Abstract
A nursing home stay towards the end of life is one of the biggest risks faced by Americans over the lifecycle. The annual cost of a nursing home stay in 2010 was $84,000. At age 50 the probability of a nursing home stay ranges between 50-59 percent and among those who have a stay, 20 percent spend more 3 years in a nursing home. Yet, only about 10 percent of U.S. retirees purchase private long-term-care (LTC) insurance. Previous research has emphasized that Medicaid crowds out the demand for private LTC insurance. However, rejection rates for private insurance are also high. Nearly 40 percent of the potential pool of purchasers would be rejected if they applied for private LTC insurance using current screening guidelines. This paper explores the possibility that high rejection rates are due to adverse selection. We propose a model in which agents have private information about their risk of a nursing home stay and model the private and public provision of LTC insurance. Our model accounts for low coverage rates and high rejections of private LTC insurance and is then used as a laboratory for considering welfare enhancing reforms of private and public provision of LTC insurance.

Keywords: Private Long-Term Care Insurance; Medicaid; Welfare; Elderly; Medical Expenses.

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1 Introduction

One of the largest risks faced by Americans is the possibility that they will have a protracted nursing home stay towards the end of their life. Some estimate that the lifetime probability of a nursing home stay for a 50 year old ranges between 50 and 59 percent. Those who experience nursing home stays spend on average about one year in a nursing home and the risk of a stay that exceeds 3 years is significant for older women. Medicare provides universal care to retirees for nursing home stays for rehabilitative purposes but, the level of coverage is reduced after the first month and then drops to zero at 100 days. Longer nursing home stays are costly. According to the U.S. Department of Health nursing home expenses averaged $205 per day in a semi-private room and $229 per day in a private room in 2010. Absent any private insurance, a nursing home stay of three years can result in out-of-pocket expenses that exceed $200,000. According to Kopecky and Koreshkova (2014) long-term care risk is the second largest risk faced by individuals over the lifecycle.

The private market for LTC insurance in the U.S. is surprisingly small. Only 10 percent of those over 65 are covered by private LTC insurance and private policies only account for 4 percent of aggregate long-term care expenditures. It is also very concentrated. Over 60 percent of new policies written in 2012 were written by two companies.

Medicaid provides a safety net for those who experience high long-term expenses. But, it is both income and asset-tested and consequently only available to those who are poor (categorically needy) or those who have exhausted their personal resources due to high medical expenses (medically needy). Recent research by De Nardi, French and Jones (2010), Braun, Kopecky and Koreshkova (2015) finds that this program is also highly valued by both poor and wealthy individuals.

Indeed Brown and Finkelstein (2008) argue that one reason for the small size of the private market of LTC insurance is Medicaid. Medicaid is the secondary insurer of LTC expenses and private LTC insurance is often redundant because it pays out in situations where individuals would qualify for Medicaid if they had no private insurance. They also find that the cost of private LTC insurance is expensive relative to an actuarially fair benchmark.

These demand-side considerations can account for low take-up rates of private LTC insurance. However, they do not provide an explanation for why many individuals are rejected for private LTC insurance. In 2013 27.8 percent of private LTCI applications were declined, withdrawn or suspended. Indeed Brown and Finkelstein (2008) argue that one reason for the small size of the private market of LTC insurance is Medicaid. Medicaid is the secondary insurer of LTC expenses and private LTC insurance is often redundant because it pays out in situations where individuals would qualify for Medicaid if they had no private insurance. They also find that the cost of private LTC insurance is expensive relative to an actuarially fair benchmark.

These demand-side considerations can account for low take-up rates of private LTC insurance. However, they do not provide an explanation for why many individuals are rejected for private LTC insurance. In 2013 27.8 percent of private LTCI applications were declined, withdrawn or suspended. Moreover, results we present below suggest that 38 percent of those aged 55-65 would be rejected if they applied for private LTC insurance. Hendren (2013) suggests that rejections are due to adverse selection and provides empirical evidence in favor of this hypothesis.

These results suggest that the private market for LTCI faces significant demand-side and supply-side distortions. In this paper we model these distortions in a general equilibrium framework that features adverse selection and Medicaid. Our model can account for the low take-up rates of private insurance and also the high rejection rates. Low income individuals are not offered private insurance and rely on Medicaid instead, those in the middle of the income distribution purchase private LTCI. For those at the top of the income distribution there are no gains to trade and they self-insure against LTC risk.

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1Life-time earnings risk is the largest risk according to their analysis.
Previous literature on optimal insurance contracts has found that the optimal contract offers the efficient level of insurance to those with the highest risk exposure Chade and Schlee (2012), Hellwig (2010) and Lester, Shourideh, Venkateswaran and Zetlin-Jones (2015). This property appears to run counter to what we see in practice, where those with the highest measured risk exposure are denied coverage. We are able to produce this empirical regularity using loads on marginal cost for the issuer and by assuming that some individuals have a risk of a nursing home that is close to 1.

