# Family Law Effects on Divorce, Fertility and Child Investment[1]

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February 14, 2015

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**Abstract**

In order to assess the child welfare impact of policies governing divorced parenting, such as child support orders, child custody and placement regulations, and marital dissolution standards, one must consider their influence not only on the divorce rate but also on spouses' fertility choices and child investments. We develop a model of marriage, fertility and parenting, with the main goal being the investigation of how policies toward divorce influence outcomes for husbands, wives and children. Estimates of preferences and the technology of child development are disciplined by data on parental time inputs, and simulations based on the model explore the effects of changes in custody allocations and child support standards on outcomes for intact and divided families. Simulations indicate that, while a small decrease in the divorce rate may be induced by a significant child support hike, the major effect of child support levels for both intact and divided households is on the distribution of welfare between parents. Simulated divorce, fertility, test scores and parental welfare all increase with a move toward shared physical placement. Finally, the simulations indicate that children's interests are not necessarily best served by minimizing divorced parenting.

JEL codes: J12, J13, J18

# 1 Introduction

Divorced parenting in the U.S. is regulated through a combination of laws controlling marital dissolution, child custody and placement, and the assignment and enforcement of child support obligations. The primary objective of these activities is to increase the well-being of children and parents, and the divorce rate is often regarded as a first order measure of the success of family law. The rationale for this focus is the preponderance of empirical evidence that suggests that children living in households without both biological parents are more likely to suffer from behavioral problems and have lower levels of a broad range of achievement indicators measured at various points over the life cycle (see, e.g., Haveman and Wolfe 1995). Recent empirical studies of unilateral divorce laws and child support enforcement have isolated the effects of changes in such legal structures on divorce rates (e.g., Friedberg 1998 and Gruber 2004, Wolfers 2006 and Nixon 1997). A complete picture of the influence of family law on family members' welfare would include an understanding of the mechanisms by which family law changes influence fertility, child outcomes, and the distribution of resources within the family, in addition to divorce rates. Toward that end, this paper models the interaction of married couples in the shadow of existing divorce regulations in terms of decisions regarding fertility, child investment and divorce.

A standing problem for research on parents' dynamic child investment activities is the frequent absence of data on fathers. Much of what we have learned about parents' dynamic decision-making, therefore, has been in the context of a mother's (or mother and father's, assuming a unitary objective) individual dynamic optimization problem, as in Bernal (2008), Bernal and Keane (2011), Blau and van der Klaauw (2008) and Liu, Mroz and van der Klaauw (2010).

Where the subject is the influence of divorce regulations on the family, however, the distinct choices of mothers and fathers are paramount. It is virtually impossible to understand the influence of potential child support, for example, on fertility, investment and divorce decisions by studying the mother's perspective in isolation. Hence we model the choices of mothers and fathers as an ongoing, simultaneous-move game. Our model and data begin from the date of marriage, which, while excluding a substantial and non-random segment of parents, has the benefit of granting access to similar information on the mother and father when early fertility, investment and divorce choices are being made.

In taking this approach, we draw on an extensive empirical literature on marriage dynamics, including Aiyagari, Greenwood and Guner (2000), Brien, Lillard and Stern (2006), Chiappori, Fortin and Lacroix (2002), and others. This literature emphasizes the repeated interaction of a husband and wife in deciding whether to continue a marriage and the allocation of household resources.[1] What we seek to add to existing analyses is the impact of the (endogenous) arrival and development of a shared child and its role, as well as the role of exposure to divorced parenting

[1]Caucutt, Guner and Knowles (2002) is a rare example of an existing study of marriage dynamics (including a marriage market), fertility and child expenditures. However, their framework is a three period overlapping generations model, and their object of interest is the life-cycle timing of fertility, where we model spouses' decisions in continuous time throughout the fertility and childrearing process, and our ultimate interest is in child outcomes.

regulation, in marital status dynamics.

A closely related paper is Tartari (2014). It includes a substantially enriched child quality production technology relative to the one that we estimate, and it focuses less on capturing the institutional structure and effects of family law.

The model allows spouses to make (simultaneous) choices regarding marriage continuation, fertility, and, where relevant, individual investments in children. A match value of marriage is drawn from a population distribution and evolves stochastically over time. Fertility choices are influenced by both the expected benefit from the presence of the child and expectations regarding the duration of the marriage, given the state of the marriage quality process. Child quality, reflected by cognitive ability in the empirical implementation of the model, progresses as a result of both endogenous parental investment and marital status choices and exogenous productivity factors. Marital dissolution may result from changes in marriage match quality, child presence and quality, and when the child reaches "independence" (in the sense of the model, which is explained below). Thus the full history of marriage values and child investments determines current marital status and child investment levels. If the history of child investments and marriage values is poorer for the marginal marriage than it is for the representative marriage, then, all else equal, the child welfare gain associated with the continuation of the marginal marriage is smaller than that associated with the continuation of the representative marriage. An important objective of our analysis is to study the welfare impacts of variations in family law, which are possible to assess under our assumptions regarding the determination of the utility levels of husbands, wives, and (potential) children.

The model is estimated utilizing data from the Panel Study of Income Dynamics (PSID) and it's Child Development Supplement (CDS), using the method of simulated moments (MSM). The model we estimate is extremely parsimonious (which is a positive way to say 'stylized'). Nevertheless, we find that the model is able to fit most of the many features of the data used in estimation to a very satisfactory degree, with a few notable exceptions. That gives us some confidence in using the estimated model to perform comparative statics exercises and welfare analysis.

An important feature of our model is the incorporation of a fertility decision. In the comparative statics exercises we find that family law potentially has an important impact not only on the achievement levels of children from intact and nonintact households, but even more fundamentally on the number of children born and the characteristics of the households having them. The effects of variations in child contact time allocations in the divorce state have a particularly strong impact on fertility, with 20 percent fewer households having children within 10 years from the date of the marriage when the father is given no time as opposed to when he is allocated 50 percent of the time. These variations also impact the (final) distribution of child quality not only through their impact on the incentive to invest in children but also due to the fact that only parents expecting high quality children and a stable marriage will have children in such an "extreme" family law environment. Conversely, we find that the effect of wide variation in child support orders has a modest impact on fertility decisions and child quality. This seems largely due to the diminishing marginal utility of consumption and the perfect substitutability of parental investments in the stochastic child quality production function.

Our analysis concludes with an attempt to determine optimal family law parameters using a Benthamite social welfare function. The main problem with employing such an approach in our modeling framework is that the set of agents is endogenous due to the presence of the fertility decision. We are able to make some progress by merging the seminal approaches to this question found in Blackorby et al. (1995) and Golosov et al. (2007). Our simulation exercise adopts an ex ante welfare criterion (evaluated at the time of marriage) which only involves the agents always present, the husband and wife. We find that the welfare objective is optimized under 50/50 physical placement, a 20 percent child support rate and a bilateral divorce standard. Custody arrangements generally dominate the welfare ordering, with the divorce standard being of minimal net welfare consequence.

The plan of the paper is as follows. In Section 2 we describe some import patterns in the data with which we hope our model is consistent. In Section 3 we develop the details of the model. Section 4 describes our estimation method and discusses the manner in which primitive parameters are identified. In Section 5 we describe the data in detail and present descriptive statistics for our sample. The estimates of the primitive parameters and assessment of model fit are found in Section 6. In Section 7 we describe comparative statics results and our attempt to determine optimal family law. Section 8 concludes.

