Risk Premia at the ZLB: a macroeconomic interpretation

Very preliminary and incomplete

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Abstract

Long-term interest rates have fallen to historically low levels since the Great Recession started. One potential contributor are low premia for inflation and interest rate risk. We show how a fairly standard New Keynesian macroeconomic model generates lower inflation and interest rate risk premia when the economy becomes close to the zero lower bound (ZLB). In particular, the inflation risk premia switches from positive to negative. We provide evidence consistent with this mechanism: since 2009, investors seem to view inflation as more positively correlated with the price of risky assets.

1 Introduction

Long-term interest rates have fallen to historically low levels since the Great Recession started. Long term interest rates can be decomposed into expected future short rates and a term premium. Lower long-term rates are certainly, in part, driven by lower expected future short rates (either because of low expected growth or low expected inflation), but one other important potential contributor is a low term premium. The basic intuition is that bonds - and in particular nominal bonds - are good hedges against macroeconomic shocks if the scary scenario for investors is low growth and low inflation - states of the world in which bonds gain value. One piece of evidence consistent with this story is the correlation of bonds with macro news and the stock market - bonds have tended to rally following bad news. Another piece of evidence comes from affine term structure model. These models (which do not use stock prices) also imply very low or negative term premia (see for instance Kim and Wright's estimates).

We show how to rationalize this simple finance intuition in a macro framework. Specifically, we show how a fairly standard New Keynesian macroeconomic model generates low risk premia when the economy becomes close to the zero lower bound (ZLB). In particular, the inflation risk premia switches from positive to negative. The intuition is that a negative TFP growth shocks leads to higher inflation when the economy is far from the ZLB, because the central bank lowers interest rates in response to bad news. But if the economy operates close to the ZLB, a negative TFP growth shock leads to

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lower inflation as the central bank is unable to respond. Hence, the covariance of economic growth and inflation *endogenously* changes depending on whether the ZLB binds.

We provide some preliminary reduced-form evidence consistent with this mechanism: since 2009, investors seem to view inflation as more positively correlated with the price of risky assets. This evidence ties together the macro and finance angles of our story.

Given the importance of long-term interest rates, which are a reference borrowing cost, understanding their movements is of primary academic interest. The reduction in term and inflation premia also has some important “practical” consequences. First, there is a wide-ranging debate about the sources of the decline of interest rates, and to what extent this decline will last. Our model suggests that an upturn in the economy or in inflation could lead to a significant increase in interest rates because these risk premia would change as the ZLB becomes less of a constraint. Second, economists and policymakers often use inflation compensation (i.e., inflation swaps or inflation breakevens (the difference between the yield of nominal and inflation-indexed government debt)) as a measure of expected inflation. It is well understood that inflation compensation may differ from expected inflation due to risk or liquidity premia; but the magnitude and even the size of this adjustment are controversial. Our model argues that breakevens *underestimate* expected inflation when the economy operates close to the ZLB. Third, our analysis is an indirect test of the widely-used ZLB New Keynesian macroeconomic model.

The remainder of the paper is organized as follows. The rest of the introduction reviews briefly the related literature. Section 2 studies a simple example that illustrates how the covariance of macroeconomic variables affects the inflation risk premium and breakeven rates. Section 3 presents some reduced form evidence that the link between the inflation and the stock market has changed since 2008. Section 4 introduces a stylized DSGE model. Section 5 provides some preliminary quantitative results. Section 6 concludes.

1.1 Related Literature

Our paper is related to several strands of literature. First, there is a large macro-asset pricing literature that attempts to account for the level and volatility of the term premium. This work typically uses endowment economies for tractability. Marshall (1992) is an early paper in this literature. Bansal and Shaliastovich (2013) is a recent state-of-the-art study in the long-run risk framework. The underlying logic of how risk premia are determined is similar to our paper (and is discussed in section 2), but our contribution relative to these papers is to study the *sources* of the correlations between inflation and growth that are taken as primitives in these studies.

Second, more closely related is a small but growing literature that considers DSGE production models with nominal rigidities and focuses on the term premium. Key contributions include Rudebusch and Swanson (2008, 2012), Li and Palomino (2014), Palomino (2012), and Swanson (2015). Especially close in spirit is the recent paper by Campbell, Pflueger and Viceira (2014) that emphasize structural breaks in monetary policy rules and how these affect asset prices and their correlations. Our contribution relative to all these papers is to introduce the ZLB and to focus on the recent changes since the Great Recession started.

Third, our paper relates to the vast macro literature on the effects of the zero lower bound (ZLB). Seminal contributions include Krugman (1997) and Eggertsson and Woodford (2003). In particular, our nonlinear methods is related to the contributions of Fernandez-Villaverde et al. (2012), Ngo (2015) and Miao and Ngo (2015).

Finally, the broader question of the relation between stock prices and inflation has long a long history dating back at least to Fama and Schwert (1977) who showed that stocks appeared to be affected negatively by inflation, a result widely viewed as “puzzling” since stocks are claims to real assets. Modigliani and Cohn (1979) argued that investors suffered from money illusion. Boudoukh and Richardson (1993) and Campbell and Vuolteenaho (2004) revisited this issue. On the empirical side, Bernanke and Kuttner (2005) and Rigobon and Sack (2014) demonstrate that monetary policy surprises have a large effect on stock prices. Gorodnichenko and Weber (2016) and Weber (2016) study the heterogeneity in price flexibility and demonstrate that it affects the responses of stocks to monetary policy shocks. Duarte (2013) also studies how inflation affects the cross-section of stock returns. Fleckenstein et al. (2014, 2015) study the pricing of TIPS and deflation.

