Endogenous Growth, Firm Heterogeneity and the Long-run Impact of Financial Crises

Tom Schmitz
Università Bocconi*

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Abstract

I propose a new endogenous growth model with heterogeneous firms and aggregate shocks. The model shows that firm heterogeneity generates several new amplification and persistence mechanisms for a transitory shock to financing conditions. This shock imposes financing constraints, which force small and young innovating firms (with low retained earnings) to reduce their R&D, and therefore leads to R&D misallocation. Furthermore, it lowers entry and persistently reduces the mass of innovating firms. Thus, even as financing constraints disappear, aggregate R&D and innovation remain persistently depressed, as the remaining large firms can only imperfectly substitute for the R&D of the missing generation of young and small ones. Finally, lower R&D during and after the shock also limits the scope for incremental follow-up innovations. My model’s main features are in line with developments in the Spanish manufacturing sector during the 2008-2013 economic and financial crisis.

Keywords: R&D, Innovation, Heterogeneous Firms, Size Distribution, Endogenous Persistence

JEL Codes: E32, O31, O33

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1 Introduction

The slow recovery of developed countries from the 2008-2009 Great Recession has revived the discussion on the long-run consequences of economic and financial crises (Ball (2014), Hall (2014)). In particular, some observers have drawn attention to the fact that such crises typically decrease private Research and Development investment (R&D). As R&D is an important driver of aggregate productivity growth, this decrease may be responsible for a permanent output and productivity loss.

Macroeconomists have argued that the decrease in R&D during crises can be explained by lower private benefits from introducing new products or processes (Comin and Gertler (2006), Barlevy (2007)) or by tighter financial constraints (Stiglitz (1993), Aghion et al. (2010)), and may indeed have permanent effects.\(^1\) However, their research on R&D fluctuations has mainly relied on aggregate, representative-firm models. In this paper, I argue that this approach overlooks relevant heterogeneities at the firm level.

In particular, I argue that an economic and financial crisis triggers important changes in the population of R&D-performing firms. This claim can be illustrated by considering the example of Spain, which suffered a severe economic and financial crisis between 2008 and 2013.\(^2\)

Figure 1: R&D dynamics in Spanish manufacturing, 2004-2014

![R&D dynamics in Spanish manufacturing, 2004-2014](image)

Figure 1 plots several R&D statistics for the Spanish manufacturing sector, using data from the Spanish Statistical Institute’s (INE) innovation survey (Encuesta sobre Innovación en las empresas). The left panel shows that manufacturing R&D fell moderately during the crisis: on average, real R&D was 3.2% lower

\(^1\)Crises may also have some positive effects on R&D. Schumpeter famously argued that they create the conditions for new innovation waves, by lowering factor prices and creating a stock of idle resources (Schumpeter (1934)). Aghion and Saint-Paul (1993), Matsuyama (1999) and Francois and Lloyd-Ellis (2003) formalised some of these ideas, emphasizing that crises lower the opportunity cost of reallocating resources from production to R&D. The procyclicality of R&D observed in the data (documented by, among others, Wilde and Woitek (2004), Barlevy (2007) and Ouyang (2011)) suggests that these countercyclical forces are generally weaker than the procyclical ones described in the main text.

\(^2\)The crisis years, marked by shaded grey areas in Figure 1, were interrupted by a short-lived recovery in 2010. Crisis dates are taken from the Spanish Business Cycle Dating Committee (http://asesec.org/CFCweb/cfs_index_e.htm).
during the crisis years 2008-2013 than in the pre-crisis year 2007. However, the right panel shows that the number of firms responsible for this R&D effort fell much more dramatically. In 2013, there were 36% less firms reporting some R&D activities than in 2007. As a result of this evolution, the overall R&D effort was shared among ever fewer firms during the crisis, and the size distribution of R&D-performing firms shifted towards the right. Indeed, Table 1 shows that the average R&D-performing firm has higher sales in 2013 than in 2007, and that the share of firms with at least 250 employees in the total population of R&D-performing firms and in aggregate R&D has substantially increased. These shifts are remarkable, as they occur against the backdrop of falling employment and sales for most firms: if all firms had behaved in the same way (as it is implicitly assumed in a representative-firm model), average sales and the shares of firms with at least 250 employees in all aggregates would have fallen. This evolution is consistent with the widespread conception that small (and young) firms suffer more from economic and financial crises (Gertler and Gilchrist (1994), Fort et al. (2013)). Using firm-level data, I show in Section 2.1 that this has indeed been the case in the Spanish manufacturing sector. However, do these firm-level developments matter for aggregate R&D and innovation?

The fall in the number of R&D-performing firms and the concentration of R&D in large firms could in principle be irrelevant for aggregate outcomes: if R&D in small firms and R&D in large firms were perfect substitutes, the latter could just substitute for the missing R&D of the former. However, empirical evidence suggests that small and large firms’ R&D is actually very different. Researchers have shown that small innovating firms generate more innovations per dollar of R&D than large ones (Cohen and Klepper (1996)), have higher ratios of R&D to sales and patents to employees, and that their patents are on average more cited and more

Table 1: Composition changes during the crisis

<table>
<thead>
<tr>
<th></th>
<th>2007</th>
<th>2013</th>
</tr>
</thead>
<tbody>
<tr>
<td>Average sales of an R&amp;D-performing firm</td>
<td>67.1m</td>
<td>81.1m</td>
</tr>
<tr>
<td>Share of firms with at least 250 employees among R&amp;D-performing firms</td>
<td>10.2%</td>
<td>12.6%</td>
</tr>
<tr>
<td>Share of firms with at least 250 employees in manufacturing R&amp;D</td>
<td>63.4%</td>
<td>70.5%</td>
</tr>
</tbody>
</table>

Source: INE and calculations of the author. m stands for millions of euros. See Appendix C for further details.

3 I focus on manufacturing because of better and more consistent coverage, and because the firm-level data used in later sections covers only manufacturing. Appendix C provides more details on the INE survey.
4 The INE surveys firms with at least 10 employees. Thus, one may be worried that this trend is due to firms reducing employment during the crisis and thereby falling out of the sampling frame, without stopping to do R&D. This concern is mitigated by the fact that if a firm has once declared to do R&D, it is kept in the survey even if it falls to less than 10 employees (INE (2014)). Moreover, the number of firms with at least 10 employees which do not perform R&D fell less (by 31% between 2007 and 2013) than the number of R&D-performing firms, even though the latter are on average larger (and thereby less likely to fall below 10 employees). Finally, note that as aggregate manufacturing R&D is also calculated for the population of firms with at least 10 employees, the claim that it is performed by less firms in 2013 than in 2007 is true in any case.
5 The population of technologically innovating firms (that is, firms which declare to have done some product or process innovations) evolves along the same lines. Their absolute number fell by 53% between 2007 and 2013, while the share of firms with at least 250 employees increased from 5.8% to 8.3%.
likely to represent major breakthroughs\textsuperscript{6} (Akçiğit and Kerr (2010, 2015)).\textsuperscript{7} I show in Section 2.2 that most of these stylized facts (which have been documented mainly with data from the United States) also hold in the Spanish manufacturing sector. This suggests that the compositional changes during the 2008-2013 crisis do have aggregate consequences.

The main contribution of my paper is to formalize this intuition by writing a new partial equilibrium endogenous growth model with heterogeneous firms and aggregate shocks. The model builds on the pioneering contribution of Klette and Kortum (2004), who developed the first endogenous growth model with heterogeneous firms, and on Akçiğit and Kerr (2015), who extended their model to introduce different types of innovations and differences in R&D intensity by firm size. However, my model also introduces aggregate shocks, while these papers only consider a balanced growth path.

The model considers an industry in which innovating and non-innovating firms of different sizes produce a fixed set of goods. Innovating firms may generate two types of innovations in every period. Radical innovations improve the firm’s productivity for a good it previously did not produce, and therefore lead to creative destruction, as the innovator overtakes the improved good from its previous non-innovating producer. They are generated by radical R&D, which has decreasing returns to scale and is subject to a negative externality: a firm’s radical R&D cost is increasing in the aggregate radical innovation level of the industry. Incremental innovations, in turn, increase the firm’s productivity for goods which it already produces, and are generated by a linear incremental R&D technology. In normal times, these assumptions imply that small firms do relatively more R&D with respect to their size, and that their innovations are on average more radical.

Firms take their R&D decisions in an environment that is subject to two exogenous aggregate shocks, affecting aggregate demand (the spending on all goods of the industry) and financing conditions (firms’ ability to borrow). My model highlights a number of novel mechanisms, relying on firm heterogeneity, which amplify the reaction of aggregate R&D and innovation to these shocks and make it persistent over time.

My main results are obtained for a shock to financing conditions. This shock hits firms asymmetrically. Small and young firms, with low retained earnings, are constrained and need to cut radical R&D. Large firms, which can self-finance, react to this by increasing their radical R&D. Therefore, they dampen the fall of aggregate R&D on impact. However, the marginal product of their additional R&D is low (while the marginal product of radical R&D at constrained firms is high), so radical R&D is misallocated and the shock is amplified: aggregate R&D during the shock period would produce more radical innovations if it were equally allocated across all innovating firms (as in a representative-firm model).

\textsuperscript{6}Importantly, these findings hold for the population of innovating firms. The numerous [often small] firms which never innovate or do R&D are in general disregarded by the literature on productivity dynamics (see, e.g., Akçiğit and Kerr (2015)).

\textsuperscript{7}Furthermore, innovative activity is negatively correlated with industry concentration (Acs and Audretsch (1990)), and patenting rates decline with firm age (Graham et al. (2015)). Kortum and Lerner (2000), Ewens and Fons-Rosen (2013) and Seru (2014) also find a disproportionate importance of small and young firms for innovation.
Most importantly, a shock to financing conditions also prevents the entry of new innovating firms. Therefore, the mass of innovating firms falls persistently, and even as financial conditions are normal again, the radical R&D effort is done by fewer and on average larger firms. This compositional change depresses radical innovation, as decreasing returns prevent large firms from fully compensating the radical R&D effort lost by the absence of small ones. Thus, productivity growth is persistently depressed. This persistent depression is further strengthened by the fact that the fall of radical innovation during and after the shock lowers the mass of goods which can be incrementally improved.

The effects of an aggregate demand shock are more limited: it lowers R&D and reduces the scope for incremental innovation, but does not trigger misallocation or a fall in the mass of innovating firms.

My model qualitatively matches the aggregate behaviour of the Spanish manufacturing sector during the 2008-2013 crisis. Furthermore, a simple quantification suggests that the fall in the number of R&D-performing firms alone may permanently lower manufacturing output by between 0.13 and 1.54 percentage points. This is a modest, but significant effect, which would have been missed by a representative-firm model.

My paper combines insights from the literature on aggregate R&D fluctuations and from the literature on heterogeneous-firm endogenous growth models. Moreover, it is closely related to a recent line of research studying the role of firm heterogeneity for the long-run impact of shocks. For instance, Sedláček and Sterk (2016), Ateş and Saffie (2014) and Bergin et al. (2014) study changes in entry composition during a crisis. Sedláček and Sterk argue that during crises, entrants choose different production technologies and therefore remain persistently smaller than entrants in normal times. In contrast, Ateş and Saffie and Bergin et al. claim that crises induce a selection of entrants which dampens their impact: entry rates fall, but the entrants who make it are on average better at doing R&D or more financially solid. Finally, Garcia-Macià (2015) studies the effect of financial shocks on intangible investment in a heterogeneous-firm model. His model, however, does not feature endogenous growth, and the persistence mechanisms it highlights are not related to the ones stressed in my paper.

The remainder of this paper is structured as follows. Section 2 provides empirical evidence for permanent and cyclical differences between small and large innovating firms in the Spanish manufacturing sector. Section 3 describes my model’s assumptions and Section 4 derives and discusses its main predictions. Section 5 confronts the model with the previously discussed empirical evidence, and Section 6 concludes.

8 Besides the research cited already, two recent contributions are Bianchi and Kung (2014) and Queraltó (2015).
9 Important contributions to this literature beyond the already cited ones include Lenz and Mortensen (2008), who introduce differences in firms’ innovation capacity into the Klette and Kortum model, and Acemoğlu et al. (2013), who analyse different possibilities to improve the resource allocation between firms with different innovation capacities. Acemoğlu and Cao (2015) propose a model in which entrants disproportionately contribute to productivity growth.
10 In García-Macià’s model, a financial shock has a persistent impact on output for two reasons. First, adjustment costs prevent the quick rebuilding of intangible capital lost through firm exit. Second, lower intangible capital at some firms spills over to all others by lowering their productivity, and therefore their incentives for intangible investment. Note that my model has exactly the opposite feature, as lower radical innovation of some firms increases the incentives of others to do radical R&D.
2 Firm-level stylized facts

This section provides some further motivating evidence. I show first that in the Spanish manufacturing sector, small firms indeed reduced their R&D significantly more than large ones during the 2008-2013 crisis, corroborating the evidence provided in the introduction. Then, I show that this is potentially important, as there are important permanent differences between small and large innovating firms, in line with those found by existing research for the United States.

