Trade Liberalization Versus Protectionism: Are the Dynamic Welfare Implications Symmetric?

Ana Maria Santacreu*  Michael Sposi†

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Abstract

We quantify changes in welfare that result from alternative trade reforms in an economy with endogenous capital accumulation. The dynamic welfare gains associated with a particular reduction in trade frictions are larger than the dynamic welfare losses associated with returning to the initial level of the frictions. This “asymmetry” occurs in the short-run, yet, permanently affects welfare. Three channels contribute to the size of the asymmetry: (i) the rate of capital depreciation, (ii) the responses of measured TFP and the marginal efficiency of investment to the trade shock, and (iii) the optimal response of the investment rate. Absent transitional dynamics, the gains from a trade liberalization are equal to the losses from returning to the initial trade frictions. The short-run asymmetries imply that the sequencing of trade reforms matters for welfare.

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*Federal Reserve Bank of St. Louis, P.O. Box 442, St. Louis, MO 63166. am.santacreu@gmail.com
†Federal Reserve Bank of Dallas, Research Department, 2200 N Pearl Street, Dallas, TX 75201. 214-922-5881. michael.sposi@dal.frb.org
1 Introduction

Standard models typically measure welfare gains from trade in static settings by computing the change in real income from an observed equilibrium to a counterfactual equilibrium, where each equilibrium is associated with a particular distribution of trade frictions. Recent examples include Arkolakis, Costinot, and Rodríguez-Clare (2012) (ACR hereafter), who compute the welfare cost of autarky, and Waugh and Ravikumar (2016), who compute the welfare gains from frictionless trade. In these models, the gains from a trade liberalization are equal to the losses from returning to the initial trade frictions; that is, the gains and losses are symmetric to each other. These symmetries also occur in models with capital as an endogenous factor of production when comparing welfare across steady states. We quantitatively explore whether these symmetries arise in models with capital accumulation by accounting for the transitional dynamics: They do not.

The dynamic welfare gains associated with a particular reduction in trade frictions are larger than the dynamic welfare losses associated with returning to the initial level of the frictions. This asymmetry occurs in the short-run, yet, permanently affects welfare. Three channels explain yield this result: (i) the rate of capital depreciation, (ii) the responses of measured total factor productivity (TFP) and the marginal efficiency of investment (MEI) to the trade shock, and (iii) the optimal response of the investment rate. We assess the quantitative importance of each channel using three different versions of our model. In each version, we calibrate the initial steady state to match several features of the data, and consider two trade reforms: a move from the the initial level of trade frictions to autarky and a return from autarky to the initial level of trade frictions.

We build on Ravikumar, Santacreu, and Sposi (2016) who develop a multicountry Ricardian model where international trade affects the capital stock in each period. The model is a version of Eaton and Kortum (2002) embedded in a two-sector neoclassical growth model. There is a continuum of tradable intermediate goods that are subject to iceberg trade frictions. Each country is endowed with an initial stock of capital. Investment goods, produced using tradables, augment the stock of capital. In the model, both the MEI and the investment rate respond to changes in trade frictions. Trade is balanced in each period.

We use the model to quantify the outcomes of two trade reforms. One in which the trade frictions are increased from the calibrated levels to infinity (autarky) and the other in which trade frictions are reduced from infinity back to the calibrated levels. That is, the change in trade frictions in the two reforms are symmetric to each other.
We first study the dynamic welfare implications of the two trade reforms using a simplified version of our model with a fixed investment rate and a fixed marginal efficiency of investment (which is the same as the price of consumption relative to investment). This version boils down to a Solow model where traded intermediate goods used as a factor of production—in addition to capital and labor—and aggregate TFP depends on trade shares.

The presence of a gradual depreciation rate generates asymmetries along the transition. In both trade reforms, changes in trade shares are symmetric, and occur only in the period of the reform; the share shares remain constant thereafter. As a result, the changes in TFP are symmetric, occur in the first period, and are constant thereafter. Since capital does not change on impact, changes in income, investment, and consumption are also symmetric on impact. However, gradual depreciation of capital implies that the rates of capital accumulation in the periods following the reform are asymmetric, leading to asymmetric changes in output and hence consumption. In particular, in the trade liberalization case, the magnitudes of increases in the capital stock exceeds the magnitudes of the decreases that occur in the autarky case. These asymmetries persist in the short run but disappear as the economy converges to the steady state. The size of the asymmetry depends on the size of the initial change in measured TFP. We illustrate the results both quantitatively in our calibrated model, and analytically taking a symmetric change to TFP as given.