2 Inflation risk premium: a simple example

In order to provide some basic intuition, it is convenient to use a stripped-down, representative agent, endowment economy model. Suppose that the representative consumer has expected CRRA utility:

$$E \sum_{t=0}^{\infty} \beta^t \frac{C_t^{1-\gamma}}{1-\gamma},$$

and that consumption growth and inflation are conditionally jointly log-normally distributed. Specifically, (log) consumption growth follows the process

$$\Delta \log C_{t+1} = \mu_{c,t} - \frac{\sigma_{c,t}^2}{2} + \sigma_{c,t} \varepsilon_{t+1},$$

while (log) inflation is

$$\pi_{t+1} = \mu_{p,t} - \frac{\sigma_{p,t}^2}{2} - \frac{\sigma_{\chi,t}^2}{2} + \sigma_{p,t} \varepsilon_{t+1} + \sigma_{\chi,t} \eta_{t+1},$$

where in these equations π_{t+1} is the (log) inflation rate, and ε_t, η_t are both *i.i.d.* $N(0, 1)$ and independent; ε_t is the shock to consumption growth or to the “state of the economy” while η_t is an (unimportant) shock to inflation that is orthogonal to ε_t . The conditional means and variances $\mu_{c,t}, \mu_{p,t}, \sigma_{p,t}, \sigma_{c,t}, \sigma_{\chi,t}$ can vary arbitrarily over time.

The critical parameter is $\sigma_{p,t}$, which may be positive or negative, and measures the exposure of inflation to consumption growth risk:

$$\rho_{c,p,t} \stackrel{\text{def}}{=} \text{Cov}_t(\Delta \log C_{t+1}, \pi_{t+1}) = \sigma_{c,t} \sigma_{p,t}.$$

Intuitively, a positive $\sigma_{p,t}$ corresponds to the case where “demand shocks” dominate: low consumption is associated with low inflation, while a negative $\sigma_{p,t}$ corresponds to the case where “supply shocks” dominate: low consumption is associated with high inflation. This in turns determines the risk premium on nominal bonds, as we now show.

For simplicity, we will focus on one-period bonds. (One may think of the time period as being 10 years.) The real log stochastic discount factor is

$$\log M_{t+1} = \log \beta - \gamma \Delta \log C_{t+1},$$

and the nominal log stochastic discount factor is

$$\log M_{t+1}^{\$} = \log M_{t+1} - \pi_{t+1}.$$

Simple calculations show that the log real risk-free rate is

$$\begin{aligned} \log R_{t+1}^f &= -\log E_t(M_{t+1}), \\ &= -\log \beta + \gamma E_t(\Delta \log C_{t+1}) - \frac{\gamma^2}{2} \text{Var}_t(\Delta \log C_{t+1}), \\ &= -\log \beta + \gamma \mu_{c,t} - \frac{\gamma^2}{2} \sigma_{c,t}^2, \end{aligned}$$

whereas the log nominal risk-free rate is

$$\begin{aligned} \log R_{t+1}^{f,\$} &= -\log E_t(M_{t+1}^{\$}), \\ &= \log R_{t+1}^f + E_t(\pi_{t+1}) - \gamma \text{Cov}_t(\pi_{t+1}, \Delta \log C_{t+1}). \end{aligned}$$

The (breakeven rate is the difference in the yields of these two bonds, or in logs:

$$\begin{aligned} \log R_{t+1}^{\$} - \log R_{t+1} &= E_t(\pi_{t+1}) - \gamma \text{Cov}_t(\pi_{t+1}, \Delta \log C_{t+1}) \\ &= E_t(\pi_{t+1}) - \gamma \sigma_{c,t} \sigma_{p,t}. \end{aligned}$$

This shows that the (log) breakeven rate is the sum of expected (log) inflation and a risk premium term which equals risk aversion γ multiplied by the covariance of consumption growth and inflation $\rho_{c,p,t}$. Hence, we see that if investors are risk-neutral ($\gamma = 0$), the breakeven measures perfectly expected inflation.¹ However, most macroeconomic models that attempt to replicate asset prices require high risk aversion, suggesting that this term may be large, and possibly time-varying.

Intuitively, if the covariance $\rho_{c,p,t} < 0$, supply shocks dominate, and breakevens overestimate inflation. Nominal bonds are risky assets, and agents require a premium to hold them, so the nominal yield is higher than it would be under risk-neutrality. On the other hand, if $\rho_{c,p} > 0$, demand shocks dominate, inflation is a hedge, and breakevens underestimate inflation. It is possible in this world that breakevens move without any change in expected inflation if risk aversion and/or the expected covariance of consumption growth and inflation changes.

The data presented in the next section suggests that since 2008, the major movements in breakevens have been positively correlated with the stock market, which suggests that $\rho_{c,p,t} > 0$. If investors currently expect this pattern to continue in the future, breakeven rates may underestimate expected inflation.

¹Technically, up to a Jensen term: the log breakeven equals the expected log inflation, hence the breakeven rate underestimates expected inflation.

Figure 1: 10-year breakeven (left-scale) and SP500 (right-scale) between July 2009 and May 2013.

3 The changing price of inflation risk

This section presents reduced-form evidence that the price of inflation risk has changed since 2008. We start by documenting the changing correlation of inflation breakevens (or inflation swaps) with stock prices.

3.1 Correlations

We start by illustrating how the correlation of breakeven inflations (the difference between the 10 year nominal and TIPS yield) became highly correlated with stock prices after 2008.² We focus on the period after mid-2009 because TIPS markets were disrupted during the peak of the financial crisis (see e.g. Fleckenstein et al. (2014)). Figure 1 shows a strong time series correlation between the two series. During that period, the US economy experienced several cycles of good news (“green shots”) that turned out to be disappointing. Both stock price and breakevens reacted in sync to these news.

Turning to a higher frequency, and comparing across subsamples, figure 2 plots the daily changes in SP500 vs. the daily changes in breakevens.³ The top left panel demonstrates that in the 2003-2007 sample, the correlation is essentially zero. The top right panel shows that the correlation becomes very strong after 2009. (The correlation then weakens after 2013, perhaps reflecting an anticipated exit from the ZLB.) Table 1 reports these correlations.

3.2 Response to Data Releases

To study further the response of stock prices to inflation, we regress the daily return on the SP500 on the “surprise” on days of macro announcements, i.e. the difference between the data as released by the

²Our findings here are illustrated with breakevens, but similar findings are obtained using inflation swaps.

