Two-sided Search in International Markets

(preliminary and incomplete)

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1 Introduction

Every new exporter must find a first client abroad. Likewise, a new importer must find a foreign supplier. This process can be costly and lengthy. We spoke with one American toy importer who spent more than a year working to find his first acceptable supplier in China. Costly search for new business partners in a foreign country is a friction in international trade which has only recently begun to be measured. As trade researchers, we would like to understand how these international relationships are formed, how large the information frictions are, and what governments might do to mitigate them. In this paper we address these issues by developing a dynamic two-sided search model with many-to-many matching between foreign exporters and domestic importers. After calibrating our model, we use it to address policy questions, including the effects of Chinese import penetration on matching patterns and the role of search and matching frictions in determining equilibria.

We motivate our analysis with a number of stylized facts. As documented in Eaton et al. (2014), international buyer-seller relationships are frequently created and destroyed. A second fact is that the distribution of foreign exporters across Colombian footwear importers is close to Pareto, as is the distribution of Colombian importers across foreign exporters of footwear to Colombia. There is also a strong downward drift in the transitions in client numbers, particularly among small firms. Colombian footwear importers with up to ten client abroad are more likely to lose clients than grow or stay the same size. Finally, the Colombian economy is tightly connected through inter-firm relationships. Think of the graph in which each node is a Colombian importer, and two nodes are connected if they buy from at least one common foreign supplier. Almost 90% of the 2012 value of Colombian footwear imports

is through the largest connected component of this graph, and more than half of Colombian footwear importers are part of the largest component.

The model we develop to capture these facts presumes that buyers and sellers actively search for each other. Taking stock of their current situation and the structure of the buyer-seller network, heterogeneous firms on each side of the market choose the hazard rates with which they meet potential business partners. When they encounter one another, they form matches that endure until they are destroyed by exogenous events. Search is subject to increasing returns, so firms with more clients find it less expensive to search. And market-wide forces help determine the return to search effort through an aggregate matching function.

This paper broadly contributes to the literature on information frictions and international trade, a literature pioneered by James Rauch and his coauthors in a series of papers a decade ago (Rauch, 2001; Rauch and Trindade, 2002; Rauch and Watson, 2003). In these studies, firms face uncertainty about the appeal of their products to foreign buyers (Rauch and Watson, 2003; Eaton *et al.*, 2014) or about the demand conditions in a location at a particular time (Jensen, 2007; Allen, 2014). Our analysis focuses on the latter—the difficulty firms have finding customers in a foreign country.

Models of search, matching, and learning have been used by trade researchers to understand both the frequent exit of new exporters and the subsequent growth of survivors (Albornoz *et al.*, 2012; Rauch and Watson, 2003; Drozd and Nosal, 2012; Li, 2013; Eaton *et al.*, 2014). In these models a firm searches for clients abroad, and updates its beliefs about market conditions after each meeting. One important policy implication of these studies is the slow reaction of economies to shocks, as the number and type of clients a firm has do not

immediately adjust when it changes its search behavior.

The models of search in the trade literature have, however, been one-sided in the sense that only either exporters or importers search and the other side of the market is passive. One of our contributions to this literature is to introduce a tractable, dynamic model of two-sided search. Two-sided search is a standard feature of search models in the labor literature, and we believe it is an important part of reality that buyers and sellers both exert effort to find clients. Moreover, a model with both sides searching is capable of generating richer exporter-importer network structures than would be possible with a one-sided search model.

By developing a model of the dynamic structure of the network of importers and exporters, we also contribute to the empirical literature on network formation. Partially due to the difficulty in modeling an individual's expectations over the future structure of an evolving network, much of the empirical network literature has employed models with statistical rules governing how agents meet and how links are created (Jackson and Rogers, 2007; Chaney, 2014; Atalay *et al.*, 2011). When agents in this literature behave optimally, they have often been modeled as myopic (Jackson and Watts, 2002; Watts, 2003; Christakis *et al.*, 2010). A notable exception is Mele (2010) in which strategic, forward-looking agents decide whether to accept or reject potential connections which arrive exogenously. We add to the network formation literature by developing a tractable two-sided search model in which forward-looking agents decide how hard they search for new connections based on the current network structure. By meeting additional business partners, firms increase their visibility

We also draw upon and add to several other network literatures. Since a collection of buyers and sellers form a naturally bipartite network, we use summary statistics developed

in the literature analyzing and measuring bipartite networks (Kranton and Minehart, 2003; Latapy *et al.*, 2008). We are also interested in the way that particular network structures affect the macroeconomic adjustment to shocks. There has been rapid and recent progress in this area, including theory papers (Gabaix, 2011) and empirical papers which take the links between firms as exogenous (Acemoglu *et al.*, 2012). We add to this literature by making the formation of network links between firms endogenous.

Finally, as we have a dynamic model of firm behavior, we contribute to the literature on the life-cycle of exporters and importers. As with the earlier literature on firm dynamics, our model is motivated in part by the presence of fat-tails in the data. In the firm dynamics literature, one way to generate a stationary distribution of firm size with fat tails is through stochastic shocks to firm productivity and demand coupled with either the possibility for entrants to imitate incumbents (Luttmer, 2007) or an exogenous drop in the productivity of old firms (Luttmer, 2011). Another possibility for generating fat tails is to use a matching model and a convenient search cost function (Klette and Kortum, 2004). We follow the latter modeling strategy. In particular, we assume that the cost for a firm to find an additional client decreases as the number of its incumbent clients increases.

2 Data and Stylized Facts

Part of the motivation for this paper is the set of stylized facts that has recently emerged concerning international links between buyers and sellers. Studies reporting such facts are now available for the United States and Colombia (Eaton *et al.*, 2008, 2014), Chile (Blum *et*

al., 2010), Mexico (Sugita *et al.*, 2014), and Norway (Bernard *et al.*, 2014).¹ Our model is additionally motivated by the data on the dynamics of buyer-seller relationships.

Below we present these facts for the population of Colombian firms that import footwear and their suppliers abroad. This choice of network reflects several considerations. First, by studying goods that are mainly supplied by foreign producers, we minimize the importance of domestic suppliers, whose connections we are unable to observe. Second, by choosing a sector in which most of the importers are wholesale/retail firms, we are able to keep the buyer side of the market relatively simple. That is, within each wholesale or retail firm, revenue functions are nearly separable across categories of consumer goods. This means that substitution across inputs essentially takes place within product categories, and firms' payoff functions can be reasonably approximated with relatively simple expressions.

2.1 Data Sources

We base our analysis on data obtained from the Colombian customs authority: Dirección de Impuestos y Aduanas Nacionales de Colombia (DIAN). These data describe all merchandise shipments to Colombia. Each record includes a ten-digit Harmonized Schedule (HS) product code, shipment value, shipment quantity, entry or exit port, date of transaction, mode of transportation (land, sea, air), and the domestic firm's tax identification number (NIT). Critically for our study, each record also includes the name and address of the foreign firm that is party to the transaction.

¹Sugita *et al.* (2014) go beyond documentation and develop a static matching model motivated by patterns in Mexican and Chinese exports to the U.S.; Bernard *et al.* (2014) perform a related exercise using Norwegian data.

In order to keep track of foreign suppliers, we construct an alphanumeric foreign exporter ID for each shipment in the database. These are based on the business names and addresses that appear in the customs records. For example, one version of this ID combines the firm's country code, first three letters of the first two main words in the firm's name, the street number, and the first three letters of the city name. Since across records the same firm may appear with slight differences in spelling, these codes are imperfect identifiers. On the other hand, two distinct firms may have similar names. If IDs are too short we may identify different firms as the same firm. There is an implicit trade off in the strictness of our matching requirements. The longer our ID variable is, the less likely we are to identify distinct firms as the same firm, and the more likely we are to misidentify transactions records of the same firm as done by two distinct firms. Robustness checks, yet to be performed, will give us a sense for the importance of this issue.

2.2 Stylized Facts

In this section we document a few of the stylized facts that are useful to motivate the specific features of our model. We start with documenting the total number of importers, exporters, and importer-exporter matches involved in footwear trade in Colombia from 2006 – 2013. We also include the total volume of imports in terms of millions of US dollars. We can observe that 2006 – 2009 is a relatively stable period for the number of buyers, sellers, and matches. Since then the Colombian imports have grown rapidly due to its government's unilateral reduction of tariffs during 2010 – 2011 and aggressive negotiation of free trade areas (FTAs), most notably with U.S. and Panama for footwear.

