Fiscal Rules and Sovereign Default*

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Abstract

We provide a quantitative analysis of fiscal rules in a standard model of sovereign debt accumulation and default, modified to incorporate quasi-hyperbolic preferences. Due to the aggregation of citizens’ preferences or to political economy reasons, government preferences are present biased, and results in over accumulation of debt. In a quantitative exercise calibrated to Brazil, we obtain that the welfare gains of the optimal fiscal policy are economically substantial, and that the optimal rule does not entail a countercyclical fiscal policy. We also analyze the cases of a simple debt rule limiting the maximum amount of debt and compare it to that of a simple deficit rule which limits the maximum amount of deficit per period. Whereas the deficit rule does not perform well, the debt rule results in welfare gains virtually equal to the optimal rule.

JEL classification: F34  
Key words: Sovereign Debt, hyperbolic discounting

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1 Introduction

The recurrent concerns over debt sustainability in emerging markets and the ongoing European debt crisis have prompted, once again, an active debate in academic and policy circles on the role of fiscal rules.

The Fiscal Affairs Department at the IMF has compiled a data set of countries and their adoption of restrictions to fiscal policy. In 1990, only 5 countries had fiscal rules in place. By 2014, the number of countries with constraints on fiscal policy had surged to more than 80. Developed countries were the first to conceive them, but about a decade later, rules were also implemented in a number of developing countries. The more common use of national fiscal rules reflects responses to pressures on public finances. In emerging economies their adoption was typically motivated by debt excesses that resulted from the debt crises of the 1980s and the banking and economic crises in the 1990s. Most recently, as countries reacted to the global financial crisis, fiscal rules were enacted to provide credible commitment to long-term fiscal discipline.

Fiscal rules also have a role in business cycle frequencies. According to standard economic theory, fiscal policy should be countercyclical (Barro, 1979). Nevertheless, in practice, most emerging countries follow pro-cyclical fiscal policies, and tend to exacerbate already pronounced cycles (Kaminsky, Reinhart and Vegh, 2005; Vegh and Vuletin, 2012). It is possible this behavior derives from their limited access to credit markets (Bauducco and Caprioli, 2014) or distorted political incentives (Alesina and Tabelini, 2008). By adopting fiscal rules, governments may attempt to impose constraints on their behavior, correcting distorted incentives to overspend, particularly in good times. This, in turn, would alleviate distress in rainy days.

In this paper we study the welfare implications of fiscal rules in the context of sovereign debt and default. Despite their widespread use, little is known about their optimality and role in

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1 See www.imf.org/external/datamapper/FiscalRules.
preventing sovereign default. Are the welfare gains of such rules significant? Are the costs of limited flexibility important? Should these rules take into account the economic cycle? How do simple rules compare with sophisticated ones in terms of their welfare implications?

We transform the traditional model of sovereign debt and default by assuming the government’s preferences are time inconsistent. In particular, our formulation of the government’s preferences corresponds to the quasi-hyperbolic consumption model (Laibson, 1997). As such, there is a conflict of the government of today and the government of tomorrow, which creates an incentive for the government to pre-commit to a certain fiscal rule. To reconcile the impatience of the country government with that of its citizens, we recall Jackson and Yariv (2014, 2015) who propose that the aggregation of preferences of time consistent citizens naturally results in time inconsistent preferences. Thus, even if benevolent ex-ante, the sovereign ends up with preferences which display an extra discount parameter, which captures the fact she is present-biased ex-post. This argument primarily motivates our use of time inconsistent government preferences.

A second, more technical motivation, relates to a prevalent problem in the calibration of models of sovereign debt. As documented in the literature (Reinhart and Rogoff, 2009), emerging countries are able to accumulate debt levels close to 60% of GDP even though they repeatedly default. In order to get the observed high amount of debt and default in an artificial economy, one needs to calibrate the intertemporal discount parameter (“beta”) to extremely low numbers. For example, Aguiar and Gopinath (2006), Alfaro and Kanczuk (2009) and Arellano (2008) employ beta equal to 0.40, 0.80 and 0.50, respectively, for annual data. These values are much lower than what would be obtained if local interest rates were used to calibrate them. Nevertheless, in spite of this unintuitive calibration, it is assumed that the government is benevolent, and maximizes households’ preferences. The use of time inconsistent government preferences removes this calibration restriction. By assuming that the government has hyperbolic preferences, even if
households’ impatience parameter is calibrated to the interest rates, the model can reproduce the level of debt of a typical emerging market and frequency of defaults.

