Competitive or Random Search?

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Abstract

We set up a search model with on-the-job search that spans random and competitive search as well as intermediate cases with partially directed search. Firms are heterogeneous in terms of productivity, and firms with different productivities are in different submarkets, with all firms in the same submarket being identical. Workers are identical, still a worker’s optimal search behavior, that is, the optimal submarket for her to visit, depends on the productivity of the current employer. The higher is the productivity of the current employer, the higher is the productivity of the firm it is optimal for the worker to target her search at. We use a discrete choice framework to model the workers’ choice of submarkets, with a single noise parameter μ governing the degree to which search is directed. As μ goes to zero, search becomes fully directed, while it becomes random as μ goes to infinity. We derive the equilibrium of the model, and simulate the equilibrium allocation. We also show how the parameter μ can be identified using a maximum likelihood approach where submarkets are identified via the Bagger and Lentz (2016) poaching rank, or in a classification step as in Lentz, Piyapromdee and Robin (2017). Alternatively, we use a simulated method of moments approach. We will identify μ on Danish and/or Norwegian matched employer-employee data.

1 Introduction

Over more than a decade, following Postel-Vinay and Robin (2002) and Christensen et al. (2005), a literature has developed in which matched employer-employee register data sets are used to structurally estimate search models of the labor market. In all the papers in this literature, it is assumed that search is random; the probability that a searching worker meets a firm is independent of the attributes (wages, productivities) of the jobs it has to offer. The underlying assumption is that workers are unable to direct their search towards the jobs they find most attractive.1

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1There is a substantial literature on job-to-job movements. First, show that job-to-job flows are huge. Christensen et al. (2005) find that reallocation of workers from low- to high productivity firms are important for economic growth. Postel-Vinay and Robin (2002) were among the first to structurally estimating a search model with on-the-job search. Other recent papers on on-the-job search include Lentz and Mortensen (2012), Bagger and Lentz (2013), Bagger et al. (2013), Lise and Robin (2013), and Lise et al. (2016).
In Garibaldi, Moen, and Sommervoll (2016) it is pointed out that random search has strong implications for the relationship between a worker’s wage prior to and after a job change (hereafter the ante and post wage). Suppose search is random, and that a worker changes jobs whenever a new employer offers a higher wage than his current one. Let \( w_a \) denote the ante wage. Then for any wage \( \bar{w} > w_a \), the distribution for the post wage \( w_p \), conditional on being above \( \bar{w} \), is independent of the ante wage \( w_a \). If we denote the conditional distribution of post wages by \( G(\bar{w} | w_a) \), random search implies that this distribution is independent of \( w_a \). As a result, the unconditional probability ratio \( g(w_1 | w_a) / g(w_2 | w_a), \) \( w_1, w_2 > w_a \), is independent of \( w_a \). If, as in Postel-Vinay and Robin, workers change jobs if and only if the new job has a higher productivity, the analogue property is true for the post productivity distribution relative to the ante productivity of the previous employer.

However, Godøy and Moen (2015) document that this independence property between ante and post wages or productivities cannot be found in the data. If jobs are ranked into percentile wage or productivity bins, workers are typically moving to the percentile bin just above the post wage in a way that is inconsistent with random search.

Godøy and Moen’s findings indicate that search to some extent is directed, as in Moen (1997). However, search does not seem to be fully directed, at least not in the wage space. Fully directed search would imply that there is a one-to-one correspondence between ante and post wages, while there is still quite a bit of randomness in the ante-post wage relationship. Godøy and Moen (2015) sets up a model where the workers direct their search fully, and independently receive random offers. Hence the post wage distribution is random but with a mass point. Their model cannot easily be estimated.

In the present paper, we set up a model in which all submarkets/jobs have a positive probability to be chosen by a searching worker, but the probability of being chosen is increasing in the attractiveness of the submarket/job in question. More specifically, we use a discrete-choice approach for searching workers’ choice of submarkets.\(^2\) We interpret the degree of noise as the degree of randomness in the search process: there is as a random component in the search process that makes workers unable to fully direct their search. With this interpretation, our model nests directed and random search as special cases. The random component may also reflect that workers have preferences over more aspects of a job than wages. Jobs come with different amenities, and that these amenities are unobservable to the econometrician. The choice of interpretation has very limited consequences for the model as such.

2 Model

The model is set in continuous time. The economy is populated by a continuum of risk neutral workers and firms with a common discount rate \( r \). All workers are identical, infinitely lived, and can costlessly search both on and off the job. The number (measure) of workers in the economy is exogenous and normalized to one. Unemployed workers receive an exogenous income of \( y_0 \). Firms

\(^2\)Another paper that uses a discrete-choice approach within a matching context is 'Cheremukhin et al. (2014).
differ in productivity, and the number (measure) of firms is endogenously determined by entry. Entrepreneurs sink a cost $K$ to open a firm. After $K$ is sunk, but before production starts, the productivity $y$ of a firm is drawn from a discrete distribution $(y_1, \tau_1), \ldots, (y_n, \tau_n)$, $y_0 < y_1, \ldots < y_{n-1} < y_n$, where $y_j$ is the productivity of a firm of type $j$ and $\tau_j = \text{prob}[y = y_j]$. Firms exit the market at an exogenous rate $\delta$. In addition, workers are laid off with an exogenous, time varying rate $s_k$. Specifically, we assume that all jobs start with a low layoff rate of $s = s_0$ and at Poisson rate $\lambda$ it increases to $s = s_1 > s_0$. Firms are endowed with a search technology, and can costlessly post one vacancy. If the firm finds a worker, it immediately repost the vacancy, and may grow over time (the “fishing line” assumption). It follows that all firm types operate in the market. Workers also have one unit of search capacity, but can only have one job at the time.