Year	Importers	Exporters	Matches	Total Volume
2006	522	892	1847	177
2007	518	1041	2066	215
2008	527	979	1992	251
2009	572	928	1922	251
2010	630	1200	2391	320
2011	822	1602	3341	481
2012	852	1699	3375	575
2013	947	1569	2989	492

Table 1: Number of Importers, Exporters, and Matches

Figure 1: Degree distribution: sellers per buyer, 2009

Figure 2: Degree distribution: buyers per seller, 2009

We now summarize the degree distributions of the sellers (a.k.a. exporters) per buyer (a.k.a. importer) and buyers per seller for the Colombian footwear’s international market.² We also report these statistics for each more detailed 4-digit HS code category. For the frequency distribution of the sellers per buyer (label “Buyer” in the table), we find that the overall distribution is quite similar to those of more detailed product categories. A substantial amount (44 – 47%) of Colombian importers have only one direct seller. Meanwhile the chance of observing multiple connected sellers is non-trivial as well. Around 35% of the buyers have between 2 – 5 connected sellers. In contrast, the frequency distribution of the buyers per seller (label “Seller” in the table) is less skewed. 74 – 77% of the foreign exporters have only one customer in Colombia, while around 18% of them have between 2 – 5 customers. To summarize the behavior of the right tail, we also plot in Figure 1 and Figure 2 the whole distribution of buyer and seller degrees. On the horizontal axis, we have the number of connections of a buyer

²We will use the word buyer and importer interchangeably, and the word exporter and seller interchangeably as well.

	2009	2013	2009	2013	2009	2013
	6402	6402	6403	6403	6404	6404
# sellers	rubber	rubber	leather	leather	textile	textile
1	0.560	0.432	0.608	0.475	0.600	0.446
2	0.163	0.180	0.161	0.176	0.158	0.189
3	0.075	0.100	0.072	0.090	0.074	0.091
4	0.049	0.064	0.046	0.061	0.049	0.062
5	0.039	0.040	0.027	0.047	0.029	0.042
6	0.025	0.035	0.021	0.026	0.025	0.033
7	0.022	0.023	0.016	0.022	0.020	0.031
8	0.017	0.027	0.014	0.022	0.012	0.017
9	0.010	0.017	0.011	0.017	0.007	0.017
10	0.008	0.014	0.005	0.013	0.005	0.013
Regression coef	-2.005	-1.873	-2.176	-1.986	-2.203	-1.949

Table 2: Degree Distribution: Seller’s per Buyer

or seller. On the vertical axis, we report inverse empirical CDF. Both axes are in a log scale, so that data distributed according to a “power law” would appear linear. The distribution of sellers per buyer is approximately power law in the tail. That is, the distribution is fat-tailed. The tail of the distribution of buyers per seller on the other hand dies more quickly than a power law. In Table 2 we report the coefficients from regressions fit to log-log plots like those in Figure 1 for different products and years. The regression slopes have become flatter, indicating that the tails of the degree distributions have become fatter over time. There are relatively more large buyers in 2013 than there were in 2009.

Given the cross-sectional degree distribution, it would also be useful to examine the over-time transitions of the number of sellers for Colombian buyers. We report that in Table 3. Several patterns are worth highlighting here. First, there is a non-trivial probability that one buyer’s connections get completely eliminated from one year to the next. Part of this, especially for buyers with large number of sellers, might reflect the exit of the buyer itself. Second, there is a quite substantial destruction rate such that the transition probability has

$t \backslash t + 1$	0	1	2	3	4	5	6	7	8	9	10	10+
1	0.529	0.316	0.105	0.015	0.010	0.010	0.003	0.004	0.000	0.004	0.001	0.003
2	0.263	0.285	0.219	0.109	0.055	0.022	0.007	0.011	0.007	0.000	0.007	0.015
3	0.227	0.174	0.167	0.152	0.061	0.114	0.008	0.045	0.015	0.008	0.000	0.030
4	0.182	0.156	0.104	0.195	0.104	0.091	0.039	0.052	0.013	0.026	0.026	0.013
5	0.120	0.067	0.107	0.107	0.147	0.107	0.133	0.067	0.027	0.027	0.027	0.067
6	0.149	0.064	0.064	0.021	0.128	0.128	0.043	0.064	0.128	0.085	0.064	0.064
7	0.085	0.021	0.085	0.064	0.043	0.106	0.170	0.128	0.128	0.064	0.043	0.064
8	0.063	0.000	0.063	0.031	0.063	0.125	0.156	0.063	0.188	0.094	0.125	0.031
9	0.069	0.000	0.069	0.103	0.034	0.034	0.138	0.034	0.138	0.103	0.034	0.241
10	0.050	0.000	0.050	0.100	0.000	0.150	0.100	0.000	0.100	0.050	0.100	0.300
10+	0.098	0.009	0.027	0.000	0.027	0.027	0.027	0.036	0.054	0.045	0.045	0.607

Table 3: Transition Matrix of Sellers per Buyer

larger mass to the left of the diagonal. Overall, this transition pattern is consistent with the cross-sectional distribution that has a large probability mass at the lower number of connections. It remains an empirical question on whether one can simultaneously account for the behavior of small firms as well as transitions in the right tail of the degree distribution.

In Table 4, we report the major countries that Colombian retailers/distributors directly purchase from. The top countries in terms of number of sellers are China, U.S, Panama, Brazil, and Hong Kong. The number of sellers have increased during 2010 – 2013 across the board, reflecting that the increase is most likely driven by Colombian unilateral tariff changes. Meanwhile focusing on the stationary period of 2006 – 2009, the value of exports reveals large differences in exporter size across countries. U.S. and Hong Kong have on average small exporters with sales around 50,000 to 100,000 USD, while China, Brazil, and Panama have larger exporters ranging from 0.25 to 0.6 million USD. One interesting outlier is Ecuador, whose exporters sell around 3 to 5 million USD. The importer side features quite substantial heterogeneity in terms both the number of sellers an importer sources from and the value of its transactions.

Year	China	USA	Panama	Brazil	HK	Ecuador
Number of Sellers						
2006	201	154	202	64	42	7
2007	276	192	184	77	46	10
2008	273	188	144	70	40	5
2009	257	195	145	51	44	7
2010	341	242	180	65	67	6
2011	386	443	234	76	69	6
2012	451	377	245	80	70	5
2013	371	319	226	69	90	13
Value of Exports (Millions of USD)						
2006	49.2	3.93	67.4	16.9	4.35	23.5
2007	71.9	5.42	67.3	18.1	4.97	27.5
2008	79.4	6.84	85.6	16.9	4.70	27.2
2009	65.9	5.58	105	16.1	7.97	27.8
2010	93.7	6.89	136	20.6	8.53	25.5
2011	130	16.0	223	30.2	13.0	27.4
2012	179	19.2	256	36.7	17.5	17.6
2013	114	14.0	234	39.8	15.1	21.1

Table 4: Major Countries of Direct Sellers

3 A model of buyer-seller networks

Motivated by the stylized facts described above, we now develop a continuous-time two-sided search model. As depicted in Figure 3, our model is populated by three types of agents: sellers, buyers, and consumers. Sellers provide goods to buyers in the wholesale market, who pass them on to consumers in the retail market. Though our model could be applied in a variety of contexts, we will think of sellers as foreign suppliers who export their goods to buyers, and we will think of buyers as domestic retailers.

Consumers acquire goods exclusively through retailers, who offer different but possibly overlapping menus of products, depending upon the set of suppliers they are currently partnered with. Retailers are also vertically differentiated in terms of the amenities they offer, like locational convenience, ambiance, and service. As a group, consumers allocate their ex-

penditures across retailers in a way that reflects their preferences for amenities and product menus.

The dimensions of retailer heterogeneity are publicly observable, so consumers' expenditure patterns are characterized by a standard static optimization problem with full information. However, buyers and sellers in the wholesale market are unable to costlessly find one another. Rather, each type of agent must invest in costly search to find business partners. Other things equal, the more intensively an agent searches, the higher the hazard rate with which she finds new partners. But these hazard rates depend upon other things as well.