We also find that interesting interaction effects between social and private LTC insurance markets. Most notably, Medicaid acts to reduce market power. All individuals face lower insurance premia when Medicaid is present. This is because Medicaid provides a floor on the losses of those who choose not to purchase private LTCI.

We then use the model as a laboratory for assessing good arrangements for insuring against LTC risk. We consider reducing the scale of Medicaid and also subsidies to those who purchase private LTCI.

Our model has two periods. Young observe a noisy public indicator their health status when young and make a consumption-savings decision. In the second period of life they observe their true risk exposure to the LTC event and decide whether to purchase private LTC insurance. After this market closes individuals realize the LTC event and consume. We assume a single issuer of private insurance. As described above this market is highly concentrated and making this assumption allows us to ascertain how market power are affected by the presence of Medicaid. We consider these questions under the assumption that the real interest rate is exogenous but we close the government’s budget constraint by taxing individuals to finance Medicaid. Profits are also paid out to households.

The remainder of the paper proceeds as follows. Section 2 describes the model. Section 3 describes how the parameterize the model and describes our baseline results. Section 4 conducts a welfare analysis and Section 5 contains our concluding remarks.

2 The model

2.1 Baseline model

We consider an endowment economy with two periods. The economy consists of two actors, a continuum of individuals who live for two periods and a monopolist insurer of private LTC insurance who issues policies in period 2.

2.1.1 Individual’s problem

At birth each individual observes its health status and endowment of the consumption good, \( w_y \), then chooses consumption \( c_y \) and savings \( a \). In period 2 the individual receives endowment \( w_o \) and discovers its true risk exposure \( \theta = \{\theta_g, \theta_b\} \). It then makes a decision about whether to purchase private LTC insurance. If so, it pays premium \( \pi \) and receives indemnity \( \iota \) if the LTC event occurs. Individuals also have access means-tested to social LTC insurance (Medicaid). Medicaid is a secondary insurer that guarantees a consumption floor of \( c \) to
those who experience a nursing home shock and have low wealth and low levels of private insurance.

After purchasing LTC insurance, individuals realize a preference shock \( \alpha(\kappa) \) and consume the fraction \( \kappa \) of their young endowment. We assume that \( \alpha(\kappa) \in [0, 1] \) with \( \alpha(0) = 1 \) and \( \alpha(1) = 0 \) where \( \kappa \in [0, 1] \) has density \( f(k) \). The preference shock is designed to capture in a parsimonious way the fact that the time when the nursing home shock occurs is random and that some individuals will live a long time have low wealth when they experience the nursing home event. Finally, the nursing home event is realized and individuals discover whether they have nursing home expenses given by \( m \).

Define \( w \equiv [w_y, w_o] \) and let \( \Pi \) denote profits of the firm, then we can express dividends from ownership in the firm as \( d(w, h)\Pi \). Allowing dividends to depend on endowments and health status allows us to capture the fact that share holdings tend to be concentrated among individuals with higher wealth and education. Given these definitions we can write the households decision problem as

\[
U(\{\pi^i_{h,w}(\cdot), \iota^i_{h,w}(\cdot)\}_{i \in \{g,b\}}, w, h) = \max_{a \geq 0} u(c_y) + \beta \left( \sum_\kappa \alpha(\kappa) [\psi_{h,w}[\theta_g u(c_{NH}^{h,w,k,g}) + (1 - \theta_g)u(c_{a}^{h,w,k,g})] + (1 - \psi_{h,w})[\theta_b u(c_{NH}^{h,w,k,b}) + (1 - \theta_b)u(c_{a}^{h,w,k,b})] f(\kappa) + \sum_\kappa (1 - \alpha(\kappa))u(\kappa w_y)f(k) \right)
\]

subject to

\[
c_y + \kappa w_y = w_y(1 - \tau) - a \tag{2}
\]

\[
c_{NH} + \kappa w_y = (w_o + ra)(1 - \tau) + a + TR(a, \pi, \iota) - \pi - m + \iota + (1 - \tau)d(w, h)\Pi \tag{3}
\]

\[
c_o = (w_o + ra)(1 - \tau) + a - \pi + (1 - \tau)d(w, h)\Pi \tag{4}
\]

where \( m \) are nursing home expenditures, \( \tau \) is a tax on the endowment that is used to finance Medicaid. Medicaid transfers are means-tested

\[
TR(a, \pi, \iota) = \max\{0, \underline{c} - [(w_o + ra)(1 - \tau) + a - \pi - m + \iota + (1 - \tau)d(w, h)\Pi]\}. \tag{6}
\]

