Optimal Delegation, Unawareness, and Financial Intermediation

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PRELIMINARY

Abstract

We study the delegation problem between an investor and a financial intermediary, who is not only privately informed about the state of the world but also has superior awareness of the available investment opportunities. Under some regularity conditions on the state distribution, we show that the intermediary has incentives to make the investor aware of investment opportunities at the extremes, e.g. very risky and very safe projects, while leaving the investor unaware of intermediate investment options.

1 Introduction

‘Financial Intermediaires are agents, or groups of agents, who are delegated the authority to invest in financial assets.’

D. W. Diamond (1996, page 51)

One of the many striking features of the recent financial crisis was the extreme exposure of investors to risk. Investment and commercial banks have been selling risky assets to investors, sometimes even hiding some of the asset characteristics (e.g., Gerardi et al., 2008). At the same time, despite the impressive amount of new financial instruments and

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the rapidly changing financial world, since the 50s a large fraction (of approx. 33% in the US) of investment demand has remained on ‘safe’ assets (e.g., Gordon et al. (2008) and Garcia (2012)).

This paper incorporates the concept of unawareness into the canonical delegation problem and applies it to financial intermediation. Consider an investor (the principal) who needs to choose how to invest his savings and delegates the task of picking the right project to a financial intermediary (the agent). We can think of a small investor who goes to the bank where she credits her monthly salary, or even a larger investor asking fur the service of an investment bank. The bank has private information about the payoffs of each investment project. In the absence of nonlinear contingent monetary transfers, the contracting problem of the investor boils down to determining a set of projects from which the intermediary chooses freely upon observing the state of the world (e.g., Alonso and Matouschek (2008)).

We assume that the interests of the investor and the intermediary are not perfectly aligned. This captures the fact that banks are non-neutral brokers. They have a portfolio of loans and other investments that generate a certain desire for risk, insurance, and liquidity. Their portfolio positions might induce desires to allocate investment opportunities to investors. Investors and intermediaries thus typically have conflicting interests in the decision of portfolio choices.

Crucially, we also depart from the standard setting of the delegation problem by assuming that the intermediary not only has private information on what the best investment choice is, but also on the actual set of available investment opportunities. This second dimension of asymmetry is captured by the assumption that the investor is only partially aware of the feasible investment projects. Before the contracting stage the financial intermediary has the possibility to make the investor aware of additional projects. We are interested in the question of how the interaction between the intermediary and the investor unfolds in the presence of this asymmetry. Will the intermediary make aware the investor of all available investment opportunities? If the investor remains partially unaware, of which projects the
investor will be made aware of? What will be the final (delegation) set of projects the intermediary will be allowed to invest in?

We address this question in an environment with a continuum of states and a continuum of investment projects, some of which the investor is unaware of. The intermediary’s and investor’s preferences are represented by a quadratic loss function, where the bliss point differs between the two agents. The bliss point can be interpreted as the investment opportunity which generates the best combination between risk, illiquidity, and return as a function of the state. The divergence between the investor’s and intermediary’s bliss point can be interpreted as banks being less risk averse, having limited liability, having a portfolio with different correlation than the investor, having different liquidity needs, etc. The uninformed investor can delegate the investment choice to the intermediary and faces a tradeoff between giving the intermediary flexibility in order to react to his private information and avoiding that the intermediary implements projects he is biased towards.

In the benchmark case of full awareness, under some regularity conditions on the distribution of states, the optimal delegation set for the investor is an interval, which can be interpreted as a cap. For example, if the intermediary is less risk-averse than the investor, the investor may impose an upper bound on the riskiness of projects the intermediary can choose. In our environment, if say the intermediary is downward biased, there thus exists a threshold project above which the intermediary is free to choose, whereas all projects below the threshold are ruled out by the investor.

Imposing the same regularity condition on the distribution of states, we characterize the equilibrium delegation set in an environment where the investor is partially unaware. Our main result shows that the intermediary makes the investor fully aware if and only if the investor is initially aware of the threshold project of the full awareness benchmark. If this is not the case, the intermediary leaves the investor unaware of an interval of projects around the threshold. The intermediary thus makes the investor aware of investment opportunities at the extremes (e.g. very safe and very risky projects), while remaining silent about intermediate
investment opportunities. This makes it optimal for the investor to include projects at both extremes into the delegation set and thus gives the intermediary the possibility of choosing among them.