First, matching hazards are influenced by market tightness. For example, when many buyers are searching for new suppliers, but not many suppliers are searching for new buyers, matching hazards will tend to be low for buyers and high for suppliers. The precise way in which search intensities on both sides of the market influence aggregate market tightness is determined by the matching function in our model, which we adopt from the labor-search literature.

Second, the ease with which agents find new business partners depends upon their previous successes. That is, agents who have already accumulated a large portfolio of business partners find it relatively easy to locate still more. This feature of our model helps us to capture the "fat-tailed" distributions of buyers across sellers and sellers across buyers discussed above.

Buyers and sellers create rents when they meet one another. These amount to the present value of the resulting retail sales, net of the costs incurred by the partnership in producing and delivering the goods transacted. Buyers and sellers bargain continuously and bilaterally over the rents they generate, and the expected outcomes of these bargaining games determine

Figure 3: Model diagram

the expected returns to successful search for each party.

3.1 The Retail Market

We now turn to model specifics. Assume the retail market is populated by a measure- B continuum of stores, and suppose consumers view these stores as imperfect substitutes, both because they offer distinct amenities and because they carry different—but not necessarily disjoint—sets of products. More precisely, indexing stores by b and products by j , let consumers' preferences over retailers be given by the utility function:

$$C = \left[\int_{b \in B} (\mu_b C_b)^{\frac{\eta-1}{\eta}} \right]^{\frac{\eta}{\eta-1}},$$

where C_b measures consumption of the set of products, J_b , offered at store b ,

$$C_b = \left[\sum_{j \in J_b} (\xi_j C_b^j)^{\frac{\alpha-1}{\alpha}} \right]^{\frac{\alpha}{\alpha-1}},$$

and μ_b and ξ_j are exogenous parameters that measure the inherent appeal of retailer b and product j , respectively.³ This characterization of preferences implies that the exact price index for retailer b is $p_b = \left[\sum \left(\frac{p_{jb}}{\xi_{jb}} \right)^{1-\alpha} \right]^{\frac{1}{1-\alpha}}$ and the exact price index for retailers as a group is $P = \left[\int_b \left(\frac{p_b}{\mu_b} \right)^{1-\eta} \right]^{\frac{1}{1-\eta}}$.

Because of search frictions, retailers cannot instantaneously adjust the set of products they offer consumers. Rather, at each point in time they take their current offerings as given and engage in Bertrand-Nash competition. It follows that the optimal retail prices at store b satisfy

$$q_{jb} + \sum_{j' \in J_b} \frac{\partial q_{j'b}}{\partial p_{jb}} (p_{j'b} - c_{j'b}) = 0 \quad \forall j \in J_b, \quad (1)$$

where $c_{j'b}$ is the marginal cost of supplying variety j' to final consumers through retailer b . This equation reflects the fact that retailer b and its suppliers set retail prices to maximize the value of the surplus generated by their business relationships. Hence $c_{j'b}$ includes the marginal production and distribution costs incurred by the supplier of product j' as well as the marginal costs incurred by retailer b in bringing this product to consumers.

As in Atkeson and Burstein (2008) and Hottman *et al.* (forthcoming), equation (1) implies the within-retailer cannibalization effect exactly offsets the cross-store substitution effect, so

³Alternative nesting structures are possible. In particular, consumers might have preferences over bundles of types of goods, each of which is a CES aggregation over the bundles available from alternative retailers. That is, consumers first allocate spending across product categories, then across retailers in each category. This formulation is used in Atkin, et al (2015). Which specification is preferable depends upon the importance of transport and shopping time costs to consumers.

the mark-up rule is simply:⁴

$$\frac{p_{jb} - c_j}{p_{jb}} = \frac{1}{\eta}.$$

That is, each retailer perceives the elasticity of demand for each of the products it offers to be the cross-store elasticity of substitution, η . It follows that the instantaneous revenue flow jointly generated by retailer b and its suppliers is

$$r_b = \frac{E}{P^{1-\eta}} \left[\sum_{j \in J_b} \left(\frac{\eta}{\eta - 1} \right)^{1-\alpha} \left(\frac{c_{jb}}{\xi_{jb}} \right)^{1-\alpha} \right]^{\frac{1-\eta}{1-\alpha}} \mu_b^{\eta-1}.$$

Further, assuming the marginal cost of each good is proportional to its quality, $\frac{c_{jb}}{\xi_{jb}} = \tilde{c} \forall j$, this expression collapses to

$$r_b = \frac{E}{P^{1-\eta}} \left[n_b \left(\tilde{c} \frac{\eta}{\eta - 1} \right)^{1-\alpha} \right]^{\frac{1-\eta}{1-\alpha}} \mu_b^{\eta-1} \quad (2)$$

where n_b is the number products carried by retailer b .

3.2 The Wholesale Market and Payoff Functions

We can now describe the flow pay-off functions for buyers (retailers) and sellers (foreign exporters) in the wholesale market. Suppose there are N^B intrinsic buyer types indexed by $i \in \{1, 2, \dots, N^B\}$, so that if buyer b is a type i retailer, $\mu_b = \mu^B(i)$. Similarly, suppose there are N^S intrinsic seller types indexed by $j \in \{1, 2, \dots, N^S\}$, and though these sellers share a common $\frac{c_{jb}}{\xi_{jb}}$ ratio, they are differentiated in terms of their product appeal, ξ_j .⁵ Finally, assume

⁴See appendix A for details.

⁵There is no conceptual difficulty with allowing heterogeneity in terms of $\frac{c_{jb}}{\xi_{jb}}$ ratios. We abstain from this generalization until later because it means keeping track of the number of each type of seller that is matched with each type of buyer, cluttering equation (2) and various expressions that follow.

that each exporter supplies a single product to each retailer it does business with, so that a buyer connected with s suppliers offers $n_b = s$ products to final goods consumers.

Given that the perceived elasticity of demand for each product is η , equation (2) then implies the total flow surplus generated by a type- i buyer and its s suppliers is:

$$\pi_s^T(i) = \frac{E}{\eta P^{1-\eta}} s^{\frac{1-\eta}{1-\alpha}} \left(\tilde{c} \frac{\eta}{\eta-1} \right)^{1-\eta} \mu^B(i)^{\eta-1} \quad (3)$$

Note that when the elasticity of substitution across retailers exceeds the elasticity of substitution across products ($\alpha > \eta > 1$), this surplus exhibits diminishing returns with respect to the number of suppliers. That is, buyers who add additional sellers reduce total surplus per supplier.

How is this flow divided up between buyers and sellers? Let $\pi_s^B(i)$ be the flow surplus per time increment that is kept by a type- i buyer with s suppliers. Also let $\tau_s(i)$ be the flow surplus that this buyer transfers to each of his s suppliers during the same time increment. Then flow balance requires that the payoffs satisfy:⁶

$$s \cdot \tau_s(i) + \pi_s^B(i) = \pi_s^T(i) \quad (4)$$

Subject to this constraint, $\tau_s(i)$ and $\pi_s^B(i)$ are determined by forward-looking, continuous, bilateral bargaining, as in Stole and Zwiebel (1996).