As with directed search, we assume that firms are allocated on submarkets in accordance with their productivity. The economy consists of $n$ submarkets, and all firms in submarket $j$ are of type $j$. In each submarket $j$, the flow of new matches is given by a concave and constant-returns to scale matching function $x(\tilde{n}_j, v_j)$, where $\tilde{n}_j$ and $v_j$ denote the number of searching workers and searching firms in this submarket, respectively. An employed worker searches so as to maximize the joint income of the worker-firm pair, reflecting that a firm and a worker are able to sort out any coordination problem that may arise between them. It is furthermore assumed that the worker’s wages in submarket $j$ are set through a bargaining process where the worker’s outside option is the full match value in the previous submarket $(i,k)$. The worker receives a fraction $\eta$ of the surplus in the negotiation. This is the same type of wage determination as in Dey and Flinn (2005) and Cahuc et al. (2006).

Denote by $m_{ijk}$ the flow value of search, that is, the flow income of a worker-firm pair in state $(i,k)$ (employed by a firm of type $i$, $i \in \{0, \ldots, N\}$, with separation rate $s_k$, $k \in \{0,1\}$), searching for a job in submarket $j$ conditional on acceptance of a match submarket $j$. Denote by $M_{ik}$ the net present value of the joint income of a worker and a type $i$ firm, when the separation rate is $s_k$.

As will be clear below, the probability that a worker ends up in a given submarket depends on the return to search in this submarket. In submarkets in which a match gives a positive surplus, the worker receives a share $\eta$ of this match surplus, and the return to search is determined accordingly. In submarkets in which the match surplus is negative, any job offer will be turned down (hence we do not assume that the agents have committed to accept a match). It is not obvious how the return to search in these markets should be determined. If we assume that workers foresee that the match will be turned down, all submarkets in which the match surplus is negative will give the same return to search, and hence receive the same number of searching workers. We find this unsatisfactory. We therefore calculate the perceived return from search, $m_{ijk}^p$, in markets with negative return to search as the return that prevails if the worker is forced to take the job, and in addition has to absorb the entire loss associated with the unprofitable move. Under these assumptions,

$$m_{ijk}^p = y_i + (\delta + s_k)(M_0 - M_{ik}) + \lambda(M_{i1} - M_{ik})$$
$$+ \eta p(\theta_j) \max(M_{j0} - M_{ik}, 0) - p(\theta_j) \max(M_{ik} - M_{j0}, 0). \quad (1)$$
Notice that the value of unemployed search, $M_{0k}$, is independent of $k$. When convenient we will remove the $k$ subscript. $m_{ijk}^p$ is defined conditional on acceptance of a match in submarket $j$, which can then involve a value loss in which case the outside firm would not share the value loss to the worker. Thus, the asymmetry in the application of the worker’s bargaining strength, $\eta$, in the expression.

At any point in time, the worker chooses which submarket to enter. The pay-off to the worker-firm pair varies between submarkets, and the worker aims at going for the one that gives the highest pay-off. However, due to un-modeled frictions in the search process, the worker may enter a search market that gives less than the optimal flow pay-off, and may even end up in submarkets with jobs that he will reject. Hence we assume that the workers “tremble”, as in the quantile response equilibrium (see McKelvey, R. D., and T. R. Palfrey (1995): “Quantal response equilibria for normal form games”, *Games and economic behavior*, 10(1), 6-38.) When modeling the choice of submarkets we apply a discrete choice approach. Let $\pi_{ijk}$ denote the probability that a worker in current state $i,k$ (employed in a firm of type $i$ with lay-off rate $s_k$ ) enters submarket $j$. We assume that $\pi_{ijk}$ can be written by the logit expression

$$\pi_{ijk} = \frac{\tau_j \exp \left[ m_{ijk}^p / \mu \right]}{\sum_{j=1}^{N} \tau_j \exp \left[ m_{ijk}^p / \mu \right]}.$$  

(2)

As mentioned above, the stochastic nature of the workers’ choice of submarkets may be given different interpretations, see Anderson et al. (1992). One interpretation is that it reflects genuine noise in the search process; a worker wants to target his search towards the submarkets that give the highest utility. However, frictions in the process imply that he is only partly successful in doing so. The worker may for instance misjudge the wage/productivity of the firm he applies to. Another interpretation may be that the stochastic choice of submarkets reflects unobserved heterogeneities,

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3The underlying assumption is that workers observe the return to search in each submarket with noise. The workers want to target their search towards the submarkets that give the highest return to search. However, the process is partly corrupted, and workers may end up in other submarkets, with a higher probability the more attractive the submarket is. We allow workers to search in submarkets with a lower NPV than their current job, and that offers from these markets will be refused. This assumption is necessary given that we want to nest the random search model as an extreme outcome.

An alternative assumption, which implies that workers only search for jobs they strictly wants, can be as follows: Let the probabilities be given by (2), but replace $m_{ijk}$ by $x = m_{ijk} - \frac{1}{m_{ijk} - m_{ijk}}$ for $m_{ijk} > m_{ijk}$, and $x = -\infty$ otherwise. Since $x$ is strictly increasing in $m_{ijk}$, this formulation implies that the worker will be more likely to go to the markets that give the highest return, and that the probability of going to a given market goes to zero as the gain from search in this market approaches zero.

Another alternative assumption is that the workers observe the productivity $y_j$ of a given firm/submarket with noise. Let $y^p$ denote the perceived productivity in a market. In this case, the NPV incomes $M_j$, and hence the flow values $m_{ijk}$ depend on $y^p$ in a non-linear way. Hence imposing the extreme value distribution on $y^p$ does not lead to simple, closed form solutions for the choice probabilities as functions of the asset values. An alternative could be as follows: Let $y^*_j$ denote the optimal output level to target for a worker in state $i,k$. Then the probability of choosing a submarket with productivity $y$ would be given by the logit expression as a function of the distance between $y^*$ and $y$, for instance $(y^*_j - y)^2$.