Next, we study the same trade reforms in a model in which the nominal investment rate is fixed (as in the Solow model) but the MEI responds to changes in trade frictions. This implies that the rate of transforming final output into investment is not fixed. Changes in the MEI depend only on changes in the trade shares. Following the trade liberalization, the MEI increases, and following the move to autarky, the MEI decreases. The changes occur on impact, are symmetric, and are permanent.

Changes in the MEI amplify the effect of changes in measured TFP and, hence, in the presence of capital depreciation, amplify the asymmetries between the two trade reforms in the short run. We illustrate this result both quantitatively in our model, and analytically taking a change to TFP as given.

Finally, we consider our full model, which is a neoclassical growth the model where the investment rate is determined optimally by an intertemporal Euler equation, and the MEI responds to changes in trade frictions. The interaction between the rate of return to capital and the optimal investment rate generates an additional amplification to the asymmetry in our previous versions of the model. Following a trade reform, TFP responds symmetrically as in previous version of the model, which initiates a corresponding change in the rate of
return to capital (an increase after a trade liberalization and a decrease following a move to autarky).

After the trade liberalization, the magnitude of the increase in the investment rate is smaller than the magnitude of the decrease in the investment rate after moving to autarky. This implies that the short-run loss in consumption from an increase in trade frictions is smaller than the short-term gain in consumption from a reduction in trade frictions. These results depend on the differences in the marginal utility of consumption in the initial steady state. In the case when trade frictions are large in the initial steady-state, consumption is low and the marginal utility of consumption is large. Therefore, relative to starting with high levels of consumption (low trade frictions), the household responds to the trade liberalization by allocating a relatively larger share of output to consumption, and hence the investment rate increases by a relatively smaller magnitude. In the case when trade frictions are low in the initial steady state, consumption is high and the marginal utility of consumption is low. As a result, after an increase in trade frictions, the household is willing to forgo some consumption, and hence allocates a larger share of output to investment; that is, has a relatively higher investment rate. These asymmetries imply asymmetries in the changes to consumption in the short run leading to asymmetries in the dynamic changes in welfare.

Since the asymmetries in welfare are materialized mainly in the short-run, our results have implications for the sequencing of trade reforms, especially if reforms are designed to be temporary.

2 Model

There are \( I \) countries indexed by \( i = 1, \ldots, I \) and time is discrete, running from \( t = 1, \ldots, \infty \). There are three sectors: consumption, investment, and intermediates, denoted by \( c, x, \) and \( m \), respectively. Neither consumption goods nor investment goods are tradable. There is a continuum of intermediate varieties that are tradable. Production of all goods is carried out by perfectly competitive firms. As in Eaton and Kortum (2002), each country’s efficiency in producing each intermediate variety is a realization of a random draw from a country-specific distribution. Trade in intermediate varieties is subject to iceberg costs. Each country purchases each intermediate variety from its lowest-cost supplier and all of the varieties are aggregated into a composite intermediate good. The composite good is used as an input along with capital and labor to produce the consumption good, the investment good, and the intermediate varieties.
Each country has a representative household. The household owns its country’s stock of capital and labor, which it inelastically supplies to domestic firms, and purchases consumption and investment goods from the domestic firms. We assume that trade is balanced period by period.

2.1 Endowments

The representative household in country \(i\) is endowed with a labor force of size \(L_i\) in each period, an initial stock of capital, \(K_{i1}\), and an initial net foreign asset (NFA) position \(A_{i1}\).

2.2 Technology

There is a unit interval of varieties in the intermediates sector. Each variety within the sector is tradable and is indexed by \(v \in [0, 1]\).

**Composite good** Within the intermediates sector, all of the varieties are combined with constant elasticity to construct a sectoral composite good according to

\[
M_{it} = \left[ \int_0^1 q_{it}(v)^{1-1/\eta} dv \right]^{\eta/(\eta-1)},
\]

where \(\eta\) is the elasticity of substitution between any two varieties. The term \(q_{it}(v)\) is the quantity of good \(v\) used by country \(i\) to construct the composite good at time \(t\) and \(M_{it}\) is the quantity of the composite good available in country \(i\) to be used as an input.

**Varieties** Each variety is produced using capital, labor, and the composite good. The technologies for producing each variety are given by

\[
Y_{mit}(v) = z_{mi}(v) \left( K_{mit}(v)^{\alpha} L_{mit}(v)^{1-\alpha} \right)^{\nu_m} M_{mit}(v)^{1-\nu_m}.
\]

The term \(M_{mit}(v)\) denotes the quantity of the composite good used by country \(i\) as an input to produce \(Y_{mit}(v)\) units of variety \(v\), while \(K_{mit}(v)\) and \(L_{mit}(v)\) denote the quantities of capital and labor used.

