Measuring the Non-Linear Effects of Monetary Policy*

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Abstract

This paper proposes a method to identify the non-linear effects of structural shocks by using Gaussian basis functions to parametrize impulse response functions. We apply our approach to monetary policy and find that the effect of a monetary intervention depends strongly on (i) the sign of the intervention, (ii) the size of the intervention, and (iii) the state of the business cycle at the time of the intervention. A contractionary policy has a strong adverse effect on output, much stronger than linear estimates suggest, but an expansionary policy has, on average, no significant effect on output. An expansionary policy can have some expansionary effect on output, but only if the intervention is large and during a recession. Even so, a contractionary policy is always more potent than its expansionary counterpart.

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1 Introduction

There now exists a broad consensus on the average effect of monetary policy on output, and it is widely accepted that a monetary contraction (expansion) leads to a persistent decline (increase) in output.

However, there is still little agreement about possible asymmetric or non-linear effects of monetary policy, and three questions at the core of monetary policy making are still unsettled. First, does monetary policy have asymmetric effects on economic activity? As captured by the string metaphor, does contractionary monetary policy have a much stronger effect –being akin to pulling on a string– than an expansionary shock –being akin to pushing on a string–? Second, does the effect of monetary policy vary with the size of the policy intervention? For instance, does one large policy change have the same effect as incremental changes of the same total magnitude? Third, does the effect of monetary policy vary with the state of the business cycle? For instance, does the central bank have more room to stimulate output (without raising inflation) during recessions?

Providing answers to these questions has been difficult in part for one important technical reason: the standard approach to identifying the effect of monetary shocks –Structural Vector-Autoregressions (SVARs)–,[1] is linear and does not allow for asymmetry or non-linearity in the effect of monetary shocks.

This paper proposes a new method to identify the (possibly non-linear) effects of monetary policy. In similar spirit to the SVAR approach, we identify exogenous policy shocks from a recursive identification scheme.[2] However, different from the SVAR approach, we allow the impulse responses to shocks to depend on the value (sign and size) of the identified structural shocks as well as on the state of the business cycle at the time of the shock. Since this generalized model does not admit a VAR representation, we instead directly estimate the impulse response functions from the data by parameterizing the impulse response functions

[1] See e.g., Christiano, Eichenbaum, and Evans (1999) and Uhlig (2005).

[2] The recursive identification scheme typically states that monetary policy does not affect the real economy instantaneously. See Christiano, Eichenbaum, and Evans (1999).

with Gaussian basis functions.

Our approach builds on two premises: (i) any mean-reverting impulse response function can be approximated to any degree of accuracy by a sum of Gaussian basis functions, and (ii), in practice, only a very small number of Gaussian functions are needed to approximate a typical impulse response function. Intuitively, the impulse response functions of stationary variables are often found to be monotonic or hump-shaped (e.g., Christiano, Eichenbaum, and Evans, 1999 or Uhlig, 2005). In such cases, a single Gaussian function can already provide an excellent approximation of the impulse response function.

Thanks to the small number of free parameters allowed by a Gaussian parametrization, it is possible to directly estimate the impulse response functions from the data using maximum likelihood or Bayesian methods.[3] The parsimony of the approach in turn allows us to estimate more general non-linear models.

Consistent with the string metaphor, our findings point towards the existence of very strong asymmetry in the effect of monetary policy: a contractionary monetary policy shock generates a large and significant reduction in output, but an expansionary shock has a small and insignificant effect. As a result, conventional (linear) estimates may substantially over-estimate the real effect of expansionary policy and under-estimate the real effect of contractionary monetary policy. This latter observation is of particular relevance for the current debate on the appropriate timing for the lift-off of the fed funds rate, currently set at the zero lower bound. Our estimates suggest that an inappropriate (i.e., too strong or too early) increase in the fed funds rate could be a lot more costly (in terms of lost output) than conventional estimates suggest.

When we allow the effects of monetary policy to depend on the size of the intervention as well as the state of the business cycle, we find strong evidence in favor of both "size dependence"

[3]Another advantage of using Gaussian basis functions is that, in the context of Bayesian estimations, prior elicitation is much easier than with Bayesian estimations of standard VARs, because the coefficients to be estimated are directly interpretable as features of impulse responses. For instance, in the context of a one Gaussian function approximation, an impulse response function is characterized by 3 parameters; one capturing the peak effect of the shock, another capturing the timing of that peak effect, and a final one capturing the persistence of the effect of the shock.

and “state dependence”. While an expansionary policy has, on average, no significant effect on output, the effect varies markedly with the state of the cycle (state dependence). An expansionary policy during a strong growth episode has no effect on output and only generates inflation. In contrast, an expansionary policy can have a stimulating effect on economic activity (at no inflationary cost) when the economy is in a recession. Moreover, the positive effect of an expansionary policy on output increases slowly with the size of the intervention (size dependence), so that an expansionary policy is most effective if the intervention (i) takes place during a recession and (ii) is relatively large. However, an expansionary policy is always considerably less potent than its contractionary counterpart. In fact, the adverse effect of a contractionary policy on output is large and relatively independent of the state of the business cycle.

Our use of Gaussian basis functions to approximate (and parametrize) impulse response functions relates to a large literature that relies on radial basis functions (of which Gaussian functions are one example) to approximate arbitrary multivariate functions (e.g., Buhmann, 2003) or to approximate arbitrary distributions using a mixture of Gaussian distributions (Alspach and Sorenson 1971, 1972, McLachlan and Peel, 2000). In economics, our parametrization of impulse responses relates to an older literature on distributed lag models and in particular on the Almon (1965) lag specification, in which the successive weights, i.e., the impulse response function in our context, are given by a polynomial function. Although Gaussian basis functions provide a more natural and more parsimonious way than polynomials to approximate mean-reverting impulse response functions, our approach is general and other basis functions are possible. For instance, the inverse quadratic function, which is also a popular radial basis function, could also be used to parametrize impulse response functions. In fact, in a different context, Jorgenson (1966) suggested that ratios of polynomials, of which the inverse quadratic function is one example, could be used to parametrize distributed lag functions. Finally, our approach shares with the non-parametric econometrics literature (Racine, 2008) the insight that mixtures of Gaussian kernels can approximate very general shapes, although we use that insight in a very different manner.

The literature has so far tackled the estimation of non-linear effects of shocks in two main ways.[4]

A first approach estimates non-linear effects by regressing a variable of interest on contemporaneous and lagged values of the structural shocks while allowing for possible non-linear effects. Naturally, this approach requires that a series of structural shocks has been previously (correctly) identified. In the context of monetary policy, Cover (1992), DeLong and Summers (1988) and Morgan (1993) proxy shocks with unanticipated money innovations (obtained from a money supply process regression, following Barro, 1977) and test whether the impulse response function depends on the sign of the shock. However, in order to recover the full non-linear impulse response function, these regressions require the estimation of many free parameters, so that the approach is quickly limited by efficiency considerations, and aside from asymmetry the identification of non-linearities is not feasible. Moreover, the approach is limited by the difficult issue of shock identification.[5] However, the approach has been recently revived thanks to the use of narratively identified shocks (Romer and Romer, 2004) and thanks to the local projection method pioneered by Jorda (2005).[6] The narrative approach was precisely developed in order to identify exogenous monetary innovations, and Jorda's method can easily accommodate non-linearities in the response function while remaining substantially less demanding in terms of free parameters.[7] However, even the local projection method is limited by efficiency consideration. Indeed, while the Jorda approach is intentionally model-free –not imposing any underlying dynamic system–, this can come at an efficiency cost (Ramey, 2012),

[4]A third non-linear approach was recently proposed by Angrist et al. (2013) who develop a semi-parametric estimator to evaluate the (possibly asymmetric) effects of monetary policy interventions. They find asymmetric effects of monetary shocks consistent with our findings.

[5]Indeed, unlike the modern VAR approach to identifying exogenous monetary shocks (and unlike our methodology), money supply regressions do not allow the monetary policy rule to respond to the state of the economy, making the mapping from innovations to structural shocks difficult.

[6]The combination of Jorda's method with narratively identified shocks was first introduced in the context of fiscal policy by Auerbach and Gorodnichenko (2013) in order to test for the existence of state dependence in the effects of fiscal policy.

[7]Tenreyro and Thwaites (2013) and Santoro et al. (2014) use the Jorda method to estimate the extent of state dependence in the effect of monetary policy.

which makes inferences on a rich set of non-linearities (e.g., sign-, size-, and state-dependence) difficult. In contrast, by positing that the response function can be approximated by one (or a few) Gaussian functions, our approach imposes strong dynamic restrictions between the parameters of the impulse response function, which in turn allow us to estimate a rich set of non-linearities.[8] Another advantage of our approach is that it can be used for model selection and model evaluation through marginal likelihood comparisons.

A second strand in the literature has relied on threshold models to allow for non-linearities.[9] However, threshold models are ill suited to identify how the impulse response function to a given structural shock depends on the value (sign or size) of that shock. Indeed, the threshold in regime-switching models applies to a "switching variable" (e.g., past unemployment, past output growth or some past unobserved variable), which is not the contemporaneous structural shock itself but instead a variable function of *past* reduced-form shocks. As a result, there is no "right" switching variable, and results are sensitive to the particular choice of switching variable (Weise, 1999). In contrast, our approach allows us to estimate how the impulse response function varies with the *contemporaneous* value of the shock. In this sense, the switching variable is the contemporaneous structural shock itself, and our framework allows the effect of a shock to change continuously with this switching variable.[10] Our model with state dependence connects to the threshold model literature, in that we allow the effect of policy to depend on the state of the business cycle, which is measured using an indicator variable similar to the threshold variable. However, by explicitly separating the roles played by sign-, size-, or state-dependence, our approach is more general and can help better identify the contribution of each type of non-linearity.

Section 2 presents the empirical model, Section 3 presents the theoretical background be-

[8]Naturally, this statement also implies that our results are valid under the assumption that response functions can be well approximated by a few Gaussian functions. In this respect, our approach is best seen as complementing the model-free approach of Jorda (2005).

[9]See, among others, Beaudry and Koop (1993), Thoma (1994), Potter (1995), Koop, Pesaran and Potter (1996), Koop and Potter, (1998), Ravn and Sola (1996, 2004), Weise (1999), Lo and Piger (2005).

[10]Note that this is not the case in regime-switching models in which regime switches are necessarily discrete events.

hind our use of Gaussian basis functions; Section 4 describes possible parameterizations with Gaussian functions; Section 5 describes the estimation routine; Section 6, 7 and 8 present evidence for the existence of, respectively, asymmetry, size dependence and state dependence; Section 9 concludes.

2 Empirical model

Our starting point is a general description of the economy, in which the behavior of a system of macroeconomic variables is dictated by its response to past and present structural shocks.

Specifically, denoting $\mathbf{Y}_t$ a vector of macroeconomic variables (e.g., output growth, inflation, policy stance), the economy is described by

$$\mathbf{Y}_t = \sum_{k=0}^{\infty} \mathbf{\Psi}_k(\varepsilon_{t-k}, \mathbf{Z}_{t-k})\varepsilon_{t-k} \tag{1}$$

where boldface letters are used to indicate vectors or matrices of variables or coefficients. ε_t is the vector of structural innovations with $E\varepsilon_t = \mathbf{0}$ and $E\varepsilon_t\varepsilon_t' = \mathbf{I}$, $\mathbf{\Psi}_k(\varepsilon_{t-k}, \mathbf{Z}_{t-k})$ is the matrix of lag coefficients and $\mathbf{Z}_t$ is a vector a macroeconomic variables. $\mathbf{Z}_t$ can either be a function of past endogenous variables $\{\mathbf{Y}_{t-j}\}_{j>0}$ or a function of exogenous variables.

Equation (1) can be seen as a non-linear vector moving average representation of the economy, in which the matrices $\{\mathbf{\Psi}_k\}$ –the coefficients of the impulse response functions to shocks– can depend on the values of the structural shocks in ε_{t-k} and on the values of $\mathbf{Z}_{t-k}$ at the time of the shocks ε_{t-k}. With $\mathbf{\Psi}_k$ a function of ε_{t-k}, the impulse response functions to a given structural shock depend on the value (sign or size) of that shock. For instance, a positive shock may trigger a different impulse response than a negative shock. With $\mathbf{\Psi}_k$ a function of $\mathbf{Z}_{t-k}$, the impulse response functions to a shock depend on the values of the variables in $\mathbf{Z}$ at the time of that shock. For instance, if Z_t captures lagged output growth, the response function may be different depending on the state of the business cycle (recession or expansion) at the time of the shock.