4On an abstract level, we may assume that a market maker allocates workers and firms to submarkets. Our assumption is that the market maker “trembles” when he allocates workers on submarkets, but not when he allocates firms on submarkets. We conjecture that having two-sided trembles qualitatively will give the same results.
like amenities. The choice of interpretation will only have limited consequences for the setup of the model. Our interpretation is that the noise reflects imperfections in the workers’ ability to direct their search. This implies that firm and job heterogeneities like differences in amenities, preferences for geographical location, changes in preferences over job characteristics that lead to job changes etc., will be counted as random search. In that sense, the model is rigged towards random search.

From (2) it is clear that with a strictly positive probability, a worker may end up in a market with jobs that give lower expected income than the current job, i.e., that $M_{ik} < M_{j0}$. If search is successful, it is assumed that the worker can freely turn down the offer. Nevertheless, the worker does crowd the submarket. Let $\bar{m}_{ik}$ denote the flow value of the joint worker-firm income if the worker does not search, i.e., $\bar{m}_{ik} = y_i + (\delta + s_k) (M_0 - M_{ik}) + \lambda (M_{i1} - M_{ik})$. The NPV of the joint income of a worker and a firm of type $i$, $i = 0, \ldots, N$ given layoff rate $k = 0, 1$ is then,

$$rM_{ik} = \sum_{j=1}^{N} \pi_{ijk} \max\{m_{ijk}^p, \bar{m}_{ik}\}$$  (3)

The tightness of submarket $j$ is given by,

$$\theta_j = \frac{\tau_j^k}{\sum_{k=0}^{1} \sum_{i=1}^{N} \pi_{ijk} n_{ik} + \pi_{0j} n_0}$$  (4)

where $n_{ik}$ is the number of workers in submarket $(i, k)$ and $n_0$ is the unemployment rate.

When a firm and a worker form a match, the terms of trade are determined by bargaining. In this bargaining game, the outside option of the worker is the NPV value of the joint income $M_{ik}$ obtained in his previous job. The firm owns zero on the job slot if the worker leaves, hence his outside option in the bargaining game is also zero. Together these assumptions ensure that a match leads to a job-to-job transition if and only if this is efficient. Finally, we assume that the Hosios condition is satisfied, so that the bargaining power of the worker is equal to $\eta = -elq(\theta)$.

\[ \text{Several issues arise with the amenity interpretation. The first issue regards whether the amenities are associated with jobs or markets. If they are associated with jobs, the value of the amenity will have to be weighted with the probability rate of finding a job, which is endogenous. Hence parametric assumptions on the probability distribution over amenities do not carry over to probability distributions on the attractiveness of submarkets. Obtaining closed-form solutions for the choice probabilities as functions of the market characteristics is therefore not trivial.} \]

\[ \text{Suppose instead that the amenities are associated with the markets as such. The markets may be more or less convenient to search in, and how convenient they are to search in is assumed to be iid over time and workers. The amenities are thus a flow value that accrue to the worker, and are gone when the worker bargain over wages with the firm. With this interpretation, the model is very similar to the model presented in the main text.} \]

\[ \text{At any point in time, the value to a worker in state } i, k \text{ of searching in market } j \text{ is given by } m_{ijk}/r + \varepsilon_j, \text{ where } \varepsilon_j \text{ is stochastic and distributed according to the and double exponential distribution. Suppose further that the worker in question choses the market with the highest total utility. Then the choice probabilities are given by equation (2) for all submarkets with jobs that the worker will accept.} \]

\[ \text{If workers foresee that they will turn down offers, the attractiveness of submarkets with jobs that will not be accepted will be independent of the firm productivities in these submarkets. Alternatively, we may introduce the concept of the perceived gain from search in this set-up as well, in which case the model will be identical to the model in the main text.} \]

\[ \text{This follows from the fact that the firm continuously searches, so that a worker does not fill a slot that alternatively could have been filled by another worker.} \]
The value of a fishing line to a firm in submarket \( j \) is,

\[
V_j = q(\theta_j)(1 - \eta) \left[ \sum_{i=1}^{N} \sum_{k=0}^{1} \max(M_{j0} - M_{ik}, 0) f_{ijk} + (M_{j0} - M_0) f_{0j} \right] / (r + \delta),
\]

where \( f_{ijk} \) is the probability that a worker who is searching in submarket \( j \) is currently in state \((i, k)\), where \( i = 0 \) is unemployment. By definition, \( \sum_{i=0}^{N} \sum_{k=0}^{1} f_{ijk} = 1 \). This probability is defined by,

\[
f_{ijk} = \frac{\pi_{ijk} n_{ik}}{\sum_{k=0}^{1} \sum_{i=1}^{N} \pi_{ijk} n_{ik} + \pi_{0j} n_0}, \forall i > 0
\]

\[
f_{0j} = \frac{\pi_{0j} n_0}{\sum_{k=0}^{1} \sum_{i=1}^{N} \pi_{ijk} n_{ik} + \pi_{0j} n_0}.
\]

In equilibrium, the total number of firms (equal to the total number of vacancies) is given by the free entry condition,

\[
E[V] = \sum_{j=1}^{N} \tau_j V_j = K.
\]

Turning to steady state, the number of workers in submarket \((i, k)\) satisfies the flow balance equations for \( i = 1, \ldots, N \),

\[
n_{i0} \left( \delta + s_0 + \lambda + \sum_{j=i+1}^{N} \pi_{ij0} p(\theta_j) \right) = p(\theta_i) \left[ \pi_{0i} n_0 + \sum_{j=1}^{i-1} \pi_{ji0} n_{j0} \right.
\]

\[
+ \sum_{j=1}^{N} \mathbb{1} \{M_{j1} < M_{i0}\} \pi_{ji1} n_{j1} \right]
\]

\[
n_{i1} \left( \delta + s_1 + \sum_{j=1}^{N} \mathbb{1} \{M_{i1} < M_{j0}\} \pi_{ij1} p(\theta_j) \right) = \lambda n_{i0}.
\]