The parameter \(\nu_m \in [0, 1]\) denotes the share of value added in total output and \(\alpha\) denotes capital’s share in value added. These parameters are constant across countries and over time.

The term \(z_{mi}(v)\) denotes country \(i\)’s productivity for producing variety \(v\). Following Eaton and Kortum (2002), the productivity draw comes from independent country-specific
Fréchet distributions with shape parameter $\theta$ and country-specific scale parameter $T_{mi}$, for $i = 1, 2, \ldots, I$. The c.d.f. for productivity draws in country $i$ is $F_{mi}(z) = \exp(-T_{mi}z^{-\theta})$.

In country $i$ the expected value of productivity across the continuum is $\gamma^{-1}T_{mi}^{\frac{1}{\theta}}$, where $\gamma = \Gamma(1 + \frac{1}{\theta}(1 - \eta))^{\frac{1}{1 - \eta}}$ and $\Gamma(\cdot)$ is the gamma function, and $T_{mi}^{\frac{1}{\theta}}$ is the fundamental productivity in country $i$. If $T_{mi} > T_{mj}$, then on average, country $i$ is more efficient than country $j$ at producing intermediate varieties. A smaller $\theta$ implies more room for specialization and, hence, more gains from trade.

**Consumption good** Each country produces a final consumption good using capital, labor, and intermediates according to

$$Y_{cit} = A_{ci} \left( K_{cit}^{\alpha} L_{cit}^{1-\alpha} \right)^{\nu_c} M_{cit}^{1-\nu_c}.$$

The terms $K_{cit}, L_{cit},$ and $M_{cit}$ denote the quantity of capital, labor, and the composite good used by country $i$ to produce $Y_{cit}$ units of consumption at time $t$. The parameters $\alpha$ and $\nu_c$ are constant across countries and over time. The term $A_{ci}$ captures country $i$’s productivity in the consumption goods sector—this term varies across countries.

**Investment good** Each country produces an investment good using capital, labor, and intermediates according to

$$Y_{xit} = A_{xi} \left( K_{xit}^{\alpha} L_{xit}^{1-\alpha} \right)^{\nu_x} M_{xit}^{1-\nu_x}.$$

The terms $K_{xit}, L_{xit},$ and $M_{xit}$ denote the quantity of capital, labor, and the composite good used by country $i$ to produce $Y_{xit}$ units of investment at time $t$. The parameters $\alpha$ and $\nu_x$ are constant across countries and over time. The term $A_{xi}$ captures country $i$’s productivity in the investment goods sector—this term varies across countries.

2.3 Trade

International trade is subject to frictions that take the iceberg form. Country $i$ must purchase $d_{ij} \geq 1$ units of any intermediate variety from country $j$ in order for one unit to arrive; $d_{ij} - 1$ units melt away in transit. As a normalization, we assume that $d_{ii} = 1$ for all $i$. 

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2.4 Preferences

The representative household’s lifetime utility is given by

$$\sum_{t=1}^{\infty} \beta^{t-1} L_i \left( \frac{C_{it}/L_i}{1 - 1/\sigma} \right)^{1-1/\sigma},$$

where $C_{it}/L_i$ is consumption per capita in country $i$ at time $t$, $\beta \in (0, 1)$ denotes the period discount factor and $\sigma$ denotes the intertemporal elasticity of substitution. Both parameters are constant across countries and over time.

Capital accumulation

The representative household enters period $t$ with $K_{it}$ units of capital, which depreciates at the rate $\delta$. Investment, $X_{it}$, adds to the stock of capital.

$$K_{it+1} = (1 - \delta)K_{it} + X_{it}.$$ 

Budget constraint

The representative household earns income by supplying capital and labor inelastically to domestic firms earning a rental rate $r_{it}$ on capital and a wage rate $w_{it}$ on labor. The household purchases consumption at the price $P_{cit}$ and purchases investment at the price $P_{xit}$. The budget constraint is given by

$$P_{cit}C_{it} + P_{xit}X_{it} = r_{it}K_{it} + w_{it}L_i.$$ 

2.5 Equilibrium

A competitive equilibrium satisfies the following conditions: (i) taking prices as given, the representative household in each country maximizes its lifetime utility subject to its budget constraint and technology for accumulating capital, (ii) taking prices as given, firms maximize profits subject to the available technologies, (iii) intermediate varieties are purchased from their lowest-cost provider subject to the trade frictions, and (iv) all domestic markets clear and trade is balanced in each period. At each point in time, we take world GDP as the numéraire: $\sum_i r_{it}K_{it} + w_{it}L_i = 1$ for all $t$. We describe each equilibrium condition in more detail in Appendix A.
2.6 Welfare Analysis

We measure changes in welfare using consumption equivalent units. In static models these changes are computed as changes in income since consumption is proportional to income. In a dynamic model, consumption might not be proportional to income along the transition path.