In this paper, we will focus on the effect of monetary shocks, although the approach is general and can be applied to studying the effects of other structural shocks. In our case, to ensure identification of the monetary policy innovations, we assume that the standard recursive identification restriction is verified: Monetary policy shocks are assumed to not affect macro variables within the current period.[11] Ordering the measure of the policy stance last in $\mathbf{Y}_t$, this implies that $\mathbf{\Psi}_0$'s last column is filled with 0 except for the last entry.[12]

If (1) admitted a VAR representation, the model could be estimated by first estimating a VAR for $\mathbf{Y}_t$ and then inverting it to get the moving average representation (1). However, this standard approach is only possible if (1) admits a VAR representation, which imposes that the model is linear, i.e., $\mathbf{\Psi}_k(\varepsilon_{t-k}, \mathbf{Z}_{t-k}) = \mathbf{\Psi}_k$,[13] and prevents the study of possible non-linear effects of monetary policy.

3 Parameterizing impulse response functions using Gaussian basis functions

In this paper, we propose an approach to estimate a non-linear model like (1). Rather than looking for a VAR representation of the dynamic system (1), our aim is to directly estimate (1), the "MA representation" of the economy. Because the number of free parameters $\mathbf{\Psi}_k$ in (1) is infinite, our strategy consists in parameterizing the impulse response functions, i.e., in positing a functional form for each impulse response function $\psi_{ij}(k)$ (with $\psi_{ij}(k)$ the row-i column-j coefficient of $\mathbf{\Psi}_k$, i.e., the impulse response function of variable i to shock j). To parametrize the impulse response functions, we will use Gaussian basis functions.

We first present the theoretical background supporting our parametrization, discuss the

[11] In the context of monetary shocks, this identification scheme is standard in the literature. See, among others, Bernanke and Blinder (1992), Christiano et al. (1999, 2005).

[12] As an alternative, we will also use our framework with the Romer and Romer (2004) shocks identified with a narrative approach.

[13] In addition, the existence of a VAR representation imposes some restrictions on the $\mathbf{\Psi}_k$ coefficients. The matrices $\mathbf{\Sigma}_k \equiv \mathbf{\Psi}_0^{-1}\mathbf{\Psi}_k$ must be such that there exists $p > 0$ and matrices $\Phi_1, ..., \Phi_p$ such that $\mathbf{\Sigma}_k$ can be written as $\mathbf{\Sigma}_k = \Phi_k\mathbf{\Sigma}_{k-1} + ... + \Phi_p\mathbf{\Sigma}_{k-p}$, $\forall k > 0$.

intuition and motivation behind our approach in practice, and finally present the different specifications that we will estimate.

3.1 Theoretical background

Our parametrization of the impulse response functions builds on the following theorem, which states that any mean-reverting impulse response function can approximated with a sum of Gaussian functions.

Theorem 1: Let f be a bounded continuous function on $\mathbb{R}$ that satisfies $\int_{-\infty}^{\infty} |f(x)|\, dx < \infty$. There exists a function f_N defined by

$$f_N(x) = \sum_{n=1}^{N} a_n e^{-(\frac{x-b_n}{c_n})^2}$$

with a_n, b_n, $c_n \in \mathbb{R}$ for $n \in \mathbb{N}$, such that the sequence $\{f_N\}$ converges pointwise to f on every interval of $\mathbb{R}$.

Proof: See Appendix.

Using Theorem 1, we can approximate any mean-reverting (in the sense that $\int_{0}^{\infty} |\psi(k)|\, dk < \infty$) impulse response function $\psi(.)$ with a sum of Gaussian functions, so that

$$\psi(k) \simeq \sum_{n=1}^{N} a_n e^{-(\frac{k-b_n}{c_n})^2}$$

with a_n, b_n, $c_n \in \mathbb{R}$ for k over some interval of $\mathbb{R}_+$.

A limitation however is that we need the impulse response functions to be bounded and integrable, which restricts our approach to stationary series (so that $\mathbf{\Psi}_k \underset{k\to\infty}{\longrightarrow} \mathbf{0}$).[14,15]

[14]Another (practical this time) limitation is that since we are estimating an *infinite* moving average model with a *finite* number of observations, the impulse responses must satisfy $\mathbf{\Psi}_k \underset{k\to\infty}{\longrightarrow} \mathbf{0}$).

[15]With non-stationary series, an option is thus to take a stationary transformation of the data (e.g., taking first-differences) so as to only include stationary series in the model.

3.2 Intuition and Motivation

The interest of our approach, and its use for studying the (possibly non-linear) effects of policy, rests on the fact that, in practice, only a very small number of Gaussian basis functions are needed to approximate a typical impulse response function.

Intuitively, impulse response functions of stationary variables are often found to be monotonic or hump-shaped (e.g., Christiano, Eichenbaum, and Evans, 1999 or Uhlig, 2005).[16] And in such cases, a single Gaussian function can already provide a good approximate description of the impulse response. To illustrate this observation, Figure 1 plots the impulse response functions of output, the price level and the fed funds rate to a monetary shock estimated from a standard VAR specification,[17] along with the corresponding Gaussian approximations with only *one* Gaussian function, i.e., using the approximation

$$\psi(k) \simeq ae^{-\frac{(k-b)^2}{c^2}}. \tag{2}$$

We can see that the one-Gaussian parametrization already does an excellent job at approximating the impulse responses implied by the VAR.

The small number of free parameters (only three per impulse response function in the one-Gaussian case), will have two important advantages. First, it will allow us to directly estimate the impulse response functions from the MA representation (1).[18] Second, it will allow us to introduce (and estimate) asymmetric or non-linear effects of monetary policy.

Another advantage of the one-Gaussian parametrization is the fact that the a, b and c coefficients can be very easily interpreted in light of the impulse-response function. With a one-Gaussian parametrization, the impulse response function is summarized by three parameters –

[16] This is also the case in theoretical models, e.g., New-Keynesian models, in which the impulse response functions are generally monotonic or hump-shaped (see e.g., Walsh 2010).

[17] See Section 6 for the exact specification of the SVAR behind Figure 1. The VAR is specified with (log) industrial production, inflation and the fed funds rate. The impulse response for the price level is calculated from the response of inflation.

[18] For instance, with 3 variables, we only have one $9 * 3 = 27$ parameters to estimate to capture the whole set of impulse response functions $\{\mathbf{\Psi}_k\}_{k=1}^{\infty}$.

peak effect, timing of peak effect, and persistence of peak effect–, which are generally considered the most relevant characteristics of an impulse response function.[19] As illustrated in Figure 3, parameter a is the height of the impulse-response, which corresponds to the maximum effect that a *unit* shock has on the variable of interest. In other words, a is the maximum marginal effect of a shock. Parameter b is the timing of this maximum effect. Parameter c captures how persistent the effect of the shock is. Specifically, for the latter term c, it is easy to show that the amount of time τ required for the effect of a shock to be 50% of its maximum value is given by

$$\tau = c\sqrt{\ln 2}.$$

A final important advantage of (2) is that, in the context of Bayesian estimation, the ease of interpretation of the a,b and c parameters makes prior elicitation easier than in standard VARs, in which the VAR coefficients have a less direct economic interpretation.

4 Examples of parametrization with Gaussian functions

In this section, we consider different parametrizations of interest, starting with linear models, using either a one-Gaussian or two-Gaussian approximation of the impulse responses, and then allowing for non-linearities with sign-dependence (asymmetry), size-dependence and state-dependence.

4.1 Linear effects of policy shocks: one-Gaussian approximation

First, as a benchmark case, consider the general model (1) in the linear case, i.e., when the economy is described with

$$\mathbf{Y}_t = \sum_{k=0}^{\infty} \mathbf{\Psi}_k \varepsilon_{t-k}. \tag{3}$$

[19]For instance, when comparing the effects of monetary shocks across different specifications, Coibion (2012) focuses on the peak effect of the monetary shock, which in the one-Gaussian parametrization is simply parameter a.

Using a one-Gaussian approximation of the impulse responses, the k lag coefficient $\psi_{ij}(k)$ (with $\psi_{ij}(k)$ the row-i column-j coefficient of $\mathbf{\Psi}_k$) is given by

$$\psi_{ij}(k) = a_{ij} e^{-\frac{(k-b_{ij})^2}{c_{ij}^2}}, \ \ \forall k > 0 \tag{4}$$

with a_{ij}, b_{ij}, c_{ij} some constants to be estimated.

In practice, to allow for more flexibility, we will let (4) only hold for $k > 0$, and we will leave the contemporaneous coefficient matrix $\mathbf{\Psi}_0$ unconstrained except that the elements above the main diagonal are zero (the structural identifying restriction) and that the diagonal elements are positive (a normalization). We also allow for intercepts in each equation.[20]

4.2 Linear effects of policy shocks: two-Gaussian approximation

To allow for a richer set of impulse-responses, in particular to allow for an overshooting (or oscillating) pattern in the impulse-response, we can consider approximating the impulse response functions with two Gaussian functions, namely

$$\psi_{ij}(k) = a_{ij,1} e^{-\frac{(k-b_{ij,1})^2}{c_{ij,1}^2}} + a_{ij,2} e^{-\frac{(k-b_{ij,2})^2}{c_{ij,2}^2}}, \ \ \forall k > 0 \tag{5}$$

with $a_{ij,1}$, $b_{ij,1}$, $c_{ij,1}$ and $a_{ij,2}$, $b_{ij,2}$, $c_{ij,2}$ some constants to be estimated. Again, the contemporaneous coefficient matrix $\mathbf{\Psi}_0$ is left unconstrained.

4.3 Asymmetric effects of policy shocks

With a parametric representation of the impulse response functions in hand, it is simple to extend the approach to allow for asymmetries by allowing $\mathbf{\Psi}_k$ to depend on the sign of the structural shock and to take two possible values $\mathbf{\Psi}_k^+$ or $\mathbf{\Psi}_k^-$. Specifically, a model that allows

[20]Note that a linear VAR has a lot more free parameters than our linear parametrized model. A VAR with 3 variables and 12 lags (as used in this paper) has $12 * 3^2 + 6 + 3 = 117$ free parameters, while the one-Gaussian model has only 3^3 terms for the Gaussian functions $+6$ free contemporaneous coefficients ($\mathbf{\Psi}_0$) $+3$ intercepts $= 36$ free parameters.

for asymmetric effect of monetary shocks would simply write

$$\mathbf{Y}_t = \sum_{k=0}^{\infty} \mathbf{\Psi}_k^+ \mathbf{1}_{\varepsilon \geq \mathbf{0}} \varepsilon_{t-k} + \sum_{k=0}^{\infty} \mathbf{\Psi}_k^- \mathbf{1}_{\varepsilon < \mathbf{0}} \varepsilon_{t-k} \tag{6}$$

with $\mathbf{\Psi}_k^+$ and $\mathbf{\Psi}_k^-$ the lag matrices of coefficients for, respectively, positive and negative shocks. For instance, in a bivariate case, we would have

$$\mathbf{\Psi}_k^+ = \begin{pmatrix} \psi_{11}^+(k) & \psi_{12}^+(k) \\ \psi_{21}^+(k) & \psi_{22}^+(k) \end{pmatrix} \tag{7}$$

the coefficient matrix for positive shocks. A similar expression holds for $\mathbf{\Psi}_k^-$. We then have $\psi_{ij}(k) = \psi_{ij}^+(k)\mathbf{1}_{\varepsilon_i \geq 0} + \psi_{ij}^-(k)\mathbf{1}_{\varepsilon_i < 0.}$

Using a one-Gaussian approximation, the impulse-response function would be parametrized as

$$\psi_{ij}^+(k) = a_{ij}^+ e^{-\left(\frac{k-b_{ij}^+}{c_{ij}^+}\right)^2}, \;\; \forall k > 0 \tag{8}$$

with a_{ij}^+, b_{ij}^+, c_{ij}^+ some constants to be estimated. A similar expression would hold for $\psi_{ij}^-(k)$.