**Definition 1.** A steady state equilibrium with wage bargaining is a matrix of net present values, \( M_{ik}, i = 0, \ldots, N, k = 1, 2 \), a matrix of perceived flow values, \( m_{ijk}^f, i = 0, \ldots, N, j = 1, \ldots, N, k = 0, 1 \), a matrix of choice probabilities, \( \pi_{ijk}, i = 0, \ldots, N, j = 1, \ldots, N, k = 0, 1 \), a vector of labour market tightnesses, \( \theta = (\theta_1, \ldots, \theta_N) \), a number of firms, \( k \), and an allocation of workers on states and firms, \( n = (n_{01}, \ldots, n_{0N}), \hat{n} = (n_{11}, \ldots, n_{1N}) \) such that

1. The matrices \( M_{ik} \) and \( m_{ijk}^f \) solve equations (1) and (3).
2. The matrix of choice probabilities, \( \pi_{ijk} \), is given by (2).
3. The vector of labour market tightnesses \( (\theta_1, \ldots, \theta_N) \) is given by equation (4).
4. The number of firms, \( k \), is implicitly defined by equations (5) and (8).
5. The allocation of workers on states and firms, \( n \) and \( \hat{n} \) solves equations (9)-(11)

**Proposition 2.** The equilibrium exists.

**Proof.** The proof is based on a fixed-point argument. We construct a mapping \( \Gamma(M, \theta, k, n, \hat{n}) \rightarrow (M', \theta', k', n', \hat{n}') \) where all the elements are bounded below by 0 and above by some constant \( S_{\text{max}} \) and that \( \theta_j \forall j \) and \( k \) are bounded below by a constant \( S_{\text{min}} \). Given that the bounds don’t bind, the mapping goes as follows

1. Given \( M \) and \( \theta \), construct \( m_{ijk} \) from equation (1).
2. Given the matrix \( m_{ijk}^f \), construct the choice probabilities \( \pi_{ijk} \).
3. Given \( \theta \) and \( M \), and \( \pi_{ijk} \), calculate \( f_{ijk} \) from (6) and (7), and then \( V \) from equation (8).
4. Update \( \theta \) by equation (4), \( \theta'_j = \frac{\tau_j^k}{\sum_{k=0}^{\infty} \sum_{i=1}^{N} \pi_{ijk} n_{ik} + \pi_{0j} n_0} \).
5. Update \( k \): \( k' = kV/K \).
6. Update \( n, \hat{n} \):

\[
n_0 \sum_{j=1}^{N} \pi_{0j} p(\theta_j) = (1 - n_0) \delta + \sum_{k=0}^{1} \sum_{j=1}^{N} n_{jk} s_k \tag{12}
\]

\[
n_{i0}' \left( \delta + s_0 + \lambda + \sum_{j=i}^{N} \pi_{ij0} p(\theta_j) \right) = p(\theta_i) \left[ \pi_{0i} n_0 + \sum_{j=1}^{i} \pi_{j0} n_{j0} + \sum_{j=1}^{N} \{ M_{j1} < M_{i0} \} \pi_{ji1} n_{j1} \right]\tag{13}
\]

\[
n_{i1}' \left( \delta + s_1 + \sum_{j=1}^{N} \{ M_{i1} < M_{j1} \} \pi_{i1j} p(\theta_j) \right) = \lambda n_{i0}. \tag{14}
\]

If the upper or the lower bound binds for a variable, then the variable is equal to the bound.

Given the lower bound on \( k \) and \( \theta_i \), it follows by construction that the mapping is continuous. Hence it follows from Brouwer’s fix-point theorem that the mapping has a fixed point, which we denote by \( (M^*, \theta^*, k^*, n^*, \hat{n}^*) \). By construction, the fix-point satisfies all the equilibrium requirements provided that bounds don’t bind. Hence we only have to show that the bounds don’t bind.

In any equilibrium constellation, \( \theta_j < \tau_j k \) for all \( i \) (the tightness obtained obtained if all workers search in market \( j \). Hence as \( k \to 0, \theta_j \to 0 \) for all \( j \), and hence \( V \to \infty \) (one can show that \( V_j \) goes to infinity for all \( j \)). Hence there exists a value \( S_{\text{min}}^m \) such that \( V > K \) for all \( k \leq S_{\text{min}}^m \). Let \( t_{\text{min}} = \min[\tau_1, \ldots, \tau_n] \). Define \( S_{\text{min}} = S_{\text{min}}^m r_{\text{min}} \). By construction, the lower bound does not bind at the fixed point neither for \( k^* \) nor \( \theta_j^*, j = 1, \ldots, n \).

An upper bound for the profit of a firm is \( \frac{y_n}{(r+\delta)k} \) (the net present value of a firm of type \( n \) that employs \( 1/k \) workers). Hence an upper bound on \( k \) is \( k_{\text{max}} = \frac{y_n}{(r+\delta)k} \). \( M_{ik} \) is bounded above by \( y_n/r \). We have only left to show that \( \theta_j \) is bounded. For any \( \mu \) this follows from the fact that all \( \pi_{ij} \) are strictly positive, so that the denominator in (4) is strictly positive (since \( k \) is bounded by \( k^m \)).

We want to look at the limit properties of the model. Consider first the equilibrium of the
model when $\mu \to \infty$. In this case, (2) implies that the probability that a worker goes to submarket $j$ is equal to $\tau_j$. Hence the worker has the same probability of meeting any firm, and search is random.

