1 \right) \right]^{-1},
\]

\[
M_{L}^{-1} = \rho_{L} \hat{\tau}_1^{-1} - \left[ \left( \frac{N}{D} \rho^{-1} \hat{\tau}_1 + M_{D,1} \right) \left( \frac{\beta_1 (2)}{B_1 + B_2} - 1 \right) + (D - 1) \left( \frac{N}{D} \rho^{-1} \hat{\tau}_2 + M_{D,2} \right) \left( \frac{\beta_2 (2)}{B_1 + B_2} - 1 \right) \right]^{-1}.
\]

Finally, solving the market clearing condition for the settle price and equating the coefficients, we obtain the coefficient expressions in the result.

**Proof of Result 3: Price when the large investor bids directly.** In this setting, the vector of shocks \( Z \) is given by

\[
Z = [ \varepsilon_1 \ldots \varepsilon_N \ \varepsilon_L \ x ]',
\]

and the vector of signals \( S \) by

\[
S = [ s_1 \ldots s_N \ s_L \ \tilde{s}_1 \ldots \tilde{s}_D \ \frac{p - A}{B + B} ]'.
\]
We can represent the vector of signals as

\[ S = 1_{N+D+2}f + \Pi^D Z, \]

where the \((N + D + 2) \times (N + 2)\) matrix \(\Pi^D\) is given by

\[
\Pi^D = \begin{bmatrix}
I_N & 0_N & 0_N \\
0'_N & 1 & 0 \\
D/N \cdot 1'_{N/D} & \cdots & 0'_N/1_D \\
\vdots & \vdots & \vdots & \vdots & \vdots \\
0'_{N/D} & \cdots & D/N \cdot 1'_{N/D} & 0 & 0 \\
1/N F B_{B+F} \cdot 1'_{N} & f_{B+F} & c_{B+F}
\end{bmatrix}.
\]

As before, each agent in the economy observes a subset of these signals, given by \(X_j\), a matrix of zeros and ones. This equilibrium has the following information structure.

- Individual investor \(i \in I_d\) bidding through dealer \(d\) observes his own private signal \(s_i\). In addition, the dealer tells them the average of the signals of the dealer’s customers, \(\bar{s}_d\). Finally, the investor can condition on the settle price \(p\). Thus, for an individual investor,

\[
X_i = \begin{bmatrix}
1'_i & 0 & 0'_D & 0 \\
0'_N & 0 & 1'_d & 0 \\
0'_N & 0 & 0'_D & 1
\end{bmatrix}, \quad i \in I_d.
\]

- Dealer \(d\) observes the average signal of his customers and conditions on price information but has no private signal of his own. Thus,

\[
X_d = \begin{bmatrix}
0'_N & 0 & 1'_d & 0 \\
0'_N & 0 & 0'_D & 1
\end{bmatrix}, \quad d = 1, \ldots, D.
\]

- The large investor on the other hand observes only his own signal and conditions on the price, so that

\[
X_L = \begin{bmatrix}
0'_N & 1 & 0'_D & 0 \\
0'_N & 0 & 0'_D & 1
\end{bmatrix}.
\]

The belief updating in this case is similar to the setting studied in the previous Section. The beliefs of an investor bidding through dealer \(d\) coincide with the beliefs of the dealer. All investors use the price to form a second unbiased signal about \(f\), which is given by:

\[
s_p = \frac{B}{B+F} \frac{1}{D} \sum_{d=1}^{D} \bar{s}_d + \frac{F}{B+F} s_L + \frac{C}{B+F} x = f + \frac{B}{B+F} \frac{1}{N} \sum_{i=1}^{N} \varepsilon_i + \frac{F}{B+F} \varepsilon_L + \frac{C}{B+F} x.
\]

As in the previous Section, the average signal of dealer \(d\)’s clients is contained in the price.
signal $s_p$. Using equation (2), we find that the weights on the two signals are

$$
\beta_D = \tau_f^{-1} |V(X_d S)|^{-1} \left[ \tau_x^{-1} \left( \frac{C}{B+F} \right)^2 + \tau_L^{-1} \left( \frac{F}{F+B} \right)^2 - \frac{1}{N} \tau_x^{-1} \left( \frac{B F}{(F+B)^2} \right) \right] \left( D - \frac{B}{F+B} \right)'.
$$