All firm-level data in this section come from the Encuesta Sobre Estrategias Empresariales (ESEE), a annual survey of the manufacturing sector carried out by the SEPI Foundation (depending on the Spanish Ministry of Finance and Public Administration) and containing several questions on R&D and innovation. The survey was set up in 1990 and initially sent to a representative sample of firms between 10 and 200 employees and to all firms with more than 200 employees. Since then, it has been periodically updated to introduce new firms and maintain representativeness. In the period 1994-2013, which I consider in this section, around 1800 firms per year answered the survey. Appendix C describes the dataset in greater detail.

2.1 R&D in small and large firms during the crisis

The large fall of the number of R&D-performing firms and the rightward shift in their size distribution suggest that small firms reduced R&D more than large ones during the 2008-2013 crisis. The ESEE data provides further evidence for this claim. Following the methodology proposed by Haltiwanger et al. (2013) to analyse the relationship between firm growth and firm size, I estimate for every year \( t \) the regression

\[
g_{i,t}^{R&D} = \alpha + \beta_t \ln \left( \frac{\text{Employment}_{i,t-1} + \text{Employment}_{i,t}}{2} \right) + \varepsilon_{i,t}. \tag{1}
\]

The dependent variable is the growth rate of R&D for a firm \( i \) between years \( t - 1 \) and \( t \), defined as

\[
g_{i,t}^{R&D} = \frac{R&D_{i,t} - R&D_{i,t-1}}{\frac{1}{2} (R&D_{i,t-1} + R&D_{i,t-1})}.
\]

This definition is widely used in the firm dynamics literature, as it delivers a growth rate bounded between \(-2\) and \(2\), and well defined for movements from or to 0. R&D growth is regressed on a measure of firm size, the average employment of firm \( i \) between years \( t - 1 \) and \( t \).

Figure 2 plots the point estimates for the coefficient \( \beta_t \), together with the 95% confidence intervals, for the years 2004-2013. In most years, there is no statistically significant difference between the R&D growth rate of small and large firms. However, during the deepest years of the crisis, 2009, 2011 and 2012, smaller firms have a significantly lower R&D growth rate than large ones. These results are generally robust to controlling

\[11\] I use the ESEE rather than the INE Encuesta sobre Innovación en las empresas for two practical reasons: the ESEE is a panel dataset (allowing me to calculate firm-level rates of change or averages) and its micro data is more easily accessible.

\[12\] Using average employment avoids a bias arising from regression to the mean. In particular, using instead employment in year \( t - 1 \) yields a spurious negative estimate for \( \beta_t \) if the firm’s R&D and employment follow correlated mean-reverting processes.
for the growth rate of firm sales (that is, for differences in firm-level shocks) and industry fixed effects.\textsuperscript{13}

\begin{figure}[h]
\centering
\includegraphics[width=0.5\textwidth]{figure2.png}
\caption{Estimates for $\beta_t$}
\end{figure}

**Notes:** Full regression tables and further details are provided in Appendix C.

These firm-level changes appear to have sizeable implications at the industry level, too: as shown in Figure 3, the industries which had the largest shares of small R&D performing firms in 2008 also saw their aggregate R&D fall most during the 2008-2013 crisis. This relationship is statistically significant at the 1\% level, and remains significant at the 2\% level when I control for the change in sales (that is, for industry-level shocks) between 2008 and 2013.\textsuperscript{14}

\begin{figure}[h]
\centering
\includegraphics[width=0.5\textwidth]{figure3.png}
\caption{Industry-level evidence}
\end{figure}

**Source:** INE and calculations of the author.

This evidence is in line with other empirical studies showing that small firms are particularly sensitive to

\textsuperscript{13}Results for these regressions are provided in Appendix C.3. They are virtually identical to the ones shown in Figure 2, except that the coefficient for the year 2009 becomes only marginally significant.

\textsuperscript{14}This industry-level data comes from the INE’s Encuesta sobre Innovación en las empresas, which has a wider coverage than the ESEE. I use 2008 as the base year because of a change in industry classifications between 2007 and 2008. Figure 3 uses data for 20 industries, listed in Appendix C. Regression tables are available upon request.
aggregate shocks. In the next section, I show that there are also permanent differences between innovation and R&D in small and large firms, suggesting that their differential crisis reaction has aggregate implications.

2.2 Permanent differences between innovation and R&D in small and large firms

To assess permanent differences in small and large firms’ behaviour, I consider three widely used innovation statistics: the ratio of R&D to sales (measuring innovation input relative to firm size), the ratio of granted patents to employment (measuring innovation output relative to firm size) and the ratio of granted patents to last year’s R&D (measuring innovation productivity). Following Akçigit and Kerr (2015), I aggregate the data into five-year periods, to account for the lumpy nature of R&D and innovation in many small firms. I then analyse the relationship between an innovation statistic \( x \) and firm size (measured by employment) by estimating

\[
x_{i,t'} = \alpha_{t'} + \alpha_k + \beta \ln \left( \text{Employment}_{i,t'} \right) + \varepsilon_{i,t'},
\]

where \( t' \) denotes a five-year period. \( \alpha_{t'} \) and \( \alpha_k \) are period and industry fixed effects.\(^{17}\) The values of the dependent variable \( x \) and of employment for a five-year period \( t' \) are calculated as simple averages of these variables over the period. Finally, the highest values of \( x \) are winsorized at the 2.5% level.

Table 2: Innovation performance and firm size

<table>
<thead>
<tr>
<th>Dependent variable</th>
<th>R&amp;D Sales (a)</th>
<th>R&amp;D Sales (b)</th>
<th>Patents Employment (a)</th>
<th>Patents Employment (b)</th>
<th>Patents R&amp;D(_{-1}) (a)</th>
<th>Patents R&amp;D(_{-1}) (b)</th>
</tr>
</thead>
<tbody>
<tr>
<td>ln (Employment)</td>
<td>-0.0008***</td>
<td>0.0000</td>
<td>-0.0051***</td>
<td>-0.0036***</td>
<td>-0.0054***</td>
<td>-0.0039***</td>
</tr>
<tr>
<td></td>
<td>(0.0003)</td>
<td>(0.0002)</td>
<td>(0.0003)</td>
<td>(0.0003)</td>
<td>(0.0001)</td>
<td>(0.0007)</td>
</tr>
<tr>
<td>Observations</td>
<td>2069</td>
<td>2780</td>
<td>398</td>
<td>675</td>
<td>290</td>
<td>480</td>
</tr>
<tr>
<td>( R^2 )</td>
<td>0.21</td>
<td>0.19</td>
<td>0.54</td>
<td>0.30</td>
<td>0.43</td>
<td>0.29</td>
</tr>
<tr>
<td>Industry Fixed Effects</td>
<td>YES</td>
<td>YES</td>
<td>YES</td>
<td>YES</td>
<td>YES</td>
<td>YES</td>
</tr>
<tr>
<td>Time Fixed Effects</td>
<td>YES</td>
<td>YES</td>
<td>YES</td>
<td>YES</td>
<td>YES</td>
<td>YES</td>
</tr>
</tbody>
</table>

Notes: *** significant at 1%, ** significant at 5%, * significant at 10%. Robust standard errors are given in parentheses.

\(^{15}\)This claim dates back to Gertler and Gilchrist (1994). Recently, Kraeger and Charnes (2011) and Siemer (2013) documented that small firms’ employment suffered disproportionately during the Great Recession in the United States, while Moscarini and Postel-Vinay (2012) defended a contrarian viewpoint. Fort et al. (2013) emphasize the role of firm age, arguing that firms which are both small and young are most sensitive to economic downturns (and particularly to the Great Recession).

This evidence focuses on employment, while there is comparatively little evidence on R&D. Paunov (2012) shows that young Latin American firms were more likely than old ones to abandon innovation investment during the Great Recession, but does not find an independent role for size. Aghion et al. (2012) show that R&D reacts more to negative sales shocks in financially constrained firms. However, even though most measures suggest that constrained firms are on average smaller than unconstrained ones (Farre-Mensa and Ijungępia (2013)), Aghion et al.’s proxy for financial constraints is uncorrelated with size.

\(^{16}\)The average time lag between R&D and the grant of a patent is difficult to measure. Therefore, I have also used the ratio of granted patents to current R&D, and to R&D two or three years earlier. In all cases, results are unchanged.

\(^{17}\)There are four time periods (1994-1998, 1999-2003, 2004-2008 and 2009-2013) and 19 industries (listed in Appendix C).
I estimate this regression on a sample of innovating firms.\textsuperscript{18} Column (a) of Table 2 reports the results for continuously innovating firms (with at least one positive observation for R&D (respectively patents) in every five-year period in which they are observed). Column (b) considers a broader sample including occasionally innovating firms (with at least 20\% of positive observations for R&D (respectively patents)).\textsuperscript{19} In both samples, the estimated coefficient on firm size is in general negative and significant: small innovating firms spend relatively more on R&D, generate relatively more patents and are more productive in their R&D than large ones.\textsuperscript{20} Furthermore, results remain unchanged when I consider the crisis period 2009-2013 and the remainder of the sample separately, suggesting that the differences between small and large firms are permanent rather than cyclical (see Appendix C).

These differences imply that a shift such as the one triggered by the Spanish 2008-2013 crisis, leaving a smaller number of larger firms in charge of aggregate R&D, can have important consequences. In the next section, I develop a model which systematically analyses those.

3 A model of R&D with heterogeneous firms and aggregate shocks

3.1 Assumptions

I write a partial equilibrium model of a small industry in discrete time ($t \in \mathbb{Z}$). The industry’s firms produce a fixed set of differentiated goods, indexed on the interval $[0, 1]$. Some firms also invest into R&D to improve their productivity. Firms’ decisions are affected by exogenous aggregate shocks to demand and financing conditions, which are described in the next section.

3.1.1 Aggregate conditions and aggregate shocks

Aggregate demand Aggregate demand in a given period $t$ is defined as the total spending on all goods of the industry in that period. It is given by an i.i.d. stochastic process $(S_t)_{t \in \mathbb{N}}$ which can take two values, $S_H$ and $S_L$ (with $S_H > S_L$). For simplicity, I assume firms can forecast demand one period ahead.

A representative consumer allocates spending across the industry’s goods. The representative consumer takes goods prices and total spending as given and maximises her utility, given by a Cobb-Douglas aggregator:

$$C_t = \exp \left( \int_0^1 \ln c_t(j) \, dj \right),$$

\textsuperscript{18}A large percentage of firms in the dataset never does R&D, and an even larger one never patents (see Table 5 in Appendix C). As I argued before (see Footnote 6), it would be misleading to include these firms in this estimation.

\textsuperscript{19}This definition includes continuously innovating firms, but is broader, as the positive observations need not be spread over all five-year periods. Results are unchanged when lowering the threshold to 15\%.

\textsuperscript{20}The ESEE also contains (noisy) information on firm age, and I find that the three innovation statistics are also negatively correlated with age. Disentangling the separate roles of size and age for firm dynamics is an important issue. However, as size and age of innovating firms are strongly correlated in my model, I do not pursue it further here: the channels I describe are valid irrespective of whether size or age is the key driver of the stylized facts documented in this section.
where $c_t(j)$ denotes the quantity of good $j$ consumed in period $t$.

**Financing conditions**  Financing conditions fluctuate between a normal and a crisis state. In the normal state, firms can borrow as much as they want at a fixed exogenous interest rate (set to 0 for simplicity), provided they use the funds for a project with a positive expected net present value (NPV). In the crisis state, firms can only borrow for projects which deliver a positive NPV with certainty.

Transitions to the crisis state are surprise events (that is, firms expect to always be in the normal state).

This defines firms’ operating environment. I now turn to their life cycle and their production technology.

