Macro-Finance Separation by Force of Habit

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Abstract

We incorporate risk premia variation arising from Campbell-Cochrane habit formation in a standard DSGE framework. We show how the simultaneous presence of consumption and labor habits can produce a separation between quantity and risk premia dynamics, and hence unite nonlinear habits and a production economy without compromising the ability of the model to fit macroeconomic variables. We can then use economic theory rather than a reduced-form approach to restrict several cashflow processes endogenously and study their pricing. First, nominal price rigidities explain an endogenous difference between aggregate consumption and market dividends and between real and nominal bonds that can rationalize two major asset pricing puzzles—an initially downward-sloping term structure of equity and an upward-sloping term structure of interest rates. Second, the model is able to explain the capital market’s reaction to a monetary policy shock documented by the extant literature.

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1. Introduction

We unite a textbook real business cycle (RBC) structure in which we allow for nominal price rigidities (Gali, 2008) with Campbell and Cochrane (1999) preferences that display a nonlinear habit in consumption and labor to incorporate risk premia variation into an otherwise standard macroeconomic model. We show how one can avoid the well-known difficulties in reconciling habit formation with business cycle facts (Jermann, 1998; Lettau and Uhlig, 2000; Boldrin, Christiano and Fisher, 2001; Uhlig, 2007; Rudebusch and Swanson, 2008; Swanson, 2012) by describing conditions under which habit formation models display business cycle dynamics that are separate
from risk premia dynamics. This result extends to the habit formation setting a macro-finance separation result analogous to one that Tallarini (2000) and Barillas, Hansen and Sargent (2009) describe in the long-run risk and robust control settings (see also Cochrane, 2008).

Under a macro-finance separation the additional source of strategic complementarities represented by the external habits does not affect quantity but only pricing decisions. This macro-finance separation implies that all decisions to save and consume are still mediated by a single return—the real risk-free rate—while it allows for explaining a set of puzzling asset pricing facts from first principles. In particular, along with the other stylized asset pricing facts that Campbell and Cochrane (1999) showed their habits could replicate, our model is able to capture simultaneously the negative slope of the term structure of dividend strip returns and volatilities as well as the upward slope of the term structure of interest rates (Lettau and Wachter, 2011; Binsbergen, Brandt and Kojien, 2012a; Binsbergen, Hueskes, Kojien and Vrugt, 2013; Lopez, 2012b), and to explain the capital market’s reaction to a monetary policy shock (e.g., Bernanke and Kuttner, 2005).

We argue that success along these dimensions is an important test of the New Keynesian framework. First, since the evidence on how a monetary impulse transmits through financial markets is evidence that a monetary model is supposed to be tested against, findings such as Bernanke and Kuttner’s suggest that the basic New Keynesian model is missing an important component and call for the incorporation of risk premia variation into the macroeconomic model (Cochrane, 2011). Second, under a macro-finance separation the only asset pricing implications of nominal rigidities are the restrictions they place on the cashflow processes. In this sense our model is particularly parsimonious; a standard New Keynesian structure restricts cashflows (and quantities more generally) by an appropriate choice of a free parameter to grant a macro-finance separation; standard Campbell-Cochrane preferences then price those cashflows.

1.1. Reconciling habit formation with business cycle facts

In a production economy habits affect equilibrium quantities by their effect on the intertemporal rate of substitution, which modifies the link between consumption-saving and consumption-investment decisions, and by their effect on the intratemporal marginal rate of substitution, which modifies the link between consumption and labor decisions. The consequence is that equilibrium quantities (consumption, output, labor, investment and the capital stock) are now driven also by an additional state (surplus consumption) that adds a nontrivial asset pricing structure onto the otherwise standard RBC model. Since the extra state explains a first-order component of asset prices, the quantity implications of the new structure are likely to generate either counterfactually large business cycle fluctuations in some real variables such as labor, the capital stock, the real wage rate, or the real risk-free rate, or small risk premia as people can absorb aggregate shocks by

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2The evidence of the capital market’s reaction to a policy shock is the reason we choose to focus on the habit formation framework rather than on a long-run risk framework building on Epstein-Zin-Weil preferences or on a variable rare disaster model. In fact, the mechanism that links the level of interest rates to the price of risk in the habit formation model works through the effect of policy on consumption, which the New Keynesian framework can model endogenously. The effect of policy on the price of risk in the Epstein-Zin-Weil or in the rare disasters frameworks would instead require modeling the effect of policy on the conditional volatility of consumption or on the disaster probability, little of which is known both theoretically and empirically.
varying labor or investment. In this sense, we argue that a crucial diagnostic test for a model that incorporates risk premia variation into a production economy is its ability to generate a macro-finance separation (and arbitrarily small departures from it).

Campbell and Cochrane (1999) engineered the consumption habit sensitivity function to control the intertemporal effect on consumption-saving decisions and to show how a particular form of habits can be used to rationalize large and volatile risk premia without producing a risk-free rate puzzle. In the same spirit, we engineer restrictions on the labor habit sensitivity function to control the intratemporal effect of habits on consumption-labor decisions so that habits can be used to rationalize large and volatile risk premia in a production economy without producing a quantity puzzle. Our microfoundation of the labor habit structure mirrors Campbell and Cochrane’s microfoundation of the consumption habit. The existence of a calibration that generates an exact separation between risk premia and quantity dynamics shows how habit formation (whether external or internal) can be reconciled with business cycle facts, independently of the presence of other intratemporal distortions such as wage rigidities or other labor market frictions (as considered for example by Uhlig, 2007 and Rudebusch and Swanson, 2008 as potential solutions to the quantity puzzle in habit formation models).

Finally, the effect of habits on the consumption-investment tradeoff is controlled by the curvature of capital adjustment costs, which determines the dependence of investment on $Q$. In the limit when capital adjustment is infinitely costly, investment is zero and $Q$ has no effect on investment.

Some financial spillovers can then be turned on by a simple change in one parameter.

1.2. Term structures of equity and bond strips

The production economy provides restrictions on cashflows based on economic theory rather than on a reduced-form modeling that may be difficult to reconcile with standard macroeconomic models. Since the macro-finance separation ensures that discount rate variation does not compromise the ability of the model to fit macroeconomic variables, we can focus on the asset pricing implications of the restrictions placed on quantities by the DSGE model.

Along with a solution to the equity premium, volatility and risk-free rate puzzles (Campbell and Cochrane, 1999), our nonlinear-habit model is able to provide an explanation for two other major asset pricing puzzles—an upward-sloping term structure of nominal (and real) interest rates and a downward-sloping term structure of the equity premium. The key properties behind the explanation are the macro-finance separation, which guarantees that habits affect the pricing kernel.

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3Our approach to unite nonlinear habits with a production economy can be applied to any RBC model of quantities to match the desired correlation properties of macro time series.

4The extant literature offers examples of habit formation in small-scale production economies but they always feature habits that are either linear (e.g., Jermann, 1998; Boldrin, Christiano and Fisher, 2001; De Paoli, Scott and Weken, 2010; Leith, Moldovan and Rossi, 2012; Challe and Giannitsarou, 2014) or that depart from the Campbell-Cochrane specification in order to grant exact exponential-affine term structures (Gallmeyer, Hollifield and Zin, 2005; Gallmeyer, Hollifield, Palomino and Zin, 2009; Bekhaert, Engstrom and Xing, 2009; Bekhaert, Cho and Moreno, 2010a; Bekhaert, Engstrom and Grenadier, 2010b; Palomino, 2012; Campbell, Pflueger and Viceira, 2013; Dew-Becker, 2013), so the financial spillovers into the intratemporal and on the intertemporal rates of substitution are left unrestrained.

5See, for example, Cochrane (2008) and Rudebusch and Swanson (2008, 2012) for a discussion of the advantages of jointly modeling both quantities and asset prices within a DSGE framework.

6While the search for a structural explanation of the positive slope of the term structure of interest rates has a rather
but not the cashflow processes, and nominal rigidities, which model a first-principled difference between consumption equity and market equity and between real and nominal bonds.

First, since an increase in the price of risk reduces holding-period returns, strip returns load negatively, and long-duration strips more heavily so, on shocks to the price of risk; these shocks are perfectly negatively correlated with the positively priced consumption shocks, so the term structures of equity and interest rates are upward-sloping in the long run. Since surplus consumption, and hence the price of risk, is a state variable that barely correlates (unconditionally) with either technology growth or the long-run technology component, investors fear long-duration equities primarily because they do poorly in recessions unrelated on average with technology risk.

Second, the ex-ante growth rates of consumption, market dividends and the price level are negative when the economy is in a bad technology state. At the same time, negative ex-ante consumption growth means low discount rates, which produce relatively higher returns. Under a unitary elasticity of intertemporal substitution, cashflow and discount-rate effects exactly offset for consumption claims, whose excess returns do not therefore load on long-run technology, while market dividend excess returns are low and bond excess returns are large in a bad technology state, and the more so the longer the claim duration. Returns on dividend claims carry long-run technology risk, while nominal bonds hedge against long-run movements in technology. In our context, technology is not a perfect random walk (there is some mean reversion), so short-run and long-run technology shocks are perfectly negatively correlated. It follows that exposure to long-run technology risk actually provides consumption insurance.

Therefore, the model generates an initially negative slope in the term structure of market equity for a sufficiently high degree of price stickiness, thus capturing the evidence by Binsbergen et al. (2012a). Moreover, the role of bonds (real and nominal) as a technology hedge does no longer produce a bond premium puzzle (Backus, Gregory and Zin, 1989; Rudebusch and Swanson, 2008; Gürkaynak and Wright, 2012; Duffee, 2013) because exposure to the state that drives the risk-free rate commands a negative price (similar to Lettau and Wachter, 2011).

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7Note that, like we do not have to rely on wage rigidities or other labor market frictions to reconcile habit formation with business cycle facts, we do not have to rely on them to capture the level and volatility of the equity premium (as in Favilukis and Lin, 2013) or its term structure properties (as in Marfè, 2013), so we remain considerably parsimonious in our modeling structure. In our framework, the main rationale for incorporating labor market imperfections would only be their potential ability to improve the time series properties of equilibrium quantities.

8To explain simultaneously time-variation in stock and bond returns in her endowment economy, Wachter (2006) makes the real rate countercyclical by activating a spillover of surplus consumption on consumption-saving decisions; as a consequence real bonds become assets with low payoffs when marginal utility is high, which helps to generate upward-sloping real and nominal yield curves. In contrast, our production economy does in general produce a departure from random-walk consumption that generates real-rate movements driven by long-run technology, which helps to generate upward-sloping yield curves endogenously, without the need to impose exogenously a countercyclicality in the real rate.
1.3. Understanding the capital market’s reaction to monetary policy

The New Keynesian elements of the economy allow for modeling the dependence of discount rates and of the different cashflows on monetary policy—in terms of their reaction to an unexpected policy movement as well as in terms of how a systematic policy rule affects the way shocks are priced. This property describes the transmission mechanism on a rich set of capital market instruments.

Moreover, since it displays time-variation in risk premia of first-order importance, the model is able to explain the documented capital market’s reaction to an unexpected interest rate movement (Rigobon and Sack, 2004; Bernanke and Kuttner, 2005; Gürkaynak, Sack and Swanson, 2005; Bjornland and Leitemo, 2009; Hanson and Stein, 2012; ?, Gertler and Karadi, 2013).

2. Macro-finance separation

The ability to preserve the quantity implications of the underlying real business cycle model is a crucial diagnostic to evaluate a macro-finance model that unites a model of the stochastic discount factor with a production economy. In making this claim we are taking to the logical extreme the critique moved by Lettau and Uhlig (2000), and later repeated by Uhlig (2007), Rudebusch and Swanson (2008) and Swanson (2012), to DSGE models with habit formation in the spirit of Campbell and Cochrane (1999). 9

2.1. Incorporating Campbell-Cochrane habit formation in a production economy

We define a spillover parameter $\xi = [\xi_1; \xi_2] \in \mathbb{R}^2$, which controls the spillover of the state that drives risk premia into the equilibrium quantities. We follow Campbell and Cochrane (1999) and Wachter (2006) in using the spillover parameter $\xi_1$ to control the effect of time-varying risk aversion on consumption-saving decisions (the intertemporal rate of substitution). Additionally, we introduce by similar reasoning a spillover parameter $\xi_2$ that controls the effect of time-varying risk aversion on consumption-labor decisions (the intratemporal rate of substitution). We then discuss what restrictions on the spillover parameter $\xi$ grant a macro-finance separation (proposition 1). We start in an economy with a deterministic capital stock; we subsequently allow for nontrivial capital accumulation and describe one last spillover, controlled by parameter $\xi_3 \in \mathbb{R}$, which affects the consumption-investment tradeoff (proposition 3).

2.1.1. Production side

The production side of the economy is characterized by a unit mass of identical firms indexed by $i \in [0, 1]$ that maximize intertemporal profits and operate with production technology

$$Y_t(i) = (\widetilde{A}_tN_t(i))^{1-\alpha}K_t(i)^\alpha$$

9We are not denying the possibility that a more volatile discount factor better fits quantity dynamics (in particular hump-shaped dynamics, as argued by Boldrin et al., 2001). However, we are arguing that the first step of the modeling exercise of incorporating volatile discount factors in a macromodel should be to keep the spillovers on quantities under control. We can then allow for an arbitrary spillover and a role of habits in the determination of quantity dynamics.
where $Y_t$ is real output, $N_t$ is the labor input, which they acquire at a unit cost equal to the nominal wage rate $W_t$, $K_t = e^{\mu t}$ is the deterministic capital stock, which grows at rate $\mu$ in a balanced-growth path, and $A_t$ denotes the exogenous labor-augmenting technology level. The $i$th good sells at the nominal price $P_t(i)$ and $P_t \equiv \left[ \int_0^1 P_t(i) \alpha(i) di \right]^{1/(1-\varepsilon)}$ is the price index. When working with a deterministic capital stock, we define for simplicity $a_t \equiv (1-\alpha)\tilde{a}_t + \alpha k_t$ as the log technology level.