Specifically, define $V_s^B(i)$ to be the expected value of the buyer's total future surplus stream, and let $V_s^S(i)$ be the expected value of the future surplus stream accruing to any one

⁶Since we are treating all suppliers as sharing the same marginal cost per unit quality delivered, $\tilde{c}$, and quality-adjusted goods enter consumer preferences symmetrically, all suppliers to a particular buyer receive the same transfer at any given point in time.

of the associated sellers. Then a partnership between a type- i retailer (buyer) and one of her s suppliers (sellers) is worth $V_s^B(i) - V_{s-1}^B(i)$ to the former and $V_s^S(i)$ to the latter. And if the seller's bargaining power allows her to capture share β of the total surplus, $V_s^S(i) = \beta [V_s^B(i) - V_{s-1}^B(i) + V_s^S(i)]$. The functions $\tau(\cdot)$ and $\pi^B(\cdot)$ must therefore must be chosen to satisfy:

$$(1 - \beta)V_s^S(i) = \beta [V_s^B(i) - V_{s-1}^B(i)], \quad s \in \{1, 2, \dots, s^{\max}\}, \quad i \in \{1, 2, \dots, N^B\} \quad (5)$$

As demonstrated in Appendix B, the solution to this bargaining problem collapses to a simple static rule:

$$(1 - \beta)\tau_s(i) = \beta (\pi_s^B(i) - \pi_{s-1}^B(i)) \quad (6)$$

Further, given that $\pi_0^B(i) = 0$, equations (4) and (6) can be solved for $\pi_s^B(i)$ as a function of $\pi_s^T(i), \pi_{s-1}^T(i), \dots, \pi_1^T(i)$:

$$\pi_s^B(i) = \sum_{\tau=1}^s \frac{s!}{(s - \tau + 1)!} \left(\frac{\beta}{1 - \beta} \right)^{\tau-1} \left[\prod_{\tau'=s-\tau+1}^s \left(1 + \tau' \frac{\beta}{1 - \beta} \right)^{-1} \right] \pi_{s-\tau+1}^T(i), \quad (7)$$

The transfer payoff function $\tau_s(i)$ is then implied by (4) or (6). Alternatively, imposing that the buyer payoffs are a constant fraction of the current surplus, one can calculate the approximate transfer function as:

$$\tau_s(i) \approx \kappa \frac{\beta}{1 - \beta} \frac{E}{\eta P^{1-\eta}} s^{\frac{1-\eta}{1-\alpha}} \left(\tilde{c} \frac{\eta}{\eta - 1} \right)^{1-\eta} \mu^B(i)^{\eta-1} \quad (8)$$

3.3 Search and Matching

3.3.1 Market aggregates and Market Slackness

Next we characterize matching patterns in wholesale markets. To do so, we introduce variables that measure agents' "visibility." The key feature of these objects is that, for any two agents or groups of agents on the same side of the market, the ratio of their visibilities is also the ratio of their hazards for meeting a new business partner.

Let $M_s^B(i)$ be the measure of type- i buyers with s sellers, and define these buyers' visibility to be:

$$H_s^B(i) = \sigma_s^B(i) M_s^B(i)$$

where $\sigma_s^B(i)$ measures the search intensity of any one of these buyers. Aggregating over types and partner counts, the overall visibility of buyers is measured by:

$$H^B = \sum_{i=1}^{N^B} \sum_{s=0}^{\infty} H_s^B(i)$$

Analogously, let $M_b^S(j)$ be the measure of type j sellers with b buyers, and suppose each of these sellers searches with intensity $\sigma_b^S(j)$. Then this group's visibility is measured by:

$$H_b^S(j) = \sigma_b^S(j) M_b^S(j),$$

and the overall visibility of sellers to buyers is:

$$H^S = \sum_{j=1}^{N^S} \sum_{b=0}^{\infty} H_b^S(j)$$

Following much of the labor search literature, we assume a matching function that is homogeneous of degree one in the visibility of buyers and sellers. Specifically we assume that

the measure of matches per unit time is given by (Petrongolo and Pissarides, 2001):⁷

$$X = f(H^S, H^B) = H^B \left[1 - \left(1 - \frac{1}{H^B} \right)^{H^S} \right] \approx H^B \left[1 - e^{-H^S/H^B} \right] \quad (9)$$

From buyers' perspective, we can then define market slackness in manner analogous to random search models:

$$\theta^B = \frac{f(H^S, H^B)}{H^B}.$$

The larger is θ^B , the more matches take place for a given amount of buyer visibility. Likewise, market slackness from sellers' perspective is:

$$\theta^S = \frac{f(H^S, H^B)}{H^S}. \quad (10)$$

Finally, assuming random matching, the share of matches involving buyers of type i with $s > 0$ sellers is:

$$\frac{\sigma_s^B(i) M_s^B(i)}{H^B} \quad (11)$$

and the share of matches involving sellers of type j with $b > 0$ buyers is:

$$\frac{\sigma_b^S(j) M_b^S(j)}{H^S}.$$

In the absence of $\tilde{c}$ heterogeneity across seller types, sellers' payoffs do not depend upon j . And if sellers' search cost functions do not depend upon their type either, we can drop the j argument from $\sigma_b^S(j)$. For the time being we do so.

⁷Other matching functions are of course feasible here. We have also experimented with $x = \frac{H^B H^S}{[(H^B)^\alpha + (H^S)^\alpha]^{1/\alpha}}$.

3.3.2 Optimal search

It remains to characterize the policy functions $\sigma_s^B(i)$ and $\sigma_b^S(j)$ that maximize the values of agents' expected payoff streams. To do so we introduce buyer and seller search cost functions, which measure the flow cost of sustaining search intensities σ^B and σ^S , respectively:

$$\begin{aligned} k_B(\sigma^B, s) &= \frac{(\sigma^B)^{\nu_B}}{(s+1)^{\gamma^B}} \\ k_S(\sigma^S, b) &= \frac{(\sigma^S)^{\nu_S}}{(b+1)^{\gamma^S}} \end{aligned}$$

By assumption, search costs are positive and convex in search intensity: $\nu_B, \nu_S > 1$. Also, network effects may reduce the costs of forming new matches as agents' partner counts grow: $\gamma^B, \gamma^S \geq 0$.

Buyer's problem: Given that type- i buyers enjoy surplus flow $\pi_s^B(i)$ when they are matched with s suppliers, such buyers choose their search intensity to solve:

$$V_s^B(i) = \max_{\sigma^B} \left\{ \frac{\pi_s^B(i) - k_B(\sigma^B) + s\delta V_{s-1}^B(i) + \sigma^B \theta^B V_{s+1}^B(i)}{\rho + s\delta + \sigma^B \theta^B} \right\} \quad (12)$$

where ρ is the rate of time preference. Intuitively, the seller reaps profit flow $\pi_s^B(i) - k_B(\sigma^B)$ until the next event occurs. With hazard $s\delta$ this event is an exogenous termination of one of the s relationships, and with hazard $\sigma^B \theta^B$ it is a new match.

The optimal search policy for type- i buyers with s sellers, $\sigma_s^B(i)$, therefore satisfies

$$\frac{\partial k_B(\sigma^B, s)}{\partial \sigma^B} = \theta^B [V_{s+1}^B(i) - V_s^B(i)]. \quad (13)$$

Sellers' problem: Since sellers have constant effective marginal costs, $\tilde{c}$, the number of buyers they currently supply does not affect their expected returns from adding another one.

On the other hand, the seller's payoff function from a particular match, $\tau_s(i)$, depends upon the buyer's type, i , and current seller count, s , so ex post, it matters whom sellers match with. The value to any seller of matching with a type- i buyer who has s suppliers is:⁸

$$V_{i,s}^S = \frac{\tau_s(i) + (s-1)\delta V_{i,s-1}^S + \sigma_s^B(i)\theta^B V_{i,s+1}^S}{\rho + s\delta + \sigma_s^B(i)\theta^B}. \quad (14)$$

Intuitively, a business relationship with a type- i buyer who has s suppliers will terminate with exogenous hazard δ , become a relationship with a type- i buyer who has $s-1$ suppliers with hazard $(s-1)\delta$, and become a relationship with a type- i buyer who has $s+1$ suppliers with hazard $\sigma_s^B(i)\theta^B$.

Taking expectations over the population of buyers that sellers might meet, the *ex ante* value of a new relationship is:

$$V^S = \sum_i \sum_{s=0}^{\infty} V_{i,s+1}^S P_s^B(i),$$

where $P_s^B(i) = H_s^B(i)/H^B$ is the relative visibility of buyers who are type- i and have s sellers.