## 2 Empirical Patterns in Divorce and Child Development

*Section forthcoming*

## 3 A Model of Child Investment and Divorce Decisions

There exist two decision-making agents in our model, spouses $s = 1, 2$. The model is set in discrete time, with one period in the model corresponding to one year in the data. The period utility function of spouse $s$ is given by

$$u_s(l_s, c_s, \chi_d, \chi_f, \theta, k) = \alpha_{s1}\ln(l_s) + \alpha_{s2}\ln(c_s) + \alpha_{s3}\chi_f(\ln(k) + \zeta) + (1 - \chi_d)\theta,\ s = 1, 2; \quad (1)$$

where $c_s$ represents the consumption of spouse $s$, $l_s$ the leisure enjoyed by spouse $s$, $\chi_d$ is an indicator variable that takes the value 1 if the spouses are divorced, $\chi_f$ is an indicator taking the value 1 if a child is present, $\theta$ is a time varying, marriage-specific match value, and $k$ is the value of child quality, which is weakly greater than 1 when the spouses have had a child and is assigned the value 1 if they have not. The parameters $\{\alpha_{s1}, \alpha_{s2}, \alpha_{s3}\}$ are the spouse-specific preference weights on leisure, consumption, and child quality, and $\zeta$ is a constant welfare cost or benefit of child presence unrelated to his or her quality.[2] We assume throughout that the price of consumption is fixed at 1.

[2]The parameter $\zeta$ will be free to take positive or negative values. It may be interpreted as a welfare cost or benefit of child presence or, equivalently in this specification, as a scaling factor relating the value of child quality to the

Marriage state consumption is assumed to be public, so that the instantaneous utilities of married childless spouses are given by

$$u_s(l_s, C, 0, 0, \theta, 1) = \alpha_{s1} \ln(C) + \alpha_{s2} \ln(l_s) + \theta, \ \ s = 1, 2.$$

In divorce, their utilities are

$$u_s(l_s, c_s, 1, 0, 0, 1) = \alpha_{s1} \ln(c_s) + \alpha_{s2} \ln(l_s), \ \ s = 1, 2.$$

Spouses earn wages $w_s$, $s = 1, 2$, and the family is subject to resource and time constraints

$$\begin{aligned} w_1 h_1 + w_2 h_2 &= C, \\ l_1 + \tau_1 + h_1 &= 1, \ l_2 + \tau_2 + h_2 = 1, \text{ and } h_s, \tau_s \geq 0 \ s = 1, 2 \end{aligned} \tag{2}$$

in marriage, where $\tau_s$ is the time invested in child quality production by spouse $s$.

With some loss of generality, we define spouse 1 to be the husband and the payer of child support in the divorced parenting state. Child custody in this state is implemented as an additional constraint imposed upon the time allocation choices by family law. We assume that parents have full access to the child in the marriage state, so that, where $\chi_d = 0$, $\tau_1$ and $\tau_2$ are unconstrained. When $\chi_d = 1$, assuming that $\overline{\tau}_s$ represents the share of custody allocated to parent $s$ in the divorce state, the custody requirement is

$$\tau_1 \leq \overline{\tau}_1 \text{ and } \tau_2 \leq \overline{\tau}_2,$$

with $\overline{\tau}_1 + \overline{\tau}_2 \leq 1$. Here time with the child is intended to represent physical placement, as opposed to legal custody. We further assume that physical custody and visitation allocations are fully anticipated and set exogenously with respect to parental behaviors.[3]

In divorce, consumption is no longer public. Divorced parents consume only their own incomes, and parent 1 may be required to pay child support to parent 2.[4] Each spouse has a baseline income flow at any moment in time of $w_s h_s$. The actual income under the control of individual $s$ is state-dependent in the following sense: the father pays share $\pi$ of his income to the mother in the divorced parenting state. Hence, the father's consumption in divorce after time decisions are made is $(1 - \pi) w_1 h_1$, and the mother's is $w_2 h_2 + \pi w_1 h_1$.[5] We denote the post-transfer resources of parent $s$ in the divorced parenting state as $y_s(w_1, w_2, h_1, h_2 | \pi)$.

The dynamics of the model are as follows.

1. The model begins at the time of marriage. Spouses are initially childless. If both spouses

value of consumption.

[3]See, for example, Fox and Kelly (1995) for details on custody determination.

[4]One would expect some relationship between the mother's share of custody and the level of child support she receives from the father. Rather than imposing a functional relationship between $\overline{\tau}_1$ and $\pi$, in the empirical exercise we impose the model custody allocation and base $\pi$ on state child support guidelines and family characteristics.

[5]By assuming that there is no transfer ordered after a divorce if the couple is childless, we are essentially assuming away alimony. Alimony is increasingly uncommon in U.S. divorce cases. According to Case et al. (2003), for example, 5 (4.2) percent of 1977 PSID (in 1997) mothers received alimony.

agree to attempt to have a child, then a child arrives in the next period with probability $\gamma_f > 0$. We define the state variable $f \in \{0, 1\}$ to indicate whether this fertility process is active, with $f = 1$ representing an active fertility process. The fertility process may only be active in the married state.

2. There are $N_\theta$ possible values of marriage quality, with $\theta \in \Theta_\theta = \{\theta_1, ..., \theta_{N_\theta}\}$ and $\theta_1 < \theta_2 < \ldots < \theta_{N_\theta}$. At the onset of marriage, there is an initial marriage quality draw. Marriage quality evolves stochastically according to a Markov process, i.e. $\theta_{t+1} \sim F_\theta(\cdot \mid \theta_t)$

3. There are $N_w$ possible baseline wage levels for each spouse, with $w_s \in \Theta_{s,w} = \{w_{s1}, ..., w_{s,N_w}\}$, where $0 < w_{s,1} < ... < w_{s,N_s}$, $s = 1, 2$. Each spouse begins marriage with a wage of $w_s$. Similarly to marriage quality, each wage evolves stochastically according to to a Markov Process, i.e. $w_{s,t+1} \sim F_{w,s}(\cdot \mid w_{s,t})$.

4. Child quality is distributed absolutely continuously with respect the positive real numbers, i.e. $k \in \mathbb{R}^+$. When born, the child has an initial child quality draw of $k_0$. This variable evolves over time according to a Cobb-Douglas production technology that depends on the child's age, $a$, parental time investments, $(\tau_1, \tau_2)$, and current quality, $k_a$:

$$k_{a+1} = \psi_a(\theta)\tau_1^{\delta_{1,a}}\tau_2^{\delta_{2,a}}k_a^{\delta_{k,a}} \tag{3}$$

   The presence of marriage quality in the child quality production function is meant to capture the impact of the home environment on the effectiveness of a given level of parental investments. We assume that divorced and married parents share the same child quality production function and thus, production in divorce is defined by an additional TFP parameter, $\psi_d$. The Cobb-Douglas shares of inputs $\delta_{s,a}$ and the TFP parameters are permitted to depend on the age, $a$, of the child. This modelling assumption reflects known developmental differences across stages of the child's developmental cycle.

