Political Specialization*

Bernardo Guimaraes†
Sao Paulo School of Economics (FGV)

Kevin D. Sheedy‡
London School of Economics

January 2016

Abstract

This paper presents a theory of political specialization in which some countries uphold the rule of law while others consciously choose not to do so, even though they are ex ante identical. This is borne out of two key insights: for incumbents in each country, (i) the first steps to the rule of law have the greatest private cost, and (ii) steps taken by some countries in the direction of the rule of law make it less attractive for others to follow the same path. The world equilibrium features a symbiotic relationship between despotic and rule-of-law economies: by producing technology-intensive goods that require protection of property rights, rule-of-law economies raise the relative price of natural resources and increase incentives for despotism in other countries; while the choice of despotism entails a positive externality because cheap oil makes the rule of law more attractive elsewhere in the world.

JEL classifications: D74; F12; F43; O43; P48.

Keywords: political specialization; international trade; rule of law; power sharing.

*We thank Cristina Terra and seminar participants at the Anglo-French-Italian Macroeconomics Workshop, HSE-ICEF Moscow, 1st International REAP Meeting, London School of Economics, Sao Paulo School of Economics (FGV), and the Stony Brook Workshop on Political Economy for their comments.

†Sao Paulo School of Economics (FGV). Email: bernardo.guimaraes@fgv.br

‡LSE, CEP, CEPR, and CfM. Email: k.d.sheedy@lse.ac.uk

1 Introduction

It is a commonplace to claim that the world has become smaller and there are fewer and fewer differences between formerly exotic places and the West. An unprecedented flow of goods and ideas has allowed emulation of faraway countries, resulting in a large increase in conformity across the globe. However, politics seem to be immune to this trend. While the rule of law can be taken for granted in large parts of the world, authoritarianism prevails in far too many places, in spite of its well-known negative consequences.

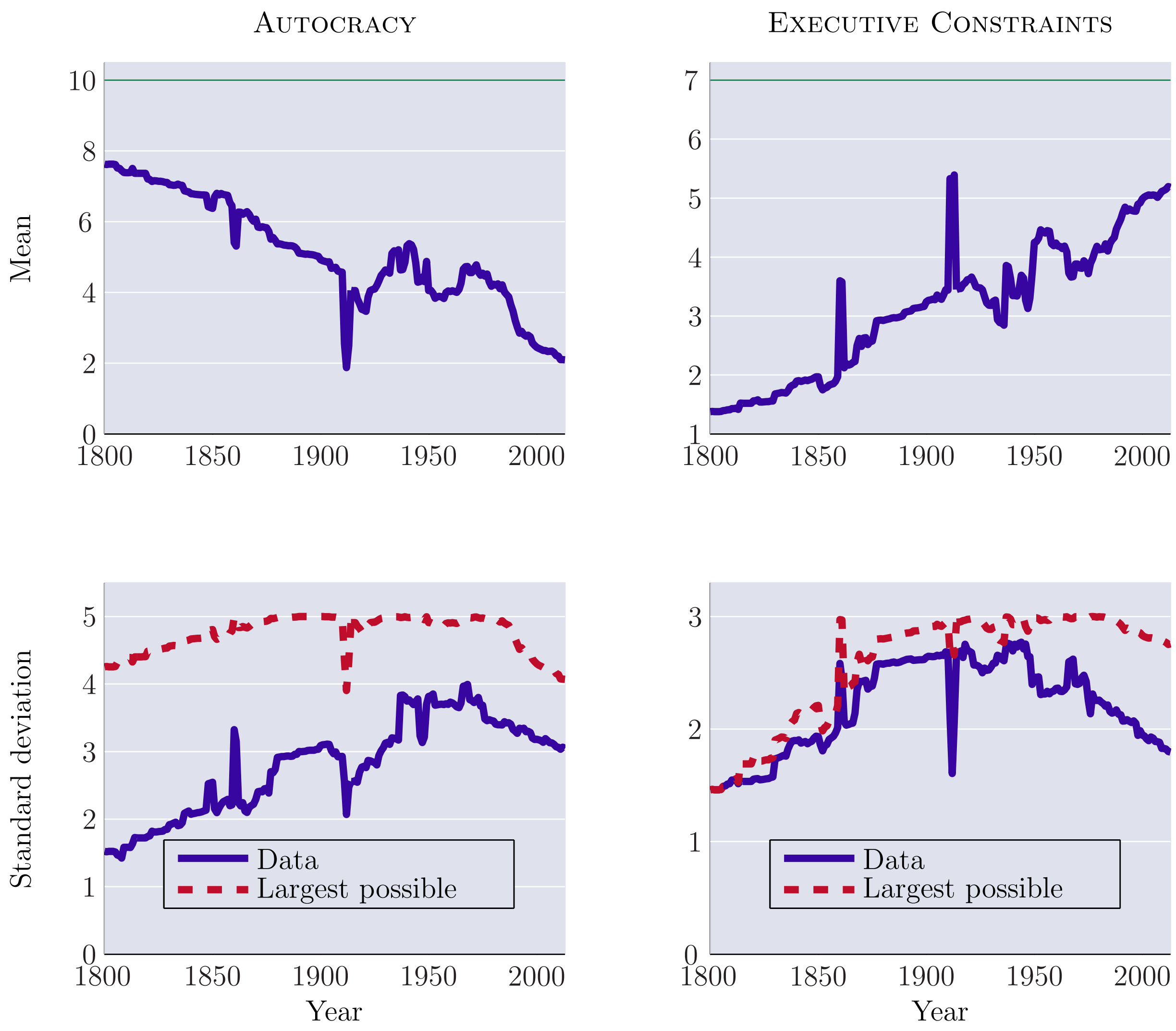
Figure 1 summarizes some key trends in governance around the world during the last two centuries. The charts show time series for two indices from the Polity IV database: 'autocracy' (an index from 0 to 10) on the left, and 'executive constraints' (an index from 1 to 7) on the right. The top panels display population-weighted cross-country means of the two Polity scores for each year. Clearly, the world as a whole now features much less autocratic government and substantially more constraints on the executive than two centuries ago.

Interestingly, Figure 1 highlights that this change has occurred largely on the extensive margin. It is not that there has been a more or less uniform decline in autocracy across countries. Instead, some countries have substantially reduced or eliminated autocratic government, while others have seen little improvement. This is illustrated in the bottom panels of the figure which plot the population-weighted cross-country standard deviations of the two Polity scores. To judge the size of these standard deviations, consider a benchmark standard deviation where all countries are divided into two blocs with the minimum and maximum scores respectively, and the fractions of countries in each bloc are set so that the average score is equal to the observed global mean. The dashed lines in the bottom panels show these maximum possible standard deviations in each particular year. For both Polity scores, the standard deviation has remained closer to the limit of full divergence rather than moving towards zero. Instead of political convergence, a persistent pattern of political specialization is observed.

This paper presents a theory of political specialization to understand how an increasingly interconnected world can nonetheless sustain diametrically opposed systems of government. According to the theory, some countries will uphold the rule of law with commitment to property rights, while others will consciously choose not to do so. Moreover, the theory implies that political specialization is to be expected even if all countries were ex ante identical. This political specialization is borne out of two key insights of the theory: (i) there is a diminishing marginal benefit of good government at the world level, but not at the country level; and (ii) there is a diminishing marginal cost of good government at the country level. 'Good government' is taken to mean the extent to which individuals appropriate the benefits of their own production.

The first key insight is due to the impact of good government on economic activity varying between different types of goods. Some production can occur even in despotic countries where individuals have no protection against expropriation, for example, extraction of natural resources. However, production of goods that require long-term investments in physical capital or research and development ('rule-of-law intensive goods' hereafter) relies on investors expecting property rights to

Figure 1: *Means and standard deviations of political regime characteristics in the world*



Notes: Annual data (1800–2013) on 'Institutionalized Autocracy' (AUTOC, score between 0 and 10) and 'Executive Constraints' (XCONST, score between 1 and 7), with the means and standard deviations calculated from the population-weighted cross-section of countries. The spikes in the graphs are due to missing observations for large countries (mainly China) in some years. The series labelled 'largest possible' in the lower panels are the maximum possible standard deviations for distributions where the mean matches the data (corresponding to distributions with two point masses at the extreme scores).
Source: Polity IV Project, Center for Systemic Peace (http://www.systemicpeace.org/inscrdata.html).

be enforced. Hence, at the world level, the marginal benefit of good government is declining owing to the diminishing marginal rate of substitution between rule-of-law intensive goods and other goods less sensitive to the political system. However, at the country level, access to world markets means that the marginal benefit of good government is constant for a small open economy that does not affect world prices.

The second key insight is that the marginal cost of good government is decreasing at the level of an individual country. What is meant by the marginal cost of good government is the marginal loss of rents received by those who hold power as a consequence of marginally better government. The idea is that while improvements in governance such as the rule of law will increase economic activity, they will also curtail the rents that incumbents are able to extract. Those in power in economies where the rule of law is pervasive will receive only little in rents, while those in autocracies will extract a substantial amount. Consequently, better government is 'cheap' at the margin to those in power in countries where the quality of governance is already high because they have only small rents to lose, while better government is 'expensive' at the margin to those in power in autocracies because they have a lot to lose.

Combined, these two insights lead to political specialization. Owing to the diminishing marginal cost of good government combined with the constant marginal benefit at the level of an individual country, countries will either fail to provide any security to investors, or will have full protection of property rights. Starting from autocracy, the first steps to better government cost incumbents more in terms of lost rents than they gain from increased economic activity. However, additional steps in that direction lead to further improvements in economic activity and progressively smaller losses of rents. If a tipping point is reached where the subsequent gains outweigh the initial losses then a country can move from autocracy to the rule of law, but if not then it is in the interests of incumbents not to take the first steps.

While this logic pushes individual countries to one of the extremes, the same reasoning does not apply to the world as a whole. The diminishing marginal benefit of good government at the world level means that the relative price of rule-of-law intensive goods is lower when good government is more widespread around the world. As the price of rule-of-law intensive goods falls in world markets, incumbents' calculations of the gains and losses from the rule of law tip in favour of autocracy. This leads to strategic substitutability in the choice of good government among countries, and implies there will be a distribution of political regimes around the world in equilibrium.

The world equilibrium features a symbiotic relationship between rule-of-law economies and despotic regimes. By producing goods requiring protection of property rights, rule-of-law economies raise the relative price of other goods such as natural resources and thus increase incentives for despotism in other countries. Conversely, despotic regimes have a positive externality on other countries because cheap oil makes the rule of law more attractive elsewhere in the world.

The theory provides a way of reconciling the claim that corruption, rent-seeking, and insecure property rights create significant barriers to development in some countries with the fact that history is replete with examples of other countries having overcome precisely these challenges. Without a theory of political specialization, one must rely on exogenous differences in countries' 'political technologies' to understand why bad regimes persist in a world where trade works to equalize the marginal benefits of good government across countries. Such an approach is unappealing in light of the widespread imitation of other successful ideas and technologies around the world. Instead, the theory of political specialization presented here provides an account of how the same political frictions can give rise to both winners and losers — and why not every country can be a winner.

While the theory predicts that political divergence arises even in a world of ex-ante identical economies, the assumption of ex-ante identical economies is not essential to the argument. When ex-ante heterogeneity across countries is considered, comparative advantage determines which countries specialize in which political system, with good government emerging in countries with a comparative advantage in rule-of-law intensive goods. A 'natural resource curse' arises owing to the effects of a comparative advantage in natural resources on the incentives for those in power to resist losing the ability to extract rents.

The theory has some important lessons on how the problem of despotic regimes should be addressed. One prediction is that exogenous improvements in one country's political institutions, perhaps brought about by well-intentioned international pressure or intervention, will tend to be counteracted by an increase in incentives for despotism in other countries. However, this does not preclude a role for international policy because the total number of despotic regimes in the world is affected by the relative price of rule-of-law intensive goods, which is in turn influenced by patterns of demand. The theory thus suggests that channelling resources to the study and development of technology-intensive alternative fuels would be more effective in curbing despotism than efforts directed to affect the political systems of particular countries.

In modelling the ideas discussed above, although the diminishing marginal benefit from producing rule-of-law intensive goods is a natural feature of any economic model, the diminishing marginal cost of good government ought not to be a primitive in any formal model of politics. In order to understand the nature and the properties of the costs of good government, it is essential to consider how questions of distribution are resolved through the use of political power. Following from this, it is appropriate to study the adoption of political institutions using a model where the distribution of power and resources is jointly determined. Upholding the rule of law requires that those who hold power are able to commit to pre-established rules and follow them even when that is not in their own interests. But how can such commitment be achieved and what costs are entailed?

An important implication of the model is that power sharing enables commitment to rules that would otherwise be time inconsistent. Power sharing supports the rule of law because it increases the number of potential losers from changes to the status quo, and thus raises the number of people willing to defend the status quo against threats from both inside and outside the group in power. However, sharing power requires sharing the rents associated with holding positions of power, which goes against the interests of incumbents and drives a wedge between the social return to investments and the return perceived by incumbents.

Power sharing is necessary for good government, but requires sharing rents among incumbents. For incumbents, the cost of sharing power is related to the difference between incumbents' own incomes and the incomes of those outside the group in power. That difference is lower in economies with more power sharing, since an additional member is less important to the overall group of incumbents when power is already shared more widely. That is why the first steps towards the rule of law have the greatest private cost to incumbents.

In equilibrium, incumbents of ex-ante identical countries are indifferent between the choice of the rule of law and despotism, and they always gain from the possibility of trading internationally. However, the ensuing political specialization leads to economic divergence as well. The economies that adopt the rule of law become substantially richer than despotisms that produce no rule-of-law intensive goods. International trade thus benefits some countries but harms others.

Since the mechanism for political specialization relies on international trade, an implication of the model is that the possibility of trade drives countries apart, economically and politically. Williamson (2011) shows that economic divergence did indeed follow the first wave of globalization in the early 19th century, when the third world 'fell behind'. While those countries with a comparative advantage in primary goods grew more than in previous centuries, the gap between rich and poor countries widened. In terms of politics, by the early 19th century, the rule of law was becoming established in the more advanced European countries as the absolute power of monarchs was eroded by increasingly powerful parliaments. However, countries in the periphery were experiencing none of this political change, despite a large increase in trade, which presumably led to greater exposure to foreign ideas.

Interestingly, this pattern of both economic and political divergence also holds for some countries that were relatively prosperous in the 18th century but which lost ground with the first wave of globalization. Russia is a case in point.[1] During the 18th and early 19th centuries, Russia was a powerful European country that went through modernizing reforms both in the reign of Peter the Great and in the age of the Russian Enlightenment.[2] However, in the early 19th century, Russia experienced strong positive terms-of-trade shocks and subsequently fell behind. By the end of the century, Russia was one of the poorest countries in Europe (Nafziger, 2008). In contrast to the expansion of power sharing in other European countries at the time, and consistent with the story of political specialization proposed here, Russia went through the 19th century without any kind of elected parliament.[3]

The plan of the paper is as follows. Section 1.1 discusses the related literature. Section 2 sets up the model, describing the environment and the assumptions on the power struggle (allocations and rebellions). The equilibrium allocation of power and resources is characterized for an individual country in section 3, and section 4 studies the benchmark case of economies in autarky. The equilibrium allocations in open economies and the world equilibrium are derived in section 5. Section 6 takes up the extension of ex-ante heterogeneity between countries. Section 7 concludes.

1.1 Related literature

The theory of political specialization builds on the framework proposed by Guimaraes and Sheedy (2015). That paper develops a model where an allocation of power and resources is established in the interests of incumbents. An allocation needs to survive rebellions from both inside and outside the group in power. The mechanism for contesting an allocation is the same no matter what that allocation prescribes, and there are no special individuals in the model: everyone is ex ante identical. The model assumes no exogenous technology to protect property rights, instead,

[1]Nafziger (2008) claims that "understanding what inhibited Russian economic development in the nineteenth century is an important task for economic historians."

[2]The expansion of the Russian empire was a sign of its power and development at the time. In the early 19th century, the Russians colonized Alaska and even founded settlements in California. Among other notable Russian sea exploration voyages, in 1820, a Russian expedition discovered the continent of Antarctica.

[3]Nicholas I ruled between 1825 and 1855, the time when Russian exports were becoming more expensive. He resisted any kind of power sharing and concentrated his existing powers even more, crushing demonstrations demanding power sharing (among other things) and abolishing several areas of local autonomy (Bessarabia, Poland, the Jewish Qahal).

power sharing allows for the endogenous emergence of the rule of law. Here, the environment is extended to a world with two goods and many economies, and the equilibrium allocation of power and resources is characterized, taking account of interactions between countries' political systems. The two main theoretical results of this paper are the diminishing marginal cost of good government and the strategic substitutability in political regimes when there is international trade.[4]

There is now a substantial body of research showing that institutional quality has an important role in explaining international trade, as surveyed by Nunn and Trefler (2014). This is consistent with the reason why countries trade with each other in the model here. Much of this work takes institutions as given. The literature studies how institutions affect trade flows (for example, Anderson and Marcouiller, 2002), the pattern of comparative advantage (Levchenko, 2007, Nunn, 2007) and its dynamic effects (Araujo, Mion and Ornelas, 2015).

The paper is also related to a literature on the effects of trade on political institutions. Milgrom, North and Weingast (1990), Greif (1993), and Greif, Milgrom and Weingast (1994) combine historical analysis and game theory to understand how institutions in medieval times allowed merchants to solve a commitment problem that arises in large-scale (international) trade.[5] Acemoglu, Johnson and Robinson (2005) and Puga and Trefler (2014) study how international trade induced institutional change by enriching and empowering merchant groups. Acemoglu, Johnson and Robinson (2005) argue that the Atlantic trade led to better institutions in European countries with good initial conditions, while Puga and Trefler (2014) show how empowering merchants in Venice led to important institutional innovations up to the 13$^{\text{th}}$ century, but also to political closure and reduced competition afterwards. In a similar vein, special-interest groups play a key role in Levchenko's (2013) analysis of the impact of international trade on institutional quality.[6] This paper takes a different perspective. It looks at the world economy as a whole and studies how international trade affects the distribution of good government around the world in a model of ex-ante identical individuals and ex-ante identical countries. The model shows that the world equilibrium is asymmetric, giving rise to winners and losers, a point that is absent from this literature.

In this sense, the paper is related to Acemoglu, Robinson and Verdier (2014), who study specialization in economic systems and also find an asymmetric world equilibrium. However, the question there is a very different one: understanding why different types of capitalism can co-exist, in particular, why we cannot all be like Scandinavians as opposed to Americans. Here, the question is why examples of good government in some countries co-exist with examples of abject failure in others — why some of us must be Venezuelans. In consequence, the model here is completely different from Acemoglu, Robinson and Verdier (2014). For example, the power struggle plays a central role here but is absent from their analysis, while here the paper abstracts from changes in the world technological frontier, which is central to their paper.

As in the 'new trade theory' models of Krugman (1979, 1980), the model here predicts a sub-

[4]There are alternative models related to power sharing (Acemoglu and Robinson, 2000, Jack and Lagunoff, 2006, Bai and Lagunoff, 2011), but power sharing in those models is an extension of the democratic franchise. In contrast, power sharing in Guimaraes and Sheedy (2015) is connected to the emergence of the rule of law.

[5]See also Greif (2006).

[6]See also Grossman and Helpman (1994) and Do and Levchenko (2009).

stantial amount of trade between ex-ante identical economies. Those papers assume production technologies with increasing returns, so countries specialize in different varieties of goods to exploit economies of scale, and trade benefits all countries. Here, there is a form of increasing returns to producing rule-of-law intensive goods (from the point of view of incumbents) because a diminishing marginal cost of good government arises endogenously from the power struggle. That leads to political specialization, and trade benefits those in power anywhere in the world irrespective of political system — but not all countries gain from trade.

The paper is also related to the large literature on the 'natural resource curse' working through political institutions.[7] Robinson, Torvik and Verdier (2006) and Mehlum, Moene and Torvik (2006) study the role of institutions in understanding the natural resource curse. While in those papers the curse is a consequence of bad institutions, here the key institutional variable (power sharing) is endogenously determined and is a consequence of having a comparative advantage in natural resources. There are models in which natural resources distort rulers' choices (for example, Acemoglu, Verdier and Robinson, 2004, Caselli and Cunningham, 2009, Caselli and Tesei, 2015), but the distinguishing and important implication of this paper is that for the world as a whole, the number of despotic countries depends on the demand for natural resources, not on the supply.

There is also a large body of work based around the idea that trade hurts economies that specialize in primary goods. One possibility is that some sectors give rise to positive externalities on the whole economy (or within an industry) through knowledge creation.[8] In this paper, trade hurts those economies that fail to institutions conductive to development, but not because of any intrinsic disadvantage of producing primary goods.[9]

Last, the paper is broadly related to discussions of democratization in the social sciences.[10] Following the demise of the Soviet Union and the end of the cold war in the early 1990s, Fukuyama (1992) famously predicted the 'end of history' arguing that the days of autocratic regimes were numbered. Reality, however, has not been so kind, and Fukuyama has since acknowledged that autocratic regimes have been stubbornly persistent (Fukuyama, 2011).[11] More systematically, the absence of convergence in Polity scores has been noted in work by Goorha (2007).

[7] For a discussion of the empirical evidence on the natural resource curse, see Sachs and Warner (2001) and Van der Ploeg (2011).

[8] See, for example, Krugman (1987), Rodrik (1996) and Melitz (2005).

[9] There is also a large literature in sociology that attempts to explain underdevelopment as the result of rich countries exploiting poor ones, so-called 'dependency theory'. See, for example, Cardoso and Faletto (1979).

[10] Huntington (1993) is an influential example.

[11] Likewise, there is work in political science on the survival of autocracies (for example, Gandhi and Przeworski, 2007), but that literature also focuses on individual countries, while this paper studies political equilibrium in the world as a whole.

2 Environment

2.1 Preferences and technologies

The world comprises a measure-one continuum of ex-ante identical countries indexed by $\jmath \in [0,1]$. Each country is an area containing a measure-one population of ex-ante identical individuals indexed by $\imath \in [0,1]$.

There are two goods in the world, an *endowment* good (E) and an *investment* good (I), the names referring to how the goods are obtained. The only use of both goods is consumption. All individuals throughout the world have the same preferences over these goods as represented by the Cobb-Douglas consumption aggregator:

$$C = \frac{c_{\mathrm{E}}^{1-\alpha} c_{\mathrm{I}}^{\alpha}}{(1-\alpha)^{1-\alpha}\alpha^{\alpha}}, \qquad [2.1]$$

where c_{E} and c_{I} are respectively an individual's consumption of the endowment and investment goods, and C is the number of units of the composite good obtained. The parameter α, satisfying $0 < \alpha < 1$, indicates the relative importance of the investment good.

Each individual in each country exogenously receives q units of the endowment good. The investment good requires effort to be produced, and there is a time lag between the effort cost being incurred and the good becoming available. In each country, a positive fraction μ of individuals exogenously receives investment opportunities at random. Each investment opportunity is the ability to produce one unit (a normalization) of the investment good if an effort cost is incurred. Whether or not an investment opportunity is received and taken by an individual is denoted by the binary variable $I \in \{0,1\}$, and the set of all individuals who invest ($I = 1$) is denoted by $\mathcal{I}$. The total amount K of the investment good produced (the capital stock) is the measure of the set $\mathcal{I}$:

$$K = |\mathcal{I}| = \mu s, \qquad [2.2]$$

where $s = |\mathcal{I}|/\mu$ denotes the fraction of investment opportunities that are taken in a country. Receiving an investment opportunity is private information, but whether or not it has been taken (the variable I) becomes common knowledge when the investment good is produced.

All individuals throughout the world have the same preferences defined over consumption C, investment I, and *rebellion effort* R, and these preferences are represented by the utility function:

$$\mathscr{U} = \log C - I\log(1+\theta) - \log(1+R). \qquad [2.3]$$

Individuals receive disutility $\log(1+\theta)$ if they exert effort in taking an investment opportunity (a binary choice, I), where θ is a positive parameter. If investing leads to an individual receiving consumption C_{k} (as a *capitalist*) while not investing leads to consumption C_{w} (as a *worker*) then investing is incentive compatible when:

$$C_{\mathrm{k}} \geq (1+\theta)C_{\mathrm{w}}. \qquad [2.4]$$

The parameter θ thus specifies the cost of taking an investment opportunity as a fraction of consumption. Note that there is no discounting of utility between the time the effort cost is incurred and consumption is received. Before individuals know whether they will receive an investment opportunity, their utility is given by the expected value of (2.3).

Individuals also receive disutility from rebellion effort R, the role of which is explained below. The substantive implication of the functional form in (2.3) is that if an individual could obtain consumption C' instead of C through a rebellion then the individual would be willing to exert no more than $R = (C' - C)/C$ units of rebellion effort. This means that the cost of R units of rebellion effort is measured as a fraction of consumption.

When the countries of the world are in contact with each other, the endowment and investment goods can be exchanged in perfectly competitive world markets. The relative price of the investment good in terms of the endowment good in those markets is denoted by π^*, which all countries take as given. A country must satisfy its international budget constraint given the world price π^*:

$$x_{\mathrm{E}} + \pi^* x_{\mathrm{I}} = 0, \qquad [2.5]$$

where x_{E} and x_{I} respectively denote the country's net exports of the endowment and investment goods. While goods can be mobile internationally, individuals cannot move between countries.

Given production K of the investment good (2.2), and net exports x_{E} and x_{I} of the two goods satisfying the international budget constraint (2.5), a country's resource constraints are:

$$\int_0^1 c_{\mathrm{E}}(\imath)\mathrm{d}\imath + x_{\mathrm{E}} = \mathrm{q}, \quad \text{and} \quad \int_0^1 c_{\mathrm{I}}(\imath)\mathrm{d}\imath + x_{\mathrm{I}} = K, \qquad [2.6]$$

where $c_{\mathrm{E}}(\imath)$ and $c_{\mathrm{I}}(\imath)$ denote the (non-negative) quantities of the endowment and investment goods consumed by individual $\imath \in [0, 1]$.

2.2 Allocations and rebellions

In each country there is an *allocation* of power and resources, which will be determined endogenously. Allocations can be contested through *rebellions*, which lead to new allocations being established, a process referred to as the power struggle. The modelling here follows Guimaraes and Sheedy (2015).

An allocation specifies the set $\mathcal{P}$ of individuals currently in *power*, referred to as the *incumbents*. Each position of power confers an equal advantage on its holder in the event of any conflict, as described below. *Power sharing* $p = |\mathcal{P}|$ is defined as the measure of the group $\mathcal{P}$. The incumbent group $\mathcal{P}$ can have any size between 0% and 50% of the population ($0 \leq p \leq 1/2$). It is assumed that investment opportunities cannot be received by those individuals currently in power, but opportunities are otherwise received at random.

An allocation also specifies how much individuals receive of each good, and how much of each good is exported or imported. Each individual's consumption of each good can depend on whether the individual is in power, and (for some or all individuals) whether an individual has taken an investment opportunity. Since receiving an investment opportunity is private information, an allocation cannot directly compel individuals to produce capital. Instead, by varying the investment-contingent consumption allocation, an allocation can determine investment through its effect on the fraction of individuals for whom investing is incentive compatible.[12]

An allocation specifies a set $\mathcal{C}$ of those individuals whose consumption is contingent on taking an investment opportunity (with $\mathcal{C} \cap \mathcal{P} = \emptyset$, as those in power do not receive investment opportunities). The set $\mathcal{K}$ of *capitalists* comprises those who have an consumption allocation that is contingent on investing and who receive and take an investment opportunity ($\mathcal{K} = \mathcal{C} \cap \mathcal{I}$). The set $\mathcal{W}$ of *workers* comprises those remaining individuals who are neither incumbents nor capitalists.

Formally, an allocation is a collection $\mathscr{A} = \{\mathcal{P}, \mathcal{C}, c_{\text{pE}}, c_{\text{pI}}, c_{\text{kE}}, c_{\text{kI}}, c_{\text{wE}}, c_{\text{wI}}, x_{\text{E}}, x_{\text{I}}\}$, where c_{pE} and c_{pI} are respectively consumptions of the endowment good and the investment good by an individual in power, c_{kE} and c_{kI} are similarly the consumption allocation of the two goods for capitalists, and c_{wE} and c_{wI} are the consumption allocation of the two goods for workers.[13] An allocation $\mathscr{A}$ must be such that the measure of $\mathcal{P}$ (power sharing p) is no more than $1/2$, net exports x_{E} and x_{I} must satisfy (2.5) given the world price π^*, and the non-negative consumption allocation must satisfy (2.6) given the capital stock K in (2.2). Individuals take investment opportunities if and only if the incentive compatibility constraint (2.4) is satisfied given the prevailing allocation, and hence the fraction s implied by an allocation is:

$$s = \begin{cases} \max\{|\mathcal{C}|/\upmu, 1\} & \text{if (2.4) holds} \\ 0 & \text{otherwise} \end{cases}, \qquad [2.7]$$

where the formula makes use of the random assignment of investment opportunities.

The timing of events is depicted in Figure 2. In a given country, an allocation is established, followed by opportunities for rebellions, with a new allocation established if a rebellion succeeds. Once there are no rebellions, there are opportunities to invest. After individuals make investments, there are again opportunities for rebellions, with new allocations established if a rebellion occurs. Finally, endowments are received, and any investment goods that have been produced become available for use. The prevailing consumption allocation is implemented with any trade between countries occurring at this point. The sequence of events occurs simultaneously in all countries: the only interaction between countries is in perfectly competitive world markets where each takes prices as given.

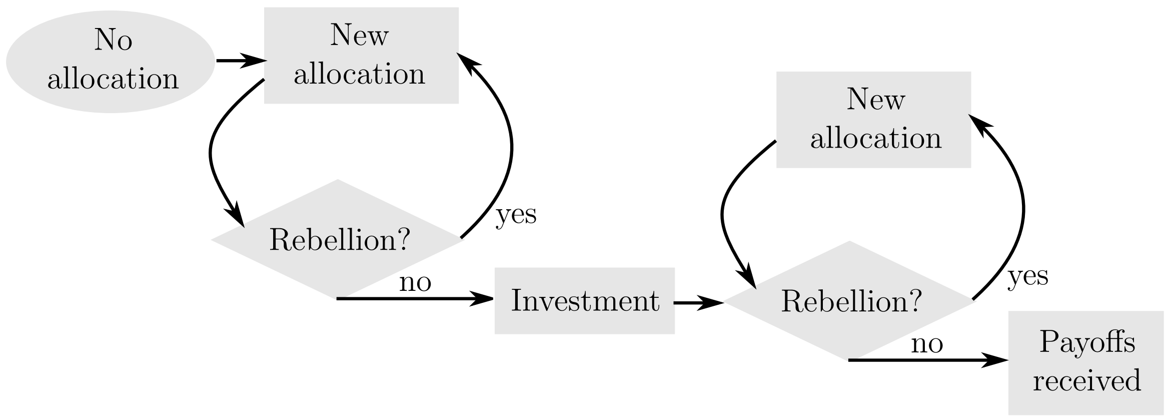
Note that utility from consumption is received at the final stage of the sequence of events in Figure 2, and there is no discounting of payoffs based on the number of rebellions that may occur. Any disutility from rebellion effort is additively separable between different rebellions.[14] At any stage of Figure 2, continuation payoffs are independent of any earlier investment effort or rebellion effort (these effort costs are sunk).

[12]With heterogeneity in the effort cost of investing between different individuals, it would be possible an allocation to determine the number of individuals for whom investing is incentive compatible through an investment-contingent consumption allocation that applies to everyone. See Guimaraes and Sheedy (2015) for an example of how this could be done.

[13]It would be possible to extend the analysis so that an allocation would specify a fully individual-specific consumption allocation, but that would add considerable complexity without necessarily affecting the results. In a related setting, Guimaraes and Sheedy (2015) allow for fully individual-specific consumption allocations but find that in equilibrium, consumption is only contingent on being in power or producing capital.

[14]An individual who exerts rebellion effort in several rebellions receives disutility $\sum_\ell \log(1 + R_\ell)$, where R_ℓ denotes effort in rebellion ℓ.

Figure 2: *Sequence of events*



Rebellions are the only mechanism for changing an established allocation. A rebellion is described by a *rebel faction* $\mathcal{R}$, an amount of (non-negative) rebellion effort $R(\imath)$ for each individual $\imath \in \mathcal{R}$ who belongs to the rebel faction, and a *loyal faction* $\mathcal{L}$ that defends the current allocation. In a given rebellion, the rebel faction can comprise those outside or inside the group currently in power, or a mixture of both. The loyal faction is drawn from those currently in power who do not join the rebel faction. Formally, a rebellion is a collection $\{\mathcal{L}, \mathcal{R}, R(\imath)\}$, where the sets $\mathcal{L}$ and $\mathcal{R}$ satisfy $\mathcal{L} \subseteq \mathcal{P}$ and $\mathcal{L} \cap \mathcal{R} = \emptyset$.

A rebellion succeeds if

$$\int_{\mathcal{R}} R(\imath) \mathrm{d}\imath > \int_{\mathcal{L}} \delta \mathrm{d}\imath, \qquad [2.8]$$

which requires that the strength of the rebel faction exceeds the strength of the loyal faction. Each faction's strength is the integral of the strengths of its members. The strength of individual $\imath \in \mathcal{R}$ in the rebel faction is the amount of rebellion effort $R(\imath)$ he exerts. Each individual $\imath \in \mathcal{L}$ in the loyal faction has strength measured by a parameter δ (the power parameter), which is obtained at no utility cost to these individuals.

Throughout the paper, the following upper bounds are imposed on the parameters μ (the measure of investment opportunities) and θ (the effort cost of investing) given the value of the power parameter δ:

$$\mu \leq \frac{\delta}{2(2+\delta)}, \quad \text{and} \;\; \theta \leq \delta \min\left\{ \frac{2(1+\delta+\delta^2)}{2+\delta^2}, \frac{4+5\delta+2\delta^2}{2(1+\delta)^2} \right\}. \qquad [2.9]$$

The interpretation of these restrictions is discussed later.