Suppose then that $\mu \to 0$. In the limit, the worker will go to job market $j$ if and only if this submarket maximizes his return from search. Let $I_{ik}$ denote the set of submarkets that a worker in state $i,k$ searches to. Then for all $j \in I_{ik}$, $M_{ijk} = \max_l M_{ikl}$. By definition it follows that $\pi_{ijk} = 0$ iff $j \notin I_{ijk}$. If $I_{ijk}$ contains a single element $j^*$, then $\pi_{ij^*k} = 1$. The definition of equilibrium is as before, except that requirement 2 is replaced by the requirement that for all $j \in I_{ik}$, $M_{ijk} = \max_l M_{ikl}$ (note that the reduction in the number of variables is exactly equal to the reduction in the number of equations).

We say that a state $i,k$ is inferior to another state $i',k'$ if $M_{ik} < M_{i'k'}$. We can then show the following

**Lemma 3.** The limit economy exists. Furthermore, if $i,k$ is inferior to $i',k'$, then the largest element in $I_{i,k}$ is smaller than or equal to the smallest element in $I_{i',k'}$.

The existence proof follows the same lines as above, and is omitted for now. The second part of the lemma states that the equilibrium leans towards a job ladder. The higher is the current state (higher NPV income), the higher submarkets the agent searches for. This follows from a single-crossing property of the indifference curves of agents in different states: the higher is the current npv income, the more “patient” is the agent, in the sense of being more willing to wait to obtain a higher wage offer. Submarkets with high firm types offer higher wages and longer queues (lower job finding rates). Hence, if a worker in a low state is indifferent between submarket $j$ and $j + 1$, a worker in a higher state will strictly prefer submarket $j + 1$. The details of the proof is omitted for now, but follows the lines of the proof of the maximum separation result in Garibaldi, Moen and Sommervoll (2016).

It follows that as long as each submarket attracts workers of one type (that is, in one particular state) only, the equilibrium with $\mu = 0$ is identical to the competitive search equilibrium with on-the-job search derived in Garibaldi, Moen and Sommervoll. If some submarkets attract more than one worker type, search is still fully directed, but not identical to the competitive search equilibrium. The reason is that in competitive search equilibrium, different submarkets would emerge for different worker types that search for the same firm type, and the different submarkets would have different labor market tightnesses.

In Garibaldi, Moen and Sommervoll, it is shown that with a continuum of firm types, the job ladder is strict, in the sense that firms of a given type will attract only one type of workers. We conjecture that this is also the case in our model with bargaining. Furthermore, we conjecture that as the measure of firms of each type goes to zero, the equilibrium with wage bargaining and $\mu=0$ converges to the competitive search equilibrium (with for instance welfare or job-to-job flows as metric).
3 Specification and solution algorithm

3.1 Specification

Let the matching function in a submarket be Cobb-Douglas, $M(s, v) = As^\alpha v^{1-\alpha}$. The worker meets a vacancy at rate $p(\theta) = M(s, v) / s = m(\theta) = A\theta^{1-\alpha}$. Hence, $p(\theta) = A\theta^{1-\alpha}$ and $q(\theta) = A\theta^{-\alpha}$.

3.2 Solution algorithm

1. The solution algorithm looks to find a fixed point in a mapping of $(\theta, k)$ into itself, $(\theta(m+1), k(m+1)) = T_{\theta k}(\theta(m), k(m))$.

2. The mapping $T_{\theta k}$ is determined as,

3. Take input $(\theta(m), k(m))$.

4. Solve for $(m^{(m)}, \pi^{(m)})$ for given $\theta^{(m)}$ using equation systems (1), (3), and (2).
   
   (a) This equation system is solved as fixed point search $M^* = T_M M^*$.

   i. Make a guess, $M^{(s,m)}$.
   ii. Solve for $m^{(s,m)}$ using equation (1) - uses $M^{(s,m)}$ and $\theta^{(m)}$.
   iii. Solve for $\pi^{(s,m)}$ using equation (2) - uses $m^{(s,m)}$.
   iv. Solve for $M^{(s+1,m)}$ using equations (3) - uses $\theta^{(m)}$ and $\pi^{(s,m)}$.

   A. This step uses that $M$ can be written as a contraction mapping,

   $$M_{ik} = \frac{y_i + (\delta + s_k) M_0 + \lambda M_{i1} + \sum_{j=1}^{n} \pi_{ijk} p(\theta_j) \eta \max[M_{j0}, M_{ik}]}{r + \delta + s_k + \lambda + \sum_{j=1}^{n} \pi_{ijk} p(\theta_j) \eta}$$

   $$M_0 = \frac{y_0 + \sum_{j=1}^{n} \pi_{ijk} p(\theta_j) \eta M_{i0}}{r + \sum_{j=1}^{n} \pi_{0jk} p(\theta_j) \eta}$$

   Thus, simply iterate until convergence to obtain solution. This happens to be quite fast.

   v. loop back to step i with $M^{(s+1,m)}$ as input. Continue to iterate until convergence...
   no convergence guaranteed.

   (b) In practice, it is faster if the algorithm does not iterate for a full solution of this system in each $(m)$ iteration.

5. Solve for $n^{(m)}$ as the solution to the linear equation system in equations (9)-(11) taking as given $\pi^{(m)}$, $M^{(m)}$ and $\theta^{(m)}$.

6. Solve for $(\theta^{(m+1)}, k^{(m+1)})$ as the solution to equations (4) and (8).
(a) This step uses a speed up from the Cobb-Douglas formulation. We have that \( q(\theta) = \theta^{-\alpha} \).