The paper makes two main contributions. First, it makes a methodological contribution in that we introduce limited awareness into the model of delegation. The potential applications are much broader than the financial market. Second, the stated framework is able to generate predictions on the equilibrium portfolio differentiation, and in particular on the demand of safe and risky assets as a function of the nature of misalignment between the investor and the financial intermediary.

**Related Literature:** This paper is first of all related to the literature on optimal delegation. Starting with Holmstrom (1984), who first defines the delegation problem and provides conditions for the existence of its solution, this literature, which includes Melamud and Shibano (1991), Martimort and Semenov (2006), Alonso and Matouschek (2008), Armstrong and Vickers (2010) and Amador and Bagwell (2013), studies optimal delegation problems in environments of increasing generality. None of them consider limited awareness in this framework. Furthermore, this paper is related to a small literature on contract theory and unawareness. The application of the concept of unawareness to contracting problems is still at its beginnings. In contrast to our setting, existing work considers contracting problems where contingent transfers are feasible and where the agent is unaware - either of possible actions (Von Thadden and Zhao, 2012 and 2014) or of possible states (Zhao, 2011; Filiz-Ozbay, 2012; Auster, 2013), while the principal is fully aware.

This paper is also related to the literature on financial intermediation which sees banks as ‘efficient’ brokers who reduce transaction and information costs. The issue of this role of financial intermediation has been studied by many authors starting from Diamond (1984). A summary of the literature can be found in Bhattacharya and Thakor (1993) and Allen and Santomero (1998). These works focus on the possibility of partially monitoring the
financial intermediaries ex-post. We here introduce the possibility of limited awareness on
the investors and find that banks can induce the investment demand at the extremes.

2 Environment

There is an investor (she) who acts as the principal and a financial intermediary (he) who
acts as the agent. The intermediary has access to a set of investment projects \( Y = [y, \bar{y}] \), the
return to which depends on the state of the world. Let \( \Theta = [0, 1] \) be the set of states and let
\( F(\theta) \) denote the cumulative distribution function on \( \Theta \), assumed to be twice differentiable
in \((0,1)\). Both the investor and the intermediary have a von-Neumann-Morgenstern utility
function that takes the quadratic form

\[
 u(y, \theta) = \frac{1}{2} (y - \theta)^2 \quad \text{and} \quad v(y, \theta) = -\frac{1}{2} (y - \theta - \delta)^2
\]

The intermediary’s preferred policy is thus \( y = \theta \), while the investor’s preferred policy is
\( y = \theta + \delta \). Hence, the intermediary has a downward bias of size \( \delta \).

As in the canonical delegation problem, we assume that the intermediary is informed
about the state of the world \( \theta \), while the investor is not. We rule out monetary transfers and
assume that the intermediary’s participation constraint is always satisfied. The contracting
problem of the investor then boils down to the decision of which projects to let the interme-
diary choose from.

In contrast to the standard setting, we assume that the investor is not aware of the whole
set \( Y \) but only of a compact subset \( Y_P \subseteq Y \). The intermediary, on the other hand, is fully

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1 We can read \( \delta \) itself as the result of contracting problem (with limitations) that generates \( \delta \) as the minimal
level of conflict of interest between the principal and the agent.

2 Formally, the investor commits to a mechanism that specifies the project which will be implemented as a
function of the intermediary’s message. Alonso and Matouschek (2008) show that this contracting problem is
equivalent to delegating a set of projects \( D \subseteq Y \) from which the investor can choose freely after observing the
state of the world.
aware and has the possibility to make the investor aware of additional projects. After updating his awareness, the investor then makes his delegation choice. The precise timing is as follows:

- The intermediary reveals a set of projects \( X \subseteq Y \) and the investor updates his awareness to \( \hat{Y} \equiv Y_P \cup X \).

- Given \( \hat{Y} \), the investor chooses a delegation set \( D \in D(\hat{Y}) \), where \( D(\hat{Y}) \) is the collection of compact subsets of \( \hat{Y} \).

- The intermediary observes the state of world \( \theta \) and chooses an action from set \( D \).

- Payoffs are realized.

In general, optimal revelation sets and optimal delegation sets will not be unique since different awareness sets may induce the same delegation set and different delegation sets may induce the same implemented actions for each state of the world. We will assume that if the investor is indifferent between two delegation sets \( D \) and \( D' \) such that \( D' \subset D \), he chooses the larger set \( D \). Similarly, we assume that if the intermediary is indifferent between two revelation strategies that yield awareness sets \( \hat{Y} \) and \( \hat{Y}' \) such that \( \hat{Y}' \subset \hat{Y} \), he expands the investor’s awareness to \( \hat{Y} \). That is, we will consider the sets that yield maximal awareness and maximal discretion.