2.3 Equilibrium definition

The requirements for an allocation to be an equilibrium of the power struggle described above are now stated. In what follows, let $U(\imath)$ denote individual $\imath$'s continuation utility under a particular allocation (that is, utility net of any sunk effort costs, and assuming the allocation prevails with no further rebellion effort exerted). The notation $'$ is used to signify any aspect of an allocation that would be established following a rebellion.

An equilibrium allocation must be optimal in the sense of maximizing the payoff of incumbents, taking into account the threat of rebellions. Any rebellions must be rational in the sense defined below.

Definition 1 *A rebellion* $(\mathcal{L}, \mathcal{R}, R(\imath))$ *against the current allocation* $\mathscr{A}$ *is rational given the subsequent allocation* $\mathscr{A}' = \{\mathcal{P}', \mathcal{C}', c'_{\mathrm{pE}}, c'_{\mathrm{pI}}, c'_{\mathrm{kE}}, c'_{\mathrm{kI}}, c'_{\mathrm{wE}}, c'_{\mathrm{wI}}, x'_{\mathrm{E}}, x'_{\mathrm{I}}\}$ *if:*

(i) All individuals in the rebel faction $\mathcal{R}$ *receive a position of power under the subsequent allocation yielding a payoff no lower than what the individual would receive under the current allocation, and the disutility of each individual's rebellion effort* $R(\imath)$ *does not exceed his utility gain from rebelling:*

$$\mathcal{R} = \{\imath \in \mathcal{P}' \mid U'(\imath) \geq U(\imath)\}, \quad \text{and} \;\; \log(1 + R(\imath)) \leq U'(\imath) - U(\imath). \qquad [2.10a]$$

(ii) The loyal faction $\mathcal{L}$ *(drawn from those currently holding positions of power who do not rebel) comprises those who would be worse off under the subsequent allocation:*

$$\mathcal{L} = \{\imath \in \mathcal{P} \backslash \mathcal{R} \mid U(\imath) > U'(\imath)\}. \qquad [2.10b]$$

(iii) Condition (2.8) *for a successful rebellion holds.* □

In a rational rebellion, the rebel faction $\mathcal{R}$ includes only individuals who would be in power under the subsequent allocation, which is an assumption designed to capture the incentive problems in inducing individuals to exert effort. The maximum amount of rebellion effort exerted by an individual in the rebel faction has disutility equal to his utility gain from changing the allocation (see the utility function, 2.3). Analogously, an individual in power will join the loyal faction $\mathcal{L}$ to defend the current allocation if this is in his own interest.[15] Note that restricting the rebels to those who would be in power under a subsequent allocation will actually restrict only the maximum number of rebels, not the current status of those who can rebel.

For an allocation established at a stage of the power struggle in Figure 2 to be an equilibrium, the allocation must be optimal from the point of view of incumbents after taking account of the threat of any rational rebellions, where any allocation established following a rebellion must itself be an equilibrium (and so on for subsequent stages of the power struggle).[16] Of those allocations satisfying these conditions, any that depend (apart from individual identities) on payoff-irrelevant histories are then deleted to leave a set of equilibrium allocations. The equilibrium conditions for an allocation in one country take as given prices in world markets.

Definition 2 *An allocation* $\mathscr{A} = \{\mathcal{P}, \mathcal{C}, c_{\mathrm{pE}}, c_{\mathrm{pI}}, c_{\mathrm{kE}}, c_{\mathrm{kI}}, c_{\mathrm{wE}}, c_{\mathrm{wI}}, x_{\mathrm{E}}, x_{\mathrm{I}}\}$ *is an equilibrium of a stage of the power struggle if the following conditions are satisfied:*

[15] See Guimaraes and Sheedy (2015) for a discussion of the rebellion mechanism used here.

[16] The equilibrium concept is related to the notion of blocking in coalitions as studied in Ray (2007). There are two important differences here. First, an allocation must be optimal for incumbents. Second, changing the allocation requires costly rebellion effort.

(i) ***Optimality for incumbents***: *The allocation $\mathscr{A}$ maximizes the utility of incumbents when it is established, subject to:*

(ii) ***Rationality of rebels***: *A rational rebellion occurs if according to Definition 1 there exists any rational rebellion against the current allocation $\mathscr{A}$ for some subsequent allocation $\mathscr{A}'$, subject to:*

(iii) ***Threats of rebellion are credible***: *Any allocation $\mathscr{A}'$ established following a rebellion is itself an equilibrium of that stage of the power struggle.*

(iv) ***Independence of irrelevant history***: *Allocations $\mathscr{A}$ and $\mathscr{A}'$ established at any two stages of the power struggle where fundamental (payoff-relevant) state variables are the same are identical up to a permutation of identities.* □

The first equilibrium condition is that an allocation must be established in the interests of incumbents after taking account of the threat of rebellions. The second condition states that there will be a rebellion if and only if there is some rebellion in the interests of those who take part in it. It is never in the interests of incumbents to have an allocation that triggers a rational rebellion, and since there is no uncertainty in the model, no rebellions occur in equilibrium. Nevertheless, the problem of finding an allocation that maximizes the payoff of incumbents is effectively constrained by the threat of rebellions. With a slight abuse of language, the term 'no-rebellion constraint' is used below to refer to the restrictions on an allocation such that there is no rational rebellion for a particular rebel faction (associated with a particular incumbent group under a subsequent allocation). The set of 'no-rebellion constraints' is the collection of these constraints for all possible compositions of the rebel faction (associated with different subsequent incumbent groups).

The nature of the threat posed by rebel factions depends on what the post-rebellion allocation would be. The third equilibrium condition is that only subsequent allocations that are themselves equilibria of the power struggle can be considered when determining whether a rebellion is rational. Essentially, subsequent allocations must be in the interests of subsequent incumbents after taking account of further threats of rebellion. This excludes the possibility that rebels make a binding commitment to an allocation that is not in their interests ex post — for example, an allocation that would give rebels an incentive to exert more effort now, but which would not be optimal once the rebellion is over. Note that because there will be many subsequent equilibrium allocations with different compositions of the group in power, and as any one of these could follow a rational rebellion, the third requirement of equilibrium does not in itself restrict who can rebel.

The fourth condition is that equilibrium allocations depend only on fundamental state variables (fundamental meaning payoff relevant, for example, variables that appear in resource constraints), with the exception of individual identities. This Markovian restriction disciplines the equilibrium concept. At the pre-investment stage in Figure 2, there are no fundamental state variables, therefore the allocation in any round of the power struggle must be the same apart from changes in the identities of those in power. At the post-investment stage of Figure 2, the capital stock will be a fundamental state variable.

In summary, Definition 2 states that an equilibrium allocation must maximize incumbents' payoff (first condition) subject to avoiding any opportunity for rational rebellion (second condition), where the range of possible rational rebellions is itself limited by the set of equilibrium allocations that could be established following a rebellion (third condition). Note that the fourth (Markovian) condition does not operate as an additional constraint on the allocations that can be chosen in the interests of incumbents. Its role is in restricting which rebellions are rational (in conjunction with the third equilibrium condition) because the rationality of a rebellion depends on what allocation rebels anticipate being established following the rebellion. When any subsequent equilibrium allocation must depend only on fundamental state variables, an optimal allocation taking account of threats of rebellion need only depend on fundamental state variables.

Since individuals are ex ante identical, there is an essential indeterminacy in the identities of the incumbents and those who are given incentives to invest. If a certain allocation $\mathscr{A}$ is an equilibrium, otherwise identical allocations with any permutation of identities will also be equilibria. Therefore, the characterization of the set of equilibrium allocations will determine power sharing p, the fraction s of investment opportunities that are taken, and the distribution of resources, but not the specific identities of incumbents, capitalists, and workers. As a shorthand, these features of the set of equilibrium allocations are referred to as 'the' equilibrium allocation in what follows.

Finally, a world equilibrium is a set of allocations for each country and a world relative price $\tilde{\pi}^*$ of the investment good such that the allocation in each country is an equilibrium taking $\tilde{\pi}^*$ as given, and where international markets clear at price $\tilde{\pi}^*$:

$$\int_0^1 x_{\mathrm{E}}(\jmath)\mathrm{d}\jmath = 0, \quad \text{and} \quad \int_0^1 x_{\mathrm{I}}(\jmath)\mathrm{d}\jmath = 0. \qquad [2.11]$$

Note that there is only one independent market-clearing condition because countries must respect their international budget constraints (2.5) at all prices π^*.

3 Equilibrium

The world equilibrium is found in several steps. Taking as given the world price π^*, the equilibrium allocation in a country is characterized. The analysis proceeds by working backwards through the sequence of events in Figure 2. Once the equilibrium allocation in any given country is known, the equilibrium world price can be determined.

The analysis begins with some basic features of the equilibrium allocation in any country. It turns out that equilibrium allocation is always consistent with free exchange of the two goods within economies and free trade between economies.

3.1 Domestic market allocation

The allocation specifies consumption levels $\{c_{\mathrm{pE}}, c_{\mathrm{pI}}, c_{\mathrm{kE}}, c_{\mathrm{kI}}, c_{\mathrm{wE}}, c_{\mathrm{wI}}\}$ of the endowment and investment goods that depends on whether an individual is an incumbent, a capitalist, or a worker. It turns out that the allocation of consumption between goods for these individuals under an equilibrium allocation is always consistent with the consumption demands each individual would choose facing perfectly competitive markets for the two goods and a given level of income. The equilibrium allocation is therefore consistent with free exchange of goods domestically.

To be precise, suppose Y is the income (in terms of the endowment good as numeraire) of a given individual who can choose consumption c_{E} and c_{I} freely subject to a budget constraint:

$$c_{\mathrm{E}} + \pi c_{\mathrm{I}} = Y, \tag{3.1}$$

where π is the relative price of the investment good in terms of the endowment good. Maximizing the consumption aggregator (2.1) subject to (3.1) implies demand functions:

$$c_{\mathrm{E}} = (1-\alpha)Y, \quad \text{and} \quad c_{\mathrm{I}} = \frac{\alpha Y}{\pi}, \tag{3.2}$$

and the maximized consumption aggregator is:

$$C = \frac{Y}{\pi^{\alpha}}. \tag{3.3}$$

The consumption allocations associated with any equilibrium are solutions (3.2) of the utility-maximization problem subject to some individual incomes $Y(\imath)$ and a price $\tilde{\pi}$ given by:

$$\tilde{\pi} = \frac{\alpha(\mathrm{q} - x_{\mathrm{E}})}{(1-\alpha)(K - x_{\mathrm{I}})}, \tag{3.4}$$

which depends on the capital stock K and the net exports x_{E} and x_{I} specified by the allocation. The individual incomes $Y(\imath)$ must sum to national income Y calculated using price $\tilde{\pi}$ from (3.4):

$$\int_0^1 Y(\imath)\mathrm{d}\imath = Y, \quad \text{where} \;\; Y = (\mathrm{q} - x_{\mathrm{E}}) + \tilde{\pi}(K - x_{\mathrm{I}}). \tag{3.5}$$

The price $\tilde{\pi}$ in (3.4) is the market-clearing price given the demand functions (3.2) (the market-clearing conditions are the resource constraints in (2.6)). The domestic resource constraints (2.6) for the two goods hold given that individual incomes satisfy (3.5) and the price is $\tilde{\pi}$ from (3.4).

3.2 International trade

The allocation specifies net exports $\{x_{\mathrm{E}}, x_{\mathrm{I}}\}$ subject to the international budget constraint (2.5) and a given world relative price π^* of the investment good. It turns out that net exports under an equilibrium allocation are always consistent with what would prevail if the country were open to international trade with no restrictions on the flow of goods, which brings the domestic market-clearing price $\tilde{\pi}$ into line with the world price π^*. The equilibrium allocation is therefore consistent with free trade internationally.

Consider a consumption allocation consistent with free exchange domestically as characterized in (3.2), (3.4), and (3.5), and suppose the domestic market-clearing price $\tilde{\pi}$ is equalized to the world price π^*. The international budget constraint (2.5) together with the equation for national income in (3.5) implies:

$$Y = \mathrm{q} + \tilde{\pi}K. \tag{3.6}$$

Using the domestic resource constraints (2.6) and the individual demand functions (3.2), net exports are:

$$x_{\mathrm{E}} = \alpha \mathrm{q} - (1-\alpha)\pi^* K, \quad \text{and} \quad x_{\mathrm{I}} = (1-\alpha)K - \frac{\alpha \mathrm{q}}{\pi^*}, \tag{3.7}$$

which depend on the capital stock K. Equivalently, note that net exports (3.7) imply that $\tilde{\pi} = \pi^*$ using equation (3.4).

3.3 The allocation following a post-investment rebellion

The next step in characterizing the equilibrium is to work backwards through the sequence of events in Figure 2. Suppose a rebellion happens after investment decisions have been made (this will be off the equilibrium path). This means that the amount K of the investment good produced is now a state variable, and continuation value $\mathcal{U} = \log C - \log(1+F)$ of the utility function (2.3) does not include any sunk effort costs of investing.

A new allocation must maximize incumbent utility subject to the resource constraints (2.5) and (2.6) and a set of 'no-rebellion constraints'. Taking as given p' and $\mathcal{U}'_{\mathrm{p}}$ that specify power sharing and the incumbent payoff following a further rebellion, for all sets $\mathcal{P}'$ with measure p', the absence of a rational rebellion according to Definition 1 requires:

$$\int_{\mathcal{P}'} \max\{\exp\{\mathcal{U}'_{\mathrm{p}} - \mathcal{U}(\imath)\} - 1, 0\} \mathrm{d}\imath \leq \delta \int_{\mathcal{P}} \mathbb{1}[\mathcal{U}_{\mathrm{p}} > \mathcal{U}'(\imath)] \mathrm{d}\imath, \tag{3.8}$$

where $\mathbb{1}[\cdot]$ is the indicator function. The problem of determining the equilibrium allocation at this stage is simplified by the results in the proposition below.

Proposition 1 *An equilibrium allocation established at the post-investment stage of the sequence of events in Figure 2 has the following features:*

(i) Free exchange of goods domestically: The consumption allocations for all individuals are solutions (3.2) of the utility-maximization problem in section 3.1 given the market-clearing price $\tilde{\pi}$ in (3.4), and incomes Y_{p}, Y_{w}, Y_{k} for incumbents, workers, and capitalists.

(ii) Free exchange of goods internationally: Net exports are given in equation (3.7) and the market-clearing price $\tilde{\pi}$ in (3.4) is equal to the world price π^*.

(iii) Full expropriation of capital: The income levels of all non-incumbents are equalized, that is, $Y_{\mathrm{k}} = Y_{\mathrm{w}} = Y_{\mathrm{n}}$.

(iv) Only one 'no-rebellion' constraint corresponding to a rebel faction comprising only non-incumbents is needed to characterize the equilibrium allocation:

$$\frac{C'_{\mathrm{p}}}{C_{\mathrm{n}}} - 1 \leq \frac{\delta p}{p'}, \tag{3.9}$$

where C_{n} is the consumption of non-incumbents.

Proof See appendix A.1. ■

The first and second statements in the proposition show that no inefficiency arises in the allocation of a given quantity of the two goods. No-rebellion constraints effectively place lower bounds on all individuals' utilities and thus Pareto-improving reallocations of goods are in the interests of the incumbents, and these are precisely the exchanges that are brought about by free markets. Crucially, these exchanges do not affect any aspect of the power struggle. Note that although the equilibrium allocation allows for free exchange of goods, this is not the same as individuals being allowed to keep all of what they produce: overall levels of income consistent with the equilibrium allocation will involve taxes and transfers between workers, capitalists, and incumbents.

The full expropriation of capital that occurs when a new allocation is established at the post-investment stage is a consequence of not needing to provide incentives for investment after the fact, alongside incumbents' desire for payoff equalization among those individuals with equal power. The intuition for payoff equalization is that the most dangerous rebel faction comprises the individuals with the greatest incentive to fight. Since any effort costs of investment are sunk at this point, the maximum rebellion effort of all non-incumbents depends only on their current consumption. As a consequence, if there were payoff inequality among non-incumbents, the most dangerous rebel faction would not include those who receive a relatively high payoff. The incumbents could then reduce the effectiveness of this rebel faction by redistributing from relatively well-off non-incumbents to the worse off, which slackens the no-rebellion constraints and allows incumbents to achieve a higher payoff for themselves.

The fourth statement shows that the only relevant no-rebellion constraint corresponds to a rebel faction comprising only non-incumbents. At this stage of the sequence of events in Figure 2, the choice of allocation is effectively not constrained by the threat of rebellion from insiders because the environment does not change between rounds of the power struggle, so in equilibrium, incumbents can do no better under a subsequent allocation. Given linearity of the conflict technology (the condition (2.8) for a successful rebellion), a rebel faction comprising a mixture of incumbents and non-incumbents cannot be more dangerous than a rebel faction formed only by non-incumbents. Note also that the no-rebellion constraint (3.9) can be stated with reference only to consumption rather than utility because no further rebellions will occur in equilibrium, hence no further rebellion effort $R(\imath)$ will be exerted by any individual.

Given consumption C_{p} and C_{n} for incumbents and non-incumbents, respectively, and given the free exchange of goods domestically and internationally, the country's resource constraint can be stated as follows using (3.3), (3.5), and (3.6):

$$pC_{\mathrm{p}} + (1-p)C_{\mathrm{n}} = C,$$

where aggregate consumption C is given by:

$$C = \frac{\mathrm{q} + \tilde{\pi}K}{\tilde{\pi}^{\alpha}}. \quad [3.10]$$

At this stage, K and $\tilde{\pi}$ have already been determined, aggregate consumption C is taken as given in this maximization problem. The resource constraint and the binding no-rebellion constraint (3.9) imply that the consumption of incumbents C_{p} can be written as:

$$C_{\mathrm{p}} = \frac{1}{p}\left(C - (1-p)\frac{C'_{\mathrm{p}}p'}{\delta p + p'}\right).$$

With the aspects p' and C'_{p} of any post-rebellion allocation taken as given, there is only one remaining choice variable, power sharing p. Thus, after taking account of the binding no-rebellion constraint, the key decision incumbents must make is how widely to share power. It can be shown that the expression for C_{p} is quasi-concave in p, so the maximum is found by taking the first-order condition. After simplification, this yields

$$C_{\mathrm{p}} = \frac{C'_{\mathrm{p}}p'}{\delta p + p'} + \frac{(1-p)C'_{\mathrm{p}}\delta p'}{(\delta p + p')^2}.$$

Imposing the equilibrium conditions $p = p'$ and $C_{\mathrm{p}} = C'_{\mathrm{p}}$ (see Definition 2) and rearranging leads to the amount of power sharing specified by the equilibrium allocation:

$$p^{\ddagger} = \frac{1}{2+\delta}. \tag{3.11}$$

The superscript ‡ denotes the equilibrium allocation following a rebellion after investment decisions have been made in the sequence of events from Figure 2. Power sharing after a post-investment rebellion is independent of the endowment q and the capital stock K. It is decreasing in the incumbent power parameter δ and always smaller than $1/2$. Sharing power strengthen the incumbent group and allows higher taxes to be extracted from non-incumbents, but also spreads the revenue more thinly among a larger group of individuals.

Aggregate consumption C is distributed among individuals so that:

$$C^{\ddagger}_{\mathrm{p}} = \frac{2+\delta}{2}C, \quad \text{and} \quad C^{\ddagger}_{\mathrm{n}} = \frac{2+\delta}{2(1+\delta)}C. \tag{3.12}$$

Half of aggregate consumption is shared among the $p^{\ddagger}$ incumbents, and the remaining $1-p^{\ddagger}$ individuals share the other half. Although a post-investment rebellion will never occur on the equilibrium path, the amount of power sharing $p^{\ddagger}$ and the consumption of incumbents $C^{\ddagger}_{\mathrm{p}}$ according to the allocation that would be established following such a rebellion are important in understanding incentives to rebel after investments have been made.

3.4 Power sharing at the pre-investment stage

At the pre-investment stage, an allocation maximizes incumbents' utility $\mathcal{U}_{\mathrm{p}}$ subject to no-rebellion constraints whenever there are opportunities for rebellions (before and after investments have been made), and taking into account that individuals will take investment opportunities only if it is incentive compatible. By respecting the incentive compatibility constraint in (2.4) and appropriately choosing the size of the set $\mathcal{C}$, incumbents can determine the fraction s of investment opportunities that will be taken.

At the pre-investment stage of Figure 2, the no-rebellion constraint (see Definition 1) is given by:

$$\int_{\mathcal{P}'} \max\{\exp\{\mathcal{U}'_{\mathrm{p}} - \mathcal{U}(\imath)\} - 1, 0\}\mathrm{d}\imath \leq \delta \int_{\mathcal{P}} \mathbb{1}[\mathcal{U}_{\mathrm{p}} > \mathcal{U}'(\imath)]\mathrm{d}\imath, \qquad [3.13]$$

for all $\mathcal{P}'$ with measure p', where $'$ denotes an allocation that would be established after the first opportunity for rebellion. Any rebellions that occur before investments have been made have no effect on fundamental state variables.

Incumbents also anticipate the threat of rebellions after investment has been made. Given the equilibrium allocation that would be established after a rebellion at the post-investment stage of Figure 2, as characterized in section 3.3, the constraint for no rebellions at the post-investment stage is given by:

$$\int_{\mathcal{P}^\ddagger} \max\{\exp\{\mathcal{U}^\ddagger_{\mathrm{p}} - \mathcal{U}(\imath)\} - 1, 0\}\mathrm{d}\imath \leq \delta \int_{\mathcal{P}} \mathbb{1}[\mathcal{U}_{\mathrm{p}} > \mathcal{U}^\ddagger(\imath)]\mathrm{d}\imath, \qquad [3.14]$$

for all $\mathcal{P}^\ddagger$ with measure $p^\ddagger$, where $p^\ddagger$ is given in (3.11).

The following proposition establishes that the equilibrium allocation features a link between the fraction s of investment opportunities undertaken, the amount of power sharing p, and the rents received by incumbents.

Proposition 2 *Irrespective of the value of s in equilibrium, the equilibrium allocation must have the following properties:*

(i) Free exchange of goods domestically and internationally, as in Proposition 1.

(ii) The incentive constraint for investment in (2.4) always binds. The post-investment no-rebellion constraint in (3.14) for workers only always binds. The post-investment no-rebellion constraint for incumbents only binds in case $\delta < 1/2$. All other no-rebellion constraints are redundant or slack.

(iii) The relationship between power sharing p and the fraction of investment opportunities that are undertaken is given by $s = \sigma(p)$, where:

$$\sigma(p) = \begin{cases} \frac{\delta(p-p^\ddagger)}{\mu\theta}\left(1 + \frac{\delta p}{\delta p + (1-\delta)p^\ddagger}\right) & \text{if } \delta < 1/2 \\ \frac{\delta(p-p^\ddagger)}{\mu\theta}\left(1 + \frac{p}{2\delta p + p^\ddagger}\right) & \text{if } \delta \geq 1/2 \end{cases}.$$

The function $\sigma(p)$ is strictly increasing and strictly convex in p in both cases, with $\sigma(p^\dagger) = 0$ and $\sigma(\bar{p}) = 1$ for $p^\dagger$ and $\bar{p}$ with $p^\dagger < \bar{p} \leq 1/2$:

$$p^\dagger = \frac{1}{2+\delta}, \quad \text{and} \quad \bar{p} = \begin{cases} \frac{\delta(\delta+(2+\delta)\mu\theta) + \sqrt{(\delta(\delta+(2+\delta)\mu\theta))^2 + \delta(1+2\delta)(\delta+(2+\delta)\mu\theta)}}{\delta(1+2\delta)(2+\delta)} & \text{if } \delta < 1/2 \\ \frac{3\delta - 1 + (2+\delta)\mu\theta + \sqrt{(3\delta-1+(2+\delta)\mu\theta)^2 + 8(1-\delta)(\delta+(2+\delta)\mu\theta)}}{4\delta(2+\delta)} & \text{if } \delta \geq 1/2 \end{cases}.$$

Note that $p^\dagger = p^\ddagger$, where the latter is given in (3.11).

(iv) A worker's share $\phi_{\mathrm{w}} = C_{\mathrm{w}}/C$ is given by the following function of p:

$$\phi_{\mathrm{w}}(p) = \frac{1}{2(\delta p + p^\ddagger)}.$$

An incumbent's share $\phi_{\mathrm{p}} = C_{\mathrm{p}}/C$ is given by the following function of p:

$$\phi_{\mathrm{p}}(p) = \begin{cases} \frac{1}{2(\delta p + (1-\delta)p^{\ddagger})} & \text{if } \delta < 1/2 \\ \frac{1+2\delta}{2(2\delta p + p^{\ddagger})} & \text{if } \delta \geq 1/2 \end{cases}.$$

Rents $\chi = (\phi_{\mathrm{p}} - \phi_{\mathrm{w}})/\phi_{\mathrm{w}}$ received by an incumbent (as a fraction of consumption) are strictly positive and strictly decreasing in p in all cases.

Proof See appendix A.2. ■

The intuition for the free exchange of goods both domestically and internationally is the same as in Proposition 1. The second statement in the proposition implies that the pre-investment no-rebellion constraint does not bind. Intuitively, at the pre-investment stage, production of the investment good requires providing incentives for investment, while at the post-investment stage, any effort costs of investment are sunk. The gains to rebels (whether currently workers or incumbents) from a change to the allocation at the post-investment stage are thus larger than at the pre-investment stage of Figure 2. In some cases, only the threat of rebellion from workers is binding at the post-investment stage, while for some values of δ, incumbents also need to consider the possibility of a coup d'etat from within their own ranks (again, at the post-investment stage).

The results in the third part of the proposition relate to the choice of power sharing p given the fraction s of investment opportunities undertaken (the equilibrium conditions will also determine s, but it is taken as given at this stage of the analysis). When $\delta < 1/2$, the amount of power sharing p is determined by the binding no-rebellion constraints and the resource constraint, given a value of s. The expressions for ϕ_{w} and ϕ_{p} are the binding no-rebellion constraints for workers and incumbents, respectively. When $\delta > 1/2$, the no-rebellion constraint for incumbents does not bind and consequently the expression for $\sigma(p)$ and ϕ_{p} is derived from the first-order condition for maximizing the incumbent payoff with respect to p, taking account of the binding no-rebellion constraint. While these two cases lead to different formulas, the qualitative features of both cases are the same and no important result depends on the value of δ.

The third statement of the proposition contains two important results. First, there is a positive relationship between the reach of the rule of law and power sharing. Second, the reach of the rule of law is a convex function of power sharing.

To understand the first result, observe that when $p = p^{\dagger}$, $\sigma(p)$ is zero, implying no investment will take place in equilibrium. Intuitively, when $p = p^{\ddagger}$, a rebellion from incumbents is costless (power sharing after the rebellion will be $p^{\ddagger}$, and since $p^{\dagger} = p^{\ddagger}$, all current incumbents can take part in the rebellion), hence there is no way to sustain a commitment to protecting property rights. A suspension of the constitution and consequent confiscation and redistribution of investment goods would benefit all but the capitalists, who have already incurred the sunk investment cost by then, and thus would not have a greater incentive to rebel than a worker. This case is referred to as *despotism*, denoted by the superscript †.

An increase in s raises incentives for rebellions for all groups. As p increases above $p^{\dagger}$, the rebellion effort required for a rebellion increases for all groups as well. This explains the increasing relationship between investment s and power sharing p. As in Guimaraes and Sheedy (2015), power sharing allows for commitment to rules that would otherwise be time inconsistent. If the increase in power sharing goes all the way to $\bar{p}$ then $s = 1$. This case is referred to as an economy with the *rule of law* because everyone who receives an investment opportunity has the fruits of their investment credibly protected by the equilibrium allocation.

The second and more important result is that $\sigma(p)$ is not only increasing, but is also a convex function as depicted in Figure 3, implying the marginal cost of good government is decreasing in the sense that extending the reach of the rule of law is less costly when power is shared more broadly. In order to understand this result, note that with p incumbents, μs capitalists, and $1 - p - \mu s$ workers, the resource constraint can be written as:

$$pC_\text{p} + \mu s C_\text{k} + (1 - p - \mu s)C_\text{w} = C,$$

where aggregate consumption C is as defined in (3.10). Using the binding incentive constraint (2.4) and the definitions of the worker and incumbent shares ϕ_w and ϕ_p leads to an equation describing the division of the total pie:

$$p\phi_\text{p} + (1 - p + \mu\theta s)\phi_\text{w} = 1.$$

Hence, the reach of the rule of law must satisfy

$$\mu\theta s = \left(\frac{1}{\phi_\text{w}} - 1\right) - \chi p,$$

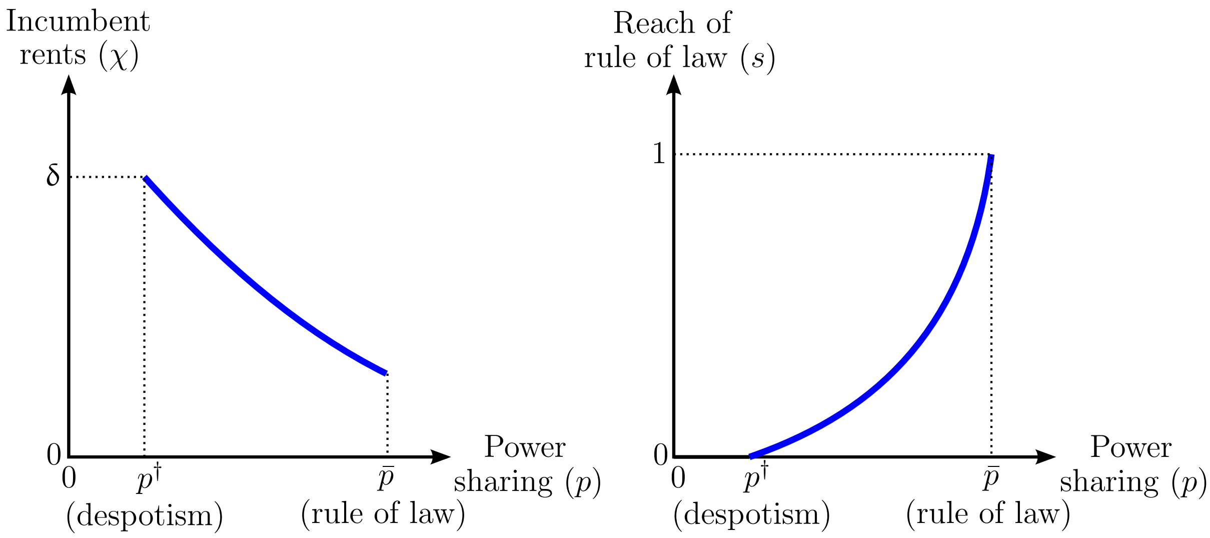
where incumbent rents are $\chi = (\phi_\text{p} - \phi_\text{w})/\phi_\text{w}$. The term $1/\phi_\text{w}$ is the output of the economy in units of worker incomes. The equilibrium allocation is such that $1/\phi_\text{w}$ is increasing in p (the fourth statement in Proposition 2) because an increase in the number of incumbents allows them to extract more from each worker. The term in brackets is the amount of output above and beyond the basic worker income, which is made up of rents to incumbents and incentives to investors (in units in a worker's income).

Owing to the utility function, the term $1/\phi_\text{w}$ is linear in p, hence the convexity of $\sigma(p)$ is due entirely to the behaviour of the total rents χp. If rents χ were constant, total rents would also be linear in p and $\sigma(p)$ would not be strictly convex. Since χ is decreasing in p, total rents $p\chi$ rise less than proportionally with p, implying $\sigma(p)$ is strictly convex. As power is shared more broadly, individuals in power are receiving a lower share of the total pie, which implies that more resources can be credibly pledged to offer incentives to investors.

The decreasing marginal cost of good government is thus explained by rents χ being diluted as power sharing p increases (as depicted in Figure 3). Inspection of the expressions for ϕ_w and ϕ_p in Proposition 2 shows that an increase in p leads to a decrease in a worker's share ϕ_w, but an even larger reduction in the share of an incumbent ϕ_p.

The reason is that an additional incumbent is less important in defending the allocation when there are already many individuals in power. For example, if there is relatively little power sharing, a coup d'etat could succeed with relatively little rebellion effort, which means that those in power would find it very worthwhile to recruit additional incumbents. Alternatively, when the power base

Figure 3: *Power sharing, incumbent rents, and the reach of the rule of law*



is narrow, the marginal gain in extracting rents from workers of an additional incumbent is large. Both of these examples point to a negative relationship between rents and power sharing, but which one is dominant turns out to depend on δ. The former is dominant when δ is low and the latter when δ is high.

In the case $\delta < 1/2$, the result comes from the relative effects of an increase in p on the no-rebellion constraints for workers and incumbents. An increase in power sharing p makes rebellions more costly for both incumbents and workers and hence decreases both ϕ_{p} and ϕ_{w}. However, the effect on rebellions launched by incumbents is relatively more important, and especially so when p is small. When p is closer to $p^{\ddagger}$, very few individuals would defend the allocation against a coup d'etat and thus the incumbents would pose a substantially greater threat to the allocation. It follows that the ratio $\phi_{\mathrm{p}}/\phi_{\mathrm{w}}$ would need to be very large. As p increases, the threat posed by incumbents would become relatively less important, leading to a smaller χ.

In the case $\delta \geq 1/2$, the incumbents are willing to accept more power sharing that required to avoid a coup d'état. In other words, the no-rebellion constraint for incumbents is not binding. Power sharing is thus extended up to the point its marginal benefit from squeezing non-incumbent income (without triggering rebellions) equates the marginal cost. From the point of view of incumbents, the marginal cost of an additional member is the difference between an incumbent payoff and a worker payoff, which is what we refer to as the rent χ. Since the marginal extraction benefit from power sharing is decreasing in p, rents and power sharing must be negatively related.