We also analyze the case in which the sovereign is simply not benevolent. In fact, in the context of fiscal policy, there are many political economy motivations of how excessive indebtedness may emerge, such as common pool problem-externalities, which lead to a deficit bias and interest groups as in Aguiar and Amador (2011), Alesina and Drazen (1991) and Persson and Svensson (1989).

We calibrate the model to the Brazilian economy, a typical emerging economy, and obtain the following results. First, by assuming that the government has hyperbolic preferences (and thus an additional parameter to be calibrated) the model can reproduce the Brazilian level of debt and frequency of defaults, even if households’ impatience parameter is calibrated to the interest rates, as usual. Second, the adoption of the optimal fiscal rule implies substantive welfare gains in comparison to a case without any rule. Third, this optimal fiscal rule does not comprise a countercyclical fiscal policy, as it is usually believed. Fourth, under this optimal fiscal rule, the country would opt to never default on its debt. Fifth, a simple deficit rule restricting the maximum amount of deficit per period produces welfare losses. Sixth, in contrast, a simple debt rule determining the maximum amount of debt implies welfare gains comparable to the gains that are obtained by the optimal fiscal rule. As such, given the simplicity and easiness of contractibility, this simpler fiscal rule seems to be the best option to be followed by emerging countries.

An essential hypothesis behind our results is that the Government benefits from borrowing, and thus have a motive to hold positive amounts of debt. This is the case of emerging countries, which will eventually catch up to developing ones. Our analysis indicate that the benefits from front loading households’ consumption is quantitative more important than the benefits from consumption smoothing. As a consequence, fiscal rules should be designed to allow consumption front loading while avoiding the costs resulting from defaults.
In addition to the literature of sovereign debt and default (see Amador and Aguiar (2015) for a recent survey of the literature), this paper is related to recent literature of rules versus discretion (Amador, Werning and Angeletos, 2006; Halac and Yared, 2014). If fiscal rules cannot define policy instructions for every possible shock or eventuality, there is a cost from lack of flexibility, and some discretion can be optimal. Differently from this literature, we explicitly consider the possibility of default and how this affects debt accumulation. We also assume that the private sector knows as much as the government about the state of the economy.

The rest of the paper is organized as follows. In Section 2 we present the model. Section 3 describes the calibration of the model and section 4 presents the results. Discussion and robustness exercises are presented in Section 5. Section 6 concludes.

2 Model

Our economy is populated by a benevolent Government (the sovereign) that borrows funds from a continuum of risk-neutral investors. Endowment is risky and, in order to smooth consumption, the Government may choose optimally to default on its commitments. As in Aguiar and Gopinath (2006) and Arellano (2008), if the Government defaults on its debts, it is assumed to be temporarily excluded from borrowing. Our environment is thus very similar with standard sovereign debt models, with the Government playing the role of the sovereign. The novelty is the assumption of quasi hyperbolic preferences.

In more precise terms, we assume the sovereign’s preferences are given by

$$U_i = E_i \left[ u(g_t) + \beta \sum_{\tau=1}^{\infty} \delta^\tau u(g_{t+\tau}) \right]$$  \hspace{1cm} (1)

with

$$u(g) = \frac{g^{1-\sigma} - 1}{(1-\sigma)},$$  \hspace{1cm} (2)
where $E$ is the expectation operator, $g$ denotes government consumption (or public spending), $\sigma > 0$ measures the curvature of the utility, and $\delta \in (0, 1)$ is the traditional discount factor and $\beta \in (0, 1)$ is an additional discount parameter.

Note, in particular, that the discount function is a discrete time function with values \{1, $\beta \delta$, $\beta \delta^2$, $\beta \delta^3$, ...\}. As such, these preferences are dynamically inconsistent, in the sense that preferences at date $t$ are inconsistent with preferences at date $(t + 1)$. As Laibson (1997) explains, this function has the advantage of mimic the qualitative property of the hyperbolic discount function, while maintaining analytical tractability. Additionally, it has been proposed by both economists and psychologists, as a characterization of animal and human behavior.

In our set up, the motivation for using such preferences is that Government preferences are an aggregation of the citizens’ preferences. As Jackson and Yariv (2014, 2015) show, with any heterogeneity in preferences, every non-dictatorial aggregation method that respects unanimity must be time inconsistent, even if the citizens' preferences are time consistent. Moreover, any such method that is time separable must lead to a present bias. Thus, we interpret the $\beta < 1$ parameter as the bias that come from this aggregation. And treat the case of $\beta = 1$ as the society’s actual preferences.