### 3.1.2 Firms

Production is carried out by two types of firms, innovating and non-innovating ones. A firm $i$ is characterized by its type (innovating or non-innovating) and by a function $a_t(i, \cdot)$, associating to every good of the industry a productivity with which the firm can produce that good in period $t$. The output of firm $i$ for a given good $j$ is given by the simple linear production function

$$y_t(i,j) = a_t(i,j)l_t(i,j),$$

where $l_t(i,j)$ stands for the labour employed by firm $i$ for the production of good $j$. Labour is the only factor of production, and there is an infinitely elastic labour supply at the constant exogenous wage $w$.\textsuperscript{21}

Firms compete à la Bertrand on the market for each differentiated good. This implies that in equilibrium, every good is produced only by the firm with the highest productivity. I denote the highest productivity for good $j$ in period $t$ by $a_t(j) \equiv \max_i a_t(i,j)$.

Firms maximize the expected net present value of profits earned over their existence. In order to make positive profits, they need to have a higher productivity than all other firms for at least some goods, in order to produce in equilibrium (and to sell at a positive mark-up over marginal cost). Firms can achieve a higher productivity than their competitors by generating innovations through successful R&D.

Only innovating firms can do R&D, and whether a firm is innovating depends on the stage of its life cycle. In every period $t$, a fixed mass $M^{\text{New}}$ of new firms appear. When these potential entrants pay an entry cost of $\varphi$ units of labour, they become innovating firms which do not produce (as they do not have the highest productivity level for any good), but which can start to invest in radical R&D (described in greater detail in the next section) in order to get the highest productivity for some good and thereby grow. An innovating firm remains able to do R&D and to grow until it is hit by an exogenous negative innovation capacity shock.

\textsuperscript{21}I do not analyse wage fluctuations, as most evidence suggests that nominal wages are acyclical (Gali (2008)).
These shocks are i.i.d. distributed across innovating firms, and hit any firm with probability \( \delta \) in any period \( t \). They turn the firm into a non-innovating firm forever: once it is hit by the shock, it cannot invest into R&D any more and starts to shrink, as the goods for which it had the highest productivity are gradually overtaken by innovating firms. Eventually, the firm loses all its goods and exits.

The next section completes the model’s assumptions by describing the R&D technologies of innovating firms.

### 3.1.3 R&D technologies

Innovating firms can improve their productivity through radical and incremental R&D.\(^\text{22}\)

#### Radical R&D

In every period \( t \), an innovating firm can generate one radical innovation with probability \( r_t \) by employing \( c_R (r_t, R_t) \) units of labour for radical R&D. \( R_t \) denotes the total mass of radical innovations generated in period \( t \), and is taken as exogenous by every individual firm. If the radical innovation is realized, it enables the firm to produce some good \( j \), randomly drawn from the set of goods produced by non-innovating firms in period \( t \),\(^\text{23}\) with productivity \( \gamma_R a_t (j) \) (where \( \gamma_R > 1 \)) from period \( t + 1 \) onwards.

The radical R&D cost function \( c_R \) is increasing and convex in \( r_t \), capturing decreasing returns to scale. Furthermore, it is increasing in \( R_t \): there is a negative externality of firms’ radical innovation efforts on the R&D costs of their competitors. This can be thought of as a shortcut to capture innovation overlaps (see, for example, Acemoglu and Cao (2015)): successful innovation gets more difficult when many other firms attempt it at the same time. I assume that \( c_R \) takes the simple functional form\(^\text{24}\)

\[
c_R (r_t, R_t) = r_t^2 R_t^\eta, \quad \text{with } \eta > 0. \tag{5}
\]

Radical innovation operates through creative destruction: by becoming the most productive producer for good \( j \), the innovating firm displaces the incumbent (non-innovating) producer and grows at its expense. Thus, as in the seminal model of Klette and Kortum (2004), innovation drives firm dynamics.

#### Incremental R&D

Apart from trying to overtake new goods, innovating firms also improve the productivity of the goods they overtook in the past through incremental R&D. Incremental R&D requires \( c_I \) units of labour in period \( t \), and improves the productivity for every good on which it is done by a factor \( \gamma_I \) (where \( 1 < \gamma_I < \gamma_R \)) in period \( t + 1 \).

An incremental innovation improves productivity less than a radical one. It does not trigger creative destruction, and does not impose negative externalities on other firms (as it is targeted to a firm’s own goods, there

\(^{22}\)This follows the terminology of Acemoglu and Cao (2015).

\(^{23}\)Assuming that innovating firms’ radical innovations only improve goods of non-innovating firms considerably simplifies the dynamic programming problem of the former, but is not crucial for my main results.

\(^{24}\)Apart from being simple, this specification is also in line with empirical evidence. Indeed, empirical research suggests that a quadratic cost function for R&D is a good approximation to reality (see Akçigit and Kerr (2015)).
is no risk of overlap). Furthermore, as firms can forecast demand one period ahead, it has a certain payoff. Thus, firms can borrow for incremental R&D even in crisis financing conditions (while they can borrow for radical R&D only in normal financing conditions).\footnote{While this is obviously a simplification, it is reasonable to think of radical R&D as riskier and less collateralizable.}

I make two final assumptions to close the model. First, I assume that one period after an innovation is introduced for a given good, imitation allows all firms to produce with a productivity that is arbitrarily close to the one of the innovator. This limits the profits from an innovation to the period in which it is introduced.\footnote{As the innovator retains an infinitesimal productivity advantage, it remains the only producer of the good as long as it is not displaced by a radical innovation. In order to simplify notation, I assume in the following that imitators can produce with the exact frontier productivity, that is, for every \(i\) and \(j\), \(a_{t+1}(i,j) = a_t(j)\).} Second, I assume that firms store all their profits (at the exogenous interest rate of 0), and that new innovating firms start out without cash. Both of these assumptions are made for simplicity and do not affect my qualitative conclusions.\footnote{I also need to impose restrictions on parameter values which guarantee that firms always choose values of \(r_t\) that are smaller than 1, and that the mass of goods produced by non-innovating firms is always larger than the mass of radical innovations. These restrictions are derived in Appendix A.}

In the next section, I solve for the model’s equilibrium.

### 3.2 Equilibrium

#### 3.2.1 Firm choices

**Pricing and profits** The representative consumer’s utility maximization problem yields for every good \(j\) the classical Cobb-Douglas demand function

\[
c_t(j) = S_t \left( \frac{p_t(j)}{p_t(j)} \right),
\]

where \(p_t(j)\) stands for the price of good \(j\) in period \(t\).

With Bertrand competition, the producer of any good (the firm with the highest productivity) sets a price equal to the marginal cost of the second most productive firm.\footnote{Precisely, the equilibrium price is the minimum between the marginal cost of the second most productive firm (the limit price) and the monopoly price, but the latter tends towards positive infinity with Cobb-Douglas preferences.} Therefore, imitation forces firms to sell all goods on which they do not introduce an innovation in period \(t\) at their marginal cost, implying zero profits.

In contrast, a firm which introduces a radical innovation on some good has a marginal cost which is by a factor \(\gamma_R\) lower than the one of the second most productive firm. This enables it to charge a mark-up \(\gamma_R\), and given the demand function in Equation (6), it is easy to verify that it earns a profit \(\left(1 - \frac{1}{\gamma_R}\right) S_t\). Likewise, goods subject to an incremental innovation in period \(t\) are sold at a mark-up \(\gamma_I\) and earn their producers a profit \(\left(1 - \frac{1}{\gamma_I}\right) S_t\). Imitation implies that these are the only profits firms ever make from these innovations.

**R&D** R&D decisions are only taken by innovating firms. In period \(t\), an innovating firm has two endogenous state variables: its number of goods produced, denoted by \(n\) (determining on how many goods it can do...}
incremental R&D) and its cash holdings after collecting period $t$ profits, denoted by $z$ (determining how much radical R&D it can do in crisis financing conditions).\(^{29}\) As shocks to financing conditions are surprises, the firm does not take this second state variable into account when assessing the future.

The firm’s incremental R&D decision is static, as it does not affect the only perceived state variable $n$. Thus, a firm does incremental R&D (on all of its $n$ goods) if and only if the innovation profit \(\left(1 - \frac{1}{\gamma_I}\right) S_{t+1}\) (which is certain, as firms can forecast next period’s aggregate demand) exceeds the R&D cost $w_{CF}$.\(^{30}\) This decision rule is not affected by financing conditions, as profitable incremental R&D delivers a positive NPV with certainty and firms can therefore always borrow for it.

To determine radical R&D decisions, I define $V_t(n)$ as the value of an innovating firm producing $n$ goods in a period $t$ with normal financing conditions, after collecting all profits from innovations introduced in that period. In a period with normal financing conditions, the firm’s Bellman equation is then

$$V_t(n) = \max_{r_t} \left[ r_t \left(1 - \frac{1}{\gamma_R}\right) S_{t+1} - wr_t^n R_t^n + n \pi_{t+1}^I + (1 - \delta) E_t (r_t V_{t+1}(n+1) + (1 - r_t) V_{t+1}(n)) \right]$$

(7)

The firm’s radical R&D choice in period $t$ generates with probability $r_t$ a radical innovation which is implemented in period $t+1$ and yields a profit \(\left(1 - \frac{1}{\gamma_R}\right) S_{t+1}\). Furthermore, its incremental R&D choices generate, for every good, a net profit $\pi_{t+1}^I \equiv \max\left(0, \left(1 - \frac{1}{\gamma_I}\right) S_{t+1} - w_{CF}\right)$. These profits are collected regardless of whether or not the firm receives the negative innovation capacity shock. However, if it receives that shock (which happens with probability $\delta$), its continuation value is 0, as it cannot innovate any more. If it does not receive the shock, it starts the next period still being an innovating firm, either with $n$ goods (if its radical R&D effort failed) or with $n+1$ goods (if it succeeded).

Equation (7) can solved with a guess-and-verify approach. Indeed, one can easily verify that the value function takes the form $V_t(n) = \mathcal{E}_t + n (\pi_{t+1}^I + \frac{1 - \delta}{\delta} \mathbb{E}\left(\pi^I\right))$, where $\mathbb{E}\left(\pi^I\right)$ is the unconditional expectation of $\pi_{t+1}^I$ and $(\mathcal{E}_t)_{t \in \mathbb{N}}$ is a stochastic process defined by the difference equation

$$\mathcal{E}_t = \frac{\left((1 - \frac{1}{\gamma_R}) S_{t+1} + \frac{1 - \delta}{\delta} \mathbb{E}\left(\pi^I\right)\right)^2} {4 wr_t^n} + (1 - \delta) \mathbb{E}_t (\mathcal{E}_{t+1}) \cdot$$

(8)

Therefore, the first-order condition for radical R&D in a period with normal financing conditions is

$$2 wr_t^n = \left(1 - \frac{1}{\gamma_R}\right) S_{t+1} + \frac{1 - \delta}{\delta} \mathbb{E}\left(\pi^I\right).$$

(9)

The optimal choice equalizes the marginal cost of radical R&D to its marginal benefit, which is the sum of

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\(^{29}\)In this section, I omit the firm index $i$ whenever this does not cause confusion.

\(^{30}\)I assume that $\left(1 - \frac{1}{\gamma_I}\right) S_L < w_{CF} < \left(1 - \frac{1}{\gamma_I}\right) S_H$, so that aggregate demand fluctuations affect incremental R&D.
the direct profit of radical innovation and the expected future profits from doing incremental R&D on the newly gained good as long as the firm remains innovating.

What happens if there are crisis financing conditions in period \( t \)? As firms expect financing conditions to be forever normal again from period \( t + 1 \) onwards, their value in that period is still given by \( V_{t+1} \), and they still want to choose the radical innovation probability defined by Equation (9). However, they may be unable to do so if their cash level \( z \) is insufficient: in that case, they either do no radical R&D at all (if \( z \) is negative) or they spend all their limited cash on radical R&D (if \( z \) is positive, but lower than the unconstrained radical R&D level).\(^{31} \)

In sum, radical R&D decisions in periods with crisis financing conditions are given by

\[
\hat{r}_t = \begin{cases} 
\tilde{r}_t & \text{if } z \geq \frac{1}{\gamma} \frac{2}{\eta} R_t \\
\sqrt{\frac{z}{\pi \hat{r}_t}} & \text{if } 0 \leq z < \frac{1}{\gamma} \frac{2}{\eta} R_t \\
0 & \text{if } z < 0
\end{cases}
\]

where

\[
\tilde{r}_t = \frac{\left(1 - \frac{1}{\gamma}\right) S_{t+1} + \frac{1 - \delta}{\gamma} \mathbb{E}(\pi_t)}{2wR_t}.
\]

\(^{10}\)

**Entry** In periods with crisis financing conditions, potential entrants cannot enter: they cannot borrow for the entry cost, as this investment does not have a positive NPV with certainty (the firm may be hit by the negative innovation capacity shock before it is able to do any profits).