4.4 Size-dependent effects of policy shocks

With a parametric representation of the impulse response functions, it is also simple to allow for non-linearities (other than asymmetry). For instance, we can allow for size dependence with $\mathbf{\Psi}_k$ becoming $\mathbf{\Psi}_k(\varepsilon_{t-k})$, i.e., allow the response functions to a monetary shock to depend on the magnitude of the shock. Specifically, with a one-Gaussian approximation, one can estimate a model of the form

$$\begin{cases} \psi_{ij}(k) = a_{ij}(\varepsilon_{t-k}) e^{-\left(\frac{k-b_{ij}}{c_{ij}}\right)^2}, \;\; \forall k > 0 \\ a_{ij}(\varepsilon_{t-k}) = \alpha_{ij} + \beta_{ij} \left|\varepsilon_{t-k}\right| \end{cases} \tag{9}$$

with α_{ij}, β_{ij}, b_{ij} and c_{ij} parameters to be estimated. In this model, the "amplitude" (or the scale) of the impulse response (the parameter a_{ij}) is a linear function of the size of the shock.

Such a specification allows us to test whether the effect of monetary policy vary with the size of the policy intervention. Naturally, other (or more general) models are possible.[21]

4.5 Asymmetry and size-dependence

One can then combine the two previous specifications to construct a model that allows for both asymmetry and size-dependence. Specifically, using a one-Gaussian approximation, the impulse response function following a positive innovation could be parametrized as

$$\begin{cases} \psi_{ij}^{+}(k) = a_{ij}^{+}(\varepsilon_{t-k}) e^{-\left(\frac{k-b_{ij}^{+}}{c_{ij}^{+}}\right)^2}, \ \forall k > 0, \, \varepsilon_{t-k} > 0 \\ a_{ij}^{+}(\varepsilon_{t-k}) = \alpha_{ij}^{+} + \beta_{ij}^{+} \left|\varepsilon_{t-k}\right| \end{cases} \tag{10}$$

with α_{ij}^{+}, β_{ij}^{+}, b_{ij}^{+} and c_{ij}^{+} parameters to be estimated. An identical functional form holds for $\psi_{k,ij}^{-}$.

In this model, the maximum effect of a shock on a given variable can depend on the size of the shock, and that relation may be different for positive and negative shocks. In other words, the response function to a monetary shock is allowed to depend on both the value and the sign of the shock. In (10), we allow the scaling factor of the impulse response (a_{ij}) to be a linear function of the value of the shock.

4.6 Asymmetry, size-dependence and state dependence

Finally, we can also consider a more general framework in which the response functions to a shock depend on the state of the business cycle. For instance, we can allow for state dependence (in addition to sign and size dependence) by estimating a model with $\mathbf{\Psi}_k(\varepsilon_{t-k}, z_{t-k})$ and z_t a business cycle indicator that is a function of past observables $\mathbf{Y}_t$ (e.g., lagged output growth or output growth over the past X months). Using a one-Gaussian approximation, the impulse

[21] For instance, one could also allow the mode of the impulse-response (i.e., b_{ij}, the time to peak effect) to depend on the value of the shock.

response function following a positive innovation can be parametrized as

$$\begin{cases} \psi_{ij}^{+}(k) = a_{ij}^{+}(\varepsilon_{t-k}) e^{-\left(\frac{k-b_{ij}^{+}}{c_{ij}^{+}}\right)^2}, \ \forall k > 0, \, \varepsilon_{t-k} > 0 \\ a_{ij}^{+}(\varepsilon_{t-k}) = \alpha_{ij}^{+} + \beta_{ij}^{+} \left|\varepsilon_{t-k}\right| + \gamma_{ij}^{+} z_{t-k} \end{cases} \tag{11}$$

with α_{ij}^{+}, β_{ij}^{+}, γ_{ij}^{+}, b_{ij}^{+} and c_{ij}^{+} parameters to be estimated. An identical functional form holds for $\psi_{k,ij}^{-}$.

In this model, the amplitude of the impulse response depends on the sign and size of the shock, as well as the state of the business cycle at the time of the shock. In (11), we allow the amplitude of the impulse response (a_{ij}) to be a linear function of the indicator variable z_t. Such a specification allows us to test whether, for instance, an expansionary policy has a more stimulative effect on output in recessions than in expansions.[22]

5 Estimation

Once parametrized, model (1) can be estimated by maximum likelihood. In this section, we first describe how we construct the likelihood function by exploiting the prediction-error decomposition, and then we discuss the estimation routine.

5.1 Constructing the conditional likelihood function

We want to build the likelihood function for a sample of size T and parameter vector θ: $p(\mathbf{Y}_1, ..., \mathbf{Y}_T | \theta)$. To do this, we decompose the likelihood using:[23]

$$p(\mathbf{Y}_1, ..., \mathbf{Y}_T | \theta) = p(\mathbf{Y}_T | \mathbf{Y}_1, ..., \mathbf{Y}_{T-1}, \theta) \dots p(\mathbf{Y}_2 | \mathbf{Y}_1, \theta) p(\mathbf{Y}_1 | \theta) \tag{12}$$

[22] Importantly, note that even the most general non-linear model discussed in this paper (with sign-, size-, and state-dependence allowed for all shocks) has "only" $5*9*2+6*2+3 = 105$ free parameters in a tri-variate model. This is still less than a linear monthly VAR (with 12 lags) that has 117 free parameters.

[23] To derive the conditional densities in decomposition (12), our parameter vector θ thus implicitly also includes initial values for the lagged innovations. We will keep those fixed throughout the estimation and discuss alternative initializations below.

To calculate the one-step-ahead conditional likelihood function needed for the prediction error decomposition, we proceed as follows. First, we assume that all innovations $\{\varepsilon_t\}$ are Gaussian with mean zero and variance one.[24] We then use the MA representation (1) and the recursive identification scheme to uniquely identify the current period value of the innovations as a function of current period data and past innovations.

As an illustration, we discuss the construction of the likelihood in the case of asymmetry, i.e., when we consider the model (6), since the approach is identical in the more general non-linear models.

We first truncate the MA(∞) representation at some level K, and we set the first K values for ε to the zero vector.[25] Conditional on knowing the first $[t-K,t]$ innovations, the $t+1$ innovation is obtained from:

$$\mathbf{\Psi}_0^{\pm}\left(\varepsilon_{t+1}\right)\varepsilon_{t+1} = \mathbf{Y}_t - \sum_{k=0}^{K} \mathbf{\Psi}_k^{+}\mathbf{1}_{\varepsilon\geq\mathbf{0}}\varepsilon_{t-k} + \sum_{k=0}^{K} \mathbf{\Psi}_k^{-}\mathbf{1}_{\varepsilon<\mathbf{0}}\varepsilon_{t-k}$$

with $\mathbf{\Psi}_0^{\pm}\left(\varepsilon_{t+1}\right)$ a function of ε_{t+1} since the elements of $\mathbf{\Psi}_0^{\pm}$, the contemporaneous lag parameter matrix, depends on the sign of the elements of the innovation vector ε_{t+1}.

To calculate ε_{t+1} as a function of the parameters, one needs to uniquely pin down the value and sign of the components of ε_{t+1} as a function of the parameters. Fortunately, this is straightforward with a recursive identifying restriction scheme, since $\Psi_0^{\pm}(\varepsilon_{t+1})$ is lower triangular.

Denoting $\psi_{0,ij}^{+}$ the element of Ψ_0^{+} and using the bivariate case as an illustration, we have

$$\Psi_0^{\pm}(\varepsilon_{t+1}) = \begin{pmatrix} \psi_{0,11}^{\pm}(\varepsilon_{1,t+1}) & 0 \\ \psi_{0,21}^{\pm}(\varepsilon_{1,t+1}) & \psi_{0,22}^{\pm}(\varepsilon_{2,t+1}) \end{pmatrix} \tag{13}$$

with $\psi_{0,11}^{\pm}(\varepsilon_{1,t+1})$ indicating that the value of $\psi_{0,11}^{\pm}$ depends on $\varepsilon_{1,t+1}$, $\psi_{0,22}^{\pm}(\varepsilon_{2,t+1})$ depends on

[24] The estimation could easily be generalized to allow for non-normal innovations such as t-distributed errors. Note that even with Gaussian structural innovations, our non-linear model implies that the reduced-form residuals need not be Gaussian.

[25] Alternatively, we could use the first K values of the shocks recovered from a structural VAR.

$\varepsilon_{2,t+1}$, etc... A similar form for would hold in a higher-dimensional case.

The fact that $\Psi_0^{\pm}(\varepsilon_{t+1})$ is lower triangular with positive entries on the diagonal allows us to uniquely identify the value (and sign) of $\varepsilon_{1,t+1}$, the first component of ε_{t+1}, from the first equation with

$$\varepsilon_{1,t+1} = \frac{\mathbf{Y}_{1,t}}{\psi_{0,11}^{\pm}(\varepsilon_{1,t+1})}$$

and then uniquely identify the sign and value of each element of ε_t by iterating on (6). For instance, for $\varepsilon_{2,t+1}$, we have

$$\varepsilon_{2,t+1} = \frac{\mathbf{Y}_{2,t} - \psi_{0,21}^{\pm}(\varepsilon_{1,t+1})\varepsilon_{1,t+1}}{\psi_{0,22}^{\pm}(\varepsilon_{2,t+1})}. \tag{14}$$

Looking at (14), once the sign and value of $\varepsilon_{1,t+1}$ is known, the first column of $\Psi_0^{\pm}$ is pinned down (and thus $\psi_{0,21}^{\pm}(\varepsilon_{1,t+1})$ is known). Since the only unknown left in (14) is $\psi_{0,22}^{\pm}$, which is positive, we know the sign of $\varepsilon_{2,t+1}$ from the sign of $\mathbf{Y}_{2,t} - \psi_{0,21}^{\pm}(\varepsilon_{1,t+1})\varepsilon_{1,t+1}$. We then get the value of $\varepsilon_{2,t+1}$ from (14).

In a higher-dimensional case, we can proceed identically to uniquely identify the sign and value of each element of ε_t.

With the series of $\{\varepsilon_t\}$ in hand, we can then construct the conditional log likelihood function from equation (6).

When the model allows for state dependence, as in specification (11), the likelihood also depends on the value of the indicator function z_t. Technically, constructing the likelihood of this specification is a straightforward extension of our previous cases, because z_t is a function of lagged values of $\mathbf{Y}_t$. To see that, note that we use the prediction-error decomposition to construct the likelihood function. Thus, we build a sequence of densities for z_t that condition on past values of z_t, which includes past values of $\mathbf{Y}_t$. Thus, conditional on past values of $\mathbf{Y}_t$, z_t is known.[26]

[26] If we wanted to use an indicator function that was not part of the vector of endogenous variables Y_t, this would also be possible by using a quasi-likelihood approach. That is, we would build a likelihood function that not only conditions on the parameters, but also the sequence of indicators z_t. This would in general not be efficient because the joint density of z_t and Y_t could carry more information about the parameters in our model

5.2 Estimation routine

We now discuss the estimation routine. To estimate our model, we use the Metropolis-Hastings algorithm (see Robert & Casella 2004, Haario et al., 2001). To be specific, we use a Metropolis-within-Gibbs algorithm with the blocks given by the different groups of parameters in our model (one block being composed of all a parameters, another composed of all b parameters and so on). As described above, we build the likelihood assuming a Normal distribution for the innovations, and we set the first K innovations to zero. We use improper uniform priors so that our results are interpretable as maximum-likelihood estimates.

Importantly, it is straightforward to incorporate prior information in the Metropolis-Hastings algorithm, and our approach makes prior elicitation easy, because our coefficients have a direct economic interpretation.

Since we have a relatively large parameter space, it is important to initialize the Metropolis-Hastings algorithm in an area of the parameter space that has substantial posterior probability. To do so, we follow a two-step procedure: first, we estimate a standard VAR using OLS on our data set, calculate the MA representation, and we use the impulse response functions implied by the VAR as our starting point. Specifically, we first fit our parametrized impulse response functions to the VAR-based impulse response functions.[27] We then use these coefficients as a starting point for our maximization routines[28] that then give us a starting value for the Metropolis-Hastings algorithm.

Finally, in order to reduce the parameter space, in our benchmark estimations, we only allow for asymmetry or non-linearity in the impulse response functions to monetary shocks. The impulse response functions to the other shocks are set at their values estimated from the corresponding SVAR.[29] However, as a robustness check, we verify that our findings are not

than the conditional density we advocate using. As long as z_t is highly correlated with elements of (functions of) Y_t, this loss in efficacy will likely be small.

[27]Specifically, we set the parameters of our model (for instance, a, b and c in the benchmark model with one Gaussian function) to minimize the discrepancy (sum of squared residuals) between the two sets of impulse responses.