We also allow for Calvo-type nominal price rigidities and monopolistic competition in the market for goods. Each firm $i$ can reset prices at any given time only with probability $1-\eta$ and faces the demand curve for the good it produces $C_t(i) = \left( \frac{P_t(i)}{\bar{P}_t} \right)^{-\varepsilon} C_t$, which arises as the cost-minimizing plan of individual consumers $j \in [0,1]$ who bundle the continuum of goods $i \in [0,1]$ via a Dixit-Stiglitz aggregate, with constant elasticity of substitution between goods, $\varepsilon$. Additionally, the government levies lump-sum taxes on each firm, $T_t$, to finance an employment subsidy $\tau = 1/\varepsilon$, which brings the unit nominal cost of labor down to $(1-\tau)W_t$ and is in place to offset any steady-state distortions caused by the monopolistic competition.

It follows that a standard New Keynesian Phillips curve describes the optimal price-setting decisions of firms, up to first order, as the forward-looking optimality condition linking inflation, $\pi_t \equiv \ln \left( \frac{P_t}{P_{t-1}} \right)$, and marginal costs, $mc_t \equiv (w_t - p_t) - \ln(\partial Y_t/\partial N_t)$,\(^10\)

$$\pi_t = \beta E_t \pi_{t+1} + \lambda m\bar{c}_t$$

where $\beta$ is the subjective discount rate and $\lambda \equiv (1-\eta)(1-\beta\eta)(1-\alpha)/\eta(1-\alpha + \alpha\varepsilon)$ controls the slope of the New Keynesian Phillips curve. Inflation is high when firms expect long-run marginal costs above the flexible-price level, in which case resetting firms choose a price above the index to realign their marginal costs to the desired level. Deviations of aggregate marginal costs from the desired level then associate with a gap in aggregate activity relative to the flexible-price equilibrium.

We define market equity as the value of the aggregate firm, which pays out per-period equilibrium profits as dividends,

$$D_t = \left( 1 - (1-\tau) \frac{W_t N_t}{P_t Y_t} \right) C_t$$

Since marginal costs fluctuate under price stickiness, the presence of nominal rigidities is entirely responsible for breaking down the equality between market dividends and (a scaled version of) consumption.

2.1.2. Consumption side

We consider the nonlinear habits of Campbell and Cochrane (1999) which we extend to allow people to develop a habit also to labor.

Preferences. Identical consumers indexed by $j \in [0,1]$ have preferences captured by the function

$$U_0(j) = E_0 \sum_{t=0}^{\infty} \beta \left( \frac{(C_t(j) - H^c_t)^{1-\gamma} - 1}{1-\gamma} - \frac{\chi (N_t(j) - H^n_t)^{1+\varphi}}{1+\varphi} \right)$$

\(^{10}\)Lower case letters denote logs and hats denote differences from the steady-state level.
where $C_t$ is real consumption, $N_t$ is the labor choice, and $H^c_t$ and $H^n_t$ represent habit levels that are a nonlinear function of past consumption and labor. Parameter $\beta$ is the subjective discount rate, $\gamma$ and $\varphi$ control the curvature of the utility function and parameter $\chi$ controls the steady-state effect of habits. We assume the calibration $\chi = \chi_0/S^{\gamma+\varphi}$, with $S$ defined below, to achieve the same steady state as under a power-utility specification ($H^c_t = H^n_t = 0$ and $\chi = \chi_0$). The case $\gamma = 1$ associates with a utility function that is consistent with balanced growth (see appendix B).

Similar preferences in which agents form a habit also to leisure have been used by Lettau and Uhlig (2000) and Uhlig (2007). People get used to an accustomed standard of living, and hence to some particular levels of consumption and labor. Like Lettau and Uhlig (2000), we use a structure that displays separability between consumption and labor to preserve the original implications of Campbell and Cochrane (1999) for the stochastic discount factor as much as possible.

Habits are endogenous state variables that induce a departure of the equilibrium dynamics from the power-utility model. As customary in the extant literature, we assume that the law of motion of habits is specified indirectly through the processes for surplus consumption $s_t \equiv \ln((C_t - H^c_t)/C_t)$ and surplus labor $z_t \equiv \ln((N_t - H^n_t)/N_t)$ in order to ensure that consumption and labor never fall below their respective habit levels, and hence well-behaved marginal utilities. The law of motion of the surplus levels is driven by aggregate log consumption and labor, $c_t \equiv \ln(\int_0^1 C_t(j) dj)$ and $n_t \equiv \ln(\int_0^1 N_t(j) dj)$; since each individual agent has zero mass, she takes the habit levels thus specified as external to her consumption and labor decisions. This structure implies the marginal utilities of consumption and labor

$$\frac{\partial U_t}{\partial C_t} = C_t^{\gamma} S_t^{-\gamma}$$
$$\frac{\partial U_t}{\partial N_t} = -\chi N_t^{\varphi} Z_t^{\varphi}$$

They compare with the marginal utilities of consumption and labor when consumers internalize the endogeneity of habits,

$$\frac{\partial U^\text{int.}}{\partial C_t} = C_t^{\gamma} S_t^{1-\gamma} + C_t^{-1}(\Delta_t - E_t - E_{t-1})M^c_t$$
$$\frac{\partial U^\text{int.}}{\partial N_t} = -\chi N_t^{\varphi} Z_t^{1+\varphi} + (1 - \alpha)\xi_t N_t^{-1}(\Delta_t - E_t - E_{t-1})M^n_t$$

where $M^c_t$ and $M^n_t$ are the shadow values of surplus consumption and surplus labor, respectively. Appendix B details the derivation.

On the one hand, a positive consumption (labor) shock means a lower marginal value of consumption (a higher marginal disutility of labor); on the other hand, a positive consumption (labor) shock increases the habit level and thereby increases the marginal value of consumption (decreases the marginal disutility of labor). When habits are internal, people take into account also the second effect as well as they become sensitive to unexpected movements in the shadow value of the surplus levels, which depend on current and future consumption and labor. Moreover, under balanced growth the two effects on the marginal utility of consumption exactly offset when habits
are internal, so the stochastic discount factor reduces to the one under power utility.\footnote{Note that this property is not true if $γ > 1$, as in the original calibration by Campbell and Cochrane.}

**Habit structure.** We assume the structure for the aggregate surplus levels

\[
\begin{align*}
    s_{t+1} &= (1 - ρ_s)s + ρ_s s_t + Λ_c(\hat{s}_t)e^c_{t+1} \\
    z_{t+1} &= (1 - ρ_z)z + ρ_z z_t + Λ_n(\hat{z}_t)e^n_{t+1}
\end{align*}
\]

where $e^x_t ≡ (E_t - E_{t-1})x_t$ denotes the one-period ahead forecast error in variable $x$ and $\tilde{n}_t ≡ \tilde{a}_t + n_t$ denotes effective labor. The mean parameter, $s$, and persistence, $ρ_s$, are common to both surplus processes. We parametrize the sensitivity functions as

\[
\begin{align*}
    Λ_c(\hat{s}_t) &= \begin{cases} 
        \frac{1}{S} \sqrt{1 - 2(s_t - s)} - 1, & s_t ≤ s + \frac{1}{2}(1 - S^2) \\
        0 & s_t > s + \frac{1}{2}(1 - S^2)
    \end{cases} \quad \text{with} \quad S = \sqrt{\frac{γ \text{var}(e^c_t)}{1 - ρ_z - ξ_1/γ}} \\
    Λ_n(\hat{z}_t) &= (1 - α)ξ_2 Λ_c(\hat{z}_t/ξ_2)
\end{align*}
\]

with $ξ = [ξ_1; ξ_2] ∈ \mathbb{R}^2$ a free parameter, where $ξ_2 = 0$ describes the case with constant surplus labor, which is equivalent to a model without labor habits.

The consumption (labor) habit indirectly specified by the surplus process is a complex nonlinear function of current and past consumption (labor), however, it is approximately a linear habit that adjusts slowly to unanticipated movements in consumption (effective labor).

Like Campbell and Cochrane (1999), we choose the consumption sensitivity function to satisfy the following conditions: (i) the consumption habit does not produce a risk-free rate puzzle; (ii) the habit coincides with the consumption level in the long run; (iii) the habit is predetermined at the steady state; and (iv) the habit moves nonnegatively with consumption near the steady state. The first condition shows how habits can be engineered in such a way that the spillover on consumption-saving decisions can be kept under control (via parameter $ξ_1$); the other conditions can be interpreted as (local) microfoundations that add to the well-behaved marginal utilities and the local slow-moving representation of habits. Appendix A proves these properties.\footnote{Moreover, in this context of a production economy both habits move strictly positively with consumption and labor, respectively, in the steady state. This property owes entirely to the endogeneization of equilibrium consumption and labor choices; in fact, in the endowment economy of Campbell and Cochrane there is only a zero relationship between habits and consumption in the steady state and a potentially strictly negative relationship near the steady state (Ljungqvist and Uhlig, 2009; see appendix A for more details).}

In this context, the real risk-free rate is determined by an intertemporal substitution motive (high discount rates give low incentives to save, so people command a high risk-free rate) and a precautionary savings motive (high discount-rate uncertainty gives high incentives to save, so people command a low risk-free rate). The spillover parameter $ξ_1$ controls whether an increase in surplus consumption drives rates up or down. As in Wachter (2006), the case $ξ_1 < 0$ has the interpretation that the precautionary savings effect wins over the intertemporal substitution effect; and vice versa for $ξ_1 > 0$.

By analogous logic, we choose the labor sensitivity function as follows: (i) the labor habit does not produce a quantity puzzle; (ii) the habit coincides with the labor level in the long run; and
(iii) the habit moves nonnegatively with labor near the steady state. While the second and third conditions can be interpreted as local microfoundations, the first condition shows how habits can be engineered in such a way that the spillover on consumption-labor decisions can be kept under control (via parameter $\xi_2$). In fact, the chosen labor sensitivity function imposes that the same state drives both surplus levels in equilibrium; in equilibrium the aggregate production function and market clearing imply at all dates\(^{13}\)

$$Z_t = S^{1-\xi_2} S_t^{\xi_2}$$

The case $\xi_2 < 0$, so high surplus labor associates with low surplus consumption, has the interpretation that people become hungrier for leisure after a bad consumption shock; the labor habit amplifies the substitution effect towards leisure after a negative consumption shock. Symmetrically, the case $\xi_2 > 0$ has the interpretation that people become more eager to work after a bad consumption shock; the labor habit amplifies the income effect of reducing leisure after a negative consumption shock. Except when $\xi_2 = 0$, the labor habit cannot in general be predetermined because, to produce the desired effects, it must react in equilibrium to the same shocks that move contemporaneous consumption. At the same time, in order not to strain the notion of habit, we require that in the long run the habit must coincide with the labor level as well as that it never moves in the opposite direction from labor. Appendix A proves these properties.

Consumers’ problem. Consumers maximize the intertemporal objective $U(j)$ subject to the sequence of budget constraints

$$C_t(j) + \frac{1}{1 + r_t} B_t(j) \leq W_t N_t(j) + B_{t-1}(j) + D_t$$

and the appropriate transversality condition, where $B_t$ denotes their time-$t$ holdings of one-period bonds and $D_t$ is the dividend they receive from owning the aggregate firm. For simplicity we include in the budget constraint only one-period non-contingent claims and the ownership of the aggregate firm; we can subsequently price an arbitrary claim in zero net supply in this setting by no-arbitrage logic (e.g., Rudebusch and Swanson, 2008; Binsbergen, Fernández-Villaverde, Koijen and Rubio-Ramírez, 2012b).

2.1.3. Equilibrium

We impose clearing of the market for each good $i$

$$Y_t(i) = C_t(i)$$

and the government runs a balanced budget, $T_t = \tau(W_t/P_t) N_t$.

When prices are sticky, optimality restricts aggregate marginal costs as

$$\hat{mc}_t = \frac{1 + \varphi}{1 - \alpha} \hat{\gamma}_t + (1 + \varphi \xi_2) \hat{\gamma}_t$$

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\(^{13}\)The equilibrium law of motion reduces to $\hat{z}_{t+1}/\xi_2 = \rho \hat{z}_t/\xi_2 + \Lambda(\hat{z}_t/\xi_2) \xi_2^{e+1} = \sum_{j=0}^{\infty} \rho^j \Lambda(\hat{z}_{t-j}/\xi_2) \xi_2^{e+1}$, which implies $\hat{z}_t/\xi_2 = \hat{s}_t$ in the appropriate equivalence class for stochastic processes.
where \( \hat{c}_t \equiv c_t - a_t \) denotes a stationary version of output. Since in this context the equilibrium level of inflation influences the equilibrium allocation, we must specify monetary policy, which we describe by a Taylor rule that reacts to inflation and a stationary transformation of output. We consider the detrended version of output \( \hat{y}_t = \hat{c}_t \) and therefore the description of policy

\[
i_t = \phi_\pi \pi_t + \phi_y \hat{y}_t
\]

which grants determinacy if \( \kappa(\phi_\pi - 1) + (1 - \beta)\phi_y > 0 \), for \( [\phi_\pi; \phi_y] \in \mathbb{R}^2_+ \), with \( \kappa \equiv \lambda(1 + \varphi)/(1 - \alpha) \) (Galí, 2008).