In periods with normal financing conditions, potential entrants enter iff \( \varphi w \leq V_t(0) \). In the baseline version of my model, I assume that this inequality always holds.\(^{32}\)

### 3.2.2 Innovation masses and the size distribution of innovating firms

Knowing firms’ policy functions, I now proceed to determine the industry-level laws of motion. The total mass of innovating firms, denoted by \( M_t \), evolves according to

\[
M_{t+1} = (1 - \delta) M_t + \begin{cases} 
M_{t}^{\text{New}} & \text{if financing conditions are normal in } t + 1 \\
0 & \text{if there are crisis financing conditions in } t + 1
\end{cases}.
\]

A fraction \( 1 - \delta \) of innovating firms receive a negative innovation capacity shock, while a mass \( M_{t}^{\text{New}} \) of potential new innovating firms appears. These firms enter if and only if financing conditions are normal.

The total mass of radical innovations created in period \( t \) is denoted \( R_t \). By definition, it is equal to

\[
R_t = \int_{M_t} r_t,
\]

\(^{31}\)It is optimal for firms to spend all their cash, because they expect that they will never need to self-finance again.

\(^{32}\)Appendix A provides a sufficient condition for this in terms of the model's parameters. Thus, in all periods with normal financing conditions, entry equals \( M_{t}^{\text{New}} \). In Section 4.4 and Appendix B, I analyse an extension of the model where the mass of potential entrants is arbitrarily large, and equilibrium entry is pinned down by a free entry condition holding with equality.
where with some abuse of notation, $M_t$ stands for the set of innovating firms, and $r_t$ is either given by Equation (9) or by Equation (10), depending on the state of financing conditions. With normal financing conditions, Equations (9) and (12) directly pin down the radical innovation mass as a function of $M_t$\textsuperscript{33}. In crisis financing conditions, the situation is more complex, and Equations (10) and (12) pin down the radical innovation mass as a function of the cash distribution across innovating firms.\textsuperscript{34} The mass of goods produced by innovating firms in period $t$, denoted $M_t$, evolves according to

$$M_{t+1} = (1 - \delta) (M_t + R_t) . \quad (13)$$

In every period, innovating firms take over a mass $R_t$ of goods from non-innovating firms. As the innovation capacity shock is independent of firm size, a fraction $1 - \delta$ of these goods as well as of the goods previously produced by innovating firms will still be produced by innovating firms in period $t + 1$. The total mass of incremental innovations created in period $t$, denoted $I_t$, is therefore given by

$$I_t = \begin{cases} M_t & \text{if } \left(1 - \frac{1}{\gamma_I}\right) S_{t+1} \geq wc, \\ 0 & \text{if } \left(1 - \frac{1}{\gamma_I}\right) S_{t+1} < wc. \end{cases} \quad (14)$$

### 3.2.3 Output and productivity growth

Defining industry output as $Y_t \equiv \exp \left(\int_0^1 \ln y_t (j) \, dj\right)$, it is easy to show that in equilibrium,

$$Y_t = \frac{S_t}{w} \exp \left(\int_0^1 \ln a_{t-1} (j) \, dj\right) . \quad (15)$$

Therefore, the growth rate of industry output is given by\textsuperscript{35}

$$\frac{Y_{t+1} - Y_t}{Y_t} = \frac{S_{t+1}}{S_t} \exp (R_{t-1} \ln \gamma_R + I_{t-1} \ln \gamma_I) - 1 . \quad (16)$$

Finally, defining the industry’s productivity as $A_t \equiv \frac{Y_t}{L_t}$ (where $L_t$ is the total labour demanded by firms for production), it comes that

$$\frac{A_{t+1} - A_t}{A_t} = \exp (R_{t-1} \ln \gamma_R + I_{t-1} \ln \gamma_I) \Theta_t - 1, \quad \text{where} \quad \Theta_t = \frac{1 - R_{t-1} \left(1 - \frac{1}{\gamma_R}\right) - I_{t-1} \left(1 - \frac{1}{\gamma_I}\right)}{1 - R_t \left(1 - \frac{1}{\gamma_R}\right) - I_t \left(1 - \frac{1}{\gamma_I}\right)} . \quad (17)$$

Industry productivity growth mainly depends on the volume of radical and incremental innovations, as

\textsuperscript{33}Precisely, it is straightforward to show that $R_t = \left[\frac{M_t}{\pi_0} \left(\left(1 - \frac{1}{\gamma_R}\right) S_{t+1} + \frac{1}{\gamma_R} E \left(\pi'\right)\right)\right]^{\frac{1}{\gamma_R}}$.

\textsuperscript{34}This distribution is analytically intractable, but it can easily be tracked numerically, as I show in Appendix A.

\textsuperscript{35}Equations (15) and (16) are derived in Appendix A.
captured by the first factor in Equation (17). However, there is also a more indirect source of variation, captured by $\Theta_t$. Indeed, innovations create mark-up dispersion (unimproved goods are sold at marginal cost, improved ones at a mark-up $\gamma_R$ or $\gamma_I$) which misallocates labour and depresses aggregate productivity.\footnote{Epifani and Gancia (2011) and Peters (2011) analyse the static and dynamic effects of mark-up dispersion.}

Thus, changes in mark-up dispersion can also lead to movements in industry productivity.

This completes the description of equilibrium. In the next section, I briefly analyse the model’s balanced growth path, which provides a useful starting point before analysing the effects of aggregate shocks.

### 3.3 The balanced growth path

I define the balanced growth path as the model’s solution when aggregate demand is constantly high (that is, the probability that demand equals $S_L$ is 0), financing conditions are always normal, industry output grows at a constant rate and the size distribution of innovating firms does not change over time.

In this case, it is easy to show the mass of innovating firms and the mass of radical innovations are given by

$$M = \frac{M_{\text{New}}}{\delta} \quad \text{and} \quad R = \left[ \frac{M_{\text{New}}}{2\delta w} \left( \left(1 - \frac{1}{\gamma_R}\right) S_H + \frac{1 - \delta}{\delta} \left( \left(1 - \frac{1}{\gamma_R}\right) S_H - wc_I \right) \right) \right]^{\frac{1}{1+\eta}}. \quad (18)$$

Furthermore, the mass of goods produced by innovating firms and the mass of incremental innovations are

$$M = \frac{(1 - \delta) R}{\delta} \quad \text{and} \quad I = M. \quad (19)$$

Industry output and productivity grow at the constant rate $\exp(R \ln \gamma_R + I \ln \gamma_I) - 1$. Finally, there is a closed-form expression for the size distribution of innovating firms. Denoting by $m_n$ the mass of innovating firms producing $n$ goods, I show in Appendix A that

$$\forall n \geq 0, \quad m_n = \left( \frac{(1 - \delta) r}{\delta + (1 - \delta) r} \right)^n \frac{M_{\text{New}}}{\delta + (1 - \delta) r}, \quad (20)$$

where $r = \frac{R}{M}$. Of course, this distribution holds $\sum_{n=0}^{+\infty} m_n = M$ and $\sum_{n=1}^{+\infty} nm_n = M$.

The balanced growth path solution matches several important firm-level stylized facts. For instance, the exit probability is decreasing in firm size,\footnote{Firms do not exit as long as they are innovating. Once they become non-innovating, larger firms have in every period larger chances to stay active, as it is less likely for them to lose all their goods at once.} and small firms have higher growth rates than large ones.

More importantly, my model also replicates the stylized facts documented in Section 2. The ratio of R&D to sales for an innovating firm producing $n$ goods is given by $\frac{\text{R&D}}{\text{Sales}}(n) = w \left( \frac{r^2 R^3}{n S_H} + \frac{c_I}{S_H} \right)$, and therefore decreases in firm size. Indeed, all firms do the same absolute amount of radical R&D (because its costs and benefits are
independent of size), while incremental R&D increases linearly with firm size. Thus, small innovating firms do more R&D relative to their size, and their average innovation is more likely to be radical. Furthermore, if radical innovations are more likely to be patented than incremental ones, the model also replicates the fact that small innovating firms have more patents per employee, and more patents per unit of R&D. These results (and my assumptions on R&D technologies which generate them) are similar to the ones of Akçigit and Kerr (2015). They depart from the classical framework of Klette and Kortum (2004), who had assumed that radical R&D costs decrease with firm size \( n \) in such a way that the optimal amount of radical R&D becomes linear in firm size, and all firms have the same ratio of R&D to sales.

Small innovating firms also have on average lower cash holdings than large ones and therefore a lower ability to self-finance. Indeed, entrants start from a negative cash position (as they need to pay the entry cost \( \varphi w \)) and accumulate cash only gradually as they grow, through successful radical and incremental innovations.

In the following section, I show that these heterogeneities among innovating firms provide novel insights on the reaction of R&D and innovation to aggregate shocks.

4 The effect of aggregate shocks

I study aggregate fluctuations in my model by analysing the impulse responses to transitory shocks. That is, I assume that the industry is hit in a crisis period \( T \) by a shock to financing conditions and/or aggregate demand. Before and after period \( T \), financing conditions are normal and aggregate demand is high.\textsuperscript{38}

4.1 Impulse responses to a shock to financing conditions

I start by considering a situation in which there are crisis financing conditions in period \( T \), but aggregate demand remains high. Figure 4 shows the impulse responses of several important variables to this shock. The vertical line in Panels 1 to 4 indicates the period in which the shock hits.

In the crisis period, several small innovating firms do not hold enough cash to finance their desired level of radical R&D. Likewise, potential entrants cannot finance the entry cost and therefore also do not do radical R&D. As a result, the mass of radical innovations (indicated by the solid line in Panel 1) falls. This fall is somewhat dampened by the reaction of unconstrained firms: the lower aggregate innovation level lowers their costs and thus increases their desired radical innovation effort (indicated by the dotted line in Panel 1).

\textsuperscript{38} The pre-crisis situation does not exactly coincide with the balanced growth path, as there is uncertainty about aggregate demand (lowering firms' radical R&D incentives). However, I assume that the shock is preceded by an arbitrary long period with high aggregate demand and normal financing conditions. Then, all formulas from Section 3.3 still apply, except that

\[
R = \frac{1}{23w} \left( \left( 1 - \frac{1}{\gamma_H} \right) S_H + \frac{1}{3} p_H \left( \left( 1 - \frac{1}{\gamma_I} \right) S_H - w_c I \right) \right)^{\frac{1}{\gamma_H}},
\]

where \( p_H \) is the probability that aggregate demand is high.
Figure 4: Impulse responses to a shock to financing conditions

Panel 1: Mass of innovating firms and radical innovation
Panel 2: Goods produced by innovating firms and incremental innovation

Note: The parameter values used for drawing these graphs are given in Table 4 of Appendix A.

Meanwhile, incremental innovation does not react on impact (see Panel 2), as firms can still borrow to finance it and aggregate demand has not changed.

Lower (radical) innovation in the crisis period permanently lowers industry output. This is shown in Panel 4, which plots actual industry output against a counterfactual path that would have prevailed in the absence of the shock. The permanent effect of transitory shocks to R&D and innovation has been repeatedly emphasized by representative-firm models. However, Figure 4 shows that firm heterogeneity adds several new features to this classical story.

First, the asymmetric impact of crisis financing conditions misallocates radical R&D: the marginal product of radical R&D at small, constrained firms is higher than the one at large, unconstrained firms. Misallocation increases the productivity losses caused by the shock beyond those which would occur if radical R&D

\[ \text{Industry output falls below its counterfactual level two periods after the shock. Indeed, innovations are implemented one period after they are produced by R&D. Furthermore, in the implementation period, the innovating firm does not lower the price of the good, keeping all the innovation surplus for itself. Only in the following period, imitation forces the innovating firm to lower prices, increase production and transmit the innovation surplus to the consumer.} \]
reductions were equal across firms (as it is implicitly assumed in a representative-firm model). This is shown in Panel 3, where the dashed line plots a counterfactual level of radical innovation obtained by allocating the actual radical R&D of my model equally across all innovating firms.\textsuperscript{40}

Moreover, my model predicts that even as financing conditions return to normal and misallocation disappears, the masses of radical and incremental innovation are persistently depressed (and thus, the permanent damage caused by the shock keeps increasing, as shown in Panel 4). This is due to two novel persistence mechanisms.

Incremental innovation falls persistently because the crisis lowers the mass of goods which innovating firms overtake from non-innovating ones, and therefore reduces the scope for incremental innovation. As radical innovation is also persistently depressed, this effect is undone only slowly.