[28]We use both a simplex algorithm and Chris Sims' csminwel routine.

[29]We did a preliminary Metropolis-Hastings MCMC run in order to explore the likelihood around the VAR-

driven by this restriction, by estimating a model with non-linearity in response to all shocks.

6 Sign dependence (asymmetry) and the effects of monetary policy

In this section, we present our empirical results on the asymmetric effects of monetary shocks. After validating our parametric estimation procedure in the symmetric case using the standard SVAR approach as a benchmark, we present the asymmetric impulse response functions.

Our benchmark analysis is based on monthly data covering the period 1959:M1 to 2006:M12. Following the literature (e.g., Bernanke and Blinder, 1992), we take the federal funds rate as our measure of policy, and use innovations in the federal funds rate as a measure of monetary policy shocks. This is the reason why we exclude the 2008-2009 recession from our analysis, since the fed funds rate stood at the zero-lower bound for the whole period and no longer captured variations in the stance of monetary policy. We consider a standard trivariate monthly model with the log of industrial production, the PCE inflation rate and the federal funds rate. All series were detrended with a quartic trend.[30] We allow for an intercept in each equation. When constructing the likelihood, we truncate the MA representation at $K = 80$ months.

6.1 The linear case: VAR versus parametric estimation

First, we evaluate how our parametric approach, either using a one-Gaussian approximation or a two-Gaussian approximation performs in fitting the impulse response functions implied by a standard VAR with a recursive identifying restriction.

As previously discussed, a one-Gaussian approximation does a very good job at capturing the impulse response functions to a monetary shock implied by a VAR (Figure 1), only missing the initially slow response of output and the price puzzle. Using two Gaussians instead of

based impulse response functions to non-monetary shocks, and we found that these impulse response functions are in an area of high posterior probability.

[30] Alternative filters such as a conventional HP filter give very similar results.

one corrects this problem. As shown in Figure 2, a two-Gaussian approximation does an excellent job at capturing the impulse response functions to a monetary shock implied by a VAR, capturing both the initially slow response of output as well as the price puzzle.

Table 1 compares the log-likelihood of the VAR model with that from our parametric model, first using a one-Gaussian approximation and second using a two-Gaussian approximation. We can see that, with substantially fewer parameters (Footnote 21), our parametric approach fits the data almost as well as the VAR.

6.2 Evidence for asymmetry

We now estimate our model allowing for asymmetry depending on the sign of the monetary shock. In our benchmark specification, we parametrize the impulse response functions with a one-Gaussian approximation.

Allowing for asymmetry provides a dramatic improvement in the model fit of the data. As shown in Table 1, the log-likelihood is about 150 log-points higher when allowing for asymmetry. Looking at the impulse responses to a monetary shock, Figure 4 plots the responses (in percentage points) of output (industrial production), the price level and the federal funds rate to a one standard-deviation monetary shock. The thick lines denote the impulse response functions implied by the maximum likelihood estimates, and the error bands cover 90 percent of the posterior probability and are centered at the median.

The evidence for asymmetry is striking: following a positive shock to the fed funds rate (a contractionary policy), output growth decreases markedly. In contrast, following a negative shock to the fed funds rate (an expansionary policy), the response of output is small and non-significantly different from zero. This evidence suggests that the contractionary effect of monetary policy may have been substantially under-estimated, as the adverse effect of a contractionary monetary shock on output is as much as two- to three-times larger than suggested by the VAR estimate.

Importantly, note that the responses of the federal funds rate are very similar after positive or negative shocks, with similar persistence levels, indicating that the asymmetric responses of output are not driven by differences in the responses of the fed funds rate. The response of prices is also somewhat asymmetric: the price level declines more following a contractionary shock than it increases following an expansionary shock.

To assess whether the impulse response functions of output to positive and negative monetary shocks are statistically significantly different, we consider the difference, in absolute value, between the peak of the impulse response function of output to a positive shock and the peak of the impulse response function to a negative shock of the same size, i.e.,

$$D = \sup_k \left|\psi_k^+\right| - \sup_k \left|\psi_k^-\right|$$

where ψ_k^+ denote the period k response to a positive shock and ψ_k^- the period k response to a negative shock. With the one-Gaussian approximation, the D statistics is simply

$$D = a^+ - a^-.$$

Table 2 shows that the D-statistics for output is large and statistically significant, as the difference between the peak effect of a positive and the peak effect of a negative shock is large; in the order of magnitude of the peak response estimated from a VAR.

6.3 Robustness to model specification

In this subsection, we present results on the robustness of our baseline specification with asymmetry.

6.3.1 Two-Gaussian parametrization

To allow for a richer set of impulse-responses, in particular to allow for an overshooting (or oscillating) pattern in the impulse-response, we consider a two-Gaussian approximation of the impulse response functions, following (5).

Figure 5 plots the estimated impulse-responses to a monetary shock and points to the same conclusion: a contractionary monetary shock has a strong effect, while expansionary shocks have no significant effect. The response of the price level is in line with that of the one-Gaussian specification: inflation's response is stronger following a contractionary shock.

6.3.2 Post-1982 period

The 1979-1982 period was a time of unusual monetary policy, as the Fed officially stopped targeting the Fed funds rate. Although this period may also be particularly instructive for identifying the effects of monetary shocks, displaying the largest monetary shocks in the sample and being a clear example of a monetary policy driven recession (Coibion, 2012), we assess the robustness of our results to this period by estimating our model using data covering only 1983-2006.

Evidence for asymmetry is still strongly present: Table 2 shows that the D-statistic, although smaller than with the full sample and less precisely estimated (given that the sample size is less than one half of the original one), is still large (in the order of magnitude of the peak response estimated from a VAR) and significantly different from 0.

6.3.3 Alternative specifications

We verified the robustness of our results to using the growth rate of industrial production, instead of the (log) level, with inflation and the fed funds rate as used as previously. In this specification, the data were not detrended with a polynomial. Figure 7 shows again strong evidence that monetary policy has asymmetric effects on output.

Finally, using the unemployment rate as our measure of real activity and the GDP deflator as our price measure does not change our conclusions.[31] The impulse response function (not shown) are qualitatively similar, and Table 2 shows that the D statistic is large (in the order of magnitude of the peak response estimated from a VAR) and significantly different from zero

[31] The GDP deflator was linearly interpolated at a monthly frequency from quarterly data.

both when using the whole 1959-2006 sample period or the 1983-2006 period.

6.3.4 Allowing for asymmetry in response to all shocks

Finally, although the baseline specifications only allowed for asymmetry in response to monetary shocks, and not in response to the other (unidentified) shocks, it is also possible to allow for asymmetry with respect to the other disturbances. Although it considerably increases the parameter space, we find evidence for asymmetry in response to monetary shocks but no evidence for asymmetry in response to the non-monetary shocks, in line with our benchmark approach. As shown in Table 2, the D-statistics is highly significant, and in line with the findings from previous specifications.

6.4 Robustness to shock identification: using the Romer and Romer monetary shocks

Our evidence for asymmetry has so far been based on using a recursive identifying restriction to recover the monetary shock. In this section, we instead use monetary shocks identified through a different approach: the narrative approach from Romer and Romer (2004). Proceeding as in Romer and Romer (2004), we use the Romer and Romer monetary shocks instead of the fed funds rate in our model (1).[32]

As pointed out by Coibion (2012), the advantage of this procedure is that one should be able to more precisely identify the effects of monetary shocks than in the previous specification with the fed funds rate, since the Romer and Romer measure controls for much of the endogenous fluctuations in the interest rate as well as the Fed's information set.

The Romer and Romer series were recently extended by Coibion et al. (2012), so that our sample covers 1966-2006. Figure 6 plots the results and shows similar results as previously with strong evidence that monetary policy has asymmetric effects on output. And again, the

[32] Alternatively, we could have used the cumulative sum of Romer and Romer monetary shocks as the measure of monetary policy. Results are very similar.

results suggest that the contractionary effect of monetary policy on output is substantially larger than suggested by VAR estimates.

7 Size dependence and the effects of monetary policy

In this section, we enrich our empirical model by allowing for size-dependence in the effect of monetary shocks, that is by allowing the effect of monetary policy to depend on the magnitude of the intervention.

We do so in a simple way; by allowing the scaling factor of the impulse response function –the a coefficient in a one-Gaussian case– to depend on the size of the shock. Given the strong evidence for asymmetry found in the previous section, we still allow for asymmetric effects by monetary policy, so that we estimate specification (10) described in Section 4.

Before discussing our parameter estimates in more details, Table 1 shows that the log-likelihood of the model with size-dependence is substantially higher than a simpler model with only asymmetry, and a likelihood-ratio test easily rejects the null in favor of the model with size-dependence.

Figure 8 presents the estimated impulse response functions as we vary the size of the monetary shock from -1.5 standard deviation to +1.5 standard deviation (roughly .70 percentage points). While there appears to be little non-linear effect of shock size on the impulse responses of inflation and the fed funds rate, the response of output depends non-linearly on the size of the shock.

To visualize this result more easily, Figure 9 plots the maximum effect of a monetary shock on output, as a function of the value (size and sign) of the shock. Going back to (1), the maximum effect of a shock ε^p on a variable of interest is given by $\max_k \psi_k(\varepsilon^p)\varepsilon^p$, which in the one-Gaussian basis function case is simply $a(\varepsilon^p)\varepsilon^p$. Intuitively, the maximum effect of a shock is the product of the size of the shock times the maximum marginal effect of that shock –the coefficient $a(\varepsilon^p)$– (as discussed in section 3.2).

Figure 9 plots the peak effect on output for shocks varying from -2 to $+2$ standard deviations, which correspond to shocks ranging from -1.1 to +.85 percentage points of fed funds rate.[33] To ease comparison with estimates from the corresponding linear VAR, Figure 9 also shows (thick dashed black line) the peak effect of a monetary shock as a function of the size of the shock. That function is linear, since in a linear framework the maximum marginal effect of a shock is constant and independent of the size of the shock.

Figure 9 confirms that monetary shocks have strongly asymmetric effects on output. Looking first at contractionary shocks, the thick solid blue line (estimated from our non-linear framework) lies substantially below the dashed black line (estimated from a linear VAR), indicating that the contractionary effects of contractionary shocks on output are substantially understated by the VAR. In contrast, looking now at expansionary shocks, the thick blue line is not significantly different from zero and lies substantially below the dashed black line, indicating that an expansionary monetary policy has little expansionary effect on output and that linear estimates may substantially overstate the effect of expansionary policy.

Turning to size dependence, the non-linearities work in opposite directions for positive and negative shocks. While small expansionary shocks are clearly neutral, large expansionary shocks may have some positive effect on output, although the error bands are large. In fact, an expansionary shock can have some positive effect on output (although still smaller than suggested by a VAR estimate), but the policy intervention has to be large (a cut in the fed funds rate of at least .50 percentage points).

Interestingly, the opposite is true for contractionary shocks. Small contractionary shocks have the strongest marginal effect on output, and as one increases the size of the contractionary shock, the marginal effect of the shock slightly declines (i.e., the thick blue line increases less fast). In fact, for shocks above +.85 percentage points, the marginal effect of the shock (the slope of the thick blue line) is equal to the VAR estimate. Importantly, this finding indicates that the strong contractionary effect of monetary policy that we uncovered is not driven by large outlier shocks.

[33]Variation across shock sizes is asymmetric, because the variance of expansionary shocks was estimated to be larger than the variance of contractionary shocks.

8 State dependence and the effects on monetary policy

We now further enrich our model by allowing the effect of monetary policy to depend on the state of the business cycle *in addition to* the sign and the size of the shock. Intuitively, we would like to test whether monetary policy is more powerful at stimulating the economy in a period of low growth than in a period of high growth, and whether the response of inflation depends on the amount of slack in the economy.

We thus estimate specification (11), where the peak effect of monetary policy is allowed to depend on (i) the sign of the shock, (ii) the magnitude of the shock, and (iii) the state of the business cycle at the time of the shock. As cyclical indicator (z_t), we use the average monthly growth rate in industrial production growth over the past 12 months.[34]

Table 1 shows that the log-likelihood of the state-dependent model is substantially higher than a simpler model with only asymmetry and size-dependence, and a likelihood-ratio test easily rejects the null in favor of the model with state-dependence.