**Remark:** It is important to point out that our model is agnostic to the question of whether or not the investor is aware of her unawareness. What matters for the investor’s expected payoff is the set of projects she permits the intermediary to implement. The investor may be well aware of the fact that there exists other projects outside her awareness but since she cannot include such projects in the delegation set, their existence does not affect her expected payoff or optimization problem.
3 Equilibrium Analysis

For this setting with full awareness, i.e. \( Y_P = Y \), Martimort and Semenov (2006) show that if the density function \( f(\theta) := F'(\theta) \) satisfies \( f'(\theta) \delta \leq f(\theta) \) for all \( \theta \), the optimal delegation set is an interval. In particular, if \( \delta < \bar{y} - E[\theta|\theta \leq \bar{y}] \), the optimal delegation set is given by \([\hat{y}, \bar{y}]\), where \( \hat{y} \) is such that

\[
\hat{y} - Ef[\theta|\theta \leq \hat{y}] = \delta. \tag{1}
\]

Otherwise the optimal delegation set is a singleton, given by \( \{\bar{y}\} \). We will adopt this condition throughout the analysis so as to have a characterization for the benchmark case of full awareness.

**Assumption 1.** \( f'(\theta) \delta \leq f(\theta) \) for all \( \theta \in (0, 1) \).

We further assume that \( \underline{y} \leq 0 \) and \( \bar{y} > \hat{y} \). If the latter condition is not satisfied, the equilibrium analysis is straightforward. In particular, if \( \bar{y} \leq \hat{y} \), the investor’s optimal delegation set is always a singleton, given by the largest element of her awareness set \( \hat{Y} \). Among all singleton delegation sets, the intermediary prefers \( \{E[\theta]\} \), as the project \( y = E[\theta] \) maximizes his ex-ante expected payoff. Hence, if \( \bar{y}_p \equiv \max Y_P \) is strictly smaller than \( E[\theta] \), the intermediary optimally expands the investor’s awareness to \( \hat{Y}^* = [\underline{y}, E[\theta]] \) so that \( D^*(\hat{Y}^*) = \{E[\theta]\} \), whereas if \( \bar{y}_p \geq E[\theta] \), the intermediary cannot improve on the delegation set \( \{\bar{y}_p\} \) by revealing additional projects.

Given these restrictions, our main result shows that if the investor is not fully aware, in particular if she is unaware of the threshold project \( \hat{y} \), it is strictly optimal for the intermediary leave the investor unaware of some projects.

**Theorem 1.** Under Assumption 1, \( \hat{Y}^* = Y \) if and only if \( \hat{y} \in Y_P \).

\(^3\)Note that the implemented project varies with the state of the world, making delegation valuable, only if \( \delta < 1 - E[\theta] \).
The statement of the theorem will be proven in a series of lemmas in the remainder of this section. In particular, the analysis will show that if the investor is aware of the threshold project \( \hat{y} \), the intermediary optimally makes the investor fully aware and the equilibrium delegation set is \([\hat{y}, \bar{y}]\). On the other hand, if the investor is unaware of \( \hat{y} \), it is optimal for the intermediary to leave the investor unaware of an interval of projects around \( \hat{y} \), while making her aware of the remaining projects. In this case the equilibrium delegation set is no longer an interval. Instead the optimal delegation set includes all projects in the investor’s awareness set that lie above the threshold \( \hat{y} \) as well as the largest project below that threshold. Formally, if \( \hat{y} \not\in Y_P \), there exists some \( \Delta \in (0, \hat{y}] \) such that the equilibrium awareness set is \([y, \hat{y} - \Delta] \cup [\hat{y} + \Delta, \bar{y}]\) if \( \hat{y} + \Delta \leq \bar{y} \) and \([y, \hat{y} - \Delta]\) otherwise. The corresponding equilibrium delegation sets are then given by \([\hat{y} - \Delta]\) if \( \hat{y} - \Delta \leq y \) and \([\hat{y} + \Delta, \bar{y}]\) otherwise. The case \( \hat{y} + \Delta \leq \bar{y} \) is illustrated in Figure 1.

To establish these properties of the equilibrium, we will proceed recursively. We will first consider the investor’s delegation problem for a given awareness set \( \hat{Y} \) and then turn to the intermediary’s problem of choosing the optimal awareness set.