The analysis so far has not considered the determination of s in equilibrium, only what implications the choice of s has for other variables. The equilibrium conditions require that s maximizes the incumbent payoff, subject to constraints, but these constraints are all incorporated in the relationships derived in Proposition 2 between s and p, and between ϕ_{p} and p. Therefore, the problem of characterizing the value of s according to the equilibrium allocation is an unconstrained choice of s to maximize the following objective function:

$$\mathcal{U}_{\mathrm{p}}(s) = \log \phi_{\mathrm{p}}(\sigma^{-1}(s)) + \log(\mathrm{q} + \mu\tilde{\pi}s) - \alpha \log \tilde{\pi}. \qquad [3.15]$$

3.5 The political distortion

In order to understand the distortions arising from the power struggle, note that the effect of a change in s on the utility of incumbents can be written as:

$$\frac{\partial \mathcal{U}_{\mathrm{p}}}{\partial s} = \frac{\mu}{Y}\left(\tilde{\pi} - (1+\gamma)\theta Y_{\mathrm{w}}\right), \qquad [3.16]$$

where γ is a strictly positive and strictly decreasing function of p. It is efficient to produce an extra unit of the investment good if its value $\tilde{\pi}$ in terms of endowments goods is larger or equal to the marginal effort cost θY_{w} of producing it. However, the incumbents factor in an extra cost in their choice of s represented by the term γ in (3.16). This cost reflects the distributional consequences of the power sharing needed to support credible protection of investors' property rights.

Power sharing enables the incumbents to commit not to expropriate a larger number of capitalists. However, sharing power requires sharing rents, which goes against the interests of the incumbents. Hence in equilibrium, there is too little commitment and too little investment.[17] The distortion does not therefore arise from restrictions on feasible tax instruments, but from the maximizing behavior of incumbents subject to the threat of rebellion. The investment wedge γ leads incumbents to restrict the number of individuals who can rely on the rule of law to uphold their property rights, which essentially means limiting the number of people permitted to produce the investment good. This is analogous to how a firm with market power would restrict the quantity of goods it sells to obtain a price above its marginal cost.

The investment wedge is decreasing in power sharing p, implying a smaller difference between the social returns to investment and the returns considered by incumbents in countries with more power sharing. Intuitively, because rents χ are smaller for larger values of p, incumbents require less compensation for expanding production of the investment good by extending the reach of the rule of law.

4 The closed-economy benchmark

This section considers the closed-economy benchmark where $x_{\mathrm{E}} = 0$ and $x_{\mathrm{I}} = 0$. All the results from section 3 hold in this setting and the domestic market-clearing relative price of the investment good (from (3.4)) is now given by:

$$\hat{\pi} = \frac{\alpha \mathrm{q}}{(1-\alpha)\mu s}.$$

[17]Production of the investment good affects incentives for rebellions, which leads to the distortion analysed here. If there were no such interactions, incumbents would choose *laissez faire*, as they do for the exchange of goods domestically and internationally.

The equilibrium allocation maximizes the incumbent payoff in (3.15) with $\tilde{\pi} = \hat{\pi}$:[18]

$$\hat{\mathcal{U}}_{\mathrm{p}}(s) = \log \phi_{\mathrm{p}}(\sigma^{-1}(s)) + \alpha \log s + \log \frac{\mathrm{q}^{1-\alpha}\mu^{\alpha}}{(1-\alpha)^{1-\alpha}\alpha^{\alpha}}. \qquad [4.1]$$

The proposition below characterizes the equilibrium allocation, in particular the amount of power sharing $\hat{p}$ and the proportion of investors $\hat{s}$.

Proposition 3 *In an economy with no access to international markets ($x_{\mathrm{E}} = 0$ and $x_{\mathrm{I}} = 0$):*

(i) The function $\hat{\mathcal{U}}_{\mathrm{p}}(s)$ is strictly quasi-concave in s.

(ii) Let $\bar{\alpha}$ be as defined below, which lies between 0 and 1:

$$\bar{\alpha} = \begin{cases} \left(1 + \frac{(1-\delta)^2+\delta(2+\delta)(3-\delta)\bar{p}}{\mu\theta(2+\delta)(1-\delta+\delta(2+\delta)\bar{p})}\right)^{-1} & \text{if } \delta < 1/2 \\ \left(1 + \frac{\delta+(2+\delta)(1+2\delta+2\delta^2)\bar{p}}{\mu\theta(2+\delta)(1+2\delta(2+\delta)\bar{p})}\right)^{-1} & \text{if } \delta \geq 1/2 \end{cases}.$$

If $\alpha < \bar{\alpha}$ then $0 < \hat{s} < 1$ and $\hat{p} < \bar{p}$, otherwise $\hat{s} = 1$ and $\hat{p} = \bar{p}$.

(iii) In the case $\alpha < \bar{\alpha}$, $\hat{p}$ is given by:

$$\hat{p} = \begin{cases} \frac{3\delta-1+4(1-\delta)\alpha+\sqrt{(3\delta-1+4(1-\delta)\alpha)^2+8(1-\delta)(\delta+(1-2\delta)\alpha)(1-\alpha)}}{4\delta(2+\delta)(1-\alpha)} & \text{if } \delta < 1/2 \\ \frac{2\delta^2+(1+2\delta)\alpha+\sqrt{(2\delta^2+(1+2\delta)\alpha)^2+4\delta^2(1+2\delta)(1-\alpha)}}{2\delta(1+2\delta)(2+\delta)(1-\alpha)} & \text{if } \delta \geq 1/2 \end{cases}.$$

The value of $\hat{p}$ is always larger than $p^{\dagger}$ and $\hat{s}$ is always positive.

Proof See appendix A.3. ■

The quasi-concavity of $\hat{\mathcal{U}}_{\mathrm{p}}$ in s is due to the effect of domestic production of the investment good on its relative price $\hat{\pi}$. On the one hand, there are increasing returns to power sharing. On the other hand, an increase in $\hat{s}$ leads to an increase in the quantity and hence a fall in the price of the investment good. The effect of the price dominates, which means there is either an interior solution ($0 < \hat{s} < 1$) or $\hat{s} = 1$. The case $s = 0$ is never an equilibrium because the Cobb-Douglas consumption aggregator implies that some of each good is essential. In order to allow for some investment, the amount of power sharing must be larger than $p^{\dagger}$.

In the case $\hat{s} = 1$, the political frictions in this paper are not binding. Henceforth, it is assumed that $\alpha < \bar{\alpha}$. One consequence is that the closed-economy equilibrium is Pareto inefficient. As reflected in (3.16), the equilibrium features a wedge between the marginal cost of producing the investment good and its price, hence a marginal increase in s would give rise to a Pareto improvement.

[18] Maximizing $\hat{\mathcal{U}}_{\mathrm{p}}(s)$ with respect to s and then substituting the price $\hat{\pi}$ yields the same result.

5 Equilibrium allocations with international trade

5.1 The equilibrium allocation in an open economy

This section considers an open economy where the world relative price of the investment good π^* is taken as given. As shown in Proposition 2, the equilibrium allocation features free international trade, so the domestic price of the investment good is equal to its international price ($\tilde{\pi} = \pi^*$). The equilibrium allocation features a choice of investment s to maximize the objective function:

$$\mathcal{U}_{\mathrm{p}}^*(s;\pi^*) = \log \phi_{\mathrm{p}}(\sigma^{-1}(s)) + \log(\mathrm{q} + \upmu\pi^* s) - \alpha \log(\pi^*). \qquad [5.1]$$

The proposition below shows there will be political specialization and that decisions on political systems are strategic substitutes across countries.

Proposition 4 (Political specialization) *In an open economy that takes the world price π^* as given:*

(i) The function $\mathcal{U}_{\mathrm{p}}^*(s;\pi^*)$ is strictly quasi-convex in s. At the critical point, the sign of $\mathcal{U}_{\mathrm{p}}^{*\prime\prime}(s;\pi^*)$ equal to the sign of $\sigma''(p)$.

(ii) In equilibrium, $s = 0$ or $s = 1$. The rule of law ($s = 1$) is the optimal choice if:

$$\frac{\mathrm{q} + \upmu\pi^*}{\mathrm{q}} \geq \frac{\phi_{\mathrm{p}}^{\dagger}}{\bar{\phi}_{\mathrm{p}}}, \qquad [5.2]$$

where $\phi_{\mathrm{p}}^{\dagger}$ and $\bar{\phi}_{\mathrm{p}}$ are the incumbent shares under despotism ($p = p^{\dagger}$) and the rule of law ($p = \bar{p}$) respectively.

(iii) Strategic substitutability: A higher world price π^* of the investment good raises the incumbent payoff from the rule of law ($s = 1$) and reduces the incumbent payoff from despotism ($s = 0$).

Proof See appendix A.4. ■

The quasi-convexity of $\mathcal{U}_{\mathrm{p}}^*(s;\pi^*)$ is a consequence of the increasing returns to power sharing demonstrated in Proposition 2 together with the world price being unaffected by the allocation adopted in the domestic economy. In a closed economy, the relative price of the investment good falls as more of it is produced. This effect is absent in an open economy, so the increasing returns to power sharing lead to political specialization. In order to see the link between the convexity of $\sigma(p)$ and the quasi-convexity of $\mathcal{U}_{\mathrm{p}}$, observe that

$$\frac{\partial \mathcal{U}_{\mathrm{p}}^*}{\partial s} = \frac{1}{Y}\left(\upmu\pi^* - \frac{1}{\sigma'(p)}\frac{\phi_{\mathrm{p}}'(p)}{\phi_{\mathrm{p}}(p)}(\mathrm{q} + \upmu\pi^* s)\right).$$

The first term inside the bracket is the marginal benefit of increasing s. The subsequent terms represent the dilution of incumbents rents owing to the extension of power sharing needed to increase s, which is a cost to incumbents. From right to left, these terms are total output, the semi-elasticity of an incumbent's share of output with respect to power sharing, and the marginal increase in power sharing required to extend the reach of the rule of law. These terms are essentially the marginal cost of extending the reach of the rule of law. The behaviour of the marginal cost is explained by the first term $1/\sigma'(p)$, which is decreasing in p and hence s because $\sigma(p)$ is a convex function of p.[19] Owing to the increasing returns to sharing power, an increase in s requires progressively smaller increases in power sharing, and thus smaller dilutions of incumbent rents.

The first steps taken by a country to the rule of law have the greatest private cost to incumbents. Hence, in equilibrium, countries will either choose a despotic regime with little power sharing and no production of the investment good, or a regime where the rule of law applies to every individual so anyone with an investment opportunity has sufficient protection to choose to invest. For a given world price π^*, the choice of allocation boils down to a comparison between incumbents' consumption in both regimes, $\bar{\phi}_{\mathrm{p}}(\mathrm{q} + \mu\pi^*)$ under the rule of law, and $\phi_{\mathrm{p}}^{\dagger}\mathrm{q}$ under despotism.

International trade allows for political specialization by allowing a despotic regime to import what its own political system precludes it from producing, and by enabling an economy with the rule of law to access to a larger market and thus fully exploit the increasing returns to power sharing. In Krugman (1979, 1980), ex-ante identical economies specialize in producing different goods and thus trade with each in order to exploit the increasing returns to production. Here, countries that are ex-ante identical economically and politically specialize in different political systems, which implies they become different ex post and thus trade with each other. Even without a predisposition to despotism, some economies will end up with a despotic political system, while others will end up with the rule of law even in the absence of any cultural or technological advantage. But there is a fundamental difference with 'new trade theory' because there are no increasing returns in production itself. This means the welfare implications of international trade are very different here.

The choices of political systems are strategic substitutes across countries. At the global level, prices depend on how much of the investment good is produced and hence on the number of economies with the rule of law. If other economies adopt the rule of law, the price of the investment good π^* falls, which lowers the marginal benefit of the rule of law, and thus tilts the balance in favour of despotism for others. At the world level, the prevalence of the rule of law is still limited by the size of the market for rule-of-law intensive goods.

5.2 The world equilibrium

The world price π^* adjusts to ensure that the market-clearing conditions (5.3) hold. Under the equilibrium allocation in each country, net exports are given by (3.7). By integrating over all countries, world markets clear at the following relative price:

$$\tilde{\pi}^* = \frac{\alpha \mathrm{q}}{(1-\alpha)K^*}, \quad \text{where } K^* = \int_0^1 K(\jmath)\mathrm{d}\jmath, \qquad [5.3]$$

where K^* is the world supply of the investment good. Since all countries are ex ante identical, the world supply of the endowment good in q. Owing to political specialization (the result of

[19] If $\sigma(p)$ were a linear function, the marginal benefit would be either above or below (or equal to) marginal cost for all values of s.

Proposition 4), the equilibrium world price $\tilde{\pi}^*$ is given by

$$\tilde{\pi}^* = \left(\frac{\alpha}{1-\alpha}\right)\left(\frac{q}{\mu\omega}\right),$$

where ω is the fraction of economies with the rule of law ($s = 1$). The measure of people holding positions of power somewhere in the world is $p^* = (1-\omega)p^\dagger + \omega\bar{p}$. The following proposition characterizes the world equilibrium.

Proposition 5 *With world market clearing (2.11) and an equilibrium allocation in each country as characterized by Proposition 4, the world equilibrium is as follows:*

(i) The world equilibrium has a fraction $\tilde{\omega}$ of rule-of-law economies with $0 < \tilde{\omega} < 1$:

$$\tilde{\omega} = \frac{\alpha}{(1-\alpha)\delta(2+\delta)\min\left\{1, \frac{2}{1+2\delta}\right\}(\bar{p} - p^\ddagger)}.$$

(ii) The equilibrium fraction of incumbents at the world level is:

$$p^* = \frac{1}{2+\delta}\left(1 + \frac{\alpha}{(1-\alpha)\delta\min\left\{1, \frac{2}{1+2\delta}\right\}}\right).$$

There are fewer incumbents in the open economy compared to the closed economy ($p^* < \hat{p}$).

(iii) Incumbents in both despotic and rule-of-law economies are strictly better off than they would be in closed economies, and they are indifferent between despotism and the rule-of-law ($\mathcal{U}_p^\dagger = \bar{\mathcal{U}}_p > \hat{\mathcal{U}}_p$).

(iv) Workers in rule-of-economies are strictly better off than workers in despotisms ($\bar{\mathcal{U}}_w > \mathcal{U}_w^\dagger$). Workers in rule-of-law economies are also strictly better off than they would be in a closed economy ($\bar{\mathcal{U}}_w > \hat{\mathcal{U}}_w$).

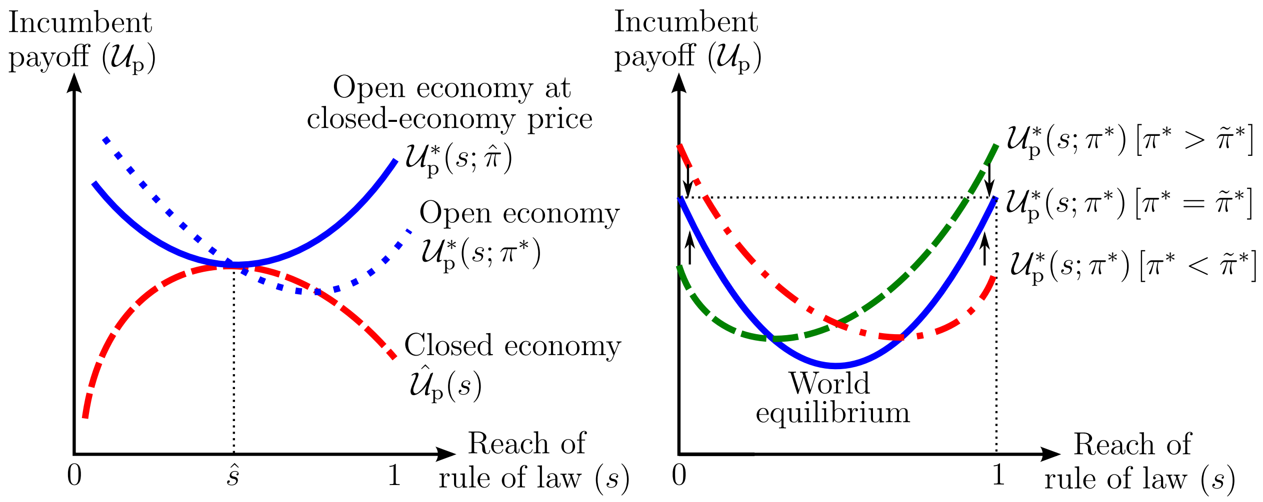
(v) Global output and prices may be larger or smaller than in closed economies depending on parameters.

Proof See appendix A.5. ■

The first panel of Figure 4 below shows the utility of an incumbent in the closed economy $\hat{\mathcal{U}}_p$, and the utility of an incumbent in an open economy as a function of s for both an arbitrary world price π^* and a world price equal to the closed-economy market-clearing price $\hat{\pi}$. At $\hat{s}$, $\hat{\mathcal{U}}_p$ is maximized. For an open economy facing that price in world markets, net exports would be zero, hence incumbents' utility (and everything else) would be the same as in the closed economy. In both the open and closed economies at $\hat{s}$, the marginal cost of power sharing (as perceived by incumbents) equals its marginal benefit. In the closed economy, that is the point of maximum, but in the open economy, owing to the world price being taken as given, this is the point at which sharing power starts to pay off.

In equilibrium, the world price adjusts in order to equate $\mathcal{U}_p^\dagger = \bar{\mathcal{U}}_p$, as illustrated in the right panel of Figure 4. A fall in the price of the investment good (caused by an increase in the amount

Figure 4: *Comparison of closed and open economies, and world equilibrium*



of economies with the rule of law) raises the payoff from despotism and lowers the payoff from the rule of law. In equilibrium, the price must adjust to make incumbents indifferent between despotism and rule of law. It can be seen from Figure 4 that all incumbents will gain from international trade. Trade enables them to exploit the increasing returns to power sharing and the possibility of choosing s allows them to obtain a higher payoff than what they can obtain in autarky.

The equilibrium fraction of economies with the rule of law is given by the simple formula in the first part of the proposition. There is a bimodal distribution of political systems. Underlying this political specialization is the symbiotic relationship between despots and incumbents in rule-of-law economies. Despotic economies keep the endowment good (say, oil) cheap, which is good for economies with the rule of law, and these contribute to keeping the price of investment goods at a relatively low level, which favours despotic regimes.

Even though incumbents are indifferent between $s = 0$ and $s = 1$, these economies will be very different. The rule-of-law economies will produce $\mathrm{q} + \mu\pi^*$, while the despotic economies will only produce q. By engendering political specialization, international trade leads to economic divergence.

The additional output in rule-of-law economies pays for greater consumption of workers and a larger fraction of incumbents (who still receive more than workers).[20] Workers in rule-of-law economies are strictly better off than workers under despotism since incumbents receive the same payoffs in both types of economies and in economies with a larger s (and p), the ratio $\phi_{\mathrm{p}}/\phi_{\mathrm{w}}$ is lower. Workers in rule-of-law economies are also strictly better off than they would be in a closed economy because $\phi_{\mathrm{p}}/\phi_{\mathrm{w}}$ is decreasing in s and incumbents gain from trade. These two factors actually imply that workers in economies with the rule of law are capturing a relatively greater share of the gains from trade than incumbents.

An economy with the rule of law is Pareto efficient while despotic regimes have inefficiently low

[20] Since capitalists obtain no surplus, in payoff terms they are no better off than workers. In a related setting, Guimaraes and Sheedy (2015) assume that investors have private information about their effort cost, hence all their surplus from investment cannot be extracted through taxes, so capitalists are then better off than workers.

production.[21] It follows that a despotic regime has a positive pecuniary externality on the rest of the world by providing cheap endowment goods. This has implications for efficiency in other countries because it makes the rule of law more attractive. Conversely, economies with the rule of law impose a negative externality on other countries by increasing incentives for other incumbents to choose despotism.

The second part of the proposition shows that there are fewer individuals in power in the open economy. While increasing returns would allow more to be produced with specialization and the same number of people in power at the global level, it is both possible and in the interests of incumbents everywhere to have fewer people in power worldwide. In consequence, under the parameter restrictions that imply $\hat{s} < 1$ in the closed economy, it is not in the interests of the incumbents to exploit the increasing returns to power sharing everywhere in the world — there will be despotism.

The effect of international trade on global output is ambiguous. The fundamental friction is that sharing power and sharing rents are inextricably linked, so incumbents have too little incentive to extend the reach of the rule of law. There are no distortions coming from trade itself, but the possibility of international trade affects how the consequences of the fundamental friction are spread across countries. The usual reasons for international trade are absent from the model and so are the usual gains from trade (in the absence of political frictions, no trade would occur in equilibrium). This means that international trade is close to a zero-sum game because it has little impact on world output, but strong distributional consequences.

Comparative advantage in institutionally-intensive goods would be seen to explain observed trade flows, consistent with the evidence in Nunn and Trefler (2014). However, comparative advantage is endogenous in the model. Those countries with low institutional quality actually lose by trading internationally. Hence, the usual argument that there are gains from exploiting comparative advantage does not apply here.

One implication of the model is that an exogenous shift of a country from despotism to the rule of law must be counteracted in equilibrium by another country moving in the opposite direction. This naturally shifts the focus to those factors determining the equilibrium fraction $\tilde{\omega}$ of rule-of-law regimes in the world. The proposition shows that $\tilde{\omega}$ depends only on the power parameter δ and the parameter α determining the demand for the rule-of-law intensive good. Intuitively, a higher value (price times quantity) of the endowment increases incentives for despotism. Lowering the demand parameter α raises the price and hence the value of the endowment, whereas a greater quantity q of the endowment has countervailing effects on price and quantity (which cancel out with Cobb-Douglas preferences). The model therefore has strong implications for policy. First, localized intervention in particular countries aiming to improve political institutions can be futile because they treat the symptoms rather than the causes. Second, reducing demand for goods with a low rule-of-law intensity would offer an effective route to curb despotism.

[21] Since workers are better off in rule-of-law regimes, at the margin the effort cost $\theta C_{\mathrm{w}}^{\dagger}$ of producing the investment good in despotic economies is lower than the effort cost $\theta \bar{C}_{\mathrm{w}}$ of producing the same investment good in a rule-of-law economy. The absence of investment in despotic economies is therefore not due to the high cost of investment — actually, incumbents would be able to extract more from a marginal investor in a despotic economy than in a rule-of-law economy if they were only able to commit not to take everything ex post.

6 Extension: Ex-ante heterogeneity between countries

The model so far has assumed ex-ante identical economies. The result that specialization arises without any ex-ante heterogeneity highlights the strength of the mechanism pushing countries towards either despotism or rule of law, but leaves open the path any particular country would take. This section presents a simple extension with one dimension of ex-ante heterogeneity that explains which countries become despotisms and which uphold the rule of law.

The assumption is that countries differ in their endowments q. Let q^* denote the average endowment across the world, and $y \equiv \mathrm{q}/\mathrm{q}^*$ the endowment of a particular country relative to the world average. It is assumed y follows a continuous distribution with cumulative distribution $G(y)$. Everything else in the model remains unchanged.

The proposition below gives the features of the world equilibrium with ex-ante heterogeneity.

Proposition 6 *(i) In a closed economy, power sharing $\hat{p}$ and investment $\hat{s}$ are independent of the endowment q and characterized in Proposition 3.*

(ii) In the open economy, the incumbent payoff $\mathcal{U}^*_{\mathrm{p}}(s;\pi^*)$ has the properties given in Proposition 4, namely that it is quasi-convex in s, and the criterion for preferring $s = 1$ over $s = 0$ remains that given in (5.2). At a given world price π^*, the condition for preferring the rule of law is equivalent to $y \leq \tilde{y}$, where the threshold relative endowment is:

$$\tilde{y} = \frac{\mu\pi^*}{\delta(2+\delta)\min\left\{1, \frac{2}{1+2\delta}\right\}(\bar{p} - p^{\ddagger})\mathrm{q}^*}.$$

(iii) The world equilibrium features a mixture of despotisms and rule-of-law economies, with the fraction $\tilde{\omega}$ $(0 < \tilde{\omega} < 1)$ of the latter being the unique solution of:

$$\tilde{\omega}G^{-1}(\tilde{\omega}) = \frac{\alpha}{(1-\alpha)\delta(2+\delta)\min\left\{1, \frac{2}{1+2\delta}\right\}(\bar{p} - p^{\ddagger})}.$$

Proof See appendix A.6. ■

First, note that adding ex-ante heterogeneity to a world of closed economies has no effect on power sharing or investment. Differences in the quantity of the endowment good are offset by opposite differences in its relative price. Thus, any effect of ex-ante heterogeneity in a world of open economies must be due to its effects on specialization and trade. In an open economy, the fundamental reason for specialization, as reflected in the quasi-convexity of the incumbent pay-off, remains unchanged. Incumbents exploit the increasing returns to power sharing through trade, and thus choose either $s = 0$ or $s = 1$ as before. However, the selection of those who will be despotic and those who will establish the rule of law is no longer arbitrary: the relative quantity of the endowment good determines which of these extremes is strictly preferred.[22] Economies with relatively low quantities of the endowment good obtain the rule of law in equilibrium; economies with high quantities of the endowment are condemned to despotism.

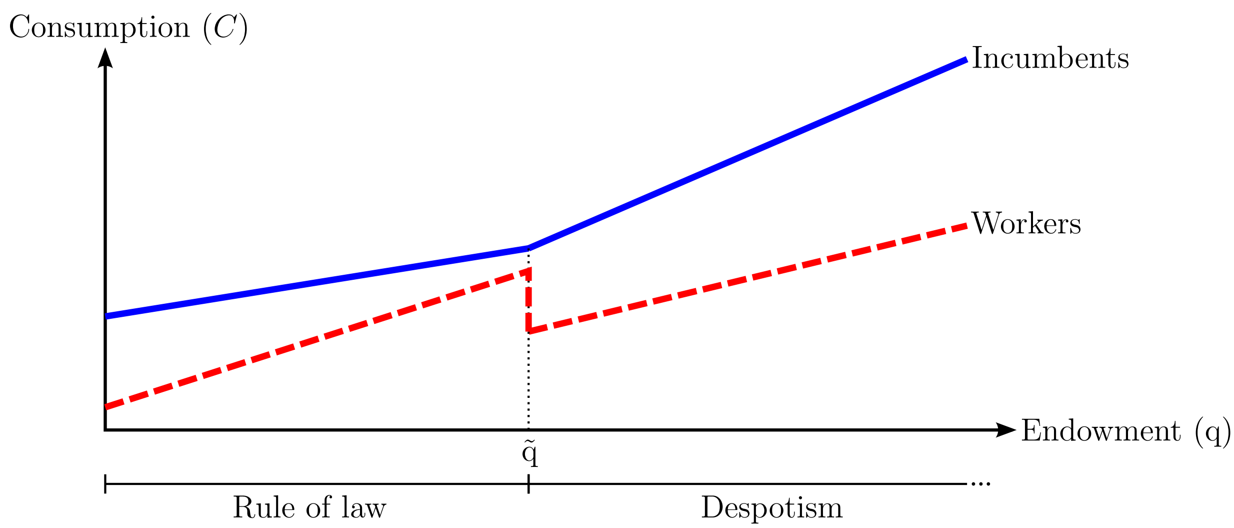
[22] The output from an investment opportunity has been normalized to one unit of the investment good. So q can also be interpreted as the quantity of the endowment good relative to the potential production of the investment good.

As before, the world equilibrium features a mixture of despotic and rule-of-law economies. Not all are able to exploit increasing returns given the size of the world market for the investment good. The equilibrium fraction of rule-of-law economies could be larger or smaller than in the case of trade between ex-ante identical economies. More precisely, $\tilde{\omega}$ in the ex-ante heterogeneous world always lies between the $\tilde{\omega}$ in the world of ex-ante identical economies and the fraction of economies with endowments below the global average (which would be one half if the distribution of endowments were symmetric). For example, if endowments were concentrated almost exclusively in a small measure of economies, the rule of law would be pervasive around the world.

The welfare effects of trade are similar to those found in Proposition 5. Incumbents always gain from trade, as do workers in rule-of-law economies. The arguments are identical to those given earlier.

Figure 5 depicts the consumption of incumbents and workers in the cross-section of economies. Consumption of incumbents is strictly increasing in the endowment q, especially so for despots because the gradient reflects the share each incumbent receives, which is greater in despotic countries. Consumption of workers is increasing in q, controlling for the political system. However, there is a discrete step down at the threshold between the rule-of-law and despotism, reflecting the smaller share of output received by workers in a despotic regime. Note also that the gradient is lower in the despotic region, again reflecting the lower worker share. Crucially, at least some, and possibly all economies with a large endowment are poorer than those with endowments consistent with the rule of law. The model thus gives rise to a natural resource curse.

Figure 5: *Ex-ante heterogeneity*



7 Concluding remarks

For social scientists grappling with the welter of autocratic regimes around the world, one particular fact is noteworthy: the stubborn resistance to adopting the rule of law in spite of its proven success elsewhere. An important policy question is what can be done to bring about positive political change. The literature in political science has focused on country-specific factors that are seen as barriers to change such as culture and history. This paper highlights the importance of thinking about the problem in general equilibrium at the world level.

The adoption of the rule of law increases the output of goods that require strong protection of property rights and thus reduces their relative price. This increases incentives for those in power in other countries to choose autocratic government. Consequently, the proportion of countries in the world with the rule of law is limited by the demand for goods that depend on the rule of law for their production. The analysis suggests those living under autocracy would be best helped by subsidies to goods that are substitutes to their own exports.

The paper also has implications for the policy debate about the merits of international trade. In spite of its well-known economic gains, international trade has its critics. Much of the opposition to free trade comes from a sense that some economies end up specializing in the wrong kinds of goods — primary goods — which is detrimental to development. The model in this paper is consistent with a negative correlation between specializing in primary goods and economic performance. However, the policy prescription is not to limit trade. This is because the distortion is not found in trade itself, but in the distributional consequences of the power sharing needed for good government.

References

Acemoglu, D., Johnson, S. and Robinson, J. (2005), "The rise of Europe: Atlantic trade, institutional change, and economic growth", *American Economic Review*, **95(3)**:546–579 (June). 6

Acemoglu, D. and Robinson, J. A. (2000), "Why did the West extend the franchise? Democracy, inequality, and growth in historical perspective", *Quarterly Journal of Economics*, **115(4)**:1167–1199 (November). 6

Acemoglu, D., Robinson, J. A. and Verdier, T. (2014), "Asymmetric growth and institutions in an interdependent world", working paper, MIT. 6

Acemoglu, D., Verdier, T. and Robinson, J. A. (2004), "Kleptocracy and divide-and-rule: A model of personal rule", *Journal of the European Economic Association*, **2(2–3)**:162–192 (April–May). 7

Anderson, J. E. and Marcouiller, D. (2002), "Insecurity and the pattern of trade: An empirical investigation", *Review of Economics and Statistics*, **84(2)**:342–352 (May). 6

Araujo, L., Mion, G. and Ornelas, E. (2015), "Institutions and export dynamics", *Journal of International Economics, forthcoming*. 6

Bai, J. H. and Lagunoff, R. (2011), "On the Faustian dynamics of policy and political power", *Review of Economic Studies*, **78(1)**:17–48. 6

Cardoso, F. H. and Faletto, E. (1979), *Dependency and Development in Latin America*, University of California Press. 7

Caselli, F. and Cunningham, T. (2009), "Leader behaviour and the natural resource curse", *Oxford Economic Papers*, **61(4)**:628–650 (October). 7

Caselli, F. and Tesei, A. (2015), "Resource windfalls, political regimes, and political stability", *Review of Economics and Statistics, forthcoming.* 7

Do, Q.-T. and Levchenko, A. A. (2009), "Trade, inequality, and the political economy of institutions", *Journal of Economic Theory*, **144(4)**:1489–1520 (July). 6

Fukuyama, F. (1992), *The End of History and the Last Man*, Free Press. 7

——— (2011), *The Origins of Political Order*, Farrar, Straus and Giroux. 7

Gandhi, J. and Przeworski, A. (2007), "Authoritarian institutions and the survival of autocrats", *Comparative Political Studies*, **40**:1279–1301. 7

Goorha, P. (2007), "Polarized futures", *Kyklos*, **60(1)**:65–75 (February). 7

Greif, A. (1993), "Contract enforceability and economic institutions in early trade: The Maghribi traders' coalition", *American Economic Review*, **83(3)**:525–548 (June). 6

——— (2006), *Institutions and the Path to the Modern Economy*, Cambridge University Press. 6

Greif, A., Milgrom, P. and Weingast, B. R. (1994), "Coordination, commitment, and enforcement: The case of the merchant guild", *Journal of Political Economy*, **102(4)**:745–776 (August). 6

Grossman, G. M. and Helpman, E. (1994), "Protection for sale", *American Economic Review*, **84(4)**:833–850 (September). 6

Guimaraes, B. and Sheedy, K. D. (2015), "Guarding the guardians", working paper, London School of Economics. 5, 6, 9, 10, 12, 21, 28

Huntington, S. (1993), *The Third Wave: Democratization in the Late Twentieth Century*, University of Oklahoma Press. 7

Jack, W. and Lagunoff, R. (2006), "Dynamic enfranchisement", *Journal of Public Economics*, **90(4–5)**:551–572 (May). 6

Krugman, P. R. (1979), "Increasing returns, monopolistic competition, and international trade", *Journal of International Economics*, **9(4)**:469–479 (November). 6, 26

——— (1980), "Scale economies, product differentiation, and the pattern of trade", *American Economic Review*, **70(5)**:950–959 (December). 6, 26

——— (1987), "The narrow moving band, the Dutch disease, and the competitive consequences of Mrs. Thatcher: Notes on trade in the presence of dynamic scale economies", *Journal of Development Economics*, **27(1–2)**:41–55 (October). 7

Levchenko, A. A. (2007), "Institutional quality and international trade", *Review of Economic Studies*, **74(3)**:791–819 (July). 6

——— (2013), "International trade and institutional change", *Journal of Law, Economics, & Organization*, **29(5)**:1145–1181 (October). 6

Mehlum, H., Moene, K. and Torvik, R. (2006), "Institutions and the resource curse", *Economic Journal*, **116(508)**:1–20 (January). 7

Melitz, M. J. (2005), "When and how should infant industries be protected?", *Journal of International Economics*, **66(1)**:177–196 (May). 7

Milgrom, P. R., North, D. C. and Weingast, B. R. (1990), "The role of institutions in the revival of trade: The Law Merchant, private judges, and the Champagne fairs", *Economics & Politics*, **2(1)**:1–23 (March). 6

Nafziger, S. (2008), "Communal institutions, resource allocation, and Russian economic development: 1861–1905", *Journal of Economic History*, **68(2)**:570–575 (June). 5

Nunn, N. (2007), "Relationship-specificity, incomplete contracts, and the pattern of trade", *Quarterly Journal of Economics*, **122(2)**:569–600 (May). 6

Nunn, N. and Trefler, D. (2014), "Domestic institutions as a source of comparative advantage", in E. Helpman, K. Rogoff and G. Gopinath (eds.), *Handbook of International Economics*, vol. 4, Elsevier. 6, 29

Puga, D. and Trefler, D. (2014), "International trade and institutional change: Medieval Venice's response to globalization", *Quarterly Journal of Economics*, **129(2)**:753–821 (May). 6

Ray, D. (2007), *A Game-Theoretic Perspective on Coalition Formation*, Oxford University Press. 12

Robinson, J. A., Torvik, R. and Verdier, T. (2006), "Political foundations of the resource curse", *Journal of Development Economics*, **79(2)**:447–468 (April). 7

Rodrik, D. (1996), "Coordination failures and government policy: A model with applications to East Asia and Eastern Europe", *Journal of International Economics*, **40(1–2)**:1–22 (February). 7

Sachs, J. D. and Warner, A. M. (2001), "The curse of natural resources", *European Economic Review*, **45(4–6)**:827–838 (May). 7

Van der Ploeg, F. (2011), "Natural resources: Curse or blessing?", *Journal of Economic Literature*, **49(2)**:366–420 (June). 7

Williamson, J. G. (2011), *Trade and Poverty: When the Third World Fell Behind*, MIT Press. 5

A Technical appendix

A.1 Proof of Proposition 1

Consider rules established at the post-investment stage of Figure 2. At this point, the fraction s of investment opportunities that were taken is predetermined, so the capital stock K (equation (2.2)) is a state variable. Rules established at the post-investment stage determine the amount of power sharing p, and the number of individuals outside the incumbent group who receive a different consumption allocation as a result of undertaking investment in the past (capitalists). The measure of capitalists is κ, which given that $\upmu s$ investment opportunities were taken, must satisfy $\kappa \leq \upmu s$. The number of workers is $1-p-\kappa$. Rules specify the amount of each good allocated to individuals in these groups, as well as the amounts exported or imported. Let $\mathcal{U}_{\mathrm{p}}$, $\mathcal{U}_{\mathrm{k}}$, and $\mathcal{U}_{\mathrm{w}}$ denote the continuation payoffs of incumbents, capitalists, and workers under the current rules.