We follow Lucas (1987) and compute the constant, $\Lambda_{i}^{dyn}$ that solves:

$$\sum_{t=1}^{\infty} \beta^{t-1} L_i \left( \Lambda_{i}^{dyn}, C_i^*/L_i \right)^{1-1/\sigma} 1 - 1/\sigma = \sum_{t=1}^{\infty} \beta^{t-1} L_i \left( \tilde{C}_{it}/L_i \right)^{1-1/\sigma}, \quad (1)$$

where $\tilde{C}_{it}$ is the consumption at time $t$ in the counterfactual.

Steady state gains are computed using steady state levels of consumption only:

$$\Rightarrow \Lambda_{i}^{ss} = \frac{C_{i}^{**}}{C_{i}^{*}}, \quad (2)$$

where $C_{i}^{*}$ is the (constant) consumption in the initial steady state in country $i$ and $C_{i}^{**}$ is the consumption in the counterfactual steady-state in country $i$. In our model, consumption is proportional to income in steady state for every country and the ratio of consumption to income, $1 - \frac{\alpha \delta}{\beta - (1 - \delta)}$, is the same across countries.

3 Calibration

We calibrate the parameters of the model to match data in 2011. Our assumption is that the world is in steady state at this time. Table B.1 provides the equilibrium conditions that describe the steady state in our model. Our technique for computing both the steady-state equilibria and the transition path between steady states is the same as in Ravikumar, Santacreu, and Sposi (2017).

Our model covers 41 countries (containing 40 individual countries plus the rest of the world). Appendix B provides the details of our data.

3.1 Common parameters

The values for the common parameters are reported in Table 1. We use recent estimates of the trade elasticity by Simonovska and Waugh (2014) and set $\theta = 4$. We set $\eta = 2$ which
satisfies the condition \(1 + \frac{1}{\theta}(1 - \eta) > 0\). This value plays no quantitative role in our results.

### Table 1: Common parameters

| \(\theta\) | Trade elasticity | 4 |
| \(\eta\) | Elasticity of substitution between varieties | 2 |
| \(\alpha\) | Capital’s share in value added | 0.33 |
| \(\beta\) | Annual discount factor | 0.96 |
| \(\delta\) | Annual depreciation rate for stock of capital | 0.06 |
| \(\sigma\) | Intertemporal elasticity of substitution | 0.67 |
| \(\nu_c\) | Share of value added in final goods output | 0.91 |
| \(\nu_x\) | Share of value added in investment goods output | 0.33 |
| \(\nu_m\) | Share of value added in intermediate goods output | 0.28 |

In line with the literature, we set the share of capital in value added to \(\alpha = 0.33\) \cite{Gollin2002}, the discount factor to \(\beta = 0.96\), so that the steady-state real interest rate is about 4 percent, and the intertemporal elasticity of substitution to \(\sigma = 0.67\).

We compute \(\nu_m = 0.28\) by taking the cross-country average of the ratio of value added to gross output of manufactures. We compute \(\nu_x = 0.33\) by taking the cross-country average of the ratio of value added to gross output of investment goods.

Computing \(\nu_c\) is slightly more involved since there is no clear industry classification for consumption goods. Instead, we infer this share by interpreting national accounts data through the lens of our model. We begin by noting that by combining firm optimization and market clearing conditions for capital and labor we get

\[
\frac{r_i K_i}{1 - \alpha} = \frac{\alpha}{1 - \alpha} w_i L_i.
\]

In steady state, the Euler equation and the capital accumulation technology imply

\[
P_{x_i} X_i = \frac{\delta \alpha}{\beta - (1 - \delta)(1 - \alpha)} \frac{w_i L_i}{1 - \alpha} = \frac{\phi_x}{1 - \alpha} \frac{w_i L_i}{1 - \alpha}.
\]