Finally, the log of technology evolves according to the process

\[
\Delta a_{t+1} = \mu + \phi u_t + \varepsilon_{a,t+1} \\
u_{t+1} = \rho u_t + \varepsilon_{u,t+1}
\]

where \( \varepsilon_{a,t} \sim \text{Niid}(0, \sigma_a^2) \) and \( \varepsilon_{a,t} = \varepsilon_{u,t} \). This structure allows for trend-stationary technology, if \( \phi = -(1 - \rho_u) \), as well as \( I(1) \) technology with a stochastic trend; if \( \phi = 0 \), the process reduces to a random walk. As it is customary in the asset pricing literature we refer to the predictable component \( \phi u_t \) as the long-run technology component and to \( (E_{t+1} - E_t) a_{t+1} \) as a short-run technology shock. To limit the number of degrees of freedom at our disposal we assume a one-shock structure, \( \varepsilon_{u,t} = \varepsilon_{a,t} \).

14 This assumption can be easily relaxed, however, the one-shock structure allows for the tractable Beveridge-Nelson decomposition of the technology process \( a_t = a_t^P + a_t^T \) into a stationary component, \( a_t^P \), and a random-walk component, \( a_t^T \), with

\[
\Delta a_t^P = \mu + \theta \varepsilon_{a,t} \\
a_t^T = (1 - \theta) u_t
\]

where \( \theta \equiv 1 + \phi/(1 - \rho_u) \). This decomposition highlights three cases of interest, which are consistent with the empirical properties of cashflows typically retained by both the real business cycle and the consumption-based asset pricing literature: (i) \( \theta = 0 \), and hence \( \phi = -(1 - \rho_u) \), characterizes the trend-stationary case—after a shock the level of technology reverts back to its previous trend; (ii) \( \theta = 1 \), and hence \( \phi = 0 \), characterizes the pure random-walk case—after a shock the level of technology remains at the new level; (iii) \( \theta \in (0, 1) \), and hence \( - (1 - \rho_u) < \phi < 0 \), characterizes a technology process that, following a shock, comes back towards its initial value but not all the way back.  

2.2. A macro-finance separation proposition

This section defines what we mean by a macro-finance separation and characterizes conditions that grant it in our context. The main results are propositions 1 and 2.

Definition (Macro-finance separation). We say that an equilibrium (a feasible allocation and a price system that solve each household’s and each firm’s problem and clear markets) is macro-financially separate if the equilibrium allocation and equilibrium inflation are the same as in the model without habits (i.e., such that \( H_n^c = H_n^h = 0 \) at all dates and \( \chi = \chi(0) \)).

In our context, risk premia are driven to first order by the price of risk, whose dynamics are fully determined by surplus consumption. Therefore, the notion of macro-finance separation boils down
to the separation between the dynamics of risk premia and the dynamics of quantities and of inflation. Time-varying risk aversion in turn spills over into a flexible-price equilibrium allocation if (and only if) it does into the intratemporal rate of substitution that determines the optimal consumption-labor decisions. In a sticky-price equilibrium, however, a spillover into consumption-saving decision would produce an additional departure from a macro-finance separation.

2.2.1. Spillover into the intratemporal rate of substitution

The dynamics of the surplus levels are engineered in such a way that their respective effects on the intratemporal marginal rate of substitution can offset for an appropriate choice of parameter $\xi_2$ because $\dot{\bar{Z}}_t = \dot{\bar{S}}^\xi_2_t$.

Parameter $\xi_2$ allows for controlling the size of the spillover into equilibrium quantities of the state, $S_t$, that adds to the power-utility model under external as well as under internal habit formation. In fact, the optimal intratemporal rate of substitution between consumption and labor, $-(\partial U_t/\partial N_t)/(\partial U_t/\partial C_t)$, reduces to the one under power utility, which is characterized by

$$\frac{-\partial U_t/\partial N_t}{\partial U_t/\partial C_t} = C_t^\gamma N_t^\varphi$$

if we assume the parametrization $\xi_2 = -\gamma/\varphi$, in the external-habit case, and the parametrization $\gamma = 1$ and $\xi_2 = 0$, when habits are internal.

Note the economic interpretation of these conditions. When habits are internal and $\gamma = 1$, labor habits are not required ($\xi_2 = 0$) to avoid an effect of surplus consumption on the consumption-labor tradeoff because the cashflow effect and the habit effect of a bad consumption shock on the marginal utility of consumption exactly offset. When habits are external, we need labor habits to have the substitution effect offset the income effect of a negative consumption shock, so people choose not to absorb the consumption movement by increasing wildly their labor effort. This is the economic interpretation of the separation condition $\xi_2 < 0$.

2.2.2. Spillover into the intertemporal rate of substitution

Time-varying risk aversion may spill over into the equilibrium allocation if it affects the rate of substitution between consumption and saving, $r_t = -\ln E_t M_{t+1}$. In our Gaussian external-habit setting, the dynamic IS equation balances an intertemporal substitution motive and a precautionary savings motive as

$$r_t = -\ln(\beta) - \gamma(1 - \rho_s - \xi_1/\gamma) \frac{2}{2} + \gamma E_t \Delta c_{t+1} - \xi_1 \dot{s}_t$$

where parameter $\xi_1$ controls the spillover into consumption-saving decisions, as in Campbell and Cochrane (1999) and Wachter (2006).

If habits are internal, people similarly balance their precautionary savings and intertemporal substitution motives only under balanced growth ($\gamma = 1$), which has the two habit effects on the

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15In the derivation we assume the conditional homoskedasticity of consumption growth, so $\text{var}(c_{t+1}) = \text{var}(\varepsilon_t)$, which is consistent, for example, with the absence of an intratemporal spillover, or with a solution for consumption based on a first-order approximation of the structural equations.
marginal utility of consumption exactly offset, so the marginal utility of consumption (and hence the stochastic discount factor) reduces to the one under power utility,

\[ \frac{\partial U_t}{\partial C_t} = C_t^{-1} \]

The Pareto optimum displays the same low and stable risk premia as under a power-utility specification; all time-variation in risk premia is symptomatic of the presence of an externality, as Pareto optimal asset prices associate with log-utility investors. It follows the Pareto optimal dynamic IS equation

\[ r^\text{int.}_t = -\ln(\beta) - \frac{1 - \rho_s - \xi_1}{2} S_t^2 + E_t \Delta c_{t+1} \]

2.2.3. Flexible prices

We formalize the last results in the following proposition:

**Proposition 1.** For any value of the preference parameter vector \([\gamma, \varphi] \in \mathbb{R}^2_+\) and given the spillover parameter \(\xi \equiv [\xi_1; \xi_2] \in \mathbb{R}^2\),

(a) there is a unique value of parameter \(\xi = [0; -\gamma/\varphi]\) such that the flexible-price competitive equilibrium (external-habit equilibrium) is macro-financially separate;

(b) there is a unique value of parameter \(\xi_2 = 0\) such that the Pareto optimum (internal-habit equilibrium) is macro-financially separate, provided \(\gamma = 1\), for any \(\xi_1 \in \mathbb{R}\).

Appendix B provides additional details on the proof of proposition 1.

Appendix F shows how under a macro-finance separation our habit structure can motivate the original consumption-based asset pricing model of Campbell and Cochrane (1999) as the outcome of a generic production economy.

2.2.4. Sticky prices

Once we activate further rigidities, such as sticky prices, and we discuss the equilibrium separation requirements a crucial question is what is the empirically relevant monetary policy in place. A natural choice is a Taylor rule that responds to inflation and some detrended version of output, in which case the competitive equilibrium is separate whenever the flexible-price equilibrium is (provided \(\xi_1 = 0\)). The following proposition formalizes this results and extends the macro-finance separation results of proposition 1 to the sticky-price setting:

**Proposition 2.** For any value of the preference parameter vector \([\gamma, \varphi] \in \mathbb{R}^2_+\), there is a unique value of parameter \(\xi = [0; -\gamma/\varphi] \in \mathbb{R}^2\) such that the sticky-price competitive equilibrium under a Taylor rule (in inflation and in detrended output) is macro-financially separate.

Appendix E describes the macro-financially separate competitive equilibrium under the Taylor rule and proves proposition 2. Intuitively, if the flexible-price equilibrium is macro-financially separate, detrended consumption, \(\hat{c}_t\), is a sufficient statistic for aggregate marginal costs; since the only remaining source of financial spillovers is the dynamic IS equation, we then require a zero spillover parameter \(\xi_1\).
2.3. Capital accumulation

In this section we allow for nontrivial capital accumulation driven by

\[ K_{t+1} = (1 - \delta)K_t + \Phi\left(\frac{I_t}{K_t}\right)K_t \]

where \( \delta \) is the depreciation rate and capital is costly adjusted according to the function

\[ \Phi\left(\frac{I_t}{K_t}\right) = \frac{e^\mu - 1 + \delta}{1 - \frac{1}{\xi_3}} I_t + \frac{1}{\xi_3} - \gamma_3 K_t^\xi_3, \]

with \( \gamma_3 \equiv \frac{e^\mu - 1 + \delta}{1 + \frac{1}{\xi_3}} \).

The parametric form of capital adjustment costs is standard (e.g., Jermann, 1998; Boldrin et al., 2001; Binsbergen et al., 2012b) and is calibrated to imply the steady-state relations \( I_t/K_t = \gamma_3 \), \( \Phi(I_t/K_t) = I_t/K_t \), \( \Phi'(I_t/K_t) = 1 \) and \( -\Phi''(I_t/K_t)I_t/K_t = 1/\xi_3 \). Through the adjustment cost curvature, \( 1/\xi_3 \), investment is determined by Tobin’s Q, which in our frictionless setting equals the expected discounted value of future market dividends (Hayashi, 1982); since the discounting is done via the Campbell-Cochrane discount factor, some spillover of surplus consumption on investment is necessary.

Therefore, the spillover of the time-varying risk aversion on quantities is now controlled by parameter \( \xi = [\xi_1; \xi_2; \xi_3] \) with \( \xi_3 \geq 0 \). As usual, parameter \( \xi_1 \) controls the spillover on the consumption-saving tradeoff and parameter \( \xi_2 \) controls the spillover on the consumption-labor tradeoff. Additionally, parameter \( \xi_3 \) controls the spillover on consumption-investment decisions; the absence of this type of spillover implies zero investment.

The only adjustment of the baseline model in this setting is the shape of the shock structure that drives surplus labor, which we now specify as

\[ z_{t+1} = (1 - \rho_s)s + \rho_z z_t + \xi_2\lambda(\tilde{z}_t/\xi_2)(E_{t+1} - E_t) f(\tilde{A}_{t+1}N_{t+1}) \]

with

\[ f(\tilde{A}_{t}N_{t}) = \ln[\tilde{A}_{t}N_{t}]^{1-\alpha} - \tilde{\delta}Q^{\xi_3}_t K_t^{1-\alpha}] \]

Surplus labor is no longer driven just by shocks to effective labor but also by shocks to the market value of the capital stock owned by consumers. This specification is arbitrarily close to the baseline specification for a curvature \( \xi_3 \) close to zero, and it is necessary to control the spillover on the intratemporal rate of substitution. In fact, market clearing, \( Y_t = C_t + I_t \), and the optimality condition for investment

\[ I_t = \tilde{\delta}Q^{\xi_3}_t K_t, \quad \xi_3 < \infty \]

imply that, in equilibrium, \( (E_{t+1} - E_t) f(\tilde{A}_{t+1}N_{t+1}) = (E_{t+1} - E_t)c_{t+1} \) and therefore

\[ \hat{\xi}_t = \xi_2\hat{s}_t \]
as required to control the intratemporal spillover.

We can therefore state in proposition 3 the requirements for macro-finance separation in the context of nontrivial capital accumulation:

**Proposition 3.** For any value of the preference parameter vector \([\gamma, \varphi] \in \mathbb{R}^2\) and given the spillover parameter \(\xi \equiv [\xi_1; \xi_2; \xi_3] \in \mathbb{R}^3\),

(a) there is a unique value of parameter \(\xi = [0; -\gamma/\varphi; 0]\) such that the flexible-price competitive equilibrium is macro-financially separate;

(b) there is a unique value of parameter \(\xi_2 = 0\) such that the Pareto optimum is macro-financially separate, provided \(\gamma = 1\), for any \(\xi_1 \in \mathbb{R}\) and for any \(\xi_3 \geq 0\);

(c) there is a unique value of parameter \(\xi = [0; -\gamma/\varphi; 0]\) such that the sticky-price competitive equilibrium under a Taylor rule in inflation and output is macro-financially separate.

Starting from a macro-financially separate equilibrium we can then allow for an arbitrarily small spillover by varying \(\xi_3\); in particular, movements in \(\xi_3\) allow for positive investment as a function of \(Q\) and for an equilibrium arbitrarily close to the equilibrium with deterministic capital (\(\xi_3 = 0\)).

2.4. Rejecting the internal-habit specification

If we take to the logical extreme the critique by Lettau and Uhlig (2000), a necessary diagnostic requirement for a model with habits (or, more generally, for any model that incorporates risk premia variation into a macromodel) to be deemed admissible is that it can be calibrated to display a macro-finance separation. In this context, internal habits have dramatically different asset pricing implications than external habits; namely, the internal-habit economy would reduce to the power-utility model not only in terms of quantity implications but also in terms of asset pricing implications.