³We start the earliest sample in January 2003 because TIPS liquidity was fairly small before that date.

	SP500	G10	G5	GF5	TIPS10	TIPS5	TIPSF5	BE 10	BE 5	BE F5
Panel A: Jan 2003 through May 2007										
SP500	1.00	0.15	0.15	0.13	0.14	0.13	0.13	0.05	0.06	0.02
G10	0.15	1.00	0.96	0.95	0.85	0.79	0.75	0.50	0.40	0.38
G5	0.15	0.96	1.00	0.83	0.84	0.81	0.70	0.45	0.44	0.27
GF5	0.13	0.95	0.83	1.00	0.79	0.70	0.74	0.51	0.33	0.46
TIPS10	0.14	0.85	0.84	0.79	1.00	0.92	0.90	-0.03	0.00	-0.04
TIPS5	0.13	0.79	0.81	0.70	0.92	1.00	0.64	-0.00	-0.16	0.15
TIPSF5	0.13	0.75	0.70	0.74	0.90	0.64	1.00	-0.05	0.19	-0.25
Breakeven 10	0.05	0.50	0.45	0.51	-0.03	-0.00	-0.05	1.00	0.77	0.80
Breakeven 5	0.06	0.40	0.44	0.33	0.00	-0.16	0.19	0.77	1.00	0.22
Breakeven F5	0.02	0.38	0.27	0.46	-0.04	0.15	-0.25	0.80	0.22	1.00
Panel B: June 2009 through Nov 2012										
SP500	1.00	0.53	0.43	0.55	0.29	0.13	0.34	0.46	0.40	0.37
G10	0.53	1.00	0.92	0.95	0.77	0.58	0.72	0.61	0.52	0.50
G5	0.43	0.92	1.00	0.76	0.73	0.67	0.60	0.53	0.52	0.37
GF5	0.55	0.95	0.76	1.00	0.71	0.45	0.74	0.61	0.46	0.54
TIPS10	0.29	0.77	0.73	0.71	1.00	0.82	0.88	-0.04	0.00	-0.07
TIPS5	0.13	0.58	0.67	0.45	0.82	1.00	0.46	-0.11	-0.29	0.08
TIPSF5	0.34	0.72	0.60	0.74	0.88	0.46	1.00	0.03	0.24	-0.17
Breakeven 10	0.46	0.61	0.53	0.61	-0.04	-0.11	0.03	1.00	0.81	0.85
Breakeven 5	0.40	0.52	0.52	0.46	0.00	-0.29	0.24	0.81	1.00	0.38
Breakeven F5	0.37	0.50	0.37	0.54	-0.07	0.08	-0.17	0.85	0.38	1.00

Table 1: Correlation of the daily changes in the SP500, and in the changes in the of yields: 10 year, 5 year, and 5 year forward Treasuries, TIPS, and breakevens.

Figure 2: Scatter plot of daily changes in SP500 (x-axis) vs. daily changes in breakevens (y-axis) for different subsample, with a regression line superimposed.

statistical agency and the forecast made by economists in the prior days. Table 2 focuses on the CPI and PPI releases and adds the employment report for comparison. Interesting, column 1 shows that before 2008, the stock market had a significant negative response to both CPI and PPI (core) releases. Column 2 shows that the stock market has a weak and insignificant response to CPI core surprises, and a positive and significant response to PPI surprises, after 2009. This is qualitatively consistent with the idea that inflation is now perceived as a positive for the economy.

4 Model

Our model follows closely Rudebusch and Swanson (2014, thereafter RS) and Swanson (2015). The main difference is that we explicitly take into account the zero lower bound.⁴ These authors themselves build closely on the standard New Keynesian model as outlined for instance in Gali (2012) and Woodford (2003). The main difference they introduce relative to the standard model is that they incorporate recursive preferences as in Epstein and Zin (1989) as well as a shock to long-run real or nominal growth and inflation target.

4.1 Household

We follow Rudebusch and Swanson (2014)'s version of Epstein and Zin (1989), except that we introduce a shock to the discount factor β_t :

$$V_t = (1 - \bar{\beta}) u(C_t, N_t) + \bar{\beta} \beta_t E_t (V_{t+1}^{1-\alpha})^{\frac{1}{1-\alpha}}.$$

⁴We also use Rotemberg rather than Calvo pricing, chiefly to economize on state variables, and consider a somewhat different structure of shocks.

	SP500		5y Treasury		5y Inflation Swap		5 y Inflation Breakeven	
Employment	-0.002**	0.005**	0.053***	0.080***	0.021	0.031***	0.020***	0.027***
	(0.001)	(0.002)	(0.006)	(0.014)	(0.024)	(0.012)	(0.007)	(0.008)
CPIcore	-2.218**	-0.435	22.489***	-8.644	31.000**	20.763***	18.144***	18.178***
	(0.949)	(1.087)	(3.894)	(9.317)	(12.052)	(6.515)	(5.333)	(6.559)
PPIcore	-0.628**	1.209**	2.565	5.883**	3.920*	10.766***	2.212	8.484***
	(0.291)	(0.538)	(1.581)	(2.614)	(2.134)	(2.805)	(2.226)	(2.381)
Observations	11,697	1,704	11,697	1,704	1,042	1,703	1,731	1,703
R-squared	0.002	0.006	0.017	0.042	0.016	0.023	0.013	0.023
Before 2009	y	n	y	n	y	n	y	n
Since 2009	n	y	n	y	n	y	n	y

Robust standard errors in parentheses
*** p<0.01, ** p<0.05, * p<0.1

Table 2: Response of asset prices to surprise in macro announcements. Daily regression in samples before and after 2009.

As RS emphasize, instantaneous utility may be negative so one may need to flip signs:

$$V_t = (1 - \bar{\beta}) u(C_t, N_t) - \bar{\beta} \beta_t E_t \left((-V_{t+1})^{1-\alpha} \right)^{\frac{1}{1-\alpha}}.$$

The per period utility is assumed to be

$$u(C_t, N_t) = \frac{C_t^{1-\sigma}}{1-\sigma} - Z_t^{1-\sigma} \frac{\chi N_t^{1+\nu}}{1+\nu},$$

where C_t is consumption, N_t is labor and Z_t is technology. The introduction of Z_t in the utility function is designed to ensure that employment remains stationary even with unit root technology. It may reflect home production or changing tastes as the economy grows.