We compute \(\phi_x\) by taking the cross-country average of the share of gross fixed capital formation in nominal GDP. Given this value and the relation \(\phi_x = \frac{\delta \alpha}{\frac{\delta \alpha}{\beta - (1 - \delta)}(1 - \delta)}\), the depreciation rate for capital is \(\delta = 0.06\). The household’s budget constraint then implies that

\[
P_{ci} C_i = \frac{w_i L_i}{1 - \alpha} - P_{x_i} X_i = (1 - \phi_x) \frac{w_i L_i}{1 - \alpha}.
\]

Consumption in our model corresponds to the sum of private and public consumption,
changes in inventories, and net exports. We use the trade balance condition together with
the firm optimality and the market clearing conditions for sectoral output to obtain

\[ P_{mi}M_i = [(1 - \nu_x)\phi_x + (1 - \nu_c)(1 - \phi_x)] \frac{w_iL_i}{1 - \alpha} + (1 - \nu_m)P_{mi}M_i, \]  

(3)

where \( P_{mi}M_i \) is total absorption of manufactures in country \( i \) and \( \frac{w_iL_i}{1 - \alpha} \) is the nominal GDP. We use a standard method of moments estimator to back out \( \nu_c \) from equation (3).

We calibrate the elasticity parameter \( \mu = 0.55 \) as in Eaton, Kortum, Neiman, and Romalis (2016). We set the adjustment cost parameter, \( \chi = \delta^{1-\mu} \), so that there is no cost to maintain the level of capital stock in steady state, i.e., \( X = \delta K \).

### 3.2 Country-specific parameters

We set the workforce, \( L_i \), equal to the population in country \( i \) documented in PWT 8.1. The remaining parameters \( A_{ci}, T_{mi}, A_{xi}, \) and \( d_{ij} \), for \( (i, j) = 1, \ldots, I \), are not directly observable. We back these out by linking structural relationships of the model to observables in the data.

The equilibrium structure relates the unobserved trade frictions for any given country pair directly to the ratio of intermediate goods prices in the two countries and the trade shares between them:

\[ \frac{\pi_{ij}}{\pi_{jj}} = \left( \frac{P_{mj}}{P_{mi}} \right)^{-\theta} d_{ij}^{-\theta}. \]  

(4)

Appendix B describes how we construct the empirical counterparts to prices and trade shares. For observations in which \( \pi_{ij} = 0 \), we set \( d_{ij} = 10^8 \). We also set \( d_{ij} = 1 \) if the inferred value of trade cost is less than 1.

Lastly, we derive three structural relationships to pin down the productivity parameters
The three equations relate observables—the price of consumption relative to intermediates, the price of investment relative to intermediates, income per capita, and home trade shares—to the unknown productivity parameters. Income per capita in country \(i\) is the total factor compensation per capita, deflated by the price of the final consumption good:

\[
y_i = \frac{r_i K_i + w_i L_i}{L_i P_{ci}}. \tag{5}
\]

Equations (5)–(7) are derived in Appendix C. We set \(A_{cU} = T_{mU} = A_{xU} = 1\) as a normalization, where the subscript \(U\) denotes the United States. For each country \(i\), system (5)–(7) yields three nonlinear equations with three unknowns: \(A_{ci}, T_{mi}, \text{and} A_{xi}\). Information about constructing the empirical counterparts to \(P_{ci}, P_{mi}, P_{xi}, \pi_{ii}\) and \(y_i\) is in Appendix B.

These equations are quite intuitive. The expression for income per capita provides a measure of aggregate productivity across all sectors: Higher income per capita is associated with higher productivity levels, on average. The expressions for relative prices boil down to two components. The first term reflects something akin to the Balassa-Samuelson effect: All else equal, a higher price of capital relative to intermediates suggests a low productivity in capital goods relative to intermediate goods. In our setup, the measured productivity for intermediates is endogenous, reflecting the degree of specialization as captured by the home trade share. The second term reflects the relative intensity of intermediate inputs. If measured productivity is high in intermediates, then the price of intermediates is relatively low and the sector that uses intermediates more intensively will have a lower relative price.
3.3 Model fit

Our model consists of 8,832 country-specific parameters: \( I(I - 1) = 1,640 \) bilateral trade frictions, \((I - 1) = 40 \) consumption-good productivity terms, \((I - 1) = 40 \) investment-good productivity terms, and \((I - 1) = 40 \) intermediate-goods productivity terms.

Calibration of the country-specific parameters uses 1,800 data points. The trade frictions use up \( I(I - 1) = 1,640 \) data points for bilateral trade shares and \((I - 1) = 40 \) for the ratio of absolute prices of intermediates. The productivity parameters use up \((I - 1) = 40 \) data points for the price of consumption relative to intermediates, \((I - 1) = 40 \) data points for the price of investment relative to intermediates, and \((I - 1) = 40 \) data points for income per capita.