We therefore favor the external-habit specification because a macro-financially separate internal-habit specification has trivial asset pricing implications.

3. Term structures of equity and interest rates

We work with the macro-financially separate equilibrium under the Taylor rule to study the asset pricing implications of our New Keynesian production economy with Campbell-Cochrane external habit formation. From this point on we impose the balanced growth requirement, \(\gamma = 1\), while the separation requirement imposes a deterministic capital accumulation (\(\xi_3 = 0\)).

3.1. Equilibrium cashflows

The New Keynesian framework models endogenously a difference between the real and the nominal term structures and a difference between aggregate consumption and market dividends. These differences follow from the New Keynesian property that high marginal costs increase inflation and reduce profits, and hence dividends (see also Lopez, 2012a). Table 1 lists the equilibrium dynamics of the main cashflow processes and summarizes their main qualitative differences. Appendix C provides the details.

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16Since under \(\xi_2 = 0\) the stochastic discount factor under internal habits reduces to the one under power-utility, parameter \(\xi_3\) can be left unrestricted to produce a macro-finance separation.
Four properties of the equilibrium cashflow processes are worth emphasizing. First, bond payoffs do not display mean growth and equity payoffs do. Second, short-run technology shocks have no effect on the bond payoffs but increase equity payoffs. Third, a negative long-run technology shock reduces the output gap, i.e., the actual level decreases more than capacity, and hence marginal costs drop. Therefore, a negative long-run technology shock reduces consumption growth and reduces marginal costs by even more, so the overall contemporaneous effect on market dividends is positive and on inflation is negative.

Finally, ex-ante consumption growth is negative when the economy is in a bad technology state (low \( \phi u_t \)) and so is the ex-ante value of inflation; moreover, expected marginal cost growth is positive, so the ex-ante value of market dividend growth is even lower than that of consumption growth. Thus, consumption and market dividends have long-run technology risk because their ex-ante payoff is low in bad technology states and even more so in the case of market dividends; on the contrary, the payoff of nominal bonds covaries positively with marginal utility.

### 3.2. Equilibrium term structures

Cashflows are discounted by means of the stochastic discount factor

\[
m_{t+1} = -\ln(\beta) - \Delta c_{t+1} - \Delta s_{t+1} = -\ln(\beta) - E_t \Delta c_{t+1} + (1 - \rho_s) \hat{s}_t - x_t D_c \varepsilon_{t+1}
\]

where \( x_t \equiv 1 + \Lambda(s_t) \) is the price of risk.

The loadings in table 1 describe the cashflow effects of the different types of shocks; to interpret the pricing of cashflow claims you must also consider the discount-rate effects of the different shocks. This is the exercise we carry out when solving for the term structures of the different cashflow claims. To do so, we set up the system in the form of Lopez et al. (2014),

\[
\begin{bmatrix}
\Delta c_{t+1} \\
\Delta d_{t+1} \\
-\pi_{t+1}
\end{bmatrix} = 
\begin{bmatrix}
\mu \\
\mu \\
0
\end{bmatrix} + 
\begin{bmatrix}
C_c \\
C_d \\
C_{nb}
\end{bmatrix} \zeta_t + 
\begin{bmatrix}
D_c \\
D_d \\
D_{nb}
\end{bmatrix} \varepsilon_{t+1}
\]

\[
\zeta_{t+1} = A \zeta_t + B \varepsilon_{t+1}
\]

where \( \zeta_t = \phi u_t, D_c = (1 + \psi_{cu}) \sigma_u, D_d = D_c - [1/(1 - \rho_a)] \psi_{cu} \sigma_u, D_{nb} = -[(\kappa)/(1 - \beta \rho_u)] \psi_{cu} \sigma_u, A = \rho_a \) and \( B = \phi \sigma_u \), and with the structural shock \( \varepsilon_t \sim N iid(0, 1) \). We then rely on their essentially-affine approximation to solve for the \( n \)th term structure component and the associated holding-period
expected excess return of consumption equity, market equity and the real and nominal interest rates,

\[ y_{d,t}^{(n)} = -\frac{1}{n} A_d^{(n)} - \frac{1}{n} B_{c,d}^{(n)} s_t - \frac{1}{n} B_{s,d}^{(n)} s_t \]

\[ r_{d,t}^{(n)} = E_{r_{d,t+1}}^{e(n)} + V_{d,n-1,t} \]

where \( E_{r_{d,t+1}}^{e(n)} = x_t D_c V'_{d,n-1,t} \) with \( V_{d,n-1,t} = D_d + B^{(n-1)} c_d + B^{(n-1)} s_d \Lambda_t D_c \), for the four different dividend processes, \( d \) (consumption, market dividends, the numeraire, and the inverse of the inflation rate). Equilibrium expressions of the term structure coefficients are detailed in Lopez et al. (2014) and are reported in appendix E.

Risk premia are the product of these systematic loadings on the structural shock and the price of a unit exposure to the structural shock,

\[ x_t D_c = x_t (1 + \psi_{c,\sigma}) \]

which is a strictly positive number for any parametrization \( \phi \in [-1-\rho_u]; 0) \). People command a strictly positive price to shoulder a unitary exposure to short-run technology shocks. By the perfectly negative correlation between short- and long-run technology shocks, systematic exposure to long-run technology shocks commands a negative price.

### 3.2.1. Properties of the term structures of equity and interest rates

To gain insight into the determinants of risk premia we analyze the three components of the loadings of risk premia on the structural shock, as reflected in variable \( V_{d,n-1,t} \) for different dividend processes and maturities. Note that a positive structural shock is a positive short-run shock and a negative long-run shock. Also, note the role of habits which, on the one hand, control the long-run term structure properties and, on the other, amplify all risk premia via the large and volatile price of risk (see also appendix G).

The loading on surplus consumption captures the properties of the premium commanded by long-duration claims; all term structures display an upward slope at the long end, a property that is driven by the perfectly negative correlation between shocks to consumption and to risk aversion. Long-duration portfolios pay off little in precisely those states of the world in which surplus consumption is low and so risk aversion is high; in fact, such loadings converge to the positive number \( B_{s,d}^{(\infty)} = 1 \) for an arbitrary dividend process (see also Lopez et al., 2014). Since shouldering surplus-consumption shocks is equivalent to shouldering consumption shocks, the strictly positive habit-related loading of infinite-duration zero-coupon cashflow claims commands a strictly positive price.

In this context, the habit persistence parameter, \( \rho_s \), controls the level of the positive risk premium commanded by infinite-duration cashflow claims because the price of risk is decreasing in the habit persistence parameter under macro-finance separation and because the habit-related loading of risk premia dominates over long horizons. It follows that a larger persistence implies lower infinite-duration risk premia and a nearly unit-root habit process produces nearly flat long-run term structures. Similar to Campbell and Cochrane (1999) and Wachter (2006), this property is key to avoid the equity premium puzzle as well as to solve the bond premium puzzle despite a positively
autocorrelated consumption growth process. Figure 1c plots these loadings under our baseline calibration listed in table 2.

The short end of the term structures depends much more on the loadings on short-term cashflow risk and on the stationary component of technology. First, the exposure of risk premia to short-run cashflow risk depends on vector $D$, and controls the level of the term structures. These loadings are depicted in figure 1a under our baseline calibration.

Second, the exposure of risk premia to long-run cashflow and discount-rate risk is able to dominate the shape of the short end of the term structures before the long-horizon effect of the habit-related loadings kicks in. In particular, we are able to generate a downward-sloping short end in the term structure of market equity for any calibration such that $B_{cd}^{(n)}B$ is sufficiently negative. In fact, since for market equity claims the exposure to long-run technology shocks commands a price of

$$\text{cov}_t(-m_{t+1}, B_{cd}^{(n)}\zeta) = x_t(1 - \rho_u^n)\frac{1}{\alpha} (1 + \psi_{cu})\psi_{cu}$$

the fact that $[1 + \psi_{cu}(\phi)]\psi_{cu}(\phi) < 0$ for any value of $\phi \in [-1 - \rho_u); 0]$ delivers the required property. Namely, the long-run technology-related loading component of zero-coupon market equities on the structural shock is a negative and convex function of maturity, and the analogous loading of zero-coupon bonds (real and nominal) is a positive and concave function of maturity, a shown in figure 1b. The respective loading of the term structure of consumption equity is flat as the cashflow and the discount-rate effects of long-run technology exactly offset under the requirement of a unitary elasticity of intertemporal substitution. Nominal price rigidities are therefore able to provide an explanation for the equity puzzle documented by Binsbergen et al. (2012a).

Therefore, on the one hand, long-run dividend strips carry long-run technology risk as their long-run technology-related loading on the structural shock is negative; a positive structural shock worsens the long-run technology state as well as equity excess returns. Since exposure to the structural shock carries a positive price there is a downward-sloping component in the term structure of equity; to carry long-run technology risk means to provide consumption insurance. This effect gets offset by the negative loading of long-run dividend strips on risk aversion but only over a long horizon—our habit model is therefore generally unable to generate a globally downward-sloping term structure of equity. On the other hand, long-run zero-coupon bonds contain a component that provides insurance against long-run technology movements; but this hedging property of bonds does no longer produce a bond premium puzzle because exposure to a long-run technology shock carries a negative price.

3.2.2. Calibration

Table 2 lists all deep parameters in the model and their calibration and highlight the parsimoniousness of our setting. We calibrate all parameters of the basic New Keynesian model using standard values in the New Keynesian literature (e.g., Galí, 2008). We pick a value of unity for $\gamma$, consistent with balanced growth. We choose a similar value for the inverse of Frisch’s labor supply elasticity, $\phi = 1$. The labor share in value added is $1 - \alpha = 2/3$. The Taylor rule coefficients are $\phi_{x} = 1.5$ and $\phi_{y} = 0.5/12$, in line with the estimates in Taylor (1993).\(^{17}\) The elasticity of

\(^{17}\)Taylor (1993) considers the yoy nominal interest rate and estimates reaction coefficients of 1.5 for yoy inflation and of 0.5 for annual deviations of output from potential; since we are working here at monthly frequency, the latter
Figure 1: Loadings of different zero-coupon cashflow claims on the sources of systematic risk.
substitution in the CES aggregator is $\varepsilon = 12$ and the average duration of prices is $(1-\eta)^{-1} = 9$ months, hence $\eta = 8/9$, which implies a slope of the New Keynesian Phillips curve at monthly frequency of $\lambda = 0.002$. Since we work under macro-finance separation, we assume a spillover parameter $\xi = [0; -1]$.

<table>
<thead>
<tr>
<th>Parameter</th>
<th>Value</th>
</tr>
</thead>
<tbody>
<tr>
<td>New Keynesian block $\gamma$</td>
<td>Utility curvature in consumption 1</td>
</tr>
<tr>
<td>$\varphi$</td>
<td>Utility curvature in labor 1</td>
</tr>
<tr>
<td>$\beta$</td>
<td>Subjective discount factor 0.9963 to match an average real interest rate, $r$ 0.01/12</td>
</tr>
<tr>
<td>$1 - \alpha$</td>
<td>Labor share in value added 2/3</td>
</tr>
<tr>
<td>$\varepsilon$</td>
<td>Elasticity of substitution in Dixit-Stiglitz aggregator 12</td>
</tr>
<tr>
<td>$1/(1-\eta)$</td>
<td>Average price duration (in months) 9</td>
</tr>
<tr>
<td>$\phi_r$</td>
<td>Policy response coefficient to inflation movements 1.5</td>
</tr>
<tr>
<td>$\phi_y$</td>
<td>Policy response coefficient to output movements 0.5/12</td>
</tr>
<tr>
<td>Habit block</td>
<td>Financial spillover into the intertemporal rate of substitution $\xi_1$ 0</td>
</tr>
<tr>
<td>$\gamma + \varphi \xi_2$</td>
<td>Financial spillover into the intratemporal rate of substitution 0</td>
</tr>
<tr>
<td>$\rho_s$</td>
<td>Habit persistence 0.91/12</td>
</tr>
<tr>
<td>Exogenous block $\mu$</td>
<td>Mean technology growth 0.0015</td>
</tr>
<tr>
<td>$\sigma^2_a$</td>
<td>Conditional volatility of technology 0.0150² to match a conditional volatility of consumption, $</td>
</tr>
<tr>
<td>$\rho_u$</td>
<td>Persistence of long-run technology 0.9</td>
</tr>
<tr>
<td>$\phi$</td>
<td>Volatility of long-run technology, $\phi$ -0.08 which implies a Beveridge-Nelson coefficient, $\theta$ 0.1</td>
</tr>
</tbody>
</table>

Table 2: Deep parameters and their calibration (monthly frequency).

We choose a habit persistence parameter, $\rho_s$, to generate a realistic average equity premium (a lower value increases the long end level of all term structures and vice versa). Finally, we calibrate $\beta$ to match an average monthly real rate $r$ of 1% per year.