Since the effects of a shock now depends on 3 parameters –the sign of the shock, the size of the shock and the state of the cycle–, there are different ways to visualize the results. We will thus present our results from two different angles: in the first one, we vary the state of the cycle holding the size of the shock fixed, and in the second one, we vary the size of the shock holding the state of the cycle fixed.

Taken together, our results will show that, during recessionary periods, expansionary monetary policy does have some stimulating effect on output with no inflationary consequences, but that, during expansionary periods, expansionary monetary policy only generates inflation. The non-significant effect of expansionary policy on output uncovered previously (Figure 4 or 9) thus seems to be the result of a "composition effect" coming from the fact that the economy alternates between expansions and recessions. Monetary policy does have some simulating effects on output but *only* in recessions. As a result, on average, the expansionary effect appears uncertain and not significantly different from zero.

[34] Using the unemployment rate as an alternative cyclical indicator gives very similar conclusions.

8.1 The distribution of shocks over the cycle

As a preliminary step, and to help put our results into perspective, Figure 10 plots the annualized monthly growth rate of IP (i.e., the indicator variable z_t) over the past 50 years, along with the identified monetary shocks, and Figure 11 plots the distribution of estimated monetary shocks over the state of the business cycle. We can see that monetary shocks are roughly equally distributed between expansions and recessions, and that shocks are observed over a wide range of values of the indicator function, increasing our confidence in the validity of our estimates.

8.2 State dependence holding the size of the shock fixed

To understand our estimates, we first vary the state of the cycle while holding the size of the shock fixed. Specifically, Figure 12 plots how the peak effects of monetary policy on output and inflation vary with the state of the cycle, both for contractionary and for expansionary shocks. In all cases, we explore the responses of output and inflation to a one standard-deviation monetary policy shock (roughly a 45 basis points change in the fed funds rate). Specifically, in the top panels, we plot the function

$$a^{+}(z) = \alpha^{+} + \beta^{+}\,|+1| + \gamma^{+} z$$

which captures the peak response of respectively output (upper-left quadrant) and inflation (upper-right quadrant) to a one-standard deviation contractionary shock (+1). Similarly, in the bottom panels, we plot the functions $a^{-}(z) = \alpha^{-} + \beta^{-}\,|-1| + \gamma^{-} z$ for a one-standard deviation expansionary shock (-1) for both output and inflation.

We first discuss the response of output. The upper-left quadrant in Figure 12 depicts how the peak effect of a contractionary shock on output varies as we move from a recession (IP growth rate of about -4 percent) to an expansion (IP growth rate of about +8 percent). The thick dashed line represents the VAR estimate. Since the VAR is linear, that latter estimate is constant and the peak effect of monetary policy is independent of the state of the business cycle. The thick blue depicts estimates from our non-linear framework. We can notice that the effect of a contractionary policy is relatively independent of the state of the business cycle, always having a large contractionary effect on output and substantially larger than estimated with a VAR.

A very different picture holds for expansionary shocks (bottom left quadrant). This time, the effect of monetary policy on output depends crucially on the state of the business cycle: the lower the level of output (IP) growth, the larger the effect of an expansionary policy. In fact, it is only in large slumps that monetary policy can have any expansionary effect on output. In fact, the effect becomes significantly positive when (annualized) monthly IP growth falls to about -4.5%. Figure 10 shows that growth rates in IP of -4.5% are not uncommon, having been observed in all past NBER recessions.[35] Thus, expansionary policy does seem to have some stimulative effect on output but only in periods of great slack. This result suggests that the small and insignificant stimulative effect of expansionary policy uncovered in previous sections is the result of a composition effect, as the economy alternates between expansions (in which expansionary policy has no effect) and recessions (in which expansionary policy has some stimulative effect).

Nonetheless, the asymmetry between expansionary and contractionary interventions remains, and an expansionary policy is always considerably less potent than its contractionary counterpart. For instance, even in a deep recession (IP growth rate of -4.5%), a 45 basis point rate cut has an effect on output (of about +0.3 percentage points) that is substantially smaller (in absolute value) than the effect of a 45 basis point rate increase (that triggers a 0.8 ppt decline in output).

We now turn to the response of inflation, depicted in the right-hand column of Figure 12. Although the estimates are more uncertain, the estimates suggest that, regardless of the sign of the shock, the stronger the state of the economy, the more responsive is inflation to monetary

[35]While the unconditional average IP growth rate over 1959-2006 is about 3.2%, the average IP growth during NBER recessions is about -3%.

shocks. In particular, while an expansionary policy has no inflationary effect in a recession, it does raise prices in an expansion, although again the point estimates are more uncertain. A similar conclusion holds for contractionary shocks; a contractionary shock has a stronger deflationary effect in a boom, with the point estimate being significantly different from zero when the economy is growing.

8.3 Size dependence holding the state of the cycle fixed

We now look at our estimates from a different angle, and we proceed as for Figure 9 in section 7 by varying the size of the monetary intervention. Specifically, we keep the state of the business cycle constant and instead vary the size of the monetary shock. We consider two opposite phases of the business cycle: an expansionary stage and a recessionary stage. We call an expansionary phase a period where IP growth is one standard-deviation above its mean (annualized growth rate of about +8%), and we call a recessionary phase a period where IP growth is one standard-deviation below its mean (growth rate of about -2%).

Similarly to Figure 9, Figure 13 and Figure 14 plot the peak effect of monetary policy on output and inflation as a function of the size of the shock, varying the size of the monetary intervention from -2 to $+2$ standard deviations, which correspond to shocks ranging -1.1 to +.85 percentage points of fed funds rate.[36] Comparing the top and bottom panels of Figure 13 shows again how expansionary monetary policy only has expansionary effects on output in recessions. And supporting our previous conclusion on size-dependence, the larger the expansionary shock, the stronger the expansionary effect of policy on output. In expansions however, expansionary policy is close to neutral, and only leads to inflation (Figure 14). In fact, the inflationary effect of an expansionary policy during a boom increases more than proportionally with the size of the shock. For an 80 basis points reduction in the fed funds rate during an expansion, the inflationary effect is already twice as large as that implied by the VAR estimate.

[36] Again, the variation in shock size is asymmetric, because the variance of expansionary shocks was estimated to be larger than the variance of contractionary shocks.

In contrast, the effects of contractionary shocks are relatively insensitive to the state of the business cycle. The effect on output are large, and much larger than estimated with a linear VAR, while the effects on inflation are not significantly different from the VAR estimates.

8.4 Robustness to shock identification

Finally, as a robustness check, we repeat our exercise using the Romer and Romer monetary shocks instead of the fed funds rate, as in sub-section 6.4. Similarly to Figure 12, Figure 15 plots how the peak effects of monetary policy on output and inflation vary with the state of the cycle, both for contractionary and for expansionary shocks.

Our main result that an expansionary policy can only have an expansionary effect during recessions is still clearly present, with a similar point estimate for the γ^- coefficient for output (the slope of the thick solid line in the bottom left quadrant). The effect of the state of the cycle on the response of inflation is also qualitatively similar, although the point estimates are smaller, meaning that the response of inflation appears less sensitive to the state of the cycle than previously estimated. Finally, the contractionary effect of policy on output (upper left quadrant) now seems to depend on the state of the cycle with a contractionary policy having a stronger adverse effect on output in a recession, although looking at the error bands, one cannot reject the null of no state dependence at the 90% confidence level.

9 Conclusion

This paper provides new evidence for the existence of asymmetry, size-dependence and state-dependence in the transmission mechanism of monetary policy. To do so, we develop a non-linear method to identify the effects of structural shocks, which consists in parameterizing the impulse response functions using Gaussian basis functions.

We find that a contractionary policy has a strong adverse effect on output, much stronger than linear estimates suggest, but an expansionary policy has, on average, no significant effect on output. An expansionary policy can have some expansionary effect on output, but only if the intervention is large and during a recession. However, an expansionary policy is always considerably less potent than its contractionary counterpart.

These results are not only important in the context of monetary policy making but may also help better understand the transmission mechanism of monetary policy and in particular the respective importance of the money channel and the credit channel. Under a credit channel, asymmetric effects of monetary policy can arise, because increasing borrowing costs may reliably slow an expansion, while cheap capital need not stimulate investment in a downturn.[37] Under a traditional money channel, the existence of downward price or wage rigidity can also generate asymmetric effects as wages (or prices) do not adjust in response to a contractionary policy. Since we find that inflation actually responds markedly (and declines) following a contractionary shock, our results point to a different mechanism than downward price rigidity and leave the possibility of a mechanism operating through credit. At the same, the state-dependent response of inflation to monetary policy (with inflation responding more strongly to shocks during booms) also points to a role for a traditional money channel with state-dependent price setting. Exploring our findings in the context of a structural macro model is thus an important task for future research.

Although this paper studies non-linearities in the effect of monetary policy, our parametric approach can be applied to other recursive identification schemes and can thus be used to estimate the non-linear effects of other important shocks where the existence of asymmetry, size-dependence or state-dependence remains an important and unresolved question; for instance, fiscal policy shocks (Romer and Romer, 2010, Auerbach and Gorodnichenko, 2012, Ramey and Zubairy, 2014), oil price shocks (Hamilton, 2003) or credit spread shocks (Gilchrist and Zakrajsek, 2012). Finally, while we presented our method in the context of a recursive identification scheme, the method is quite general and can also be applied to other population identification schemes, such as sign-restrictions (Uhlig, 2005) or long-run restrictions (Blanchard and Quah, 1989, Gali 1999), which we are currently exploring in on-going work.

[37]See Morgan (1993) for a review of possible reasons for asymmetry.

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Appendix: Proof of Theorem 1

Following Alspach and Sorenson (1971, 1972) in the context of approximating distributions, the problem of approximating a function f can be considered within the context of delta families of positive types.

Delta families are families of functions which converge to a delta function as a parameter characterizing the family converges to a limit value.

Let $\{\delta_\lambda\}$ be a family of functions on the interval $]-\infty, +\infty[$ which are integrable over every interval. $\{\delta_\lambda\}$ forms a delta family of positive type if the following conditions are satisfied:

1. For every constant $\gamma > 0$, δ_λ tends to zero uniformly for $\gamma \leq |x| \leq \infty$ as $\lambda \to \lambda_0$
2. There exist s in $\mathbb{R}$ so that $\int_{-s}^{s} \delta_\lambda(x)dx \longrightarrow 1$ as λ tends to some limit value λ_0
3. $\delta_\lambda(x) \geq 0$ for all x and λ

Defining

$$\delta_\lambda(x) \equiv G_\lambda(x) = \frac{1}{\sqrt{2\pi\lambda^2}} e^{-\frac{x^2}{\lambda^2}}, \tag{15}$$

it is easy to see that the Gaussian functions $\{G_\lambda\}$ form a delta family of positive type as $\lambda \to 0$ (i.e., $\lambda_0 = 0$). That is, the Gaussian function tends to the delta function as the variance tends to zero.[38]

We can then make use of the following theorem.

Theorem: The sequence $\{f_\lambda\}$ which is formed by the convolution of δ_λ and f

$$f_\lambda(x) = \int_{-\infty}^{+\infty} \delta_\lambda(x-u) f(u) du \tag{16}$$

converges uniformly to f as $\lambda \to \lambda_0$ for x on every interval $[x_0, x_1]$ of $\mathbb{R}$.

Proof: see Korevaar (1968).

[38] Note that this proof can be easily applied to other functions (such as the inverse quadratic function $x \to \frac{1}{1+(\frac{x}{\lambda})^2}$) that form a delta family of a positive type, so that our approach is not restricted to Gaussian functions.

Using (15) in (16), the function f_λ given by

$$f_\lambda(x) = \int_{-\infty}^{+\infty} G_\lambda(x-u)f(u)du \tag{17}$$

converges uniformly to f as $\lambda \to 0$ for x in some arbitrary interval $[x_0, x_1]$ of $\mathbb{R}$.

Next, we want to approximate (17) with a Riemann sum. To do so, first rewrite f_λ as

$$f_\lambda(x) = \underbrace{\int_{-\infty}^{-s} G_\lambda(x-u)f(u)du}_{=A(\lambda,x)} + \int_{-s}^{+s} G_\lambda(x-u)f(u)du + \underbrace{\int_{s}^{+\infty} G_\lambda(x-u)f(u)du}_{=B(\lambda,x)} \tag{18}$$

for $s > 1$.