5. Finally, the child attains functional independence at age 17, in which case the child quality production process ends. The parents enjoy a terminal value that increases with the current child quality level and continues to depend on the parents' marital status and wages.

   state variable $\chi_i \in \{0, 1\}$ indicates the current investment condition, and equals 1 when the investment process has been terminated.

The child quality production function described by dynamic elements 4-5 can be related to leading models of child investment. Cunha and Heckman (2007) and Cunha, Heckman, Lochner, and Masterov (2006) argue that a variety of skills that children must develop are subject to "critical periods" early in life, and hence much of intellectual development is accomplished by the time the child reaches school age. Hopkins and Bracht (1975), for example, demonstrate that IQ is stable by the age of 10 or so, suggesting that the critical period for intellectual development occurs by this time. Further, Cunha and Heckman, Cunha et al., and Cunha, Heckman and Schennach (2010) emphasize the importance of both cognitive and non-cognitive skill acquisition to child outcomes, along with the importance of "dynamic complementarity" and "self-productivity" of skill levels in ongoing skill production. Todd and Wolpin (2003, 2007) consider cognitive skill formation, and argue from a different perspective for the importance of both current and lagged inputs to the ongoing production process. They demonstrate the importance of allowing for unobserved endowment effects and the endogeneity of inputs to child skill production.

Like Todd and Wolpin, we restrict attention to cognitive skill.[6] In our empirical implementation, we relate $k_t$ to cognitive test scores from the CDS in a way that permits noisy measurement. The initial conditions that we specify when estimating the model directly address the need to account for unobserved endowment heterogeneity, and the model accounts for endogeneity of investments in determining absolute skill level in a specific manner. Finally, the investment problem that we model begins at birth. Our empirical implementation focuses on progress from birth through a set of tests that are completed for most sample children before the age of ten, befitting an analysis of cognitive skill production under the prescriptions of the literature.

In modeling the behavior of married and divorced parents an important specification choice is the manner in which spouses interact. One may assume that spouses interact cooperatively or noncooperatively.[7] It is unclear that ex-spouses are able to interact in a manner that achieves the Pareto frontier. In a model that moves though married and divorced states, if cooperation is ever attained in marriage it is unclear how spouses' mode of interaction might transition from such cooperation in marriage to the potential cooperation failures of divorce, or how the presence of children might influence interactions in the divorce state. One might assume cooperation throughout, though this is unsatisfying for childless divorced spouses and, moreover, rules out any effect of marital dissolution standards on divorce rates under conventional specifications. Instead we assume noncooperative interaction throughout. The public consumption in the marriage state, along with the public good nature of the child, brings the marriage state noncooperative equilibrium closer to a cooperative equilibrium, and yet this approach still allows us to model divorce state interactions noncooperatively. More complex approaches include allowing spouses to choose the current mode of interaction as events progress, following Flinn (2000), or specifying population heterogeneity in spouses' mode of interaction, following Eckstein and Lifshitz (2009) and Del Boca and Flinn (2012). Though the latter approaches are appealing, they would add a great deal of complexity to an already unwieldy model. For the above reasons, and given standing evidence of marriage dissolution standard effects on divorce rates in Friedberg (1998), Gruber (2004) and elsewhere, we choose to assume noncooperative interaction throughout. Finally, we assume that spouses' investment strategies constitute a Markov Perfect Equilibrium.[8]

[6]Our empirical meausure, discussed below in Section 5, is in fact more narrow than theirs in the space of cognitive skills.

[7]For examples of the cooperative and non-cooperative approaches, respectively, see Browning and Chiappori (1998), Lundberg and Pollak (1994), and Del Boca and Flinn (2011).

[8]See, for example, Pakes and McGuire (2000).

### 3.1 Divorced Parents

Given the absence of a remarriage market, divorce is an absorbing state. Since the value of the future is independent of current labor supply, an ex-spouse $s$ who has a child of quality $k_t$ at the termination of the investment process need only solve the equivalent of static labor supply problem. Let $V_{md,s}$ indicate the value to a divorced spouse $s$, whose child has matured. The relevant state variables in this case are the wage of individual $s$, $w_s$, and the final attributes of the child, $k$. Thus the value function is given by:

$$V_{md,s}(w_s, k) = \max_{h_s} \left\{ \alpha_{1s} \log(w_s h_s) + \alpha_{2s} \log(1 - h_s) + \alpha_{3s} \log(k) + \zeta + \beta \mathbb{E}_{\hat{w} \mid w} V_{md,s}(\hat{w}, k) \right\} \quad (4)$$

In the case of divorce with an ongoing child quality improvement process, each parent's time allocation problem now includes the question of how much to invest in the child, with the evident interdependence and dynamic repercussions. We therefore look for an equilibrium in parental time investments and labor supplies, which may be determined by the state of the marriage, child quality and parental wages. In addition, since child development is a finite-stage process, the age $a$ of the child is a state variable for the parental investment problem. Following the typical logic of strategic equilibrium, each spouse $s$ takes the strategies of the other, $(\tau_{k,-s}, h_{k,-s})_{k=a}^{17}$, as given and solves the dynamic problem:

$$V_{d,s}(w_1, w_2, k_a, a) = \max_{h_2, \tau_2} \{ u(c, 1 - h_s - \tau_s) + \alpha_{3s} \log(k) + \beta \mathbb{E}[V_{d,s}(\hat{w}_1, \hat{w}_2, k_{a+1}, a+1)] \} \quad (5)$$

$$k_{a+1} = \psi_{a,d} \tau_1^{\delta_{1,16}} \tau_2^{\delta_{2,16}} k_a^{\delta_{k,16}} \quad (6)$$

$$c = y_s(w_1, w_2, h_1, h_2 | \pi) \quad (7)$$

$$h_s \geq 0 \quad (8)$$

$$\tau_s \leq \overline{\tau}_s \quad (9)$$

Here, $V_{d,s}$ represents the value to spouse $s$, which depends on the wages of each parent, $(w_1, w_2)$, the child's current attributes, $k_a$, and the age, $a$, of the child. The expectation next period is taken with respect to the markov transition kernels described above. Solution of this dynamic program produces a pair of strategies $(\tau_{s,a}, h_{s,a})$ that are functions of the state. Markov Perfect Equilibrium requires that the strategies of the other spouse, $(\tau_{-s,a}, h_{-s,a})$ taking this pair as given, solve the equivalent problem.

### 3.2 Married Parents

The experiences they will have if they enter the divorce state can meaningfully affect the investment decisions of forward-looking married parents. In particular, currently married parents who believe that divorce is likely in the near future will make investment decisions that look more like those made by divorced parents than will couples who believe that divorce is a remote possibility.

We must specify the manner in which divorce decisions are made. Under our assumption of noncooperative behavior, these decisions are not, in general, efficient. The nature of the decisions depends critically on legal statutes. There are two plausible cases: one in which it is enough for one of the parents to ask for a divorce for the couple to enter the divorce state and the second in which both parents must agree to the divorce for it to occur. These cases are commonly termed unilateral and bilateral divorce regimes. Given the time frame of the sample we use, we develop the model under the assumption of unilateral divorce, however the solution can easily be generalized to the bilateral case.