To calibrate the remaining parameters, which pin down the model’s dynamics, we turn to a parametrization similar to those routinely used in the consumption-based asset pricing literature (e.g., Campbell and Cochrane, 1999; Bansal and Yaron, 2004; Croce et al., 2014). Note how, in calibrating the exogenous dynamics, one has to be careful in maintaining an empirically reasonable volatility of the different cashflow processes relative to the consumption process. For example, Campbell and Cochrane (1999) work with random-walk processes with ratios $||D_d||/||D_c|| = \text{var}(\Delta d)/\text{var}(\Delta c)$ of about 60. The calibration of Bansal and Yaron (2004) includes nontrivial predictable components and implies ratios $\text{var}(\Delta d)/\text{var}(\Delta c)$ and $||D_d||/||D_c||$ that are both around 20. The calibration of Croce et al. (2014) imposes instead ratios around 35.¹⁸

¹⁸Note how, in choosing a perfect correlation between short- and long-run technology shocks, we are sacrificing coefficient must be divided by 12.
Accordingly, we calibrate the mean monthly growth rate, $\mu$, the conditional volatility of technology, $\sigma_a$, and the other technology parameters, $[\phi, \rho_u]$, to match the respective values of the consumption process in Bansal and Yaron (2004), using the equilibrium property
\[
\|D_c\|^2 = (1 + \psi_{cu})^2 \sigma_a^2
\]
and to generate cashflow volatility ratios at the higher end of the values found in the literature for high degrees of price rigidity (an average price duration of 12 months) and at the lower end for low degrees of price rigidity (an average price duration of 6 months). Namely, our calibration implies a ratio of conditional volatilities, $\|D_d\|/\|D_c\|$, and a ratio of unconditional volatilities, $\text{var}(\Delta d)/\text{var}(\Delta c)$, that vary between 20 and 60 as the degree of price stickiness varies from 6 months to 12 months.

3.2.3. Results

Figure 2 reports the average term structure of equilibrium risk premia, volatilities and Sharpe ratios of consumption and market equities and of real and nominal interest rates. Figure 2 also shows how only long-duration cashflow strips are unconditionally mean-variance efficient, even though all zero-coupon claims reach the conditional Hansen and Jagannathan (1991) bound and are therefore on the conditional mean-variance frontier.

Figure 3 shows the effect of price stickiness and highlights its role in generating an initially downward-sloping term structure of market equity. Moreover, equilibrium risk premia, volatilities and Sharpe ratios shift upwards as the degree of nominal price rigidity increases. The case without nominal rigidities (price duration = 1 month) associates with the case in which there is no endogenous difference between the real and nominal bond term structures and between the term structures of consumption and market equity.

Figure 4 plots the term structures conditional on different values of the state that drives them (surplus consumption). Bad surplus consumption states, which associate with high risk aversion, scale up level, slope and curvature of the term structures. Good surplus consumption states associate with virtually flat term structures. The initially negative slope of the term structure of market equity is similarly a property that holds in any region of the state space. Against this background, note how our simple model—which in particular relies on a one-shock structure—is unable to predict a sign shift in the slope of the term structures in any region of the state space.

3.2.4. Diagnostics

To gain further insight into the properties of our model of the stochastic discount factor we study the diagnostic decompositions of the discount factor proposed by Alvarez and Jermann (2005) and simplicity for the replication of the imperfect correlations seen in the data between consumption and market dividends and between consumption growth and market returns (e.g., Albuquerque, Eichenbaum and Rebelo, 2014).

As discussed in subsection 3.2.4, the reason we depart in our baseline calibration from a trend-stationary technology process, $\phi = -(1 - \rho_u)$, is that it implies a trivial martingale discount-factor component $m_{t+1} = 0$ because there are no permanent shocks to the marginal utility of wealth. Alvarez and Jermann (2005) argued forcefully for a model of the
Figure 2: Unconditional term structures of equity and interest rates under macro-finance separation. Different lines associate with term structures of different cashflow claims: real bonds (dotted), nominal bonds (dash-dotted), consumption equity (dashed) and market equity (solid line).
Figure 3: Term structure of market equity for different degrees of price stickiness under macro-finance separation. Different lines represent different calibrations for the average price duration: one month (dotted), six months (dash-dotted), nine months (dashed) and twelve months (solid line).
Figure 4: Term structures for different values of surplus consumption, $S$, the state that drives risk premia. Colorbars depict the probability of occurrence of the different values of the state variable.
Hansen and Scheinkman (2009). In the context of an essentially-affine approximation, Lopez et al. (2014) show how the transient component of the stochastic discount factor in the external-habit model has nontrivial properties as long as \( \phi \neq 0 \), in which case \( B_{b,\xi}^{(\infty)} = 1 \) is strictly positive, while in the random-walk technology case, \( \phi = 0 \), the unstable asymptotic solution \( B_{b,s}^{(\infty)} = 0 \) realizes. To help visualize this property, figure 5 plots the phase map \( B_{n-1}^{s} \rightarrow B_{n}^{s} - B_{n-1}^{s} \) for the term structure loading on surplus consumption of equilibrium real bonds under our baseline calibration. Since \( B_{s}^{(0)} = 0 \) is not in the left cluster point and that cluster point is unstable, the equilibrium coefficients \( \{B_{s}^{(n)}\} \) must converge to the right cluster point.

Lopez et al. (2014) show how the martingale component of the stochastic discount factor,

\[
m_{t+1}^p = \begin{cases} \frac{1}{2}x_t^2\|D_c\|^2 - x_tD_c\epsilon_t+1, & \text{if } \phi = \xi_1 = 0 \\ \frac{1}{2}\theta^2\|B\|^2 - \theta BE_{t+1} & \text{elsewhere} \end{cases}
\]

which is discontinuous at \( \phi = \xi_1 = 0 \) (in which case the equilibrium coefficients always equal the unstable solution), has trivial properties only under trend-stationary technology, \( \theta = 0 \), and implies marginal utility of wealth to include both a nontrivial permanent component and a nontrivial transient component.

22See Hansen, Heaton and Li (2008) and Koijen, Lustig and Nieuwerburgh (2010a); Koijen, Lustig, Nieuwerburgh and Verdelhan (2010b) for applications.
the approximate entropy ratio

\[ \omega_t = \frac{\text{var}(m_{t+1}^P)}{\text{var}(m_{t+1})} = \begin{cases} 1 & \text{if } \phi = \xi_1 = 0 \\ \frac{\rho_0^2 \| b_0 \|^2}{(1-\rho_s)(1-2\rho_s)}, & \text{elsewhere} \end{cases} \quad (1) \]

Consider the two extreme cases, \( \theta = 0, 1 \). The case of trend-stationary technology, \( \theta = 0 \), implies \( m_{t+1}^P = 0 \) because there are no permanent shocks to the marginal utility of wealth. The case of random-walk technology, \( \theta = 1 \), combined with a zero spillover parameter \( \xi_1 = 0 \), implies a variance ratio (1) constant at unity. This property is appealing in that it satisfies a diagnostic property forcefully advocated by Alvarez and Jermann (2005); however, it would predict no time-variation in the relative importance of the permanent and transient components, which the return forecastability literature would reject (see Koijen et al., 2010a; Lopez et al., 2014).

Therefore, intermediate parametrizations such that \( 0 < \theta < 1 \) produce a model of the stochastic discount factor that displays three key realistic features: a time-varying permanent component, a time-varying transient component, and time-variation in the relative importance of the permanent and transient components.\(^{23}\)

A final diagnostic exercise concerns the property of our simple setting that the price of risk is univariate, so stock and bond risk premia are nearly perfectly correlated with each other. This property is contrary to some return predictability evidence (see also Koijen et al., 2010a; Lettau and Ludvigson, 2010; Duffee, 2013) in which case the external habit should be modeled so as to allow for a multivariate price of risk.

4. Transmission mechanism

The simplicity of the Taylor rule induces a monetary policy disturbance, \( v_t = -\phi u_t \), that equals the inverse of long-run technology. Therefore, we can equivalently characterize the competitive equilibrium as

\[
\begin{align*}
    c_t &= a_t + \psi_{cv} v_t \\
    \pi_t &= \psi_{\pi v} v_t
\end{align*}
\]

with \( \psi_{cv} = -\psi_{cu}/\phi \) and \( \psi_{\pi v} = -\psi_{\pi u}/\phi \). We switch freely between the representation in terms of technology shocks (which we use in section 3 to understand how different contingent claims are

\( ^{23} \) However, if the real bond loadings on surplus consumption are on the stable path, it is extremely difficult to have an average entropy ratio close to one, as advocated by Alvarez and Jermann (2005). The finding in Alvarez and Jermann of real and nominal variance ratios close to one rests however on proxies for the unobservable infinite-horizon zero-coupon bonds; in our model one can show that using a 20-year bond as a proxy for the infinite-duration bond associates with entropy ratios much closer to unity; the high persistence of surplus consumption is responsible for the low speed of convergence of the loadings, as shown in figure 1c. The same is true if we consider nominal payoffs and a decomposition of the nominal stochastic discount factor.
priced) and the representation in terms of policy shocks (which we use in this section to understand how a monetary policy shock transmits through capital markets).

As long as we work under a macro-finance separation the absence of spillovers implies that the transmission mechanism of a policy shock on quantities and on inflation does not change relative to the basic New Keynesian model. The effect of policy shocks on consumption-saving decisions remains mediated by the risk-free rate as made clear by the dynamic IS equation. However, since the positive implications of external habits are on asset prices we can rationalize in a parsimonious framework the evidence about the capital market’s reaction to a policy shock, and hence gain a more complete understanding of the effect of policy on capital markets as well as empirical support for the model’s predictions.

4.1. Exogenous effect of a monetary policy shock on capital markets

The effect of an unexpected exogenous movement in monetary policy allows for a comparison with the evidence gathered by the extant literature (see Bernanke and Kuttner, 2005 as well as Gurkaynak et al., 2005; Hanson and Stein, 2012; Gertler and Karadi, 2013).

We rely on the notion of generalized impulse reponse (e.g., Koop et al., 1996) of an arbitrary process \( Y \) after a shock \( \sigma \epsilon_t \) that hits the economy at time \( t \),

\[
GI_y(k, t-1, \sigma) \equiv E_{t-1}[Y_{t+k}|\sigma \epsilon_t] - E_{t-1}[Y_{t+k}] \quad \text{for } n = 0, 1, \ldots
\]

This definition in terms of conditional expectations constructs a response of the system as an average of what might happen given the past history and the present shock that allows for nonlinear laws of motion. We use Monte Carlo methods to construct the generalized impulse responses of nonlinear transformations of surplus consumption such as the price of risk. Namely, for a given history up to period \( t-1 \) we draw \( L \) sequences of \( K \) shocks \( \{\epsilon_{\ell,t+k}\}_{k=0}^{K-1} \) for \( \ell = 1, \ldots, L \) and construct the associated simulated processes \( \{Y_{\ell,t+k}\}_{k=0}^{K-1} \), similarly we construct the shocked sequence

\[
\tilde{\epsilon}_{\ell,t+k} = \begin{cases} 
(1 + \sigma)\epsilon_{\ell,t}, & k = 0 \\
\epsilon_{\ell,t+k}, & \text{otherwise}
\end{cases} \quad \text{for } \ell = 1, \ldots, L
\]

and the associated simulated process \( \{\tilde{Y}_{\ell,t+k}\}_{k=0}^{K-1} \). We rely on a law of large numbers to grant

\[
\frac{1}{L} \sum_{\ell=1}^{L} (\tilde{Y}_{\ell,t+k} - Y_{\ell,t+k}) \xrightarrow{a.s.} GI_y(k, t-1, \sigma)
\]

as the number of simulations \( L \) grows to infinity.

Finally, we repeat this exercise for different histories of surplus consumption, \( s_{t-1} \), and report a fan chart that plots the different impulse responses and codes them with a color scheme that weights each history by its unconditional frequency. Figure 6 depicts the generalized impulse responses of some stock and bond market variable to a monetary shock.
Figure 6: Generalized impulse responses of equity and bond market variables to a 100-basis-point monetary policy shock. Different shaded areas denote different quantiles of the history up to time $t-1$ that conditions the financial market’s response. We maintain the calibration for the persistence parameter $\rho_v = \rho_u = 0.9$. The plots are based on $L = 10,000$ simulations.
To interpret the results note how the equilibrium growth rate depends on the different shocks via
\[
\frac{\partial c_{t+k}}{\partial (\varepsilon_t, \varepsilon_t^u, \varepsilon_t^v)} = \left[ 1 \phi^{1-\rho_v} \rho_v \psi_{cv} \right]
\]
where
\[
\psi_{cv} = -\frac{1 - \beta \rho_v}{(1 - \rho_v + \phi_v)(1 - \beta \rho_v) + \kappa(\phi - \rho_v)} < 0
\]
with \( \rho_v = \rho_u \). A short-run technology shock has a permanent effect on output and the effect of short- and long-run technology shocks is independent of monetary policy.

Since the predictable component of consumption loads negatively on the monetary disturbance, a policy shock depresses consumption and thereby surplus consumption. This effect then increases risk aversion and risk premia. The term structures then shift as described in figure 4 as we move to a lower value of surplus consumption, which exacerbates their curvature by acting on people’s willingness to shoulder risk, \( x_t D_t \), as well as on the quantity of risk exposure in the term structure claims, \( V_n \). The depression in surplus consumption after a monetary tightening is then expected to revert to its mean.

The theoretical response function is consistent with the evidence of Bernanke and Kuttner (2005). They use Federal funds futures data to extract a measure of structural shocks to monetary policy and find that the monthly excess return on a broad-based value-weighted stock market index increases by about 16 basis points after a 100-basis-point monetary policy shock and decays asymptotically at a highly persistent rate. Habit persistence close to unity generates this persistence. Bernanke and Kuttner also show a positive response of the real rate following the unexpected increase in interest rates, with a response on impact of 0.4% that subsequently slightly persists before decaying to zero, as well as a downward shift in the yield curve. A relatively low parameter \( \rho_v \), such as \( \rho_v = 0.9 \)—as we used in section 3 to understand the term structures of equity and interest rates—is necessary to match such a low persistence and the shape of the response of the change in the bill rate. Thus, Bernanke and Kuttner’s conclusion that most of the capital market’s reaction to a policy shock is a reaction of a broad spectrum of equity and bond risk premia is in line with our model’s predictions, both qualitatively and quantitatively.