The labor supply equation is simply

$$w_t = \frac{u_2(C_t, N_t)}{u_1(C_t, N_t)} = Z_t^{1-\sigma} C_t^\sigma \chi N_t^\nu. \quad (1)$$

The real stochastic discount factor is

$$M_{t,t+1} = \bar{\beta} \beta_t \left(\frac{C_{t+1}}{C_t} \right)^{-\sigma} \left(\frac{V_{t+1}}{E_t (V_{t+1}^{1-\alpha})^{\frac{1}{1-\alpha}}} \right)^{-\alpha},$$

and the nominal SDF is

$$M_{t,t+1}^{\$} = \frac{M_{t,t+1}}{\Pi_{t+1}}.$$

The first order condition links the nominal short-term interest rate to the nominal SDF:

$$1 = E_t \left[Y_t^{\$, (1)} M_{t+1}^{\$} \right],$$

where $Y_t^{\$, (1)}$ is the gross nominal yield on a 1-period asset.

4.2 Production and optimal price-setting

There is a number of identical monopolistically competitive firms, each of which operates a production function that is constant return to scale in labor:

$$Y_t = Z_t N_t. \quad (2)$$

Each firm faces a downward-sloping demand curve coming from the Dixit Stiglitz aggregator with elasticity of demand ε . We use the Rotemberg (1982) assumption of quadratic price setting.⁵ A standard derivation for the optimal price yields a nonlinear version of the forward-looking Phillips curve:

$$0 = \left(1 - \varepsilon + \varepsilon \left(\frac{w_t}{Z_t}\right) - \phi(\Pi_t - \Pi)\Pi_t\right) Y_t + \phi E_t (M_{t+1}(\Pi_{t+1} - \Pi)\Pi_{t+1} Y_{t+1}),$$

where Π is the steady-state inflation, and ϕ is the price adjustment cost parameter.

The resource constraint reads

$$C_t = \left(1 - \frac{\phi}{2}(\Pi_t - \Pi)^2\right) Y_t, \quad (3)$$

since we need to subtract price adjustment costs from output. The definition of gross domestic product similarly takes into account that price adjustment is an intermediate input:

$$GDP_t = \left(1 - \frac{\phi}{2}(\Pi_t - \Pi)^2\right) Y_t = C_t.$$

4.3 Monetary Policy Rule

Following Swanson (2015), we assume that the central bank uses the following “Taylor” policy rule:

$$Y_t^{\$, (1)} = \max \left\{ 1, Y^{\$, (1)} \left(\frac{\Pi_t}{\Pi} \right)^{\phi_\pi} \left(\frac{GDP_t}{YPOT_t} \right)^{\phi_y} \right\} \quad (4)$$

where $YPOT_t$ is the measured potential GDP, which we assume the central bank estimates as

$$YPOT_t = (e^{\mu_z} YPOT_{t-1})^{\phi_{ybar}} GDP_t^{1-\phi_{ybar}},$$

where μ_z is the trend growth of the economy (equal to the trend log growth of TFP in this model). Hence, the central bank simply calculates a moving average of past output to estimate potential. We view this as a reasonable assumption since central banks (and other economists) often in practice estimate potential GDP by some filtering procedure that smooths over past data. As a result, the central bank will, in this model (as in reality) recognize changes in potential only with a lag.

4.4 Fundamental Shocks

We assume that:

- (1) The preference shock follows an AR(1) process:

$$\log \beta_t = \rho_\beta \log \beta_{t-1} + \varepsilon_{\beta, t},$$

⁵Miao and Ngo (2015) illustrate that some results (such as the size of the fiscal multiplier) may be affected by the price setting assumptions at the ZLB.

with $\varepsilon_{\beta,t}$ iid $N(0, \sigma_\beta^2)$

(2) The growth rate of TFP follows an AR(1) process:

$$\Delta \log Z_t = \rho_z \Delta \log Z_{t-1} + (1 - \rho_z) \mu_z + \varepsilon_{z,t},$$

with $\varepsilon_{z,t}$ iid $N(0, \sigma_z^2)$.

4.5 Asset prices: Bonds

In order to simplify the numerical computation of the model, we study the prices of geometric consols rather than zero-coupon bonds. A nominal geometric consol with parameter λ 1\$ next period, then λ \$ the period after, then λ^2 \$, and so on. A real consol with parameter λ has the same payoffs, but in units of final goods rather than in \$.⁶

Hence the price for a (nominal or real) consol with parameter λ satisfies the recursion:

$$q_t^{i,\lambda} = E \left[M_{t+1}^i \left(1 + \lambda q_{t+1}^{i,\lambda} \right) \right] \quad (5a)$$

for $i \in \{\$, \text{real}\}$. The yield is defined as

$$Y_t^{i,\lambda} = \frac{1}{q_t^{i,\lambda}} + \lambda \quad (6)$$

where $i \in \{\$, \text{real}\}$.

We now calculate the risk-neutral price and yield, i.e. the price/yield that would occur if agents were risk-neutral. The difference between the yield and the risk-neutral yield is a measure of term premium. (It coincides with the standard definition for zero-coupon bonds.)

The risk-neutral price for a (nominal or real) consol with parameter λ satisfies:

$$q_t^{i,\lambda,RN} = E_t \left[\left(1 + \lambda q_{t+1}^{i,\lambda,RN} \right) \right] E_t \left[M_{t+1}^i \right], \quad (7)$$

and we can define the risk-neutral yield similar to equation 6.

The returns on consols is given by the standard formula:

$$R_{t+1}^{i,\lambda} = \frac{1 + \lambda q_{t+1}^{i,\lambda}}{q_t^{i,\lambda}} \quad (8)$$

where $i \in \{\$, \text{real}\}$.