Note that for any $s > 1$, we have

$$\begin{aligned} 0 &\le \int_s^{+\infty} G_\lambda(u)du \\ &\le \frac{1}{\sqrt{2\pi\lambda^2}} \int_s^{+\infty} e^{-\frac{u}{\lambda^2}} du \text{ since } u^2 > u \text{ for any } u \text{ in } [s, +\infty[,\ s > 1 \\ &\le \left[\frac{-\lambda^2}{\sqrt{2\pi\lambda^2}} e^{-\frac{u}{\lambda^2}} \right]_s^{+\infty} = \frac{|\lambda|}{\sqrt{2\pi}} e^{-\frac{s}{\lambda^2}} \underset{\lambda \to 0}{\longrightarrow} 0 \end{aligned}$$

which shows that $\forall s > 1$, $\lim_{\lambda -> 0} \int_s^{+\infty} G_\lambda(u)du = 0$. Symmetrically, we can show $\lim_{\lambda -> 0} \int_{-\infty}^{-s} G_\lambda(u)du = 0$.

Going back to (18), we have

$$0 \le |B(\lambda, x)| \le M \int_{-\infty}^{x-s} G_\lambda(t)dt$$

where $M = \sup_{x \in \mathbb{R}} |f(x)|$. Since $x \in [x_0, x_1]$, we can choose an $s > 1$ such that $x - s < -1$, so that we can apply the previous result and get

$$\lim_{\lambda \to 0} |B(\lambda, x)| = 0. \tag{19}$$

Proceeding symmetrically, we have $\lim_{\lambda \to 0} |A(\lambda, x)| = 0$.

Finally, since the function $u \mapsto G_\lambda(x-u)f(u)$ is continuous over $[-s, s]$, we can approximate $\int_{-s}^{+s} G_\lambda(x-u)f(u)du$ with a Riemann sum. Denoting

$$f_{\lambda,N}(x) = \sum_{n=1}^{N} G_\lambda(x - \xi_n) f(\xi_n) (\xi_n - \xi_{n-1})$$

where $\xi_n = -s + n\frac{2s}{N}$, we get that

$$\lim_{N \to \infty} f_{\lambda,N}(x) = \int_{-s}^{+s} G_\lambda(x-u)f(u)du. \tag{20}$$

Denoting $a_n = f(\xi_n) (\xi_n - \xi_{n-1})$, $b_n = \xi_n$ and $c_n = \lambda$, using (20) and (19) in (18) and combining with (17), we get that

$$\lim_{\lambda \to 0} \left(\lim_{N \to \infty} f_{\lambda,N}(x) \right) = f(x)$$

which completes the proof.

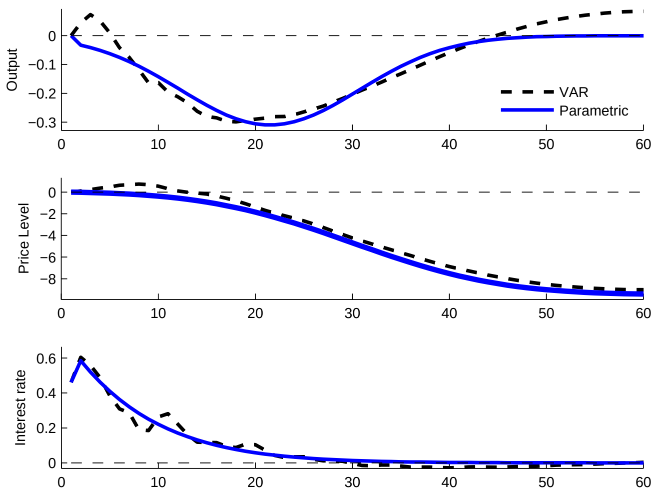

Figure 1: Impulse response functions (in ppt) of output (IP), the price level and the federal funds rate to a one standard-deviation monetary shock. Estimation from a standard VAR (dashed-line) or from a parametric (one Gaussian) specification of the impulse-responses (plain line). Estimation using data covering 1959-2006.

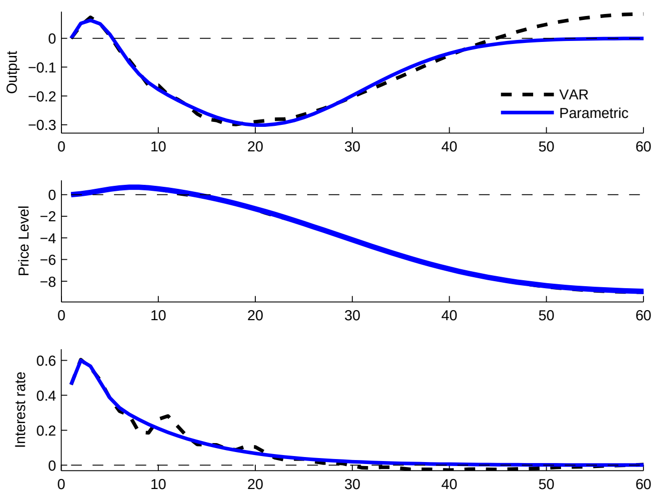

Figure 2: Impulse response functions (in ppt) of output (IP), the price level and the federal funds rate to a one standard-deviation monetary shock. Estimation from a standard VAR (dashed-line) or from a parametric (two Gaussian) specification of the impulse-responses (plain line). Estimation using data covering 1959-2006.

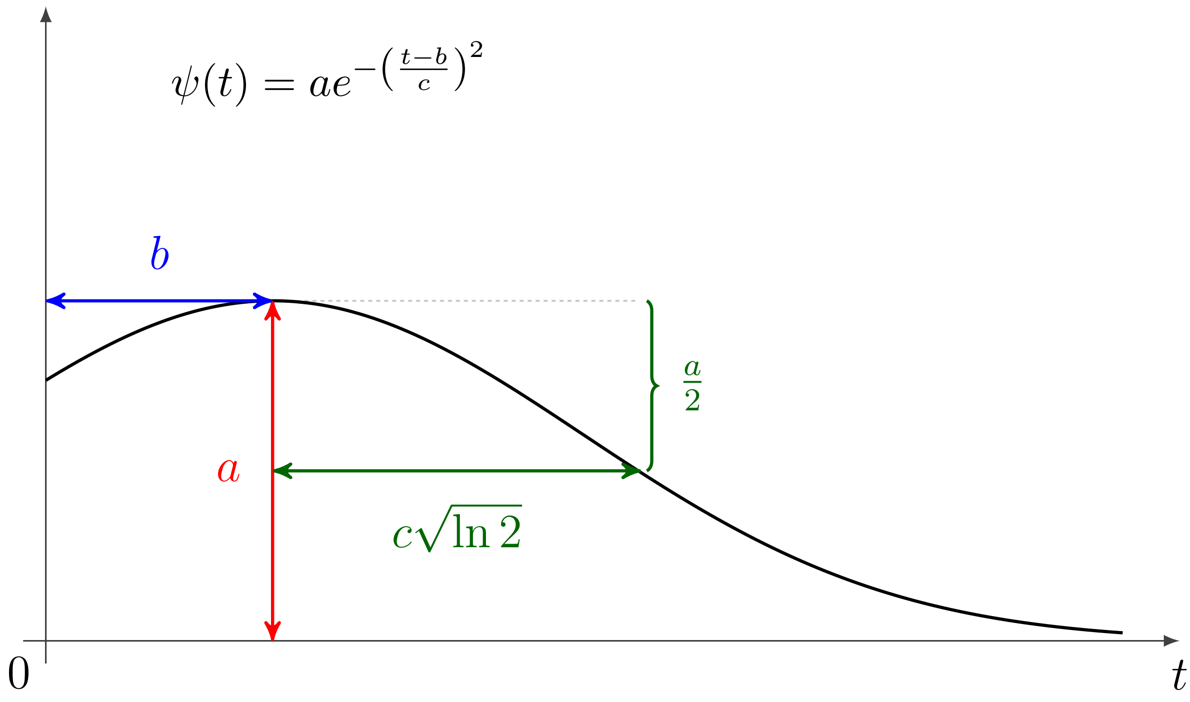

Figure 3: Interpreting an impulse response function with a Gaussian kernel.

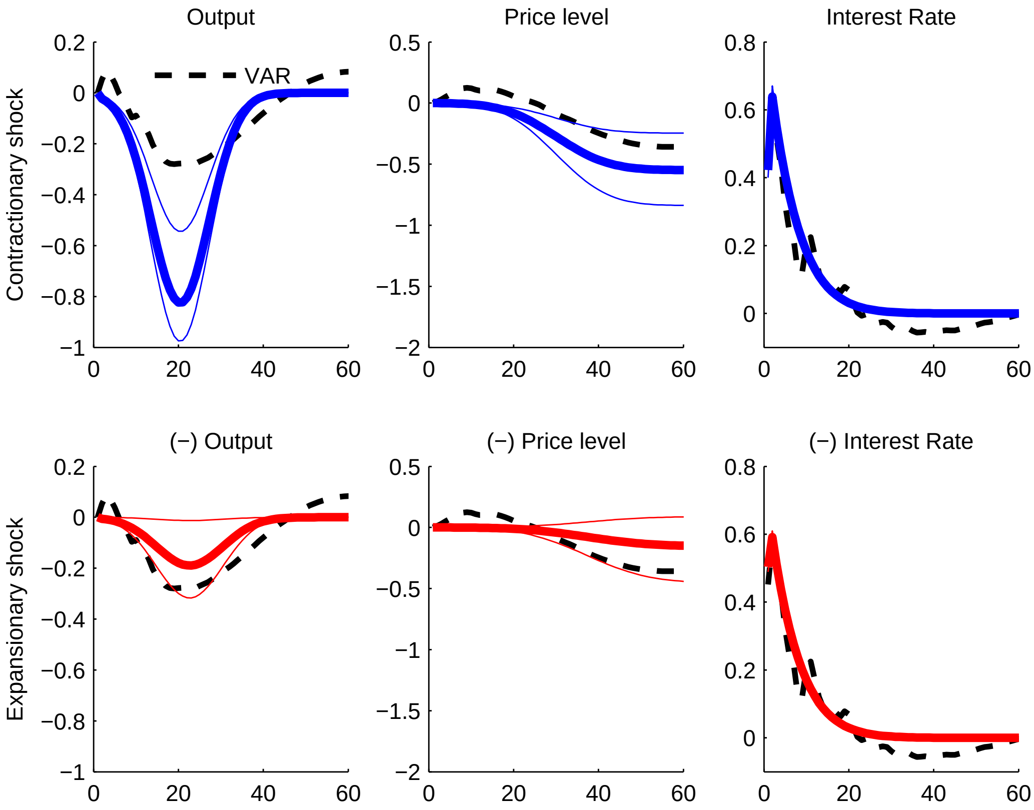

Figure 4: Impulse response functions (in percentage points) of output, the price level and the federal funds rate to a one standard-deviation monetary shock. Estimation from a standard VAR (dashed-line) or from a parametric (one-Gaussian function) specification of the impulse-responses (plain line). For ease of comparison, responses to the expansionary shock are multiplied by -1. Estimation using data covering 1959-2006.

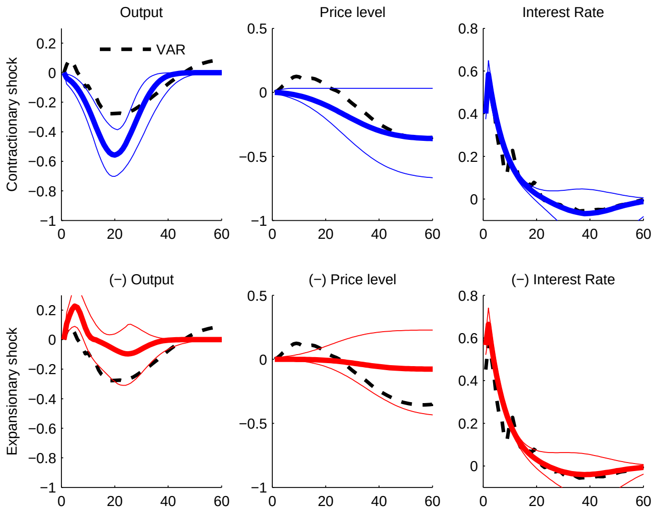

Figure 5: Impulse response functions (in ppt) of output, the price level and the federal funds rate to a one standard-deviation monetary shock. Estimation from a standard VAR (dashed-line) or from a parametric (two Gaussian functions) specification of the impulse-responses (plain line). Responses to expansionary shock multiplied by -1. 1959-2006.