If the government chooses to repay its debt, its budget constraint is given by

$$g_t = \tau \exp(z_t) - d_t + q_t d_{t+1},$$

(3)

where $d_t$ denotes the Government debt level in period $t$, $\tau$ is the exogenous and constant tax rate, and $z_t$ is the technology state in this period, which determines the output level, $\exp(z_t)$. The debt price functions, $q(s_t, d_{t+1})$ is endogenously determined in the model, and potentially depend on all the states of the economy, $s_t$, as well as the government’s decisions.

We assume the technology state $z_t$ can take a finite number of values and evolves over time according to a Markov transition matrix with elements $\pi(z_i, z_j)$. That is, the probability that $z_{t+1} = z_j$ given that $z_t = z_i$ is given by the matrix $\pi$ element of row $i$ and column $j$. 
When the government chooses to default, the economy’s constraint is

\[ g_t = \tau (1 - \phi) \exp(z_t) \]  

where the parameter \( \phi \) governs the additional loss of output in autarky, a common feature in sovereign debt models (see Alfaro and Kanczuk (2005)). After defaulting, the Government is temporarily excluded from issuing debt. We assume that \( \theta \) is the probability that it regains full access to the credit markets.

Investors are risk neutral and have an opportunity cost of funds given by \( \rho \), which denotes the risk-free rate. The investors’ actions are to choose the debt price \( q_t \), which depends on the perceived likelihood of default. For these investors to be indifferent between the riskless asset and lending to a country, it must be that

\[ q_t = \frac{(1 - \psi_t)}{(1 + \rho)} \]  

where \( \psi_t \) is the probability of default, which is endogenously determined and depends on the Government incentives to repay debt.

We are intentionally not specifying if these investors are international or domestic. Therefore, \( d_t \) stands for total Government debt level, both domestic and international. The assumption that investors are risk neutral implies they compete away their profits, and thus do not affect Government’s utility. Another implicit assumption is that household’s private consumption does not affect the utility households derive from public expenditures. This occurs, for example, when these two types of consumption are separable in households’ preferences.

The timing of the decisions is as follows. In the beginning of each period, the Government starts with debt level \( d_t \) and receives the tax revenues endowment \( \tau \exp(z_t) \). It faces the bond price schedule \( q(s_t, d_{t+1}) \). Taking this schedule as given, the Government simultaneously makes two decisions: (i) it decides whether to default on its debt or not, and (ii) if it decides not to default, it chooses the next level of debt, \( d_{t+1} \).
The model described is a stochastic dynamic game played by a large agent (the Government) against many small agents (the continuum of investors). We focus exclusively on the Markov perfect equilibria. In these equilibria, the sovereign (the Government) does not have commitment, and players act sequentially and rationally. This definition of equilibrium is identical to that of Arellano (2008) and Alfaro and Kanczuk (2009), among many others. The only difference is that it was adapted to deal with the time inconsistency problem that results from the sovereign preferences. But the quasi-hyperbolic assumption implies that its solution can be written as a recursive problem.

In order to describe the equilibrium, notice first that investors are passive, and their actions can be completely described by equation (5). In order to write the Government problem recursively, let \( w^G \) denote the value function of the Government if it decides to maintain a good credit history this period (\( G \) stands for good credit history). Similarly let \( w^B \) denote the value function of the Government once it defaults (\( B \) stands for bad credit history). The value of being in a good credit standing at the start of a period can then be defined as,

\[
 w = \max\{w^G, w^B\} \tag{6}
\]

This indicates that the Government defaults if \( w^G < w^B \). The “good credit” value function \( w^G \) and the policy function \( D^G \) can be written as,

\[
 w^G(s_t) = \max\{u(g_t) + \beta \delta E v^G(s_{t+1})\} \tag{7a}
\]

\[
 D^G(s_t) = \text{ArgMax}\{u(g_t) + \beta \delta E v^G(s_{t+1})\} \tag{7b}
\]

subject to (3). The “bad credit” value function \( w^B \) can be written by

\[
 w^B(s_t) = u(g_t) + \beta \delta [\theta E v^G(s_{t+1}) + (1-\theta)E v^B(s_{t+1})] \tag{8}
\]

subject to (4). In their turn, the “good credit” continuation value \( v^G \) is written by

\[
 v^G(s_t) = u(g_t) + \beta \delta E v^G(s_{t+1}) \tag{9}
\]

subject to (3) and to
\[ d_{t+1} = D^G(s_t) \]  

That is, it is evaluated by using the policy function obtained in the good credit optimization. 