We define the term premium as the difference between the log yield and the log risk-neutral yield:

$$TP_t^{i,\lambda} = y_t^{i,\lambda} - y_t^{i,\lambda,RN} \quad (9)$$

where $i \in \{\$, \text{real}\}$. Note that $TP_t^{i,0} = 0$.

We define the inflation term premium as the difference between the nominal term premium and the real term premium:

$$ITP_t^\lambda = TP_t^{\$, \lambda} - TP_t^{\text{real}, \lambda}. \quad (10)$$

⁶For our simulations, we calculate these yields and prices for three values of λ . These values are chosen so that $\lambda_1 = 0$, and λ_2 and λ_3 are such that these consols have the durations of actual 5 and 10 year bonds.

The slope of the yield curve is the difference between the log yield of a λ -consol and the log yield of a 0-consol:

$$SL_t^{i,\lambda} = y_t^{i,\lambda} - y_t^{i,0} \quad (11)$$

where $i \in \{\$, \text{real}\}$.

4.6 Asset prices: Breakevens and Inflation premia

Breakevens are the difference between the log nominal yield and the log real yield

$$BE_t^\lambda = y_t^{\$, \lambda} - y_t^\lambda. \quad (12)$$

We define expected log inflation (over the lifetime of a consol) recursively as:

$$ELI_t^\lambda = (1 - \lambda)E_t(\log \Pi_{t+1}) + \lambda E_t(ELI_{t+1}^\lambda).$$

For $\lambda = 0$, this is simply the expected inflation next period, and for $\lambda \rightarrow 1$, this is the long-run average inflation in the future.

Last, the inflation risk premium is the difference between breakevens and expected inflation:

$$IRP_t^\lambda = BE_t^\lambda - ELI_t^\lambda.$$

Inflation risk premia are closely related to inflation term premia.⁷

4.7 Asset Prices: Stocks

Following Abel (1999), we define a stock as an asset with payoff $D_t = C_t^\xi$, where $\xi > 1$ reflects leverage.

The real stock price satisfies the recursion

$$P_t^s = E_t [M_{t+1} (P_{t+1}^s + D_{t+1})],$$

so that if we define the P/D ratio as $q_t^s = \frac{P_t^s}{D_t^s}$, we simply need to solve the recursion for the P-D ratio:

$$q_t^s = E_t \left[M_{t+1} (q_{t+1}^s + 1) \frac{D_{t+1}}{D_t} \right],$$

and the return on equity from t to $t + 1$ is

$$R_{t+1}^e = \frac{P_{t+1}^s + D_{t+1}}{P_t^s} = \frac{q_{t+1}^s + 1}{q_t^s} \frac{D_{t+1}}{D_t}.$$

⁷The appendix discusses the relation between the two concepts in the more common case of zero-coupon bonds.

Parameter	Description and source	Value
β	Subjective discount factor, RS (AEJ 2012), Swanson (2015), Woodford (2003)	0.99
α	Risk aversion parameter, (standard and no recursive, $\alpha = 0$)	-5
χ_c	IES is 0.5, RS (AEJ 2012)	2
χ_n	Frisch labor supply elasticity is 2/3, RS (AEJ 2012)	3/2
χ_0	Normalized to 1, i.e. Fernandez-Villaverde et al (JEDC 2015)	1
ε	Gross markup is 1.15, i.e. Fernandez-Villaverde et al (JEDC 2015)	7.66
ϕ_π	Weight on inflation in the Taylor rule, Swanson (2015)	1.53
ϕ_y	Weight on output in the Taylor rule , 0.75 in Swanson (2015)	0.9
$\rho_{\bar{y}}$	Persistence of output target, Swanson (2015)	0.9
ϕ	Adjustment cost, corresponding to the Calvo parameter of 0.85	238
ρ_A	Persistence of technology shock, 0.95 in SR12, 0 in Swanson (2015)	0.9
ρ_β	Persistence of preference shock, in Nakata '12, Ngo '14, Fernandez-Villaverde et al	0.8
σ_A^2	Std. dev. of the technology shock	0.00075 ²
σ_β^2	Std. dev. of the preference shock	0.0035 ²

Table 3: Parameter values used and sources.

5 Quantitative Results

This section studies the quantitative implications of the model studied in the previous section. These results are still preliminary - we are not yet at the stage where a “best-fitting” calibration can be presented. Rather, the goal for now is more to illustrate “comparative statics” - how some effects vary depending on (i) the level of risk aversion and (ii) whether the economy is close to the ZLB.

Due to the presence of the ZLB, we need to solve carefully the model using nonlinear method. This is especially important because asset prices can be highly sensitive to nonlinearities. We use projection methods that build on the methods used in Ngo (2015) and Miao and Ngo (2015).

5.1 Parameter choices

Table 3 presents the parameters that we use together with the source of the value we use. Most of these parameters are standard in the literature. We will present results both for a “high risk aversion” calibration and a “low risk aversion” calibration to illustrate the effect of risk aversion on asset prices and business cycles.

5.2 Policy functions

Figure 3 plots the policy functions of the model. The top left corner shows tht low TFP growth or a high preference shock lead the ZLB to bind. This is associated with low ELI (expected log inflation) unsurprisingly. Figure 4 depicts the conditional correlation between stock returns, inflation and breakeven, as

Figure 3: Policy functions in the model, as a function of the value of the preference and TFP shocks.

a function of the current shock. These shows that when the preference shock is low (and the ZLB does not bind), stock returns are negatively correlated with inflation. This is because in that regime, TFP growth leads to higher stock prices and lower inflation. On the other hand, if the preference shock is high, stock returns are positively correlated with inflation (and with breakevens). This is because TFP growth now leads to both to higher stock prices and to higher inflation.

5.3 Simulated moments

We now simulate the model assuming that the economy is driven by both TFP shocks and preference shocks. Tables 4 and 5 report the moments (mean and standard deviation of filtered macro quantities and annualized asset prices) under high and low risk aversion respectively. We also report the moments for subsamples when the economy is “close” to the ZLB. We operationalize this by simply splitting the sample between dates where the nominal short-term interest rate is above or below 2% per year (the steady-state value is 4% per year).