Define,

\[
\tilde{\theta}_j^{(m)} = \frac{\tau_j}{\sum_{k=0}^{N} \sum_{i=1}^{N} \pi_{ijk} \pi_{ik}^{(m)} + \pi_{0j}^{(m)} \pi_{0k}^{(m)}}.
\]

With this, the free entry condition can then be written as,

\[
k^{-\alpha} \sum_{j=1}^{N} \tau_j \tilde{\theta}_j^{(m)} (1 - \eta) \left[ \sum_{i=1}^{N} \sum_{k=0}^{1} \max(M_{j0} - M_{ik}, 0) f_{ijk} + (M_{j0} - M_{0}) f_{0j} \right] = K.
\]

Hence,

\[
k^{(m+1)} = \left[ \sum_{j=1}^{N} \tau_j \left( \tilde{\theta}_j^{(m)} \right)^{-\alpha} (1 - \eta) \left[ \sum_{i=1}^{N} \sum_{k=0}^{1} \max(M_{j0}^{(m)} - M_{ik}^{(m)}, 0) f_{ijk} + (M_{j0}^{(m)} - M_{0}^{(m)}) f_{0j} \right] \right]^{1/\alpha}.
\]

And with this, it immediately follows that,

\[\theta_j^{(m+1)} = \tilde{\theta}_j^{(m)} k^{(m+1)}.
\]

7. With \((\theta^{(m+1)}, k^{(m+1)})\) loop back to step 3. Repeat to convergence.

### 3.3 Simulation

A worker \( i \)'s history consists of a sequence of spells \( \{\gamma_{il}, w_{il}, j_{il}, k_{il}\}_{l=1}^{L_i} \), where \( \gamma \) is the duration, \( w \) is the wage, \( j_{il} \) is the firm type of the spell and \( k_{il} \) is the layoff rate state. Denote by \( h_{jk} \) the hazard of spell with a \( j \) type firm and layoff rate state \( k \). Conditional on an employment spell, \( j > 0 \), the spell hazard is given by

\[
h_{jk} = \lambda (1 - k) + \delta + s_k + \sum_{j' = 1}^{N} \pi_{j'k} p(\theta_{j'}) \mathbb{I} [M_{jk} < M_{j'0}].
\]

If the spell is an unemployment spell, \( j = 0 \), the spell is (where the layoff state notation is suppressed),

\[
h_0 = \sum_{j' = 1}^{N} \pi_{0j'} p(\theta_{j'}).\]
Denote by \( \eta_{jj'kk'} \) the transition conditional probability that the state of the next spell is \((j', k')\) given that the current spell is state \((j, k)\). It follows directly that,

\[
\eta_{jj01} = \frac{\lambda}{h_{j0}}, \quad \forall j > 0 \\
\eta_{j0k} = \frac{\delta + s_k}{h_{jk}}, \quad \forall j > 0, \ k \in \{0, 1\} \\
\eta_{jj'kk} = \frac{\pi_{jj'k}p(\theta_{j'})1_{M_{jk} < M_{j'k}0}}{h_{jk}}, \quad \forall j, j' > 0, \ k \in \{0, 1\} \\
\eta_{0j} = \frac{\pi_{0j}p(\theta_{j})}{h_{0}}, \quad \forall j > 0,
\]

and \( \eta_{jj'kk'} = 0 \) for all other \((j, j', k, k')\) combinations.

The simulation initializes by steady state \( n_{jk} \). Thus, from this distribution draw an initial state for worker \( i \). Then simulate the duration \( \gamma_{il} \sim \text{Exp}(h_{jikl}) \). The state of the next spell is then drawn according to \( \eta_{jj'kk'} \). Let the length of the simulated panel of workers be \( T \) periods. Stop simulating spells for worker \( i \) when \( \sum_{l=1}^{L_i} \gamma_{il} > T \). Let there be \( I \) workers in the panel.

### 4 Estimation

The estimation is based on MEE data meaning we observe mobility patterns close to perfectly. Specifically, write a worker’s employment history as a series of spells \( X_i = \{\gamma_{il}, \phi_{il}\}_{l=1}^{L_i} \) where \( \gamma_{il} \) is worker \( i \)’s, \( l \)’th spell duration and \( \phi_{il} \) is the firm id of the \( l \)’th spell. The last spell is necessarily right censored meaning that the duration of the spell is greater than \( \gamma_{iL_i} \) and we do not know the destination firm. A nonemployment spell is associated with \( \phi_{il} = 0 \).

In the model there are \( N \) submarkets with a single firm type in each market. This suggests a simple discrete firm classification as in Bonhomme et al. (2015) and Lentz et al. (2016). There are \( J \) firms and each firm has a type that is drawn from the set \( j \in \{1, 2, \ldots, N\} \).

#### 4.1 The mobility likelihood for a known firm classification

To start with, suppose we have a known firm classification \( \mathcal{J} = \{j_1, j_2, \ldots, j_J\} \). Denote by \( j_{il} = \phi(i, l) \) the firm type that worker \( i \) is matched with in her \( l \)’th spell. Denote the job spell hazards conditional on firm type and layoff rate state by \( h_{j0} = \delta + s_0 + \lambda + \sum_{j'=j+1}^{N} \pi_{jj'0}p(\theta_{j'}) \) and \( h_{j1} = \delta + s_1 + \sum_{j'=1}^{j-1} 1_{\{M_{j1} < M_{j'0}\}} \pi_{jj'1}p(\theta_{j'}) \). The likelihood of the data is based on the spell duration probability and the origin firm type conditional probability distribution over the destination firm type. The destination firm distribution depends on the layoff state. Therefore, we must condition on the layoff state at the end of the spell.

Consider first the probability that a spell ends at duration \( \gamma \). This probability can be split into the sum of the joint probabilities that the spell ends at duration \( \gamma \) and that the worker’s layoff state at duration \( \gamma \) is \( k = 0 \), plus the joint probability that the spell ends at duration \( \gamma \) and that
the worker’s layoff state at duration $\gamma$ is $k_\gamma = 1$,

$$
\Pr(\gamma_{it} = \gamma | j_{it} = j) = \Pr(\gamma_{it} = \gamma \cap k_\gamma = 0 | j_{it} = j) + \Pr(\gamma_{it} = \gamma \cap k_\gamma = 1 | j_{it} = j)
$$

$$
= h_{j0}e^{-h_{j0}\gamma} \frac{h_{j0} - \lambda}{h_{j0}} + \int_0^{\gamma} h_{j0}e^{-h_{j0}\gamma'} \frac{\lambda}{h_{j0}} h_{j1}e^{-h_{j1}(\gamma - \gamma')} d\gamma'
$$

$$
= e^{-h_{j0}\gamma} (h_{j0} - \lambda) + \lambda \frac{h_{j1}}{h_{j1} - h_{j0}} \left[ e^{-h_{j0}\gamma} - e^{-h_{j1}\gamma} \right].
$$