It is necessary to understand what rebellions would be rational according to Definition 1 to determine the features of the equilibrium rules. Since the equilibrium conditions (Definition 2) restrict rebellions to be followed by the establishment of rules that are themselves an equilibrium, and given the Markovian requirement and the absence of any further changes to fundamental state variables at the post-investment stage, there is a unique amount of power sharing p' and a unique continuation payoffs $\mathcal{U}'_{\mathrm{p}}$, $\mathcal{U}'_{\mathrm{k}}$, and $\mathcal{U}'_{\mathrm{w}}$ associated with rules established following a rebellion. But the identities of the individuals in the groups are not determined by the requirements of equilibrium, so it is necessary to consider different compositions of groups subject to these groups having sizes consistent with equilibrium.

The set of conditions required for the absence of any rational rebellion (Definition 1) is given in (3.8), which must hold for all permutations of identities under subsequent rules. Let $\mathcal{U}(\imath)$ denote the continuation payoff of individual $\imath$ under the current rules. The inequality in (3.8) is the complement of the successful rebellion condition (2.8) with the maximum participation in the rebel faction and the maximum rebellion effort given the rationality requirements for rebellions in Definition 1, and the size of the opposing loyal faction, given the incentives of incumbents to defend the rules. The left-hand side of (3.8) is the integral over individuals in $\mathcal{P}'$, membership of which is a requirement for joining the rebel faction. The integrand is the maximum over zero (where there no incentive to participate in the rebellion) and the maximum individually rational rebellion effort from (2.10a). The right-hand side the integral over the strength $\updelta$ of each incumbent who belongs to the loyal faction, as determined by (2.10b). This reduces to an integral over current incumbents $\mathcal{P}$ who expect a worse payoff under subsequent rules. Incumbents fail to fight in the loyal faction if they have a place in the rebel faction (which must be individually rational for them, so they must gain from the rebellion), or if they gain from the rebellion even though they will lose power. It is conjectured that the latter requirement will automatically be satisfied, and this will be confirmed later (which is sufficient because alternative rules where this requirement is relevant would imply a tighter set of no-rebellion conditions). The collection of no-rebellion conditions is therefore

$$\int_{\mathcal{P}'} \max\{\exp\{\mathcal{U}'_{\mathrm{p}} - \mathcal{U}(\imath)\} - 1, 0\}\mathrm{d}\imath \leq \updelta \left(\int_{\mathcal{P}} \mathrm{d}\imath - \mathbb{1}[\mathcal{U}_{\mathrm{p}} \leq \mathcal{U}'_{\mathrm{p}}] \int_{\mathcal{P}\cap\mathcal{P}'} \mathrm{d}\imath \right), \qquad \text{[A.1.1]}$$

for all sets $\mathcal{P}'$ with measure p'.

The equilibrium conditions leave open the identities of those $\mathcal{P}'$ individuals subsequently in power following a rebellion. Let σ_{p}, σ_{k}, and σ_{w} denote the fractions of the subsequent incumbent group drawn from current incumbents, capitalists, and workers respectively. These non-negative numbers must sum to one and satisfy the natural restrictions $\sigma_{\mathrm{p}} \leq p/p'$, $\sigma_{\mathrm{k}} \leq \kappa/p'$, and $\sigma_{\mathrm{w}} \leq (1-p-\kappa)/p'$. Since incumbents will have incentives to design current rules that avoid rebellion, all members of a group will receive the same continuation payoff. The conditions (A.1.1) to avoid any rational rebellions can be stated as follows:

$$\sigma_{\mathrm{w}} p' \max\{\exp\{\mathcal{U}'_{\mathrm{p}} - \mathcal{U}_{\mathrm{w}}\} - 1, 0\} + \sigma_{\mathrm{k}} p' \max\{\exp\{\mathcal{U}'_{\mathrm{p}} - \mathcal{U}_{\mathrm{k}}\} - 1, 0\} + \sigma_{\mathrm{p}} p' \max\{\exp\{\mathcal{U}'_{\mathrm{p}} - \mathcal{U}_{\mathrm{p}}\} - 1, 0\} \leq \updelta \left(p - \mathbb{1}[\mathcal{U}_{\mathrm{p}} \leq \mathcal{U}'_{\mathrm{p}}]\sigma_{\mathrm{p}} p'\right),$$

which can be written more concisely as:

$$\sigma_{\mathrm{w}} p' \max\{\exp\{\mathcal{U}'_{\mathrm{p}} - \mathcal{U}_{\mathrm{w}}\} - 1, 0\} + \sigma_{\mathrm{k}} p' \max\{\exp\{\mathcal{U}'_{\mathrm{p}} - \mathcal{U}_{\mathrm{k}}\} - 1, 0\} \\ + \sigma_{\mathrm{p}} p' \mathbb{1}[\mathcal{U}_{\mathrm{p}} \leq \mathcal{U}'_{\mathrm{p}}] \left(\exp\{\mathcal{U}'_{\mathrm{p}} - \mathcal{U}_{\mathrm{p}}\} - 1 + \delta\right) \leq \delta p, \quad \text{[A.1.2]}$$

for all feasible σ_{p}, σ_{k}, and σ_{w}. Since it will not be optimal for the incumbents to choose rules that lead to them losing power through rebellions, these conditions are treated as a set of constraints on the rules. With no further rebellion effort incurred, and any past effort costs sunk, using the consumption aggregator (2.1) and utility function (2.3), the continuation payoffs are:

$$\mathcal{U}_{\mathrm{p}} = (1-\alpha)\log c_{\mathrm{pE}} + \alpha \log c_{\mathrm{pI}} - \log\left((1-\alpha)^{1-\alpha}\alpha^{\alpha}\right); \quad \text{[A.1.3a]}$$

$$\mathcal{U}_{\mathrm{k}} = (1-\alpha)\log c_{\mathrm{kE}} + \alpha \log c_{\mathrm{kI}} - \log\left((1-\alpha)^{1-\alpha}\alpha^{\alpha}\right); \quad \text{[A.1.3b]}$$

$$\mathcal{U}_{\mathrm{w}} = (1-\alpha)\log c_{\mathrm{wE}} + \alpha \log c_{\mathrm{wI}} - \log\left((1-\alpha)^{1-\alpha}\alpha^{\alpha}\right). \quad \text{[A.1.3c]}$$

Given the sizes p, κ, and $1-p-\kappa$ of the groups of incumbents, capitalists, and workers, the resource constraints in (2.6) can be written as follows:

$$p c_{\mathrm{pE}} + \kappa c_{\mathrm{kE}} + (1-p-\kappa) c_{\mathrm{wE}} = \mathrm{q} - x_{\mathrm{E}}, \quad \text{and} \quad p c_{\mathrm{pI}} + \kappa c_{\mathrm{kI}} + (1-p-\kappa) c_{\mathrm{wI}} = K - x_{\mathrm{I}}. \quad \text{[A.1.4]}$$

Free markets and no taxes that distort the allocation of consumption between goods

Equilibrium rules must maximize the incumbent payoff $\mathcal{U}_{\mathrm{p}}$ subject to the resource constraints and the no-rebellion constraints (A.1.2). Taking as given the sizes of the different groups, the no-rebellion constraints (A.1.2) depend only on the continuation utilities, which are given in (A.1.3). Taking as given the stock of capital K and net exports x_{E} and x_{I}, consider the allocation of goods consistent with the resource constraints (A.1.4) that maximizes $\mathcal{U}_{\mathrm{p}}$ subject to providing payoffs $\mathcal{U}_{\mathrm{k}}$ and $\mathcal{U}_{\mathrm{w}}$ above thresholds sufficient to satisfy the no-rebellion constraints. The first-order conditions for the consumption allocation $\{c_{\mathrm{pE}}, c_{\mathrm{pI}}, c_{\mathrm{kE}}, c_{\mathrm{kI}}, c_{\mathrm{wE}}, c_{\mathrm{wI}}\}$ are:

$$\frac{\alpha/c_{\mathrm{pI}}}{(1-\alpha)/c_{\mathrm{pE}}} = \frac{\alpha/c_{\mathrm{kI}}}{(1-\alpha)/c_{\mathrm{kE}}} = \frac{\alpha/c_{\mathrm{wI}}}{(1-\alpha)/c_{\mathrm{wE}}}.$$

This requires equating the marginal rates of substitution between goods across all individuals, which given the consumption basket (2.1), means equating the consumption ratios of the two goods $c_{\mathrm{I}}/c_{\mathrm{E}}$ across all individuals. Using the resource constraints (A.1.4), this common ratio is:

$$\frac{c_{\mathrm{pI}}}{c_{\mathrm{pE}}} = \frac{c_{\mathrm{kI}}}{c_{\mathrm{kE}}} = \frac{c_{\mathrm{wI}}}{c_{\mathrm{wE}}} = \frac{K - x_{\mathrm{I}}}{\mathrm{q} - x_{\mathrm{E}}}. \quad \text{[A.1.5]}$$

If there were a market for exchanging the two goods (with relative price π of the investment good) then individual demand functions (3.2) subject to a given income imply a common consumption ratio equal to:

$$\frac{c_{\mathrm{I}}}{c_{\mathrm{E}}} = \frac{\alpha}{(1-\alpha)\pi}. \quad \text{[A.1.6]}$$

Since markets would clear at price $\pi = \tilde{\pi}$ from (3.4), the consumption ratio above would be exactly the same as the one in (A.1.5). Let Y_{p}, Y_{k}, and Y_{w} denote the incomes (in terms of the endowment good) of incumbents, capitalists, and workers. Suppose these satisfy the aggregate budget constraint given in (3.5):

$$p Y_{\mathrm{p}} + \kappa Y_{\mathrm{k}} + (1-p-\kappa) Y_{\mathrm{w}} = Y, \quad \text{where} \quad Y = (\mathrm{q} - x_{\mathrm{E}}) + \tilde{\pi}(K - x_{\mathrm{I}}). \quad \text{[A.1.7]}$$

Observe that the market-clearing price $\tilde{\pi}$ from (3.4) implies:

$$Y = \frac{\mathrm{q} - x_{\mathrm{E}}}{1-\alpha}, \quad \text{and} \ \ Y = \frac{\tilde{\pi}(K - x_{\mathrm{I}})}{\alpha}, \tag{A.1.8}$$

and therefore both resource constraints in (A.1.4) are satisfied given incomes consistent with (A.1.7), individual demands (3.2), and the market-clearing price (3.4). This means that markets can deliver exactly the same consumption allocation subject to some individual incomes as would be chosen to maximize incumbent payoffs. The equilibrium rules will therefore allow such markets to be established and will not interfere with their operation, confirming the claim in the proposition.

The aggregate budget constraint (A.1.7) can also be expressed in terms of consumption by dividing both sides by $\tilde{\pi}^{\alpha}$:

$$pC_{\mathrm{p}} + \kappa C_{\mathrm{k}} + (1-p-\kappa)C_{\mathrm{w}} = C, \quad \text{where} \ \ C \equiv \frac{Y}{\tilde{\pi}^{\alpha}} = \frac{(\mathrm{q} - x_{\mathrm{E}})^{1-\alpha}(K - x_{\mathrm{I}})^{\alpha}}{(1-\alpha)^{1-\alpha}\alpha^{\alpha}}, \tag{A.1.9}$$

where the left-hand side follows from equation (3.3), and the expression for C uses the market-clearing price (3.4) and the equations in (A.1.8). The consumption levels appearing in this constraint are those that directly enter the utility function (2.3), and the variable C summarizes all information about a country's resource constraints.

Free international trade

Consider a case where the economy can access world markets with relative price π^*. The continuation utility $\mathcal{U} = \log C$ of all individuals can be expressed only in terms of the amount of the consumption basket C received, so the bounds on utility necessary to satisfy the no-rebellion constraints (A.1.2) can also be expressed in terms of lower bounds for consumption. Since net exports x_{E} and x_{I} enter only through the expression for C in (A.1.9), the equilibrium rules must select these variables to maximize the value of C subject to the international budget constraint (2.5). The first-order condition for this constrained maximization problem is:

$$\frac{K - x_{\mathrm{I}}}{\mathrm{q} - x_{\mathrm{E}}} = \frac{\alpha}{(1-\alpha)\pi^*}.$$

Comparison with equations (A.1.5) and (A.1.6) shows that this first-order condition can only be satisfied (given the optimality of domestic free exchange) when the domestic relative price $\tilde{\pi}$ is equal to the international relative price π^*. It follows that the equilibrium rules will permit international trade in goods with no interference and no distorting tariffs, confirming the claim in the proposition.

With free international trade ($\tilde{\pi} = \pi^*$), note that by using (2.5), aggregate income and consumption from (A.1.8) and (A.1.9) are:

$$pY_{\mathrm{p}} + \kappa Y_{\mathrm{k}} + (1-p-\kappa)Y_{\mathrm{w}} = Y = \mathrm{q} + \tilde{\pi}K, \quad \text{and} \ \ pC_{\mathrm{p}} + \kappa C_{\mathrm{k}} + (1-p-\kappa)C_{\mathrm{w}} = C = \frac{\mathrm{q} + \tilde{\pi}K}{\tilde{\pi}^{\alpha}}. \tag{A.1.10}$$

Note that these expressions are also valid when the economy is not in contact with world markets (x_{E} and x_{I} are both constrained to be zero).

A no-rebellion constraint must bind for a rebel faction comprising a positive measure of non-incumbents

Substituting continuation utilities $\mathcal{U} = \log C$ into the set of no-rebellion constraints (A.1.2) leads to:

$$\sigma_{\mathrm{w}} p' \max\{C'_{\mathrm{p}}/C_{\mathrm{w}} - 1, 0\} + \sigma_{\mathrm{k}} p' \max\{C'_{\mathrm{p}}/C_{\mathrm{k}} - 1, 0\} + \sigma_{\mathrm{p}} p' \mathbb{1}[C_{\mathrm{p}} \le C'_{\mathrm{p}})] \left(C'_{\mathrm{p}}/C_{\mathrm{p}} - 1 + \delta\right) \le \delta p,$$

for all feasible weights σ_{p}, σ_{k}, and σ_{w}. Rules cannot satisfy optimality if $C_{\mathrm{w}} > C'_{\mathrm{p}}$ or $C_{\mathrm{k}} > C'_{\mathrm{p}}$ otherwise strictly less consumption could be allocated to workers or capitalists, allowing incumbent consumption to be raised, yet still ensuring that the no-rebellion constraints above hold. Attention can be restricted to rules specifying $C_{\text{w}} \le C'_{\text{p}}$ and $C_{\text{k}} \le C'_{\text{p}}$, which means that the no-rebellion constraints can be stated as follows:

$$\sigma_{\text{w}} p' \left(\frac{C'_{\text{p}}}{C_{\text{w}}} - 1\right) + \sigma_{\text{k}} p' \left(\frac{C'_{\text{p}}}{C_{\text{k}}} - 1\right) + \sigma_{\text{p}} p' \mathbb{1}[C_{\text{p}} \le C'_{\text{p}}] \left(\frac{C'_{\text{p}}}{C_{\text{p}}} - 1 + \delta\right) \le \delta p,$$

for all feasible σ_{p}, σ_{k}, and σ_{w}. It is convenient to reformalate these as:

$$(1-\sigma) p' \left(\left(\frac{1-\varkappa}{C_{\text{w}}} + \frac{\varkappa}{C_{\text{k}}}\right) C'_{\text{p}} - 1\right) + \sigma p' \mathbb{1}[C_{\text{p}} \le C'_{\text{p}}] \left(\frac{C'_{\text{p}}}{C_{\text{p}}} - 1 + \delta\right) \le \delta p, \qquad \text{[A.1.11]}$$

which must hold for all feasible σ and $\varkappa$, where $\sigma \equiv \sigma_{\text{p}}$ is the fraction of current incumbents included in the post-rebellion incumbent group, and $\varkappa \equiv \sigma_{\text{k}}/(\sigma_{\text{w}} + \sigma_{\text{k}})$ is the fraction of capitalists among current non-incumbents who are included in the post-rebellion incumbent group. Since both p and p' must be less than $1/2$, feasible values of σ lie between 0 and $\min\{p/p', 1\}$.

The constraint (A.1.11) must bind for some $\sigma < 1$. Otherwise, the term multiplying $1-\sigma$ is too high, which allows C_{w} or C_{k} to be reduced, increasing the incumbent payoff $\mathcal{U}_{\text{p}}$ and yet still satisfying all no-rebellion constraints.

Equalization of non-incumbent payoffs: full expropriation of capital

Suppose the rules were to allocate different amounts of consumption to capitalists and workers: $C_{\text{k}} \ne C_{\text{w}}$ (when there are both some capitalists and workers, so $\kappa > 0$ and $\kappa < 1-p$). Consider first the case where $C_{\text{k}} > C_{\text{w}}$, which would mean the no-rebellion constraint with the largest value of the left-hand side would be the one with the smallest possible value of $\varkappa$ (for each σ). Now suppose that consumption is redistributed equally between workers and capitalists to give both consumption C_{n}:

$$C_{\text{n}} = \frac{(1-p-\kappa)C_{\text{w}} + \kappa C_{\text{k}}}{1-p},$$

which is feasible given the resource constraint (A.1.10). Since $1/C$ is a convex function, Jensen's inequality implies that:

$$\frac{1}{C_{\text{n}}} < \left(\frac{1-p-\kappa}{1-p}\right)\frac{1}{C_{\text{w}}} + \left(\frac{\kappa}{1-p}\right)\frac{1}{C_{\text{k}}}.$$

Note that the smallest feasible value of $\varkappa$ (for a given σ) is always smaller than $\kappa/(1-p)$ because there are $(1-\sigma)p' \le 1-p$ places for current non-incumbents in the post-rebellion incumbent group, and some workers are available to join this group. Since the smallest feasible $\varkappa$ is such that $\varkappa < \kappa/(1-p)$ and as $1/C_{\text{w}} > 1/C_{\text{k}}$:

$$\left(1 - \frac{\kappa}{1-p}\right)\frac{1}{C_{\text{w}}} + \left(\frac{\kappa}{1-p}\right)\frac{1}{C_{\text{k}}} < \frac{1-\varkappa}{C_{\text{w}}} + \frac{\varkappa}{C_{\text{k}}}.$$

Putting these results together, it follows that:

$$\frac{1}{C_{\text{n}}} < \frac{1-\varkappa}{C_{\text{w}}} + \frac{\varkappa}{C_{\text{k}}}.$$

If the no-rebellion constraints hold for all values of σ and $\varkappa$ initially, it follows that all no-rebellion constraints with $\sigma < 1$ are slackened by redistribution between capitalists and workers. Since one such constraint must be binding, consumption inequality between capitalists and workers is not consistent with rules being optimal. An exactly analogous argument holds in the case where $C_{\text{w}} > C_{\text{k}}$. This establishes that all non-incumbents share a common level of consumption $C_{\text{n}} = C_{\text{w}} = C_{\text{k}}$, confirming the claim in the proposition. The labelling of individuals as capitalists or workers becomes irrelevant, and the set of no-rebellion constraints reduces to:

$$(1-\sigma)p'\left(\frac{C'_{\mathrm{p}}}{C_{\mathrm{n}}}-1\right)+\sigma p'\mathbb{1}[C_{\mathrm{p}}\leq C'_{\mathrm{p}}]\left(\frac{C'_{\mathrm{p}}}{C_{\mathrm{p}}}-1+\delta\right)\leq \delta p, \qquad \text{[A.1.12]}$$

for all feasible values of σ, that is, all $\sigma \in [0, \min\{p/p', 1\}]$.

The equilibrium rules can be characterized by considering only no-rebellion constraints for workers

The claim is that the equilibrium rules can be found by solving a simpler problem where it is assumed that rebel factions can only include those not currently in power (formally, Definition 2 is unchanged, but Definition 1 requires that $\mathcal{R}$ does not include any individuals in $\mathcal{P}$). In other words, taking account only of the no-rebellion constraint (A.1.12) when $\sigma = 0$:

$$p'\left(\frac{C'_{\mathrm{p}}}{C_{\mathrm{n}}}-1\right)\leq \delta p. \qquad \text{[A.1.13]}$$

First, let $\{p^\ddagger, C^\ddagger_{\mathrm{p}}, C^\ddagger_{\mathrm{n}}\}$ denote values of these variables under the equilibrium rules in the simple problem subject only to (A.1.13). With no change in the state variable K following rebellions, the equilibrium conditions require $p' = p^\ddagger$ and $C'_{\mathrm{p}} = C^\ddagger_{\mathrm{p}}$. Since (A.1.13) must hold, it follows that $C^\ddagger_{\mathrm{p}}/C^\ddagger_{\mathrm{n}} \leq 1+\delta$.

Now consider the optimal rules in the full problem, taking $p' = p^\ddagger$ and $C'_{\mathrm{p}} = C^\ddagger_{\mathrm{p}}$ as given. With $p = p^\ddagger$, optimal rules must satisfy (A.1.12) for all $\sigma \in [0,1]$. If the equilibrium rules of the simpler problem are to satisfy these constraints then the following must hold:

$$(1-\sigma)p^\ddagger\left(\frac{C^\ddagger_{\mathrm{p}}}{C^\ddagger_{\mathrm{n}}}-1\right)+\delta\sigma p^\ddagger\leq \delta p^\ddagger,$$

which does indeed follow from $C^\ddagger_{\mathrm{p}}/C^\ddagger_{\mathrm{n}} \leq 1+\delta$. Since the rules satisfy all the no-rebellion constraints and maximize the incumbent payoff subject to constraint (A.1.13), which is weaker than the general constraint (A.1.12), it follows that these rules are optimal subject to the full set of no-rebellion constraints. They are thus an equilibrium of the original problem.

Now consider the converse. Take rules that are an equilibrium of the original problem with $\{p^\ddagger, C^\ddagger_{\mathrm{p}}, C^\ddagger_{\mathrm{n}}\}$. With $p' = p^\ddagger$ and $C'_{\mathrm{p}} = C^\ddagger_{\mathrm{p}}$, these rules clearly satisfy the single no-rebellion constraint (A.1.13), which is a special case of (A.1.12). Now suppose they are not optimal subject only to (A.1.13). This means there are alternative rules with $\{p, C_{\mathrm{p}}, C_{\mathrm{n}}\}$ satisfying (A.1.13) that yield a higher incumbent payoff $C_{\mathrm{p}} > C^\ddagger_{\mathrm{p}}$. For any $\sigma \in [0,1]$:

$$(1-\sigma)p^\ddagger\left(\frac{C^\ddagger_{\mathrm{p}}}{C_{\mathrm{n}}}-1\right)+\sigma p^\ddagger\mathbb{1}[C_{\mathrm{p}}\leq C^\ddagger_{\mathrm{p}}]\left(\frac{C^\ddagger_{\mathrm{p}}}{C_{\mathrm{p}}}-1+\delta\right)=(1-\sigma)p^\ddagger\left(\frac{C^\ddagger_{\mathrm{p}}}{C_{\mathrm{n}}}-1\right)\leq(1-\sigma)\delta p\leq \delta p,$$

where the first equality follows from $C_{\mathrm{p}} > C^\ddagger_{\mathrm{p}}$, the first inequality follows from the rules satisfying (A.1.13), and the second inequality follows from $0 \leq 1-\sigma \leq 1$. This shows that the alternative rules satisfy all the no-rebellion constraints (A.1.12) of the original problem and yield a higher incumbent payoff, contradicting the optimality of the rules with $\{p^\ddagger, C^\ddagger_{\mathrm{p}}, C^\ddagger_{\mathrm{n}}\}$. This contradiction shows that they must be optimal subject only to the single no-rebellion constraint (A.1.13), and thus an equilibrium of the simpler problem. This confirms the claim in the proposition.

Equilibrium rules subject to the binding no-rebellion constraint for workers

With payoff equalization for all non-incumbents ($C_{\mathrm{w}} = C_{\mathrm{k}} = C_{\mathrm{n}}$), the resource constraint in (A.1.10) implies that incumbents' consumption is:

$$C_{\mathrm{p}} = \frac{C-(1-p)C_{\mathrm{n}}}{p}, \qquad \text{[A.1.14]}$$

where the level of aggregate resources C available for consumption is given in (A.1.10) (which depends only on the predetermined capital stock K). There is only one relevant no-rebellion constraint (A.1.13) (where the post-rebellion incumbent group would comprise only those currently not incumbents), and this constraint must be binding in equilibrium. Rearranging the constraint shows that consumption of all non-incumbents is given by:

$$C_{\mathrm{n}} = \frac{C'_{\mathrm{p}}}{1 + \delta \frac{p}{p'}}. \tag{A.1.15}$$

Now define $\phi_{\mathrm{p}} \equiv C_{\mathrm{p}}/C$ and $\phi_{\mathrm{n}} \equiv C_{\mathrm{n}}/C$ to be the per-person shares of consumption of incumbents and non-incumbents respectively. Dividing both sides of (A.1.14) by C implies:

$$\phi_{\mathrm{p}} = \frac{1 - (1-p)\phi_{\mathrm{n}}}{p}. \tag{A.1.16}$$

Given the state variable K, the optimality of domestic free exchange and international free trade already determine the value of C (equation (A.1.10)). It follows that the optimality condition requires that the remaining aspects of the rules maximize the incumbent share ϕ_{p} subject to the no-rebellion constraint (A.1.15) (the resource constraint has already been accounted for in (A.1.16)). Using the Markovian property of equilibrium rules, since any rules established after a subsequent rebellion would face the same value of the only fundamental state variable K, and since domestic free exchange and international free trade will remain necessary for optimality of those rules, aggregate resources available for consumption will remain the same after a rebellion ($C' = C = C^{\ddagger}$). Dividing the binding no-rebellion constraint (A.1.16) by this common total amount of resources implies:

$$\phi_{\mathrm{n}} = \frac{\phi'_{\mathrm{p}}}{1 + \delta \frac{p}{p'}}. \tag{A.1.17}$$

This can be substituted into the expression for ϕ_{p} in (A.1.16) to obtain:

$$\phi_{\mathrm{p}} = \frac{1 - (1-p)\frac{\phi'_{\mathrm{p}}}{1+\delta\frac{p}{p'}}}{p}. \tag{A.1.18}$$

Optimality of the equilibrium rules requires that power sharing p maximizes ϕ_{p} above (subject to $0 \leq p \leq 1/2$), taking p' and ϕ'_{p} as given, with the Markovian restrictions $p = p'$ and $\phi_{\mathrm{p}} = \phi'_{\mathrm{p}}$ in equilibrium because fundamental state variables (and the level of C) are the same following any further rebellion. Finally, it remains to confirm that incumbents who would lose power following a rebellion are willing to defend the rules, that is, $C_{\mathrm{p}} > C'_{\mathrm{w}}$, or equivalently, $\phi_{\mathrm{p}} > \phi'_{\mathrm{w}}$.

The first derivative of the expression for ϕ_{p} in (A.1.18) with respect to p is:

$$\frac{\partial \phi_{\mathrm{p}}}{\partial p} = \frac{1}{p}\left(\frac{\phi'_{\mathrm{p}}}{1+\delta\frac{p}{p'}}\left(1 + \frac{\frac{\delta(1-p)}{p'}}{1+\delta\frac{p}{p'}}\right) - \frac{1}{p}\left(1 - \frac{(1-p)\phi'_{\mathrm{p}}}{1+\delta\frac{p}{p'}}\right)\right), \tag{A.1.19}$$

and the first-order condition is:

$$\frac{1 - (1-p)\frac{\phi'_{\mathrm{p}}}{1+\delta\frac{p}{p'}}}{p} = \left(1 + \frac{\frac{\delta(1-p)}{p'}}{1+\delta\frac{p}{p'}}\right)\frac{\phi'_{\mathrm{p}}}{1+\delta\frac{p}{p'}}. \tag{A.1.20}$$

Using (A.1.19), the second derivative of ϕ_{p} with respect to p evaluated at a point where the first-order condition (A.1.20) holds is:

$$\left.\frac{\partial^2 \phi_{\mathrm{p}}}{\partial p^2}\right|_{\frac{\partial \phi_{\mathrm{p}}}{\partial p}=0} = -\frac{2\delta}{pp'}\left(1+\frac{\frac{\delta(1-p)}{p'}}{1+\delta\frac{p}{p'}}\right)\frac{\phi'_{\mathrm{p}}}{\left(1+\delta\frac{p}{p'}\right)^2}.$$

This is unambiguously negative, so it follows that ϕ_{p} is a quasi-concave function of p. The first-order condition (A.1.20) is necessary and sufficient for the global maximum (assuming $0 \leq p \leq 1/2$ is satisfied).

Given the equilibrium conditions $p = p' = p^\ddagger$ and $\phi_{\mathrm{p}} = \phi_{\mathrm{p}} = \phi^\ddagger_{\mathrm{p}}$, the objective function in equation (A.1.18) and the first-order condition in equation (A.1.20) imply:

$$\phi^\ddagger_{\mathrm{p}} = \frac{1-(1-p^\ddagger)\frac{\phi^\ddagger_{\mathrm{p}}}{1+\delta}}{p^\ddagger}, \quad \text{and} \tag{A.1.21a}$$

$$\frac{1-(1-p^\ddagger)\frac{\phi'_{\mathrm{p}}}{1+\delta}}{p^\ddagger} = \left(1+\frac{\frac{\delta(1-p^\ddagger)}{p^\ddagger}}{1+\delta}\right)\frac{\phi^\ddagger_{\mathrm{p}}}{1+\delta}. \tag{A.1.21b}$$

Substituting (A.1.21a) into (A.1.21b) and cancelling the non-zero term $\phi^\ddagger_{\mathrm{p}}$ from both sides leads to:

$$\frac{1}{1+\delta}\left(1+\frac{\frac{\delta(1-p^\ddagger)}{p^\ddagger}}{1+\delta}\right) = 1,$$

and solving this equation yields an expression for equilibrium power sharing $p^\ddagger$:

$$p^\ddagger = \frac{1}{2+\delta}.$$

This confirms the expression for $p^\ddagger$ given in the proposition. Observe that since $\delta > 0$, $0 < p^\ddagger < 1/2$, so the constraint $0 \leq p \leq 1/2$ is satisfied. Substituting the expression for $p^\ddagger$ back into equation (A.1.21a) leads to an expression for $\phi^\ddagger_{\mathrm{p}}$:

$$\phi^\ddagger_{\mathrm{p}} = \frac{2+\delta}{2}.$$

In equilibrium, the binding no-rebellion constraint (A.1.17) implies $\phi^\ddagger_{\mathrm{n}} = \phi^\ddagger_{\mathrm{p}}/(1+\delta)$, and hence:

$$\phi^\ddagger_{\mathrm{n}} = \frac{2+\delta}{2(1+\delta)}.$$

Note that since $\delta > 0$, it follows immediately that $\phi_{\mathrm{p}} > \phi'_{\mathrm{w}}$ in equilibrium, confirming the earlier conjecture. The expressions for $p^\ddagger$, $\phi^\ddagger_{\mathrm{p}}$, and $\phi^\ddagger_{\mathrm{n}}$ are verified, completing the proof.

A.2 Proof of Proposition 2

The equilibrium rules at the pre-investment stage are characterized taking as given that incentives must be provided for a fraction s of investment opportunities to be taken, even though the equilibrium conditions will determine s as well (the Markovian restriction applies to s here). Without loss of generality, s is assumed to be strictly positive. In cases where $s = 0$, fundamental state variables are the same before and after the investment stage, so the whole sequence of events in Figure 2 becomes identical to the post-investment stage, and the resulting equilibrium rules are those characterized in Proposition 1.