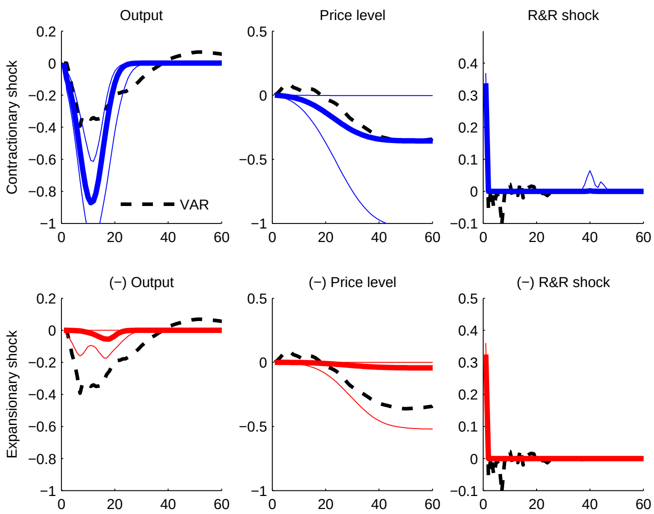

Figure 6: Impulse response functions (in ppt) of output, the price level and the Romer and Romer shock to a one standard-deviation monetary shock as estimated from a standard VAR (dashed-line) or from a parametric (one Gaussian function) specification of the impulse-responses (plain line). Responses to expansionary shock are multiplied by -1. 1959-2006.

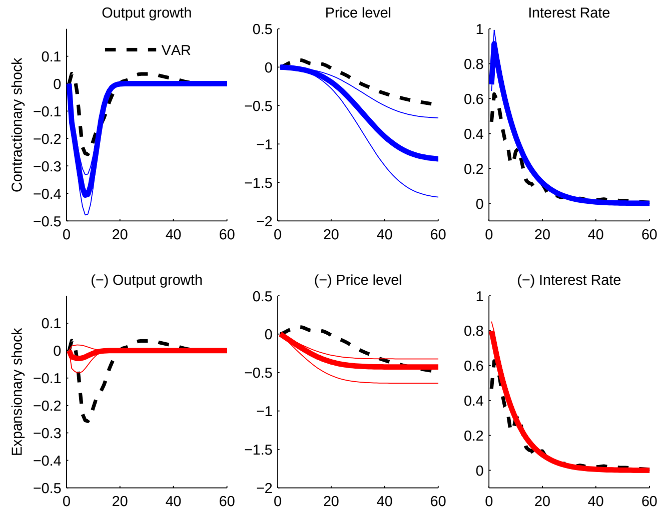

Figure 7: Impulse response functions (in ppt) of output growth, the price level and the federal funds rate to a one standard-deviation monetary shock. Estimation from a standard VAR (dashed-line) or from a parametric (one Gaussian function) specification of the impulse-responses (plain line). Responses to expansionary shock multiplied by -1. 1959-2006.

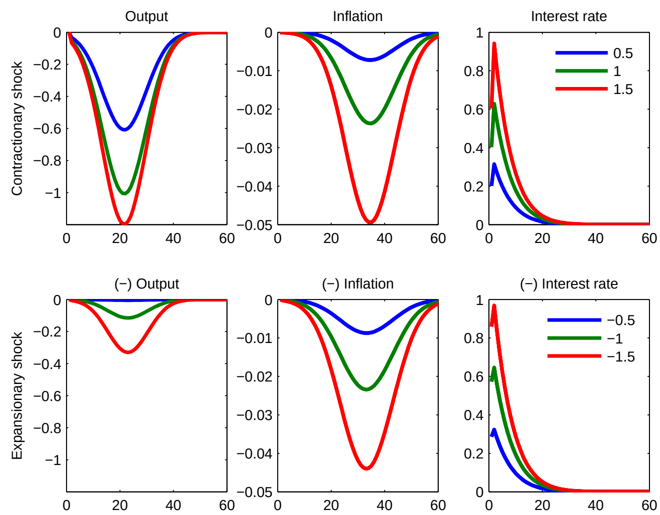

Figure 8: Impulse response functions (in ppt) of output, inflation and the federal funds rate to monetary shocks of different sizes (0.5, 1 and 1.5 standard-deviations) and different signs. Responses to expansionary shock multiplied by -1. Estimation using a one-Gaussian parametrization with data covering 1959-2006.

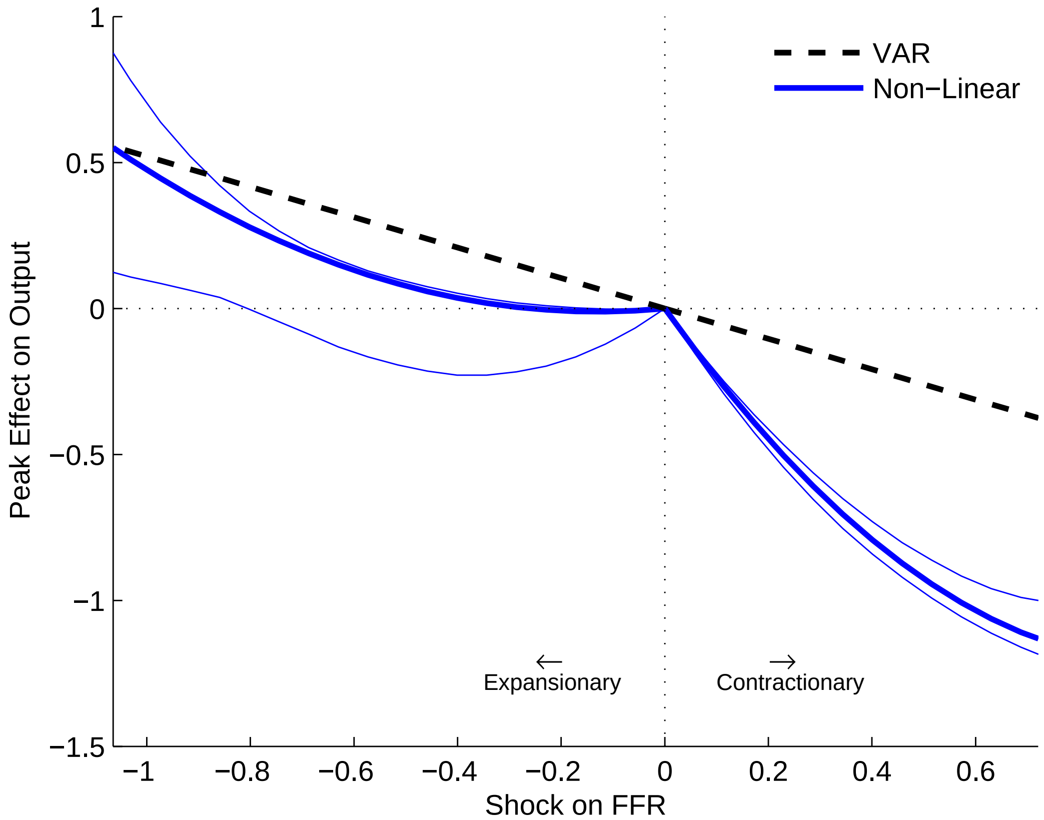

Figure 9: Peak effect of monetary policy on output (in percentage points) as a function of the size of the monetary shock (in percentage points of Fed Funds Rate, FFR). The thick blue line depicts the maximum likelihood estimate, and the error bands cover 90 percent of the posterior probability and are centered at the median. Estimation using data covering 1959-2006.

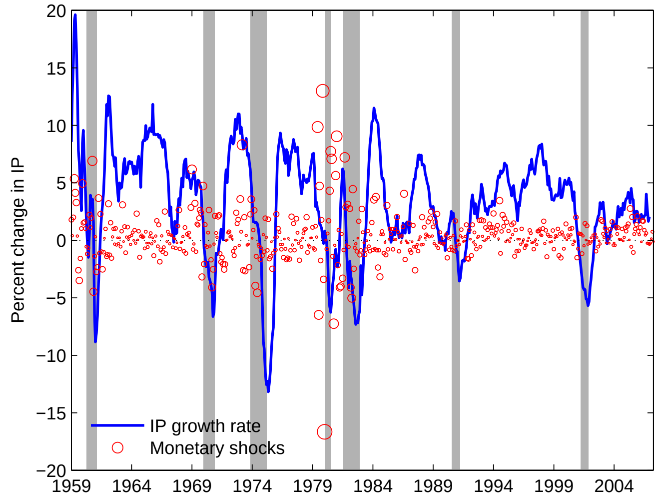

Figure 10: Annualized monthly growth rate of Industrial Production (12-months moving average) used as the business cycle indicator (solid line, left scale), and estimated monetary shocks (circles, right scale) with larger circles indicating larger shocks.

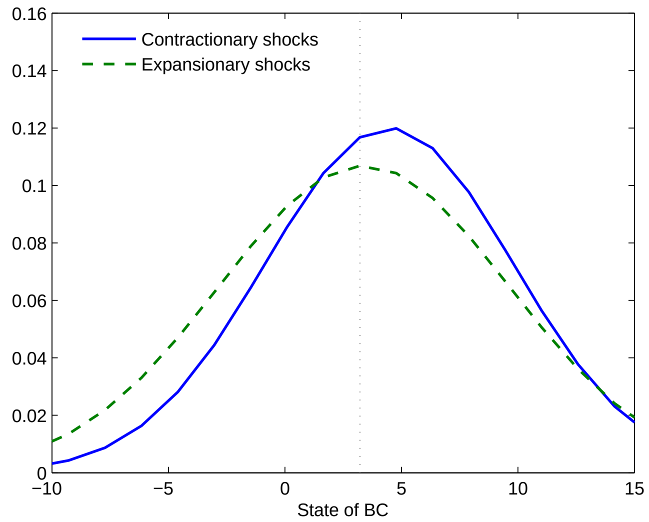

Figure 11: Distribution of expansionary shocks (dashed line) and contractionary shocks (solid line) over the state of the business cycle, defined as the annualized monthly growth rate of Industrial Production. Distributions interpolated from a Gaussian distribution. Estimation using data covering 1959-2006.

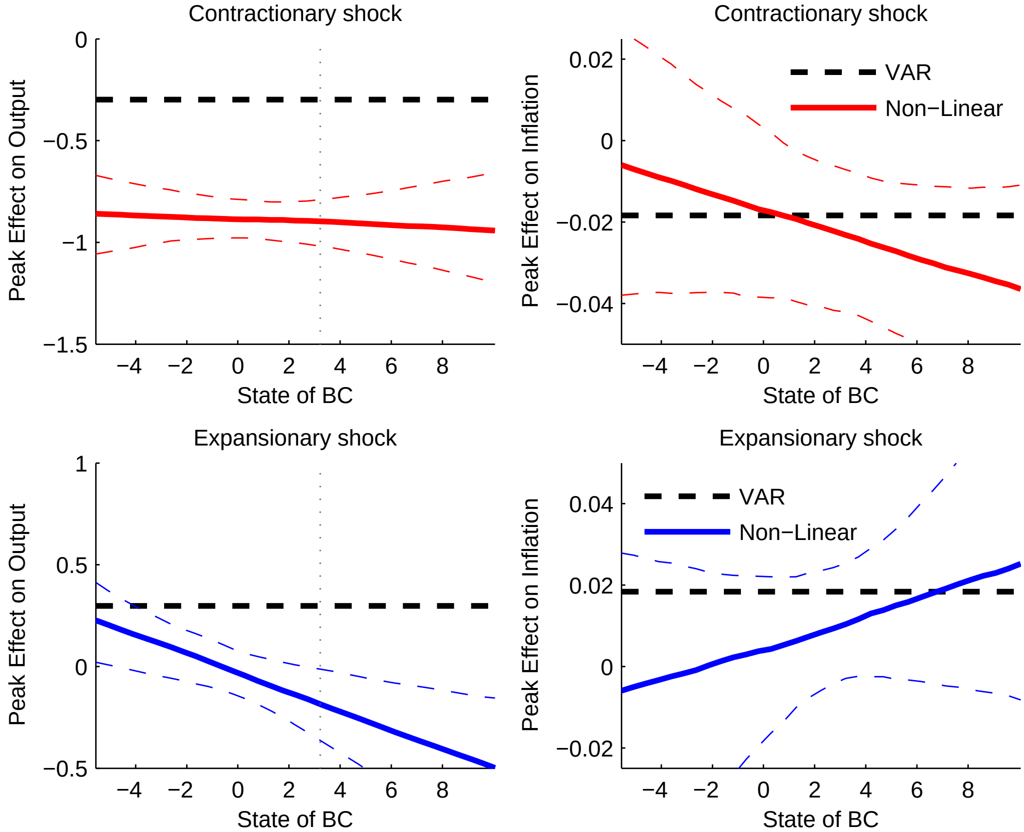

Figure 12: Peak effect of monetary policy on output and inflation (in percentage points) as a function of the state of the business cycle (in units of annualized IP growth rate) for 1 standard deviation contractionary monetary shocks (upper panel) and expansionary monetary shocks (lower panel). The dashed line indicate the 90% confidence interval. The thick-dashed line is the linear VAR estimate. The vertical dotted line indicates the average IP growth rate over the sample period. Estimation using data covering 1959-2006.