The derivation of the married parents' equilibrium is similar to that of the divorced parents' equilibrium, with one major difference being the search for an equilibrium in divorce decisions as well as in investments and values. As before, we begin with the value of a terminated child investment process at $k_{17}$ for spouse $s$. Spouses solve the same static labor supply problem as in the divorce state with terminated investment, except that their earned income contributes to the public household consumption $C$.

Let the value function to spouse $s$ in this stage be given by $V_{mm,s}(w_1, w_2, \theta, k)$. Due to the non-excludable nature of consumption, spouses play a simulataneous move game on labor supply:

$$V_{mm,s}(w_1, w_2, \theta, k_{17}) = \max_{h_s} \left\{ u_s(C, 1-h_s, \theta, k) + \beta \mathbb{E}\left[ dV_{md,s}(\hat{w}_1, k_{17}) + (1-d)V_{mm,s}(\hat{w}_1, \hat{w}_2, \hat{\theta}, k_{17}) \right] \right\}, \tag{10}$$

subject to the constraints:

$$C = w_0 h_0 + w_1 h_1 \tag{11}$$

$$d = \begin{cases} 0 & \text{if } V_{mm,s}(\hat{w}_1, \hat{w}_2, \hat{\theta}, k_{17}) \geq V_{md,s}(\hat{w}_s, k_{17}) \text{ for } s = 0, 1 \\ 1 & \text{otherwise} \end{cases} \tag{12}$$

Note the transition rule for divorce, $d$, implies that divorce is unilateral.

When investment is active, parents make time investment as well as labor supply decisions. Much like the case in divorce, this introduces a dynamic element to the non-cooperative game. At age $a$, each spouse takes the current and future strategies of the other as given. These are restricted to be a function of the relevant state variables. We can therefore define these equilibrium conditions recursively. Let $V_s(w_1, w_2, \theta, k_a, a)$ indicate the value to spouse $s$ in a marriage of quality $\theta$, with wages $(w_1, w_2)$ and an $a$-aged child of quality $k_a$. This value is defined as follows:

$$V_{fm,s}(w_1, w_2, \theta, k_a, a) = \max_{h_s, \tau_s} \Big\{ u(C, 1 - h_s - \tau_s, \theta, k_a) + \beta \mathbb{E}[d V_{fd,s}(\hat{w}_1, \hat{w}_2, k_{a+1}, a+1) + (1-d) V_{fm,s}(\hat{w}_1, \hat{w}_2, \hat{\theta}, k_{a+1}, a+1) \Big\} \quad (13)$$

$$k_{a+1} = \psi_a(\theta) \tau_1^{\delta_{1,a}} \tau_2^{\delta_{2,a}} k_a^{\delta_{k,a}} \quad (14)$$

$$C = w_1 h_1 + w_2 h_2 \quad (15)$$

$$h_s \geq 0 \quad (16)$$

$$d = \begin{cases} 0 & \text{if } V_{fm,s}(\hat{w}_1, \hat{w}_2, \hat{\theta}, k_{a+1}, a+1) \geq V_{fd,s}(\hat{w}_1, \hat{w}_2, k_{a+1}, a+1) \text{ for } s = 1, 2 \\ 1 & \text{otherwise} \end{cases} \quad (17)$$

Once again, the divorce decision rule satisfies the conditions of a unilateral standard. In addition, it is restricted to be a function of state variables. This optimization is performed taking the current choices of the other spouse, $(h_{-s}, \tau_{-s})$ which must themselves be the solution to the other spouse's equivalent maximization problem, taking $(h_s, \tau_s)$ as given.

It should be apparent that without further inspection into the properties of the solution to this game, each state variable is a potential influence on the divorce decisions of the couple. Naturally, this means that investment and labor supply decisions themselves may be influenced by the extent to which they influence the probability of divorce in future periods. Thus, the general specification of this model permits a highly complex set of causal interactions. In the appendix, we will show that the solution to this Markov Perfect Equilibrium given our modelling assumptions admits a far simpler solution, which facilitates analysis and estimation.

### 3.3 Childless Couples and the Fertility Decision

Since divorce is an absorbing state and the fertility process is only active in the married state, divorced childless ex-spouse $s$ solves an equivalent static labor supply problem to a divorced spouse with a mature child. Let $V_{d,s}(w_s)$ indicate the value to such a person, with current wage $w_s$. This function satisfies the recursion:

$$V_{d,s}(w_s) = \max_{h_s} \{ u_s(w_s h_s, 1 - h_s) + \beta \mathbb{E}[V_{d,s}(\hat{w}_s)] \} \quad (18)$$

Childless married couples, on the other hand, must jointly choose to continue in the marriage and attempt to conceive a child, to continue in the marriage and not attempt to conceive a child, and whether to divorce or not. This requires us to set a timing convention, which we do as follows. In period $t$, parents decide whether or not to attempt to have a child. If they do make such an attempt, a child arrives with probability $\gamma_f$. We allow parents to make their divorce decision in $t+1$ *after* the realization of the fertility shock. Thus, we can write the dynamic program as the following series of equalities:

$$V_{m,s}(w_1, w_2, \theta) = \max_{h_s} \Big\{ u(C, 1 - h_s) + \beta \left[ \chi_f F_{1,s}(w_1, w_2, \theta) + (1 - \chi_f) F_{0,s}(w_1, w_2, \theta) \right] \Big\} \tag{19}$$

$$F_{0,s}(w_1, w_2, \theta) = \mathbb{E}\left[ d_{f,0} V_{d,s}(\hat{w}_s) + (1 - d_{f,0}) V_s(\hat{w}_1, \hat{w}_2, \hat{\theta}) \right] \tag{20}$$

$$F_{1,s}(w_1, w_2, \theta) = \mathbb{E}\Big\{ \gamma_f \left[ d_{f,1} V_{fd,s}(\hat{w}_1, \hat{w}_2, k_0, 0) + (1 - d_{f,1}) V_{fm,s}(\hat{w}_1, \hat{w}_2, \hat{\theta}, k_0, 0) \right] + (1 - \gamma_f) F_{0,s}(w_1, w_2, \theta) \Big\} \tag{21}$$

$$\chi_f = \begin{cases} 1 & \text{if } F_{1,s}(w_1, w_2, \theta) \geq F_{0,s}(w_1, w_2, \theta) \text{ for } s = 1, 2 \\ 0 & \text{otherwise} \end{cases} \tag{22}$$

$$d_{f,0} = \begin{cases} 0 & \text{if } V_{m,s}(\hat{w}_1, \hat{w}_2, \hat{\theta}) \geq V_{d,s}(\hat{w}_1, \hat{w}_2) \text{ for } s = 1, 2 \\ 1 & \text{otherwise} \end{cases} \tag{23}$$

$$d_{f,1} = \begin{cases} 0 & \text{if } V_{fm,s}(\hat{w}_1, \hat{w}_2, \hat{\theta}, k_0, 0) \geq V_{fd,s}(\hat{w}_1, \hat{w}_2, k_0, 0) \text{ for } s = 1, 2 \\ 1 & \text{otherwise} \end{cases} \tag{24}$$

$$C = w_1 h_1 + w_2 h_2 \tag{25}$$

$$h_s \geq 0 \tag{26}$$

Each spouse $s$ solves a maximization problem over labor supply, $h_s$, that takes the decision of the the other as given. As discussed previously, we require that this pair of choices $(h_1, h_2)$ satisfies the constraints of strategic equilibrium. The continuation value (found in equation 21) from a successful fertility attempt integrates over possible realizations of child quality, future wages and marriage quality, subject to a unilateral divorce rule (which is a function of these variables), described in (24). A similar integration is performed in (20) over possible state realizations in the case of an unsuccessful fertility attempt, which coincides with the case in which no such attempt is made. Finally, fertility itself is the result of a *bilateral* decision: it is chosen if the continuation value (moderated by the probability of success) is greater in the case of fertility for both spouses, as is described in (22).