Thus,

$$
\mathbb{E} [f | X_i S] = (1 - \beta_D (1) - \beta_D (2)) \mu + \beta_D (1) \bar{s}_d + \beta_D (2) \left( \frac{p - A}{B + F} \right), \quad i \in I_d.
$$

Using (4), we find that each investor’s conditional variance of beliefs is

$$
\mathbb{V} [f | X_i S] = \tau_f^{-1} - \tau_f^{-2} |V(X_d S)|^{-1} \left[ \tau_x^{-1} \left( \frac{C}{B+F} \right)^2 + \tau_L^{-1} \left( \frac{F}{F+B} \right)^2 + \tau_x^{-1} \frac{1}{N} \left( D - \frac{B (B + 2F)}{(B + F)^2} \right) \right]
$$

$$
\equiv \tau_D^{-1}.
$$

Similarly, the direct bidder averages his signal and the signal from the price using

$$
\beta_L = \tau_f^{-1} |V(X_L S)|^{-1} \left[ \left\{ \frac{1}{N} \tau_x^{-1} \left( \frac{B}{F+B} \right)^2 + \tau_L^{-1} \left( \frac{C}{F+B} \right)^2 - \frac{B F}{(F+B)^2} \tau_L^{-1} \right\} \right] \left( \frac{B}{F+B} \right) \tau_L^{-1}',
$$

so that

$$
\mathbb{E} [f | X_L S] = (1 - \beta_L (1) - \beta_L (2)) \mu + \beta_L (1) s_L + \beta_L (2) \left( \frac{p - A}{B + F} \right).
$$

Using (4), we find that the direct bidder’s conditional variance of beliefs is

$$
\mathbb{V} [f | X_L S] = \tau_f^{-1} - \tau_f^{-2} |V(X_L S)|^{-1} \left( \frac{1}{N} \tau_x^{-1} \left( \frac{B}{F+B} \right)^2 + \tau_L^{-1} \left( \frac{C}{F+B} \right)^2 + \left( \frac{B}{F+B} \right)^2 \right) \equiv \tau_L^{-1}.
$$

Turn now to solving for the price impact of dealer $d$. Substituting demand functions into the market clearing constraint, we obtain

$$
1 = x + q_d + \sum_{d=1}^{D} \frac{N}{D^\rho - 1} \tau_D \left[ (1 - \beta_D (1) - \beta_D (2)) \mu + \beta_D (1) \bar{s}_d + \beta_D (2) \left( \frac{p - A}{B + F} \right) - p \right]
$$

$$
+ \sum_{j \neq d} M_D \left[ (1 - \beta_D (1) - \beta_D (2)) \mu + \beta_D (1) \bar{s}_j + \beta_D (2) \left( \frac{p - A}{B + F} \right) - p \right]
$$

$$
+ M_L \left[ (1 - \beta_L (1) - \beta_L (2)) \mu + \beta_L (1) s_L + \beta_L (2) \left( \frac{p - A}{B + F} \right) - p \right].
$$

Taking the derivative with respect to the demand of dealer $d$, we obtain:

$$
\frac{dp}{dq_d} = - \left( N \rho^{-1} \tau_D + M_D (D - 1) \left( \frac{\beta_D (2)}{B + F} - 1 \right) + M_L \left( \frac{\beta_L (2)}{B + F} - 1 \right) \right)^{-1}.
$$
Thus, $M_D$ is given implicitly as the solution to
\[
M_D^{-1} = \rho_D \hat{\tau}_D^{-1} - \left[ (N \rho^{-1} \hat{\tau}_D + M_D (D - 1)) \left( \frac{\beta_D (2)}{B + F} - 1 \right) + M_L \left( \frac{\beta_L (2)}{B + F} - 1 \right) \right]^{-1}.
\]

Similarly, taking the derivative with respect to the direct bidder’s demand, we obtain
\[
\frac{dp}{dq_L} = - \left[ (N \rho^{-1} \hat{\tau}_D + D M_D) \left( \frac{\beta_D (2)}{B + F} - 1 \right) \right]^{-1},
\]
so that $M_L$ is given implicitly by
\[
M_L^{-1} = \rho_L \hat{\tau}_L^{-1} - \left[ (N \rho^{-1} \hat{\tau}_D + D M_D) \left( \frac{\beta_D (2)}{B + F} - 1 \right) \right]^{-1}.
\]