So the optimal search intensity for any seller with b buyers satisfies:

$$\frac{\partial k_S(\sigma^S, s)}{\partial \sigma^S} = \theta^S V^S. \quad (15)$$

3.3.3 Equilibria and Transition Dynamics

Equations of motion: Finally, we assume that relationships end with a hazard δ regardless of number of contacts or type. Then the equation of motion for the measure of buyers of type i with s sellers:

⁸We multiply the destruction event δ by $(s-1)$ to adjust for the fact that the seller's own relationship with the buyer dies, in which case the continuation value of this relationship for this seller is zero. Of course V_s^S makes sense only if $s > 0$, as a seller can't have a connection with a buyer with zero sellers.

$$\begin{aligned}
\dot{M}_s^B(i) &= \sigma_{s-1}^B(i)\theta^B M_{s-1}^B(i) + \delta(s+1)M_{s+1}^B(i) \\
&\quad - (\sigma_s^B(i)\theta^B M_s^B(i) + \delta s M_s^B(i)). \\
s &= 1, \dots, s_{\max}; \quad i = 1, \dots, N_B
\end{aligned} \tag{16}$$

This group gains a member whenever any of the $M_{s-1}^B(i)$ buyers with $s-1$ suppliers adds a supplier, and the hazard of this happening is $\sigma_{s-1}^B(i)\theta^B$. Similarly, it gains a member whenever any of the $M_{s+1}^B(i)$ buyers with $s+1$ suppliers loses a supplier because of exogenous attrition, and this occurs with hazard $\delta(s+1)$. By analogous logic, the group loses existing members that either successfully add a supplier (with hazard $\sigma_s^B(i)\theta^B$) or loses one (with hazard δ). Finally, the measure of buyers of type i with $s=0$ sellers evolves according to:

$$\dot{M}_0^B(i) = \delta M_1^B(i) - \sigma_0^B(i)\theta^B M_0^B(i) \quad i = 1, \dots, N_B \tag{17}$$

Replacing B with S and s with b in (16) and (17), the equations of motion for seller measures $M_b^S(j)$ obtain.

Steady state: To characterize the steady state of this system, we set $\dot{M}_s^B(i) = \dot{M}_b^S(j) = 0$ and solve the system of $N_B(s_{\max}+1) + N_S(b_{\max}+1)$ equations implied by both versions of (16) and (17)—for buyers and sellers. In doing so we, treat the measures of each type of intrinsic type as exogenous constants and impose the adding-up constraints:

$$M^B(i) = \sum_{s=0}^{s_{\max}} M_s^B(i) \tag{18}$$

$$M^S(j) = \sum_{b=0}^{b_{\max}} M_b^S(j), \tag{19}$$

Transition dynamics: Solving for transition dynamics is more involved. Suppose we wish to find the transition path from one market environment to a new one under perfect

foresight. We begin by finding the steady distribution of buyers and sellers across types for the new regime, as well as the associated value functions. We then guess the trajectory of endogenous market-wide aggregates $\{\theta^B(t), \theta^S(t), P(t)\}$ from the initial state to this steady state, and solve for buyer and seller distribution functions using backward induction and finite differencing. Appendix C provides details.

3.4 Introducing compatibility

Thus far, our model does not allow for the possibility that some retailers specialize in athletic shoes, while others are more about dress shoes, and still others do both types of business. Nor it does it provide a mechanism through which assortative matching on the basis of product quality might be accommodated. We now generalize our formulation to address this shortcoming.

3.4.1 Modifications to the basic structure

To begin, continue to assume that buyers and sellers encounter each other through an undirected search process. But now suppose that shipments only take place between *compatible* buyers and sellers who meet, and let any randomly selected pair of type- i buyer and type- j seller be compatible with probability $d(i, j) \in [0, 1]$. Finally, assume that buyers and sellers know these probabilities and choose their search intensities accordingly. With these additional assumptions, we are able keep the random search aspects of the model while accommodating the fact that we observe particular types of businesses doing business with one other with greater or lesser frequency than pure randomness would imply.

Success rates: For type- i buyers, the expected share of encounters that result in business

partnerships is now:

$$a^B(i) = \frac{\sum_j \sum_{b=0}^{\infty} d(i, j) \sigma_b^S(j) P_b^S(j)}{\sum_j \sum_{b=0}^{\infty} \sigma_b^S(j) P_b^S(j)} \quad (20)$$

where $P_b^S(i) = H_b^S(i)/H^S$ is the share of matches that involve type- i sellers with b buyers.

Similarly, for type j sellers, the expected share of meetings that result in business partnerships is:

$$a^S(j) = \frac{\sum_i \sum_{s=0}^{\infty} d(i, j) \sigma_s^B(i) P_s^B(i)}{\sum_i \sum_{s=0}^{\infty} \sigma_s^B(i) P_s^B(i)} \quad (21)$$

where, recall, $P_s^B(i) = H_s^B(i)/H^B$ is the share of matches that involve type- i buyers who have s sellers. Thus, for a type- i buyer with s suppliers, the hazard of finding another compatible seller is $\sigma_b^B a^B(i) \theta^B$. Likewise, for a type- j seller with b buyers, the hazard of finding another compatible buyer is $\sigma_b^S a^S(j) \theta^S$.

Policy functions: Incorporating compatibility, the programming problem for a type- i buyer with s sellers becomes:

$$V_s^B(i) = \max_{\sigma_s^B} \left\{ \frac{\pi_s^B(i) - c_B(\sigma^B) + s\delta V_{s-1}^B(i) + \sigma_s^B a^B(i) \theta^B V_{s+1}^B(i)}{\rho + s\delta + \sigma_s^B a^B(i) \theta^B} \right\} \quad (22)$$

Accordingly, the new buyer policy functions, $\sigma_s^B(i)$, solve the first order conditions:

$$c'_B(\sigma_s^B(i)) = a^B(i) \theta^B [V_{s+1}^B(i) - V_s^B(i)]. \quad (23)$$

Similar modifications apply on the sellers' side. The value to a seller of an existing compatible relationship with a buyer in state (i, s) now depends on $a^B(i)$. This is because the hazard of this buyer adding another seller depends upon her compatibility:

$$V_{i,s}^S = \frac{\pi_s^S(i) + (s-1)\delta V_{i,s-1}^S + \sigma_s^B(i)a^B(i)\theta^B V_{i,s+1}^S}{\rho + s\delta + \sigma_s^B(i)a^B(i)\theta^B} \quad (24)$$

And the ex-ante potential value of a new relationship with a compatible buyer is:

$$V^S(j) = \sum_i \sum_{s=0}^{\infty} V_{i,s+1}^S \frac{d(i,j)P_s^B(i)}{\sum_{i,s} d(i,j)P_s^B(i)}$$

The associated seller policy functions, $\sigma_b^S(j)$, therefore solve:

$$c'_S(\sigma_b^S(j)) = a^S(j)\theta^S V^S(j). \quad (25)$$

Appendix D provides further details on implementing this version of the model.

4 Fitting the model to data

In this section, we calibrate the model to the data and assess the quality of the fit.

4.1 Payoff functions

In our data, we observe the transfer from the Colombian buyers to the foreign sellers. We modify the theoretical transfer equation approximation (8) by taking logs, adding time dummies d_t , and adding a stochastic match-specific shock $\varepsilon_{j,i}$:

$$\ln \tau_s(j, i) = \text{const.} + \left(\frac{\eta - 1}{\alpha - 1} - 1 \right) \ln s + (\eta - 1) \ln \mu^B(i) + d_t + \varepsilon_{j,i} \quad (26)$$

The time dummies d_t are meant to proxy for changes in overall expenditures on shoes and the market-wide price level for shoes. The number of clients of each buyer s is observed,

	6401	6402	6403	6404
Description	waterproof	rubber	leather	textile
$\frac{1-\eta}{1-\alpha}-1$	-0.182 (0.154)	-0.146** (0.026)	-0.182** (0.029)	-0.114** (0.029)
$(\eta - 1) \ln \mu^B(i)$				
std.	2.7	1.6	1.5	1.4
p95-p5	7.2	4.9	4.5	4.5
p75-p25	4.1	2.1	1.8	1.8

Table 5: Estimation of transfer equation

and the buyer specific quality $(\eta - 1) \ln \mu^B(i)$ is treated as a fixed effect. We estimate (26) separately for each of the four digit shoe categories, and results are reported in Table 5.

We calibrate $\alpha = 4.35$ based on estimates in Hottman *et al.* (forthcoming), which gives us an η of 3.71 - 3.98. We discretize the fixed effect distribution using a method suggested in Kennan (2006). We assume that the discount rate $\rho = 0.05$, and impose symmetry in search cost function, $k_0^B = k_0^S = k_0$, and we assume that search costs are convex, $\nu_V = \nu_S = 2$. We are left with the search cost level parameter k_0 , the network effects in search cost γ^B and γ^S , and the match death hazard δ . To calibrate these parameters we minimize the sum of differences between the model and 2009 data in the degree distribution of sellers per buyer and buyers per seller as well as the partner transition matrices. Loosely speaking, the degree distributions pin down the search cost parameters, and the transition matrices will identify the match death hazard.