Given a set of rules $\mathscr{A}$, the conditions required for the absence of any rational rebellion either before or after investment opportunities are given in (3.13) and (3.14). These conditions depend on the rules that would be established following a successful rebellion, which the equilibrium conditions (Definition 2) require to be equilibrium rules. For a post-investment rebellion, the features of the subsequent equilibrium rules have been uniquely determined in Proposition 1: power sharing would be $p^{\ddagger}$ and the continuation payoff of incumbents would be $\mathcal{U}_{\mathrm{p}}^{\ddagger}$. For a pre-investment rebellion, the amount of power sharing p' and the incumbent continuation payoff $\mathcal{U}_{\mathrm{p}}'$ are taken as given, but will be determined in equilibrium given the requirement that subsequent rules are an equilibrium. Note that no changes in fundamental state variables can occur until investment decisions are made.

Starting from the point where new rules are established, either before or after investment, the equilibrium conditions place no restrictions on the identities of incumbents and non-incumbents, only the sizes of these groups. Thus, in assessing which rebellions are rational according to Definition 1, even though members of the rebel faction are required to anticipate a position in the subsequent incumbent group, all permutations of the identities of those in that group are consistent with equilibrium. The no-rebellion conditions are derived by considering all possible compositions of the subsequent incumbent group and checking that the successful rebellion condition (2.8) does not hold, given the conditions on participation and maximum rebellion effort necessary for a rational rebellion (Definition 1). The left-hand sides of (3.13) and (3.14) are derived from the individually rational rebel-faction participation condition and maximum rebellion effort level in (2.10a), while the right-hand sides are derived from the incumbent-faction participation condition (2.10b).

It is assumed for now that current incumbents will receive more utility in power in the current rules survive than if they lose power following a rebellion. This means that optimality of the rules requires that the no-rebellion conditions hold, so these are treated as constraints on the design of the rules (no-rebellion constraints). Conditions (3.13) and (3.14) can be simplified by noting that the incumbent-faction participation condition (2.10b) is automatically satisfied for incumbents who lose power. The pre- and post-investment no-rebellion constraints are therefore:

$$\int_{\mathcal{P}'} \max\{\exp\{\mathcal{U}_{\mathrm{p}}' - \mathcal{U}(\imath)\} - 1, 0\}\mathrm{d}\imath \leq \delta\left(\int_{\mathcal{P}} \mathrm{d}\imath - \mathbb{1}[\mathcal{U}_{\mathrm{p}} \leq \mathcal{U}_{\mathrm{p}}'] \int_{\mathcal{P}\cap\mathcal{P}'} \mathrm{d}\imath\right); \qquad \text{[A.2.1a]}$$

$$\int_{\mathcal{P}^{\ddagger}} \max\{\exp\{\mathcal{U}_{\mathrm{p}}^{\ddagger} - \mathcal{U}(\imath)\} - 1, 0\}\mathrm{d}\imath \leq \delta\left(\int_{\mathcal{P}} \mathrm{d}\imath - \mathbb{1}[\mathcal{U}_{\mathrm{p}} \leq \mathcal{U}_{\mathrm{p}}^{\ddagger}] \int_{\mathcal{P}\cap\mathcal{P}^{\ddagger}} \mathrm{d}\imath\right), \qquad \text{[A.2.1b]}$$

for any sets $\mathcal{P}'$ with measure p' and $\mathcal{P}^{\ddagger}$ with measure $p^{\ddagger}$, where $\mathcal{U}(\imath)$ denotes the payoff of individual $\imath$ under the current rules. Note that since the constraints in (A.2.1) are looser than the originals (3.13) and (3.14), it suffices to work with (A.2.1) first and then verify the conjecture.

Given that the no-rebellion constraints (A.2.1) hold, the consumption allocation prescribed by the current rules will be realized, so incentive compatibility (2.4) for investors is assessed using this contingent consumption allocation. To achieve $s > 0$, the current rules must place a fraction s of the total measure $1-p$ of non-incumbents in the set $\mathcal{C}$, for whom the consumption allocation is contingent on being seen to take an investment opportunity. The contingent consumption allocation must satisfy the incentive constraint:

$$C_{\mathrm{k}} \geq (1+\theta)C_{\mathrm{w}}. \qquad \text{[A.2.2]}$$

The current rules specify the identities of those p individuals in $\mathcal{P}$, and the $(1-p)s$ individuals in $\mathcal{C}$. There will be three groups of individuals (payoffs the same within groups) after investment decisions have been made: incumbents, capitalists, and workers. There are p incumbents, and since there are μ investment opportunities randomly distributed among non-incumbents, there will be $\kappa = \mu s$ capitalists ex post. The remaining measure $1-p-\kappa$ of individuals will be workers. The continuation payoffs (ignoring sunk costs) of these individuals at the post-investment stage under the current rules are:

$$\mathcal{U}_{\mathrm{p}} = \log C_{\mathrm{p}}, \quad \mathcal{U}_{\mathrm{k}} = \log C_{\mathrm{k}}, \quad \text{and } \mathcal{U}_{\mathrm{w}} = \log C_{\mathrm{w}}, \qquad \text{[A.2.3]}$$

using the utility function (2.3). There are also three groups of individuals (payoffs the same within groups) at the pre-investment stage: incumbents, those whose consumption allocation is contingent on taking an investment opportunity (those in $\mathcal{C}$), and those with a consumption allocation that does not depend on investing. The size of the incumbent group is the amount of power sharing p. The group $\mathcal{C}$ has size

$(1-p)s$, and the group not in $\mathcal{P}$ or $\mathcal{C}$ has size $(1-p)(1-s)$. Since there are $1-p$ non-incumbents in total, and μ investment opportunties distributed at random among them, each non-incumbent has probability $\gamma = \mu/(1-p)$ of receiving an investment opportunity (this is a well-defined probability because $\mu < 1/2$ and $1-p \geq 1/2$). Those in the third group have no incentive to invest even if they receive an opportunity, and thus will become workers. Those in $\mathcal{C}$ invest with probability γ given that (A.2.2) holds, and become capitalists subsequently. With probability $1-\gamma$ no opportunity is received, and they become workers. The continuation payoffs at the pre-investment stage are:

$$\mathcal{U}_{\mathrm{p}} = \log C_{\mathrm{p}}, \quad \mathcal{U}_{\mathrm{l}} = (1-\gamma)\log C_{\mathrm{w}} + \gamma\left(\log C_{\mathrm{k}} - \log(1+\theta)\right), \quad \text{and } \mathcal{U}_{\mathrm{o}} = \log C_{\mathrm{w}}. \qquad \text{[A.2.4]}$$

Since individuals in a group all receive the same continuation payoffs (A.2.3) and (A.2.4) under the prevailing rules, the collection of no-rebellion constraints (A.2.1) can be stated more concisely in terms of the fraction of the post-rebellion incumbents that would be drawn from each current group. For a rebellion at the pre-investment stage, let σ_{o}, σ_{l}, and σ_{p} denote respectively the fractions of the size p' post-rebellion incumbent group drawn respectively from those not in $\mathcal{P}$ or $\mathcal{C}$, those in $\mathcal{C}$, and those in $\mathcal{P}$. These proportions must be non-negative and sum to one, as well as satisfying the natural restrictions $\sigma_{\mathrm{o}} \leq (1-p)(1-s)/p'$, $\sigma_{\mathrm{l}} \leq (1-p)s/p'$, and $\sigma_{\mathrm{p}} \leq p/p'$. Similarly, for a rebellion at the post-investment stage, let σ_{w}, σ_{k}, and σ_{p} denote respectively the fractions of the size $p^{\ddagger}$ post-rebellion incumbent group drawn from current workers, capitalists, and incumbents. The natural restrictions on these fractions are $\sigma_{\mathrm{w}} \leq (1-p-\kappa)/p^{\ddagger}$, $\sigma_{\mathrm{k}} \leq \kappa/p^{\ddagger}$, and $\sigma_{\mathrm{p}} \leq p/p^{\ddagger}$. Using this notation, the pre-investment no-rebellion constraints (A.2.1a) can be stated as:

$$\sigma_{\mathrm{o}} p' \max\{\exp\{\mathcal{U}_{\mathrm{p}}' - \mathcal{U}_{\mathrm{o}}\} - 1, 0\} + \sigma_{\mathrm{l}} p' \max\{\exp\{\mathcal{U}_{\mathrm{p}}' - \mathcal{U}_{\mathrm{l}}\} - 1, 0\} \\ + \sigma_{\mathrm{p}} p' \mathbb{1}[\mathcal{U}_{\mathrm{p}} \leq \mathcal{U}_{\mathrm{p}}'] \left(\exp\{\mathcal{U}_{\mathrm{p}}' - \mathcal{U}_{\mathrm{p}}\} - 1 + \delta\right) \leq \delta p, \qquad \text{[A.2.5a]}$$

and the post-investment no-rebellion constraints (A.2.1b) as:

$$\sigma_{\mathrm{w}} p^{\ddagger} \max\{\exp\{\mathcal{U}_{\mathrm{p}}^{\ddagger} - \mathcal{U}_{\mathrm{w}}\} - 1, 0\} + \sigma_{\mathrm{k}} p^{\ddagger} \max\{\exp\{\mathcal{U}_{\mathrm{p}}^{\ddagger} - \mathcal{U}_{\mathrm{l}}\} - 1, 0\} \\ + \sigma p^{\ddagger} \mathbb{1}[\mathcal{U}_{\mathrm{p}} \leq \mathcal{U}_{\mathrm{p}}^{\ddagger}] \left(\exp\{\mathcal{U}_{\mathrm{p}}^{\ddagger} - \mathcal{U}_{\mathrm{p}}\} - 1 + \delta\right) \leq \delta p. \qquad \text{[A.2.5b]}$$

Free markets and no taxes that distort the allocation of consumption between goods

The equilibrium rules must maximize the payoff $\mathcal{U}_{\mathrm{p}}$ of incumbents subject to satisfying the no-rebellion constraints, and resource and incentive constraints. Given the minimum levels of utility of non-incumbents needed to satisfy all these constraints, and given the resource constraints (2.6), the consumption allocation prescribed by the rules must satisfy:

$$\frac{\alpha/c_{\mathrm{pI}}}{(1-\alpha)/c_{\mathrm{pE}}} = \frac{\alpha/c_{\mathrm{kI}}}{(1-\alpha)/c_{\mathrm{kE}}} = \frac{\alpha/c_{\mathrm{wI}}}{(1-\alpha)/c_{\mathrm{wE}}}. \qquad \text{[A.2.6]}$$

This allocation of consumption is supported by what would be chosen by individuals who can exchange goods in perfectly competitive domestic markets subject to some levels of income, with the market-clearing price being (3.4). The claim in the proposition is confirmed.

Free international trade

Given domestic free exchange, if the country can access world markets with price π^* then the maximum value of the resources available for consumption is obtained by choosing net exports x_{E} and x_{I} such that the following first-order condition holds:

$$\frac{K - x_{\mathrm{I}}}{q - x_{\mathrm{E}}} = \frac{\alpha}{(1-\alpha)\pi^*}. \qquad \text{[A.2.7]}$$

This is satisfied if and only if the domestic market price $\tilde{\pi}$ is equal to the world price π^*, confirming the claim in the proposition. The resource constraint of the country is therefore:

$$pC_{\mathrm{p}} + \kappa C_{\mathrm{k}} + (1 - p - \kappa)C_{\mathrm{w}} = C = \frac{\mathrm{q} + \tilde{\pi}K}{\tilde{\pi}^{\alpha}}, \tag{A.2.8}$$

where $\tilde{\pi} = \pi^*$ if the country is in contact with world markets, and K is the capital stock given in (2.2). The no-rebellion constraints (A.2.5) can be written in terms of consumption using the expressions for the continuation payoffs in (A.2.3) and (A.2.4):

$$\sigma_{\mathrm{o}}p' \max\left\{\frac{C'_{\mathrm{p}}}{C_{\mathrm{w}}} - 1, 0\right\} + \sigma_{\mathrm{l}}p' \max\left\{\frac{C'_{\mathrm{p}}}{C_{\mathrm{w}}^{1-\gamma}\left(\frac{C_{\mathrm{k}}}{1+\theta}\right)^{\gamma}} - 1, 0\right\} + \sigma_{\mathrm{p}}p' \mathbb{1}[C_{\mathrm{p}} \leq C'_{\mathrm{p}}]\left(\frac{C'_{\mathrm{p}}}{C_{\mathrm{p}}} - 1 + \delta\right) \leq \delta p; \tag{A.2.9a}$$

$$\sigma_{\mathrm{w}}p^{\ddagger} \max\left\{\frac{C^{\ddagger}_{\mathrm{p}}}{C_{\mathrm{w}}} - 1, 0\right\} + \sigma_{\mathrm{k}}p^{\ddagger} \max\left\{\frac{C^{\ddagger}_{\mathrm{p}}}{C_{\mathrm{k}}} - 1, 0\right\} + \sigma_{\mathrm{p}}p^{\ddagger} \mathbb{1}[C_{\mathrm{p}} \leq C^{\ddagger}_{\mathrm{p}}]\left(\frac{C^{\ddagger}_{\mathrm{p}}}{C_{\mathrm{p}}} - 1 + \delta\right) \leq \delta p, \tag{A.2.9b}$$

which must hold for all valid σ_{o}, σ_{l}, and σ_{p}, and for all valid σ_{w}, σ_{k}, and σ_{p} at the pre- and post-investment stages.

The time inconsistency problem

The equilibrium conditions require that the rules maximize the incumbent payoff $\mathcal{U}_{\mathrm{p}}$. This is subject to the resource constraint (A.2.8), the incentive constraint (A.2.2), and the no-rebellion constraints (A.2.9). Given s, and hence K, the equilibrium rules established following a rebellion at the post-investment stage would maximize the incumbent payoff $\mathcal{U}_{\mathrm{p}}$ subject only to the resource constraint (A.2.8) and the post-investment no-rebellion constraints (A.2.9b). The solution of that constrained maximization problem is characterized in Proposition 1. In that problem, it is optimal to give all non-incumbents the same level of consumption. Here, the equilibrium rules must also satisfy the incentive constraint (A.2.2), which requires granting a positive measure μs of non-incumbents a consumption level strictly greater than workers (since $\theta > 0$). Given that the resource constraint is the same in both cases, satisfying the incentive constraint strictly reduces the payoff of incumbents. Furthermore, the pre-investment no-rebellion constraints (A.2.9a) must also be satisfied. With these additional constraints, the incumbent payoff $\mathcal{U}_{\mathrm{p}}$ must be strictly lower than the constrained-maximum payoff $\mathcal{U}^{\ddagger}_{\mathrm{p}}$ following a rebellion at the post-investment stage.

Since $\mathcal{U}^{\ddagger}_{\mathrm{p}} > \mathcal{U}_{\mathrm{p}}$ (and $C^{\ddagger}_{\mathrm{p}} > C_{\mathrm{p}}$), members of the incumbent group have an incentive to rebel against the rules after investment has occurred, even though the rules were established to maximize their payoff at the pre-investment stage. Rules prescribing credible incentives for investors are therefore subject to a time-inconsistency problem.

There must be more power sharing

Suppose that the rules specified power sharing p no more than the equilibrium level of power sharing $p^{\ddagger}$ following a rebellion at the post-investment stage. With $p \leq p^{\ddagger}$, it is feasible to have a post-investment rebellion with $\sigma_{\mathrm{p}} = p/p^{\ddagger}$ and values of σ_{k}, and σ_{w} such that $\sigma_{\mathrm{p}} + \sigma_{\mathrm{k}} + \sigma_{\mathrm{w}} = 1$ in (A.2.9b). Since $\sigma_{p}p^{\ddagger} = p$ and $C_{\mathrm{p}} < C^{\ddagger}_{\mathrm{p}}$, the left-hand side is strictly greater than the right-hand side, violating the no-rebellion constraint. The equilibrium rules must therefore feature $p > p^{\ddagger}$, and increase in power sharing relative to what would be optimal ex post for incumbents.

No capitalists in a rebel faction

The natural limit on the fraction of workers σ_{w} included in the incumbent group following a rebellion at the post-investment stage is $\sigma_{\mathrm{w}} \leq (1-p-\kappa)/p^{\ddagger}$. Since $p \leq 1/2$ and $\kappa \leq \mu$, a sufficient condition for $\sigma_{\mathrm{w}} = 1$ to be feasible is:

$$1 - \frac{1}{2} - \mu \geq p^{\ddagger}, \quad \text{or equivalently} \;\; \mu \leq \frac{1}{2} - \frac{1}{2+\delta} = \frac{\delta}{2(2+\delta)},$$

which uses the expression for $p^{\ddagger}$ in (3.11). This condition holds under the parameter restriction on μ in (2.9).

The incentive constraint (A.2.2) requires $C_{\text{k}} > C_{\text{w}}$, which implies $\max\{C_{\text{p}}^{\ddagger}/C_{\text{k}} - 1, 0\} \leq \max\{C_{\text{p}}^{\ddagger}/C_{\text{w}} - 1, 0\}$. Since $\sigma_{\text{w}} = 1$ is feasible in the post-investment no-rebellion constraint (A.2.9b), it follows that σ_{k} can be set to zero because if the no-rebellion constraint holds for all feasible values of σ_{w} and σ_{p} given $\sigma_{\text{k}} = 0$, it must hold any positive value of σ_{k} as well. The most dangerous rebel faction does not include capitalists because workers have a greater incentive to change the rules, and there is no shortage of workers in filling the places in the incumbent group following a rebellion.

With $\sigma_{\text{k}} = 0$ and $C_{\text{p}} < C_{\text{p}}^{\ddagger}$, the set of relevant post-investment no-rebellion constraints can be written as follows:

$$(1-\sigma)p^{\ddagger} \max\left\{\frac{C_{\text{p}}^{\ddagger}}{C_{\text{w}}} - 1, 0\right\} + \sigma p^{\ddagger}\left(\frac{C_{\text{p}}^{\ddagger}}{C_{\text{p}}} - 1 + \delta\right) \leq \delta p, \qquad \text{[A.2.10]}$$

which must hold for all $\sigma \in [0,1]$ (where $\sigma = \sigma_{\text{p}}$, and all values of σ in the unit interval are feasible, which follows from $p > p^{\ddagger}$ and $1 - p - \kappa \geq p^{\ddagger}$).

A no-rebellion constraint must bind for a rebel faction comprising a positive measure of non-incumbents

Given the Markovian condition (Definition 2), the equilibrium rules will feature $\mathcal{U}_{\text{p}} = \mathcal{U}_{\text{p}}'$ and $\mathcal{U}_{\text{w}} = \mathcal{U}_{\text{w}}'$, and if it is in interests of incumbents who lose power at the pre-investment stage to defend the rules ($\mathcal{U}_{\text{p}} > \mathcal{U}_{\text{w}}'$) then $\mathcal{U}_{\text{p}}' > \mathcal{U}_{\text{w}}$, and hence $C_{\text{p}}' > C_{\text{w}}$ (workers would be willing to exert some positive rebellion effort in a rebel faction). Since the incentive constraint (A.2.2) requires $C_{\text{k}}/(1+\theta) \geq C_{\text{w}}$, and as C_{k} does not appear in the post-investment no-rebellion constraints (A.2.10), it follows that if $C_{\text{w}}^{1-\gamma}(C_{\text{k}}/(1+\theta))^{\gamma} > C_{\text{p}}'$ then C_{k} can be reduced by some positive amount without violating any constraint. This would allow the incumbent payoff to be increased, so optimal rules must feature $C_{\text{w}}^{1-\gamma}(C_{\text{k}}/(1+\theta))^{\gamma} \leq C_{\text{p}}'$. It follows that the pre-investment no-rebellion constraints can be stated as:

$$(1-\sigma)p'\left(\left(\frac{1-\varkappa}{C_{\text{w}}} + \frac{\varkappa}{C_{\text{w}}^{1-\gamma}\left(\frac{C_{\text{k}}}{1+\theta}\right)^{\gamma}}\right)C_{\text{p}}' - 1\right) + \sigma p' \mathbb{1}[C_{\text{p}} \leq C_{\text{p}}']\left(\frac{C_{\text{p}}'}{C_{\text{p}}} - 1 + \delta\right) \leq \delta p, \qquad \text{[A.2.11a]}$$

where the notation $\sigma = \sigma_{\text{p}}$ and $\varkappa = \sigma_{\text{l}}/(\sigma_{\text{o}} + \sigma_{\text{l}})$ is used, and the constraints must hold for all feasible values of σ and $\varkappa$. Since $C_{\text{w}} \leq C_{\text{p}} < C_{\text{p}}^{\ddagger}$, the post-investment no-rebellion constraints (A.2.10) can also be written in a simpler form:

$$(1-\sigma)p^{\ddagger}\left(\frac{C_{\text{p}}^{\ddagger}}{C_{\text{w}}} - 1\right) + \sigma p^{\ddagger}\left(\frac{C_{\text{p}}^{\ddagger}}{C_{\text{p}}} - 1 + \delta\right) \leq \delta p, \qquad \text{[A.2.11b]}$$

which must hold for all $\sigma \in [0,1]$. One of the constraints (A.2.11a) or (A.2.11b) must bind for some $\sigma < 1$. Otherwise the terms multiplying $1-\sigma$ in these constraints are both too low, allowing C_{w} to be reduced by some positive amount without violating any no-rebellion constraint (and reducing C_{w} does not violate the incentive constraint (A.2.2)).

The incentive compatibility constraint is binding

Now suppose the incentive compatibility constraint (A.2.2) is slack, that is, $C_{\text{k}} > (1+\theta)$. Consider a redistribution of contingent consumption between workers and capitalists such that workers will receive C_{n} and capitalists will receive $(1+\theta)C_{\text{n}}$, which means that the incentive constraint becomes binding. Since there will be κ capitalists and $1-p-\kappa$ workers, the resource constraint requires:

$$C_{\text{n}} = \frac{(1-p-\kappa)C_{\text{w}} + \kappa C_{\text{k}}}{(1-p-\kappa)+\kappa(1+\theta)} = \left(\frac{1-p-\kappa}{1-p+\kappa\theta}\right)C_{\text{w}} + \left(\frac{\kappa(1+\theta)}{1-p+\kappa\theta}\right)\frac{C_{\text{k}}}{1+\theta}. \qquad \text{[A.2.12]}$$

This shows that C_{n} is a weighted average of C_{w} and $C_{\text{k}}/(1+\theta)$, where $C_{\text{k}}/(1+\theta) > C_{\text{w}}$. Using Jensen's inequality, the convexity of $1/C$ implies:

$$\frac{1}{C_{\text{n}}} < \left(\frac{1-p-\kappa}{1-p+\kappa\theta}\right)\frac{1}{C_{\text{w}}} + \left(\frac{\kappa(1+\theta)}{1-p+\kappa\theta}\right)\frac{1+\theta}{C_{\text{k}}}. \qquad \text{[A.2.13]}$$

Since $1/C_{\text{w}} > (1+\theta)/C_{\text{k}}$ and $(1-p-\kappa)/(1-p+\kappa\theta) < (1-p-\kappa)/(1-p)$, it follows that:

$$\left(\frac{1-p-\kappa}{1-p+\kappa\theta}\right)\frac{1}{C_{\text{w}}} + \left(\frac{\kappa(1+\theta)}{1-p+\kappa\theta}\right)\frac{1+\theta}{C_{\text{k}}} < \left(\frac{1-p-\kappa}{1-p}\right)\frac{1}{C_{\text{w}}} + \left(\frac{\kappa}{1-p}\right)\frac{1+\theta}{C_{\text{k}}}. \qquad \text{[A.2.14]}$$

Finally, by putting together (A.2.13) and (A.2.14) and noting $C_{\text{w}}^{1-\gamma}(C_{\text{k}}/(1+\theta))^{\gamma} < C_{\text{k}}/(1+\theta)$:

$$\frac{1}{C_{\text{n}}} < \frac{1-\left(\frac{\kappa}{1-p}\right)}{C_{\text{w}}} + \frac{\left(\frac{\kappa}{1-p}\right)}{C_{\text{w}}^{1-\gamma}\left(\frac{C_{\text{k}}}{1+\theta}\right)^{\gamma}}. \qquad \text{[A.2.15]}$$

Observe that with $C_{\text{w}} < C_{\text{k}}/(1+\theta)$, the value of $\varkappa$ with the highest value of the left-hand side of (A.2.11a) (given a σ) is the lowest feasible value of $\varkappa$. This value of $\varkappa$ must be such that $\varkappa \leq \kappa/(1-p)$ since there are κ capitalists and $1-p$ non-incumbents. By using (A.2.15), it must be the case that:

$$\frac{1}{C_{\text{n}}} < \frac{1-\varkappa}{C_{\text{w}}} + \frac{\varkappa}{C_{\text{w}}^{1-\gamma}\left(\frac{C_{\text{k}}}{1+\theta}\right)^{\gamma}},$$

and it follows that the redistribution specified in (A.2.12) slackens the pre-investment no-rebellion constraint (A.2.11a) if this is binding for some $\sigma < 1$. This would allow the payoff of incumbents to be increased. If (A.2.11a) is not binding for any $\sigma < 1$ then C_{k} does not appear in any relevant no-rebellion constraint (it is absent from (A.2.11b)), and so C_{k} can be reduced until the incentive constraint (A.2.2) is binding. Therefore, in either case, the equilibrium rules must necessarily have:

$$C_{\text{k}} = (1+\theta)C_{\text{w}}. \qquad \text{[A.2.16]}$$

This confirms the claim in the proposition.

There are only two independent no-rebellion constraints at the post-investment stage

With $C_{\text{k}}/(1+\theta) = C_{\text{w}}$, the pre-investment no-rebellion constraints simplify to:

$$(1-\sigma)p'\left(\frac{C'_{\text{p}}}{C_{\text{w}}}-1\right) + \sigma p' \mathbb{1}[C_{\text{p}} \leq C'_{\text{p}}]\left(\frac{C'_{\text{p}}}{C_{\text{p}}}-1+\delta\right) \leq \delta p, \qquad \text{[A.2.17a]}$$

which must hold for all feasible $\sigma \in [0, \min\{p/p', 1\}]$ (note that $\sigma = 0$ is feasible because $p \leq 1/2$ and $p' \leq 1/2$). The set of post-investment no-rebellion constraints (A.2.11b) is linear in σ and must be verified for all σ in the fixed interval $[0,1]$ (all of which are feasible). If (A.2.11b) holds for all $\sigma \in [0,1]$ then it must hold in particular at $\sigma = 0$ and $\sigma = 1$:

$$p^{\ddagger}\left(\frac{C^{\ddagger}_{\text{p}}}{C_{\text{w}}}-1\right) \leq \delta p; \qquad \text{[A.2.17b]}$$

$$p^{\ddagger}\left(\frac{C^{\ddagger}_{\text{p}}}{C_{\text{p}}}-1+\delta\right) \leq \delta p. \qquad \text{[A.2.17c]}$$

Furthermore, if both of the individual no-rebellion constraints (A.2.17b) and (A.2.17c) hold then taking a linear combination with weights $1-\sigma$ and σ implies that (A.2.11b) holds for any $\sigma \in [0,1]$. The relevant set of no-rebellion constraints has therefore been reduced to (A.2.17a), (A.2.17b) and (A.2.17c).

Only a single no-rebellion constraint involving non-incumbents is relevant at the pre-investment stage

Consider a simpler constrained maximization problem for the incumbent payoff $\mathcal{U}_{\mathrm{p}} = \log C_{\mathrm{p}}$ where the pre-investment no-rebellion constraint is only required to hold for $\sigma = 0$:

$$p'\left(\frac{C'_{\mathrm{p}}}{C_{\mathrm{w}}} - 1\right) \leq \delta p. \qquad [\text{A.2.18}]$$

All other relevant no-rebellion constraints must hold ((A.2.17b) and (A.2.17c)), as must the resource constraint (A.2.8) and the (binding) incentive constraint (A.2.16). Equivalents of the equilibrium conditions in Definition 2 are required to hold for this simpler problem.

Let $\{\dot{p}, \dot{C}_{\mathrm{p}}, \dot{C}_{\mathrm{w}}\}$ denote an equilibrium of this simpler problem. Since there are no changes to fundamental state variables following any pre-investment rebellion, the Markovian conditions are $p' = \dot{p}$ and $C'_{\mathrm{p}} = \dot{C}_{\mathrm{p}}$. Since (A.2.18) must hold, it follows that $\dot{C}_{\mathrm{p}}/\dot{C}_{\mathrm{w}} \leq 1 + \delta$.

Now consider the original constrained maximization problem, taking $p' = \dot{p}$ and $C'_{\mathrm{p}} = \dot{C}_{\mathrm{p}}$ for an equilibrium of the simpler problem. When $p = \dot{p}$, any value of σ between zero and one is feasible, and rules must therefore satisfy the no-rebellion constraint (A.2.17a) for all $\sigma \in [0, 1]$. If $p = \dot{p}$, $C_{\mathrm{w}} = \dot{C}_{\mathrm{w}}$, and $C_{\mathrm{p}} = \dot{C}_{\mathrm{p}}$ are to satisfy these constraints then the following must hold:

$$(1 - \sigma)\dot{p}\left(\frac{\dot{C}_{\mathrm{p}}}{\dot{C}_{\mathrm{w}}} - 1\right) + \delta\sigma\dot{p} \leq \delta\dot{p},$$

which does indeed follow for any $\sigma \in [0, 1]$ given that $\dot{C}_{\mathrm{p}}/\dot{C}_{\mathrm{w}} \leq 1 + \delta$. Since the rules satisfy this and all other no-rebellion constraints (they are an equilibrium of the simpler problem), and all other resource and incentive constraints, and because they maximize the incumbent payoff subject to the weaker no-rebellion constraint (A.2.18), it follows that they are optimal subject to the full set of no-rebellion constraints (A.2.17a). The rules are thus an equilibrium of the original problem.

Now consider the converse. Take rules with $\{\dot{p}, \dot{C}_{\mathrm{p}}, \dot{C}_{\mathrm{w}}\}$ that are an equilibrium of the original problem. When $p' = \dot{p}$ and $C'_{\mathrm{p}} = \dot{C}_{\mathrm{p}}$, these rules clearly satisfy the simpler no-rebellion constraint (A.2.18) (a special case of (A.2.17a) for $\sigma = 0$, which is always feasible). But now suppose that they were not optimal subject only to (A.2.18) instead of (A.2.17a) (with $p' = \dot{p}$ and $C'_{\mathrm{p}} = \dot{C}_{\mathrm{p}}$) for all feasible σ values. This means there are alternative rules with $\{p, C_{\mathrm{p}}, C_{\mathrm{w}}\}$ satisfying (A.2.18) (with $p' = \dot{p}$ and $C'_{\mathrm{p}} = \dot{C}_{\mathrm{p}}$) that yield a higher incumbent consumption level $C_{\mathrm{p}} > \dot{C}_{\mathrm{p}}$. Now take any $\sigma \in [0, 1]$ and observe that:

$$(1 - \sigma)\dot{p}\left(\frac{\dot{C}_{\mathrm{p}}}{C_{\mathrm{w}}} - 1\right) + \sigma\dot{p}\mathbb{1}[C_{\mathrm{p}} \leq \dot{C}_{\mathrm{p}}]\left(\frac{\dot{C}_{\mathrm{p}}}{C_{\mathrm{p}}} - 1 + \delta\right) = (1 - \sigma)\dot{p}\left(\frac{\dot{C}_{\mathrm{p}}}{C_{\mathrm{w}}} - 1\right) \leq (1 - \sigma)\delta p \leq \delta p,$$

where the first equality follows from $C_{\mathrm{p}} > \dot{C}_{\mathrm{p}}$, the first inequality follows from the alternative rules satisfying (A.2.18) (with $p' = \dot{p}$ and $C'_{\mathrm{p}} = \dot{C}_{\mathrm{p}}$), and the second inequality follows from $0 \leq \sigma \leq 1$. This shows that the alternative rules satisfy (A.2.17a) for all feasible σ (a subset of $[0, 1]$), and thus satisfy all the constraints of the original problem, yet yield a higher payoff than the equilibrium rules, contradicting the optimality requirement of the rules with $\{\dot{p}, \dot{C}_{\mathrm{p}}, \dot{C}_{\mathrm{w}}\}$. This contradiction that these rules must also be optimal when the constraints (A.2.17a) are weakened to (A.2.18) (and other constraints are the same). The rules are thus an equilibrium of the simpler problem.

This logic shows that the set of equilibrium rules is not affected by imposing only (A.2.18) instead of (A.2.17a). Therefore, the only relevant no-rebellion constraints are (A.2.17b), (A.2.17c), and (A.2.18).