The model described in this section includes fertility decisions through the arrival of the first child only. Subsequent fertility is unmodeled, though estimation of the model will include those families that eventually have more than one child. We have chosen subsequent fertility as a dimension on which to limit the scope of the modeling exercise, as the modeling of ongoing fertility equilibria, and investment alocation many siblings, complicates the specification more than it adds to available insights on marial status and child investment dynamics. However, the resources invested and the decision to divorce are surely influenced by the number of children. In section 2, as in other research, marital stability shows a strong positive association with fertility. A primary channel through which unmodeled subsequent fertility is likely to affect our results is the larger family size, and therefore likely smaller investment per child, in intact households. The omission of investment in second and later childen is likely to mute any effect of divorced parenting on firstborn children's attainment implied by our model.

### 3.4 Solving and Characterizing the Equilibrium of the Model

As discussed above, in its general form this model allows for a complex set of strategic and dynamic interactions. This may, in principle, make the equilibrium decision rules difficult to characterize. However, under the specifications that we use here, we will see that this model permits considerable simplification. In this section we will discuss the nature of these decision rules and present some intuition for the results. We relegate the technical details of the solution to the appendix.

First, it can be shown that for couples with children, the value functions $V_{mm,s}, V_{md,s}, V_{fm,s}, V_{mm,s}$ for $s = 1, 2$ are additively separable in log-child quality, $\log(k)$. Specifically, we have the following:

$$V_{mm,s}(w_1, w_2, \theta, k) = (1-\beta)^{-1}\alpha_{s3}[\log(k) + \zeta] + \mathcal{V}_{mm,s}(w_1, w_2, \theta) \tag{27}$$

$$V_{md,s}(w_1, w_2, k) = (1-\beta)^{-1}\alpha_{s3}[\log(k) + \zeta] + \mathcal{V}_{md,s}(w_1, w_2) \tag{28}$$

$$V_{fm,s}(w_1, w_2, k, \theta, a) = (1-\beta)^{-1}\zeta + \alpha_{V,a}\log(k) + \mathcal{V}_{mm}(w_1, w_2, \theta, a) \tag{29}$$

$$V_{fd,s}(w_1, w_2, k, a) = (1-\beta)^{-1}\zeta + \alpha_{V,s,a}\log(k) + \mathcal{V}_{mm}(w_1, w_2, a) \tag{30}$$

$$\alpha_{V,s,a} = \alpha_{s3} + \beta\delta_{3,a}\alpha_{V,s,a+1} \tag{31}$$

$$\alpha_{V,s,17} = (1-\beta)^{-1}\alpha_{s3} \tag{32}$$

An immediate consequence of this result is that, in Markov Perfect Equilibrium, labor supply, divorce and investment strategies are not a function of child quality. In addition, the closed form solution of this component of the value function greatly simplifies solution of the model, since $k$ is a continuously distributed state variable. This allows for a much richer set of empirical implications than would otherwise be possible. In Appendix A we show how this result can be derived in addition to the form of component value functions $\mathcal{V}_{mm,s}, \mathcal{V}_{md,s}, \mathcal{V}_{fm,s}, \mathcal{V}_{fd}$. Intuitively, the derivation proceeds as follows. First, when the child reaches maturity, $\log(k)$ is static, and hence enters as an additive constant, as is shown in equations (27) and (28). While the child is developing, there is a period return to $\log(k_a)$ represented by $\alpha_{3s}$. In addition, $\log(k_a)$ enters additively in the production of $\log(k_{a+1})$, which recursively preserves additive separability. These two components represent the value to $\log(k_a)$ in the current period, which is shown in (31).

One important consequence of this simplification is that the return to time investments is itself log-linear, which greatly simplifies the invesment problem. At each stage of the dynamic program, we can take first order conditions with respect to $h_s$ and $\tau_s$ (when applicable) to produce a low-dimensional set of equations that characterize equilbrium choices. First, when spouses are divorced without a child or with a mature child, labor supply solves a static choice:

$$h_s = \frac{\alpha_{1s}}{\alpha_{1s} + \alpha_{2s}} \tag{33}$$

In the case of married couples without a child or with a mature child there are also no dynamic considerations, hence labor supply is also a static choice, but must constitute a strategic equilibrium. First order conditions produce the pair of equations:

$$h_1 = \max\left\{1 - \phi_1 - \phi_1\left(\frac{w_2 h_2}{w_1}\right), 0\right\} \tag{34}$$

$$h_2 = \max\left\{1 - \phi_2 - \phi_2\left(\frac{w_1 h_1}{w_2}\right), 0\right\} \tag{35}$$

$$\phi_s = \frac{\alpha_{2s}}{\alpha_{1s} + \alpha_{2s}} \tag{36}$$

When parents have an actively developing child, each must choose $\tau_s$ in addition to $h_s$, factoring in the future returns in child quality to investments. As mentioned above, simplifying the value functions makes the first order conditions to this problem more tractable. When parents are divorced, the problem is no longer symmetric, since the Father must pay a fraction $\pi$ of his income in child support. The Father's first order conditions yield the solution:

$$h_1 = \frac{\alpha_{11}}{\alpha_{11} + \alpha_{12}}(1 - \tau_1) \tag{37}$$

$$\tau_1 = \min\left\{\overline{\tau}_1, \frac{\delta_1 \beta \alpha_{1,V,t+1}}{\alpha_{11} + \alpha_{12} + \delta_1 \beta \alpha_{1,V,t+1}}\right\}, \tag{38}$$

while, given this choice, first order conditions for the Mother produce the solution:

$$h_2 = \max\left\{\frac{\alpha_{21}}{\alpha_{21} + \alpha_{22}}(1 - \tau_2) - \frac{\alpha_{22}}{\alpha_{21} + \alpha_{22}} \frac{\pi h_1 w_1}{w_2}, 0\right\} \tag{39}$$

$$\tau_2 = \min\left\{\frac{\delta_2 \beta \alpha_{2,V,t+1}}{\alpha_{22} + \delta_2 \beta \alpha_{2,V,t+1}}(1 - h_2), \overline{\tau}_2\right\} \tag{40}$$