We estimate a search cost level parameter $k_0 = 0.9$, and search cost network effects of $\gamma^S = 0.65$ and $\gamma^B = 0.7$. We get a match death hazard of $\delta = 0.6$. The match death hazard is easy to interpret. It implies that 45% of matches die in the first year, and 70% of matches die by their second year. The fit with the data degree distributions is reported in Table 6. The fit is quite good, considering we only have four parameters to play with. The number of buyers

	6402	6403	6404	Model
# sellers	rubber	leather	textile	
1	0.560	0.608	0.600	0.512
2	0.163	0.161	0.158	0.165
3	0.075	0.072	0.074	0.078
4	0.049	0.046	0.049	0.045
5	0.039	0.027	0.029	0.030
6	0.025	0.021	0.025	0.021
7	0.022	0.016	0.020	0.016
8	0.017	0.014	0.012	0.012
9	0.010	0.011	0.007	0.008
10	0.008	0.005	0.005	0.006
Regression coef.	-2.005	-2.176	-2.216	-2.243

Table 6: Estimated Degree Distribution

	Data		Model	
	1	2	1	2
1	0.680	0.201	0.797	0.122
2	0.404	0.260	0.507	0.366
3	0.242	0.267	0.300	0.393
4	0.144	0.200	0.169	0.323
5	0.080	0.160	0.092	0.231
6	0.088	0.100	0.049	0.152
7	0.065	0.065	0.025	0.094
8	0.021	0.021	0.013	0.056
9	0.000	0.024	0.006	0.032
10	0.026	0.000	0.003	0.018

Table 7: Estimated Transition Matrix

Figure 4: Transitions, sellers per buyer, data vs model

with one seller is a bit underestimated. The fit with the partner transition matrix is reported in Table 7, and displayed graphically in Figure 4. With only four free parameters, the model is not capable of matching the noisy mountains and valleys of the observed transition matrix. The general shape is replicated, however, including the general drift toward zero clients.

5 Putting the model to work

In this section we run two experiments with the model. In the first experiment we double the mass of sellers. This is more or less what we observe between 2009 and 2013. We report the steady state change in the degree distribution of sellers per buyer in Table 8. The model adjusts to the increase in sellers by increasing the fatness of the tail in the sellers per buyer distribution. This is exactly what we see in the data, reported in Table 8 only for leather shoes. This change leads to a non-trivial increase in consumer surplus of 5.51%.

Our second experiment is to reduce the search cost level k_0 by 30%. In this experiment, we show the full transition from the estimated steady state to eight years after the shock. One feature of our model is that the transition takes time. In Figure 5 we see that again there is a non-trivial increase in welfare, rising more than 10% by the eighth year. In Figure 6 again

	2009	2013	2009	2013
# sellers	model	model	leather	leather
1	0.512	0.497	0.608	0.475
2	0.165	0.166	0.161	0.176
3	0.078	0.079	0.072	0.090
4	0.045	0.046	0.046	0.061
5	0.030	0.031	0.027	0.047
6	0.021	0.027	0.021	0.026
7	0.016	0.017	0.016	0.022
8	0.012	0.013	0.014	0.022
9	0.008	0.010	0.011	0.017
10	0.006	0.008	0.005	0.013
Regression coef.	-2.243	-1.781	-2.176	-1.986

Table 8: Degree distribution, doubling number of sellers

we see that the tail of the degree distribution of sellers per buyer gets fatter. In both of the experiments we run, we see large gains to consumer surplus. These gains are driven by the increase in varieties which each retailer offers.

6 Further work and Conclusion

We have developed a relatively parsimonious theoretical model, but we have yet to push the model too far on the empirical side. There is still a fairly lengthy to-do list for this paper. As hinted at in the exercises we ran in Section 5, we are interested in teasing out how much of the change we have observed in network structure over the last decade or so can be explained by the composition of the market, say the entry of China on the world stage, and how much can be explained by the reduction in search costs due to advances in information technology. On the more technical side, we would like to weaken the assumption that sellers are all identical in terms of quality adjusted cost.

In this paper we present some stylized facts relating to observed trade networks. We

Figure 5: 30% reduction in search cost: Welfare

Figure 6: 30% reduction in search cost: Sellers per buyer

develop a theoretical model which is rich but still tractable enough to estimate on our data. We calibrate the model and find that it is able to match relationships from the data with relatively few free parameters. Finally, using the estimated model we show that reducing the cost to search can lead to significant gains to consumer surplus, and that search leads to slow transitions between steady states. Through the lens of our model, search frictions appear to both have sizable welfare effects and to also comprise an important part of the dynamics of international trade.

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Appendices

A Demand and Pricing

Using standard CES results, we begin by characterizing prices and market shares for a particular retailer b offering a particular subset of product varieties in the group, $j \in J_b$:

$$\begin{aligned} C_b &= \left(\sum_{j \in J_b} (\xi_j C_b^j)^{\frac{\alpha-1}{\alpha}} \right)^{\frac{\alpha}{\alpha-1}}, & C &= \left(\int_b (\mu_b C_b)^{\frac{\eta-1}{\eta}} \right)^{\frac{\eta}{\eta-1}} \\ P_b &= \left[\sum_{j \in J_b} \left(\frac{P_{jb}}{\xi_j} \right)^{1-\alpha} \right]^{1/(1-\alpha)}, & P &= \left[\int_b \left(\frac{P_b}{\mu_b} \right)^{(1-\eta)} \right]^{1/(1-\eta)} \\ s_b &= \frac{\left(\frac{P_b}{\pi_b} \right)^{1-\eta}}{\int_b \left(\frac{P_{b'}}{\mu_{b'}} \right)^{1-\eta}}, & s_{j|b} &= \frac{\left(\frac{P_{jb}}{\xi_j} \right)^{1-\alpha}}{\sum_{j' \in J_b} \left(\frac{P_{j'b}}{\xi_{j'}} \right)^{1-\alpha}} \end{aligned}$$

Define $R_{jb} = P_{jb} q_{jb} = s_{j|b} s_b E$ to be the revenue generated by sales of product j at store b .

Then, using the share and price expressions above, one can show that Bertrand-Nash pricing implies:

$$\begin{aligned} \frac{\partial \ln R_{jb}}{\partial \ln P_{jb}} &= (1 - \alpha) + s_{j|b} (\alpha - \eta) \\ \frac{\partial \ln R_{jb}}{\partial \ln P_{j'b}} &= s_{j|b} (\alpha - \eta) \quad \forall j' \neq j \end{aligned}$$

Also since $\frac{\partial \ln R_{jb}}{\partial \ln P_{jb}} = \frac{\partial \ln q_{jb}}{\partial \ln P_{jb}} + 1$ and $\frac{\partial \ln R_{jb}}{\partial \ln P_{j'b}} = \frac{\partial \ln q_{jb}}{\partial \ln P_{j'b}} \quad \forall j' \neq j$, the own elasticity and cross-price elasticities of demand for product j at store b are

$$\begin{aligned} \frac{\partial \ln q_{jb}}{\partial \ln P_{jb}} &= \frac{\partial \ln R_{jb}}{\partial \ln P_{jb}} - 1 = -\alpha + (\alpha - \eta) s_{j|b}, \\ \frac{\partial \ln q_{jb}}{\partial \ln P_{j'b}} &= (\alpha - \eta) s_{j'|b} \end{aligned}$$