Finally, we use the market clearing condition
\[
1 = x + \sum_{d=1}^{D} \left( \frac{N D}{D} \rho^{-1} \hat{\tau}_D + M_D \right) \left[ (1 - \beta_D (1) - \beta_D (2)) \mu + \beta_D (1) \bar{s}_d + \beta_D (2) \left( \frac{p - A B}{B + F} \right) - p \right]
+ M_L \left[ (1 - \beta_L (1) - \beta_L (2)) \mu + \beta_L (1) \bar{s}_L + \beta_L (2) \left( \frac{p - A B}{B + F} \right) - p \right],
\]
we can solve for the equilibrium settle price:
\[
\left[ \left( \frac{\beta_D (2)}{B + F} - 1 \right) (N \rho^{-1} \hat{\tau}_D + D M_D) + M_L \left( \frac{\beta_L (2)}{B + F} - 1 \right) \right] p = 1 - x
- \left( N \rho^{-1} \hat{\tau}_D + D M_D \right) \left[ (1 - \beta_D (1) - \beta_D (2)) \mu - \beta_D (2) \frac{A}{B + F} \right] - \sum_{d=1}^{D} \left( \frac{N D}{D} \rho^{-1} \hat{\tau}_D + M_D \right) \beta_D (1) \bar{s}_d
- M_L \left[ (1 - \beta_L (1) - \beta_L (2)) \mu - \beta_L (1) \bar{s}_L - \beta_L (2) \frac{A}{B + F} \right].
\]

Equating coefficients, we obtain the system of equations in the result. This verifies the price conjecture. Since supply of the asset is one, price and revenue are equal.

**Proof of Result 4:** A large investor who bids directly and a dealer with minimum bidding requirements. All the conditional expectations and variances are the same as in the previous model (proof of Result 3 holds up until this point).

Turn now to solving for the price impact of dealer $d$. Substituting demand functions into
the market clearing constraint, we obtain

\[ 1 = x + q_d + \sum_{d=1}^{D} \frac{N}{D} \rho^{-1} \tau_D \left[ (1 - \beta_D (1) - \beta_D (2)) \mu + \beta_D (1) \tilde{s}_d + \beta_D (2) \left( \frac{p - A}{B + F} \right) - p \right] \\
+ \sum_{j \neq d} M_D \left[ (1 - \beta_D (1) - \beta_D (2)) \mu + \beta_D (1) \tilde{s}_j + \beta_D (2) \left( \frac{p - A}{B + F} \right) - p \right] \\
+ M_L \left[ (1 - \beta_L (1) - \beta_L (2)) \mu + \beta_L (1) s_L + \beta_L (2) \left( \frac{p - A}{B + F} \right) - p \right]. \]

where the first line is the demand of the small investors (unchanged), the second line is the other dealers (where the minimum bidding penalty \( \chi \) enters), and the third line is the demand of the large investor (unchanged).

Using the implicit function theorem to differentiate the price with respect to the demand of dealer \( d \), we obtain:

\[ \frac{dp}{dq_d} = - \left[ (N \rho^{-1} \tilde{\tau}_D + M_D (D - 1)) \left( \frac{\beta_D (2)}{B + F} - 1 \right) + M_D (D - 1) \chi + M_L \left( \frac{\beta_L (2)}{B + F} - 1 \right) \right]^{-1}. \]

Thus, \( M_D \) is given implicitly as the solution to

\[ M_D^{-1} = \rho_D \tilde{\tau}_D^{-1} - (1 - \chi) \left[ (N \rho^{-1} \tilde{\tau}_D + M_D (D - 1)) \left( \frac{\beta_D (2)}{B + F} - 1 \right) + M_D (D - 1) \chi + M_L \left( \frac{\beta_L (2)}{B + F} - 1 \right) \right]^{-1}. \]