The model matches the targeted data well.

4 Quantitative exercise

We consider three versions of our model: (i) a Solow-type model, (ii) A model where the MEI responds to changes in trade frictions, and (iii) our neoclassical growth model where the investment rate is determined optimally and the MEI depends on trade frictions.

In each version, we compute two counterfactual transition paths. First, we compute the equilibrium transition when moving from the calibrated trade frictions to autarky. Second, we compute the equilibrium transition from autarky to the calibrated trade frictions. In both cases, we begin the world in steady state and consider a one-time, permanent, uniform and unanticipated change in trade frictions.

We then compute the steady-state and dynamic welfare gains from trade using equations (2) and (1) and decompose the short-run versus long-run gains.

In each of our models, the steady-state gains are symmetric with respect to the two trade reforms. We will make use of the expression for aggregate income, which is given by:

\[
Y_{it} \propto A_{cit} \left( \frac{T_{imi}}{\pi_{iit}} \right)^{\frac{1-\nu_c}{\nu_c - \nu_m}} K_{it}^\alpha L_{it}^{1-\alpha}
\]

4.1 Solow-type model with balanced trade

In the Solow-type model we fix the nominal investment rate and the relative price of investment. To do this, we change a few equations and recalibrate the model to match the same
target moments as in the benchmark calibration.

First, to impose a fixed nominal investment rate we set \( P_{xit} X_{it} = \rho Y_{it} \), where \( Y_{it} \) is aggregate income:

\[
C_{it} = (1 - \rho) Y_{it}.
\]

Second, in addition to a fixed nominal investment rate, we fix the relative price of investment, \( \frac{P_{xit}}{P_{cit}} \), to 1. To do this, we restrict the technologies for consumption and investment goods to be the same. That is, we set \( A_{xi} = A_{ci} \) for every \( i \) (but allow for variation across countries) and set \( \nu_x = \nu_c \). We choose \( A_{xi} = A_{ci} \) to match the price of GDP relative to intermediates in country \( i \) and choose \( \nu_x = \nu_c = 0.88 \) to satisfy the national income accounts equation \( \text{[3]} \). We recalibrate all other parameters to match the same targets as in the benchmark calibration. The restriction on the relative price implies that, \( Y = C + X \), where \( Y = \frac{(rK + wL)}{P_c} \) is real income.

**Quantitative magnitude of symmetries in the Solow-type model** First, note that the steady state in this model can be solved for independently from the transition. In other words, for any world distribution of trade barriers there is a unique steady-state level of consumption. Therefore, it is trivial that the steady-state changes in welfare are symmetric with respect to the direction of changes in trade costs.

The interesting exercise it to compare the transition path (i) from baseline trade frictions to autarky and (ii) from autarky back to baseline trade frictions. We find that the changes in welfare are indeed asymmetric. In particular, the dynamic welfare cost of moving to autarky is smaller than the dynamic gain of moving from autarky to the baseline frictions (see Figure \[1\]). However, if we impose full depreciation, the changes in welfare are actually symmetric. On the other hand, as \( \delta \to 0 \), the welfare change approach symmetry. Note that the steady state is not well-defined in the limiting case of \( \delta = 0 \) since the steady-state level of investment \( X^* \to 0 \).

The asymmetry arises as a result of the short-run responses to the shock. In particular, in the short run, welfare gains accumulate more quickly following a trade liberalization, than the losses accrue following a move to autarky. This can be seen in Figure \[1\] where we define the asymmetry, in each period, as the product of the change in consumption in each model; \( \frac{C^1_t}{C^1_{t-1}} \times \frac{C^2_t}{C^2_{t-1}} \) (superscripts 1 and 2 denotes the two trade reforms).

**Analytical results** In this version of the model we can analytically characterize the asymmetry that occurs immediately following the trade shock at time \( t = 1 \). For the sake of
intuition, we abstract from international trade all together, and consider the impact of (i) an increase in TFP and (ii) a symmetric reduction in TFP. This is approximately true in our model of trade following a shock to trade frictions; measured TFP jumps on impact roughly to its new steady state level and changes very little thereafter (see Figure 2). As in our trade model, the aggregate income in the economy can be expressed as $Y_i = Z_i K_i^\alpha L_i^{1-\alpha}$, where $Z_i$ denotes aggregate TFP. (In our model of trade, $Z_i \propto (T_{mi}/\pi_{ii})^{1/\theta}$ is measured TFP.)