Finally, the “bad credit” continuation value \( v^B \) is written by,

\[ v^B(s_t) = u(g_t) + \delta[\theta Ev^G(s_{t+1}) + (1 - \theta)Ev^B(s_{t+1})] \]  

subject to (4).

To compute the equilibrium, it is useful to define a default set as the states of the economy in which the Government chooses to default. The default set in turn determines the price \( q_t \) through expression (5). With these prices at hand, one can solve the Government problem (7) to (11). The solution for (6) determines a default set, which can be used in the next iteration.

The recursive equilibrium is defined by the set of policy functions for Government asset holdings and default choice such that, (i) taking the price functions as given, the Government policy functions satisfy the government optimization problem, and (ii) the price of bonds is consistent with the Government decisions.

3 Calibration

We calibrate the model to Brazilian annual data since 1955. We set the risk-free (international) interest rate \( \rho = 0.04 \) and inter-temporal substitution parameter \( \sigma = 2 \), as it is usual in real-business-cycle research in which each period corresponds to one year. We set tax rate \( \tau = 30\% \) which is the average tax burden over the period.

We calibrate the technology state \( z^T \) by considering the (logarithm) of the GDP to follow an \( AR(1) \) process; that is, \( z_{t+1}^T = \alpha \epsilon_t^T + \epsilon_{t+1}^T \) where \( \epsilon_t \approx N(0, \sigma^2) \). We obtain \( \alpha = 0.85 \) and \( \sigma = 0.044 \). We discretize this technology state and use the Quadrature Method to calculate transition probabilities. We also discretize the space state of debt sufficiently to avoid affecting the decision rules.
We set the probability of redemption at $\theta = 0.2$, which implies an average stay in autarky of five years, in line with estimates by Reinhart (2010) for Brazil. Direct output costs are modeled from default episodes, and equal $\phi = 10\%$.

We calibrate $\delta = 0.90$ using the Brazilian average real interest rate. The fact that the country’s impatience is higher than the investors patience is motivated by the fact that growth in emerging markets are higher than in Developed countries. Since poorer countries should catch up with richer ones, they have incentives to frontload consumption.

Finally, to obtain reasonable levels of debt in equilibrium, we set the additional intertemporal factor, related to the Government present bias, at $\beta = 0.70$. Table 1 summarizes the parameter values.

4 Simulation Results

In this model, the Government has two instruments to affect his time path of consumption: default and borrowing. By choosing to default, the Government opts to have a higher level of consumption today, in exchange for being excluded from capital markets and suffering output costs. Thus, default may make sense as a means to escape from a situation where high indebtedness and low technology would result in very low consumption levels.

Debt, the second instrument of the government, can be potentially used in two ways. First, it could be used to smooth income fluctuations relative to the mean level of income, in the same way default is used. Second, given that the country is more impatient than investors, debt can be used to tilt the consumption profile towards the present.

Because, in this model, financial contracts available are state dependent, front loading consumption is easier in high income shocks when debt is in fact cheaper and borrowing limits are loose. Thus, the two objectives of the debt instrument tend to conflict with each other. When the technology shock is good, it is cheaper to frontload consumption, but it also makes sense to save
for rainy days. And the converse is true when the technology shock is bad. The policy rule that obtained by the solution of the model reflects which objective is quantitatively more important. Under which circumstances should debt be use to smooth consumption or to frontload its profile.

4.1 No Rule

We start by considering the case in which the Government is not subject any to fiscal rules. Figures 1a and 1b represent the policy functions obtained by the solution of the model described and depict, respectively, the Government’s default decision and the choice of the next period debt level contingent on the Government not defaulting during this period. The curve denoted by “high technology” refers to a favorable situation, in which the country is hit by a good shock, that is, a “boom”. The curve denoted by “low technology” refers to the case where the country suffers a bad shock, that is, a “recession”.

Figure 1a indicates that when the technology shock is high the Government defaults when the debt level is higher than 70% of GDP. If the technology shock is low, the maximum amount of sustainable debt is 58% of GDP. Since, as expected, default is used as a means to smooth consumption, default is more likely the lower the output levels.