4.2. Endogenous effect of a monetary policy shock on capital markets

As shown by equation (2), the transmission mechanism of monetary policy depends crucially on the way a systematic policy rule affects how the endogenous variables react to the shocks that hit the economy.

In fact, the presence of nominal rigidities is entirely responsible for the endogenous differences between consumption and market equity strips and between real and nominal bond strips, so a larger degree of price stickiness exacerbates the differences in the term structure properties of the four cashflow claims and therefore the way the prices of different claims respond to an exogenous shock. For example, if prices are more rigid people command a higher premium to hold short-duration equities as shown in figure 3.
Similarly, a stronger anti-inflationary stance (a higher coefficient of reaction to either inflation or detrended output, which reduces in modulus the policy-dependent coefficient, \( \psi_{cv} \)) reduces the effect of nominal rigidities and thereby the differences in the four term structures.

5. Conclusion

We incorporate risk premia variation arising from Campbell-Cochrane external habits into a standard macro model. We propose a method to break the apparent tradeoff between fitting macroeconomic variables or asset price data in nonlinear habit models. The notion of macro-finance separation (and arbitrarily small departures from it) is key to incorporating large discount-rate variation in a DSGE framework while preserving the model’s ability to fit quantities. For example, we are able to motivate the original consumption-based asset pricing model by Campbell and Cochrane (1999) as the outcome of a production economy. By adding nominal rigidities, we derive testable implications for the term structures of equity and interest rates as well as for the reaction of capital markets to unexpected monetary news that conform with recent capital market evidence, including a downward-sloping term structure of equity and an upward-sloping term structure of interest rates.

Our framework remained parsimonious along many dimensions that can easily be generalized. In particular, further work could relax the one-shock structure and the univariate price of risk. The essentially-affine method for solving the equilibrium term structures of Lopez et al. (2014) that we used here can be easily applied to such a generalized setting. The introduction of demand shocks may help to mitigate the correlation puzzle (e.g., Albuquerque et al., 2014, for a recent exposition).

Finally, we work under a full macro-finance separation, which is likely an unnecessarily strong requirement; namely, small departures from macro-finance separation are most likely empirically valid descriptions of the data, as they would for example include the case of nontrivial capital accumulation. An estimated model that builds on our modeling framework could identify such spillovers, and hence the parameter vector \( \xi \). In this context, the essentially-affine approximation by Lopez et al. (2014) is particularly convenient in estimation in that it permits the use of linear filtering techniques.

References

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Appendix

A. Campbell-Cochrane habit specification in a production economy

The law of motion of surplus consumption assumed by Campbell and Cochrane (1999) in their endowment economy with random-walk consumption can be cast in three equivalent specifications:

\[
\hat{s}_{t+1} = \rho_s \hat{s}_t + \Lambda(\hat{s}_t)(E_{t+1} - E_t)c_{t+1}
\]

(A.1a)

\[
\hat{s}_{t+1} = \rho_s \hat{s}_t + \Lambda(\hat{s}_t)(\Delta c_{t+1} - \mu_c)
\]

(A.1b)

\[
\hat{s}_{t+1} = \rho_s \hat{s}_t + \Lambda E_t(\Delta c_{t+1} - \mu_c) + \Lambda(\hat{s}_t)(E_{t+1} - E_t)c_{t+1}
\]

(A.1c)

where \(\mu_c \equiv E(\Delta c)\). The equality breaks down however once we allow for a predictable component in consumption growth, consistent with a generic production economy. To understand what specification we should retain in a production economy, note how there is a strong reason to prefer specification (A.1a) owing to its implications for the risk-free rate and for the relationship between consumption and the habit level.

A.1. Local structure and predeterminedness

As shown by Campbell and Cochrane (1999) and Lynch and Randall (2011), specifications (A.1b) and (A.1c) imply the local habit structure

\[
h^c_{t+1} = h + c_{t+1} - \sum_{j=0}^{\infty} \rho^j_s \Delta c_{t-j+1}
\]

\[
h^c_{t+1} = h + \sum_{j=0}^{\infty} \theta^j \Delta c_{t-j+1}
\]

where \(\theta^j = 1 - \rho^j_s\) and \(h = \ln(1 - S)\), so the consumption habit is a slow moving average of past consumption growth such that consumption growth moves transitorily consumption away from habits. Specification (A.1a) implies a habit structure that also depends on what people expect to consume,

\[
h'^c_{t+1} = h + c_{t+1} - \sum_{j=0}^{\infty} \rho^j_s \hat{e}^c_{t-j+1}
\]

\[
h'^c_{t+1} = h + \sum_{j=0}^{\infty} E_{t-j} \Delta c_{t-j+1} + \sum_{j=0}^{\infty} \theta^j \hat{e}^c_{t-j+1}
\]

so the consumption habit is the sum of past anticipated consumption movements and of a slow moving average of past consumption shocks, which receive their full weight only asymptotically.

\[\text{Footnote 24}\text{For example, in a recent study of Campbell-Cochrane habits with non-random-walk cashflows, Lynch and Randall (2011) adopt specification (A.1c).}\]
(lim \(j\to\infty\) \(\theta_j^c = 1\)); only unanticipated movements in consumption move consumption away from habits.

Since \(\theta_0^c = 0\), the habit level is locally predetermined, \(h_{t+1}^c = E_t h_{t+1}^c\), under all specifications.

### A.2. Relationship between consumption and the habit level

Specifications (A.1a), (A.1b) and (A.1c) imply the respective relationship between consumption and the habit level (see appendix C for more details)

\[
\frac{\partial h_t^c}{\partial c_t} = 1 - \frac{\Lambda(\delta_{t-1})}{\exp(-s_t) - 1} \frac{(E_t - E_{t-1}) M_t^c}{M_t^c}
\]

\[
\frac{\partial h_t^c}{\partial c_t} = 1 - \frac{\Lambda(\delta_{t-1})}{\exp(-s_t) - 1} \frac{(E_t - E_{t-1}) M_t^c}{M_t^c}
\]

\[
\frac{\partial h_t^c}{\partial c_t} = 1 - \frac{\Lambda(\delta_{t-1})}{\exp(-s_t) - 1} + \frac{\Lambda(\delta_{t-1}) - \Lambda}{\exp(-s_t) - 1} \frac{E_t - E_{t-1}}{M_t^c}
\]

with \(M_t^c\) the shadow value of surplus consumption.

It follows that, in the steady state, consumption habits move strictly positively with consumption, \(\partial h_t^c / \partial c = 1\), under specification (A.1a) but they are unrelated with consumption, \(\partial h_t^c / \partial c = 0\), under specifications (A.1b) and (A.1c), a property that leads to the critique by Ljungqvist and Uhlig (2009), who look at the second derivative \(\partial^2 h_t^c / \partial c_t^2\) and note that in a neighborhood of the steady state the habit process can move strictly negatively with consumption. The reason specification (A.1a) bypasses Ljungqvist and Uhlig’s critique is that neither the social planner nor the internal-habit consumer can take into account in their optimization the equilibrium expression for the ex-ante value of consumption because the law of motion of consumption, when consumption is endogenous, is not a structural relation but an outcome of optimization.

### A.3. No risk-free rate puzzle

The respective equilibrium risk-free rates under specifications (A.1a), (A.1b) and (A.1c) are \(^{25}\)

\[
r_t = r + \gamma E_t(\Delta c_{t+1} - \mu_c) \quad \text{(A.2a)}
\]

\[
r_t = r + x_t E_t(\Delta c_{t+1} - \mu_c) \quad \text{(A.2b)}
\]

\[
r_t = r + x E_t(\Delta c_{t+1} - \mu_c) \quad \text{(A.2c)}
\]

where \(x_t \equiv \gamma(1 + \Lambda_t)\) is the price of risk. As shown by equations (A.2b) and (A.2c), specifications (A.1b) and (A.1c) imply a distorted dynamic IS equation relative to a power-utility specification that would imply a risk-free rate puzzle. Note in fact how a large price of risk \(x = \gamma/S\) is necessary to generate a large equity premium; the parametrization \(S < 1\) is the element that amplifies the coefficient of risk aversion (see appendix G) while remaining neutral on the risk-free rate, and that thereby allows for breaking the tradeoff between solving the equity premium and the risk-free rate puzzles in the habit framework. We therefore discard specifications (A.1b) and (A.1c) on the ground that they would kill the central idea of the Campbell-Cochrane habits. We thus retain specification (A.1a) and the associated dynamic IS equation (A.2a).

\(^{25}\)For simplicity, we turn off the spillover parameter \(\xi_1\) as it adds nothing to the argument.
A.4. Labor habits

Our labor habits can produce a macro-finance separation, and hence break the quantity puzzle, because the same state drives both surplus consumption and surplus labor, so the respective effects on consumption-labor decisions can offset.

The local microfoundations of our labor habit parallel those of the consumption habit. First, the labor habit (expressed in efficiency units) can be written locally as

$$h^\pi_{t+1} = h + \bar{n}_{t+1} - (1 - \alpha)\xi_2 \sum_{j=0}^{\infty} \rho_j \delta^\pi_{t-j+1}$$

$$= h + \sum_{j=0}^{\infty} E_{t-j} \Delta \bar{n}_{t-j+1} + \sum_{j=0}^{\infty} \theta_j \delta^\pi_{t-j+1}$$

where $h^\pi_{t+1} = h^\pi_{t+1} + \bar{n}_{t+1}$ and $\theta_j \equiv 1 - (1 - \alpha)\xi_2 \rho_j^i$, so the labor habit is the sum of past anticipated effective labor movements and of a slow moving average of past effective labor innovations, which receive their full weight only asymptotically ($\lim_{j \to \infty} \theta_j = 1$); only unexpected movements in effective labor move labor away from habits, which coincide with labor in the long run. In this sense, our labor habit expressed in efficiency units can be interpreted as a habit to effective labor.

Second, the labor habit maintains the appealing local representation as a slow moving average of past labor and labor expectations but is not in general locally predetermined.\(^{26}\) Note in fact how

$$(E_{t+1} - E_t)(h^\pi_{t+1} - n_{t+1}) = -\xi_2 \delta^\pi_{t+1}$$

is true in equilibrium; this property captures locally the idea that people become used to more (if $\xi_2 > 0$) or less (if $\xi_2 < 0$) work after bad consumption news.

Third, labor habits relate with labor via

$$\frac{\partial h^\pi_t}{\partial n_t} = 1 - \frac{(1 - \alpha)\xi_2 \Lambda(\delta_{t-1}) (E_t - E_{t-1}) M^\pi_t}{\exp(-z_t) - 1}$$

and hence the habit moves strictly positively with labor in the steady state, $\partial h^\pi / \partial n = 1$.

B. Flexible-price equilibria

This section characterizes the Pareto optimum and the flexible-price equilibrium.

B.1. Pareto optimum (internal habits, flexible prices)

The Pareto optimum can be characterized as the solution to a social planner problem. However, we appeal to the welfare theorems and decentralize the economy to build intuition and gain insight into the consumption and labor margins.

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\(^{26}\)The labor habit is predetermined in the macro-financially separate Pareto optimum, where $n_{t+1} = E_t n_{t+1}$ and $\xi_2 = 0$, and hence $h^\pi_{t+1} = E_t h^\pi_{t+1}$. Outside this equilibrium, contemporaneous shocks may move labor habits; for example, in the Pareto optimum labor habits cease to be predetermined under nontrivial investment ($\xi_3 > 0$) but they remain arbitrarily close to being predetermined for a parameter $\xi_3$ close to zero.
B.1.1. Consumers

Internal-habit consumers maximize the intertemporal objective

\[ \max U_0 = E_0 \sum_{t=0}^{\infty} \beta^t \left( \frac{(C_t - H_t^c)^{1-\gamma} - 1 - \frac{\chi(N_t - H_t^m)^{1+\varphi}}{1 + \varphi}}{1 - \gamma} \right) \]

subject to the budget constraint and the structural habit equations

\[ C_t - H_t^c = C_t S_t, \quad s_{t+1} = (1 - \rho_s)s + \rho_s s_t + \Lambda(s_t)(E_{t+1} - E_t)c_{t+1} \]
\[ N_t - H_t^m = N_t Z_t, \quad z_{t+1} = (1 - \rho_z)z + \rho_z z_t + (1 - \alpha)\xi_2 \Lambda(\hat{z}_t)z_2)(E_{t+1} - E_t)(n_{t+1} + \bar{a}_{t+1}) \]

Optimality requires that the joint evolution of the processes satisfies

\[ \frac{\partial U_t^\text{int}}{\partial C_t} = C_t^{-\gamma} S_t^{1-\gamma} - \frac{\Lambda(s_t)}{C_t}(E_t - E_{t-1})M_t^{c} \tag{B.1} \]
\[ M_t^{c} = C_t^{1-\gamma} S_t^{1-\gamma} + \beta E_t M_{t+1}^{c}[\rho_s + \Lambda'(s_t)\epsilon_t] \tag{B.2} \]
\[ \frac{\partial U_t^\text{int}}{\partial N_t} = -\chi N_t^\varphi S_t^{1+\varphi} + (1 - \alpha)\xi_2 \Lambda(s_t)(E_t - E_{t-1})M_t^{\varphi} \tag{B.3} \]
\[ M_t^{\varphi} = -\chi N_t^{1+\varphi} S_t^{1+\varphi} + \beta E_t M_{t+1}^{\varphi}[\rho_s + \Lambda'(s_t)\epsilon_t] \tag{B.4} \]

where \( M_t^{c} \) and \( M_t^{\varphi} \) are Lagrange multipliers associates with the consumption and labor habit equations, and thereby implies the intertemporal and intratemporal rates of substitution

\[ M_{t+1}^{\text{int}} = \beta \left( \frac{C_{t+1}}{C_t} \right)^{-1} \left( C_t^{1-\gamma} S_t^{1-\gamma} + \frac{\Lambda(s_t)(E_{t+1} - E_t)}{C_t} M_{t+1}^c \right)^{C_t^{1-\gamma} S_t^{1-\gamma} + \Lambda(s_{t-1})(E_t - E_{t-1})M_t^{c}} \]
\[ \frac{\partial U_t^\text{int}}{\partial N_t} = \frac{C_t \chi N_t^{1+\varphi} S_t^{1+\varphi} - (1 - \alpha)\xi_2 \Lambda(s_t)(E_t - E_{t-1})M_t^{\varphi}}{N_t \left( C_t^{1-\gamma} S_t^{1-\gamma} + \Lambda(s_{t-1})(E_t - E_{t-1})M_t^{c} \right)} \]

Relative to a deterministic case, unexpected movements in the Lagrange multipliers \( M_t^{c} \) and \( M_t^{\varphi} \) affect the marginal utility of wealth with a time-varying loading.