Regarding macroeconomic quantities, we see that the volatility of consumption (equal to GDP) is somewhat higher if the economy is close to the ZLB. This is a property that has been emphasized in several studies such as Plante et al. (2012). However, employment is no more volatile close to the ZLB. Turning to interest rates, we see that nominal and real rates are less volatile close to the ZLB while the slope is steeper (due to anticipated mean-reversion of the shocks).

Both nominal and real term premia are negative on average in this calibration. This reflects the offsetting effects of TFP shocks and preference shocks. TFP shocks lead (if the economy is far from the ZLB) to a positive term premium because positive TFP shocks generate growth and low inflation. Hence, the key covariance of consumption growth and inflation is negative. On the other hand, preference

Figure 4: This figure plots the conditional correlation of stock returns with inflation and with breakevens of different maturities, as a function of the current preference shock.

shock are akin to demand shock - they lead to lower growth and lower inflation. Their presence leads to a negative term and inflation premium. In the current calibration their effect dominates. The term premium volatility is also not large enough compared to the data. We plan to address these issues by changing the volatility of the shocks and by increasing risk aversion.

Consistent with our general argument, the inflation risk premium is more negative when the economy is close to the ZLB. The effect is fairly small however.

Last, when risk aversion is low, term and inflation risk premia are obviously lower. More interestingly, macro quantities are also smoother.

6 Conclusion

TBA

High risk aversion									
	All sample		Interest rate < 2		Interest rate > 2		Diff mean	Diff std	
	Mean	Std	Mean	Std	Mean	Std			
C	0.0009	0.1849	-0.0245	0.2367	0.0072	0.1689	-0.032	0.068	
N	-0.0002	0.2394	-0.0178	0.1297	0.0042	0.2596	-0.022	-0.130	
W	0.0013	0.0738	0.0392	0.0591	-0.0082	0.074	0.047	-0.015	
II	0.9999	0.0054	0.9942	0.002	1.0013	0.005	-0.007	-0.003	
y_\$1	4.0229	2.3583	0.9184	0.7084	4.802	1.9506	-3.884	-1.242	
y_\$20	3.8762	1.3700	2.3078	0.4511	4.2698	1.2347	-1.962	-0.784	
y_\$40	3.8762	1.3700	2.3078	0.4511	4.2698	1.2347	-1.962	-0.784	
y_Real1	4.0789	1.1222	3.1266	0.7396	4.3178	1.0741	-1.191	-0.335	
y_Real20	3.9402	0.3966	3.7085	0.2887	3.9984	0.3988	-0.290	-0.110	
y_Real40	3.9402	0.3966	3.7085	0.2887	3.9984	0.3988	-0.290	-0.110	
Slope_\$	-0.1467	1.1786	1.3894	0.5116	-0.5322	0.9649	1.922	-0.453	
Slope_Real	-0.1386	0.7543	0.5818	0.4756	-0.3194	0.7015	0.901	-0.226	
Term Premium_\$	-0.1487	0.0036	-0.1452	0.0051	-0.1496	0.0024	0.004	0.003	
Term Premium_Real	-0.1071	0.0052	-0.0993	0.0063	-0.1091	0.0022	0.010	0.004	
Break Even_1	-0.0559	2.0639	-2.2082	0.7817	0.4842	1.9291	-2.692	-1.147	
Break Even_20	-0.0640	1.4287	-1.4006	0.6298	0.2714	1.3759	-1.672	-0.746	
Break Even_40	-0.0640	1.4287	-1.4006	0.6298	0.2714	1.3759	-1.672	-0.746	
Infl term premium 20	-0.0416	0.0045	-0.046	0.0045	-0.0405	0.0037	-0.006	0.001	
Infl term premium 40	-0.0416	0.0045	-0.046	0.0045	-0.0405	0.0037	-0.006	0.001	
Infl Risk Premium 1	-0.0147	0.0033	-0.0184	0.0052	-0.0138	0.0016	-0.005	0.004	
Infl Risk Premium 20	-0.0874	0.071	-0.113	0.0216	-0.081	0.0774	-0.032	-0.056	
Infl Risk Premium 40	-0.0874	0.0710	-0.113	0.0216	-0.081	0.0774	-0.032	-0.056	

Table 4: Simulated moments under high risk aversion. Columns 1 and 2 give the mean and standard deviation. Columns 3-6 give the mean and standard deviation by subsamples (low interest rates and high interest rates). Columns 7 and 8 calculate the differences across the two samples.

Low risk aversion									
	All sample		Interest rate < 2		Interest rate > 2		Diff mean	Diff std	
	Mean	Std	Mean	Std	Mean	Std			
C	0.001	0.179	-0.025	0.216	0.007	0.169	-0.032		0.048
N	0.000	0.249	-0.018	0.126	0.004	0.270	-0.022		-0.144
W	0.000	0.587	-0.116	0.571	0.028	0.588	-0.144		-0.017
II	1.000	0.005	0.994	0.002	1.001	0.005	-0.007		-0.003
y_\$1	4.034	2.351	0.954	0.694	4.771	1.980	-3.817		-1.286
y_\$20	4.015	1.359	2.478	0.443	4.383	1.242	-1.905		-0.799
y_\$40	4.015	1.359	2.478	0.443	4.383	1.242	-1.905		-0.799
y_Real1	4.056	1.124	3.055	0.719	4.295	1.070	-1.240		-0.351
y_Real20	4.008	0.398	3.754	0.289	4.069	0.397	-0.315		-0.108
y_Real40	4.008	0.398	3.754	0.289	4.069	0.397	-0.315		-0.108
Slope_\$	-0.019	1.178	1.524	0.491	-0.388	0.978	1.912		-0.487
Slope_Real	-0.048	0.754	0.699	0.452	-0.226	0.701	0.925		-0.249
Term Premium_\$	-0.011	0.001	-0.012	0.001	-0.011	0.001	-0.001		0.000
Term Premium_Real	-0.006	0.000	-0.006	0.000	-0.006	0.000	0.000		0.000
Break Even_1	-0.022	2.055	-2.101	0.758	0.475	1.953	-2.577		-1.195
Break Even_20	0.007	1.415	-1.276	0.616	0.314	1.380	-1.590		-0.763
Break Even_40	0.007	1.415	-1.276	0.616	0.314	1.380	-1.590		-0.763
Infl term premium 20	-0.005	0.001	-0.006	0.001	-0.005	0.001	-0.001		0.001
Infl term premium 40	-0.005	0.001	-0.006	0.001	-0.005	0.001	-0.001		0.001
Infl Risk Premium 1	-0.001	0.001	-0.002	0.001	0.000	0.001	-0.001		0.000
Infl Risk Premium 20	-0.036	0.069	-0.055	0.023	-0.032	0.075	-0.023		-0.052
Infl Risk Premium 40	-0.036	0.069	-0.055	0.023	-0.032	0.075	-0.023		-0.052