### 3.1 Delegation Choice

Let \( y^*(D, \theta) = \arg \max_{y \in D} u(y, \theta) \) denote the implemented action when the delegation set is \( D \) and the realized state of the world is \( \theta \) and let \( D^*(\hat{Y}) \equiv \arg \max_{D \in D(\hat{Y})} \mathbb{E}[v(y^*(D, \theta), \theta)] \).
denote the optimal delegation set when the awareness set is \( \widehat{Y} \). Alonso and Matouschek (2008) derive conditions under which for \( y_1, y_2 \in D \) increasing the agent’s discretion by adding a project \( y \in (y_1, y_2) \) is profitable for the investor. In particular, they show that this crucially relies on the curvature of the so-called backward bias, in our setting defined as

\[
T(y) \equiv F(y) (y - E[\theta | \theta \leq y] - \delta)
\]

The backward bias \( T(y) \) measures the difference between the intermediary’s preferred decision in state \( y \) and the investor’s preferred decision if she believes that the state is smaller than \( y \), weighted by the probability that the state is smaller than \( y \). Alonso and Matouschek (2008) show that if \( T \) is convex, the investor can increase his expected payoff by including \( y \) into the delegation set, whereas if \( T \) is concave, adding \( y \) to the delegation set decreases the investor’s expected payoff. As the following lemma asserts, in our setting convexity of the backward bias corresponds to the condition \( f'(\theta)\delta < f(\theta) \).

**Lemma 2** (Alonso and Matouschek, 2008). Assume Assumption 1 and consider \( y_1, y_2 \in \widehat{Y} \) with \( y_1 < y_2 \). If \( y_1, y_2 \in D^*(\widehat{Y}) \), then all \( y \in \widehat{Y} \cap (y_1, y_2) \) belong to \( D^*(\widehat{Y}) \).

**Proof** See Appendix A.1.

We can next show that whenever the optimal delegation set includes some project \( y \) that is weakly smaller than the threshold project \( \hat{y} \), it includes no project that is smaller than \( y \). In other words, the investor permits the agent to implement at most one project in the interval \([y, \hat{y}]\).

**Lemma 3.** Assume Assumption 1. If there exist some \( y \in D^*(\widehat{Y}) \) such that \( y \leq \hat{y} \), then \( y = \min D^*(\widehat{Y}) \).

**Proof** See Appendix A.2.

Finally, we can show that the investor optimally includes all projects that are weakly greater than \( \hat{y} \).
Lemma 4. Assume Assumption 1. For all \( y \geq \hat{y}, \) if \( y \in \hat{Y}, \) then \( y \in D^*(\hat{Y}). \)

Proof See Appendix A.3.

Taken together, this implies that given awareness set \( \hat{Y}, \) the optimal delegation set includes all projects \( y \in \hat{Y} \) that lie weakly above the threshold project \( \hat{y}, \) whereas it includes at most one action that lies below that threshold.

3.2 Awareness Choice

We can now turn to the optimal revelation strategy of the intermediary. As a first observation, note that if the investor is aware of the threshold project \( \hat{y}, \) the intermediary optimally reveals all other projects. This follows directly from Lemma 3, which shows that if \( \hat{y} \in \hat{Y}, \) the optimal delegation set will never include any project \( y < \hat{y}. \) The intermediary thus maximizes his choice set by revealing all projects to the investor, who then optimally chooses delegation set \( [\hat{y}, 1]. \) This argument further implies that the optimal awareness set \( \hat{Y}^* \) is such that \( \min D^* (\hat{Y}^*) \leq \hat{y}. \)

If the investor is unaware of the threshold project \( \hat{y}, \) the intermediary may have incentives to leave her unaware, provided that this makes it optimal for the investor to include some project \( y < \hat{y} \) into the delegation set. Whether or not it is optimal for the investor to do so is determined by the smallest element of \( \hat{Y} \cap [\hat{y}, 1], \) the project that is implemented instead of \( y \) if \( y \) is not available. The following lemma shows that including \( y < \hat{y} \) is optimal for the investor only if the distance between the smallest element of \( \hat{Y} \cap [\hat{y}, 1] \) and \( \hat{y} \) is at least as large as the distance between \( y \) and \( \hat{y}. \)

Lemma 5. Assume Assumption 1 and let \( \Delta \in (0, \hat{y}). \) Then \( \hat{y} - \Delta = \min D^* (\hat{Y}) \) if and only if \( (\hat{y} - \Delta) \in \hat{Y} \) and \( \hat{Y} \cap (\hat{y} - \Delta, \hat{y} + \Delta) = \emptyset. \)