B.1.2. Firms

Firms maximize \( Y_t - W_t N_t \) subject to the production technology \( Y_t = A_t N_t^{1-\alpha} \), which results in the optimality condition

\[ W_t = (1 - \alpha) \frac{Y_t}{N_t} \]

B.1.3. Equilibrium

After imposing market clearing, \( Y_t = C_t \), we can characterize the Pareto optimum by the equality between the intratemporal rate of substitution and the marginal product of labor,

\[ -\frac{\partial U_t^\text{int}/\partial N_t}{\partial U_t^\text{int}/\partial C_t} = (1 - \alpha) \frac{C_t}{N_t} \]

35
It is straightforward to verify how the balanced-growth requirement, $\gamma = 1$, produces a constant shadow value of surplus consumption, while $\xi_2 = 0$ kills the role of the shadow value of surplus labor in determining the intratemporal rate of substitution. Under this parametrization we have

$$M_{t+1}^{\text{int.}} = \beta \left( \frac{C_{t+1}}{C_t} \right)^{-1}$$

$$-\frac{\partial U_t^{\text{int.}}/\partial N_t}{\partial U_t^{\text{int.}}/\partial C_t} = \chi_0 C_t N_t^e$$

so all intertemporal and intratemporal effects of the habit are absent. The conditions $\xi_2 = 0$ and $\gamma = 1$ are necessary and sufficient to grant a macro-finance separation when habits are internal.

**B.2. Flexible-price equilibrium (external habits)**

Optimality requires that the joint evolution of the processes satisfies

$$\frac{\partial U_t}{\partial C_t} = C_t^{-\gamma} S_t^{-\gamma}$$

$$\frac{\partial U_t}{\partial N_t} = -\chi N_t^e Z_t^e$$

$$-\frac{\partial U_t/\partial N_t}{\partial U_t/\partial C_t} = (1 - \alpha) \frac{C_t}{N_t}$$

Thus, the competitive equilibrium is characterized by

$$c_t = \frac{1 + \varphi}{\gamma(1 - \alpha) + \alpha + \varphi} a_t - \frac{(1 - \alpha)(\gamma + \varphi \xi_2)}{\gamma(1 - \alpha) + \alpha + \varphi} s_t$$

$$n_t = \frac{1 - \gamma}{\gamma(1 - \alpha) + \alpha + \varphi} a_t - \frac{\gamma + \varphi \xi_2}{\gamma(1 - \alpha) + \alpha + \varphi} s_t$$

up to an irrelevant constant. The competitive equilibrium is macro-financially separate if and only if $\xi_2 = -\gamma/\varphi$.

**B.3. Balanced growth**

Provided a nonzero drift in technology $\mu$, it follows that, for both the competitive equilibrium and the Pareto optimum, the requirement for a balanced growth path, $E(\Delta n) = 0$, holds if and only if $\gamma = 1$.

In the case of external habits, the equilibrium condition $-(\partial U_t/\partial N_t)/(\partial U_t/\partial C_t) = (1 - \alpha)(\partial Y_t/\partial N_t)$, along with the market clearing condition implies

$$(1 - \gamma) \Delta c_{t+1} + (\gamma + \varphi \xi_2) \Delta s_{t+1} = (1 + \varphi) \Delta n_{t+1}$$

$$= \frac{1 + \varphi}{1 - \alpha} \Delta c_{t+1} - (1 + \varphi) \Delta a_{t+1} - \frac{(1 + \varphi) \alpha}{1 - \alpha} \Delta k_{t+1}$$

From $\mu > 0$ follow the steady-state growth rates $\Delta c = \mu(1 + \varphi)/[\gamma(1 - \alpha) + \alpha + \varphi] > 0$ and $\Delta n = \mu(1 - \gamma)/[\gamma(1 - \alpha) + \alpha + \varphi]$, which is zero for $\gamma = 1$. 

36
In the case of internal habits, the equilibrium condition \(- (\partial U_t / \partial N_t) / (\partial U_t / \partial C_t) = (1-\alpha)(\partial Y_t / \partial N_t)\), along with the market clearing condition implies

\[
\frac{\chi_0 N_t^{1+\psi} E_t^{1+\psi}}{\chi_0 N_t^{1+\psi} E_t^{1+\psi} - \xi_2 S^{1-\gamma} N_t(E_t - E_t) M_{t+1}^c} = \frac{C_t^{1-\gamma} E_t^{1-\gamma} - S^{1-\gamma} N_t(E_t - E_t) M_{t+1}^c}{C_t^{1-\gamma} E_t^{1-\gamma} - S^{1-\gamma} N_t(E_t - E_{t-1}) M_{t}^c}
\]

\[
\Delta N_{t+1} = \frac{\Delta C^{t+1}}{\Delta A_{t+1} \Delta K^{t+1}}
\]

It follows that in the steady state \(\Delta c = \mu(1+\varphi)/[\gamma(1-\alpha) + \alpha + \varphi] > 0\) and \(\Delta n = \mu(1-\gamma)/[\gamma(1-\alpha) + \alpha + \varphi]\), which is zero for \(\gamma = 1\).

C. Relationship between consumption, labor and habit levels

C.1. Consumption habits

As shown by Campbell and Cochrane (1999), we can write the derivative of utility with respect to consumption as

\[
\frac{\partial U_t}{\partial C_t} = C_t^{-\gamma} F_t^c
\]

\[
F_t^c = S_t^{-\gamma} \left[ 1 - E_t \sum_{j=0}^{\infty} \beta^j \left( \frac{C_{t+j} S_{t+j}}{C_t S_t} \right)^{-\gamma} \frac{\partial H_{t+j}^c}{\partial C_t} \right] \tag{C.1}
\]

\[
= S_t^{-\gamma} \left[ 1 - \frac{\partial H_t^c}{\partial C_t} - E_t \beta \left( \frac{C_{t+1}}{C_t} \right)^{-\gamma} \left( \frac{\partial H_{t+1}^c}{\partial C_t} + \frac{\partial H_{t+1}^c}{\partial H_t} \frac{\partial H_t^c}{\partial C_t} \right) W_{t+1}^c \right] \tag{C.2}
\]

with \(W_{t+1}^c = S_{t+1}^{-\gamma} + \beta E_{t+1} \left( \frac{C_{t+2}}{C_{t+1}} \right)^{-\gamma} \frac{\partial H_{t+2}^c}{\partial H_{t+1}^c} W_{t+2}^c\)

where we used

\[
\frac{\partial H_{t+j}^c}{\partial C_t} = \frac{d H_{t+j}}{d C_t} \prod_{h=2}^{j} \frac{\partial H_{t+h}}{\partial H_{t+h-1}}
\]

\[
\frac{d H_{t+1}^c}{d C_t} = \frac{\partial H_{t+1}^c}{\partial C_t} + \frac{\partial H_{t+1}^c}{\partial H_t} \frac{\partial H_t^c}{\partial C_t}
\]

because the law of motion of surplus consumption defines an implicit function \(H_{t+1}^c = H_{t+1}(H_t, C_{t+1}, C_t)\), so \(C_t\) affects \(H_{t+1}^c\) directly and via \(H_t\). From the law of motion

\[
\ln \left(1 - \frac{H_{t+1}^c}{C_{t+1}}\right) = \rho_s \ln \left(1 - \frac{H_t^c}{C_t}\right) + \Lambda \left[ \ln \left(1 - \frac{H_t^c}{C_t}\right) \right] \left( \ln C_{t+1} - E_t \ln C_{t+1} \right)
\]

37
we can easily derive

\[ \frac{\partial H_{i+1}}{\partial H_i} = \frac{C_i S_{i+1}}{C_i S_i} [\rho_s + \Lambda'(s_i)\epsilon^c_{i+1}] \]  

(C.3)

\[ \frac{\partial H_{i+1}}{\partial C_i} = -(1 - S_i) \frac{C_i S_{i+1}}{C_i S_i} [\rho_s + \Lambda'(s_i)\epsilon^c_{i+1}] \]  

(C.4)

and we can verify that \( M_i^c = C_i^{1-\gamma} S_i W_i^c \) in equation (B.2). Thus, we can plug equations (C.3) and (C.4) in expression (C.2) and, since \( \partial U_i^{\text{int.}}/\partial C_i = C_i^{-\gamma} F_i^c \), we use equation (B.1) to deduce

\[ \frac{\partial h^c_i}{\partial c_i} = 1 - \frac{\Lambda(s_{t-1})}{\exp(-s_t) - 1} \times \frac{(E_t - E_{t-1}) M_i^c}{M_i^c} \]  

(C.5)

It follows that consumption habits move strictly positively with consumption in the steady state, \( \partial h_i^c / \partial c = 1 \).

C.2. Labor habits

Analogously for the internal labor habits,

\[ \frac{\partial U_i}{\partial N_i} = \chi N_i^{1-\gamma} F_i^n \]

\[ F_i^n = \chi N_i^{1-\gamma} \left[ 1 - E_i \sum_{j=0}^{\infty} \beta^j \left( \frac{N_{i+j} Z_{i+j}}{N_i Z_i} \right) \frac{\partial H_{i+j}}{\partial N_i} \right] \]

\[ = Z_i^{1-\gamma} \left[ 1 - \frac{\partial H_{i+1}}{\partial N_i} \right] - E_i \beta \left( \frac{N_{i+1}}{N_i} \right)^{1-\gamma} \left[ \frac{\partial H_{i+1}}{\partial H_{i+1}} + \frac{\partial H_{i+1}}{\partial N_i} \right] W_{i+1}^{n+1} \]

with

\[ W_{i+1}^{n+1} = Z_i^{1-\gamma} + E_i + \beta \left( \frac{N_{i+2} Z_{i+2}}{N_{i+1} Z_{i+1}} \right) \frac{\partial H_{i+2}}{\partial H_{i+1}} W_{i+2}^{n+2} \]

From the law of motion

\[ \ln \left( 1 - \frac{H_{i+1}}{N_{i+1}} \right) = \rho_s \ln \left( 1 - \frac{H_{i}}{N_{i}} \right) + \Lambda \left[ \ln \left( 1 - \frac{H_{i}}{N_{i}} \right) \right] \left( \ln N_{i+1} - E_t \ln N_{i+1} + \tilde{\epsilon}_{i+1}' \right) \]

we can easily derive

\[ \frac{\partial H_{i+1}}{\partial H_i} = \frac{N_{i+1} Z_{i+1}}{N_i Z_i} [\rho_s + \tilde{\epsilon}_{i+1}' \Lambda'(s_i) \epsilon^c_{i+1}] \]

\[ \frac{\partial H_{i+1}}{\partial N_i} = -(1 - Z_i) \frac{N_{i+1} Z_{i+1}}{N_i Z_i} [\rho_s + \tilde{\epsilon}_{i+1}' \Lambda'(s_i) \epsilon^c_{i+1}] \]

and we can verify that \( M_i^c = -\chi N_i^{1-\gamma} Z_i W_i^c \) in equation (B.4). Thus, since \( -\partial U_i^{\text{int.}}/\partial N_i = \chi N_i^{1-\gamma} F_i^n \),
we use equation (B.3) to find
\[
\frac{\partial h^n}{\partial n_t} = 1 - \frac{(1 - \alpha)\xi_2\Lambda(s_{t-1})}{\exp(-z_t) - 1} \times \frac{(E_t - E_{t-1}) M_t^c}{M_{t-1}}.
\]

It follows that labor habits move strictly positively with labor in the steady state, \(\partial h^n/\partial n = 1\).

D. Capital accumulation

We provide results for the macro-financially separate external-habit equilibrium; the analogous results for the internal-habit equilibrium are straightforward but tedious.