The mechanism behind the persistent fall in radical innovation is the most novel feature of my model. That fall is due to a persistent fall in the mass of innovating firms. Indeed, zero entry during the crisis creates a “missing generation” of innovating firms, which is only slowly replaced in the aftermath (as entry returns to its pre-crisis level, but does not overshoot it). In the meantime, the remaining firms increase their radical R&D (and therefore, radical innovation after the crisis is less depressed than the mass of innovating firms itself). However, as they face decreasing returns, their additional R&D effort is lower and less efficient than the one which could have been carried out by an additional innovating firm.\textsuperscript{41} Thus, in an environment with fewer innovating firms than before the shock, radical innovation is depressed even with normal financing conditions.

It is important to stress what role the permanent differences between small and large firms play for this persistence channel. After the crisis period, the average innovating firm is larger than before, both because of a selection effect (firms which were kept from entering would have been small) and because of the higher radical R&D effort of large firms in the crisis period. However, precisely because small firms are relatively more R&D and innovation-intensive than large ones, the greater size of the remaining innovating firms does not compensate for their lower number. This would not be the case in a Klette and Kortum model: in their framework, halving the mass of innovating firms would not affect aggregate R&D, because it would automatically make the remaining firms twice as large and therefore twice as efficient at R&D.\textsuperscript{42} Therefore, departing from the Klette and Kortum assumptions is key for my model’s prediction that the fall in the mass of innovating firms and the shift in their size distribution have aggregate effects.

\textsuperscript{40}Panel 3 also shows that the ratio between the mass of radical innovations and aggregate radical R&D increases in the crisis period. This is because constrained firms get to a flatter region of their convex cost curve, where their marginal R&D effort is more efficient. This may be interpreted as firms dropping their least promising R&D projects first.

\textsuperscript{41}Note that this effect is already present during the crisis period $T$, and therefore also contributes to the fall of radical innovation on impact.

\textsuperscript{42}This statement needs to be qualified: the composition of the mass of innovating firms is irrelevant in the Klette and Kortum model as long as the mass of goods they produce, $M_t$, is constant. In my model, $M_t$ actually falls after the shock, exacerbating its negative effects. However, the persistence mechanism I describe here would be unchanged if $M_t$ were held constant (for instance, by randomly distributing goods of non-innovating firms among innovating firms every time $M_t$ would tend to fall).
4.2 Impulse responses to an aggregate demand shock

I now assume aggregate demand falls to its low level $S_L$ in period $T$, but financing conditions remain normal. The impulse responses to this shock are shown in Figure 5.

Figure 5: Impulse responses to an aggregate demand shock

Panel 1: Mass of innovating firms and radical innovation

Panel 2: Goods produced by innovating firms and incremental innovation

Panel 3: Misallocation during the crisis

Panel 4: Industry output

Notes: The parameter values used for drawing these graphs are given in Table 4 of Appendix A.

Panel 1 shows that radical R&D falls one period before the shock (as firms forecast aggregate demand and anticipate that innovations implemented during the crisis period have a low return). Incremental innovation (shown in Panel 2) falls for the same reason. There is no misallocation of R&D, as the shock hits all firms symmetrically, and no persistent fall in the mass of innovating firms, as potential entrants are not prevented from entering. However, lower radical innovation still persistently reduces the scope for incremental innovation in the post-crisis periods. Finally, Panel 4 shows that the shock also has a direct effect on output, as falling demand reduces production. This effect is reversed in the period after the shock, but lower innovation in the pre-crisis period and the persistent drop of incremental innovation still trigger a (small) permanent loss in industry output.
4.3 Impulse responses to a joint shock

Figure 6 shows impulse responses when both an aggregate demand shock and a shock to financing conditions hit in period $T$.

Figure 6: Impulse responses to a joint shock to aggregate demand and financing conditions

Panel 1: Mass of innovating firms and radical innovation

Panel 2: Goods produced by innovating firms and incremental innovation

Panel 3: Misallocation during the crisis

Panel 4: Industry output

Notes: The parameter values used for drawing these graphs are given in Table 4 of Appendix A.

A joint shock combines the effects described so far. Moreover, there is some interaction: the fall in aggregate demand reinforces the effect of crisis financing conditions, by lowering firms’ cash holdings and therefore increasing the mass of constrained firms during the crisis.\footnote{The role of financial constraints in my model is very distinct from their role in classical financial accelerator models such as Bernanke and Gertler [1989] or, in a setup with heterogeneous firms and creative destruction, Caballero and Hammour [2005]. In these models, financial constraints are permanent. A transitory aggregate demand (or productivity) shock then generates persistence because it lowers firms’ profits and thereby persistently reduces their cash holdings. This makes financial constraints more likely to bind in the following periods and persistently depresses investment. In contrast, once financing conditions have normalised in my model, firms’ cash holdings become irrelevant, and all persistence is due to other factors.}

Summing up, my analysis highlights a number of channels through which firm heterogeneity can amplify the effect of transitory aggregate shocks and increase their persistence. In the next section, I briefly discuss the role of some key assumptions in generating my results.
4.4 Key assumptions and robustness

My model makes several assumptions which preserve its tractability without affecting its qualitative results. First, it is specified in partial equilibrium, which considerably simplifies calculations. Moreover, in the case of a small open economy such as Spain, this setup is arguably realistic, as a large part of the demand for manufacturing goods comes from other European countries and interest rates are determined on international capital markets. Second, assuming that shocks to financing conditions are surprises prevents precautionary savings. Allowing for these would complicate the model, but would not affect the mechanisms I have described. Indeed, the constant entry of innovating firms implies that there will always be a range of small firms which, due to their youth, have not had the time to build up sufficient cash buffers. Finally, there is no endogenous exit of innovating firms in my model. When introducing endogenous exit (for instance, through a fixed cost of production), the mass of innovating firms would fall even further during periods with crisis financing conditions.

One important feature of my model deserves a more detailed discussion. My most important result is that a shock to financing conditions persistently lowers the mass of innovating firms, creating a missing generation and a persistent drag on productivity growth. The key reason for persistence is that entry of innovating firms in the periods after the shock can never exceed its pre-shock level $M^{\text{New}}$, as the mass of potential entrants is exogenously fixed. Instead, if entry were allowed to overshoot and would overshoot sufficiently, the missing generation of innovating firms may be immediately replaced.

In Appendix B, I therefore analyse an extension of my model with an arbitrarily large mass of potential entrants, where entry is pinned down (in periods with normal financing conditions) by a free entry condition. I show that there are indeed greater incentives for entry after a shock to financing conditions, as the depressed level of radical innovation lowers radical R&D costs for all firms. However, as long as the marginal entrant’s entry cost is increasing in the aggregate mass of entrants, overshooting is limited and does not prevent the persistent fall in the mass of innovating firms. Assuming increasing entry costs for innovating firms appears plausible. Indeed, when there is a limited and heterogeneous pool of innovative entrepreneurs, higher entry must mean that the marginal entrant is less efficient.\footnote{I assume that the entry cost schedule is the same in every period. This implicitly implies that potential entrants which did not enter during the crisis period cannot defer entry, as this would potentially change the composition of the pool of entrants over time. In the Spanish case, one may actually argue that the massive emigration of high-skilled individuals during the economic and financial crisis reduced the pool of potential innovative entrants.} Alternatively, when some industry-specific resources needed for entry are in fixed supply, a higher mass of entrants increases their price for every firm.

This completes the discussion of my model. In the next section, I briefly confront its main predictions with the empirical evidence provided in Sections 1 and 2.
5 Confronting the model with the data

My model qualitatively matches the developments in the Spanish manufacturing sector during the 2008-2013 economic and financial crisis, which were documented earlier in this paper. Indeed, it predicts that after a joint shock to aggregate demand and financing conditions, R&D and the number of R&D-performing firms fall, with small firms reacting more strongly than large ones. Furthermore, the recovery begins with a lower number of R&D-performing firms and a right-shifted size distribution, as in the data. My model suggests that these changes add persistence to the crisis, and may depress Spanish innovation and productivity growth for several years to come. However, what is the quantitative impact of these changes?

For simplicity, I focus in this section on the fall in the number of R&D-performing firms, and do not attempt to quantify the effects of R&D misallocation and of persistently depressed incremental innovation. Thus, my calculation represents a lower bound for the impact of the novel mechanisms highlighted by my model.

As in most developed economies, the manufacturing sector in Spain has been shrinking over time. One part of the fall in the number of R&D-performing firms can be interpreted as a mechanical consequence of this shrinkage, and therefore does not affect productivity growth. Indeed, my model should not be misread as suggesting that the productivity growth of an industry depends on the absolute number of innovating firms or on the absolute mass of innovations produced. Instead, productivity growth depends on the mass of innovations relative to the mass of goods produced by the industry. Intuitively, an industry producing half as many goods only needs half as many innovations to achieve the same rate of productivity growth. In a Cobb-Douglas model, a sector’s share in the total mass of goods produced equals its GDP share. Thus, one can expect the 10% fall in Spanish manufacturing’s GDP share between 2007 and 2014 (according to Eurostat, it fell from 13.5% to 12.1%) to result in a 10% lower number of R&D-performing firms in 2014.

For example, a simple example may illustrate this point further. On the balanced growth path, my model’s industry does a mass $R$ and $I$ of radical and incremental innovations, achieving a productivity growth rate $\exp (R \ln \gamma_R + I \ln \gamma_I) - 1$. Now, divide this industry into two sub-industries which both receive an equal share of spending. Then, overall output can be written

$$ Y = Y_1 + Y_2 = \left( \exp \left( 2 \int_0^1 \ln y(t) \, dt \right) \right)^{\frac{1}{2}} \left( \exp \left( 2 \int_0^1 \ln y(t) \, dt \right) \right)^{\frac{1}{2}}. $$

Proceeding as in Section 3.2, I can show $Y = S \exp \left( 2 \int_0^1 \ln y(t) \, dt \right)$. Therefore, it is easy to verify that sub-industry 1 achieves the same productivity growth rate as the (twice as large) aggregate industry with innovation masses $\frac{R}{2}$ and $\frac{I}{2}$.

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45 The model's focus is on innovating firms (that is, firms with the capacity to do R&D). However, I only observe firms which actually spend on R&D in the data. This difference is irrelevant for comparisons in periods with normal financing conditions, where all innovating firms in my model also do R&D. In periods with crisis financing conditions, however, there is a slight difference between the two concepts, as some innovating firms with 0 goods and negative cash holdings do not do R&D.

46 This pattern is not unique to Spain. In a previous version of this paper [Schmitz (2015)], I document that Germany experienced a similar evolution during the 2008-2009 Great Recession. However, both the fall in the number of R&D-performing manufacturing firms (-10% between 2008 and 2010) and the increase in the share of firms with more than 250 employees among them (from 8.3% to 8.7%) were substantially more modest than in Spain (German figures are from ZEW (2009, 2011)). Furthermore, in Germany, changes were driven by firms which reported only "occasional" R&D. The number of German manufacturing firms with continuous R&D activity actually increased during the Great Recession (while it fell 31% in Spain). Given the much higher intensity of the Spanish crisis, these differences are not surprising.

47 In my model, this mass is constant and equal to 1, which is why it does not show up in any formula.

48 A simple example may illustrate this point further. On the balanced growth path, my model’s industry does a mass $R$ and $I$ of radical and incremental innovations, achieving a productivity growth rate $\exp (R \ln \gamma_R + I \ln \gamma_I) - 1$. Now, divide this industry into two sub-industries which both receive an equal share of spending. Then, overall output can be written

$$ Y = Y_1 + Y_2 = \left( \exp \left( 2 \int_0^1 \ln y(t) \, dt \right) \right)^{\frac{1}{2}} \left( \exp \left( 2 \int_0^1 \ln y(t) \, dt \right) \right)^{\frac{1}{2}}. $$

Proceeding as in Section 3.2, I can show $Y = S \exp \left( 2 \int_0^1 \ln y(t) \, dt \right)$. Therefore, it is easy to verify that sub-industry 1 achieves the same productivity growth rate as the (twice as large) aggregate industry with innovation masses $\frac{R}{2}$ and $\frac{I}{2}$. 

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23
without lowering productivity growth. However, the number of R&D-performing firms fell by 37% during that period, leaving a fall of roughly 30 percentage points which can be attributed to the crisis and which does lower productivity growth, by depressing radical innovation.