The resource constraint (A.2.8) implies the following expression for incumbent consumption C_{p} in terms of total resources C available for consumption, and the consumption levels C_{w} and C_{k} of workers and capitalists:

$$C_{\mathrm{p}} = \frac{C - (1 - p - \mu s)C_{\mathrm{w}} - \mu s C_{\mathrm{k}}}{p},$$

where C is given in (A.2.8), and the measure of capitalists is $\kappa = \upmu s$. Eliminating C_{k} using the binding incentive compatibility constraint (A.2.16):

$$C_{\mathrm{p}} = \frac{C - (1 - p + \upmu\uptheta s)C_{\mathrm{w}}}{p}.$$

Let $\phi_{\mathrm{p}} \equiv C_{\mathrm{p}}/C$ and $\phi_{\mathrm{w}} \equiv C_{\mathrm{w}}/C$ denote the per-person consumption shares of incumbents and workers of total resources C. Dividing both sides of the expression for C_{p} by C:

$$\phi_{\mathrm{p}} = \frac{1 - (1 - p + \upmu\uptheta s)\phi_{\mathrm{w}}}{p}. \qquad \text{[A.2.19]}$$

Since s is taken as given here, which using (2.2) also means K is taken as given, total resources C is also taken as given (the optimality of free exchange and free international trade have already been shown, which means that the price $\tilde{\pi}$ is also taken as given). The optimality condition (Definition 2) then requires that ϕ_{p} (equation (A.2.19)) be maximized subject to the no-rebellion constraints. All other constraints have already been accounted for in (A.2.19). The no-rebellion constraints are (A.2.18), (A.2.17b), and (A.2.17c), which can be written as follows:

$$C_{\mathrm{w}} \geq \frac{C'_{\mathrm{p}}}{1 + \updelta\frac{p}{p'}}, \quad C_{\mathrm{w}} \geq \frac{C^{\ddagger}_{\mathrm{p}}}{1 + \updelta\frac{p}{p^{\ddagger}}}, \quad \text{and } C_{\mathrm{p}} \geq \frac{C^{\ddagger}_{\mathrm{p}}}{1 + \updelta\frac{p - p^{\ddagger}}{p^{\ddagger}}}.$$

The capital stock K is predetermined at the post-investment stage, and it has been shown here and in Proposition 1 that free exchange domestically and free international trade are required in equilibrium. Together with the Markovian restriction on s following a pre-investment stage rebellion, this means that there would be no change to the total amount of resources C available for investment after any rebellion ($C = C' = C^{\ddagger}$). Dividing both sides of all the no-rebellion constraints above by this common level of total output leads to the following set of no-rebellion constraints written in terms of ϕ'_{p} and $\phi^{\ddagger}_{\mathrm{p}}$:

$$\phi_{\mathrm{w}} \geq \frac{\phi'_{\mathrm{p}}}{1 + \updelta\frac{p}{p'}}; \qquad \text{[A.2.20a]}$$

$$\phi_{\mathrm{w}} \geq \frac{\phi^{\ddagger}_{\mathrm{p}}}{1 + \updelta\frac{p}{p^{\ddagger}}}; \qquad \text{[A.2.20b]}$$

$$\phi_{\mathrm{p}} \geq \frac{\phi^{\ddagger}_{\mathrm{p}}}{1 + \updelta\frac{p - p^{\ddagger}}{p^{\ddagger}}}. \qquad \text{[A.2.20c]}$$

The values of $p^{\ddagger}$, $\phi^{\ddagger}_{\mathrm{p}}$, and $\phi^{\ddagger}_{\mathrm{w}}$ are obtained from Proposition 1:

$$p^{\ddagger} = \frac{1}{2 + \updelta}, \quad \phi^{\ddagger}_{\mathrm{p}} = \frac{2 + \updelta}{2}, \quad \text{and} \quad \phi^{\ddagger}_{\mathrm{w}} = \frac{2 + \updelta}{2(1 + \updelta)}. \qquad \text{[A.2.21]}$$

The values of p', ϕ'_{p}, and ϕ'_{w} are subject to the Markovian restrictions $p' = p$, $\phi'_{\mathrm{p}} = \phi_{\mathrm{p}}$, and $\phi'_{\mathrm{w}} = \phi_{\mathrm{w}}$ in equilibrium. There is also a size constraint on the incumbent group, namely $0 \leq p \leq 1/2$, and it remains to be confirmed that incumbents who would lose power in any rebellion are willing to defend the rules, that is, $\phi_{\mathrm{p}} > \phi'_{\mathrm{w}}$ and $\phi_{\mathrm{p}} > \phi^{\ddagger}_{\mathrm{w}}$.

It has already been established that one of (A.2.20a) or (A.2.20b) must be binding, that $\phi_{\mathrm{p}} < \phi^{\ddagger}_{\mathrm{p}}$, and that $p > p^{\ddagger}$ in equilibrium.

The pre-investment no-rebellion constraint for workers cannot be the only binding constraint

Suppose for contradiction that (A.2.20a) is the only binding no-rebellion constraint, where ϕ'_{p} and p' are taken as given. Substituting the binding constraint (A.2.20a) into (A.2.19), the optimality condition requires that p maximize ϕ_{p} (with s taken as given):

$$\phi_{\text{p}} = \frac{1 - (1 - p + \mu\theta s)\frac{\phi'_{\text{p}}}{1+\delta\frac{p}{p'}}}{p}. \qquad \text{[A.2.22]}$$

The first derivative with respect to p is:

$$\frac{\partial \phi_{\text{p}}}{\partial p} = \frac{1}{p}\left(\left(1 + \frac{\frac{\delta(1-p+\mu\theta s)}{p'}}{1+\delta\frac{p}{p'}}\right)\frac{\phi'_{\text{p}}}{1+\delta\frac{p}{p'}} - \frac{1 - (1 - p + \mu\theta s)\frac{\phi'_{\text{p}}}{1+\delta\frac{p}{p'}}}{p}\right), \qquad \text{[A.2.23]}$$

and the second derivative evaluated at a point where the first derivative is zero is:

$$\left.\frac{\partial^2 \phi_{\text{p}}}{\partial p^2}\right|_{\frac{\partial \phi_{\text{p}}}{\partial p}=0} = -\frac{2\delta}{pp'}\left(1 + \frac{\frac{\delta(1-p+\mu\theta s)}{p'}}{1+\delta\frac{p}{p'}}\right)\frac{\phi'_{\text{p}}}{\left(1+\delta\frac{p}{p'}\right)^2}.$$

This is unambiguously negative, which demonstrates that ϕ_{p} is a quasi-concave function of p. Using (A.2.23), the optimal value of p therefore satisfies (assuming $0 \le p \le 1/2$):

$$\frac{1 - (1 - p + \mu\theta s)\frac{\phi'_{\text{p}}}{1+\delta\frac{p}{p'}}}{p} = \frac{\phi'_{\text{p}}}{1+\delta\frac{p}{p'}}\left(1 + \frac{(1-p+\mu\theta s)\frac{\delta}{p'}}{1+\delta\frac{p}{p'}}\right).$$

The equilibrium conditions $p' = p$ and $\phi'_{\text{p}} = \phi_{\text{p}}$ must hold, so this equation reduces to:

$$\left(\frac{\delta(1-p+\mu\theta s)}{(1+\delta)^2 p} + \frac{1}{1+\delta} - 1\right)\phi_{\text{p}} = 0,$$

and since $\phi_{\text{p}} > 0$, the equations simplifies to:

$$\frac{\delta(1-p+\mu\theta s)}{(1+\delta)p} = \delta.$$

After multiplying both sides by $p > 0$, this is a linear equation in p and can be solved as follows:

$$p = \frac{1+\mu\theta s}{2+\delta}, \quad \text{and hence} \quad p - p^{\ddagger} = \frac{\mu\theta s}{2+\delta}, \qquad \text{[A.2.24]}$$

where the second equation uses the expression for $p^{\ddagger}$ in (3.11). Observe that $p > p^{\ddagger}$ for any $s > 0$. It is assumed here that $p \le 1/2$; the case where the constraint $p \le 1/2$ binds is considered later.

Using the equilibrium value of p in (A.2.24) and $p = p'$ and $\phi_{\text{p}} = \phi'_{\text{p}}$, the implied equilibrium income share ϕ_{p} can be obtained from (A.2.22):

$$p\phi_{\text{p}} = 1 - \left(1 - p + (2+\delta)(p - p^{\ddagger})\right)\frac{\phi_{\text{p}}}{1+\delta},$$

which can be simplified by noting $p^{\ddagger} = 1/(2+\delta)$:

$$(1+\delta)p\phi_{\text{p}} = (1+\delta) - (1+\delta)p\phi_{\text{p}}.$$

The per-person income shares of incumbents and workers are therefore:

$$\phi_{\text{p}} = \frac{1}{2p}, \quad \text{and} \quad \phi_{\text{w}} = \frac{1}{2(1+\delta)p}, \qquad \text{[A.2.25]}$$

where the latter follows from the former together with the binding no-rebellion constraint (A.2.20a), which implies $\phi_{\mathrm{w}} = \phi_{\mathrm{p}}/(1+\delta)$ in equilibrium.

For this case to be an equilibrium it is necessary that the other no-rebellion constraints (A.2.20b) and (A.2.20c) are satisfied. By using (A.2.25), the constraint (A.2.20b) requires:

$$\frac{1}{2(1+\delta)p} \geq \frac{1}{2(\delta p + p^{\ddagger})},$$

where the right-hand side follows from the expressions for $p^{\ddagger}$ and $\phi_{\mathrm{p}}^{\ddagger}$ by noting $p^{\ddagger}\phi_{\mathrm{p}}^{\ddagger} = 1/2$. Rearranging the inequality above reveals that it is equivalent to $p^{\ddagger} \geq p$. However, this is violated for any $s > 0$ because that led to $p > p^{\ddagger}$ using (A.2.24).

Now consider the case where the no-rebellion constraint (A.2.20a) binds in conjunction with the constraint $p \leq 1/2$ on power sharing. Since ϕ_{p} in (A.2.22) is quasi-concave in p, the constraint $p \leq 1/2$ binds when the p value where the first derivative of ϕ_{p} is zero is found to the right of $1/2$. Using (A.2.24), this occurs when

$$\frac{1}{2} < p^{\ddagger} + \frac{\mu\theta s}{2+\delta} = \frac{1+\mu\theta s}{2+\delta}, \quad \text{or equivalently} \;\; \mu\theta s > \frac{\delta}{2}. \tag{A.2.26}$$

With $p = p' = 1/2$, the expression for ϕ_{p} in (A.2.22) can be used to deduce the following in equilibrium ($\phi_{\mathrm{p}} = \phi'_{\mathrm{p}}$):

$$\frac{\phi_{\mathrm{p}}}{2} = 1 - \left(1 - \frac{1}{2} + \mu\theta s\right)\frac{\phi_{\mathrm{p}}}{1+\delta},$$

and solving this linear equation in ϕ_{p} and using $\phi_{\mathrm{w}} = \phi_{\mathrm{p}}/(1+\delta)$ (when (A.2.20a) binds in equilibrium):

$$\phi_{\mathrm{p}} = \frac{1+\delta}{1+\delta/2+\mu\theta s}, \quad \text{and} \;\; \phi_{\mathrm{w}} = \frac{1}{1+\delta/2+\mu\theta s}. \tag{A.2.27}$$

With $p = 1/2$, $p^{\ddagger} = 1/(2+\delta)$, and $p^{\ddagger}\phi_{\mathrm{p}}^{\ddagger}$, the post-investment no-rebellion constraint for workers (A.2.20b) requires:

$$\phi_{\mathrm{w}} \geq \frac{1}{2(\delta/2 + 1/(2+\delta))} = \frac{1}{\delta + 1/(1+\delta/2)}. \tag{A.2.28}$$

However, under the conditions from (A.2.26) when $p \leq 1/2$ is binding, it follows from the expression for ϕ_{w} in (A.2.27) that:

$$\phi_{\mathrm{w}} < \frac{1}{1+\delta} < \frac{1}{\delta + 1/(1+\delta/2)},$$

which contradicts (A.2.28), so the no-rebellion constraint (A.2.20b) is violated when both (A.2.20a) and $p \leq 1/2$ are binding. Hence, (A.2.20a) cannot be the only binding no-rebellion constraint, irrespective of whether $0 \leq p \leq 1/2$ is also binding or not.

The no-rebellion constraints for workers at the pre-investment stage and incumbents at the post-investment stage cannot bind together

Suppose no-rebellion constraints (A.2.20a) and (A.2.20c) are both binding. In equilibrium, with $p = p'$ and $\phi_{\mathrm{p}} = \phi'_{\mathrm{p}}$, having (A.2.20a) bind requires:

$$\phi_{\mathrm{w}} = \frac{\phi_{\mathrm{p}}}{1+\delta}, \tag{A.2.29}$$

and having (A.2.20c) bind requires:

$$\phi_{\mathrm{p}} = \frac{1}{2(\delta p + (1-\delta)p^{\ddagger})},$$

which is obtained using $p^{\ddagger}\phi_{\mathrm{p}}^{\ddagger} = 1/2$. Using this equation to substitute for ϕ_{p} in (A.2.29) yields an expression for ϕ_{w}:

$$\phi_{\mathrm{w}} = \frac{1}{2(1+\delta)(\delta p + (1-\delta)p^{\ddagger})}.$$

The no-rebellion constraint (A.2.20b) must also be satisfied. Given the expression for ϕ_{w} above, and noting $p^{\ddagger}\phi_{\mathrm{p}}^{\ddagger} = 1/2$, this requires:

$$\frac{1}{2(1+\delta)(\delta p + (1-\delta)p^{\ddagger})} \geq \frac{1}{2(\delta p + p^{\ddagger})}.$$

Rearranging this inequality shows that it is equivalent to $\delta^2(p^{\ddagger} - p) \geq 0$. However, since $p > p^{\ddagger}$ is known to be required in any equilibrium with $s > 0$, the no-rebellion constraint (A.2.20b) cannot hold. Therefore, this configuration of binding no-rebelllion constraints is not possible in equilibrium.

The case where the no-rebellion constraint for workers at the post-investment stage is the only binding constraint

Suppose that (A.2.20b) is the only binding no-rebellion constraint. The per-person worker share ϕ_{w} is thus given by:

$$\phi_{\mathrm{w}} = \frac{1}{2(\delta p + p^{\ddagger})}, \tag{A.2.30}$$

where this expression uses $p^{\ddagger}\phi_{\mathrm{p}}^{\ddagger} = 1/2$. Substituting this into (A.2.19) yields an expression for the per-person incumbent share ϕ_{p}:

$$\phi_{\mathrm{p}} = \frac{1 - \frac{(1-p+\mu\theta s)}{2(\delta p + p^{\ddagger})}}{p}, \tag{A.2.31}$$

and the optimality condition for equilibrium requires that p maximizes ϕ_{p} (given s). The first derivative of ϕ_{p} with respect to p is:

$$\frac{\partial \phi_{\mathrm{p}}}{\partial p} = \frac{1}{p}\left(\left(1 + \frac{\delta(1-p+\mu\theta s)}{\delta p + p^{\ddagger}}\right)\frac{1}{2(\delta p + p^{\ddagger})} - \frac{1 - \frac{(1-p+\mu\theta s)}{2(\delta p + p^{\ddagger})}}{p}\right), \tag{A.2.32}$$

which can be used to find the second derivative and to evaluate it at a point where the first derivative is zero:

$$\left.\frac{\partial^2 \phi_{\mathrm{p}}}{\partial p^2}\right|_{\frac{\partial \phi_{\mathrm{p}}}{\partial p}=0} = -\frac{\delta}{p(\delta p + p^{\ddagger})^2}\left(1 + \frac{\delta(1-p+\mu\theta s)}{\delta p + p^{\ddagger}}\right).$$

This is unambiguously negative, demonstrating that ϕ_{p} is a quasi-concave function of p (given an s), so the first-order condition is necessary and sufficient for a maximum. Setting the first derivative in (A.2.32) to zero and rearranging terms to have common denominators on both sides:

$$\frac{1}{2(\delta p + p^{\ddagger})}\left(\frac{\delta p + p^{\ddagger} + \delta - \delta p + \delta\mu\theta s}{\delta p + p^{\ddagger}}\right) = \frac{(1+2\delta)p - (1-2p^{\ddagger}) - \mu\theta s}{2(\delta p + p^{\ddagger})p}.$$

Cancelling common terms, using $1 - 2p^\ddagger = \delta p^\ddagger$, and simplifying leads to:

$$\frac{\delta + p^\ddagger + \delta\mu\theta s}{\delta p + p^\ddagger} = \frac{((1+2\delta)p - \delta p^\ddagger) - \mu\theta s}{p},$$

and therefore by multiplying both sides by $p(\delta p + p^\ddagger)$ and expanding the brackets:

$$(\delta + p^\ddagger + \delta\mu\theta s)p = \delta(1+2\delta)p^2 + (1+2\delta)p^\ddagger p - \delta^2 p^\ddagger p - \delta {p^\ddagger}^2 - (\delta p + p^\ddagger)\mu\theta s.$$

Collecting terms in $\mu\theta s$ on the left-hand side and simplifying:

$$(2\delta p + p^\ddagger)\mu\theta s = \delta(1+2\delta)p^2 + (1 + 2\delta - \delta^2 - 1 - \delta(2+\delta))p^\ddagger p - \delta {p^\ddagger}^2 = \delta((1+2\delta)p^2 - 2\delta p^\ddagger p - {p^\ddagger}^2),$$

where $\delta p = \delta(2+\delta)p^\ddagger p$ has been used. Note that the term of the right-hand side can be factorized:

$$(2\delta p + p^\ddagger)\mu\theta s = \delta(p - p^\ddagger)((1+2\delta)p + p^\ddagger),$$

and therefore the relationship between s and p is given by:

$$s = \frac{\delta(p - p^\ddagger)}{\mu\theta}\left(1 + \frac{p}{2\delta p + p^\ddagger}\right). \qquad \text{[A.2.33]}$$

It can be seen that $s > 0$ is consistent with $p > p^\ddagger$. Under the parameter restrictions in (2.9), it will be shown below that the constraint $p \leq 1/2$ is satisfied for any $0 \leq s \leq 1$.

The next step is to derive an expression for the maximized per-person incumbent share ϕ_{p} given the optimal p and s relationship in (A.2.33). Taking a common denominator of the expression for ϕ_{p} in (A.2.31):

$$\phi_{\mathrm{p}} = \frac{(1+2\delta)p - (1 - 2p^\ddagger) - \mu\theta s}{2(\delta p + p^\ddagger)p} = \frac{((1+2\delta)p - \delta p^\ddagger) - \mu\theta s}{2(\delta p + p^\ddagger)p},$$

which uses $1 - 2p^\ddagger = \delta p^\ddagger$. Substituting for $\mu\theta s$ using (A.2.33):

$$\phi_{\mathrm{p}} = \frac{(1+2\delta)p - (1 - 2p^\ddagger) - \mu\theta s}{2(\delta p + p^\ddagger)p} = \frac{((1+2\delta)p - \delta p^\ddagger) - \frac{\delta(p-p^\ddagger)((1+2\delta)p+p^\ddagger)}{2\delta p + p^\ddagger}}{2(\delta p + p^\ddagger)p},$$

and taking a common denominator:

$$\phi_{\mathrm{p}} = \frac{((1+2\delta)p - \delta p^\ddagger)(2\delta p + p^\ddagger) - \delta(p - p^\ddagger)((1+2\delta)p + p^\ddagger)}{2(2\delta p + p^\ddagger)(\delta p + p^\ddagger)p}.$$

Multiplying out the brackets in the numerator leads to:

$$\phi_{\mathrm{p}} = \frac{2\delta(1+2\delta)p^2 - 2\delta^2 p^\ddagger p + (1+2\delta)p^\ddagger p - \delta {p^\ddagger}^2 - \delta((1+2\delta)p^2 - (1+2\delta)p^\ddagger p + p^\ddagger p - {p^\ddagger}^2)}{2(2\delta p + p^\ddagger)(\delta p + p^\ddagger)p},$$

which can be simplified as follows:

$$\phi_{\mathrm{p}} = \frac{\delta(1+2\delta)p^2 + (1+2\delta)p^\ddagger p}{2(2\delta p + p^\ddagger)(\delta p + p^\ddagger)p} = \frac{(1+2\delta)\left(\delta p + p^\ddagger\right)p}{2(2\delta p + p^\ddagger)(\delta p + p^\ddagger)p}.$$

Therefore, setting p to maximize ϕ_{p} (given s) implies that:

$$\phi_{\mathrm{p}} = \frac{1+2\delta}{2(2\delta p + p^\ddagger)}. \qquad \text{[A.2.34]}$$

The two other no-rebellion constraints must also be satisfied, namely (A.2.20a) for workers at the pre-investment stage, and (A.2.20c) for incumbents at the post-investment stage. Taking the first of these, in equilibrium with $p' = p$ and $\phi'_{\mathrm{p}} = \phi_{\mathrm{p}}$, (A.2.20a) requires $\phi_{\mathrm{w}} \geq \phi_{\mathrm{p}}/(1+\delta)$, and by using the expressions for ϕ_{w} and ϕ_{p} in (A.2.30) and (A.2.34), this is equivalent to:

$$\frac{1}{2(\delta p + p^{\ddagger})} \geq \frac{1+2\delta}{2(1+\delta)(2\delta p + p^{\ddagger})}.$$

Rearranging the inequality above leads to $2\delta(1+\delta)p + (1+\delta)p^{\ddagger} \geq \delta(1+2\delta)p + (1+2\delta)p^{\ddagger}$, which simplifies to $\delta(p - p^{\ddagger}) \geq 0$. This is satisfied because $p > p^{\ddagger}$ when $s > 0$, so the no-rebellion constraint (A.2.20a) is therefore slack in this case. For the no-rebellion constraint (A.2.20c), using $p^{\ddagger}\phi^{\ddagger}_{\mathrm{p}} = 1/2$ and the expression for ϕ_{p} in (A.2.34), the constraint requires:

$$\frac{1+2\delta}{2(2\delta p + p^{\ddagger})} \geq \frac{1}{2(\delta p + (1-\delta)p^{\ddagger})}.$$

Rearranging shows that this inequality is equivalent to $\delta(1+2\delta)p + (1+2\delta)(1-\delta)p^{\ddagger} \geq 2\delta p + p^{\ddagger}$, which simplifies to $\delta(2\delta-1)(p-p^{\ddagger}) \geq 0$. With $\delta > 0$, this is in turn equivalent to $(2\delta - 1)(p - p^{\ddagger}) \geq 0$. Since $p > p^{\dagger}$ when $s > 0$, (A.2.20c) holds in this case if and only if $\delta \geq 1/2$.

Assuming $\delta \geq 1/2$, it must also be confirmed that incumbents would be willing to defend the rules against a rebellion that would result in them losing power. At the pre-investment stage, in equilibrium with $\phi'_{\mathrm{w}} = \phi_{\mathrm{w}}$ this requires $\phi_{\mathrm{p}} > \phi_{\mathrm{w}}$. Since it has been shown the no-rebellion constraint (A.2.20c) is satisfied when $\delta \geq 1/2$, by using the expression for ϕ_{w} in (A.2.30) observe that:

$$\phi_{\mathrm{p}} \geq \frac{1}{2(\delta p + (1-\delta)p^{\ddagger})} > \frac{1}{2(\delta p + p^{\ddagger})} = \phi_{\mathrm{w}},$$

where the middle inequality follows from $\delta p + (1-\delta)p^{\ddagger} < \delta p + p^{\ddagger}$. This confirms $\phi_{\mathrm{p}} > \phi'_{\mathrm{w}}$ at the pre-investment stage. At the post-investment stage, the requirement is $\phi_{\mathrm{p}} > \phi^{\ddagger}_{\mathrm{w}}$, and by using the expression for ϕ_{p} in (A.2.34) and $\phi^{\ddagger}_{\mathrm{w}} = (2+\delta)/(2+2\delta)$, this is equivalent to:

$$\frac{1+2\delta}{2(2\delta p + p^{\ddagger})} > \frac{2+\delta}{2(1+\delta)}.$$

Using $(2+\delta)p^{\ddagger} = 1$ and rearranging this inequality leads to $1 + 3\delta + 2\delta^2 > 2\delta(2+\delta)p + 1$, which simplifies to $p < (3+2\delta)/(4+2\delta)$. The right-hand side of the inequality is increasing δ, so with $\delta \geq 1/2$, it is sufficient to verify it for the case of $\delta = 1/2$. The requirement is $p < 4/5$, which is necessarily satisfied because $p \leq 1/2$, hence $\phi_{\mathrm{p}} > \phi^{\ddagger}_{\mathrm{w}}$ is confirmed.

In summary, in the case $\delta \geq 1/2$, all constraints are satisfied when only the no-rebellion constraint (A.2.20b) binds. But if $\delta < 1/2$, this configuration of binding constraint is not possible.

The case where both no-rebellion constraints at the post-investment stage are binding

Now suppose that both no-rebellion constraints (A.2.20b) and (A.2.20c) at the post-investment stage are binding. This means that the per-person worker and incumbent shares are:

$$\phi_{\mathrm{w}} = \frac{1}{2(\delta p + p^{\ddagger})}, \quad \text{and} \quad \phi_{\mathrm{p}} = \frac{1}{2(\delta p + (1-\delta)p^{\ddagger})}. \qquad \text{[A.2.35]}$$

Using these to substitute for ϕ_{w} and ϕ_{p} in equation (A.2.19) leads to:

$$\frac{1}{2(\delta p + (1-\delta)p^{\ddagger})} = \frac{1 - \frac{1-p+\mu\theta s}{2(\delta p + p^{\ddagger})}}{p},$$

and multiplying both sides by p and simplifying:

$$\frac{p}{2(\delta p+(1-\delta)p^{\ddagger})}=1-\frac{1-p+\mu\theta s}{2(\delta p+p^{\ddagger})}=\frac{(1+2\delta)p-(1-2p^{\ddagger})-\mu\theta s}{2(\delta p+p^{\ddagger})}=\frac{((1+2\delta)p-\delta p^{\ddagger})-\mu\theta s}{2(\delta p+p^{\ddagger})}.$$

This equation can be rearranged to yield an expression for $\mu\theta s$ in terms of p:

$$\mu\theta s=((1+2\delta)p-\delta p^{\ddagger})-\frac{(\delta p+p^{\ddagger})p}{\delta p+(1-\delta)p^{\ddagger}}=\frac{((1+2\delta)p-\delta p^{\ddagger})(\delta p+(1-\delta)p^{\ddagger})-(\delta p+p^{\ddagger})p}{\delta p+(1-\delta)p^{\ddagger}},$$

and expanding the brackets in the numerator leads to:

$$\mu\theta s=\frac{\delta(1+2\delta)p^2-\delta^2p^{\ddagger}p+(1-\delta)(1+2\delta)p^{\ddagger}p-\delta(1-\delta){p^{\ddagger}}^2-\delta p^2-p^{\ddagger}p}{\delta p+(1-\delta)p^{\ddagger}}.$$

This expression can be simplified as follows:

$$\mu\theta s=\frac{\delta\left(2\delta p^2+(1-3\delta)p^{\ddagger}p-(1-\delta){p^{\ddagger}}^2\right)}{\delta p+(1-\delta)p^{\ddagger}},$$

and note that the numerator can be factorized, hence:

$$\mu\theta s=\frac{\delta(p-p^{\ddagger})(2\delta p+(1-\delta)p^{\ddagger})}{\delta p+(1-\delta)p^{\ddagger}}.$$

Therefore, the relationship between s and p in this case is given by the equation:

$$s=\frac{\delta(p-p^{\ddagger})}{\mu\theta}\left(1+\frac{\delta p}{\delta p+(1-\delta)p^{\ddagger}}\right). \qquad \text{[A.2.36]}$$

It can be seen that $s>0$ is consistent with $p>p^{\ddagger}$. Under the parameter restrictions in (2.9) it will be shown below that the constraint $p\leq 1/2$ is satisfied for any $0\leq s\leq 1$.

The remaining no-rebellion constraint (A.2.20a) for workers at the pre-investment stage must also hold. In equilibrium with $p'=p$ and $\phi'_{\mathrm{p}}=\phi_{\mathrm{p}}$, this requires $\phi_{\mathrm{w}}\geq\phi_{\mathrm{p}}/(1+\delta)$. Using the expressions for ϕ_{p} and ϕ_{w} in (A.2.35), this condition is equivalent to:

$$\frac{1}{2(\delta p+p^{\ddagger})}\geq\frac{1}{2(1+\delta)(\delta p+(1-\delta)p^{\ddagger})}.$$

Rearranging this inequality leads to $\delta(1+\delta)p+(1+\delta)(1-\delta)p^{\ddagger}\geq\delta p+p^{\ddagger}$, which simplifies to $\delta^2(p-p^{\ddagger})\geq 0$. Given that $p>p^{\ddagger}$ whenever $s>0$, it is confirmed that the no-rebellion constraint (A.2.20a) holds in this case.

It is known that when $\delta\geq 1/2$, the choice of p that maximizes ϕ_{p} subject only to the no-rebellion constraint (A.2.20b) for workers at the post-investment also satisfies all other constraints. Therefore, both no-rebellion constraints (A.2.20b) and (A.2.20c) can only bind when $\delta\leq 1/2$.

Assuming $\delta\leq 1/2$, it must also be confirmed that incumbents are willing to defend the rules against a rebellions that would see them lose power subsequently. At the pre-investment stage, in equilibrium with $\phi'_{\mathrm{w}}=\phi_{\mathrm{w}}$, this requires $\phi_{\mathrm{p}}>\phi_{\mathrm{w}}$. Using the expressions for ϕ_{w} and ϕ_{p} in (A.2.35), since $\delta p+(1-\delta)p^{\ddagger}<\delta p+p^{\ddagger}$, it immediately follows that:

$$\phi_{\mathrm{p}}=\frac{1}{2(\delta p+(1-\delta)p^{\ddagger})}>\frac{1}{2(\delta p+p^{\ddagger})}=\phi_{\mathrm{w}}.$$

At the post-investment stage, the requirement is $\phi_{\mathrm{p}}>\phi^{\ddagger}_{\mathrm{w}}$, and by using the expression for ϕ_{p} from (A.2.35) and $\phi^{\ddagger}_{\mathrm{w}} = (2+\delta)/(2+2\delta)$, this is equivalent to:

$$\frac{1}{2(\delta p + (1-\delta)p^{\ddagger})} > \frac{2+\delta}{2(1+\delta)}.$$

Using $(2+\delta)p^{\ddagger} = 1$, and rearranging this inequality leads to $1+\delta > \delta(2+\delta)p + (1-\delta)$, which simplifies to $p < 2/(2+\delta)$. The right-hand side of the inequality is decreasing in δ, and since $\delta \leq 1/2$ in this case, it suffices to verify this for $\delta = 1/2$. The requirement is $p < 4/5$, which is necessarily satisfied since $p \leq 1/2$, hence $\phi_{\mathrm{p}} > \phi^{\ddagger}_{\mathrm{w}}$ is confirmed.

In summary, when $\delta \leq 1/2$, all the equilibrium conditions are satisfied in this case and the minimum number of constraints is binding.

Power sharing, rents, and the rule of law

For any value of δ, it has been shown that no-rebellion constraint (A.2.20b) for workers at the post-investment stage is always binding. When $\delta < 1/2$, the no-rebellion constraint (A.2.20c) for incumbents at the post-investment stage is also binding. All other no-rebellion constraints are redundant or slack, confirming the claims in the proposition.

In both cases $\delta < 1/2$ and $\delta \geq 1/2$, the equilibrium rules feature a relationship $s = \sigma(p)$ between power sharing p and the reach of the rule of law s given respectively by (A.2.36) or (A.2.33). These equations confirm the functional form of $\sigma(p)$ given in the proposition. Note that in either case, $s = 0$ corresponds to $p = p^{\dagger} \equiv 1/(2+\delta)$, where $p^{\dagger} = p^{\ddagger}$.

Taking the function $\sigma(p)$ in the case where $\delta \leq 1/2$ (equation (A.2.36)):

$$\sigma(p) = \frac{\delta(p-p^{\ddagger})}{\mu\theta}\left(1 + \frac{\delta p}{\delta p + (1-\delta)p^{\ddagger}}\right). \tag{A.2.37}$$

The first derivative is

$$\begin{aligned}\sigma'(p) &= \frac{\delta}{\mu\theta}\left(1 + \frac{\delta p}{\delta p + (1-\delta)p^{\ddagger}} + \frac{\delta(p-p^{\ddagger})}{\delta p + (1-\delta)p^{\ddagger}} - \frac{\delta^2(p-p^{\ddagger})p}{(\delta p + (1-\delta)p^{\ddagger})^2}\right) \\ &= \frac{\delta\left((\delta p + (1-\delta)p^{\ddagger})^2 + \delta p(\delta p + (1-\delta)p^{\ddagger}) + \delta(p-p^{\ddagger})(\delta p + (1-\delta p^{\ddagger}) - \delta^2(p-p^{\ddagger})p\right)}{\mu\theta(\delta p + (1-\delta)p^{\ddagger})^2} \\ &= \frac{\delta\left(2\delta^2p^2 + (4\delta - 4\delta^2)p^{\ddagger}p + (1-3\delta+2\delta^2){p^{\ddagger}}^2\right)}{\mu\theta(\delta p + (1-\delta)p^{\ddagger})^2} = \frac{\delta\left(2\delta^2p^2 + 4\delta(1-\delta)p^{\ddagger}p + (1-\delta)(1-2\delta){p^{\ddagger}}^2\right)}{\mu\theta(\delta p + (1-\delta)p^{\ddagger})^2}.\end{aligned} \tag{A.2.38}$$

Since $\delta \leq 1/2$ in this case, the terms $(1-\delta)$ and $(1-2\delta)$ are non-negative, so the first derivative is strictly positive, confirming that $\sigma(p)$ is an increasing function.

With $\sigma(p)$ being an increasing function, the constraint $p \leq 1/2$ can be verified for all $s \in [0,1]$ by checking $\sigma(1/2) \geq 1$. Using (A.2.37) and $p^{\ddagger} = 1/(2+\delta)$, this requires:

$$1 \leq \frac{\delta\left(\frac{1}{2} - \frac{1}{2+\delta}\right)}{\mu\theta}\left(1 + \frac{\frac{\delta}{2}}{\frac{\delta}{2} + \frac{(1-\delta)}{(2+\delta)}}\right),$$

which is equivalent to:

$$\mu\theta \leq \frac{\delta((2+\delta)-2)}{2(2+\delta)}\left(1 + \frac{\delta(2+\delta)}{\delta(2+\delta) + 2(1-\delta)}\right) = \frac{\delta^2(2+2\delta+2\delta^2)}{2(2+\delta)(2+\delta^2)} = \frac{\delta}{2(2+\delta)}\,\frac{2\delta(1+\delta+\delta^2)}{2+\delta^2}.$$

It is clear that the parameter restrictions on μ and θ in (2.9) are sufficient for this to hold, confirming that $p \leq 1/2$ is never binding. Let $\bar{p}$ denote the value of p associated with $s = 1$ (the rule of law). Using

(A.2.37), $\bar{p}$ is the solution (with $\bar{p} > p^\ddagger$) of the equation:

$$\frac{\delta(\bar{p} - p^\ddagger)}{\mu\theta}\left(1 + \frac{\delta\bar{p}}{\delta\bar{p} + (1-\delta)p^\ddagger}\right) = 1, \tag{A.2.39}$$

which is equivalent to:

$$\delta(\bar{p} - p^\ddagger)(2\delta p + (1-\delta)p^\ddagger) = \mu\theta(\delta\bar{p} + (1-\delta)p^\ddagger).$$

Using $p^\ddagger = 1/(2+\delta)$ and multiplying both sides of the equation by $(2+\delta)^2$ leads to the following quadratic in $\bar{p}$:

$$2\delta^2(2+\delta)^2\bar{p}^2 - \delta(2+\delta)(3\delta - 1 + (2+\delta)\mu\theta)\bar{p} - (1-\delta)(\delta + (2+\delta)\mu\theta) = 0.$$

Since $0 < \delta \leq 1/2$ in this case, the quadratic has only one positive root, so the solution for $\bar{p}$ is:

$$\bar{p} = \frac{3\delta - 1 + (2+\delta)\mu\theta + \sqrt{(3\delta - 1 + (2+\delta)\mu\theta)^2 + 8(1-\delta)(\delta + (2+\delta)\mu\theta)}}{4\delta(2+\delta)},$$

which confirms the expression for $\bar{p}$ given in the proposition.