Finally, when parents are married, the solution to the problem bears strong resemblance to the solution without active development:

$$h_1 = \max\left\{\frac{\alpha_{11}}{\alpha_{11} + \alpha_{12} + \delta_{1,a}\beta\alpha_{V,1,a+1}} - \frac{\alpha_{12} + \delta_{1,a}\beta\alpha_{V,1,a+1}}{\alpha_{11} + \alpha_{12} + \delta_{1,a}\beta\alpha_{V,1,a+1}} \frac{h_2 w_2}{w_1}, 0\right\} \tag{41}$$

$$\tau_1 = \frac{\delta_{1,a}\beta\alpha_{V,1,a+1}}{\alpha_{12} + \delta_{1,a}\beta\alpha_{V,1,a+1}}(1 - h_1) \tag{42}$$

$$h_2 = \max\left\{\frac{\alpha_{21}}{\alpha_{21} + \alpha_{22} + \delta_{2,a}\beta\alpha_{V,2,a+1}} - \frac{\alpha_{22} + \delta_{2,a}\beta\alpha_{V,2,a+1}}{\alpha_{21} + \alpha_{22} + \delta_{2,a}\beta\alpha_{V,2,a+1}} \frac{h_1 w_1}{w_2}, 0\right\} \tag{43}$$

$$\tau_2 = \frac{\delta_{2,a}\beta\alpha_{V,2,a+1}}{\alpha_{22} + \delta_{2,a}\beta\alpha_{V,2,a+1}}(1 - h_2) \tag{44}$$

The solution for labor supply is functionally identical to the childless case, with coefficients adjusted for the extra marginal value of a unit of time. In addition, time investment is allocated according to its proportional value with leisure.

## 4 Data

*Section forthcoming*

## 5 Estimation and Results

*Section forthcoming*

## 6 Policy Experiments

*Section forthcoming*

## 7 Conclusion

We have developed and estimated a model that allows for strategic behavior between parents in making fertility, child quality investment, and divorce decisions. An important component of the behavioral model is the family law environment, which has a large impact on the rewards attached to the marital states and, in turn, the returns to investment in child quality. We use data from the PSID and the PSID-CDS to estimate model parameters using a Method of Simulated Moments estimation procedure. We find that the parameter estimates are roughly in accord with our priors, and that the correspondence between simulated and sample moments varies between "adequate" to "good."

The most important contribution of our work is to the understanding of the dynamic relationship between divorce decisions and the evolution of fertility and child quality, and the dependence of this process on family law parameters. While there is a well-established empirical relationship between child outcomes and the characteristics of the household in which she or he lives, we have attempted to disentangle the simultaneous relationships between divorce, fertility, and child development using a behavioral model of these decisions. To our knowledge, this is one of the first studies to link the family law environment to the fertility decisions of intact families, and, in some instances, we find the link to be substantial.[9] While our estimated model is based on a number of restrictive and ultimately untestable assumptions, our view is that this type of framework is the only way to begin to understand the complex dynamics present within married households.

We have conducted some initial investigations of how substantial changes in the parameters characterizing the family law environment - those reflecting contact time between divorced parents and the child and the child support transfers between parents - impact the parental welfare distribution and child outcomes, which include birth. To date, our experiments suggest small to moderate impacts of changing the family law environment on the average value of child quality in the population (though the impact on the number of children born can be great in some circumstances). Instead, the concurrent impact on the welfare distribution of parents is substantially

[9] The other that we are aware of is Aizer and McLanahan (2006).

greater, with custody arrangements dominating child support in terms of their impact on spouses' welfare. Such a result may suggest a rationale for why changes in family law tend to occur very gradually over time. While "better" family law environments may favorably impact the child outcome distribution, the gains are slight compared to the shifts in the parental welfare distribution. It follows that it may be difficult to attain the wide-spread support from both mothers and fathers that radical changes in family law require.

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# A Details of the Model Solution

### Divorced at Maturity

When the child has reached maturity and the marriage has dissolved, parents choose only their private leisure and consumption. The value function can be written as:

$$V_{md,s}(w,k) = \max_{h_s} \left\{ \alpha_{1s} \log(w_s h_s)) + \alpha_{2s} \log(1-h_s) + \alpha_{3s} \log(k) + \zeta + \beta \mathbb{E}_{\hat{w} \mid w} V_{md,s}(\hat{w},k) \right\} \quad (45)$$

Since $h_s$ has the closed form solution:

$$h_s = \frac{\alpha_{1s}}{\alpha_{1s} + \alpha_{2s}}, \quad (46)$$

utility in each period is a closed form expression involving wages. The value function in this case can be solved simply as a matrix inversion problem using the transition matrix for wages, $\Pi_{ws}$. The value from children enters as an additive constant. Thus, we can write the value function as:

$$V_{md,s}(w,k) = \frac{\alpha_{3s} \log(k) + \zeta}{1-\beta} + \mathcal{V}_{md,s}(w), \quad (47)$$

where

$$\mathcal{V}_{md,s}(w) = \max_{h_s} \left\{ \alpha_{1s} \log(w_s h_s) + \alpha_{2s} \log(1-h_s) + \beta \mathbb{E}_{\hat{w} \mid w} [\mathcal{V}_{md,s}(\hat{w}) \right\} \quad (48)$$

Thus, modulo an additive constant, the value function of divorced couples at maturity is the same as for those who divorce without children.

### Married at Maturity

In this case, both spouses make labor supply decisions $(h_1, h_2)$ and enjoy public consumption, as well as the utility derived from their adult child. Let the value function to spouse $s$ in this stage be given by $V_{mm,s}(w,\theta,k)$. Since, as before, the utility from children essentially enters as a constant, we can write:

$$V_{mm,s}(w,\theta,k) = \frac{\alpha_{3s} \log(k) + \zeta}{1-\beta} + \mathcal{V}_{mm,s}(w,\theta), \quad (49)$$

where:

$$\mathcal{V}_{mm,s}(w,\theta) = \max_{h_s} \left\{ u_s(c, 1-h_s) + \beta \mathbb{E} \left[ d\mathcal{V}_{d,s}(\hat{w}) + (1-d)\mathcal{V}_{m,s}(\hat{w},\theta) \right] \right\}, \quad (50)$$

subject to the constraints:

$$c = w_0 h_0 + w_1 h_1 \quad (51)$$

$$d = \begin{cases} 0 & \text{if } \mathcal{V}_{mm,s}(\hat{w},\theta) \geq \mathcal{V}_{d,s}(\hat{w}) \text{ for } s = 0,1 \\ 1 & \text{otherwise} \end{cases} \quad (52)$$

Note the transition rule for divorce, $d$, implies that divorce is unilateral. Taking first-order conditions, the equilibrium is characterized as follows:

$$h_1 \geq 1 - \phi_1 - \phi_1 \left(\frac{w_2 h_2}{w_1}\right) \tag{53}$$

$$h_2 \geq 1 - \phi_2 - \phi_2 \left(\frac{w_1 h_1}{w_2}\right) \tag{54}$$

$$h_1 \geq 0 \tag{55}$$

$$h_2 \geq 0 \tag{56}$$

$$\phi_s = \frac{\alpha_{2s}}{\alpha_{1s} + \alpha_{2s}} \tag{57}$$

Uniqueness of this solution is simple to verify. We can solve by first checking each corner condition, then solving for the internal solution. The interior solution is:

$$h_1 = \frac{1 - \phi_1 - \phi_1 \frac{w_2}{w_1}(1 - \phi_2)}{1 + \phi_1 \phi_2} \tag{58}$$

$$h_2 = \frac{1 - \phi_2 - \phi_2 \frac{w_1}{w_2}(1 - \phi_1)}{1 + \phi_2 \phi_1} \tag{59}$$

As before, the value from the child enters as an additive constant. The other component of the value function can be solved either by value function iteration or gradient based solution.