Similarly, taking the derivative with respect to the direct bidder’s demand, we find the the minimum bidding cost \( \chi \) also moderates the strategic effect of the large investor on the price:

\[ \frac{dp}{dq_L} = - \left[ (N \rho^{-1} \tilde{\tau}_D + D M_D) \left( \frac{\beta_D (2)}{B + F} - 1 \right) + D M_D \chi \right]^{-1}, \]

so that \( M_L \) is given implicitly by

\[ M_L^{-1} = \rho_L \tilde{\tau}_L^{-1} - \left[ (N \rho^{-1} \tilde{\tau}_D + D M_D) \left( \frac{\beta_D (2)}{B + F} - 1 \right) + D M_D \chi \right]^{-1}. \]

Finally, we use the market clearing condition again to solve for price \( p \).

\[ 1 = x + \sum_{d=1}^{D} \left( \frac{N}{D} \rho^{-1} \tilde{\tau}_D + M_D \right) \left[ (1 - \beta_D (1) - \beta_D (2)) \mu + \beta_D (1) \tilde{s}_d + \beta_D (2) \left( \frac{p - A}{B + F} \right) - p \right] \\
+ D M_D \chi p + M_L \left[ (1 - \beta_L (1) - \beta_L (2)) \mu + \beta_L (1) s_L + \beta_L (2) \left( \frac{p - A}{B + F} \right) - p \right]. \]
we can solve for the equilibrium settle price:

\[
\left[ \left( \frac{\beta_D (2)}{B + F} - 1 \right) (N \rho^{-1} \tau_D + DM_D) + DM_D\chi + M_L \left( \frac{\beta_L (2)}{B + F} - 1 \right) \right] p = 1 - x \\
- (N \rho^{-1} \tau_D + DM_D) \left[ (1 - \beta_D (1) - \beta_D (2)) \mu - \beta_D (2) \frac{A}{B + F} \right] - \sum_{d=1}^{D} \left( \frac{N \rho^{-1} \tau_D + M_D}{D} \right) \beta_D (1) \bar{s}_d \\
- M_L \left[ (1 - \beta_L (1) - \beta_L (2)) \mu + \beta_L (1) s_L - \beta_L (2) \frac{A}{B + F} \right].
\]

Equating coefficients, we obtain the system of equations in the result. This verifies the price conjecture. Since supply of the asset is one, price and revenue are equal.

**Proof of Result 5: Auction price with minimum bidding requirements and a large bidder who bids through a dealer.** When the large investor bids indirectly through a dealer, dealer 1 has a different market impact than the other dealers. In particular, from the market clearing constraint, we have:

\[
1 = x + \left( \frac{N}{D} \rho^{-1} \tau_1 + M_{D,1} + M_L \right) \left[ (1 - \beta_D (1) - \beta_D (2)) \mu + \beta_D (1) \bar{s}_1 + \beta_D (2) \left( \frac{p - A}{B_1 + B_2} \right) - p \right] \\
+ \sum_{d=2}^{D} \left( \frac{N}{D} \rho^{-1} \tau_2 + M_{D,d} \right) \left[ (1 - \beta_D (1) - \beta_D (2)) \mu + \beta_D (1) \bar{s}_d + \beta_D (2) \left( \frac{p - A}{B_1 + B_2} \right) - p \right] \\
+ (M_{D,1} + (D - 1)M_{D,2}) \chi p.
\]

where the last term is the new term that arises because of the minimum bidding penalty \( \chi \).

Taking the derivative with respect to \( q_1 \), we obtain the price impact of dealer 1:

\[
\frac{dp}{d q_1} = - \left[ \left( \frac{N}{D} \rho^{-1} \tau_1 + M_L \right) \left( \frac{\beta_D (2)}{B_1 + B_2} - 1 \right) + (D - 1) \left( \frac{N}{D} \rho^{-1} \tau_2 + M_{D,2} \right) \left( \frac{\beta_D (2)}{B_1 + B_2} - 1 \right) \right] \\
+ \chi (D - 1) M_{D,2}^{-1}.
\]

Using the dealer’s first order condition, we find the constant \( M_{D,1} \) that maps expected profit into demand solves

\[
M_{D,1}^{-1} = \rho_D \bar{s}_1^{-1} - (1 - \chi) \left[ \left( \frac{N}{D} \rho^{-1} \tau_1 + M_L \right) \left( \frac{\beta_D (2)}{B_1 + B_2} - 1 \right) \right] \\
+ (D - 1) \left( \frac{N}{D} \rho^{-1} \tau_2 + M_{D,2} \right) \left( \frac{\beta_D (2)}{B_1 + B_2} - 1 \right) + \chi (D - 1) M_{D,2}^{-1}.
\]