In the model, two crucial parameters govern the strength of this effect: \( \eta \), the curvature of the negative externality which firms impose on their competitors by doing radical innovations, and \( \delta \), the firm-level probability of a negative innovation capacity shock. Indeed, in periods with normal financing conditions, aggregate radical innovation \( R_t \) is proportional to \( M_t^{1+\eta} \). Thus, the higher is \( \eta \), the more the innovating firms remaining after a shock to financing conditions can compensate for the missing generation of small and young ones. The parameter \( \delta \), in turn, pins down the average number of periods during which a firm can do innovations. Therefore, it determines how long the fall in the mass of innovating firms persists: when \( \delta \) is high, the missing generation of firms would not have been innovating for a long time and is therefore quickly replaced, while when \( \delta \) is low, this replacement takes more time.\(^9\)

What are reasonable values for these two parameters? The literature unfortunately does not provide precise guidance on this point. Acemoglu and Cao (2015), who use a similar specification, set \( \eta = 1 \) in their calibration (see Appendix A for details), but stress that their choice is somewhat arbitrary. Therefore, I also consider a more indirect approach to pin down this parameter. In my stylized model, radical R&D costs are independent of firm size. Using US firm-level data, Alçigit and Kerr (2015) however estimated that larger firms do have some cost advantages, even though they are weaker the ones suggested by the Klette and Kortum model (and therefore do not overturn the fact that small firms do relatively more R&D). My model’s innovation overlap externality (which is not present in Alçigit and Kerr) introduces an indirect link between radical R&D costs and (average) firm size. Indeed, imagine adding a new innovating firm to the industry. All else equal, that firm’s R&D costs are decreasing in average firm size: when the average firm is larger, there are fewer firms, and therefore there is less radical innovation and less overlap. The parameter \( \eta \) determines the strength of this relationship. Thus, I can set \( \eta \) such that the elasticity of a new firm’s radical R&D cost to average innovating firm size in my model equals the elasticity of radical R&D costs to firm size uncovered by Alçigit and Kerr’s structural estimation. As I show in Appendix A, this implies \( \eta = 4 \).

To the best of my knowledge, there is no empirical study which could be used to pin down the value of the parameter \( \delta \). Therefore, I consider a number of different values for \( \delta \), ranging from 0.1 to 0.5. Interpreting one period in my model as one year, this corresponds to the average firm retaining its innovation capacity for between two (\( \delta = 0.5 \)) and ten (\( \delta = 0.1 \)) years.

\(^9\)This section refers to my baseline model, where entry returns to its pre-crisis level once financing conditions are normal again. Given the stagnation of the number of R&D-performing firms during the first post-crisis year in Spain (see Figure 1) it seems that this assumption may be even too optimistic, and therefore, that I underestimate the true persistence of the fall of the number of R&D-performing firms.
Given values for $\eta$, $\delta$ and the mass of innovating firms in the post-crisis period (70% of its pre-crisis value), I can now use my model to calculate the mass of radical innovations (again, relative to their pre-crisis value) in every period after the crisis shock. To translate these values into an estimate for the permanent output loss due to lower radical innovation, I need to know the pre-crisis productivity growth rate, as well as the contribution of radical innovation to this growth rate. According to Gopinath et al. (2015), the pre-crisis productivity growth rate in Spanish manufacturing, abstracting from changes in the efficiency of the resource allocation, has been 1.25% per year.\(^{50}\) Finally, according to Akçigit and Kerr (2015), radical innovation accounts for 80% of all productivity growth in the United States. I assume the same is true for Spain. Under these assumptions, Table 3 shows the permanent losses of manufacturing output obtained for different values of the parameters $\eta$ and $\delta$.

<table>
<thead>
<tr>
<th>$\delta$ \ $\eta$</th>
<th>1</th>
<th>2</th>
<th>3</th>
<th>4</th>
</tr>
</thead>
<tbody>
<tr>
<td>0.10</td>
<td>1.54%</td>
<td>1.04%</td>
<td>0.79%</td>
<td>0.63%</td>
</tr>
<tr>
<td>0.25</td>
<td>0.62%</td>
<td>0.42%</td>
<td>0.32%</td>
<td>0.26%</td>
</tr>
<tr>
<td>0.50</td>
<td>0.31%</td>
<td>0.21%</td>
<td>0.16%</td>
<td>0.13%</td>
</tr>
</tbody>
</table>

**Notes:** These numbers indicate the total permanent fall in industry output triggered by a one-time 30% decrease in the mass of innovating firms. Details on the calculation can be found in Appendix A.

Being based on a very stylized model, these numbers should be interpreted with caution. Nevertheless, they suggest that the aggregate consequences of a fall in the number of R&D-performing firms are potentially important: in some cases, this permanently lowers output by more than 1%. Even when setting $\eta = 4$, in line with the estimates of Akçigit and Kerr (2015), the effects are modest, but not negligible.

### 6 Conclusion

I have proposed a new endogenous growth model with heterogeneous firms and aggregate shocks, showing that heterogeneity creates a line of novel amplification and persistence mechanisms for transitory aggregate shocks. Asymmetrically applying financing constraints misallocate R&D during a financial crisis, and the fall in the mass of innovating firms lowers R&D and innovation even in subsequent periods, as the remaining firms cannot fully compensate for a missing generation of small and young innovating firms. Furthermore,\(^{50}\) Gopinath et al. (2015) show that an increasingly worse allocation of resources (especially capital) has greatly lowered Spanish total factor productivity between 1999 and 2007 (Garcia-Santana et al. (2015) make the same point). However, their work also implies that without this misallocation, manufacturing TFP would have grown by 1.25% per year [see Figure 5, page 17 of their paper]. This change in the efficient productivity level is commonly attributed to the effects of R&D and innovation, and is therefore the relevant unit of comparison for my model. Obviously, if there are large changes in the efficiency of the resource allocation in the aftermath of the Spanish crisis, observed productivity growth may differ from my predictions.
the scope for incremental follow-up innovations is persistently reduced.

These predictions are in line with developments in the Spanish manufacturing sector during the 2008-2013 economic and financial crisis, and suggest that the changes in the number and the composition of its R&D-performing firms may lead to sizeable permanent output losses. However, my model’s predictions are of course not limited to Spain: the mechanisms it describes may also have contributed to the slow pace of output and productivity growth after the Great Recession in other developed economies.

There are several directions for future research, including making the model’s quantitative implications more precise and sharpening its policy prescriptions. Indeed, my model implies that a government trying to limit the long-run damage from a financial crisis should subsidize young and small innovating firms during the crisis. However, in practice, the government may have trouble to differentiate between innovating and non-innovating firms, so that there may be a trade-off between the cost of funds wasted on non-innovating firms and the benefits of funds well spent on innovating ones.

Finally, my model provides a new perspective on the link between growth and volatility at the industry level. In particular, it suggests that structural factors which affect an industry’s size distribution of innovating firms or its cash distribution affect both the level and the volatility of productivity growth. A deeper investigation of these relationships may yield further interesting results.

References


In my model, all volatility is due to aggregate shocks. However, depending on their characteristics, different industries may react more or less strongly to these aggregate shocks.


[31] Instituto Nacional de Estadística, Encuesta sobre Innovación en las Empresas, Metodología general 2014.


A Additional proofs and details

Derivation of Equations (15) and (16)

Cobb-Douglas preferences imply \( Y_t = C_t = \frac{S_t}{P_t} \), where \( P_t = \exp \left( \int_0^1 \ln p_t(j) \, dj \right) \). Every good is sold at the marginal cost of the second most productive firm, so \( p_t(j) = \frac{w_{t-1}(j)}{v_{t-1}(j)} \), which directly gives Equation (15).

From period \( t-1 \) to period \( t \), the frontier productivity of a mass \( R_{t-1} \) of goods is multiplied by \( \gamma_R \), the frontier productivity of a mass \( I_{t-1} \) by \( \gamma_I \), while the remainder’s productivity does not change. Therefore,

\[
Y_{t+1} = \frac{S_{t+1}}{w} \exp \left( \int_0^1 \ln a_{t-1}(j) \, dj + R_{t-1} \ln \gamma_R + I_{t-1} \ln \gamma_I \right) = \frac{S_{t+1}}{S_t} Y_t \exp \left( R_{t-1} \ln \gamma_R + I_{t-1} \ln \gamma_I \right)
\]

which gives Equation (16).

The size distribution of innovating firms on the balanced growth path

Under normal financing conditions, the transition law for the size distribution of innovating firms is given by

\[
m_{n,t+1} = (1 - \delta) \left( (1 - r_t)m_{n,t} + r_t m_{n-1,t} \right) \quad \text{if } n \geq 1
\]

\[
m_{0,t+1} = (1 - \delta) \left( (1 - r_t)m_{0,t} + M_{\text{New}} \right)
\]

Innovating firms do not lose goods, and can gain at most one good per period. Thus, firms producing \( n \) goods in period \( t+1 \) must have produced either \( n \) or \( n-1 \) goods in period \( t \). For both of these categories, a fraction \( 1 - \delta \) of firms is not hit by the negative innovation capacity shock and remains innovating.\(^{52}\) Among those, a fraction \( 1 - r_t \) of firms with \( n \) goods does not do a radical innovation and stays with \( n \) goods, and a fraction \( r_t \) of the firms with \( n-1 \) goods does a radical innovation and progresses to \( n \) goods.

On the balanced growth path, the size distribution of innovating firms does not change over time and the radical innovation probability is constant. This implies that \( m_0 = \frac{M_{\text{New}}}{\delta + (1 - \delta)r_f} \), and shows that \( (m_n)_{n \in \mathbb{N}} \) is the simple geometric sequence specified in Equation (20).

\(^{52}\)As there is a continuum of firms in each category and shocks are i.i.d. across firms, the law of large numbers implies that the fraction of firms hit by the shock equals the probability that the shock hits.
Parameter conditions and parameter values

To ensure that a firm’s radical innovation probability is never larger than 1 on the balanced growth path, it is sufficient to impose

$$\frac{R}{M} < 1 \Leftrightarrow \left[ \frac{1}{2w} \left( \left( 1 - \frac{1}{\gamma R} \right) S_H + \frac{1 - \delta}{\delta} \left( \left( 1 - \frac{1}{\gamma I} \right) S_H - w c_I \right) \right) \right]^{1+\eta} < \left( \frac{M^{\text{New}}}{\delta} \right)^{1+\eta}. \quad (21)$$

There is, however, no generic restriction guaranteeing that the optimal radical innovation probability $\tilde{r}_t$ remains smaller than 1 after a shock to financing conditions. Indeed, its value in this case depends on the cash distribution, for which there is no analytical expression. Therefore, I just verify ex post in all my impulse response analyses that $\tilde{r}_t$ always remains smaller than 1.

The aggregate mass of radical innovations $R_t$ must be smaller than the mass of goods produced by non-innovating firms $1 - M_t$. On the balanced growth path, this holds under the sufficient condition

$$R < 1 - \frac{(1 - \delta) R}{\delta} \Leftrightarrow \left[ \frac{M^{\text{New}}}{2\delta w} \left( \left( 1 - \frac{1}{\gamma R} \right) S_H + \frac{1 - \delta}{\delta} \left( \left( 1 - \frac{1}{\gamma I} \right) S_H - w c_I \right) \right) \right]^{1+\eta} < \delta. \quad (22)$$

This condition is also sufficient to guarantee that $R_t$ is smaller than $1 - M_t$ after any transitory shock, as a shock can only lower the values of $R_t$ and $M_t$.

Finally, I can derive a sufficient condition for every potential entrant wanting to pay the entry cost, as mentioned in Footnote 32. The value of an innovating firm which does not produce any goods, denoted by $V_t(0)$, is equal to $E_t$. Solving Equation (8) forward (and assuming $\lim_{s \to +\infty} (1 - \delta)^s E_t(E_t + s) = 0$) gives

$$E_t = E_t \sum_{s=0}^{+\infty} (1 - \delta)^s \frac{\left( \left( 1 - \frac{1}{\gamma R} \right) S_{t+s+1} + \frac{1 - \delta}{\delta} E \left( \pi^I \right) \right)^2}{4w R_{t+s}^\eta}. \quad \text{for every } s, R_{t+s} \leq R \text{ and } S_{t+s} \geq S_L.$$ 

For every $s$, $R_{t+s} \leq R$ and $S_{t+s} \geq S_L$. Therefore, a lower bound for $E_t$ is given by $\sum_{s=0}^{+\infty} (1 - \delta)^s \left( \left( 1 - \frac{1}{\gamma R} \right) S_L + \frac{1 - \delta}{\delta} E \left( \pi^I \right) \right)^2 = \frac{\left( \left( 1 - \frac{1}{\gamma R} \right) S_L + \frac{1 - \delta}{\delta} E \left( \pi^I \right) \right)^2}{4w R^\eta}$. Thus, a sufficient condition for the entry cost being always smaller than the value of an incumbent innovating firm producing 0 goods is

$$\varphi w \leq \frac{\left( \left( 1 - \frac{1}{\gamma R} \right) S_L + \frac{1 - \delta}{\delta} E \left( \pi^I \right) \right)^2}{4w R^\eta \delta}. \quad (23)$$

The actual parameter values chosen for the analysis in Section 4 are given in Table 4.