## Divorced during Development

So far, the value function has been additively separable in terms of child quality; a component that is linear in $\log(k)$. We will show that this property holds as we move backwards through each stage of the parental investment problem. This provides a huge simplification of the model's solution. First, we state the problem. Then, we'll show that the property holds in the penultimate period before maturity. This should be sufficient to demonstrate that it holds in all previous periods, as well as to show the recursion that defines the coefficents on $\log(k)$ each period.

### Mother's Problem

We start with the mother's problem. To do this, we take the father's choices, $(h_1, \tau_1)$, as given. The value function at $a = 16$ can be written:

$$V_{fd,2}(w, k, 16) = \max_{h_2, \tau_2} \left\{ u(c, 1 - h_2 - \tau_2) + \alpha_{23} \log(k) + \beta \mathbb{E}[V_{md,2}(\hat{w}, \hat{k})] \right\} \tag{60}$$

$$\hat{k} = \psi_{16}(0) \tau_1^{\delta_{1,16}} \tau_2^{\delta_{2,16}} k^{\delta_{k,16}} \tag{61}$$

$$c = \pi w_1 h_1 + w_2 h_2 \tag{62}$$

$$h_2 \geq 0 \tag{63}$$

$$\tau_2 \leq \overline{\tau}_2 \tag{64}$$

Note the introduction of two divorce law parameters. $\overline{\tau}_2 = 1 - \overline{\tau}_1$ sets the allocation of custody time, while $\pi$ is the marginal tax rate on the Father's earnings that is transferred to the Mother.

Now, substituting the form for $V_{md,s}$ given in (47) and using the production function (3), we get

$$V_{fd,2}(w,k,16) = \max_{h_2,\tau_2}\Big\{u(c, 1-h_2-\tau_2) + \alpha_{23}\log(k) + \frac{\beta\alpha_{23}}{1-\beta}[\log(\psi_{16}(0)) + \delta_{1,16}\log(\tau_1) + \delta_{2,16}\log(\tau_2) + \delta_{k,16}\log(k)] + \beta\mathbb{E}[\mathcal{V}_{d,2}(\hat{w})]\Big\} \quad (65)$$

Finally, collecting terms gives the following:

$$V_{md,2}(w,k,16) = \alpha_{V,2,16}\log(k) + \mathcal{V}_{md,2}(w,16) \quad (66)$$

$$\alpha_{V,2,16} = \alpha_{23} + \beta\delta_{k,16}\frac{\alpha_{23}}{1-\beta} \quad (67)$$

$$\mathcal{V}_{fd,2}(w,16) = \max_{h_2,\tau_2}\Big\{u(c,1-h_2-\tau_2) + \frac{\beta\alpha_{23}}{1-\beta}[\delta_{1,16}\log(\tau_1) + \delta_{2,16}\log(\tau_2)] + \beta\mathbb{E}[\mathcal{V}_{md,s}(\hat{w})]\Big\} \quad (68)$$

So we see that the additive structure is preserved in this case. By induction then, we can write the value function at age $a$ as:

$$V_{fd,2}(w,k,a) = \mathcal{V}_{fd,2}(w,a) + \alpha_{V,2,a}\log(k) \quad (69)$$

$$\alpha_{V,2,a} = \alpha_{23} + \beta\delta_{k,a}\alpha_{V,2,a+1} \quad (70)$$

$$\mathcal{V}_{fd,2}(w,a) = \max_{h_2,\tau_2}\{u(c,1-h_2-\tau_2) + \beta\mathbb{E}[\mathcal{V}_{fd,2}(\hat{w},a+1) + \alpha_{V,2,a+1}(\log(\psi_a(0)) + \delta_1\log(\tau_1) + \delta_2\log(\tau_2))]\} \quad (71)$$

$$c = \pi w_1 h_1 + w_2 h_2 \quad (72)$$

$$h_2 \geq 0 \quad (73)$$

$$\tau_2 \leq \overline{\tau}_2 \quad (74)$$

These decisions are made taking the father's choices of $(h_1, \tau_1)$ as given. The first order conditions yield

$$h_2 = \max\left\{\frac{\alpha_{21}}{\alpha_{21}+\alpha_{22}}(1-\tau_2) - \frac{\alpha_{22}}{\alpha_{21}+\alpha_{22}}\frac{\pi h_1 w_1}{w_2}, 0\right\} \quad (75)$$

$$\tau_2 = \min\left\{\frac{\delta_2\beta\alpha_{2,V,t+1}}{\alpha_{22}+\delta_2\beta\alpha_{2,V,t+1}}(1-h_2), \overline{\tau}_2\right\} \quad (76)$$

Given $h_1$, we can solve the system above by solving for $h_2$ assuming that the constraint on $\tau_1$ does not bind:

$$h_2 = \max\left\{\frac{\alpha_{21}}{\alpha_{21}+\alpha_{22}+\delta_2\beta\alpha_{V,2,t+1}} - \frac{\alpha_{22}+\delta_2\beta\alpha_{V,2,t+1}}{\alpha_{21}+\alpha_{22}+\delta_2\beta\alpha_{V,2,t+1}}\frac{\pi h_1 w_1}{w_2}, 0\right\} \quad (77)$$

If the corresponding solution to $\tau_2$ violates the constraint, then we set $\tau_2 = \overline{\tau}_2$ and solve:

$$h_2 = \max\left\{\frac{\alpha_{21}}{\alpha_{21}+\alpha_{22}}(1-\overline{\tau}_2) - \frac{\alpha_{22}}{\alpha_{21}+\alpha_{22}}\frac{\pi h_1 w_1}{w_2}, 0\right\} \tag{78}$$

This solution method is valid since we know only one interior solution exists. Thus the constraint must bind if it is violated by this single interior solution.

### Father's Problem

Logic when solving for the Father's problem is identical. Therefore we skip verification of additive separability and state the problem in this separable form:

$$\mathcal{V}_{fd,1}(w,a) = \max_{h_1,\tau_1}\{u(c, 1-h_1-\tau_1) + \beta\mathbb{E}[\mathcal{V}_{fd,1}(w,a+1) + \alpha_{V,1,t+1}(\log(\psi_a(0) + \delta_1\log(\tau_1) + \delta_2\log(\tau_2)]\} \tag{79}$$

$$\alpha_{V,1,a} = \alpha_{13} + \beta\delta_{k,a}\alpha_{V,1,a+1} \tag{80}$$

$$c = (1-\pi)w_1 h_1 \tag{81}$$

$$h_1 \geq 0 \tag{82}$$

$$\tau_1 \leq \overline{\tau}_1 \tag{83}$$

The father's decisions are made taking the mother's choice of $\tau_2$ as given. His first order conditions yield:

$$h_1 = \frac{\alpha_{11}}{\alpha_{11}+\alpha_{12}}(1-\tau_1) \tag{84}$$

$$\tau_1 = \min\left\{\overline{\tau}_1, \frac{\delta_1\beta\alpha_{1,V,t+1}}{\alpha_{12}+\delta_1\beta\alpha_{1,V,t+1}}(1-h_1)\right\} \tag{85}$$