Figure 1b shows that the next period debt level is higher the better the technology state and the higher this period debt. The positive relationship between consecutive debt levels was anticipated because, for a given technology shock, the Government would attempt to avoid sharp changes in the level of debt, as these imply higher consumption volatility.

The relationship between technology and debt is a fairly surprising result, but obtained previously (Arellano (2008), Kanczuk and Alfaro (2009)). It means that the Government does not use debt as a way to smooth consumption, a departure from the “pure” Eaton and Gersovitz (1981) framework. Instead, debt is predominantly used to front-load consumption, given that the discount factor $\delta$ is lower than the inverse of the risk-free interest rate. As mentioned, consumption
smoothing is mostly achieved through default, as in contingent debt service models, such as Grossman and Van Huyck (1988).

Calculating the invariant distribution of the states, we determine that the Government is excluded from the market 3.2% of the time, and the average debt is 60.1% of output, as reported in the first line of Table 2. These results are broadly consistent with the stylized facts.

4.2 Optimal Fiscal Rule

Next, we consider the case of the optimal fiscal rule. This corresponds simply to solve the model for $\beta = 1$, in which case the time inconsistency problem and present bias of the Government disappears.

In our set up, in contrast with those of Halac and Yared (2014), there is no private information about the technology shocks that hit the economy. In Halac and Yared the optimal rule has to balance discretion with commitment, and is shown to be history dependent, as it provides dynamic incentives. In our simpler case, due to the absence of private information, the first best policy can be implemented with full commitment, that is, by giving Government a pre-determined contingent path of efficient consumption.

We report the invariant distribution properties of the economy under the optimal fiscal rule in the second row of Table 2. Under the optimal rule the invariant distribution displays no default, and the average level of debt drops to 50.2% of GDP. In other words, the Government present bias is responsible for a debt over-accumulation of about 10% of GDP, and for the occurrence of default episodes.

Table 2 also compares the welfare level of the economy under the optimal rule with that of the economy with no rule, in terms of consumption. For that we consider, for both economies, the transition path from a starting point with no debt to their respective invariant distribution. We
obtained that the adoption of optimal rule results in welfare gains of 0.277% of GDP. This is very substantial when compared to typical business cycles welfare gains.

Figures 2a and 2b depict, respectively, the Government’s default decision and the choice of the next period debt level contingent on the Government not defaulting during this period. Figure 2a looks like figure 1a, but it has different threshold axis values. Again, as expected, default is more likely the lower the output levels. The maximum amount of sustainable debt is 63% and 50% of GDP, respectively when the technology is high and low. Thus, although there is no default in equilibrium, the threshold values for default are lower under the optimal rule than under the case without any rule.

By comparing Figure 2b with Figure 1b one notices that the optimal policy also has similar qualitative properties to the solution without fiscal rules. Again, contrary to the usual intuition, debt accumulation does not increase when the economy is hit by a bad shock. In other words, whereas default is (potentially) used to smooth consumption, debt is used only to tilt the consumption profile. And this holds regardless of the fact that default does not happen in equilibrium, at least after the economy converged to its invariant distribution.

From figure 2b one sees that when debt is relatively small, the economy saves the same regardless of the output shock. But when debt is large, the economy borrows more in booms than in recessions because of the countercyclical interest rate schedules. In other words, when debt is larger (and the incentives to default are also larger), the borrower would like to borrow heavily during bad shocks. But it cannot because such financial contracts are too expensive or just not available. As a consequence, when debt is large, the optimal fiscal policy is pro-cyclical.

4.3 Debt Rules

Next, we consider the case of simple debt rules, under the hypothesis that the Government is presently biased ($\beta = 0.70$). Under these rules, the Government is forbidden to choose debt levels
above a certain threshold, which is previously set. We solve for many such thresholds, and report the corresponding invariant distribution properties in Table 2.

Notice that when the debt threshold is set to 65% or 60% of GDP, the average debt level is lower than under the no rule case, but it is smaller than the threshold itself. Thus, the threshold is constraining debt accumulation, but the Government has yet some margin. Additionally, for these threshold levels, the frequency of default is as high as for the economy without any rule.

In contrast, when the debt threshold is set to 55% of GDP or lower, the invariant distribution debt level is exactly equal to the threshold. Under these rules, the debt accumulation becomes, in fact, binding all the time. In addition, there is no more default in equilibrium (after converging to the invariant distribution).

