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January 7, 2015

Abstract

To be written.

*Thanks to... All errors are our own.
†The views expressed here are those of the authors and do not necessarily reflect the views of the Federal Reserve Bank of Philadelphia or the Federal Reserve System.
1 Introduction

To be written.

2 Environment

We consider a two-period economy that is populated by three types of agents. First, there is a measure $\lambda$ of non-depository financial institutions which we will refer to as “lenders,” as this is representative of their primary role in the model. Second, there is a large measure of depository institutions which we will refer to as “DIs” for brevity. Finally, there is a central bank which, for obvious reasons, we will call the “Fed.” All agents are risk neutral and do not discount between $t = 1$ and $t = 2$.

Lenders. At $t = 1$, each lender is endowed with a single, indivisible unit of reserves, with which it can engage in one of three different types of activity. First, the lender can safely store this unit of reserves for $t = 2$, in which case it neither depreciates nor earns any interest. Second, the lender can engage in an “Overnight Reverse Repurchase” (ON RRP) agreement with the Fed, in which case the lender uses its reserves to purchase a security from the Fed at $t = 1$, and then resells the security to the Fed at $t = 2$ at a pre-specified price. We denote the net rate of return on this investment by $r$, and assume this is chosen by the Fed. Finally, the lender can deposit its unit of reserves at a DI at $t = 1$ in exchange for some promised rate of return at $t = 2$; we discuss this type of agreement in greater detail below.

Depository Institutions. Depository institutions differ from lenders along two dimensions. First, we assume they are not endowed with any reserves at $t = 1$; this assumption is mostly for convenience. Second, we assume that DIs have access to a facility at the Fed that allows them to deposit reserves at $t = 1$ and earn a net interest rate $R > r$ at $t = 2$. We refer to $R$ as the Interest on Excess Reserves (IOER) rate, and assume that this rate is also chosen by the Fed. Hence, the Fed here has two instruments: the ON RRP rate, $r$, and the IOER rate, $R$.

We assume that each DI, which we index by $j$, can accept up to one unit of reserves from a lender, and that doing so imposes a “balance sheet cost” $c_j$. This balance sheet cost captures both the direct costs of a DI expanding its balance sheet, like FDIC fees, as well as the indirect costs associated with requirements on capital and leverage ratios.$^1$

DIs are heterogeneous with respect to their balance sheet costs: formally, we denote by $G(c_j)$ the distribution of costs across DIs, for $c_j \in [0, \infty)$. Given its balance sheet cost, each DI decides whether or not to enter the interbank market and, if they enter, they choose an interest rate that they will pay to borrow a unit of reserves, which we denote by $\rho_j$. We assume that DIs will enter the interbank market if, and only if, they can make non-negative profits in equilibrium.

Matching in the Interbank Market. Once DIs have made entry decisions and posted interest rates, each lender observes the interest rates that have been posted and chooses one to approach. We will often refer to the set of DIs that have chosen a particular interest rate as a

$^1$See, e.g., [?].
“sub-market.” Conditional on choosing a sub-market, a lender may or may not be paired with a DI—there are matching frictions in the interbank market. In particular, suppose a measure $d$ of DIs post a particular interest rate and a measure $\ell$ of lenders choose that rate. Then, letting $q = \ell/d$ denote the market tightness or “queue length” in that sub-market, the probability that each DI receives a deposit is $1 - e^{-q}$ and, symmetrically, the probability that each lender matches with a DI is $\frac{1-e^{-q}}{q}$. For now, we assume that a lender who is not successful matching with a DI can always engage in an ON RRP agreement with the Fed.

**Summary of Timeline and Payoffs.** The following timeline depicts the sequence of events.

<table>
<thead>
<tr>
<th>$t = 1$</th>
<th>$t = 2$</th>
</tr>
</thead>
<tbody>
<tr>
<td>DIs enter, post $\rho_j$’s</td>
<td>Fed pays $R$ to DIs, $r$ to lenders</td>
</tr>
<tr>
<td>Matching occurs</td>
<td>Unmatched lenders go to ON RRP facility</td>
</tr>
<tr>
<td>Lenders choose $\rho_j$ to approach</td>
<td>Matched DIs deposit reserves to earn IOER, incur cost $c_j$</td>
</tr>
</tbody>
</table>

![Figure 1: Timeline](image)

A DI who is matched with a lender at $t = 1$ incurs the balance sheet cost $c_j$ and deposits the reserves at the Fed. Then, at $t = 2$, the DI earns the IOER rate $R$ and pays the lender the promised interest rate $\rho_j$. Hence, a DI with balance sheet cost $c_j$ that posts interest rate $\rho_j$ and matches with a lender will earn a net profit of $R - c_j - \rho_j$. An unmatched DI, on the other hand, earns zero. Meanwhile, a lender earns a payoff of $\rho_j$ if he successfully matches with a DI who has posted an interest rate $\rho_j$, and otherwise earns $r$ from an ON RRP transaction with the Fed.

### 3 Equilibrium

To characterize the equilibrium in the environment described above, it is helpful to think of three different stages of decision-making. First, DIs have to decide whether or not to enter the interbank market given their balance sheet cost, $c_j$. Second, those DIs that enter have to choose an interest rate, $\rho_j$. Finally, given the interest rates that have been posted, lenders have to choose which one to approach. In order to describe optimal behavior at each stage, we work backwards.