The above can be rearranged to yield

$$\tau_1 = \min\left\{\overline{\tau}_1, \frac{\delta_1\beta\alpha_{1,V,t+1}}{\alpha_{11}+\alpha_{12}+\delta_1\beta\alpha_{1,V,t+1}}\right\}. \tag{86}$$

## Married during Development

Once again, we wish to show that the additive separability of the value function is preserved when married. We will do this for the mother in the final period of development, which should be sufficient to establish the intuition. Once again, take the labor supply and investment decisions of the husband, $(h_1, \tau_1)$, as given. The value function at $a = 16$ can be written:

$$V_{fm,2}(w, \theta, k, 16) = \max_{h_2, \tau_2} \left\{ u(c, 1 - h_2 - \tau_2, \theta) + \alpha_{23} \log(k) + \beta \mathbb{E}[d V_{md,2}(\hat{w}, \hat{\theta}, \hat{k}) + (1 - d) V_{mm,2}(\hat{w}, \hat{\theta}, k) \right\} \quad (87)$$

$$\hat{k} = \psi_{16}(\theta) \tau_1^{\delta_{1,16}} \tau_2^{\delta_{2,16}} k^{\delta_{k,16}} \quad (88)$$

$$c = w_1 h_1 + w_2 h_2 \quad (89)$$

$$h_2 \geq 0 \quad (90)$$

$$d = \begin{cases} 0 & \text{if } V_{mm,s}(\hat{w}, \theta) \geq V_{md,s}(\hat{w}) \text{ for } s = 1, 2 \\ 1 & \text{otherwise} \end{cases} \quad (91)$$

Now, substituting the form for $V_{md,s}$ given in (47), the form for $V_{mm,s}$ given in (49) and using the production function (3), we get

$$V_{fm,2}(w, k, 16) = \max_{h_2, \tau_2} \left\{ u(c, 1 - h_2 - \tau_2) + \alpha_{23} \log(k) + \frac{\beta \alpha_{23}}{1 - \beta} [\log(\psi_{16}(\theta)) + \delta_{1,16} \log(\tau_1) + \delta_{2,16} \log(\tau_2) + \delta_{k,16} \log(k)] + \beta \mathbb{E}[d \mathcal{V}_{md,2}(\hat{w}) + (1 - d) \mathcal{V}_{mm,2}(\hat{w}, \hat{\theta})] \right\} \quad (92)$$

The additional step in the case of marriage requires us to verify that child quality $k$ does not affect the divorce decision. Inspecting equations (47) and (49) we see that:

$$V_{md,s}(\hat{w}, k) \geq V_{mm,s}(\hat{w}, \theta, k) \Leftrightarrow \mathcal{V}_{md,s}(\hat{w}) \geq \mathcal{V}_{mm,s}(\hat{w}, \theta) \quad (93)$$

This follows by the fact that the coefficient on $\log(k)$, $(1 - \beta)^{-1}[\alpha_{s3} + \zeta]$, is unaffected by marital status. Thus, finally, we can collect terms and write:

$$V_{fm,2}(w, \theta, k, 16) = \alpha_{V,2,16} \log(k) + \mathcal{V}_{fm,2}(w, \theta, 16) \quad (94)$$

$$\alpha_{V,2,16} = \alpha_{23} + \beta \delta_{k,16} \frac{\alpha_{23}}{1 - \beta} \quad (95)$$

$$(96)$$

And the value function $\mathcal{V}_{fm,2}$ can be written as:

$$\mathcal{V}_{fm,2}(w, \theta, 16) = \max_{h_2, \tau_2} \left\{ u(c, 1 - h_2 - \tau_2) + \frac{\beta \alpha_{23}}{1 - \beta} [\log(\psi_{16}(\theta)) + \delta_{1,16} \log(\tau_1) + \delta_{2,16} \log(\tau_2)] + \beta \mathbb{E}[d \mathcal{V}_{d,2}(\hat{w}) + (1 - d) \mathcal{V}_{mm,2}(\hat{w}, \theta)] \right\}, \quad (97)$$

with the divorce rule given by

$$d = \begin{cases} 0 & \text{if } \mathcal{V}_{mm,s}(\hat{w}, \theta) \geq \mathcal{V}_{d,s}(\hat{w}) \text{ for } s = 1, 2 \\ 1 & \text{otherwise} \end{cases} \quad (98)$$

Importantly, note that the coefficients on $\log(k)$ are the same as in the case for divorce. This is crucial since it implies that the divorce decision is unaffected by child quality. This logic is preserved by further induction, hence we can write the marriage problem more generally as:

$$V_{fm,2}(w, \theta, k, a) = \alpha_{V,2,a} \log(k) + \mathcal{V}_{fm,2}(w, \theta, k, a) \tag{99}$$

Where $\mathcal{V}_{fm,2}$ is given by:

$$\begin{aligned}\mathcal{V}_{fm,2}(w, \theta, a) = \max_{h_2, \tau_2} \{ & u(c, 1 - h_2 - \tau_2) + \beta\alpha_{2,V,a+1}[\log(\psi_a(\theta)) + \delta_{1,a}\log(\tau_1) + \delta_{2,a}\log(\tau_2)] \\ & + \beta\mathbb{E}[d\mathcal{V}_{fd,2}(\hat{w}, a+1) + (1-d)\mathcal{V}_{fm,2}(\hat{w}, \theta, a+1)]\Big\},\end{aligned} \tag{100}$$

subject to:

$$c = w_1 h_1 + w_2 h_2 \tag{101}$$

$$h_2 \geq 0 \tag{102}$$

$$d = \begin{cases} 0 & \text{if } \mathcal{V}_s(\hat{w}, \theta, a+1) \geq \mathcal{V}_s(\hat{w}, a+1) \text{ for } s = 0, 1 \\ 1 & \text{otherwise} \end{cases} \tag{103}$$

In similar fashion to the marriage game at maturity, the mother's solution can be written as:

$$h_2 = \max\left\{\frac{\alpha_{21}}{\alpha_{21} + \alpha_{22} + \delta_2\beta\alpha_{V,2,t+1}} - \frac{\alpha_{22} + \delta_2\beta\alpha_{V,2,a+1}}{\alpha_{21} + \alpha_{22} + \delta_2\beta\alpha_{V,2,a+1}}\frac{h_1 w_1}{w_2}, 0\right\} \tag{104}$$

$$\tau_2 = \frac{\delta_2\beta\alpha_{V,2,a+1}}{\alpha_{22} + \delta_2\beta\alpha_{V,2,t+1}}(1 - h_2) \tag{105}$$

The father's solution and the expression for $\mathcal{V}_1$ is symmetric. Note that the solution to this game can be computed in an identical fashion to the marriage game at maturity. We have the interior solution:

$$h_1 = \frac{1 - \phi_1 - \phi_1\frac{w_2}{w_1}(1 - \phi_2)}{1 + \phi_1\phi_2} \tag{106}$$

$$h_2 = \frac{1 - \phi_2 - \phi_2\frac{w_1}{w_2}(1 - \phi_1)}{1 + \phi_2\phi_1} \tag{107}$$

$$\phi_s = \frac{\alpha_{s2} + \delta_s\beta\alpha_{V,s,a+1}}{\alpha_{s1} + \alpha_{s2} + \delta_2\beta\alpha_{V,s,a+1}} \tag{108}$$

We are free to let the production parameters vary by marriage quality.