Proof See Appendix A.4.
Lemma 5 demonstrates that a necessary and sufficient condition for project \( \hat{y} - \Delta \) to belong to the delegation set is that the investor is aware of \( \hat{y} - \Delta \) and is unaware of all projects in the interval \((\hat{y} - \Delta, \hat{y} + \Delta)\). In fact if \( \hat{y} + \Delta > \overline{y} \), the investor must be unaware of all projects \( y > \hat{y} - \Delta \), making the optimal delegation set a singleton, whereas if \( \hat{y} + \Delta \leq \overline{y} \), the investor’s awareness set may include projects in the interval \([\hat{y} + \Delta, \overline{y}]\). In the latter case, it is easy to see that, conditional on the investor being aware of \( \hat{y} - \Delta \), it is optimal for the intermediary to reveal all projects in \([\hat{y} + \Delta, \overline{y}]\) since excluding any project in that interval would strictly reduce his choice set. The equilibrium delegation set is thus of the form \( \{\hat{y} - \Delta\} \cup [\hat{y} + \Delta, \overline{y}] \) or \( \{\hat{y} - \Delta\} \). Let the set of such delegation sets be \( \mathcal{G} = \{G(\Delta)\}_{\Delta \in [0, \overline{y}]} \), defined by

\[
G(\Delta) \equiv \begin{cases} 
\{\hat{y} - \Delta\} \cup [\hat{y} + \Delta, \overline{y}] & \text{if } \hat{y} + \Delta \leq \overline{y} \\
\{\hat{y} - \Delta\} & \text{if } \hat{y} + \Delta > \overline{y}
\end{cases}
\]

When deciding which projects to reveal to the investor, the intermediary implicitly chooses a delegation set in \( \mathcal{G} \). This choice is constrained by the initial awareness of the investor, \( Y_P \). Lemma 3 shows that if the investor is aware of some project \( y \leq \hat{y} \), the optimal delegation set will never include any project \( y' < y \). If \( Y_P \cap [0, \hat{y}] \neq \emptyset \), feasible delegation sets for the intermediary thus need to satisfy \( \hat{y} - \Delta \geq \max Y_P \cap [0, \hat{y}] \). Similarly, Lemma 4 shows that if the investor is aware of some \( y \geq \hat{y} \), this project will always belong to the optimal delegation set, implying that if \( Y_P \cap [\hat{y}, 1] \neq \emptyset \), feasible delegation sets need to satisfy \( \hat{y} + \Delta \leq \min Y_P \cap [\hat{y}, 1] \). Letting \( \bar{\Delta}(Y_P) = \min_{\hat{y} \in Y_P} |y - \hat{y}| \), the intermediary thus chooses a delegation set \( G(\Delta) \) such that \( \Delta \in [0, \min \{\bar{\Delta}(Y_P), \overline{y} - \hat{y}\}] \).

Suppose first \( \bar{\Delta}(Y_P) \leq \overline{y} - \hat{y} \). Then the intermediary’s choice amounts to the optimization problem

\[
\max_{\Delta \in [0, \bar{\Delta}(Y_P)]} \mathbb{E}[u^*(G(\Delta), \theta)] = -\frac{1}{2} \int_{0}^{\hat{y}} (\hat{y} - \Delta - \theta)^2 f(\theta) d\theta - \frac{1}{2} \int_{\hat{y}}^{\hat{y} + \Delta} (\hat{y} + \Delta - \theta)^2 f(\theta) d\theta
\]

If \( \hat{y} \in Y_P \), we have \( \bar{\Delta}(Y_P) = 0 \) and the only feasible delegation set in \( \mathcal{G} \) is \( G(0) = [\hat{y}, 1] \). On the other hand, if \( \hat{y} \not\in Y_P \), then \( \bar{\Delta}(Y_P) \) is strictly positive. The first derivative of the
intermediary’s expected payoff with respect to $\Delta$ is given by

$$\frac{\partial E[u(y^*(G(\Delta), \theta), \theta)]}{\partial \Delta} = \int_0^{\hat{y}} (\hat{y} - \Delta - \theta) f(\theta) d\theta - \int_{\hat{y}}^{\hat{y} + \Delta} (\hat{y} + \Delta - \theta) f(\theta) d\theta = [2F(\hat{y}) - F(\hat{y} + \Delta)] \delta - T(\hat{y} + \Delta)$$

Evaluated at $\Delta = 0$, this derivative is strictly positive:

$$\left. \frac{\partial E[u(y^*(G(\Delta), \theta), \theta)]}{\partial \Delta} \right|_{\Delta=0} = F(\hat{y}) \delta > 0$$