(15)

The first term on the right hand side is the probability that the first event to hit the match happens at duration $\gamma$ and that this event is not a layoff shock. In this case, the match ends at duration $\gamma$ while being in the low layoff rate state. The second term collects the events where the match is hit by a layoff shock at duration $\gamma' \in (0, \tau)$ and has a subsequent duration of $(\gamma - \gamma')$ until the match is hit by another event while in the high layoff state, which can do nothing but end the match. All these different events result in an overall match duration $\gamma$ and that the match is in the high layoff state at the end of its duration.

The decomposition in equation (15) allows a conditioning on the layoff state at the end of the match. With this we can formulate the joint probability that the spell is initially in layoff state $k = 0, 1$ and ends at remainder duration $\gamma$ and that the worker transitions to firm type $j'$. By the same logic as above, this probability is for $j, j' > 0$ given by,

$$
\Pr(\gamma_{it} = \gamma \cap j_{i(t+1)} = j' | j_{it} = j, s = s_k) = p(\theta_{j'}) \left[ e^{-h_{jk}\gamma} \pi_{jj'k} 1 [M_{jk} < M_{j'0}] + (1 - k)\lambda \frac{e^{-h_{j0}\gamma} - e^{-h_{j1}\gamma}}{h_{j1} - h_{j0}} \pi_{jj'1} 1 [M_{jk}] \right]
$$

$$
\equiv \alpha_k (\gamma, j, j').
$$

The first term on the right hand side is the probability that the spell ends while in the $k$ layoff state and the worker transitions to $j'$. The second term is present only when $k = 0$ initially and collects the combinations of duration $\gamma'$ in the low layoff state and duration $(\gamma - \gamma')$ in the high layoff state and a subsequent transition to $j'$. Furthermore, consider the transitions in and out of unemployment,

$$
\Pr(\gamma_{it} = \gamma \cap j_{i(t+1)} = 0 | j_{it} = j) = e^{-h_{jk}\gamma} (\delta + s_k) + (1 - k)\lambda \frac{e^{-h_{j0}\gamma} - e^{-h_{j1}\gamma}}{h_{j1} - h_{j0}} (\delta + s_1)
$$

$$
\equiv \alpha_k (\gamma, j, 0)
$$

$$
\Pr(\gamma_{it} = \gamma \cap j_{i(t+1)} = j' | j_{it} = 0) = e^{-h_{j0}\gamma} \pi_{0j'} p(\theta_{j'}) \equiv \alpha_k (\gamma, 0, j'),
$$

where $h_0 = \sum_{j=1}^N \pi_{0j} p(\theta_j)$. Finally, the individual likelihood contains the possibility that the spell
is right censored,
\[
\Pr (\gamma_{it} > \gamma \mid j_{it} = j) = e^{-h_{jk}\gamma} + (1 - k) \int_0^\gamma h_{j0}e^{-h_{j0}\gamma'} \frac{\lambda}{h_{j0}}e^{-h_{j1}(\gamma-\gamma')} d\gamma' \\
= e^{-h_{jk}\gamma} + (1 - k) \frac{\lambda}{h_{j1} - h_{j0}} \left[ e^{-h_{j0}\gamma} - e^{-h_{j1}\gamma} \right]
\]
\[\equiv \alpha_k(\gamma, j)\]
\[
\Pr (\gamma_{it} > \gamma \mid j_{it} = 0) = e^{-h_{0}\gamma} \equiv \alpha_k(\gamma, 0).
\]
Initialize the likelihood by steady state. That is, the worker is initially observed in submarket \(j = 0, 1, \ldots, L\). Then the probability of this observation is \(\Pr (j_{i1} = j) = n_j\), where \(n_j = n_{j0} + n_{j1}\) for any \(j > 0\). With this, the likelihood for a given set of model parameters \(\Omega\) and worker \(i\)'s mobility pattern for \(L_i > 1\) is,
\[
\mathcal{L}_i (X_i, \Omega) = \alpha(\gamma_{i1}, j_{i1}, j_{i2})\alpha_0(\gamma_{iL_i}, j_{iL_i}) \prod_{l=2}^{L_i-1} \alpha_0(\gamma_{il}, j_{il}, j_{i(l+1)})
\]
where,
\[
\alpha (\gamma, j, j') = \begin{cases} 
  n_{j0}\alpha_0 (\gamma, j, j') + n_{j1}\alpha_1 (\gamma, j, j') & \text{if } j > 0 \\
  n_0\alpha_k (\gamma, 0, j) & \text{if } j = 0.
\end{cases}
\]
For \(L_i = 1\) worker \(i\)'s likelihood simplifies to,
\[
\mathcal{L}_i (X_i, \Omega) = \alpha(\gamma_{i1}, j_{i1}),
\]
where,
\[
\alpha (\gamma, j) = \begin{cases} 
  n_{j1}\alpha_0 (\gamma_{i1}, j_{i1}) + n_{j11}\alpha_1 (\gamma_{i1}, j_{i1}) & \text{if } j > 0 \\
  n_0\alpha_k (\gamma, 0) & \text{if } j = 0.
\end{cases}
\]
The total log likelihood of the data and the given firm classification is,
\[
\mathcal{L} (\Omega, p \mid \mathcal{J}, X) = \sum_{i=1}^{l} \ln \mathcal{L}_i (X_i, \theta) . \tag{16}
\]