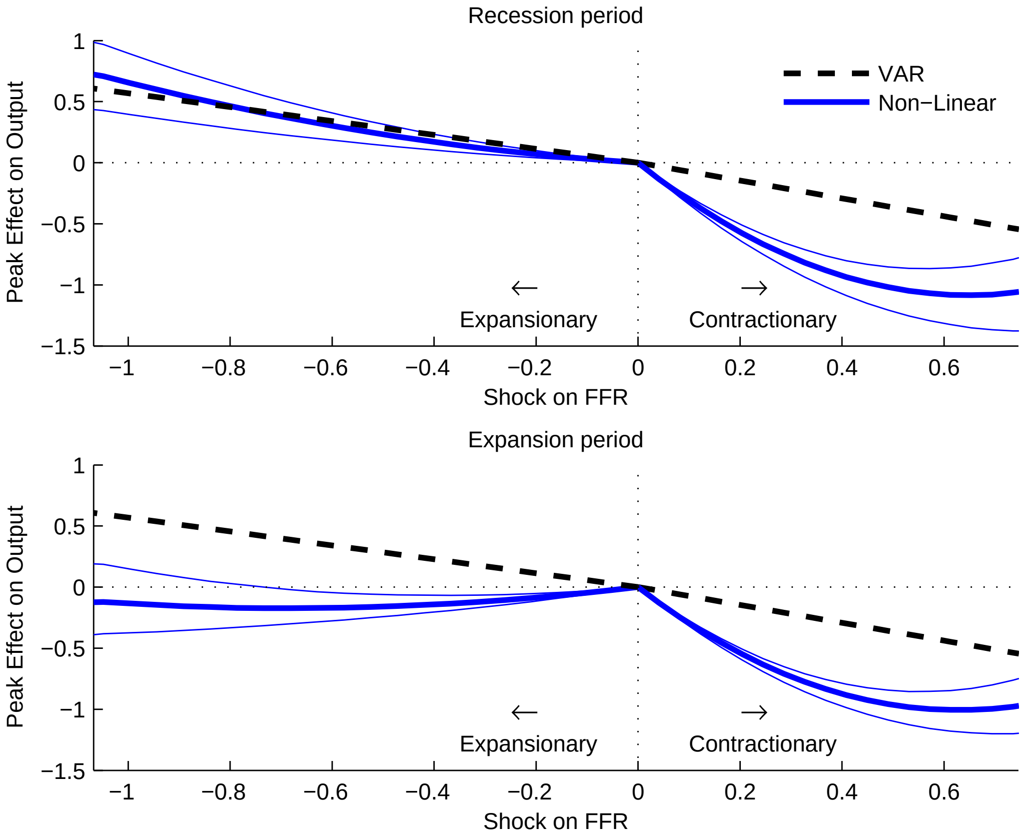

Figure 13: Peak effect of monetary policy on output (in ppt) as a function of the size of the monetary shock (in ppt of Fed Funds Rate, FFR) in a recession period (upper panel) and an expansionary period (lower panel). The dashed line indicate the 90% confidence interval. The thick-dashed line is the linear VAR estimate. Estimation using data covering 1959-2006.

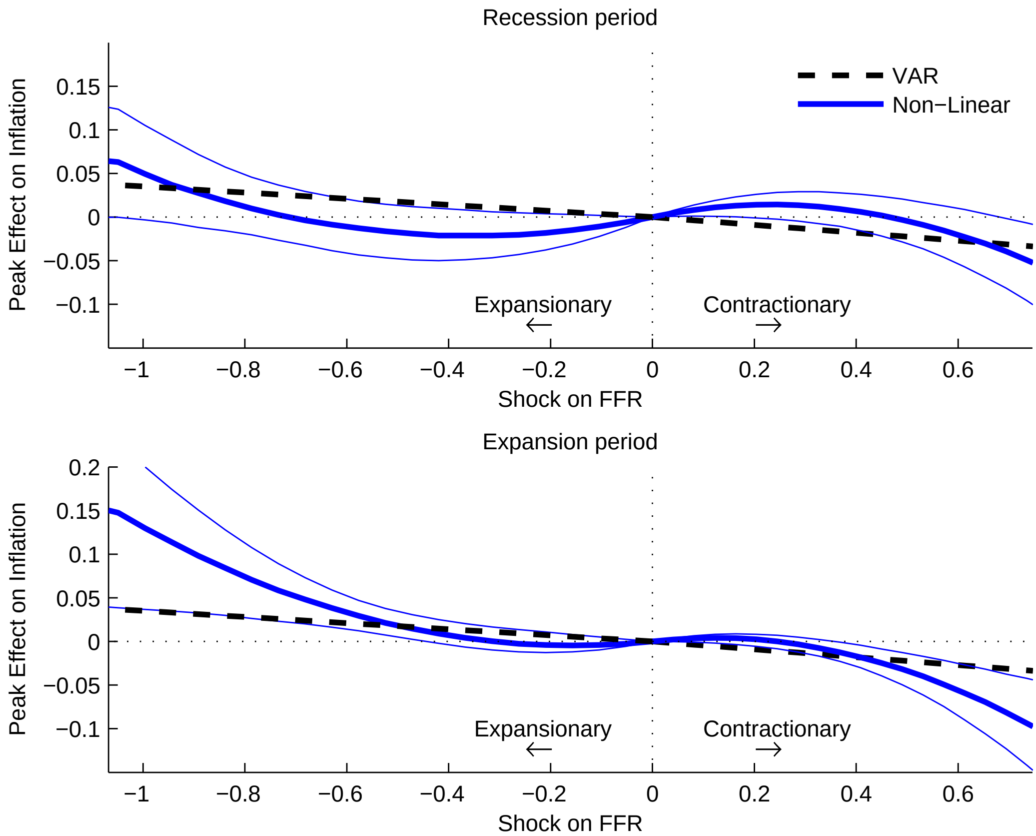

Figure 14: Peak effect of monetary policy on inflation (in ppt) as a function of the size of the monetary shock (in ppt of Fed Funds Rate, FFR) in a recession period (upper panel) and an expansionary period (lower panel). The dashed line indicate the 90% confidence interval. The thick-dashed line is the linear VAR estimate. Estimation using data covering 1959-2006.

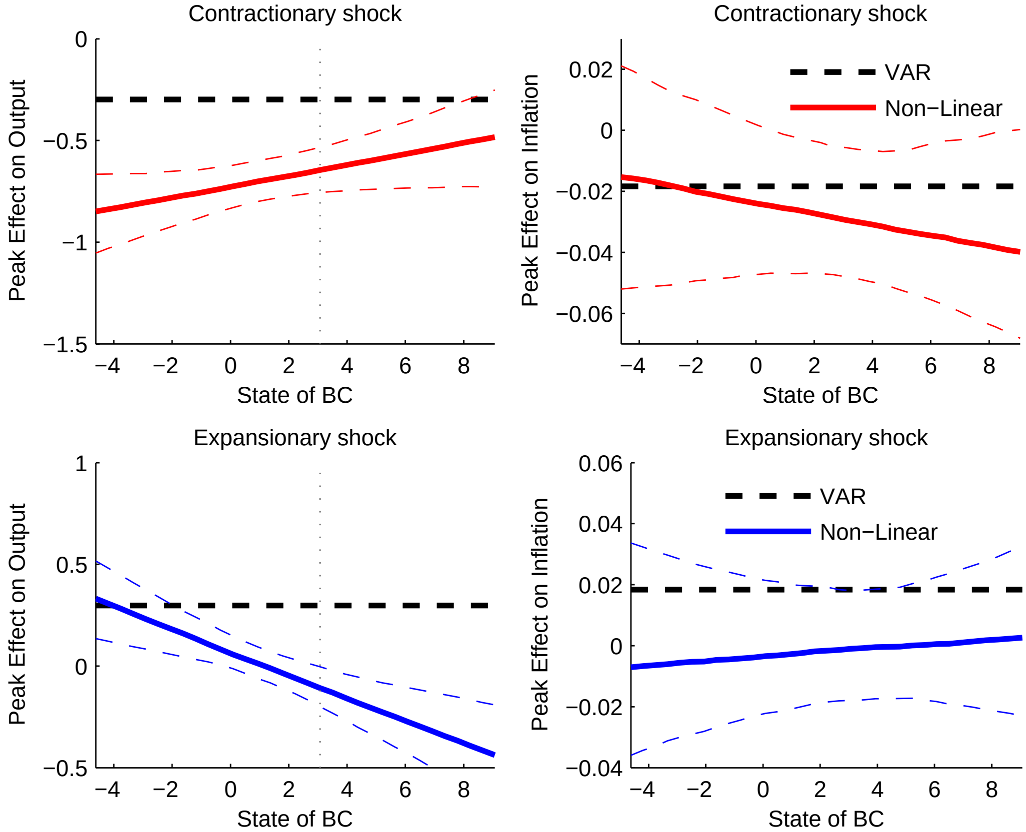

Figure 15: Peak effect of monetary policy on output and inflation (in percentage points) as a function of the state of the business cycle (in units of annualized IP growth rate) for 1 standard deviation contractionary monetary shocks (upper panel) and expansionary monetary shocks (lower panel) using the Romer and Romer shocks as the policy measure. The dashed line indicate the 90% confidence interval. The thick-dashed line is the linear VAR estimate. The vertical dotted line indicates the average IP growth rate over the sample period. Estimation using data covering 1966-2006.

Table 1: Log-likelihood of alternative models

	VAR	BM Symmetric One Gaussian	BM Symmetric Two Gaussian	BM Asymmetric	BM Asymmetric Size depend.	BM Asymmetric Size depend. State depend.
	(1)	**(2)**	**(3)**	**(4)**	**(5)**	**(6)**
(-) log-likelihood	2359	2409	2390	2247	2128	2108
Likelihood ratio test (p value)	--	--	(3) vs. (2) .90	(4) vs. (2) <.001	(5) vs. (4) <.001	(6) vs. (5) .001

Note: Specification with (log(Y),dlog(P),r) estimated over 1959m1-2006m12. Model (1) denotes the structural VAR, Model (2) is based on a parameterization of the impulse response functions (IRFs) with one Gaussian kernel, Model (3) is identical to (2) but using two Gaussian kernels and the LR test is between models (3) and (2), Model (3) is identical to (2) but allowing for asymmetric IRFs to monetary shocks and the LR test is between (4) and (2), Model (5) is identical to (4) but also allowing for the scale of the IRF to depend linearly on the size of the shock and the LR test is between models (5) and (4), Model (6) is identical to (5) but also allowing for the scale of the IRF to depend linearly on the state of the business cycle and the LR test is between models (6) and (5).

Table 2: Differences in impulse responses of real variable to positive and negative monetary shocks

	Baseline 59-06	Baseline 59-06 Full asymmetry	Baseline 83-06	U and PGDP 59-06	U and PGDP 83-06
	(1)	**(2)**	**(3)**	**(4)**	**(5)**
D statistics for *Y* (*100)	0.59	0.45	0.16	0.09	0.07
(95% conf. int.)	(.44, .72)	(.29, .57)	(.01, .30)	(.04, .13)	(.01, .12)

Note: Model (1) uses the variables *(log(Y),dlog(P),r)* with *Y* denoting industrial production, *P* the PCE price index and *r* the federal funds rate. (1) is estimated over 1959m1-2006m12 with a parameterization of the IRFs with one Gaussian basis function and allowing for asymmetric IRFs in response to monetary shocks, Model (2) is identical to (1) but allowing for asymmetric IRFs in response to all shocks, Model (3) is identical to (1) but estimated over 1983m1-2006m12, Model (4) is identical to (1) but using the Unemployment rate in level and the and the GDP deflator to measure inflation, Model (5) is identical to (4) but estimated over 1983m1-2006m12.