In what follows it will be helpful to define

\[
u_2(\theta, a, \pi, \iota) \equiv \theta u(c_{NH}) + (1 - \theta)u(c_o). \tag{8}
\]

2.1.2 Insurer’s problem

The insurer sees endowments \( w \), health \( h \) and assets \( a \). However, the insurer does not observe the true NH entry probability \( \theta_i, i \in \{g,b\} \). It also does not recognize that asset holdings depend on \( w, h \) and \( a \) via household optimization. We believe that this is realistic because individuals purchase private LTC insurance relatively late in life.\(^2\) The monopolist insurer

\(^2\)Although we do not model the optimal age of when to purchase LTCI here, evidence from the LTCI industry suggests that concerns about adverse selection among younger individuals are so strong, that actuaries discourage insurers from offering policies younger individuals Add citation.
creates a menu of contracts \((\pi^i_{h,w}(a), \iota^i_{h,w}(a)), i \in \{g, b\}\) to maximize expected revenues for each observable type. The insurer faces two costs: a load on contracts, \(\lambda\) that is proportional to the total payout and a fixed cost, \(k\) of processing claims. Note also that \(f(h, w, a)\) is the distribution of agents with health status \(h\), endowment \(w\) and asset holdings \(a\) and that \(\psi_{h,w}\) is the probability of \(\theta^g\) for given \(h\) and \(w\).

\[
\Pi = \max_{\{\pi^i_{h,w}(a), \iota^i_{h,w}(a)\}_{i \in \{g, b\}}} \sum_w \sum_h \sum_a \left[ \psi_{h,w}[\pi^g_{h,w}(a) - \theta^g(\lambda \iota^g_{h,w}(a) + kI(\iota^g_{h,w} > 0))] + (1 - \psi_{h,w})[\pi^b_{h,w}(a) - \theta^b(\lambda \iota^b_{s,w}(a) + kI(\iota^b_{h,w} > 0))] \right] f(h, w, a) \tag{9}
\]

subject to

\[
(IC_i) \quad u_2(\theta_i, a, \pi^i_{h,w}(a), \iota^i_{h,w}(a)) \geq u_2(\theta_i, a, \pi^j_{h,w}(a), \iota^j_{h,w}(a)) \quad \forall h, w, a \quad i, j \in \{g, b\}, i \neq j \tag{10}
\]

\[
(PC_i) \quad u_2(\theta_i, a, \pi^i_{h,w}(a), \iota^i_{h,w}(a)) \geq u_2(\theta_i, a, 0, 0) \quad \forall h, w, a \quad i \in \{g, b\}. \tag{11}
\]

### 2.1.3 Solution algorithms

With MTSI for given \(w, h\), guess \(a\), then solve the monopolists problem for

1. Case 1. Solve under the assumption that the household receives no MTSI when choosing the optimal contract. But, when checking the participation constraint assume that they can receive MTSI.

2. Case 2. Solve under the assumption that the household is on MTSI for both realizations of \(\theta\).

3. Case 3. Solve under the assumption that the household is on MTSI only when \(\theta = \theta^b\).

4. Case 4. Solve under the assumption that the household is on MTSI only when \(\theta = \theta^g\).

For each case after solving the monopolists problem, go to the household problem and find the optimal \(a\). Finally, compare utility under the two cases to find the optimal solution. It is possible that these two cases do not occur in equilibrium.

When no MTSI is available for given \(w, s\), guess \(a\), then solve the monopolists problem. Use these contracts to compute the maximum utility of the household via golden section.

### 2.2 Baseline market structure with no private information

In order to isolate the role of private information and adverse selection, we also report results for the same model with no private information. The individual’s problem does not change. But, the insurer now observes each individual’s realization of \(\theta\) at the start of the second period and no longer faces the incentive compatibility constraint.
\[ \Pi = \max_{(\pi_{h,w}^i(a), \iota_{h,w}^i(a))_{i \in \{g,b\}}} \sum_h \sum_a \sum_w [\psi_{h,w}[\pi_{h,w}^g(a) - \theta_g^i(\lambda_{h,w}^g(a) - k)] + (1 - \psi_{h,w})[\pi_{h,w}^b(a) - \theta_b^i(\lambda_{h,w}^b(a) - k)]] f(h, w, a) \]

subject to

\[ (PC_i) \quad u_2(\theta_i, a, \pi_{h,w}^i(a), \iota_{h,w}^i(a)) \geq u_2(\theta_i, a, 0, 0) \quad \forall h, w, a, \quad i \in \{g, b\}. \]

### 2.3 Efficient Allocations

The focus of our analysis is on insurance of long-term care risk and one way to assess the effectiveness of long-term care insurance is according to the metric of how well these risks are mutualized within a group. However, under pooling there is cross subsidization across groups with different risk exposures. It follows that these insurance arrangements may also act to equalize consumption across groups. Medicaid, as discussed in Braun et al. (2015), provides valuable insurance against LTC risk and also helps to insure against the risk of a low endowment. It thus makes sense to consider two different planner problems. The social planner’s problem delivers the Pareto efficient allocations. The ex post planner’s problem, delivers the best allocations that can be achieved after agents learn about their type and type is public information. We discuss each problem in turn.