Begin by considering the capital accumulation equation as it applies from period 1 to 2 and define the change in capital stock as $g_k$:

$$g_k = K_2/K_1 = (1 - \delta) + X_1/K_1$$

In period 1, the capital is unchanged from the steady-state value ($K_1 = K^*$, where $K^*$ is the steady state level of capital). Define $\Gamma_z \equiv Z_1/Z^*$ as the change in TFP in period 1, relative to its steady state level. Note that the change in investment relative to its steady-state level equals the change in TFP, hence $\Gamma_z$. Therefore,

$$g_k = (1 - \delta) + X_1/K^* = (1 - \delta) + (X^*/K^*)\Gamma_z.$$  

Because we are considering the Solow model, investment is proportional to consumption, so $\Gamma_z = C_1/C^*$ also describes the change in welfare in period 1 relative to the initial steady
state.

Now consider the two counterfactual exercises: (i) Double TFP and (ii) Half TFP. We will explore whether the welfare outcomes from these two counterfactuals are symmetric to each other.

Denote variables in the first (second) counterfactual with the superscript “1” (“2”). Note that $\Gamma^1 \Gamma^2 = 1$, i.e, the change in consumption in the first period relative to the steady state is symmetric; that is, the increase in consumption in counterfactual 1 is exactly the reciprocal of the decline in consumption in counterfactual 2. This is because total income is given by $C_t = (1 - \rho)Y_t = Z_t K_t^{\alpha} L^{1-\alpha}$ and, hence, since capital does not change on impact, the change in output equals the change in TFP.

Next, consider what happens to changes in welfare in period 2 relative to period 1. This will depend on the change in capital. We do not allow TFP to change at any point after period 1.

$$\frac{C_2}{C_1} = (K_2/K^*)^\alpha = (g_k)^\alpha. \quad (9)$$

Changes in consumption (and welfare) from period 1 to 2 are symmetric if and only if
\( g_k^1 g_k^2 = 1 \). Using the definition of \( g_k \):

\[
g_k^1 g_k^2 = (1 - \delta)^2 + X^*/K^* (1 - \delta)(\Gamma_z + 1/\Gamma_z) + (X^*/K^*)^2
\]

Clearly, when \( \delta = 1 \), \( g_k^1 g_k^2 = 1 \), since \((X^*/K^*)^2 = \delta^2 = 1\).

In this model there is no well-defined steady state when \( \delta = 0 \) so we resort to the limiting case. As \( \delta \to 0 \), \( X^* \to 0 \), and so \( g_k^1 g_k^2 = 1 \).

In the intermediate cases with \( \delta \in (0, 1) \), \( g_k^1 g_k^2 = (1 - \delta)^2 + \delta (1 - \delta)(\Gamma_z + 1/\Gamma_z) + \delta^2 > 1 \). This implies that, for every \( \delta \in (0, 1) \), the change in welfare from period 1 to 2 is greater following an increase in TFP than is the loss following a symmetric decline in TFP. Moreover, the magnitude of the asymmetry is higher for larger TFP shocks (i.e., higher \( \Gamma_z \), see Figure 3). For any value of \( \Gamma_z \), the asymmetry is maximized when \( \delta = 1/2 \).

Figure 3 shows that the two trade reforms have a symmetric effect on TFP, since they have a symmetric effect on the home trade share. However, as we have shown, symmetric changes in TFP under the presence of gradual depreciation rate translate into asymmetric changes in welfare, especially in the short-run.

Figure 3: Asymmetry in dynamic welfare gains in Solow-type model increases with change in TFP

We extend the previous model to allow for the MEI (the price of consumption relative to investment) to respond to changes in trade frictions. We recalibrate the model to a two-sector neoclassical growth model in which the productivity and the share of intermediate inputs into final production differs in the investment and consumption sectors.
In this model, changes in the marginal efficiency of investment depend only on changes in the home trade share. In turn, changes in the MEI amplify the effect of changes in measured TFP and, hence, in the presence of capital depreciation, the asymmetries between the two trade reforms are amplified (see Figure 4). After the decline in trade frictions, the increase in MEI implies a faster rate of capital accumulation and hence an increase in the investment rate. The opposite is true after an increase in trade frictions. In the presence of gradual depreciation of capital the effect of the decline in investment has a lower effect on capital, since the household can use the stock of capital that has not depreciated yet. Hence, the gain after a trade liberalization is larger than the loss after the increase in trade frictions.