This property implies that whenever $\hat{\Delta}(Y_P) > 0$, the optimal value of $\Delta$ is strictly positive, establishing the claim of Theorem 1. Note further that the second derivative of the intermediary’s expected payoff is strictly negative:

$$\frac{\partial^2 E[u(y^*(G(\Delta), \theta), \theta)]}{\partial \Delta^2} = -F(\hat{y} + \Delta) < 0$$

Hence, the intermediary’s expected payoff as a function of $\Delta$ is strictly concave on the interval $[0, \hat{\Delta}(Y_P)]$. The interior solution of the intermediary’s optimization problem is characterized by the first-order condition

$$[2F(\hat{y}) - F(\hat{y} + \Delta)] \delta = T(\hat{y} + \Delta) \quad (2)$$

The following proposition summarizes the characterization of the equilibrium for the case when $\bar{y}$ is sufficiently large.

**Proposition 6.** Assume Assumption 1 and $\bar{y} = +\infty$. The equilibrium awareness and delegation sets are given by

$$\hat{Y}^* = [y, \hat{y} - \text{Min}\{\Delta^*, \hat{\Delta}(Y_P)\}] \cup [\hat{y} + \text{Min}\{\Delta^*, \hat{\Delta}(Y_P)\}, \bar{y}]$$

$$D^*(\hat{Y}^*) = \{\hat{y} - \text{Min}\{\Delta^*, \hat{\Delta}(Y_P)\}\} \cup [\hat{y} + \text{Min}\{\Delta^*, \hat{\Delta}(Y_P)\}, \bar{y}]$$

where $\Delta^*$ solves condition (2).\[^4\]

Given that $\bar{y} - \hat{y} > 0$, an analogous argument applies for the case $\hat{\Delta} > \bar{y} - \hat{y}$.
Finally, suppose $\bar{\Lambda}(Y_P) \geq \bar{y} - \hat{y}$. Note that a necessary condition for this inequality to be satisfied is that the investor is unaware of all projects above the threshold $\hat{y}$. If the interior solution is feasible, that is $\Delta^* \leq \bar{y} - \hat{y}$, it characterizes the optimal awareness set. If the interior solution is not feasible, the optimal delegation set for the intermediary is either $G(\bar{y} - \hat{y}) = \{2\hat{y} - \bar{y}\} \cup \{\bar{y}\}$ or the singleton $\{\max\{E[\theta], \hat{y} - \bar{\Lambda}(Y_P)\}\}$. The latter is optimal whenever $\delta$ is sufficiently large.

Example: Suppose the state of the world $\theta$ is uniformly distributed on the unit interval, that is $f(\theta) = 1$. The optimal threshold in the benchmark case of full awareness is then given by $\hat{y} = 2\delta$ (see Martimort and Semenov) and the interior solution for the optimal awareness set, characterized by condition (2), is given by $\Delta^* = (2\sqrt{2} - 2)\delta$. Provided that $\max\{\bar{\Lambda}(Y_P), \bar{y} - 2\delta\} > (2\sqrt{2} - 2)\delta$, the intermediary thus leaves the investor unaware of all projects in the interval $((4 - 2\sqrt{2})\delta, 2\sqrt{2}\delta)$. Hence, the equilibrium awareness set and delegation set, respectively, are given by $\hat{Y}^* = [0, (4 - 2\sqrt{2})\delta] \cup [2\sqrt{2}\delta, \bar{y}]$ and $D^*(\hat{Y}^*) = \{(4 - 2\sqrt{2})\delta\} \cup [2\sqrt{2}\delta, \bar{y}]$.

4 Conclusion

We study the delegation problem between an investor and a financial intermediary, who is not only privately informed about the state of the world but also has superior awareness of the available investment opportunities. We find that the intermediary makes the investor aware of investment opportunities at the extremes (e.g. very safe and very risky projects), while remaining silent about intermediate investment opportunities.

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Note that if $\delta \to \bar{y} - E[\theta | \theta \leq \bar{y}]$, then $\hat{y} \to \bar{y}$, implying that given $\hat{y} \notin Y_P$ the condition $\bar{\Lambda}(Y_P) > \bar{y} - \hat{y}$ is satisfied. The intermediary’s payoff associated to delegation set $G(\bar{y} - \hat{y})$ converges to $E[u(\bar{y}, \theta)]$, which is strictly smaller than the payoff $E[u(E[\theta], \theta)]$ for the case $\hat{y} - \bar{\Lambda}(Y_P) \leq E[\theta]$ and $E[u(\hat{y} - \bar{\Lambda}(Y_P), \theta)]$ for the case $\hat{y} - \bar{\Lambda}(Y_P) > E[\theta]$. If the intermediary is restricted to one project, his preferred project is $y = E[\theta]$. However, due to the constraint imposed by $Y_P$, this project may not be feasible. This is the case if $\hat{y} - \bar{\Lambda}(Y_P) > E[\theta]$. 