4.1.1 A smoothed likelihood

The likelihood function in equation (16) can be irregular in \(\Omega\) in that the model may consider moves between particular submarkets zero likelihood. Of course for the true \(\Omega\) this does not happen, but it does pose a practical problem in the search for the maximum. For the purpose of smoothing the likelihood, consider that the firm classification is measured with noise. Specifically let \(a(j, j')\) be the probability that the firm truly belongs to submarket \(j\) given that it is measured to belong to \(j'\).
Hence, $\sum_{j=1}^{L} a(j, j') = 1$. In the next section, we consider a firm specific posterior over the type of the firm. For now, the measurement error serves the technical purpose of smoothing the likelihood function. For this purpose, consider the specification,

$$a(j, j') \propto (1 + (|j - j'| + \varepsilon)^\rho)^{-1},$$

where $\rho > 0$. For $\rho \to \infty$ one obtains the likelihood in equation (16). With this, denote by $\hat{\alpha}_k (\gamma, j, j')$ the probability that a spell that is initially in layoff state $k$ has remainder duration $\gamma$ with a firm that is observed to be in submarket $j$ and moves to another firm that is observed in submarket $j'$. For the case where $j, j' > 0$ it follows that,

$$\hat{\alpha}_k (\gamma, j, j') = \sum_{i=1}^{L} \sum_{i' = 1}^{L} a(i, j) a(i', j') \alpha_k (\gamma, i, i').$$

Similarly,

$$\hat{\alpha}_k (\gamma, j) = \sum_{i=1}^{L} a(i, j) \alpha_k (\gamma, i)$$

$$\hat{\alpha} (\gamma, j, j') = \sum_{i=1}^{L} \sum_{i' = 1}^{L} a(i, j) a(i', j') \alpha (\gamma, i, i')$$

$$\hat{\alpha} (\gamma, 0, j') = \sum_{i' = 1}^{L} a(i', j') \alpha (\gamma, 0, i')$$

$$\hat{\alpha}_k (\gamma, 0, j) = \sum_{i=1}^{L} a(i, j) \alpha_k (\gamma, 0, i)$$

$$\hat{\alpha}_k (\gamma, j, 0) = \sum_{i=1}^{L} a(i, j) \alpha_k (\gamma, i, 0).$$

With this the smoothed likelihood can be written as,

$$\hat{L}_i (X_i, \Omega | \rho) = \hat{\alpha}(\gamma_{i1}, j_{i1}, j_{i2})\hat{\alpha}_0(\gamma_{iL_i}, j_{iL_i}) \prod_{l=2}^{L_i-1} \hat{\alpha}_0(\gamma_{il}, j_{il}, j_{i(l+1)}).$$

And the total log likelihood is,

$$\hat{\mathcal{L}} (\Omega | J, X, \rho) = \sum_{i=1}^{I} \ln \hat{L}_i (X_i, \theta | \rho).$$

The following shows plots of basic performance of the likelihood in recovering the true parameters based on simulated data from the model.
Figure 1: Partial identification on simulated data
4.2 A feasible estimation

The firm classification is unobserved. However, following the logic of Bagger and Lentz (2013) where firms are classified by the poaching rank, we can obtain a ranking through the composition of the firm’s inflow over hires from other firms relative to unemployment. We conjecture this is a valid identifier as long as search is not perfectly directed. Alternative firm rankings include ranking by firm productivity, firm profits, and firm wages, or a combination of the above.

Denote by ˆJ the estimated firm rank classification, where the number of types has been taken as known. Notice that the directed search model requires cardinal information about firm productivity to determine mobility patterns (as opposed to the random search model). This means that ˆθ contains productivity parameters for each firm type. In the extreme case where search is perfectly random, the mobility likelihood will not identify the productivity parameters in Ω. Additional information can be brought to bear in the likelihood through cardinal measures like wages.

The model parameters are then estimated through the maximization of the log likelihood of the data and the estimated firm classification, L(Ω | ˆJ, X).

5 Examples

This shows some model solution examples. The model is parameterized by, δ = 0.05, η = 0.5, r = 0.05, α = 0.5, λ = 0.2, s₀ = 0.1 and s₁ = 2.0. In addition, the output function and firm type distributions are presented in Table 1. The parameterization is set to match annual rates. In the regular layoff state, layoffs happen on average every 10 years. But at rate λ = 0.2, the job enters the high layoff rate, where if nothing else happens, the job only lasts for about 6 months, on average.
Table 1: Output and firm type distribution

<table>
<thead>
<tr>
<th>y_i</th>
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<td>0.900</td>
</tr>
<tr>
<td>10</td>
<td>1.000</td>
</tr>
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</table>

Table 2: Equilibrium outcomes for high precision search case.

<table>
<thead>
<tr>
<th>( M_{i0} )</th>
<th>( M_{i1} )</th>
<th>( n_{i0} )</th>
<th>( n_{i1} )</th>
<th>( p(\theta_j) )</th>
<th>spell hazard, ( i )</th>
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</tr>
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</table>

5.1 High search precision case

In this case, search is quite precise, \( \mu = 0.15 \). The parameters have been picked to produce a roughly 5% unemployment rate. In particular \( K = 1.5 \). The equilibrium outcomes are given in Table 2. The direction of search is illustrated in Figure 3.

5.2 Low search precision case

This case illustrates the case where search is almost random. Again, parameters have been picked to produce a 5% unemployment rate. In this case \( \mu = 10.00 \) and \( K = 0.2 \). The equilibrium outcomes in this case are given in Table 3. The search directions are illustrated in Figure 4.
Figure 3: Market conditional searcher composition and search directions.
Figure 4: Market conditional searcher composition and search directions (high $\mu$).
Table 3: Equilibrium outcomes for low precision search case.

<table>
<thead>
<tr>
<th>$M_{i0}$</th>
<th>$M_{i1}$</th>
<th>$n_{i0}$</th>
<th>$n_{i1}$</th>
<th>$p(\theta_i)$</th>
<th>spell hazard$_i$</th>
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References