Table 5: Simulated moments under low risk aversion. Columns 1 and 2 give the mean and standard deviation. Columns 3-6 give the mean and standard deviation by subsamples (low interest rates and high interest rates). Columns 7 and 8 calculate the differences across the two samples.

7 References

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8 Appendix

8.1 Detrended system

We denote with a $\sim$ the variables detrended by Z_t . That is,

$$\begin{aligned}\widetilde{C}_t &= \frac{C_t}{Z_t}, \\ \widetilde{Y}_t &= \frac{Y_t}{Z_t},\end{aligned}$$

and so on. The system of equations to solve is hence:

(1) Taylor rule

$$Y_t^{\$, (1)} = \max \left\{ 1, Y^{\$, (1)} \left(\frac{\Pi_t}{\Pi} \right)^{\phi_\pi} \left(\frac{\widetilde{GDP}_t}{\widetilde{YPOT}_t} \right)^{\phi_y} \right\} \quad (13)$$

(2) Resource constraint:

$$\widetilde{C}_t = \left(1 - \frac{\phi}{2} (\Pi_t - \Pi)^2 \right) \widetilde{Y}_t \quad (14)$$

(3) Production Function

$$\widetilde{Y}_t = N_t \quad (15)$$

(4) Phillips curve:

$$0 = (1 - \varepsilon + \varepsilon \widetilde{w}_t - \phi(\Pi_t - \Pi)\Pi_t) \widetilde{Y}_t + \phi E_t \left[e^{\Delta \log Z_{t+1}} M_{t+1} (\Pi_{t+1} - \Pi) \Pi_{t+1} \widetilde{Y}_{t+1} \right] \quad (16)$$

(5) Euler equation

$$1 = E_t \left[Y_t^{\$, (1)} M_{t+1}^{\$} \right] \quad (17)$$

(6) Labor supply

$$\widetilde{w}_t = \chi N_t^\nu \widetilde{C}_t^\sigma \quad (18)$$

(7) Utility - in this case we define

$$\widetilde{V}_t = \frac{V_t}{Z_t^{1-\sigma}}$$

and we now have:

$$\widetilde{V}_t = (1 - \bar{\beta}) \left(\frac{\widetilde{C}_t^{1-\sigma}}{1 - \sigma} - \frac{\chi N_t^{1+\nu}}{1 + \nu} \right) + \bar{\beta} \beta_t E_t \left(\widetilde{V}_{t+1}^{1-\alpha} e^{(1-\alpha)(1-\sigma)\Delta \log Z_{t+1}} \right)^{\frac{1}{1-\alpha}} \quad (19)$$

(8) Real SDF:

$$\begin{aligned}M_{t,t+1} &= \bar{\beta} \beta_t \left(\frac{C_{t+1}}{C_t} \right)^{-\sigma} \left(\frac{V_{t+1}}{E_t (V_{t+1}^{1-\alpha})^{\frac{1}{1-\alpha}}} \right)^{-\alpha} \\ &= \bar{\beta} \beta_t \left(\frac{\widetilde{C}_{t+1}}{\widetilde{C}_t} \right)^{-\sigma} e^{-\sigma \Delta \log Z_{t+1}} \left(\frac{\widetilde{V}_{t+1} e^{(1-\sigma)\Delta \log Z_{t+1}}}{E_t \left(\widetilde{V}_{t+1}^{1-\alpha} e^{(1-\alpha)(1-\sigma)\Delta \log Z_{t+1}} \right)^{\frac{1}{1-\alpha}}} \right)^{-\alpha} \\ &= \bar{\beta} \beta_t \left(\frac{\widetilde{C}_{t+1}}{\widetilde{C}_t} \right)^{-\sigma} e^{-(\sigma + \alpha(1-\sigma))\Delta \log Z_{t+1}} \left(\frac{\widetilde{V}_{t+1}}{E_t \left(\widetilde{V}_{t+1}^{1-\alpha} e^{(1-\alpha)(1-\sigma)\Delta \log Z_{t+1}} \right)^{\frac{1}{1-\alpha}}} \right)^{-\alpha} \quad (20)\end{aligned}$$

(9) Nominal SDF:

$$M_{t,t+1}^{\$} = \frac{M_{t,t+1}}{\Pi_{t+1}} \quad (21)$$

(10) Definition of potential as measured by the Central Bank:

$$\widetilde{YPOT}_t = e^{\phi_{ybar}(\mu_z - \Delta \log Z_t)} \widetilde{YPOT}_{t-1}^{\phi_{ybar}} \widetilde{C}_t^{1-\phi_{ybar}} \quad (22)$$

The state variables are $YPOT_t$, $\Delta \log Z_t$, and $\log \beta_t$. This is a system of 12 equations (once we add the shock law of motion) in 12 unknowns:

$$Y_t^{\$, (1)}; \widetilde{YPOT}_t; \widetilde{Y}_t; \widetilde{C}_t; N_t; M_{t+1}, M_{t+1}^{\$}, \widetilde{V}_t; \widetilde{w}_t, \Delta \log Z_t \log \beta_t.$$