To show convexity of the $\sigma(p)$ function, note that using (A.2.38) and $(\delta p + (1-\delta)p^\ddagger)^2 = \delta^2 p^2 + 2\delta(1-\delta)p^\ddagger p + (1-\delta)^2 {p^\ddagger}^2$, $\sigma'(p)$ can be written as follows:

$$\sigma'(p) = \frac{\delta\left(2(\delta p + (1-\delta)p^\ddagger)^2 - 2(1-\delta)^2{p^\ddagger}^2 + (1-\delta)(1-2\delta){p^\ddagger}^2\right)}{\mu\theta(\delta p + (1-\delta)p^\ddagger)^2}$$
$$= \frac{\delta\left(2(\delta p + (1-\delta)p^\ddagger)^2 - (1-\delta){p^\ddagger}^2\right)}{\mu\theta(\delta p + (1-\delta)p^\ddagger)^2} = \frac{\delta}{\mu\theta}\left(2 - \frac{(1-\delta){p^\ddagger}^2}{(\delta p + (1-\delta)p^\ddagger)^2}\right).$$

The second derivative of $\sigma(p)$ is thus:

$$\sigma''(p) = \frac{2\delta^2(1-\delta){p^\ddagger}^2}{\mu\theta(\delta p + (1-\delta)p^\ddagger)^2},$$

which is always positive, confirming that $\sigma(p)$ is a convex function.

Now consider the case with $\delta \geq 1/2$. The relevant expression for $\sigma(p)$ in this case is from (A.2.33):

$$\sigma(p) = \frac{\delta(p - p^\ddagger)}{\mu\theta}\left(1 + \frac{p}{2\delta p + p^\ddagger}\right), \tag{A.2.40}$$

which has first derivative:

$$\sigma'(p) = \frac{\delta}{\mu\theta}\left(1 + \frac{p}{2\delta p + p^\ddagger} + \frac{p - p^\ddagger}{2\delta p + p^\ddagger} - \frac{2\delta(p - p^\ddagger)p}{(2\delta p + p^\ddagger)^2}\right)$$
$$= \frac{\delta\left((2\delta p + p^\ddagger)^2 + p(2\delta p + p^\ddagger) + (p - p^\ddagger)(2\delta p + p^\ddagger) - 2\delta(p - p^\ddagger)p\right)}{\mu\theta(2\delta p + p^\ddagger)^2}$$
$$= \frac{\delta\left((2\delta + 4\delta^2)p^2 + (2 + 4\delta)p^\ddagger p\right)}{\mu\theta(2\delta p + p^\ddagger)^2} = \frac{2\delta(1+2\delta)(\delta p + p^\ddagger)p}{\mu\theta(2\delta p + p^\ddagger)^2}. \tag{A.2.41}$$

This is positive, confirming that $\sigma(p)$ is an increasing function.

With $\sigma(p)$ being increasing in p, the constraint $p \leq 1/2$ can be verified for all $s \in [0, 1]$ by checking

$\sigma(1/2) \geq 1$. Using (A.2.40) and $p^\ddagger = 1/(2+\delta)$, this requires:

$$1 \leq \frac{\delta\left(\frac{1}{2} - \frac{1}{2+\delta}\right)}{\mu\theta}\left(1 + \frac{\frac{1}{2}}{\frac{2\delta}{2} + \frac{1}{2+\delta}}\right),$$

which is equivalent to:

$$\mu\theta \leq \frac{\delta((2+\delta)-2)}{2(2+\delta)}\left(1 + \frac{2+\delta}{2\delta(2+\delta)+2}\right) = \frac{\delta^2(4+5\delta+2\delta^2)}{4(2+\delta)(1+2\delta+\delta^2)} = \frac{\delta}{2(2+\delta)}\frac{\delta(4+5\delta+2\delta^2)}{2(1+\delta)^2}.$$

It is clear that the parameter restrictions in (2.9) are sufficient for this to hold, confirming that $p \leq 1/2$ is never binding. Let $\bar{p}$ denote the value of p associated with $s = 1$ (the rule of law). Using (A.2.40), $\bar{p}$ is the solution (with $\bar{p} > p^\ddagger$) of the equation:

$$\frac{\delta(\bar{p} - p^\ddagger)}{\mu\theta}\left(1 + \frac{\bar{p}}{2\delta\bar{p} + p^\ddagger}\right), \tag{A.2.42}$$

which is equivalent to:

$$\delta(\bar{p} - p^\ddagger)((1+2\delta)\bar{p} + p^\ddagger) = \mu\theta(2\delta\bar{p} + p^\ddagger).$$

Using $p^\ddagger = 1/(2+\delta)$ and multiplying both sides of the equation by $(2+\delta)^2$ leads to the following quadratic in $\bar{p}$:

$$\delta(1+2\delta)(2+\delta)^2\bar{p}^2 - 2\delta(2+\delta)(\delta + (2+\delta)\mu\theta) - (\delta + (2+\delta)\mu\theta) = 0.$$

This quadratic equation has one positive root, so $\bar{p}$ is given by:

$$\bar{p} = \frac{\delta(\delta + (2+\delta)\mu\theta) + \sqrt{(\delta(\delta + (2+\delta)\mu\theta))^2 + \delta(1+2\delta)(\delta + (2+\delta)\mu\theta)}}{\delta(1+2\delta)(2+\delta)},$$

which confirms the expression for $\bar{p}$ given in the proposition.

To show convexity of $\sigma(p)$, note that $(2\delta p + p^\ddagger)^2 = 4\delta(\delta p + p^\ddagger)p + {p^\ddagger}^2$ and hence $2\delta(\delta p + p^\ddagger) = (2\delta p + p^\ddagger)^2/2 - {p^\ddagger}^2/2$. Substituting this into (A.2.41) shows that $\sigma'(p)$ can be written as follows:

$$\sigma'(p) = \frac{(1+2\delta)\left((2\delta p + p^\ddagger)^2 - {p^\ddagger}^2\right)}{2\mu\theta(2\delta p + p^\ddagger)^2} = \frac{1+2\delta}{2\mu\theta}\left(1 - \frac{{p^\ddagger}^2}{(2\delta p + p^\ddagger)^2}\right).$$

The second derivative of $\sigma(p)$ is thus:

$$\sigma''(p) = \frac{2\delta(1+2\delta){p^\ddagger}^2}{\mu\theta(2\delta p + p^\ddagger)^3},$$

which is always positive, confirming the claim that $\sigma(p)$ is a convex function.

The no-rebellion constraint (A.2.20b) is binding for all parameter values, and thus the per-person worker share is:

$$\phi_{\mathrm{w}} = \frac{1}{2(\delta p + p^\ddagger)}. \tag{A.2.43}$$

In the case $0 < \delta \leq 1/2$, both post-investment no-rebellion constraints are binding and the per-person incumbent share is given in (A.2.35). In the case, $\delta \geq 1/2$, only the post-investment no-rebellion constraint for workers is binding, and the optimal choice of p subject to this constraint is given in (A.2.34):

$$\phi_{\mathrm{p}} = \begin{cases} \frac{1}{2(\delta p+(1-\delta)p^{\ddagger})} & \text{if } \delta \leq 1/2 \\ \frac{1+2\delta}{2(2\delta p+p^{\ddagger})} & \text{if } \delta \geq 1/2 \end{cases}. \tag{A.2.44}$$

These confirm the expressions for ϕ_{w} and ϕ_{p} given in the proposition. Now consider the extra consumption received by an incumbent relative to a worker, that is, rents $\chi \equiv (\phi_{\mathrm{p}} - \phi_{\mathrm{w}})/\phi_{\mathrm{w}}$. In the case of $0 < \delta \leq 1/2$, equations (A.2.43) and (A.2.44) imply:

$$\chi = \frac{\delta p + p^{\ddagger}}{\delta p + (1-\delta)p^{\ddagger}} - 1 = \frac{\delta p^{\ddagger}}{\delta p + (1-\delta)p^{\ddagger}},$$

which (since $p > p^{\ddagger}$) is positive and decreasing in p. In the case of $\delta \geq 1/2$, equations (A.2.43) and (A.2.44) imply that rents are:

$$\chi = \frac{(1+2\delta)(\delta p + p^{\ddagger})}{2\delta p + p^{\ddagger}} - 1 = \frac{\delta(1+2\delta)p + \delta(1+2\delta)p^{\ddagger} - 2\delta p - p^{\ddagger}}{2\delta p + p^{\ddagger}} = \frac{\delta(2\delta-1)p + (2\delta^2+\delta-1)p^{\ddagger}}{2\delta p + p^{\ddagger}}$$
$$= \frac{(2\delta-1)(\delta p + (1+\delta)p^{\ddagger})}{2\delta p + p^{\ddagger}},$$

using $2\delta^2 + \delta - 1 = (2\delta-1)(1+\delta)$. The derivative with respect to p is:

$$\frac{\partial \chi}{\partial p} = \frac{(2\delta-1)\left(\delta(2\delta p + p^{\ddagger}) - 2\delta(\delta p + (1+\delta)p^{\ddagger})\right)}{(2\delta p + p^{\ddagger})^2} = -\frac{\delta(2\delta-1)(1+2\delta)p^{\ddagger}}{(2\delta p + p^{\ddagger})^2},$$

which is negative, showing that rents χ are decreasing in p in this case too. All the claims of the proposition are thus confirmed, completing the proof.

A.3 Proof of Proposition 3

The results of Proposition 2 show that for a given value of s, the equilibrium conditions for rules established at the pre-investment stage are met if power sharing p satisfies the equation $s = \sigma(p)$ for the function $\sigma(p)$ defined in Proposition 2. This is a strictly increasing function, so it has a well-defined inverse $p = \sigma^{-1}(s)$. Given s, for the value of p implied by this equation, the per-person incumbent share ϕ_{p} is expressed as a function of p in Proposition 2, denoted here by $\phi_{\mathrm{p}}(p)$. Optimality of the rules further requires that s maximizes the incumbent payoff $\mathcal{U}_{\mathrm{p}}$, subject only to $p = \sigma^{-1}(s)$, and with $\phi_{\mathrm{p}} = \phi_{\mathrm{p}}(\sigma^{-1}(s))$.

Proposition 2 also establishes that the equilibrium rules must feature domestic free exchange of goods. This means that aggregate consumption C is as given in (3.10), with relative price $\tilde{\pi}$ from (3.4). The market-clearing price depends on the capital stock K, which is determined by s according to (2.2). In autarky, there is no choice of net exports x_{E} and x_{I} to make. With $x_{\mathrm{E}} = 0$ and $x_{\mathrm{I}} = 0$, the autarky market-clearing price is $\hat{\pi}$, which depends on K, and hence on s, and this effect must be accounted for when determining s. The incumbent payoff $\hat{\mathcal{U}}_{\mathrm{p}}(s)$ taking account of all these features of the equilibrium rules for a particular value of s is given in equation (4.1).

The derivative of $\hat{\mathcal{U}}_{\mathrm{p}}(s)$ with respect to s is:

$$\hat{\mathcal{U}}_{\mathrm{p}}'(s) = \frac{\phi_{\mathrm{p}}'(\sigma^{-1}(s))}{\sigma'(\sigma^{-1}(s))\phi_{\mathrm{p}}(\sigma^{-1}(s))} + \frac{\alpha}{s}. \tag{A.3.1}$$

In the case where $\delta \leq 1/2$, Proposition 2 shows that the function $\phi_{\mathrm{p}}(p)$ is given by:

$$\phi_{\mathrm{p}}(p) = \frac{1}{2(\delta p + (1-\delta)p^{\ddagger})}, \tag{A.3.2}$$

which has the following derivative with respect to p:

$$\phi_{\mathrm{p}}'(p) = -\frac{\delta}{2(\delta p + (1-\delta)p^{\ddagger})^2} = -2\delta\phi_{\mathrm{p}}(p)^2. \tag{A.3.3}$$

In the case where $\delta \geq 1/2$, Proposition 2 shows that the function $\phi_{\mathrm{p}}(p)$ is given by:

$$\phi_{\mathrm{p}}(p) = \frac{1+2\delta}{2(2\delta p + p^{\ddagger})}, \tag{A.3.4}$$

which has the following derivative with respect to p:

$$\phi_{\mathrm{p}}'(p) = -\frac{\delta(1+2\delta)}{(2\delta p + p^{\ddagger})^2} = -\frac{4\delta}{1+2\delta}\phi_{\mathrm{p}}(p)^2. \tag{A.3.5}$$

Since $\min\{1, 2/(1+2\delta)\}$ equals 1 when $\delta \leq 1/2$ and $2/(1+2\delta)$ when $\delta \geq 1/2$, equations (A.3.3) and (A.3.5) together show that:

$$\frac{\phi_{\mathrm{p}}'(p)}{\phi_{\mathrm{p}}(p)} = -2\delta \min\left\{1, \frac{2}{1+2\delta}\right\}\phi_{\mathrm{p}}(p). \tag{A.3.6}$$

Substituting this into (A.3.1) shows that the derivative of $\hat{\mathcal{U}}_{\mathrm{p}}(s)$ is:

$$\hat{\mathcal{U}}_{\mathrm{p}}'(s) = \frac{1}{s}\left(\alpha - 2\delta\min\left\{1, \frac{2}{1+2\delta}\right\}\frac{s\phi_{\mathrm{p}}(\sigma^{-1}(s))}{\sigma'(\sigma^{-1}(s))}\right). \tag{A.3.7}$$

Now consider the expression for $\sigma'(p)$ in the case $\delta \leq 1/2$ in equation (A.2.38) from the proof of Proposition 2. Combined with (A.3.2), this implies:

$$\frac{\sigma'(p)}{\phi_{\mathrm{p}}(p)} = \frac{2\delta\left(2\delta^2p^2 + 4\delta(1-\delta)p^{\dagger}p + (1-\delta)(1-2\delta){p^{\dagger}}^2\right)}{\mu\theta(\delta p + (1-\delta)p^{\dagger})}. \tag{A.3.8}$$

Since Proposition 2 shows that the function $\sigma(p)$ in the case $\delta \leq 1/2$ is

$$\sigma(p) = \frac{\delta(p-p^{\ddagger})}{\mu\theta}\left(\frac{2\delta p + (1-\delta)p^{\ddagger}}{\delta p + (1-\delta)p^{\ddagger}}\right),$$

equation (A.3.8) can be used to deduce the following:

$$2\delta\min\left\{1, \frac{2}{1+2\delta}\right\}\frac{\sigma(p)\phi_{\mathrm{p}}(p)}{\sigma'(p)} = \frac{\delta(p-p^{\ddagger})(2\delta p + (1-\delta)p^{\ddagger})}{2\delta^2p^2 + 4\delta(1-\delta)p^{\ddagger}p + (1-\delta)(1-2\delta){p^{\ddagger}}^2}. \tag{A.3.9}$$

When $\delta \geq 1/2$, the relevant expression for $\sigma'(p)$ is given in equation (A.2.41) of the proof of Proposition 2. Combined with (A.3.4), this implies:

$$\frac{\sigma'(p)}{\phi_{\mathrm{p}}(p)} = \frac{4\delta(\delta p + p^{\dagger})}{\mu\theta(2\delta p + p^{\dagger})}. \tag{A.3.10}$$

Since Proposition 2 shows that the function $\sigma(p)$ in the case $\delta \geq 1/2$ is

$$\sigma(p) = \frac{\delta(p-p^{\ddagger})}{\mu\theta}\left(\frac{(1+2\delta)p + p^{\ddagger}}{2\delta p + p^{\ddagger}}\right),$$

equation (A.3.10) can be used to deduce the following:

$$2\delta\min\left\{1, \frac{2}{1+2\delta}\right\}\frac{\sigma(p)\phi_{\mathrm{p}}(p)}{\sigma'(p)} = \frac{\delta(p-p^{\ddagger})((1+2\delta)p + p^{\ddagger})}{(1+2\delta)(\delta p + p^{\ddagger})p}. \tag{A.3.11}$$

Combining equation (A.3.7) with equations (A.3.9) and (A.3.11) respectively for the cases of $\delta \leq 1/2$ and $\delta \geq 1/2$, and using $s = \sigma(p)$ and $p = \sigma^{-1}(s)$, it follows that:

$$\hat{\mathcal{U}}'_{\mathrm{p}}(s) = \frac{1}{s}\left(\alpha - \frac{2\delta^2(\sigma^{-1}(s))^2 + \delta(1-3\delta)p^{\ddagger}\sigma^{-1}(s) - \delta(1-\delta){p^{\ddagger}}^2}{2\delta^2(\sigma^{-1}(s))^2 + 4\delta(1-\delta)p^{\ddagger}\sigma^{-1}(s) + (1-\delta)(1-2\delta){p^{\ddagger}}^2}\right)$$
$$= \frac{2\delta^2(\alpha-1)(\sigma^{-1}(s))^2 + \delta(3\delta-1+4(1-\delta)\alpha)p^{\ddagger}\sigma^{-1}(s) + (1-\delta)(\delta+(1-2\delta)\alpha){p^{\ddagger}}^2}{\left(2\delta^2(\sigma^{-1}(s))^2 + 4\delta(1-\delta)p^{\ddagger}\sigma^{-1}(s) + (1-\delta)(1-2\delta){p^{\ddagger}}^2\right)s}, \quad \text{[A.3.12a]}$$

for the case $\delta \leq 1/2$, and for $\delta \geq 1/2$:

$$\hat{\mathcal{U}}'_{\mathrm{p}}(s) = \frac{1}{s}\left(\alpha - \frac{\delta(1+2\delta)(\sigma^{-1}(s))^2 - 2\delta^2p^{\ddagger}\sigma^{-1}(s) - \delta{p^{\ddagger}}^2}{\delta(1+2\delta)(\sigma^{-1}(s))^2 + (1+2\delta)p^{\ddagger}\sigma^{-1}(s)}\right)$$
$$= \frac{\delta(1+2\delta)(\alpha-1)(\sigma^{-1}(s))^2 + (2\delta^2+(1+2\delta)\alpha)p^{\ddagger}\sigma^{-1}(s) + \delta{p^{\ddagger}}^2}{(1+2\delta)(\delta\sigma^{-1}(s)+p^{\ddagger})\sigma^{-1}(s)s}. \quad \text{[A.3.12b]}$$

Inspection of equations (A.3.12a) and (A.3.12b) shows that these expressions for the first derivative of $\hat{\mathcal{U}}_{\mathrm{p}}(s)$ can only be zero if the numerators are zero. Thus, the second derivative of $\hat{\mathcal{U}}_{\mathrm{p}}(s)$ evaluated at a value of s where the first derivative is zero is:

$$\hat{\mathcal{U}}''_{\mathrm{p}}(s) = \begin{cases} \frac{4\delta^2(\alpha-1)\sigma^{-1}(s)+\delta(3\delta-1+4(1-\delta)\alpha)p^{\ddagger}}{\left(2\delta^2(\sigma^{-1}(s))^2+4\delta(1-\delta)p^{\ddagger}\sigma^{-1}(s)+(1-\delta)(1-2\delta){p^{\ddagger}}^2\right)s\sigma'(\sigma^{-1}(s))} & \text{for } \delta \leq 1/2 \\ \frac{2\delta(1+2\delta)(\alpha-1)\sigma^{-1}(s)+(2\delta^2+(1+2\delta)\alpha)p^{\ddagger}}{(1+2\delta)(\delta\sigma^{-1}(s)+p^{\dagger})\sigma^{-1}(s)s\sigma'(\sigma^{-1}(s))} & \text{for } \delta \geq 1/2 \end{cases}, \quad \text{if } \hat{\mathcal{U}}'_{\mathrm{p}}(s) = 0. \quad \text{[A.3.13]}$$

Now define the following quadratic function of p:

$$\mathcal{Q}(p) \equiv \begin{cases} 2\delta^2(2+\delta)^2(1-\alpha)p^2 - \delta(2+\delta)(3\delta-1+4(1-\delta)\alpha)p - (1-\delta)(\delta+(1-2\delta)\alpha) & \text{for } \delta \leq 1/2 \\ \delta(1+2\delta)(2+\delta)^2(1-\alpha)p^2 - (2+\delta)(2\delta^2+(1+2\delta)\alpha)p - \delta & \text{for } \delta \geq 1/2 \end{cases}, \quad \text{[A.3.14]}$$

noting that $\mathcal{Q}(0) < 0$ and $\mathcal{Q}''(p) > 0$ for all parameter values. It follows that the quadratic equation $\mathcal{Q}(p) = 0$ has one positive and one negative root. The function $\mathcal{Q}(p)$ is negative for p values between 0 and the positive root, and positive for p values above the positive root, which means that the derivative $\mathcal{Q}'(p)$ is positive at the positive root. A formula for the derivative of $\mathcal{Q}(p)$ is:

$$\mathcal{Q}'(p) = \begin{cases} 4\delta^2(2+\delta)^2(1-\alpha)p - \delta(2+\delta)(3\delta-1+4(1-\delta)\alpha) & \text{for } \delta \leq 1/2 \\ 2\delta(1+2\delta)(2+\delta)^2(1-\alpha)p - (2+\delta)(2\delta^2+(1+2\delta)\alpha) & \text{for } \delta \geq 1/2 \end{cases}. \quad \text{[A.3.15]}$$

Comparing (A.3.12) to (A.3.14) and noting $p^{\ddagger} = 1/(2+\delta)$ shows that the numerator of $\hat{\mathcal{U}}'_{\mathrm{p}}(s)$ is proportional to $-{p^{\ddagger}}^2\mathcal{Q}(\sigma^{-1}(s))$. Similarly, comparing (A.3.13) and (A.3.15) shows that the numerator of $\hat{\mathcal{U}}''_{\mathrm{p}}(s)$ where $\hat{\mathcal{U}}'_{\mathrm{p}}(s) = 0$ is proportional to $-{p^{\ddagger}}^2\mathcal{Q}'(\sigma^{-1}(s))$. Since the denominators of (A.3.12) and (A.3.13) are strictly positive for $s > 0$ and zero for $s = 0$, it follows from the properties of $\mathcal{Q}(p)$ that any $s \in [0,1]$ with $\hat{\mathcal{U}}'_{\mathrm{p}}(s) = 0$ must feature $\hat{\mathcal{U}}''_{\mathrm{p}}(s) < 0$. Therefore, the function $\hat{\mathcal{U}}_{\mathrm{p}}(s)$ is strictly quasi-concave in $s \in [0,1]$.

Given the quasi-concavity of $\hat{\mathcal{U}}_{\mathrm{p}}(s)$, it follows that the optimal value of s, denoted by $\hat{s}$, is the solution of $\mathcal{Q}(\sigma^{-1}(\hat{s})) = 0$ if $\hat{s} < 1$, or $\hat{s} = 1$ if $\mathcal{Q}(\sigma^{-1}(1)) \geq 0$. Let $\hat{p} = \sigma^{-1}(\hat{s})$ denote the associated value of power sharing p. The solution for p is either the positive root of $\mathcal{Q}(\hat{p}) = 0$ when $\hat{p} < \bar{p}$, or $\hat{p} = \bar{p}$ if $\mathcal{Q}(\bar{p}) \leq 0$. The condition for an interior solution is $\mathcal{Q}(\bar{p}) > 0$, where $\bar{p}$ is the value of p associated with $s = 1$. In the case $\delta \leq 1/2$, it is shown in the proof of Proposition 2 that $\bar{p}$ satisfies the quadratic equation (A.2.39), and hence:

$$2\delta^2(2+\delta)^2\bar{p}^2 = \delta(2+\delta)(3\delta-1+(2+\delta)\mu\theta)\bar{p} + (1-\delta)(\delta+(2+\delta)\mu\theta).$$

Evaluating the quadratic $\mathcal{Q}(p)$ in (A.3.14) at $p = \bar{p}$ and using the equation above to deduce:

$$\mathcal{Q}(\bar{p}) = (1-\alpha)\left(\delta(2+\delta)(3\delta - 1 + (2+\delta)\mu\theta)\bar{p} + (1-\delta)(\delta + (2+\delta)\mu\theta)\right)$$
$$- \delta(2+\delta)(3\delta - 1 + 4(1-\delta)\alpha)\bar{p} - (1-\delta)(\delta + (1-2\delta)\alpha)$$
$$= (1-\alpha)\mu\theta(2+\delta)(1-\delta+\delta(2+\delta)\bar{p}) - \alpha((1-\delta)^2 + \delta(2+\delta)(3-\delta)),$$

from which it follows that $\mathcal{Q}(\bar{p}) > 0$ is equivalent to:

$$\frac{\alpha}{1-\alpha} < \frac{\mu\theta(2+\delta)(1-\delta+\delta(2+\delta)\bar{p})}{(1-\delta)^2 + \delta(2+\delta)(3-\delta)\bar{p}}.$$

This inequality can be stated as $\alpha < \bar{\alpha}$, where $\bar{\alpha}$ is given by:

$$\bar{\alpha} = \left(1 + \frac{(1-\delta)^2 + \delta(2+\delta)(3-\delta)\bar{p}}{\mu\theta(2+\delta)(1-\delta+\delta(2+\delta)\bar{p})}\right)^{-1}, \qquad \text{[A.3.16]}$$

where this number lies strictly between 0 and 1. Similarly, in the case $\delta \geq 1/2$, the proof of Proposition 2 shows that $\bar{p}$ satisfies the quadratic equation (A.2.42), and hence:

$$\delta(1+2\delta)(2+\delta)^2\bar{p}^2 = 2\delta(2+\delta)(\delta + (2+\delta)\mu\theta)\bar{p} + (\delta + (2+\delta)\mu\theta).$$

Evaluating the quadratic $\mathcal{Q}(p)$ in (A.3.14) at $p = \bar{p}$ and using the equation above to deduce:

$$\mathcal{Q}(\bar{p}) = (1-\alpha)\left(2\delta(2+\delta)(\delta + (2+\delta)\mu\theta)\bar{p} + (\delta + (2+\delta)\mu\theta)\right) - (2+\delta)(2\delta^2 + (1+2\delta)\alpha)\bar{p} - \delta$$
$$= (1-\alpha)\mu\theta(2+\delta)(1+2\delta(2+\delta)\bar{p}) - \alpha(\delta + (2+\delta)(1+2\delta+2\delta^2)\bar{p}),$$

from which it follows that $\mathcal{Q}(\bar{p}) > 0$ is equivalent to:

$$\frac{\alpha}{1-\alpha} < \frac{\mu\theta(2+\delta)(1+2\delta(2+\delta)\bar{p})}{\delta + (2+\delta)(1+2\delta+2\delta^2)\bar{p}}.$$

This inequality can be stated as $\alpha < \bar{\alpha}$, where $\bar{\alpha}$ is given by:

$$\bar{\alpha} = \left(1 + \frac{\delta + (2+\delta)(1+2\delta+2\delta^2)\bar{p}}{\mu\theta(2+\delta)(1+2\delta(2+\delta)\bar{p})}\right)^{-1}, \qquad \text{[A.3.17]}$$

where this number lies strictly between 0 and 1. This verifies the claim that $\alpha < \bar{\alpha}$ is necessary and sufficient for an interior solution ($\hat{s} < 1$), and equations (A.3.16) and (A.3.17) confirm the expression for $\bar{\alpha}$ given in the proposition.

Note that the derivative (A.3.12) can never be zero at $s = 0$, so $\hat{s} > 0$, implying that $\hat{p} > p^\dagger$ ($p^\dagger$ is the value of p associated with $s = 0$). When $\alpha \geq \bar{\alpha}$, the solution is at the corner $\hat{s} = 1$. For $\alpha < \bar{\alpha}$, the value of $\hat{p}$ associated with the interior solution $\hat{s}$ must be the positive root of the quadratic equation $\mathcal{Q}(\hat{p}) = 0$, where $\mathcal{Q}(p)$ is as defined in (A.3.14). Solving the equation in the case $\delta \leq 1/2$ leads to:

$$\hat{p} = \frac{3\delta - 1 + 4(1-\delta)\alpha + \sqrt{(3\delta - 1 + 4(1-\delta)\alpha)^2 + 8(1-\delta)(\delta + (1-2\delta)\alpha)(1-\alpha)}}{4\delta(2+\delta)(1-\alpha)},$$

and in the $\delta \geq 1/2$ case:

$$\hat{p} = \frac{2\delta^2 + (1+2\delta)\alpha + \sqrt{(2\delta^2 + (1+2\delta)\alpha)^2 + 4\delta^2(1+2\delta)(1-\alpha)}}{2\delta(1+2\delta)(2+\delta)(1-\alpha)}.$$

These equations confirm the expression for $\hat{p}$ given in the proposition, completing the proof.

A.4 Proof of Proposition 4

The results of Proposition 2 show that for a given value of s, the equilibrium conditions for rules established at the pre-investment stage are met if power sharing p satisfies the equation $s = \sigma(p)$ for the function $\sigma(p)$ defined in Proposition 2. This is a strictly increasing function, so it has a well-defined inverse $p = \sigma^{-1}(s)$. Given s, for the value of p implied by this equation, the per-person incumbent share ϕ_p is expressed as a function of p in Proposition 2, denoted here by $\phi_\mathrm{p}(p)$. Optimality of the rules further requires that s maximizes the incumbent payoff $\mathcal{U}_\mathrm{p}$, subject only to $p = \sigma^{-1}(s)$, and with $\phi_\mathrm{p} = \phi_\mathrm{p}(\sigma^{-1}(s))$.

Proposition 2 also establishes that the equilibrium rules must feature domestic free exchange of goods and free international trade. This means that aggregate consumption C is as given in (3.10), with relative price $\tilde{\pi}$ equal to the world relative price π^*, which is taken as given by individual countries. The capital stock K is determined by s according to (2.2), and net exports are as given in (3.7). The incumbent payoff $\mathcal{U}_\mathrm{p}^*(s;\pi^*)$ taking account of all these features of the equilibrium rules for a particular value of s is given in equation (5.1).

The partial derivative of $\mathcal{U}_\mathrm{p}^*(s;\pi^*)$ with respect to s is:

$$\mathcal{U}_\mathrm{p}^{*\prime}(s;\pi^*) = \frac{\phi_\mathrm{p}'(\sigma^{-1}(s))}{\sigma'(\sigma^{-1}(s))\phi_\mathrm{p}(\sigma^{-1}(s))} + \frac{\mu\pi^*}{\mathrm{q} + \mu\pi^* s}. \quad \text{[A.4.1]}$$

For the first term in this expression, it is shown in the proof of Proposition 3 (equation (A.3.6)) that:

$$\frac{\phi_\mathrm{p}'(p)}{\phi_\mathrm{p}(p)} = -2\delta \min\left\{1, \frac{2}{1+2\delta}\right\}\phi_\mathrm{p}(p), \quad \text{[A.4.2]}$$

and this is also valid in the open-economy case since the functions $\phi_\mathrm{p}(p)$ and $\sigma(p)$ are the same. Substituting this expression into equation (A.4.1) shows that the first partial derivative of $\mathcal{U}_\mathrm{p}^*(s;\pi^*)$ with respect to s is:

$$\mathcal{U}_\mathrm{p}^{*\prime}(s;\pi^*) = \frac{\mu\pi^*}{\mathrm{q} + \mu\pi^* s} - 2\delta \min\left\{1, \frac{2}{1+2\delta}\right\}\frac{\phi_\mathrm{p}(\sigma^{-1}(s))}{\sigma'(\sigma^{-1}(s))}. \quad \text{[A.4.3]}$$

Differentiating (A.4.3) with respect to s again (holding the international relative price π^* constant) leads to:

$$\mathcal{U}_\mathrm{p}^{*\prime\prime}(s;\pi^*) = -\left(\frac{\mu\pi^*}{\mathrm{q} + \mu\pi^* s}\right)^2 - 2\delta \min\left\{1, \frac{2}{1+2\delta}\right\}\left(\frac{\phi_\mathrm{p}'(\sigma^{-1}(s))}{(\sigma'(\sigma^{-1}(s)))^2} - \frac{\phi_\mathrm{p}(\sigma^{-1}(s))\sigma''(\sigma^{-1}(s))}{(\sigma'(\sigma^{-1}(s)))^3}\right),$$

which can be written as follows using equation (A.4.2):

$$\mathcal{U}_\mathrm{p}^{*\prime\prime}(s;\pi^*) = 2\delta \min\left\{1, \frac{2}{1+2\delta}\right\}\frac{\sigma''(\sigma^{-1}(s))\phi_\mathrm{p}(\sigma^{-1}(s))}{(\sigma'(\sigma^{-1}(s)))^3} + \left(2\delta \min\left\{1, \frac{2}{1+2\delta}\right\}\frac{\phi_\mathrm{p}(\sigma^{-1}(s))}{\sigma'(\sigma^{-1}(s))}\right)^2 - \left(\frac{\mu\pi^*}{\mathrm{q} + \mu\pi^* s}\right)^2. \quad \text{[A.4.4]}$$

Using equation (A.4.3), if the second partial derivative in (A.4.4) is evaluated at a value of s where the first partial derivative $\mathcal{U}_\mathrm{p}^{*\prime}(s;\pi^*)$ is zero then:

$$\mathcal{U}_\mathrm{p}^{*\prime\prime}(s;\pi^*) = 2\delta \min\left\{1, \frac{2}{1+2\delta}\right\}\frac{\sigma''(\sigma^{-1}(s))\phi_\mathrm{p}(\sigma^{-1}(s))}{(\sigma'(\sigma^{-1}(s)))^3}, \quad \text{if } \mathcal{U}_\mathrm{p}^{*\prime\prime}(s;\pi^*) = 0.$$

Since $\phi_\mathrm{p} > 0$ and $\sigma'(p) > 0$ (according to Proposition 2), it is established that the sign of the second derivative $\mathcal{U}_\mathrm{p}^{*\prime\prime}(s;\pi^*)$ depends only on the sign of $\sigma''(\sigma^{-1}(s))$. Proposition 2 demonstrates the strictly convexity of $\sigma(p)$ for all parameter values, hence it follows that the second partial derivative of $\mathcal{U}_\mathrm{p}^*(s;\pi^*)$ is strictly negative whenever the first partial derivative is zero. Therefore, $\mathcal{U}_\mathrm{p}^*(s;\pi^*)$ is a strictly quasi-convex function of s, confirming the claim in the proposition.