Similarly, the price impact of any other dealer is:

\[
\frac{dp}{d q_d} = - \left[ \left( \frac{N}{D} \rho^{-1} \tau_1 + M_L + M_{D,1} \right) \left( \frac{\beta_D (2)}{B_1 + B_2} - 1 \right) \right] \\
+ (D - 1) \left( \frac{N}{D} \rho^{-1} \tau_2 + \frac{D - 2}{D - 1} M_{D,2} \right) \left( \frac{\beta_D (2)}{B_1 + B_2} - 1 \right) + (M_{D,1} + (D - 2)M_{D,2}) \chi^{-1},
\]

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and the price impact of the large investor is:

\[
\frac{dp}{dq_L} = - \left[ \left( \frac{N}{D} \right)^{\rho^{-1} \hat{\tau}_1 + M_{D,1}} \left( \frac{\beta_1 (2)}{B_1 + B_2} - 1 \right) + (D - 1) \left( \frac{N}{D} \right)^{\rho^{-1} \hat{\tau}_2 + M_{D,2}} \left( \frac{\beta_2 (2)}{B_1 + B_2} - 1 \right) + (M_{D,1} + (D - 1)M_{D,2}) \chi \right]^{-1}.
\]

Thus, \(M_{D,2}\) and \(M_L\) are given implicitly as solutions to

\[
M_{D,2}^{-1} = \rho D \hat{\tau}_2^{-1} - (1 - \chi) \left[ \left( \frac{N}{D} \right)^{\rho^{-1} \hat{\tau}_1 + M_L + M_{D,1}} \left( \frac{\beta_1 (2)}{B_1 + B_2} - 1 \right) + (D - 1) \left( \frac{N}{D} \right)^{\rho^{\hat{\tau}_2 + M_{D,2}}} \left( \frac{\beta_2 (2)}{B_1 + B_2} - 1 \right) + (M_{D,1} + (D - 2)M_{D,2}) \chi \right]^{-1}
\]

\[
M_L^{-1} = \rho L \hat{\tau}_1^{-1} - \left[ \left( \frac{N}{D} \right)^{\rho^{-1} \hat{\tau}_1 + M_{D,1}} \left( \frac{\beta_1 (2)}{B_1 + B_2} - 1 \right) + (D - 1) \left( \frac{N}{D} \right)^{\rho^{-1} \hat{\tau}_2 + M_{D,2}} \left( \frac{\beta_2 (2)}{B_1 + B_2} - 1 \right) + (M_{D,1} + (D - 1)M_{D,2}) \chi \right]^{-1}.
\]

Finally, solving the market clearing condition for the settle price and equating the coefficients, we obtain the following coefficients.

**C Simulation Details**
Table 3: Descriptive statistics for the calibrated, simulated model with direct and indirect bidding and low-bid penalty.

<table>
<thead>
<tr>
<th></th>
<th>Full sample</th>
<th>Bidding indirectly</th>
<th>Bidding directly</th>
<th>$s_L &lt; \mu - \frac{2}{\sqrt{\tau f + \tau L}}$</th>
<th>$s_L &gt; \mu + \frac{2}{\sqrt{\tau f + \tau L}}$</th>
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</thead>
<tbody>
<tr>
<td></td>
<td>Revenue</td>
<td>Dealer allocation</td>
<td>Large investor allocation</td>
<td>Large investor utility</td>
<td>Revenue</td>
</tr>
<tr>
<td>Mean</td>
<td>9.5775</td>
<td>44.352</td>
<td>40.187</td>
<td>0.40091</td>
<td>75.073</td>
</tr>
<tr>
<td>Std. Dev.</td>
<td>61.629</td>
<td>42.707</td>
<td>41.295</td>
<td>0.41108</td>
<td>50.226</td>
</tr>
<tr>
<td>Skew</td>
<td>-1.0485</td>
<td>0.351</td>
<td>2.4001</td>
<td>2.3909</td>
<td>-1.168</td>
</tr>
</tbody>
</table>

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