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53 Equations (21) and (22) are also sufficient parameter conditions when, as in the pre-crisis situations of my impulse response analysis, there are aggregate demand fluctuations. This is because the latter always lower radical innovation.
Table 4: Parameter values

<table>
<thead>
<tr>
<th>Parameter</th>
<th>Value</th>
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<tbody>
<tr>
<td>$\gamma_R$</td>
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</tr>
<tr>
<td>$\gamma_I$</td>
<td>1.005</td>
</tr>
<tr>
<td>$\eta$</td>
<td>0.45</td>
</tr>
<tr>
<td>$\delta$</td>
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<tr>
<td>$S_H$</td>
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<tr>
<td>$c_I$</td>
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</tr>
<tr>
<td>$p_H$</td>
<td>0.9</td>
</tr>
<tr>
<td>$M^{\text{New}}$</td>
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</tr>
<tr>
<td>$S_L$</td>
<td>4.8</td>
</tr>
<tr>
<td>$\varphi$</td>
<td>0.43</td>
</tr>
<tr>
<td>$w$</td>
<td>1</td>
</tr>
</tbody>
</table>

These parameter values are chosen for illustrative purposes, and have no pretension to be realistic. Thus, in particular, the value for the parameter $\eta$ differs from the estimates discussed in Section 5.

**Tracking the joint distribution of size and cash holdings**

I denote by $m_t (n, z)$ the mass of innovating firms which in period $t$ produce $n$ goods and have cash holdings $z$ after collecting their period $t$ profits. The transition law from $m_t (\cdot)$ to $m_{t+1} (\cdot)$ is easy to characterize. For every $n$ and $z$, a fraction $1 - \delta$ of the mass $m_t (n, z)$ of firms remains innovating in period $t + 1$. Among those, a fraction $r_t$ produces $n + 1$ goods and holds $z - w r_t^2 R_t^\eta + \left(1 - \frac{1}{\gamma_R}\right) S_{t+1} + n \pi_{t+1}^I$ units of cash, while the remaining fraction $1 - r_t$ still produces $n$ goods and holds $z - w r_t^2 R_t^\eta + n \pi_{t+1}^I$ units of cash. The total mass of innovating firms in period $t + 1$ is made up by these incumbents and, if financing conditions are normal, by a mass $M^{\text{New}}$ of entrants producing 0 goods and holding $-\varphi w$ units of cash.

When aggregate demand is high and financing conditions are normal for an arbitrary high number of periods, $m_t (\cdot)$ converges to an invariant distribution.\(^{55}\) This invariant distribution is the one with which the industry enters the crisis period $T$ in case of a shock to financing conditions (Section 4.1). For a joint shock to financing conditions and aggregate demand (Section 4.3), the situation is slightly more complex. In that case, the invariant distribution prevails in period $T - 1$. The distribution in period $T$ can be recovered by applying the transition law described above, taking into account the fall in radical and incremental innovation in period $T - 1$ triggered by the anticipation of lower aggregate demand, and the lower profits in period $T$.

**Estimates for the parameter $\eta$**

Both Acemoglu and Cao (2015) and Akçigit and Kerr (2015) use continuous time models, in which firms spend on radical (in Akçigit and Kerr’s terminology, “external”) R&D to generate a Poisson arrival rate of radical innovations. Even though my model is specified in discrete time, its cost function is directly comparable to

\(^{54}\) $r_t$ may depend on $z$ if there are crisis financing conditions in period $t$.

\(^{55}\) This distribution is most conveniently computed by taking into account a third dimension, firm age. In every period, there is a mass $(1 - \delta)^a M^{\text{New}}$ of innovating firms of age $a$. Firms of age 0 produce 0 goods and hold $-\varphi w$ units of cash. Firms of age 1 must have had age 0 in the previous period, so I can determine all possible combinations of goods and cash holdings they may have, and the mass of firms in each of these “states”, by using the same logic as for the general transition law. Proceeding this way, I can calculate the mass of firms of any size and cash level for every age, and then recover $m_t (\cdot)$ by integrating over age.
the ones of these continuous-time models.\textsuperscript{56}

In Acemoğlu and Cao’s model, innovation overlap does not appear in the firm’s radical R&D cost function, but instead lowers its Poisson arrival rate to \( r_t R^{-\tilde{g}}_t \) (to avoid confusion, I continue using my model’s notation). Using this specification does not change my model substantially, and it is easy to show that Acemoğlu and Cao’s calibration choice of \( \tilde{g} = 0.5 \) corresponds to setting \( \eta = 1 \) in my model.\textsuperscript{57}

In Akçigit and Kerr’s model, a firm producing \( n \) goods needs to spend \( r_t^2 n^{-2\sigma} \) units of resources to receive a Poisson arrival rate \( r_t \) for radical innovations.\textsuperscript{58} In my model, in a period with normal financing conditions, the radical R&D cost function of a new firm entering the industry can be written as

\[
r_t^2 n_t^{-2\sigma} \tilde{\Upsilon}_t, \quad \text{where } n_t = \frac{M_t}{M_t} \text{ is the average number of goods produced by an innovating firm and } \tilde{\Upsilon}_t = w_t M_t \left[ \frac{1}{\alpha} \left( \left( 1 - \frac{1}{\gamma_t} \right) S_{t+1} + \frac{1-\delta}{\sigma} E \left( \pi^I \right) \right) \right]^{\frac{1}{1+2\sigma}}.\textsuperscript{59} \]

Akçigit and Kerr’s structural estimation yields \( \sigma = 0.4 \).

Thus, choosing \( \eta = 4 \), the elasticity of a new firm’s radical R&D cost to average innovating firm size in my model equals the elasticity of radical R&D costs to firm size in theirs.\textsuperscript{60}

### Calculations for determining permanent losses of industry output

For any post-crisis period \( T + s \), if the mass of innovating firms is equal to a fraction \( x_{T+s} \) of its pre-crisis level, then, all else equal, the mass of radical innovations is equal to a fraction \( x_{T+s}^{\frac{1}{1+2\sigma}} \) of its pre-crisis level. The Spanish data suggest \( x_{T+1} = 0.7 \), and Equation (13) implies, under the assumption that there are no further shocks to financing conditions, the law of motion \( x_{T+1+s} = 1 - (1 - \delta)^s (1 - x_{T+1}) \). This allows me to calculate the mass of radical innovations (expressed as a fraction of its pre-crisis level) for every period starting from \( T + 1 \).

Given that radical innovation contributes 80% to overall productivity growth (and assuming that incremental innovation has not changed), productivity growth generated in period \( T + s \) is then given by

\[
g_{T+s} = \left( 0.2 + 0.8 x_{T+s}^{\frac{1}{1+2\sigma}} \right) g_{T-1}, \tag{24} \]

where \( g_{T-1} \) stands for the growth rate generated by the pre-crisis innovation level.\textsuperscript{61} Then, the permanent

\textsuperscript{56}Indeed, suppose I divide the time between period \( t \) and period \( t+1 \) into subperiods of length \( \Delta \), and assume firms need to spend \( c_R \) (\( r_t, \Delta \) to do a radical innovation with probability \( r_t \Delta \) between time \( t \) and \( t + \Delta \). Then, when \( \Delta \rightarrow 0 \), my model converges to a continuous-time model in which firms spend \( c_R \) (\( r_t, \Delta \) to receive a Poisson arrival rate \( r_t \) of radical innovations.

\textsuperscript{57}Equation (9) still applies with this alternative specification. Furthermore, now \( R_t = \int_{M_t} r_t R_t^{-\tilde{g}} \), and combining these two

\textsuperscript{58}This already assumes that the cost function is quadratic in \( r_t \), as Akçigit and Kerr do for all their quantitative analysis.

\textsuperscript{59}This follows from directly from the expression for aggregate radical innovation in periods with normal financing conditions, given in Footnote 33.

\textsuperscript{60}As noted previously, in my model, fluctuations in \( M_t \) affect aggregate radical innovation. Calculations in Section 5 abstract from this effect and instead focus on quantifying the effect of a fall in \( M_t \) keeping all other variables constant. Including the effects of the fall in \( M_t \) in the calculations would naturally only increase the estimate for the productivity loss.

\textsuperscript{61}This equation uses the approximation \( g_t \approx (1 + g_t) \). Indeed, \( (1 + g_t) = R_{t-1} \ln (1 + g_t) \). Thus, when assuming \( \frac{R_{t-1} \ln \gamma_R}{g_t} = 0.8 \) and \( \frac{I_{t-1} \ln \gamma_I}{g_t} = 0.2 \), Equation (24) follows directly.
output loss $L$ induced by the fall in the mass of innovating firms is given by

$$L = \prod_{s=1}^{+\infty} \frac{(1 + g_{T+s})}{\prod_{s=1}^{+\infty} (1 + g_{T-1})}. \quad (25)$$

## B A model with free entry

### B.1 Assumptions

I consider here an extension of my model in which there is an arbitrary large mass of potential entrants in every period. Potential entrants face an entry cost of $\psi E_t^\nu$ units of labour, where $\psi$ and $\nu$ are positive constants, and $E_t$ denotes the aggregate mass of entrants.\(^{62}\) This specification nests the baseline case presented in the main text: when $\psi = \varphi (M^{\text{New}})^{-\nu}$ and $\nu$ is arbitrarily large, entry costs are equal to $\varphi$ as long as entry does not exceed $M^{\text{New}}$, and tend towards positive infinity if entry exceeds $M^{\text{New}}$. Thus, under the parameter restriction given by Equation (23), entry always equals $M^{\text{New}}$ in periods with normal financing conditions. For simplicity, I abstract from aggregate demand shocks in this extension (that is, I assume $p_H = 1$).\(^{63}\)

### B.2 Equilibrium conditions and balanced growth path

In every period with normal financing conditions, free entry implies $\psi w E_t^\nu = V_t(0)$. Using Equations (8), (13) and the expression for $R_t$ given in Footnote 33, this gives

$$\psi w E_t^\nu = \left( \frac{A}{2} \right)^{2+\eta} \frac{1}{w ((1-\delta) M_{t-1} + E_t)^\eta} + (1-\delta) \psi w E_{t+1}^\nu, \quad (26)$$

where $A = \left( 1 - \frac{1}{\gamma_R} \right) S_H + \frac{1-\delta}{\delta} \left( \left( 1 - \frac{1}{\gamma_I} \right) S_H - \omega c_I \right)$.

On the balanced growth path, entry and the mass of innovating firms are constant and hold $M = \frac{E}{\delta}$. Therefore, Equation (26) implies

$$E = \left( \frac{A}{2} \right)^{2+\eta} \left( \frac{1}{\delta w} \right)^{\frac{1+\eta}{\eta}} \frac{1}{\psi w} \frac{1}{(1+\eta)^\frac{1+\eta}{\eta}}. \quad (27)$$

From this, I can easily deduce the balanced growth path values of all other variables, just as in Section 3.3.

\(^{62}\)This implies that the entry cost of all firms [and not only the one of the marginal entrant] increases in $E_t$. Alternatively, I could assume heterogeneous entry costs, given by $\psi e^\nu$ (where $e$ denotes a potential entrant’s index on the real line). This model generates the same results, but is computationally more intensive, as it adds a further layer of firm heterogeneity.

\(^{63}\)Note, however, that aggregate demand shocks would have richer effects in this extension than in the baseline model, as they affect entry incentives. Thus, a fall in aggregate demand can now also persistently lower the mass of innovating firms.
B.3 Impulse responses to a shock to financing conditions

What does this alternative model imply for the reaction to a shock to financing conditions?