Using these expressions to re-state the first-order conditions for pricing,

$$q_{jb} + \sum_{j' \in J_b} \frac{\partial q_{j'b}}{\partial p_{jb}} (p_{j'b} - c_{j'b}) = 0 \quad \forall j \in J_b,$$

we obtain:

$$\begin{aligned} \frac{q_{jb}}{E} + \frac{\partial q_{jb}}{\partial p_{jb}} \frac{1}{q_{jb}} \left(\frac{p_{jb} q_{jb}}{E} \right) \left(\frac{p_{jb} - c_{jb}}{p_{jb}} \right) + \sum_{j' \in J_b, j' \neq j} \frac{\partial q_{j'b}}{\partial p_{jb}} \frac{1}{q_{j'b}} \left(\frac{q_{j'b} p_{j'b}}{E} \right) \left(\frac{p_{j'b} - c_{j'b}}{p_{j'b}} \right) &= 0 \\ \frac{p_{jb} q_{jb}}{E} + \frac{\partial q_{jb}}{\partial p_{jb}} \frac{p_{jb}}{q_{jb}} \left(\frac{p_{jb} q_{jb}}{E} \right) \left(\frac{p_{jb} - c_{jb}}{p_{jb}} \right) + \sum_{j' \in J_b, j' \neq j} \frac{\partial q_{j'b}}{\partial p_{jb}} \frac{p_{jb}}{q_{j'b}} \left(\frac{q_{j'b} p_{j'b}}{E} \right) \left(\frac{p_{j'b} - c_{j'b}}{p_{j'b}} \right) &= 0 \\ s_{jb} + \frac{\partial \ln q_{jb}}{\partial \ln p_{jb}} (s_{jb}) \left(\frac{p_{jb} - c_{jb}}{p_{jb}} \right) + \sum_{j' \in J_b, j' \neq j} \frac{\partial \ln q_{j'b}}{\partial \ln p_{jb}} s_{jb} \left(\frac{p_{j'b} - c_{j'b}}{p_{j'b}} \right) &= 0 \\ s_{jb} + (-\alpha + (\alpha - \eta) s_{j|b}) (s_{jb}) \left(\frac{p_{jb} - c_{jb}}{p_{jb}} \right) + \sum_{j' \in J_b, j' \neq j} ((\alpha - \eta) s_{j'|b}) s_{jb} \left(\frac{p_{j'b} - c_{j'b}}{p_{j'b}} \right) &= 0 \\ 1 - \alpha \left(\frac{p_{jb} - c_{jb}}{p_{jb}} \right) + (\alpha - \eta) \sum_{j' \in J_b} s_{j'|b} \left(\frac{p_{j'b} - c_{j'b}}{p_{j'b}} \right) &= 0 \end{aligned}$$

Since this relationship holds for all $j \in J_b$, the mark-up for each product must be the same.

Call it $\mu = \frac{p_{jb} - c_{jb}}{p_{jb}}$ and reduce this equation to $1 - \alpha\mu + (\alpha - \eta)\mu = 0$, or

$$\mu = \frac{1}{\eta}.$$

Essentially the same result can be found in Atkeson and Burstein (2008) and Hottman *et al.*

(forthcoming).

B Bargaining

As discussed in the text, the transfer function $\tau_s^S(i)$ and buyer payoff function $\pi_s^B(i)$ are chosen to satisfy the surplus sharing rule

$$(1 - \beta) V_s^S(i) = \beta [V_s^B(i) - V_{s-1}^B(i)], \quad s \in \{1, 2, \dots, s^{\max}\}, \quad i \in \{1, 2, \dots, N^B\}, \quad (\text{A-1})$$

subject to the flow balance condition (4). This appendix derives closed-form expressions for each function. The logic is similar to that found in Bertola and Garibaldi (2001), though it is adapted to our discrete state space.

Suppressing buyer type indices, the flow value of a buyer who is currently matched with s sellers is:

$$\rho V_s^B = \pi_s^B - k_s^B(\sigma_s^B) + \sigma_s^B \theta^B (V_{s+1}^B - V_s^B) + s\delta(V_{s-1}^B - V_s^B),$$

Likewise the value of a buyer with $s - 1$ sellers is:

$$\rho V_{s-1}^B = \pi_{s-1}^B - k_{s-1}^B(\sigma_{s-1}^B) + \sigma_{s-1}^B \theta^B (V_s^B - V_{s-1}^B) + (s - 1)\delta(V_{s-2}^B - V_{s-1}^B)$$

Differencing these equations, and approximating the first-order condition $k'_{s-1}(\sigma_{s-1}^B) = \theta^B(V_s^B - V_{s-1}^B)$ with

$$\frac{k_s^B(\sigma_s^B) - k_{s-1}^B(\sigma_{s-1}^B)}{\sigma_s^B - \sigma_{s-1}^B} \approx \theta^B(V_s^B - V_{s-1}^B),$$

one obtains:

$$\begin{aligned} (\rho + \delta)(V_s^B - V_{s-1}^B) &= (\pi_s^B - \pi_{s-1}^B) + \sigma_s^B \theta^B [(V_{s+1}^B - V_s^B) - (V_s^B - V_{s-1}^B)] \\ &\quad + (s - 1)\delta[(V_{s-1}^B - V_s^B) - (V_{s-2}^B - V_{s-1}^B)] \end{aligned} \quad (\text{A-2})$$

Also, on the seller side, equation (14) implies:

$$(\rho + \delta)V_s^S = \tau_s^S + \sigma_s^B \theta^B [V_{s+1}^S - V_s^S] + (s - 1)\delta[V_{s-1}^S - V_s^S] \quad (\text{A-3})$$

Substituting (A-2) and (A-3) into (A-1), cancelling terms, and bringing back i indices, we are left with equation (6) of the text:

$$(1 - \beta)\tau_s(i) = \beta [\pi_s^B(i) - \pi_{s-1}^B(i)]$$

It remains to solve for $\tau_s(i)$ and $\pi_s^B(i)$. Using equation (4), this equation can be re-stated

as:

$$(1 - \beta) \left(\frac{\pi_s^T(i) - \pi_s^B(i)}{s} \right) = \beta [\pi_s^B(i) - \pi_{s-1}^B(i)] \quad (\text{A-4})$$

Or, rearrranging:

$$\pi_s^B(i) = \pi_s^T(i) \left(1 + s \frac{\beta}{1 - \beta} \right)^{-1} + s \frac{\beta}{1 - \beta} \left(1 + s \frac{\beta}{1 - \beta} \right)^{-1} \pi_{s-1}^B(i)$$

So given that $\pi_0^B(i) = 0$, the solutions for $\pi_s^B(i)$, $s = 1, \dots, 4$ are:

$$\pi_1^B(i) = \left(1 + \frac{\beta}{1 - \beta} \right)^{-1} \pi_1^T(i)$$

$$\pi_2^B(i) = \left(1 + 2 \frac{\beta}{1 - \beta} \right)^{-1} \pi_2^T(i) + 2 \frac{\beta}{1 - \beta} \left(1 + 2 \frac{\beta}{1 - \beta} \right)^{-1} \left(1 + \frac{\beta}{1 - \beta} \right)^{-1} \pi_1^T(i)$$

$$\begin{aligned} \pi_3^B(i) = & \left(1 + 3 \frac{\beta}{1 - \beta} \right)^{-1} \pi_3^T(i) + 3 \frac{\beta}{1 - \beta} \left(1 + 3 \frac{\beta}{1 - \beta} \right)^{-1} \left(1 + 2 \frac{\beta}{1 - \beta} \right)^{-1} \pi_2^T(i) \\ & + 3 \frac{\beta}{1 - \beta} 2 \frac{\beta}{1 - \beta} \left(1 + 3 \frac{\beta}{1 - \beta} \right)^{-1} \left(1 + 2 \frac{\beta}{1 - \beta} \right)^{-1} \left(1 + \frac{\beta}{1 - \beta} \right)^{-1} \pi_1^T(i) \end{aligned}$$

$$\begin{aligned} \pi_4^B(i) = & \pi_4^T(i) \left(1 + 4 \frac{\beta}{1 - \beta} \right)^{-1} + 4 \frac{\beta}{1 - \beta} \left(1 + 4 \frac{\beta}{1 - \beta} \right)^{-1} \left(1 + 3 \frac{\beta}{1 - \beta} \right)^{-1} \pi_3^T(i) \\ & + 4 \frac{\beta}{1 - \beta} 3 \frac{\beta}{1 - \beta} \left(1 + 4 \frac{\beta}{1 - \beta} \right)^{-1} \left(1 + 3 \frac{\beta}{1 - \beta} \right)^{-1} \left(1 + 2 \frac{\beta}{1 - \beta} \right)^{-1} \pi_2^T(i) \\ & + 4 \frac{\beta}{1 - \beta} 3 \frac{\beta}{1 - \beta} 2 \frac{\beta}{1 - \beta} \left(1 + 4 \frac{\beta}{1 - \beta} \right)^{-1} \left(1 + 3 \frac{\beta}{1 - \beta} \right)^{-1} \left(1 + 2 \frac{\beta}{1 - \beta} \right)^{-1} \left(1 + \frac{\beta}{1 - \beta} \right)^{-1} \pi_1^T(i) \end{aligned}$$

And by induction, the general form of the buyer payoffs is:

$$\pi_s^B(i) = \sum_{\tau=1}^s \frac{s!}{(s - \tau + 1)!} \left(\frac{\beta}{1 - \beta} \right)^{\tau-1} \left[\prod_{\tau'=s-\tau+1}^s \left(1 + \tau' \frac{\beta}{1 - \beta} \right)^{-1} \right] \pi_{s-\tau+1}^T(i).$$

C Transition dynamics

Coming soon!