The external-habit equilibrium with nontrivial capital accumulation is characterized by

\[
C_t = (1 - \phi_t)Y_t, \quad N_t^{(1-\alpha)+\alpha+\varphi} = \frac{(1 - \alpha)A_t^{(1-\gamma)(1-\alpha)} K_t^{\gamma(1-\gamma)}}{\chi_0(1 - \phi_t)\tilde{S}_t^2\tilde{Z}_t^2}, \quad Y_t = (\tilde{A}, N_t)^{1-\alpha} K_t^\alpha
\]

\[
I_t = \begin{cases} \tilde{\delta} Q_t^{\xi_3} K_t, & \xi_3 < \infty \\
K_{t+1} - (1 - \delta) K_t, & \xi_3 \to \infty \end{cases}
\]

\[
K_{t+1} = \begin{cases} e^\mu K_t + \frac{1 - \delta}{1 - \delta} K_t + \frac{\beta\gamma}{1 - \delta} (\frac{d_t}{K_t})^{1-\delta} K_t, & \xi_3 < \infty \\
\sum_{j=1}^{\infty} E_t M_{t+j} D_{t+j} & \xi_3 \to \infty \end{cases}
\]

\[
Q_t = \frac{\sum_{j=1}^{\infty} E_t M_{t+j} D_{t+j}}{K_{t+1}}, \quad D_t = (\alpha - \phi_t) Y_t
\]

where \(\phi_t \equiv I_t/Y_t\) and \(\tilde{Z}_t = \tilde{S}_t^{\xi_3}\).

In the steady state, \(Q = 1, \phi = I/Y, I/K = \tilde{\delta}, \) and hence, from \(Q_t = E_t M_{t+1}(\frac{Q_{t+1} K_{t+2}}{K_{t+1}} + \frac{D_{t+1}}{K_{t+1}}),\)

\[
\frac{Y}{K} = \frac{e^{\gamma t} - \beta(e^{\mu t} - \tilde{\delta})}{\alpha \beta}, \quad \phi = \frac{\alpha \beta \tilde{\delta}}{e^{\gamma t} - \beta(e^{\mu t} - \tilde{\delta})}, \quad N = \left( \frac{1 - \alpha}{\chi_0(1 - \phi)} \right) \tilde{S}_t^2
\]

E. Competitive equilibrium under a Taylor rule

The full model driving quantities is, to a first-order approximation,

\[
\pi_t = \beta E_t \pi_{t+1} + \kappa(c_t - c_t^n) + \lambda(1 + \varphi \xi_2)(s_t - s_t^n)
\]

\[
r_t - r_t^n = E_t(\Delta c_{t+1} - \Delta c_{t+1}^n) - \xi_1(s_t - s_t^n)
\]

\[
r_t - r_t^n = i_t - r_t^n - E_t \pi_{t+1}
\]

\[
i_t = \phi_t \pi_t + \phi_s \tilde{c}_t + \phi_s \tilde{s}_t
\]

\[
= r_t^n + \phi_t \pi_t + \phi_s(c_t - c_t^n) + \phi_s(s_t - s_t^n) + \nu_t
\]
where \( v_t = -r^a_t + \phi_\gamma (c^a_t - a_t) + \phi_\delta s^a_t \) and where we allow for a hypothetical reaction to risk premia by monetary policy to gain better insight into the role of the Taylor rule. The state equations are

\[
\begin{align*}
    a_{t+1} &= a_t + \mu + \phi u_t + \varepsilon^a_{t+1} \\
    u_{t+1} &= \rho_a u_t + \varepsilon^u_{t+1} \\
    s_{t+1} &= (1 - \rho_s) + \rho_s s_t + \Lambda_t (E_{t+1} - E_t) c_{t+1}
\end{align*}
\]

We assume at first that the policy disturbance is absent for simplicity. To verify the guessed stationarity of \( \bar{m}c_t \) and hence of \( [\pi_t; \hat{c}_t] \), pose the linear parametric forms \( c_t - c^a_t = \psi_c (s_t - s^a_t) \) and \( \pi_t = \psi_\pi (s_t - s^a_t) \), and verify them as

\[
\begin{align*}
    \psi_c &= -\lambda (1 + \phi_\xi_2 (\phi_\pi - \rho_s) + (\xi_1 + \phi_\delta) (1 - \beta \rho_s)) \\
    \psi_\pi &= \lambda (1 + \phi_\xi_2 (1 - \rho_s + \phi_\delta) - (\xi_1 + \phi_\delta) \kappa) \\
    \psi_{cu} &= \phi \frac{1 - \beta \rho_u}{(1 - \beta \rho_u) (1 - \rho_a + \phi_\delta) + \kappa (\phi_\pi - \rho_u)} \\
    \psi_{\pi u} &= \phi \frac{1}{(1 - \beta \rho_u) (1 - \rho_a + \phi_\delta) + \kappa (\phi_\pi - \rho_u)}
\end{align*}
\]

(E.1)

which is the unique solution of the model economy as long as the system is determined. Macroeconomic requirements are robust to activating the policy disturbance, which under \( \xi_1 = -\phi_\delta \) and \( \xi_2 = -1/\phi \) reduces to \( v_t = -E_t \Delta c^a_{t+1} = -\phi u_t \) and therefore \( c_t = a_t + \psi_{cu} u_t \) and \( \pi_t = \psi_{\pi u} u_t \) with

\[
\begin{align*}
    \psi_{cu} &= \phi \frac{1 - \beta \rho_u}{(1 - \beta \rho_u) (1 - \rho_a + \phi_\delta) + \kappa (\phi_\pi - \rho_u)} \\
    \psi_{\pi u} &= \phi \frac{1}{(1 - \beta \rho_u) (1 - \rho_a + \phi_\delta) + \kappa (\phi_\pi - \rho_u)}
\end{align*}
\]

Thus, surplus consumption is left to drive only asset prices, while consumption, hours, output, and inflation are driven only by technology.

E.1. Equilibrium cashflows

Consumption equity. Equilibrium aggregate consumption is

\[
\Delta c_{t+1} = \Delta a_{t+1} + \psi_{cu} \Delta u_{t+1} = \mu_c + C_c \phi u_t + \varepsilon^a_{t+1} + \psi_{cu} \varepsilon^u_{t+1}
\]

where \( C_c \equiv [\phi_\delta (1 - \beta \rho_u) + \kappa (\phi_\pi - \rho_u)] / [(1 - \rho_a + \phi_\delta) (1 - \beta \rho_u) + \kappa (\phi_\pi - \rho_u)] \in (0, 1) \).

Market equity. Equilibrium aggregate profits, \( P_t D_t = P_t C_t - (1 - \tau) W_t N_t - T_t \), which firms pay out as dividends, are, up to a first-order approximation around the steady state,

\[
d_t = c_t - \frac{1 - \alpha}{\alpha} \bar{m}c_t
\]

40
where average marginal costs are the inverse of average markups.27 Dividends are income plus an increasing function of markups. Therefore,

$$\Delta d_{t+1} = \Delta c_{t+1} - \frac{1 + \varphi}{\alpha} [\Delta c_{t+1} - \Delta a_{t+1}]$$

$$= \mu_c + C_d \phi u_t + \epsilon_{t+1} - \frac{1 + \varphi - \alpha}{\alpha} \psi_c u \epsilon_{t+1}$$

where $C_d \equiv \{(1 + \varphi)(1 - \rho_a)/(\alpha + \phi_x)(1 - \beta \rho_a) + \kappa(\phi_x - \rho_a))/[(1 - \rho_a + \phi_x)(1 - \beta \rho_a) + \kappa(\phi_x - \rho_a)] > 1$. **Nominal bonds.** The payoff at time $t + n$ for a $n$-period zero-coupon nominal bond is a unit of money, whose real value is $1/P_{t+n}$, i.e., the dividend grows at rate $\Delta d_{t+1} = \ln(1/P_{t+1}) - \ln(1/P_t) = -\pi_{t+1}$ with

$$-\pi_{t+1} = C_{nb} \phi u_t - \frac{\kappa}{1 - \beta \rho_a} \psi_c u \epsilon_{t+1}$$

with $C_{nb} \equiv -\kappa \rho_a / [(1 - \rho_a + \phi_x)(1 - \beta \rho_a) + \kappa(\phi_x - \rho_a)] < 0$. **Real bonds.** The payoff at time $t + n$ for a $n$-period zero-coupon real bond is a unit of numeraire, i.e., the dividend is $d_t = 0$.

### E.2. Equilibrium term structures

Lopez et al. (2014) show how to identify the equilibrium coefficients in an essentially-affine approximation of the $n$th term structure component of an arbitrary cashflow process, $d$,

$$y_{d,t}^{(n)} = -\frac{1}{n} A_d^{(n)} - \frac{1}{n} B_{d,s,t}^{(n)} \xi_s - \frac{1}{n} B_{s,t}^{(n)} \xi_s$$

as

$$A_d^{(n)} = A_d^{(n-1)} + \ln(\beta) + \frac{1}{2} ||D_d - D_c + B_{\xi}^{(n-1)} B - (1 - B_{\xi}^{(n-1)}) D_c ||^2$$

$$B_{\xi}^{(n)} = \frac{1 - \rho_s^{(n)}}{1 - \rho_u^{(n)}} (C_d - C_c)$$

$$B_{s,t}^{(n)} = B_{s,t}^{(n-1)} \rho_s + (1 - \rho_s)[1 - (1 - e^s)(1 - B_{s,t}^{(n-1)})^2] + e^{-s}(1 - B_{s,t}^{(n-1)}) (D_d - D_c + B_{s,t}^{(n-1)} B) D_c'$$

### F. Replicating Campbell and Cochrane in a production economy

In line with Campbell and Cochrane, consider a random-walk specification, $\phi = 0$, and build on our external-habit framework under macro-finance separation, $\xi = [0; -\gamma/\phi; 0]$. The absence of

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27Note that even if the output level inherits the nonstationarity from technology, the competitive, the flexible-price and the efficient output levels are all cointegrated, so average marginal costs are always stationary. It follows that the consumption-dividend ratio is stationary, so consumption and market dividends are cointegrated.
investment and the random walk specification of technology reduce the production economy to a particularly simple structure. The full model is

\[
\Delta c_{t+1} = \mu c_{t} + \frac{1 + \varphi}{\gamma(1 - \alpha) + \alpha + \varphi} \varepsilon_{t+1}^c
\]

\[
r_t = -\ln(\beta) - \frac{\gamma(1 - \rho_s)}{2}
\]

\[
m_{t+1} = -\ln(\beta) - \mu c_{t} + \gamma(1 - \rho_s) \hat{s}_t - x_t \varepsilon_{t+1}^c
\]

where \(\mu_c \equiv \mu (1 + \varphi) / [\gamma(1 - \alpha) + \alpha + \varphi] \) and \(x_t = \gamma [1 + \Lambda(\hat{s}_t)]\). The resulting model is observationally equivalent to the model by Campbell and Cochrane.\(^{28}\)

G. Coefficient of risk aversion

We follow Swanson (2012) and compute the coefficient of risk aversion of a consumer faced with a mean-zero, variance-\(\sigma\), state-independent gamble, which she can avoid by paying a one-time fee \(\mu_1(\sigma)\). In our context, the indirect utility function of a consumer with generic budget constraint \(A_{t+1} = (1 + r_t)A_t + W_tN_t + D_t - C_t\), where \(A_t\) is the household’s beginning-of-period asset, is

\[
V(A_t; \zeta_t) = \max_{[C_t; N_t] \mid C_t + A_{t+1} = (1 + r_t)A_t + W_tN_t + D_t} \frac{(C_t - H_t)^{1-\gamma} - 1}{1 - \gamma} - \frac{\chi(N_t - H^n_t)^{1+\varphi}}{1 + \varphi} + \beta E_t V(A_{t+1}; \zeta_{t+1})
\]

where \(\zeta_t\) denotes all state variables that drives the economy. Swanson (2012) shows how in a context of expected utility the household’s coefficient of absolute risk aversion to the gamble, \(R(A_t; \zeta_t) \equiv \lim_{\sigma \to 0} \frac{\mu(\lambda_t; \zeta_t; \sigma)}{\sigma^2/2}\), equals

\[
R(A_t; \zeta_t) = \frac{-E_t V_{11}(A_{t+1}; \zeta_{t+1})}{E_t V_1(A_{t+1}; \zeta_{t+1})}
\]

G.1. External habits

By the optimality conditions and the envelope theorem, the steady-state coefficient of risk aversion is

\[
\frac{-V_{11}}{V_1} = \frac{1}{S} \frac{r/C}{\frac{1}{\gamma} + \frac{1 - \alpha}{\varphi(1 - \phi)}}
\]

Relative to the case without habits \((S = 1)\), the coefficient of absolute risk aversion scales up by

\(^{28}\)Note how Campbell and Cochrane adopt the calibration \(\gamma = 2\), which is incompatible with balanced growth (since \(\mu > 0\)). A better choice would be a unitary curvature, \(\gamma = 1\), consistent with balanced growth. Since risk premia are directly proportional to \(\gamma\), to generate risk premia of the same size as under a curvature \(\gamma = 2\) the persistence coefficient, \(\rho_s\), can be slightly increased, which reduces parameter \(S\) (see appendix G).
$1/S > 1$. Relative to the case without labor habits, i.e.,

$$\frac{-V_{11}}{V_1} = \frac{1}{S} \frac{r/C}{\frac{1}{\gamma} + \frac{(1-\alpha)}{\varphi(1-\delta)}}$$

the risk aversion coefficient, and hence risk premia, is strictly larger.

**G.2. Internal habits**

By the optimality conditions and the envelope theorem, we have

$$\frac{-V_{11}}{V_1} = \frac{r/C}{\frac{1}{\gamma} + \frac{1-\alpha}{\varphi(1-\delta)}}$$

which is equivalent to the steady-state coefficient of absolute risk aversion in the model without habits.