Figure 3 depicts, for the case in which output is neither high nor low (i.e., $z = 0$), the policy function for the cases without rule, with optimal fiscal rule and with a simple rule with threshold $d \leq 50\%$ of GDP. Note how the simple rule implies an average debt level similar to the optimal rule. But the constraint affects consumption smoothing, making the Government accumulate the same level of debt regardless of its previous indebtedness.

Welfare gains vary widely depending on the debt threshold. When the threshold is too high or too low, these simple rules imply in substantial welfare losses, even compared with the no rule case. This is somewhat surprising. More interesting, when the threshold is set to 50%, the welfare gains are virtually equal to that of the optimal rule. That is, a very simple rule can lead to gains comparable to a fairly complex one.

In our view, this finding may have a relevant policy implication. Given that the threshold rule is easily contractible, and it leads to welfare levels virtually equal to the optimal fiscal rule, they seem to be the best alternative to be implemented in practice.
4.4 Deficit Rules

Next, we consider the case of simple deficit rules, again under the hypothesis that the Government is presently biased \((\beta = 0.70)\). Under these rules, the Government is forbidden to choose deficit (or changes in debt, \(\Delta d = d_{t+1} - d_t\)) above a certain threshold, which is previously set. We solve for many such thresholds, and report the corresponding invariant distribution properties in Table 3.

Notice that as the maximum amount of deficit is reduced, and thus the constraint gets binding, the economy displays less defaults and higher amounts of debt. It is unexpected, but promising, that the deficit constraint can generate high amounts of debt, and thus more frontloading, without suffering the costs from defaulting. However, by looking at the welfare implications, one realizes that the arrangement is not working as expected.

Figure 4 displays the debt policy \((d_{t+1} \text{ as a function of } d_t)\) for the deficit policy \((\Delta d < 2\%)\) compared with the optimal policy (always when technology is neutral, i.e., \(z = 0\)). As expected, in the relevant region the deficit rule results in a policy very close to the 45 degrees line. That is, debt is restricted to change very slower.

To grasp the practical implication of the deficit rule, Figure 5 displays the consumption profile for an economy that starts with zero debt in the beginning of time, and always faces neutral technology shocks \((z = 0)\). Recall that zero debt is the case at hand after a sovereign defaults and it is also the initial condition adopted in our welfare calculations.

Under the optimal policy the household frontloads consumption, which starts in a much higher level and rapidly converge to its steady state, in about 6 years. In contrast, under the deficit rule, the Government is restricted to increase debt by small amounts, cannot increase initial consumption, and converge to the steady state very slowly. In other words, it cannot seize the benefits from frontloading consumption.
An indirect effect of the deficit rule is that the potential benefits from defaulting are greatly reduced. By defaulting, the country gets rid of the debt and can thus increase consumption. But under a deficit rule, consumption can only increase slowly, thus deriving little utility. Therefore, as a consequence, the Government resists default even for larger amounts of debt. This creates the apparent contradiction of being able to hold large amounts of debt without defaulting but, at the same time, achieving lower welfare levels.

5 Discussions and Robustness

5.1 Self-Interested (Non-Benevolent) Government

The analysis of the last sections was based on the hypothesis that the Government is benevolent but displays quasi-hyperbolic preferences, which in turn motivated the adoption of fiscal rules. An alternative motivation for fiscal rules occurs when the Government is simply not benevolent. There is a vast literature Political Economy literature that analyses why self-interested Governments overaccumulate debt.

A simple way to capture this possibility in our framework is to set $\beta = 1$ and to calibrate $\delta^{gov}$ such that our artificial economy displays levels of debt consistent with those observed in reality. That is, by supposing Government preferences are not time inconsistent, but they are different from the country citizens’, who are less impatient.

Performing such procedure, by setting $\delta^{gov} = 0.80$, we get that the invariant distribution displays debt equal to 62% of GDP, and that default happens 3.2% of the time. That is broadly consistent with the stylized facts, and with our basic calibration. The policy functions are also very similar, as expected.
If the Government discount factor is \( \delta^{\text{gov}} = 0.80 \), but the citizens’ discount factor is \( \delta = 0.90 \), as before, the same policy implications would apply. The optimal rule and the results from simple threshold rules would also be the same as before.