D Implementing the compatibility function

This appendix describes implementation of the compatibility function, $d(i, j)$ introduced in section 3.4. To keep things simple, we assume that the only exogenous dimension of heterogeneity is the menu of products that these buyers or sellers offer.⁹ Let there be k products of possible interest to importers of shoes, and let $\mathbf{a}_i$ be a $1 \times k$ vector of HS category dummies that summarize the subset of products that buyer i is interested in carrying. Similarly, let $\mathbf{a}_j$ be a $1 \times k$ vector of HS category dummies that seller j is willing to offer. The set of 2^k possible realizations of values for this vector corresponds to the set of possible buyer types, and the set of 2^k possible realizations of values for this vector corresponds to the set of possible seller types.

Clearly, when many HS product are considered, the set of possible combinations of buyer and seller types will be unworkably large. However, we can dramatically reduce the dimensionality of the problem by using a clustering algorithm to group sellers and buyers into N^S and N^B groups, respectively, based on the similarities of their product vectors.¹⁰ Then we

⁹Generalizations to incorporate continuous variables, like productivity, can be accomplished using copulas specific to each $i - j$ combination.

¹⁰Experiments will reveal how many clusters provide reasonable partitions of the sets of buyers and sellers. Of course, the partitions will reflect sampling error, particularly when small samples are used, and this may have implications for estimates of the model's deep parameters. Monte Carlo experiments should reveal whether this is a concern.

can use sample frequencies to approximate the probabilities of any particular $i - j$ buyer-seller match will be observed in a random sample of matches. Hereafter we refer to this probability as $f(i, j)$.

Dividing this probability by the probability that a type- i buyer and type- j seller will randomly meet gives the function $d(i, j)$. To do this, note the probability that a randomly matching buyer will encounter a type- i seller (compatible or otherwise) is

$$\phi_i^B = \frac{\sum_{s=0}^{\infty} \sigma_s^B(i) P_s^B(i)}{\sum_{i'} \sum_{s=0}^{\infty} \sigma_s^B(i') P_s^B(i')}$$

Likewise, the probability that a randomly matching buyer will encounter a type- j seller is

$$\phi_j^S = \frac{\sum_{b=0}^{\infty} \sigma_b^S(j) P_b^S(j)}{\sum_{j'} \sum_{b=0}^{\infty} \sigma_b^S(j') P_b^S(j')}.$$

So under simple random matching, the probability that any given match—successful or otherwise— involves a type- i seller and a type- j buyer is $\phi_i^B \phi_j^S$. The probability of observing a successful $i - j$ match is thus $f(i, j) = \phi_i^B \phi_j^S d(i, j)$, and the compability function is related to the distribution of types among successful matches by:

$$d(i, j) = \frac{f(i, j)}{\phi_i^B \phi_j^S}.$$

Substituting this expression back into the success rate functions yields:

$$a^B(i) = \frac{\sum_j \sum_{b=0}^{\infty} d(i, j) \sigma_b^S(j) P_b^S(j)}{\sum_{j'} \sum_{b=0}^{\infty} \sigma_b^S(j') P_b^S(j')} = \frac{\sum_j f(i, j)}{\phi_i^B} \quad (\text{A-5})$$

and

$$a^S(j) = \frac{\sum_i \sum_{s=0}^{\infty} d(i, j) \sigma_s^B(i) P_s^B(i)}{\sum_{i'} \sum_{s=0}^{\infty} \sigma_s^B(i') P_s^B(i')} = \frac{\sum_i f(i, j)}{\phi_j^S} \quad (\text{A-6})$$

Unsurprisingly, the success rate is just the ratio of the probability of observing a viable business relationship for a type j buyer to the probability that matching sellers meet type j buyers.

Note that the denominators of (A-5) and (A-6) depend only on parameters that appear in the non-assortative matching version of the model, so given that $f(i, j)$ comes directly from the data, no new parameters are involved. The only extra computational burden here comes from the fact that ϕ_i^B and ϕ_j^S depend upon policy functions and buyer type shares ($P_b^{S'}$ s and P_s^B). This necessitates a change in the solution algorithm.

Before moving to estimation, a few comments are in order concerning the limitations of our specification. First, as particular types of firms add clients, their preferences over the types of business partners they deal with do not change. This does not preclude the possibility that large firms will deal with different types of business partners than small firms, but it does imply that systematic size differences trace to associated differences in firm type. Second, our payoff functions are also limited in the sense that, given type, the only arguments are client counts. Thus importers don't care how their inventory breaks down between the categories of goods by buy, and exporters don't care which goods they export. At the expense of greater complexity and computational burden, more general payoff functions could presumably be used. Exploratory work on the nature of the payoff functions should reveal whether these properties should be relaxed

E A simple statistical model (not yet updated)

Finally, we assume that relationships end with a hazard δ regardless of number of contacts or type.

Putting these assumptions together gives us equations of motion for the measure of buyers of type i with s sellers:

$$\begin{aligned} \dot{M}_s^B(i) = & \frac{s^{\gamma^B} \sigma^B(i) M_{s-1}^B(i)}{H^B} \frac{H^B H^S}{[(H^B)^\alpha + (H^S)^\alpha]^{1/\alpha}} + \delta(s+1) M_{s+1}^B(i) \\ & - \left(\frac{(s+1)^{\gamma^B} \sigma^B(i) M_s^B(i)}{H^B} \frac{H^B H^S}{[(H^B)^\alpha + (H^S)^\alpha]^{1/\alpha}} + \delta_s M_s^B(i) \right). \end{aligned}$$

The measure of buyers of type i with $s = 0$ sellers evolves according to:

$$\dot{M}_0^B(i) = \delta M_1^B(i) - \frac{\sigma^B(i) M_0^B(i)}{H^B} \frac{H^B H^S}{[(H^B)^\alpha + (H^S)^\alpha]^{1/\alpha}}$$

The measure of sellers of type j with $b > 0$ buyers evolves according to:

$$\begin{aligned} \dot{M}_b^S(j) = & \frac{b^{\gamma^S} \sigma^S(j) M_{b-1}^S(j)}{H^S} \frac{H^B H^S}{[(H^B)^\alpha + (H^S)^\alpha]^{1/\alpha}} + \delta(b+1) M_{b+1}^S(j) \\ & - \left(\frac{(b+1)^{\gamma^S} \sigma^S(j) M_b^S(j)}{H^S} \frac{H^B H^S}{[(H^B)^\alpha + (H^S)^\alpha]^{1/\alpha}} + \delta_b M_b^S(j) \right). \end{aligned}$$

The measure of sellers of type j with $b = 0$ buyers evolves according to:

$$\dot{M}_0^S(j) = \delta M_1^S(j) - \frac{\sigma^S(j) M_0^S(j)}{H^S} \frac{H^B H^S}{[(H^B)^\alpha + (H^S)^\alpha]^{1/\alpha}}$$

It's useful to examine what happens in a stationary state in which the distribution of contacts for both buyers and sellers is constant. To do so define the ratios:

$$\lambda_s^B(i) = \frac{M_s^B(i)}{M_{s-1}^B(i)}$$

and:

$$\lambda_b^S(j) = \frac{M_b^S(j)}{M_{b-1}^S(j)}$$