#### 2.3.1 The Social Planner’s Problem

The Pareto efficient allocations for our economy are found taking the perspective of a social planner who operates under a veil of ignorance. All agents in our economy have the same utility function and are thus ex ante identical. It follows that all agents will have identical consumption in the set of (equal treatment) Pareto efficient allocations. In other words, individuals’ consumption in period one and two will be independent of their endowment, their health status and whether or not they experience a LTC event in their second period of life. The social planner’s problem delivers the Pareto efficient allocations. The ex post planner’s problem, delivers the best allocations that can be achieved after agents learn about their type and type is public information. We discuss each problem in turn.
where \( f(w) = \sum_h \sum_a f(h, w, a) \) is the marginal density of endowments.

### 2.3.2 Ex post Planners problems

In all of the market arrangements we consider markets open after individuals learn about \( w \) and \( h \) it is assumed that these realizations are public information. It is known since Arrow that the allocations under this assumption are generally speaking are not Pareto Efficient. Still, these allocations are also a useful benchmark for understanding what markets can achieve under these informational assumptions. We find them by solving a separate social planners problem for each combination of \( w, h \):

\[
SP^{w,h} = \max_{c^{w,h}, c^{w,0}} u(c^{w,h}) + \beta [\psi_{h,w} [\theta_g u(c^{w,NH}) + (1 - \theta_g)u(c^{w,h})] \\
+ (1 - \psi_{h,w})[\theta_b u(c^{w,h}) + (1 - \theta_b)u(c^{w,h})]]
\]

subject to

\[
c^{w,h}_y + \left[\psi_{h,w} (m + c^{w,0}_N) + (1 - \theta_g) c^{w,h}_0 \right] + (1 - \psi_{h,w}) \left[\theta_b (m + c^{w,0}_N) + (1 - \theta_b) c^{w,h}_0 \right] / (1 + r) = \left[ w \begin{array}{c} 1 \\ 1/(1 + r) \end{array} \right] f(w, h)
\]

Given these allocations average welfare for the entire economy is given by

\[
SP^{expost} = \sum_w \sum_h SP^{w,h} f(w, h)
\]

### 3 Parameterization and results

#### 3.1 Calibration

- Joint distribution of income and health status (young).

<table>
<thead>
<tr>
<th>Group</th>
<th>Fraction of Total Wealth</th>
</tr>
</thead>
<tbody>
<tr>
<td>Top 1%</td>
<td>34.6%</td>
</tr>
<tr>
<td>96-98%</td>
<td>27.3%</td>
</tr>
<tr>
<td>91-95%</td>
<td>11.6%</td>
</tr>
<tr>
<td>81-90%</td>
<td>12%</td>
</tr>
<tr>
<td>61-80%</td>
<td>10.9%</td>
</tr>
<tr>
<td>41-60%</td>
<td>4%</td>
</tr>
<tr>
<td>0-40%</td>
<td>0.2%</td>
</tr>
</tbody>
</table>
2. Health status by wealth TBA.

- Probability of a nursing home event when old. Hurd, Michaud and Rohwedder (2014) find that the probability ranges from 0.56-0.577 for all individuals aged 56+. Life-time Long-term stay probabilities are about 0.27 (stay of longer than 100 days.) Women higher 0.375 than males 0.211.

- At a 0% discount rate Hurd et al. (2014) estimate that expected lifetime OOP NH costs as follows P50 $0, P90, $18,039, P95 $43,421 at age 57 with a 3% discount rate. At a zero discount rate they rise to P90 $36,900 and P95 $87,974. Women have more nights (360) than males (186).

- Medicaid consumption floor. We estimate recipiency rates to be 13%. Asset tests are 14% of average earnings. Income thresholds range from 33-43% of average earnings.

- Load on private insurance. Brown and Finkelstein (2007) estimate that the gap between actuarially fair and the actual cost to individuals is 0.18. However, there are a large number of policy forfeitures. Recognizing these increases the load to individuals to 0.51. We can derive numbers like this from our solution.

Assessment

- Individual loads.
- Private insurance rejection rates.
- Cost of private insurance.
- Extent of private insurance.
- Fraction of individuals insured by income group.

References