8.2 Calculation of asset prices

Recall that the stock is an asset with payoff $D_t = C_t^{\xi}$, and that the real stock price satisfies the recursion

$$P_t^s = E_t [M_{t+1} (P_{t+1}^s + D_{t+1})],$$

so that if we define the P/D ratio as $q_t^s = \frac{P_t^s}{D_t^s}$, then we need to solve the recursion for the P-D ratio:

$$q_t^s = E_t \left[M_{t+1} (q_{t+1}^s + 1) \frac{D_{t+1}}{D_t} \right],$$

and the return on equity from t to $t+1$ is

$$R_{t+1}^e = \frac{P_{t+1}^s + D_{t+1}}{P_t^s} = \frac{q_{t+1}^s + 1}{q_t^s} \frac{D_{t+1}}{D_t}.$$

We can define detrended dividend as

$$\widetilde{D}_t = \frac{D_t}{Z_t^{\xi}},$$

and hence

$$\widetilde{D}_t = \widetilde{C}_t^{\xi},$$

and hence we can solve for the P-D ratio q_t^s (which is stationary, so no need for detrending) using the recursion

$$q_t^s = E_t \left[M_{t+1} (q_{t+1}^s + 1) \frac{\widetilde{D}_{t+1}}{\widetilde{D}_t} e^{\xi \Delta \log Z_{t+1}} \right].$$

We could also define the detrended price

$$\widetilde{P}_t^s = \frac{P_t^s}{Z_t^{\xi}},$$

and use the recursion

$$\widetilde{P}_t^s = E_t \left[M_{t+1} (\widetilde{P}_{t+1}^s + \widetilde{D}_{t+1}) \right] e^{\xi \Delta \log Z_{t+1}}.$$

All the quantities defined for bonds do not require detrending since interest rates and inflation are stationary.

8.3 Definition of asset price moments using zero-coupon bonds

In the paper we define asset pricing object (e.g., the term premium) using our “geometric consol” assets. For clarity and completeness, this section defines the same object for the more standard zero-coupons assets. Let $P_t^{(n)}$ the price of a zero coupon real bond (in real terms). We have

$$P_t^{(n)} = E_t \left(M_{t+1} P_{t+1}^{(n-1)} \right)$$

and $P_t^{(0)} = 1$, where M_{t+1} is the real SDF. We define the yield as

$$\frac{1}{\left(1 + Y_t^{(n)}\right)^n} = P_t^{(n)}$$

In log, let $p_t^{(n)} = \log P_t^{(n)}$ and $Y_t^{(n)} = \log \left(1 + Y_t^{(n)}\right)$, then

$$y_t^{(n)} = -\frac{1}{n} p_t^{(n)}.$$

The holding period return of a bond of maturity n is defined as

$$R_{t+1}^{(n)} = \frac{P_{t+1}^{(n-1)}}{P_t^{(n)}}$$

We can have the same exact relationships with nominal yields. We denote them with a \$. Of course we need to use the nominal SDF, and the bond price is now the \$ price of a nominal bond.

The breakeven is defined as the difference of log yields:

$$BE_t^{(n)} = y_t^{\$(n)} - y_t^{(n)},$$

and the inflation risk premium is defined as the difference between breakevens and expected log inflation:

$$IRP_t^{(n)} = BE_t^{(n)} - ELI_t^{(n)}$$

where, denoting Q the price level:

$$ELI_t^{(n)} = E_t \left(\log \frac{Q_{t+n}}{Q_t} \right) = E_t \sum_{k=1}^n \pi_{t+k}$$

where $\pi_{t+1} = \log \left(\frac{Q_{t+1}}{Q_t} \right)$ is log inflation.

The term premium is defined as the difference between the log yield of a n -period bond and the expected short rate over n periods, for $n \geq 2$:

$$TP_t^{(n)} = y_t^{(n)} - \frac{1}{n} \sum_{k=0}^{n-1} E_t y_{t+k}^{(1)}.$$

We can use this definition for nominal or for real term premia.

The inflation term premium is defined as be the difference between the nominal and real term premium. This is close, but not exactly equal, to the inflation risk premium. To see this, note that

$$\begin{aligned} ITP_t^{(n)} &= TP_t^{\$(n)} - TP_t^{(n)}, \\ &= y_t^{\$(n)} - y_t^{(n)} - \frac{1}{n} \sum_{k=0}^{n-1} E_t \left(y_{t+k}^{\$(1)} - y_{t+k}^{(1)} \right), \\ &= BE_t^{(n)} - \frac{1}{n} \sum_{k=0}^{n-1} E_t \left(BE_{t+k}^{(1)} \right), \end{aligned}$$

and since $y_{t+k}^{(1)} - y_{t+k}^{(1)} = BE_{t+k}^{(1)} = IRP_{t+k}^{(1)} + ELI_{t+k}^{(1)}$, we have

$$ITP_t^{(n)} = BE_t^{(n)} - \frac{1}{n} \sum_{k=0}^{n-1} E_t \left(IRP_{t+k}^{(1)} + ELI_{t+k}^{(1)} \right)$$

and since $ELI_t^{(n)} = E_t \sum_{k=1}^n \pi_{t+k} = E_t \sum_{k=1}^n ELI_{t+k}^{(1)}$, we have

$$\begin{aligned} ITP_t^{(n)} &= BE_t^{(n)} - ELI_t^{(n)} - \frac{1}{n} \sum_{k=0}^{n-1} E_t \left(IRP_{t+k}^{(1)} \right) \\ &= IRP_t^{(n)} - \frac{1}{n} \sum_{k=0}^{n-1} E_t \left(IRP_{t+k}^{(1)} \right). \end{aligned}$$

In the case where the inflation risk premium is constant, then $ITP_t^{(n)} = IRP_t^{(n)} - IRP_t^{(1)}$. Generally, since the inflation risk premium is thought to be small at short horizons $IRP_t^{(1)} \simeq 0$, and the difference between the inflation term premium and the inflation risk premium ought to be small.

8.4 Numerical method

To be added.