Given strict quasi-convexity of $\mathcal{U}_{\mathrm{p}}^{*}(s;\pi^{*})$ in s, and since s is confined to the unit interval, it follows that either $s=0$ or $s=1$ must maximize $\mathcal{U}_{\mathrm{p}}^{*}(s;\pi^{*})$ (for a particular international relative price π^{*}, which is taken as given). Proposition 2 shows that the choice of $s=0$ is associated with power sharing $p=p^{\dagger}$ (despotism) in equilibrium, while $s=1$ is associated with power sharing $p=\bar{p}$ (the rule of law). The rule of law is (weakly) optimal if $\mathcal{U}_{\mathrm{p}}^{*}(1;\pi^{*})\geq\mathcal{U}_{\mathrm{p}}^{*}(1;\pi^{*})$, and equation (5.1) shows that this is equivalent to:

$$\log\phi_{\mathrm{p}}(\sigma^{-1}(1))+\log(\mathrm{q}+\mu\pi^{*})-\alpha\log\pi^{*}\geq\log\phi_{\mathrm{p}}(\sigma^{-1}(0))+\log\mathrm{q}-\alpha\log\pi^{*}.$$

Cancelling common terms and defining $\phi_{\mathrm{p}}^{\dagger}\equiv\phi_{\mathrm{p}}(p^{\dagger})=\phi_{\mathrm{p}}(\sigma^{-1}(0))$ and $\bar{\phi}_{\mathrm{p}}\equiv\phi_{\mathrm{p}}(\bar{p})=\phi_{\mathrm{p}}(\sigma^{-1}(1))$, the inequality above becomes:

$$\log\bar{\phi}_{\mathrm{p}}+\log(\mathrm{q}+\mu\pi^{*})\geq\log\phi_{\mathrm{p}}^{\dagger}+\log\mathrm{q},$$

and rearranging this condition shows that it is equivalent to:

$$\frac{\mathrm{q}+\mu\pi^{*}}{\mathrm{q}}\geq\frac{\phi_{\mathrm{p}}^{\dagger}}{\bar{\phi}_{\mathrm{p}}}.$$

This confirms the claim in the proposition.

Now suppose that given s, the world relative price π^{*} were equal to the closed-economy market-clearing price $\hat{\pi}$:

$$\hat{\pi}=\frac{\alpha\mathrm{q}}{(1-\alpha)\mu s}.$$

Evaluating the partial first derivative of $\mathcal{U}_{\mathrm{p}}^{*}(s;\pi^{*})$ (from (A.4.1)) at $\pi^{*}=\hat{\pi}$:

$$\mathcal{U}_{\mathrm{p}}^{*\prime}(s;\hat{\pi})=\frac{\phi_{\mathrm{p}}^{\prime}(\sigma^{-1}(s))}{\sigma^{\prime}(\sigma^{-1}(s))\phi_{\mathrm{p}}(\sigma^{-1}(s))}+\frac{\mu\left(\frac{\alpha\mathrm{q}}{(1-\alpha)\mu s}\right)}{\mathrm{q}+\mu\left(\frac{\alpha\mathrm{q}}{(1-\alpha)\mu s}\right)s}=\frac{\phi_{\mathrm{p}}^{\prime}(\sigma^{-1}(s))}{\sigma^{\prime}(\sigma^{-1}(s))\phi_{\mathrm{p}}(\sigma^{-1}(s))}+\frac{\alpha}{s}.$$

Comparison with the first derivative of $\hat{\mathcal{U}}_{\mathrm{p}}(s)$ for the closed-economy in the proof of Proposition 3 (equation (A.3.1)) establishes that $\mathcal{U}_{\mathrm{p}}^{*\prime}(s;\hat{\pi})=\hat{\mathcal{U}}_{\mathrm{p}}(s)$ for all s (with the price $\hat{\pi}$ varying with s). In particular, at the closed-economy equilibrium $\hat{s}$ and with a world price equal to the closed-economy equilibrium price $\hat{\pi}$, the partial first derivative of $\mathcal{U}_{\mathrm{p}}^{*}(\hat{s};\hat{\pi})$ with respect to s is equal to zero.

Now consider the partial derivative of the incumbent payoff $\mathcal{U}_{\mathrm{p}}^{*}(s;\pi^{*})$ from (5.1) with respect to the international relative price π^{*}:

$$\frac{\partial\mathcal{U}_{\mathrm{p}}^{*}(s;\pi^{*})}{\partial\pi^{*}}=\frac{\mu s}{\mathrm{q}+\mu\pi^{*}s}-\frac{\alpha}{\pi^{*}}=\frac{(1-\alpha)\mu\pi^{*}s-\alpha\mathrm{q}}{\pi^{*}(\mathrm{q}+\mu\pi^{*}s)}. \quad [\text{A.4.5}]$$

Evaluating this at $s=0$ leads to:

$$\frac{\partial\mathcal{U}_{\mathrm{p}}^{*}(s;\pi^{*})}{\partial\pi^{*}}=-\frac{\alpha}{\pi^{*}},$$

which is strictly negative. Evaluating (A.4.5) at $s=1$:

$$\frac{\partial\mathcal{U}_{\mathrm{p}}^{*}(s;\pi^{*})}{\partial\pi^{*}}=\frac{(1-\alpha)\mu\pi^{*}-\alpha\mathrm{q}}{\pi^{*}(\mathrm{q}+\mu\pi^{*})}=\frac{(1-\alpha)\mu}{\pi^{*}(\mathrm{q}+\mu\pi^{*})}\left(\pi^{*}-\frac{\alpha\mathrm{q}}{(1-\alpha)\mu}\right),$$

which is strictly positive if $\pi^{*}>\alpha\mathrm{q}/(1-\alpha)\mu$. Therefore, $\mathcal{U}_{\mathrm{p}}^{*}(0;\pi^{*})$ is strictly decreasing in π^{*}, and $\mathcal{U}_{\mathrm{p}}^{*}(1;\pi^{*})$ is strictly increasing in π^{*} (for π^{*} values above the threshold). This confirms the claim in the proposition and completes the proof.

A.5 Proof of Proposition 5

Proposition 4 shows that the equilibrium rules in a country that has access to international markets must specify either $s = 0$ (despotism, with $p = p^\dagger$) or $s = 1$ (the rule of law, with $p = \bar{p}$). Given an international relative price π^*, the rule of law is weakly optimal if and only if:

$$\frac{\mathrm{q} + \mu\pi^*}{\mathrm{q}} \geq \frac{\phi_{\mathrm{p}}^\dagger}{\bar{\phi}_{\mathrm{p}}},$$

where $\phi_{\mathrm{p}}^\dagger \equiv \phi_{\mathrm{p}}(p^\dagger)$ and $\bar{\phi}_{\mathrm{p}} \equiv \phi_{\mathrm{p}}(\bar{p})$ are respectively the per-person incumbent shares under despotism and the rule of law. The inequality above is equivalent to:

$$\frac{\mu\pi^*}{\mathrm{q}} \geq \frac{\phi_{\mathrm{p}}^\dagger - \bar{\phi}_{\mathrm{p}}}{\bar{\phi}_{\mathrm{p}}}. \qquad \text{[A.5.1]}$$

An expression for the right-hand side of (A.5.1) can be obtained from the formula for the function $\phi_{\mathrm{p}}(p)$ given in Proposition 2 and $\phi_{\mathrm{p}}^\dagger = \phi_{\mathrm{p}}^\ddagger = (2+\delta)/2$ (since $p^\dagger = p^\ddagger$). For the case of $\delta \leq 1/2$:

$$\frac{\phi_{\mathrm{p}}^\dagger - \bar{\phi}_{\mathrm{p}}}{\bar{\phi}_{\mathrm{p}}} = \frac{\frac{2+\delta}{2}}{\frac{1}{2(\delta\bar{p}+(1-\delta)p^\ddagger)}} - 1 = (2+\delta)(\delta\bar{p} + (1-\delta)p^\ddagger) - (2+\delta)p^\ddagger = \delta(2+\delta)(\bar{p} - p^\ddagger), \qquad \text{[A.5.2a]}$$

which uses $p^\ddagger = 1/(2+\delta)$, and in the case of $\delta \geq 1/2$:

$$\frac{\phi_{\mathrm{p}}^\dagger - \bar{\phi}_{\mathrm{p}}}{\bar{\phi}_{\mathrm{p}}} = \frac{\frac{2+\delta}{2}}{\frac{1+2\delta}{2(2\delta\bar{p}+p^\ddagger)}} - 1 = \frac{(2+\delta)(2\delta\bar{p} + p^\ddagger) - (1+2\delta)(2+\delta)p^\ddagger}{1+2\delta} = \frac{2\delta(2+\delta)}{1+2\delta}(\bar{p} - p^\ddagger). \qquad \text{[A.5.2b]}$$

Since $\min\{1, 2/(1+2\delta)\}$ equals 1 when $\delta \leq 1/2$ and $2/(1+2\delta)$ when $\delta \geq 1/2$, it follows from (A.5.2a) and (A.5.2b) that a general formula for $(\phi_{\mathrm{p}}^\dagger - \bar{\phi}_{\mathrm{p}})/\bar{\phi}_{\mathrm{p}}$ valid for all parameter values is:

$$\frac{\phi_{\mathrm{p}}^\dagger - \bar{\phi}_{\mathrm{p}}}{\bar{\phi}_{\mathrm{p}}} = \delta(2+\delta)\min\left\{1, \frac{2}{1+2\delta}\right\}(\bar{p} - p^\ddagger). \qquad \text{[A.5.3]}$$

The criterion (A.5.1) for $s = 1$ being weakly optimal is therefore:

$$\frac{\mu\pi^*}{\mathrm{q}} \geq \delta(2+\delta)\min\left\{1, \frac{2}{1+2\delta}\right\}(\bar{p} - p^\ddagger). \qquad \text{[A.5.4]}$$

If the inequality above is reversed, the equilibrium rules in a country must feature $s = 0$. If the inequality is strict, the rules must feature $s = 1$. Finally, if the above holds as an equality, then either $s = 0$ or $s = 1$ are consistent with optimality.

Since the equilibrium rules in each country must feature $s = 0$ or $s = 1$, let ω denote the measure of countries with the rule of law ($s = 1$), and given a unit continuum of countries, $1 - \omega$ is the measure of countries with despotism ($s = 0$). Despotic countries produce none of the investment good ($K^\dagger = 0$), while countries with the rule of law produce the maximum feasible amount ($\bar{K} = \mu$). The world supply of the investment good is $K^* = (1-\omega)K^\dagger + \omega\bar{K} = \mu\omega$. All countries have the same endowment q, so the world supply of the endowment good is q. Therefore, given the fraction ω of rule-of-law countries, the equilibrium world price $\tilde{\pi}^*$ of the investment good is:

$$\tilde{\pi}^* = \frac{\alpha \mathrm{q}}{(1-\alpha)\mu\omega}, \quad \text{and hence} \quad \frac{\mu\tilde{\pi}^*}{\mathrm{q}} = \frac{\alpha}{(1-\alpha)\omega}. \qquad \text{[A.5.5]}$$

It follows using (A.5.4) that given the fraction ω of rule-of-law countries, the rule of law ($s = 1$) is weakly optimal in an individual country if and only if:

$$\frac{\alpha}{(1-\alpha)\omega} \geq \delta(2+\delta)\min\left\{1, \frac{2}{1+2\delta}\right\}(\bar{p}-p^{\ddagger}). \quad \text{[A.5.6]}$$

The variables ω must satisfy $0 \leq \omega \leq 1$. Consider the possibility of a world equilibrium $\tilde{\omega}$ such that $0 < \tilde{\omega} < 1$. In this case, some countries would be despotic ($s = 0$), and others would have the rule of law ($s = 1$). Since this case requires that both $s = 0$ and $s = 1$ are optimal in any individual country, (A.5.6) must hold with equality:

$$\frac{\alpha}{(1-\alpha)\tilde{\omega}} = \delta(2+\delta)\min\left\{1, \frac{2}{1+2\delta}\right\}(\bar{p}-p^{\ddagger}). \quad \text{[A.5.7]}$$

With $p^{\ddagger}$ and $\bar{p}$ being independent of ω (Proposition 2 gives expressions for these variables solely in terms of parameters), this equation can be easily solved for a unique value of $\tilde{\omega}$:

$$\tilde{\omega} = \frac{\alpha}{(1-\alpha)\delta(2+\delta)\min\left\{1, \frac{2}{1+2\delta}\right\}(\bar{p}-p^{\ddagger})}. \quad \text{[A.5.8]}$$

The right-hand side is strictly positive (since $\bar{p} > p^{\ddagger}$), so $\tilde{\omega} > 0$ as required. It will also be confirmed below that $\tilde{\omega} < 1$.

Let p^* denote the fraction of the world population holding positions of power in some country. Note that with $p^{\dagger} = p^{\ddagger}$ (Proposition 2), equation (A.5.8) implies:

$$\tilde{\omega}(\bar{p}-p^{\dagger}) = \frac{\alpha}{(1-\alpha)\delta(2+\delta)\min\left\{1, \frac{2}{1+2\delta}\right\}},$$

and since $p^* = p^{\dagger} + \tilde{\omega}(\bar{p}-p^{\dagger})$ at $\omega = \tilde{\omega}$, the world equilibrium features the following value of p^*:

$$p^* = p^{\dagger} + \frac{\alpha}{(1-\alpha)\delta(2+\delta)\min\left\{1, \frac{2}{1+2\delta}\right\}} = \frac{1}{2+\delta}\left(1 + \frac{\alpha}{(1-\alpha)\delta\min\left\{1, \frac{2}{1+2\delta}\right\}}\right), \quad \text{[A.5.9]}$$

which uses $p^{\dagger} = 1/(2+\delta)$. With $\tilde{\omega} > 0$, p^* must be larger than $p^{\dagger}$. Under the parameter restriction $\alpha < \bar{\alpha}$, it is shown in Proposition 3 that closed-economy equilibrium power sharing $\hat{p}$ is such that $\hat{p} < \bar{p}$, and that $\hat{p}$ is the root of the quadratic equation $\mathcal{Q}(\hat{p}) = 0$ given in (A.3.14). Note that the formula for p^* from (A.5.9) can be broken down into two cases for $\delta \leq 1/2$ and $\delta \geq 1/2$:

$$p^* = \begin{cases} \frac{1}{\delta(2+\delta)}\left(\delta + \frac{\alpha}{1-\alpha}\right) & \text{if } \delta \leq 1/2 \\ \frac{1}{2+\delta}\left(1 + \frac{(1+2\delta)\alpha}{2\delta(1-\alpha)}\right) & \text{if } \delta \geq 1/2 \end{cases}. \quad \text{[A.5.10]}$$

First consider the case where $\delta \leq 1/2$. Using the formula for the quadratic $\mathcal{Q}(p)$ in (A.3.14) for this case together with (A.5.10):

$$\begin{aligned}
\mathcal{Q}(p^*) &= 2(1-\alpha)\left(\delta+\frac{\alpha}{1-\alpha}\right)^2-(3\delta-1+4(1-\delta)\alpha)\left(\delta+\frac{\alpha}{1-\alpha}\right)-(1-\delta)(\delta+(1-2\delta)\alpha)\\
&= 2\delta^2-2\delta^2\alpha+4\delta\alpha+2\frac{\alpha^2}{1-\alpha}-(3\delta-1)\delta-4\delta(1-\delta)\alpha-(3\delta-1+4(1-\delta)\alpha)\frac{\alpha}{1-\alpha}-\delta(1-\delta)-(1-\delta)(1-2\delta)\alpha\\
&= -\alpha\left(2\delta^2-4\delta-\frac{2\alpha}{1-\alpha}+4\delta(1-\delta)+\frac{3\delta-1}{1-\alpha}+\frac{4(1-\delta)\alpha}{1-\alpha}+(1-\delta)(1-2\delta)\right)\\
&= -\alpha\left(1+\frac{(2-4\delta)\alpha}{1-\alpha}-3\delta+\frac{3\delta-1}{1-\alpha}\right)=-\alpha\left((3\delta-1)\frac{\alpha}{1-\alpha}+(2-4\delta)\frac{\alpha}{1-\alpha}\right)=-(1-\delta)\frac{\alpha^2}{1-\alpha}.
\end{aligned} \quad \text{[A.5.11a]}$$

Now consider the case where $\delta \geq 1/2$. Again, evaluating the quadratic $\mathcal{Q}(p)$ in (A.3.14) at p^* from (A.5.10):

$$\begin{aligned}
\mathcal{Q}(p^*) &= \delta(1+2\delta)(1-\alpha)\left(1+\frac{(1+2\delta)\alpha}{2\delta(1-\alpha)}\right)^2-(2\delta^2+(1+2\delta)\alpha)\left(1+\frac{(1+2\delta)\alpha}{2\delta(1-\alpha)}\right)-\delta\\
&= \delta(1+2\delta)-\delta(1+2\delta)\alpha+(1+2\delta)^2\alpha+\frac{(1+2\delta)^3\alpha^2}{4\delta(1-\alpha)}-2\delta^2-(1+2\delta)\alpha-\frac{\delta(1+2\delta)\alpha}{1-\alpha}-\frac{(1+2\delta)^2\alpha^2}{2\delta(1-\alpha)}-\delta\\
&= -(1+2\delta)\alpha\left(1+\delta-(1+2\delta)+\frac{\delta}{1-\alpha}+\frac{(1+2\delta)(2-(1+2\delta))\alpha}{4\delta(1-\alpha)}\right)\\
&= -(1+2\delta)\alpha\left(\frac{\delta}{1-\alpha}-\delta+\frac{(1+2\delta)(1-2\delta)\alpha}{4\delta(1-\alpha)}\right)=-\frac{(1+2\delta)\alpha}{4\delta}\left(\frac{4\delta^2\alpha}{1-\alpha}+\frac{(1+2\delta-2\delta-4\delta^2)\alpha}{1-\alpha}\right)\\
&= -\frac{(1+2\delta)\alpha^2}{4\delta(1-\alpha)}.
\end{aligned} \quad \text{[A.5.11b]}$$

Observe from (A.5.11a) and (A.5.11b) that $\mathcal{Q}(p^*)<0$ in both cases. It is shown in the proof of Proposition 3 that $\mathcal{Q}(p)$ is strictly negative for values of p between $p^\dagger$ and $\hat{p}$, and strictly positive for values of p above $\hat{p}$. With $p^*>p^\dagger$ and $\mathcal{Q}(p^*)<0$, it follows that $p^\dagger<p^*<\hat{p}$. Note that $p^*=(1-\tilde{\omega})p^\dagger+\tilde{\omega}\bar{p}$ can be written as:

$$1-\tilde{\omega}=\frac{\bar{p}-p^*}{\bar{p}-p^\dagger}.$$

Since $p^\dagger<p^*<\hat{p}$ and $\hat{p}<\bar{p}$ (under the parameter restriction $\alpha<\bar{\alpha}$), it follows that $p^*<\bar{p}$ and hence $\tilde{\omega}<1$. This confirms that the value of $\tilde{\omega}$ in (A.5.8) satisfies $0<\tilde{\omega}<1$, and it is therefore a world equilibrium. Given the uniqueness of the solution of equation (A.5.7), it is also the only equilibrium with $0<\tilde{\omega}<1$.

Now consider the possibility of equilibria with $\omega=0$ or ω. Note that the left-hand side of the condition (A.5.6) for the weak optimality of $s=1$ is strictly decreasing in ω, while the right-hand side is independent of ω. Since (A.5.6) holds with equality at $\omega=\tilde{\omega}$, it follows that the inequality (A.5.6) is a strict inequality at $\omega=0$, and is violated at $\omega=1$. This means that $s=0$ is not optimal for rules in an individual country when $\omega=0$ (all countries have $s=0$), and $s=1$ is not optimal when $\omega=1$ (all countries have $s=1$). Hence, neither $\omega=0$ nor $\omega=1$ can be a world equilibrium, so the equilibrium $\tilde{\omega}$ in (A.5.8) is unique. This confirms the expression for $\tilde{\omega}$ given in the proposition, and equation (A.5.9) confirms the formula for p^* given in the proposition. It has also been shown that the equilibrium features $p^*<\hat{p}$, confirming the claim in the proposition.

The equilibrium price in the world of open economies is:

$$\tilde{\pi}^*=\frac{\alpha q}{(1-\alpha)\mu\tilde{\omega}}>\frac{\alpha q}{(1-\alpha)\mu},$$

where the lower bound on the price is obtained because $\tilde{\omega}<1$. In autarky, every country would have the same domestic equilibrium price:

$$\hat{\pi} = \frac{\alpha q}{(1-\alpha)\mu \hat{s}} > \frac{\alpha q}{(1-\alpha)\mu},$$

where the lower bound on the price is obtained because $\hat{s} < 1$ (Proposition 3, under the parameter restriction $\alpha < \bar{\alpha}$). Given the lower bound on both $\tilde{\pi}^*$ and $\hat{\pi}$, it follows from the results of Proposition 4 that $\mathcal{U}^*_{\mathrm{p}}(0;\pi^*)$ is decreasing in π^*, and $\mathcal{U}^*_{\mathrm{p}}(1;\pi^*)$ is increasing in π^*, for any π^* in the range between $\tilde{\pi}^*$ and $\hat{\pi}$. Note that in the world equilibrium, since $0 < \tilde{\omega} < 1$, (A.5.6) must hold with equality, so the incumbent payoffs $\mathcal{U}^*_{\mathrm{p}}(0;\tilde{\pi}^*)$ and $\mathcal{U}^*_{\mathrm{p}}(1;\tilde{\pi}^*)$ must be equal. Take the case where $\tilde{\pi}^* \leq \hat{\pi}$. It follows that:

$$\mathcal{U}^*_{\mathrm{p}}(0;\hat{\pi}) \leq \mathcal{U}^*_{\mathrm{p}}(0;\tilde{\pi}^*) = \mathcal{U}^*_{\mathrm{p}}(1;\tilde{\pi}^*) \leq \mathcal{U}^*_{\mathrm{p}}(1;\hat{\pi}). \tag{A.5.12a}$$

Similarly, in the case where $\tilde{\pi}^* \geq \hat{\pi}$:

$$\mathcal{U}^*_{\mathrm{p}}(1;\hat{\pi}) \leq \mathcal{U}^*_{\mathrm{p}}(1;\tilde{\pi}^*) = \mathcal{U}^*_{\mathrm{p}}(0;\tilde{\pi}^*) \leq \mathcal{U}^*_{\mathrm{p}}(0;\hat{\pi}). \tag{A.5.12b}$$

Together, equations (A.5.12a) and (A.5.12a) imply for any possible values of $\tilde{\pi}^*$ and $\hat{\pi}$:

$$\mathcal{U}^*_{\mathrm{p}}(0;\tilde{\pi}^*) = \mathcal{U}^*_{\mathrm{p}}(1;\tilde{\pi}^*) \geq \min\left\{\mathcal{U}^*_{\mathrm{p}}(0;\hat{\pi}), \mathcal{U}^*_{\mathrm{p}}(1;\hat{\pi})\right\}. \tag{A.5.13}$$

Proposition 4 has demonstrated that $\mathcal{U}^*_{\mathrm{p}}(s;\hat{\pi})$ is a strictly quasi-convex function of s. Since $0 < s < 1$, it follows immediately that:

$$\min\left\{\mathcal{U}^*_{\mathrm{p}}(0;\hat{\pi}), \mathcal{U}^*_{\mathrm{p}}(1;\hat{\pi})\right\} > \mathcal{U}^*_{\mathrm{p}}(\hat{s};\hat{\pi}). \tag{A.5.14}$$

Now note that when $s = \hat{s}$, if the world price were equal to $\hat{\pi}$, the payoff to incumbents would be the same as when $s = \hat{s}$ in the case of autarky:

$$\mathcal{U}^*_{\mathrm{p}}(\hat{s};\hat{\pi}) = \hat{\mathcal{U}}_{\mathrm{p}}(\hat{s}).$$

Hence, using this equation and chaining together the inequalities in (A.5.13) and (A.5.14), it follows that:

$$\mathcal{U}^*_{\mathrm{p}}(0;\tilde{\pi}^*) = \mathcal{U}^*_{\mathrm{p}}(1;\tilde{\pi}^*) > \hat{\mathcal{U}}_{\mathrm{p}}(\hat{s}), \tag{A.5.15}$$

which confirms the claim $\mathcal{U}^\dagger_{\mathrm{p}} = \bar{\mathcal{U}}_{\mathrm{p}} > \hat{\mathcal{U}}_{\mathrm{p}}$ in the proposition.

Finally, note that given the definition of incumbent rents $\chi \equiv (\phi_{\mathrm{p}} - \phi_{\mathrm{w}})/\phi_{\mathrm{w}}$, it follows that the incumbent and worker payoffs are related according to:

$$\mathcal{U}_{\mathrm{w}} = \mathcal{U}_{\mathrm{p}} - \log(1+\chi). \tag{A.5.16}$$

Comparing workers in rule-of-law and despotic economies (both open economies):

$$\bar{\mathcal{U}}_{\mathrm{w}} - \mathcal{U}^\dagger_{\mathrm{w}} = \left(\bar{\mathcal{U}}_{\mathrm{p}} - \log(1+\bar{\chi})\right) - \left(\mathcal{U}^\dagger_{\mathrm{p}} - \log(1+\chi^\dagger)\right) = (\bar{\mathcal{U}}_{\mathrm{p}} - \mathcal{U}^\dagger_{\mathrm{p}}) + \log\left(\frac{1+\chi^\dagger}{1+\bar{\chi}}\right) = \log\left(\frac{1+\chi^\dagger}{1+\bar{\chi}}\right),$$

where the first equality follows from (A.5.16) and the second uses the result $\bar{\mathcal{U}}_{\mathrm{p}} = \mathcal{U}^\dagger_{\mathrm{p}}$. Since $\bar{p} > p^\dagger$ and as Proposition 2 shows that rents χ are strictly decreasing in p, it follows that $\chi^\dagger > \bar{\chi}$, and hence $\bar{\mathcal{U}}_{\mathrm{w}} > \mathcal{U}^\dagger_{\mathrm{w}}$, confirming the claim in the proposition. Now compare workers in rule-of-law open economies to workers in autarkic economies:

$$\bar{\mathcal{U}}_{\mathrm{w}} - \hat{\mathcal{U}}_{\mathrm{w}} = \left(\bar{\mathcal{U}}_{\mathrm{p}} - \log(1+\bar{\chi})\right) - \left(\hat{\mathcal{U}}_{\mathrm{p}} - \log(1+\hat{\chi})\right) = (\bar{\mathcal{U}}_{\mathrm{p}} - \hat{\mathcal{U}}_{\mathrm{p}}) + \log\left(\frac{1+\hat{\chi}}{1+\bar{\chi}}\right),$$

which uses equation (A.5.16). Since $\bar{p} > \hat{p}$, it follows that $\hat{\chi} > \bar{\chi}$. Together with the earlier result that $\bar{\mathcal{U}}_{\mathrm{p}} > \hat{\mathcal{U}}_{\mathrm{p}}$, the equation above establishes that $\bar{\mathcal{U}}_{\mathrm{w}} > \hat{\mathcal{U}}_{\mathrm{w}}$, confirming the claim in the proposition. This completes the proof.

A.6 Proof of Proposition 6

Assume there is a distribution of endowments q across countries. In closed economies, using the proof of Proposition 3, it follows immediately that neither power sharing $\hat{p}$ nor the reach of the rule of law $\hat{s}$ depend on a country's particular value of q.

In the case of open economies, the results in Proposition 4 continue to apply given a world relative price π^*. This means that $\mathcal{U}_{\mathrm{p}}^*(s;\pi^*)$ is a strictly quasi-convex function of s, so the equilibrium rules must feature either $s=0$ or $s=1$. The criterion for $s=1$ to be weakly optimal is:

$$\frac{\mathrm{q}+\mu\pi^*}{\mathrm{q}} \geq \frac{\phi_{\mathrm{p}}^{\dagger}}{\bar{\phi}_{\mathrm{p}}}.$$

Rearranging shows that this inequality is equivalent to:

$$\mathrm{q} \leq \mu\pi^* \left(\frac{\bar{\phi}_{\mathrm{p}}}{\phi_{\mathrm{p}}^{\dagger} - \bar{\phi}_{\mathrm{p}}} \right),$$

and stating this in terms of the relative endowment $y \equiv \mathrm{q}/\mathrm{q}^*$ (where q^* is the average world endowment):

$$y \leq \frac{\mu\pi^*}{\mathrm{q}^*} \left(\frac{\bar{\phi}_{\mathrm{p}}}{\phi_{\mathrm{p}}^{\dagger} - \bar{\phi}_{\mathrm{p}}} \right). \quad \text{[A.6.1]}$$

Following the same steps as in the proof of Proposition 5 (equation (A.5.3)), it can be shown that:

$$\frac{\phi_{\mathrm{p}}^{\dagger} - \bar{\phi}_{\mathrm{p}}}{\bar{\phi}_{\mathrm{p}}} = \delta(2+\delta) \min\left\{1, \frac{2}{1+2\delta}\right\} (\bar{p} - p^{\ddagger}).$$

Substituting this into (A.6.1) shows that the condition for $s=1$ being weakly optimal is:

$$y \leq \frac{\mu\pi^*}{\mathrm{q}^*} \left(\frac{1}{\delta(2+\delta) \min\left\{1, \frac{2}{1+2\delta}\right\} (\bar{p} - p^{\ddagger})} \right), \quad \text{[A.6.2]}$$

which confirms the claim in the proposition.

Since either $s=0$ or $s=1$ must be optimal, let ω denote the fraction of countries with $s=1$. This means the world supply of the investment good is $\mu\omega$. The world supply of the endowment good is equal to the country average q^*. Given the fraction ω, it follows that the equilibrium world price is:

$$\tilde{\pi}^* = \frac{\alpha \mathrm{q}^*}{(1-\alpha)\mu\omega}, \quad \text{and hence} \quad \frac{\mu\tilde{\pi}^*}{\mathrm{q}^*} = \frac{\alpha}{(1-\alpha)\omega}.$$

Using (A.6.2), the criterion for $s=1$ to be weakly optimal given ω is:

$$y \leq \frac{1}{\omega} \left(\frac{\alpha}{(1-\alpha)\delta(2+\delta) \min\left\{1, \frac{2}{1+2\delta}\right\} (\bar{p} - p^{\ddagger})} \right). \quad \text{[A.6.3]}$$

If $G(y)$ is the cumulative distribution function of y, the requirement above for $s=1$ to be weakly optimal implies that for $\tilde{\omega}$ to be a world equilibrium, the following equation must hold:

$$G^{-1}(\tilde{\omega}) = \frac{1}{\tilde{\omega}} \left(\frac{\alpha}{(1-\alpha)\delta(2+\delta) \min\left\{1, \frac{2}{1+2\delta}\right\} (\bar{p} - p^{\ddagger})} \right),$$

where $G^{-1}(\omega)$ is the inverse of the cumulative distribution function (which is a well-defined function). Since $G^{-1}(0)$ must be finite, it is not possible to have a solution with $\tilde{\omega}=0$, therefore an equivalent equation can be obtained by multiplying both sides by $\tilde{\omega}$:

$$\tilde{\omega}G^{-1}(\tilde{\omega}) = \frac{\alpha}{(1-\alpha)\delta(2+\delta)\min\left\{1, \frac{2}{1+2\delta}\right\}(\bar{p}-p^{\ddagger})}. \qquad \text{[A.6.4]}$$

This confirms the equation given in the proposition.

Define the function $\mathcal{H}(\omega)$ as follows:

$$\mathcal{H}(\omega) \equiv \omega G^{-1}(\omega). \qquad \text{[A.6.5]}$$

Since $G^{-1}(\omega)$ is weakly increasing, $\mathcal{H}(\omega)$ is seen to be strictly increasing. Note that $G^{-1}(0)$ is finite, so $\mathcal{H}(0) = 0$. With y having a mean of one by construction, it must be the case that $G^{-1}(1) \geq 1$, and thus $\mathcal{H}(1) \geq 1$. Now define $\omega^{\#}$ as follows:

$$\omega^{\#} \equiv \frac{\alpha}{(1-\alpha)\delta(2+\delta)\min\left\{1, \frac{2}{1+2\delta}\right\}(\bar{p}-p^{\ddagger})}, \qquad \text{[A.6.6]}$$

observing that this is the same as the equilibrium value of $\tilde{\omega}$ in the world equilibrium with homogeneous endowments (Proposition 5). It is shown in Proposition 5 that $0 < \omega^{\#} < 1$ under the parameter restriction $\alpha < \bar{\alpha}$. Using (A.6.5) and (A.6.6), equation (A.6.4) for the equilibrium value of $\tilde{\omega}$ can be stated as follows:

$$\mathcal{H}(\tilde{\omega}) = \omega^{\#}.$$

The feasible range of $\tilde{\omega}$ is between zero and one. At $\tilde{\omega} = 0$, the left-hand side is below the right-hand side ($\mathcal{H}(0) = 0 < \omega^{\#}$), while at $\tilde{\omega} = 1$, this is reversed ($\mathcal{H}(1) \geq 1 > \omega^{\#}$). The left-hand side is strictly increasing in $\tilde{\omega}$, while the right-hand side is independent of $\tilde{\omega}$. Therefore, there exists a unique solution $\tilde{\omega}$. This solution satisfies $0 < \tilde{\omega} < 1$, confirming the claim in the proposition and completing the proof.