An important omission from the analysis is why the optimal rule (or the threshold rules) would be applied. The self-interested Government would certainly be against the adoption of such rules. Although we do not analyze how, by means of elections citizens need somehow to discipline politicians and force them to adopt the rules.

5.2 Private Information

As mentioned previously, in our framework there is no private information about the technology shocks that hit the economy. In other words, we assumed households know about the state of the economy as much as the Government. Although this is the natural hypothesis to entertain, there are some possible rationales to consider the private information case, where the Government is better informed than the households. One such rationale is that private information is an indirect way to capture the fact that some fiscal contingencies are not easily contractible.

How would our results change if only the Government observed the technology shock? The optimal policy would be a lot more complex, as shown by Halac and Yared (2014). But it would result in welfare gains (to the households) smaller or equal to the optimal rule with complete information. More precisely, the welfare gains of the private information optimal rule would be somewhere between the welfare gains under the public information optimal rule and under the simple threshold rules.

Our analysis pointed out that the welfare gains of the simple threshold rule are almost identical to those of the adequately chosen threshold sure. That is a good reason for using the simple threshold rule, given that is easily contractible. One can then conclude that the same policy implication also holds when there is private information.
5.3 Distortionary Taxes, Risk Aversion and Fiscal Pro-cyclicality

A surprising result of our simulations is that optimal fiscal policy is not countercyclical. It is surprising because at least since Barro (1979) we are used to the idea that since tax distortion costs are convex, debt should fluctuate in order to keep tax rates constant. As discussed, in our model, this logic does not hold. We obtained that,

In principle, our economy already has the ingredients that should make countercyclical fiscal policy optimal. The model assumes output is endowed, but it already considers the extreme case in which taxes rates are constant. Thus, even if the model contemplated production and tax distortions, it could not achieve any more tax smoothing than what it already has, by straight assumption.

As for the expenditure, rather than assumed constant, it is chosen to maximize utility. But since preferences are concave in consumption, the Government does have the incentives to use debt to smooth consumption. In other words, it has the motive to hold expenditures constant and thus implement a countercyclical fiscal policy. However, our results indicated that this motive is dominated by other motivations. In particular, under the optimal fiscal rule, debt is used to frontload consumption, rather than smooth it.

To investigate how quantitative robust this result is, we modify the model to increase the gains from consumption smoothing. A simple way to do that is to modify the calibration by increasing the risk aversion parameter $\sigma$. Since this parameter also plays the role of intertemporal elasticity, it controls the benefits from smoothing consumption.

Figure 6 displays the correlation between (the logarithm of) output and Government savings (the change in debt) for various risk aversion parameters. Note that for $\sigma$ smaller of equal than 4 the model displays procyclical fiscal policy. When $\sigma = 5$, fiscal policy is completely acyclical. For higher values of $\sigma$, when the benefits from consumption smoothing become really
large, fiscal policy turns countercyclical. However, these are calibrations that contradict much evidence about consumers’ preferences.

We take this result as an indication that, from the quantitative point of view, the gains from frontloading consumption seem to be more important than tax smoothing for a wide range of economies.

6 Conclusion

Emerging countries, as they catch up to developed ones, benefit from borrowing in order to frontload their consumption profile. However, in practice, they tend to borrow too much, and often resort to defaulting on their debt. Government’s preferences may display a present bias, either because of aggregation of heterogeneous citizens’ preference, or due to political economy reasons.

Fiscal rules are a potentially useful commitment technology to solve this problem. In the context of a traditional model of sovereign debt and default, we analyze the welfare gains from alternative fiscal rules. We obtain the gains from the optimal fiscal rule are economically relevant and that the optimal fiscal rule does not display procyclical fiscal policy. In addition, a simple threshold rule, which is easily contractible, can generate gains virtually as high as an optimal rule.

7 References


**Table 1: Calibration**

<table>
<thead>
<tr>
<th>Parameter</th>
<th>Value</th>
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<tbody>
<tr>
<td>Technology autocorrelation</td>
<td>$\alpha = 0.85$</td>
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<tr>
<td>Technology standard deviation</td>
<td>$\sigma = 0.044$</td>
</tr>
<tr>
<td>Probability of redemption</td>
<td>$\theta = 0.20$</td>
</tr>
</tbody>
</table>
Output costs $\phi = 0.10$
Risk aversion $\sigma = 2$
Risk free interest rate $\rho = 0.04$
Tax rate $\tau = 0.30$
Discount factor $\delta = 0.90$
Hyperbolic Discount factor $\beta = 0.70$