As before, potential entrants cannot borrow to finance the entry cost in the crisis period $T$. Thus, the mass of innovating firms falls on impact to $M_T = (1 - \delta) M$. In the next period, Equation (26) defines $E_{T+1}$ as a function of the (known) mass of incumbent innovating firms $M_T$ and the (unknown) mass of future entrants, $E_{T+2}$. I can then determine the equilibrium value of $E_{T+1}$ using a “shooting algorithm” (Ljungqvist and Sargent (2012)). Indeed, for every guess for $E_{T+1}$, I can solve forward for the entire path of entry. As Figure 7 illustrates, there is a unique guess which puts entry on a stable path, converging back to its balanced growth path value $E$. This must be the equilibrium path: on all others, entry either becomes negative (which is impossible) or tends to positive infinity (which is also impossible, as this would drive the aggregate mass of radical innovations to infinity, and thereby every firm’s radical innovation probability and the value of an entrant to 0).

Figure 7: Entry paths for different guesses of $E_{T+1}$

Notes: Each dotted line shows a path of entry corresponding to an initial guess of $E_{T+1}$. The solid line marks the balanced growth path level of entry. I assume $\psi = 3$, $\nu = 0.5$, and all other parameter values are still given by Table 4.

Figure 8 plots the impulse responses to a shock to financing conditions under three specifications: constant entry costs ($\nu = 0$), increasing entry costs ($\nu = 0.5$), and the baseline model.\(^{64}\)

When entry costs are constant, $E_{T+1}$ overshoots so much that the mass of innovating firms immediately returns to its balanced growth path level. However, under the more realistic assumption of increasing entry costs, overshooting is not sufficient to immediately compensate for the missing generation. Then, as in the baseline model, the mass of innovating firms, radical R&D and radical innovation remain persistently depressed.\(^{65}\)

\(^{64}\)In the first case, $\psi = 0.4$, and in the second one, $\psi = 3$. All other parameter values are still the ones of Table 4.

\(^{65}\)In all cases, the other mechanisms described in Section 4.1 (the misallocation of radical R&D during the crisis and the persistent fall in incremental R&D) also remain present.
Figure 8: Impulse responses to a shock to financing conditions, models with free entry

Panel 1: Mass of innovating firms

Panel 2: Entry

Panel 3: Radical innovation mass ($R_t$)

Panel 4: Incremental innovation mass ($I_t$)

C Data sources and additional empirical results

C.1 INE innovation data

The Encuesta sobre Innovación en las empresas is a survey carried out annually by the INE since 2002, targeting all Spanish firms with 10 employees or more in a wide range of sectors. Its industry-level results are publicly available at http://www.ine.es.

Definitions  Total R&D, shown in Figure 1, is the sum of total internal and external R&D. Real R&D is calculated by deflating the original nominal series with a deflator for manufacturing value added, taken from the Spanish National Accounts in the Eurostat database. R&D-performing firms are defined as all firms which report, in a given year’s survey, to spend either “continuously” or “occasionally” on R&D. Technologically innovating firms are defined as all firms which report in the survey for a given year $t$ to have carried out a
product innovation, a process innovation, or both during the years between \( t - 2 \) and \( t \).

**Industries and changes in the industry classification**  In 2008, the INE changed its industry classification, passing from NACE Revision 1.1 to NACE Revision 2. This slightly changed the definition of the manufacturing sector, as it moved Recycling (NACE 1.1 code 37) out of manufacturing. Furthermore, one part of Publishing, printing and reproduction of recorded media (NACE 1.1 code 22) moved out of manufacturing, while the other part was affected to the new manufacturing industry Printing and reproduction of recorded media (NACE 2 code 18). To maintain comparability, I exclude for all aggregate statistics (Figure 1 and Table 1) the recycling and the publishing and printing industry from all pre-2008 data, and the printing industry from all post-2008 data.\(^{66}\)

Finally, in the industry-level analysis of Figure 3, which focuses on the post-2008 period, there is data for twenty manufacturing industries (NACE 2 code in brackets): Food, beverages and tobacco (10-12), Textiles (13), Wearing Apparel (14), Wood and wood products (16), Paper and Paper products (17), Printing and Publishing (18), Chemicals (20), Pharmaceuticals (21), Rubber and plastics (22), Other non-metallic minerals (23), Basic metals (24), Fabricated metals (25), Computer, electronic and optical products (26), Electrical equipment (27), Machinery (28), Motor vehicles (29), Air and Spacecraft (303), Other transport equipment (30 without 303 and 301 (Shipbuilding)), Furniture (31) and Other manufacturing (32).

**C.2 ESEE data**

The ESEE targets all manufacturing industries, with the exception of the Coke and Refined Petroleum industry (NACE 2 code 19). In total, the dataset has information for 4593 different firms over the period I consider. I define R&D as the sum of internal and external R&D spending (variables GI ID and GEID), and granted patents as the sum of granted patents in Spain and in the rest of the world (variables PATESP and PATEXT).\(^{67}\) Table 5 reports some summary statistics for the main variables used in my analysis. It shows, in particular, that R&D and patenting are relatively rare events: only around 35% of observations have positive R&D, and only 6% have patents.

\(^{66}\)In Table 1, I moreover need to omit some industries for which the INE does not report a split between firms with more or less than 250 employees for confidentiality reasons. These industries are, in 2013, Textiles, Apparel and Leather (NACE 2 codes 13-15), Wood and Paper (16-17), Petroleum (19), Ship construction (301) and Repair [33]. They account for less than 10% of manufacturing R&D. In 2007, I omit the equivalent industries for the NACE 1.1 classification, except for Ship Construction and Repair, for which there is no direct equivalent.

\(^{67}\)Further information on survey design and variable definitions can be found at https://www.fundacionsepi.es/investigacion/eesee/en/spresentacion.asp.
### Table 5: ESEE Summary Statistics

<table>
<thead>
<tr>
<th></th>
<th>Observations†</th>
<th>Average</th>
<th>10th percentile</th>
<th>Median</th>
<th>90th percentile</th>
</tr>
</thead>
<tbody>
<tr>
<td>Employment</td>
<td>36145</td>
<td>239</td>
<td>12</td>
<td>50</td>
<td>493</td>
</tr>
<tr>
<td>Sales (thousands of Euros)</td>
<td>36126</td>
<td>64014</td>
<td>634</td>
<td>6120</td>
<td>112771</td>
</tr>
<tr>
<td>R&amp;D</td>
<td>12797</td>
<td>2653</td>
<td>18</td>
<td>273</td>
<td>2949</td>
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<tr>
<td>Patents</td>
<td>2258</td>
<td>7</td>
<td>1</td>
<td>2</td>
<td>12</td>
</tr>
</tbody>
</table>

Notes: † This column lists the number of observations with positive values for the considered variable.

The ESEE’s industry classification roughly corresponds to the NACE Rev. 2 two-digit level, and distinguishes 19 industries: Meat products, Food and tobacco, Beverages, Textiles and clothing, Leather, fur and footwear, Timber, Paper, Printing, Chemicals and pharmaceuticals, Plastic and rubber products, Nonmetal mineral products, Basic metal products, Fabricated metal products, Machinery and equipment, Computer products, electronics and optical equipment, Electric materials and accessories, Vehicles and accessories, Other transport equipment, Furniture, and Other manufacturing.

### C.3 Additional regression tables

Table 6 summarizes the results of the regressions described in Section 2.1. Note that I exclude in these regressions observations for firms which report to have undergone a merger, acquisition or spin-off in year $t$, as well as observations for firms in which the reporting unit changed in year $t$. Furthermore, by definition, firms which exit the sample in year $t$ are also not included. Defining their employment and R&D in year $t$ as 0 and including them would probably strengthen my results.

Table 7 shows the results for the regression of innovation statistics on firm size in two sub-samples: a non-crisis sample consisting of the periods 1994-1998, 1999-2003, 2004-2008 and a crisis sample consisting of the period 2009-2013. As in the main text, regressions are still estimated on the population of continuously (Column (a)) and occasionally innovating firms (Column (b)), defined with respect to the entire sample. Note that the results in Table 7 (just as the ones in Table 2) exclude all firms which once during their observation in the sample report a merger, acquisition, spin-off, or change of the reporting unit.

---

68 That is, I drop observations for which the ESEE variable CAMBIO equals 1, 2, 3 or 4, and observations for which the variable IDREF equals 2.
Table 6: R&D growth rates and firm size

<table>
<thead>
<tr>
<th>Year</th>
<th>2004</th>
<th>2005</th>
<th>2006</th>
<th>2007</th>
<th>2008</th>
</tr>
</thead>
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<tr>
<td><strong>Firm Size</strong></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td></td>
<td>−0.013</td>
<td>−0.026</td>
<td>−0.061*</td>
<td>−0.068*</td>
<td>0.049</td>
</tr>
<tr>
<td></td>
<td>(0.030)</td>
<td>(0.032)</td>
<td>(0.034)</td>
<td>(0.038)</td>
<td>(0.032)</td>
</tr>
<tr>
<td><strong>Sales growth</strong></td>
<td>0.290</td>
<td>0.503*</td>
<td>0.466*</td>
<td>−0.044</td>
<td>−0.027</td>
</tr>
<tr>
<td></td>
<td>(0.187)</td>
<td>(0.257)</td>
<td>(0.262)</td>
<td>(0.217)</td>
<td>(0.233)</td>
</tr>
<tr>
<td><strong>Observations</strong></td>
<td>487</td>
<td>482</td>
<td>481</td>
<td>476</td>
<td>670</td>
</tr>
<tr>
<td><strong>R²</strong></td>
<td>0.001</td>
<td>0.087</td>
<td>0.007</td>
<td>0.063</td>
<td>0.004</td>
</tr>
<tr>
<td><strong>Industry FE</strong></td>
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<td>NO</td>
<td>YES</td>
<td>NO</td>
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</tbody>
</table>

<table>
<thead>
<tr>
<th>Year</th>
<th>2009</th>
<th>2010</th>
<th>2011</th>
<th>2012</th>
<th>2013</th>
</tr>
</thead>
<tbody>
<tr>
<td><strong>Firm Size</strong></td>
<td>0.069**</td>
<td>0.050</td>
<td>−0.007</td>
<td>−0.013</td>
<td>0.111***</td>
</tr>
<tr>
<td></td>
<td>(0.034)</td>
<td>(0.036)</td>
<td>(0.033)</td>
<td>(0.036)</td>
<td>(0.032)</td>
</tr>
<tr>
<td><strong>Sales growth</strong></td>
<td>0.377**</td>
<td>0.216</td>
<td>0.129</td>
<td>0.327*</td>
<td>0.264</td>
</tr>
<tr>
<td></td>
<td>(0.175)</td>
<td>(0.199)</td>
<td>(0.202)</td>
<td>(0.197)</td>
<td>(0.196)</td>
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<tr>
<td><strong>Observations</strong></td>
<td>645</td>
<td>640</td>
<td>674</td>
<td>670</td>
<td>655</td>
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<tr>
<td><strong>R²</strong></td>
<td>0.007</td>
<td>0.042</td>
<td>0.000</td>
<td>0.041</td>
<td>0.019</td>
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<tr>
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<td>NO</td>
<td>YES</td>
<td>NO</td>
</tr>
</tbody>
</table>

**Notes:** ***significant at 1%, **significant at 5%, * significant at 10%. Robust standard errors are given in parentheses.
Table 7: Innovation performance and firm size: Subsample analysis

Non-crisis sample

<table>
<thead>
<tr>
<th>Dependent variable</th>
<th>R&amp;D Sales</th>
<th>R&amp;D</th>
<th>Employment</th>
<th>Employment</th>
<th>R&amp;D</th>
<th>R&amp;D</th>
</tr>
</thead>
<tbody>
<tr>
<td>ln (Employment)</td>
<td>-0.0007**</td>
<td>-0.0006</td>
<td>-0.0049***</td>
<td>-0.0035***</td>
<td>-0.0060***</td>
<td>-0.0052***</td>
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<tr>
<td>Observations</td>
<td>1427</td>
<td>1980</td>
<td>260</td>
<td>474</td>
<td>189</td>
<td>414</td>
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<tr>
<td>$R^2$</td>
<td>0.22</td>
<td>0.19</td>
<td>0.59</td>
<td>0.32</td>
<td>0.53</td>
<td>0.33</td>
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<tr>
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<td>YES</td>
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<tr>
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Crisis sample

<table>
<thead>
<tr>
<th>Dependent variable</th>
<th>R&amp;D Sales</th>
<th>R&amp;D</th>
<th>Employment</th>
<th>Employment</th>
<th>R&amp;D</th>
<th>R&amp;D</th>
</tr>
</thead>
<tbody>
<tr>
<td>ln (Employment)</td>
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<td>-0.0057****</td>
<td>-0.0039***</td>
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<td>NA</td>
</tr>
</tbody>
</table>

Notes: ***(significant at 1%), **(significant at 5%), *(significant at 10%. Robust standard errors are given in parentheses.