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The paper has implications for bank regulation and brokerage practices. Given that small investors are those more likely to have limited awareness, banks may have incentives to eliminate investment opportunities with intermediate levels of risk so as to induce small investors to invest in risky assets. If financial authorities forbid banks to offer very risky investments, banks are induced to reveal investment opportunities more suitable to the needs of investors. A similar effect can be obtained by actually offering public assets of intermediate risk.

References


A Appendix

A.1 Proof of Lemma 2

Suppose not and let \( y_1, y_2 \in D^*(\hat{Y}) \). Then there exists an action \( y \in (y_1, y_2) \cap \hat{Y} \) such that \( y \not\in D^*(\hat{Y}) \). With slight abuse of notation, let \( y_1 \) be the largest element of \( D^*(\hat{Y}) \) such that \( y_1 < y \) and let \( y_2 \) be the smallest element of \( D^*(\hat{Y}) \) such that \( y_2 > y \). Define \( r = \frac{y_1 + y_2}{2}, s = \frac{y_1 + y_2}{2} \) and \( t = \frac{y_1 + y_2}{2} \). Alonso and Matouschek (2008) show that the difference in the investor’s payoff between delegation sets \( D^*(\hat{Y}) \{y\} \) and \( D^*(\hat{Y}) \) can be written as

\[
\frac{1}{2} \int_r^s (y_1 - \theta - \delta)^2 f(\theta) d\theta + \frac{1}{2} \int_s^t (y_2 - \theta - \delta)^2 f(\theta) d\theta - \frac{1}{2} \int_r^t (y - \theta - \delta)^2 f(\theta) d\theta
\]

Letting \( y = (1 - \lambda)y_1 + \lambda y_2 \) for some \( \lambda \in (0,1) \) so that \( y \) and \( y_2 - y = (1 - \lambda)(y_2 - y_1) \), the payoff difference can be written as

\[
E[v(D^*(\hat{Y}) \{y\}, \theta)] - E[v(D^*(\hat{Y}), \theta)] = (y_2 - y_1)[T(s) - \lambda T(r) - (1 - \lambda)T(t)]
\]

Given that

\[
T''(y) = f(\theta) - \delta f'(\theta) \geq 0
\]

\( T(y) \) is weakly convex, implying that the payoff difference is weakly negative. A contradiction. \( \square \)

A.2 Proof of Lemma 3

Suppose not. Let \( y_1 = \min D^*(\hat{Y}) \) and suppose first that \( y_1 \) is an isolated point. Letting \( y_2 \) denote the smallest element of \( D^*(\hat{Y}) \) such that \( y_2 > y_1 \), the change in the investor’s expected payoff when excluding \( y_1 \), \( E[v(y^*(D^*(\hat{Y}) \{y_1\}, \theta), \theta)] - E[v(y^*(D^*(\hat{Y})\theta), \theta)] \), is given by

\[
-\frac{1}{2} \int_0^{\frac{y_1 + y_2}{2}} (y_2 - \theta - \delta)^2 f(\theta) d\theta + \frac{1}{2} \int_0^{\frac{y_1 + y_2}{2}} (y_1 - \theta - \delta)^2 f(\theta) d\theta
\]

\[= -(y_2 - y_1) \int_0^{\frac{y_1 + y_2}{2}} \left( \frac{y_1 + y_2}{2} - \theta - \delta \right) f(\theta) d\theta
\]

\[= -(y_2 - y_1) F \left( \frac{y_1 + y_2}{2} - \left[ \frac{y_1 + y_2}{2} - E[\theta | \theta \leq \frac{y_1 + y_2}{2}] - \delta \right] \right)
\]

\[= T(\frac{y_1 + y_2}{2})
\]
Given that $T(y)$ is strictly convex and $T(0) = -\delta; T(\hat{y}) = 0$, we know that $T(y) < 0$ for all $y < \hat{y}$ and $T(y) > 0$ for all $y > \hat{y}$. Given $y_2 \leq \hat{y}$, we thus have $T\left(\frac{y_1 + y_2}{2}\right) < 0$, implying that $D^*(\hat{y}) \setminus \{y_1\}$ yields a strictly larger payoff for the investor than $D^*(\hat{y})$.