Table 2: Invariant Distributions for Alternative Fiscal Rules

<table>
<thead>
<tr>
<th>Model Specification</th>
<th>Exclusion from Market (% time)</th>
<th>Debt if not excluded (% GDP)</th>
<th>Welfare (% GDP)</th>
</tr>
</thead>
<tbody>
<tr>
<td>No Rule</td>
<td>3.2</td>
<td>60.1</td>
<td>0</td>
</tr>
<tr>
<td>Optimal Rule</td>
<td>0</td>
<td>50.2</td>
<td>0.277</td>
</tr>
<tr>
<td>Rule $d \leq 65%$</td>
<td>3.2</td>
<td>59.8</td>
<td>-0.214</td>
</tr>
<tr>
<td>Rule $d \leq 60%$</td>
<td>3.2</td>
<td>57.8</td>
<td>-0.163</td>
</tr>
<tr>
<td>Rule $d \leq 55%$</td>
<td>0</td>
<td>55.0</td>
<td>0.259</td>
</tr>
<tr>
<td>Rule $d \leq 50%$</td>
<td>0</td>
<td>50.0</td>
<td>0.276</td>
</tr>
<tr>
<td>Rule $d \leq 45%$</td>
<td>0</td>
<td>45.0</td>
<td>0.275</td>
</tr>
<tr>
<td>Rule $d \leq 40%$</td>
<td>0</td>
<td>40.0</td>
<td>0.212</td>
</tr>
<tr>
<td>Rule $d \leq 35%$</td>
<td>0</td>
<td>35.0</td>
<td>0.129</td>
</tr>
<tr>
<td>Rule $d \leq 30%$</td>
<td>0</td>
<td>30.0</td>
<td>0.024</td>
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<tr>
<td>Rule $d \leq 20%$</td>
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<td>20.0</td>
<td>-0.257</td>
</tr>
<tr>
<td>Rule $d \leq 10%$</td>
<td>0</td>
<td>10.0</td>
<td>-0.656</td>
</tr>
<tr>
<td>Rule $d \leq 0$</td>
<td>0</td>
<td>0</td>
<td>-1.141</td>
</tr>
</tbody>
</table>

Table 3: Invariant Distributions for Deficit Rules

<table>
<thead>
<tr>
<th>Model Specification</th>
<th>Exclusion from Market (% time)</th>
<th>Debt if not excluded (% GDP)</th>
<th>Welfare (% GDP)</th>
</tr>
</thead>
</table>


<table>
<thead>
<tr>
<th>Rule</th>
<th>$\Delta d \leq 20%$</th>
<th>$\Delta d \leq 10%$</th>
<th>$\Delta d \leq 5%$</th>
<th>$\Delta d \leq 4%$</th>
<th>$\Delta d \leq 3%$</th>
<th>$\Delta d \leq 2%$</th>
<th>$\Delta d \leq 1%$</th>
</tr>
</thead>
<tbody>
<tr>
<td>No Rule</td>
<td>3.2</td>
<td>3.2</td>
<td>3.2</td>
<td>3.2</td>
<td>0</td>
<td>0</td>
<td>0</td>
</tr>
<tr>
<td>$\Delta d \leq 20%$</td>
<td>60.1</td>
<td>56.8</td>
<td>57.4</td>
<td>61.3</td>
<td>-0.184</td>
<td>-0.511</td>
<td>-0.946</td>
</tr>
<tr>
<td>$\Delta d \leq 10%$</td>
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<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>$\Delta d \leq 5%$</td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>$\Delta d \leq 4%$</td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>$\Delta d \leq 3%$</td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>$\Delta d \leq 2%$</td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>$\Delta d \leq 1%$</td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
</tbody>
</table>

*Figure 1a: Default Policy Function for No Rule Economy*
Figure 1b: Debt Policy Function for No Rule

Figure 2a: Default Policy Function for Optimal Rule
Figure 2b: Debt Policy Function for Optimal Rule
Figure 3: Debt Policy Function for different fiscal rules ($z = 0$)

![Graph showing Debt Policy Function for different fiscal rules ($z = 0$)](image)

Figure 4: Debt Policy Function for Deficit Rule ($z = 0$)

![Graph showing Debt Policy Function for Deficit Rule ($z = 0$)](image)
Figure 5: Consumption Profile ($z = 0$)

Figure 6: Fiscal policy counter-cyclicality