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2This nomenclature is borrowed from the competitive search literature, where each posted price (wage) is associated with a sub-market. In these models, sellers (firms) first choose a sub-market to sell their good (post their vacancy), and then buyers (workers) choose a sub-market in attempt to buy the good (get a job).
Optimal Search by Lenders. Once interest rates have been posted, each lender must choose the sub-market (or mix between sub-markets) that offers the maximum expected payoff, taking into account both the interest rate being offered in that sub-market and the probability of being matched. In particular, the expected payoff from a lender choosing a sub-market with interest rate $\rho_j$ and queue length $q_j$ is

$$U(\rho_j, q_j) = \left[ 1 - e^{-q_j(c_j)} \right] \frac{1}{q(c_j)} \rho_j + \left[ 1 - \frac{1 - e^{-q_j(c_j)}}{q(c_j)} \right] r.$$ 

Let $\overline{U}$ denote the maximum expected payoff that a lender can obtain, or what we will call the “market utility.” In equilibrium, then, any sub-market with $q_j > 0$ must satisfy

$$U(\rho_j, q_j) = \overline{U}. \quad (1)$$

That is, in equilibrium, any DI that is able to attract lenders must deliver an expected payoff equal to the market utility: if a DI posts a relatively low interest rate, lenders must be compensated with a high probability of being matched (i.e., a short queue length), and vice versa.

Optimal Interest Rate Posting by DIs. The relationship described in equation (1) is akin to a typical demand curve: DIs that post low interest rates attract a smaller queue length and are matched with a low probability, while those who post high interest rates attract more lenders, on average, and hence are matched with a high probability. Taking this relationship as given, a DI with balance sheet cost $c_j$ who has entered the interbank market solves the following maximization problem:

$$\max_{\rho_j, q_j} \left[ 1 - e^{-q_j(c_j)} \right] (R - c_j - \rho_j) \quad (2)$$

$$\text{sub to } U(\rho_j, q_j) = \overline{U},$$

where the market utility $\overline{U}$ is taken parametrically by each DI. From the objective function, (2), it’s clear that a DI’s profits are equal to the product of the probability of being matched, $1 - e^{-q_j}$, and the revenue from accepting a deposit, $R - c_j - \rho_j$, taking as given the positive relationship between interest rates and queue lengths.

One can solve the constraint $U(\rho_j, q_j) = \overline{U}$ for the interest rate,

$$\rho_j = r + \left[ \frac{q_j}{1 - e^{-q_j(c_j)}} \right] (\overline{U} - r)$$

and plug this into (2). The result is an objective function that is strictly concave over its single variable, $q_j$, and hence the first order condition delivers the optimal queue length for any $c_j$ and any market utility $\overline{U}$:

$$q_j \equiv q(c_j; \overline{U}) = \log \left( \frac{R - c_j - r}{\overline{U} - r} \right). \quad (3)$$

The optimal interest rate then follows immediately from (3), i.e., the optimal interest rate for a DI with balance sheet cost $c_j$, given $\overline{U}$, is given by

$$\rho_j \equiv \rho(c_j; \overline{U}) = r + \log \left( \frac{R - c_j - r}{\overline{U} - r} \right) \left[ \frac{(R - c_j - r)(\overline{U} - r)}{R - c_j - \overline{U}} \right]. \quad (4)$$
Optimal Entry by DIs. A DI enters the interbank market if, and only if, its expected profits from doing so are nonnegative. One can easily show that a DI’s profits are decreasing in $c_j$ for any $\mathcal{U}$, so that the optimal entry decision is determined by a cutoff rule: for any $\mathcal{U} > r$, there exists a unique $c^* > 0$ such that profits are nonnegative if, and only if, $c_j \leq c^*$. Substituting (3) and (4) into (2) and solving reveals that this cutoff satisfies

$$c^*(\mathcal{U}) = R - \mathcal{U}.$$  \hfill (5)

Market Clearing. The analysis above describes the optimal decisions by lenders and borrowers, taking as given the market utility $\mathcal{U}$. The final condition requires that markets clear:

$$\int_{0}^{c^*(\mathcal{U})} q(c_j; \mathcal{U}) dG(c_j) = \lambda.$$ \hfill (6)

In words, (6) requires that aggregating the queue lengths (or expected number of lenders per DI) across the active DIs yields the total measure of lenders in the market, $\lambda$.

Definition of Equilibrium. Given the results above, an equilibrium is a market utility $\mathcal{U}$, a cutoff $c^*$, queue lengths $q(c_j) > 0$ and interest rates $\rho(c_j) \in (r, R)$ for all $c_j < c^*$ such that

1. Lenders are indifferent between all active DIs, i.e., (1) is satisfied.
2. Given lender behavior, DIs choose interest rates to maximize profits, i.e., (4) is satisfied.
3. DIs enter the market if, and only if, it is profitable to do so, i.e., (5) is satisfied.
4. Markets clear, i.e., (6) is satisfied.

\footnote{In other words, we are assuming that a DI will stay out of the interbank market if it would not attract any lenders by entering. One could motivate this assumption by assuming that there was a cost $\epsilon > 0$ associated with posting an interest rate, where $\epsilon$ was arbitrarily close to zero.}