Suppose now that $y_1$ is an accumulation point so that there exists an interval $[y_1, y_2] \subseteq D^*(\hat{y})$. The first derivative of the investor’s payoff with respect to $y_1$ is given by

\[
\frac{\partial E[v(y^*(D^*(\hat{y}), \theta), \theta)]}{\partial y_1} = -\int_0^{y_1} (y_1 - \theta - \delta) f(\theta) d\theta = -F(y_1) \left[ y_1 - E[f(\theta | \theta \leq y_1) - \delta \right] = -T(y_1)
\]

Since $y_1 < \hat{y}$ and thus $T(y_1) > 0$, this derivative is strictly positive, implying that the investor can increase his expected payoff by marginally increasing the lower bound of the delegation set. A contradiction.

\section*{A.3 Proof of Lemma 4}

Suppose not. Let $y_1 \in \hat{Y} \cap [\hat{y}, 1]$ be such that $y_1 \not\in D^*(\hat{y})$. By Lemma 2, we must have either $y_1 < \min D^*(\hat{y})$ or $y_1 > \max D^*(\hat{y})$. Suppose first $y_1 < \min D^*(\hat{y})$. Letting $y_2 = \min D^*(\hat{y})$, the proof of Lemma 3 showed that the change in the investor’s expected payoff when including $y_1$ is given by

\[
E[v(y^*(D^*(\hat{y}) \cup \{y_1\}, \theta), \theta)] - E[v(y^*(D^*(\hat{y}), \theta), \theta)] = (y_2 - y_1) T\left(\frac{y_1 + y_2}{2}\right)
\]

Since $\frac{y_1 + y_2}{2} > \hat{y}$, we have $T\left(\frac{y_1 + y_2}{2}\right) > 0$, making the inclusion of $y_1$ into the delegation set strictly optimal.

Suppose now $y_1 > \max D^*(\hat{y})$. Letting $y_0 = \max D^*(\hat{y})$, the change in the investor’s expected payoff when including $y_1$, $E[v(y^*(D^*(\hat{y}) \cup \{y_1\}, \theta), \theta)] - E[v(y^*(D^*(\hat{y}), \theta), \theta)]$, is given by

\[
-\frac{1}{2} \int_{y_1}^{y_0} (y_1 - \theta - \delta)^2 f(\theta) d\theta + \frac{1}{2} \int_0^{y_0 - y_1} (y_0 - \theta - \delta)^2 f(\theta) d\theta
\]

\[
= - (y_1 - y_0) \int_{y_1}^{y_0} \left( y_0 + y_1 - \theta - \delta \right) f(\theta) d\theta
\]

\[
= - (y_1 - y_0) F\left(\frac{y_0 + y_1}{2}\right) \left[ \frac{y_0 + y_1}{2} - E\left[ \theta | \theta \geq \frac{y_0 + y_1}{2} \right] - \delta \right]
\]

Given that $\frac{y_0 + y_1}{2} < E\left[ \theta | \theta \geq \frac{y_0 + y_1}{2} \right]$, this difference is strictly positive, implying that the inclusion of $y_1$ into the delegation set is strictly optimal. A contradiction.
A.4 Proof of Lemma 5

Let $y_l < \hat{y}$ and $y_h = \max \hat{Y} \cap [\hat{y}, 1]$. The investor weakly prefers to include $y_l$ into the delegation set if

\[
-\frac{1}{2} \int_{0}^{y_l + y_h} (y_l - \theta - \delta)^2 f(\theta) \, d\theta + \frac{1}{2} \int_{0}^{y_l + y_h} (y_h - \theta - \delta)^2 f(\theta) \, d\theta \geq 0
\]

\[\iff (y_h - y_l) \int_{0}^{y_l + y_h} \left[ \frac{y_l + y_h}{2} - \theta - \delta \right] f(\theta) \, d\theta \geq 0\]

\[\iff (y_h - y_l) F\left( \frac{y_l + y_h}{2} \right) \left[ \frac{y_l + y_h}{2} - E\left[ \theta \mid \theta \leq \frac{y_l + y_h}{2} \right] - \delta \right] \geq 0\]

\[= T\left( \frac{y_l + y_h}{2} \right)\]

From the argument in the proof of Lemma 3, we know that this inequality is satisfied if and only if $\frac{y_l + y_h}{2} \geq \hat{y}$. Setting $y_l = \hat{y} - \Delta$, this condition corresponds to $y_h \geq \hat{y} + \Delta$. \qed