4.1.1 Solow-type model with endogenous MEI

Figure 4: Asymmetry in dynamic welfare gains in Solow-type model with endogenous relative price

Analytical results  Begin with

\[ K_{t+1} = (1 - \delta)K_t + q_tX_t \]

with \( q_t \) is the inverse of the relative price of investment (i.e., the MEI). Now the trade shocks causes both an change in TFP, \( \Gamma_z = Z_2/Z_1 \), and a change in the efficiency of invest-
ment, $\Gamma_q = q_2/q_1$. Therefore, the change in investment is now the product of the change in TFP and the change in the efficiency of investment.

$$g_k g_e = (1 - \delta)^2 + X^*/K^*(1 - \delta)(\Gamma_k \Gamma_q + 1/(\Gamma_2 \Gamma_q)) + (X^*/K^*)^2$$

The relative price of investment (that is, the inverse of the MEI) can be expressed as

$$\frac{P_{xit}}{P_{cit}} \propto \left( \frac{A_{ci}}{A_{xi}} \right) \left( \frac{T_{mi}}{\pi_{iit}} \right)^{\frac{\nu_p - \nu_G}{\theta \nu_m}}$$

Note that changes in the relative price of investment are entirely driven by changes in the home trade share, and hence the effect of the two trade reforms are symmetric on the MEI.

Figure 5: TFP and relative price of investment in Solow-type model with endogenous relative price

### 4.1.2 Endogenous relative price and endogenous investment rate

We consider a version of the model in which the investment rate is the determined optimally by an intertemporal Euler equation. The interaction between the rate of return to capital and the optimal investment rate generates an additional amplification to the asymmetry in our previous versions of the model. Following a trade reform, TFP responds symmetrically as in previous versions of the model. This is followed by a corresponding change in the rate
of return to capital (an increase after a trade liberalization and a decrease following a move to autarky). After the trade liberalization, the magnitude of the increase in the investment rate is smaller than the magnitude of the decrease in the investment rate after moving to autarky (see Figure 6).

Figure 6: Asymmetry in real investment rate

This implies that the short-run loss in consumption from an increase in trade frictions is smaller than the short-term gain in consumption from a reduction in trade frictions (see Figure 7). These results depend on the differences in the marginal utility of consumption in the initial steady state. In the case when trade frictions are large in the initial steady-state, consumption is low and the marginal utility of consumption is large. Therefore, after a decrease in trade frictions the household responds by allocating a larger share of output to consumption, and hence the investment rate does not increase as much. In the case when trade frictions are low in the initial steady state, consumption is high and the marginal utility of consumption is low. As a result, after an increase in trade frictions, the household is willing to forgo some consumption, and hence allocates a larger share of output to investment.

Note that in all cases analyzed, the asymmetries are amplified in the short run and they disappear as the economy converges to the steady-state. In all cases, both trade reforms have a symmetric effect on the endogenous variables when the economy reaches the counterfactual steady-state.
5 Conclusion

References

Affendy, Arip M., Lau Sim Yee, and Madono Satoru. 2010. “Commodity-industry Classification Proxy: A Correspondence Table Between SITC Revision 2 and ISIC Revision 3.” MPRA Paper 27626, University Library of Munich, Germany.


Appendix

A Equilibrium conditions in the balanced trade model

We describe each equilibrium condition in detail below.

Household optimization The representative household chooses a path for consumption that satisfies the following Euler equation:

$$\frac{C_{it+1}}{C_{it}} = \beta^\sigma \left( 1 + \frac{r_{it+1}}{P_{xit+1}} - \delta \right)^\sigma \left( \frac{P_{xit+1}/P_{cit}+1}{P_{xit}/P_{cit}} \right)^\sigma, \quad (A.1)$$

Combining the representative household’s budget constraint, together with capital accumulation technology and rearranging, implies the following:

$$C_{it} = \left( 1 + \frac{r_{it+1}}{P_{xit+1}} - \delta \right) \left( \frac{P_{xit}}{P_{cit}} \right) K_{it} + \left( \frac{w_{it}}{P_{cit}} \right) L_i - \left( \frac{P_{xit}}{P_{cit}} \right) K_{it+1}. \quad (A.2)$$

Firm optimization Markets are perfectly competitive, so firms set prices equal to marginal costs. Denote the price of variety $v$, produced in country $j$ and purchased by country $i$, as $p_{mi}(v)$. Then $p_{mi}(v) = p_{mjj}(v)d_{ij}$, where $p_{mjj}(v)$ is the marginal cost of producing variety $v$ in country $j$. Since country $i$ purchases each variety from the country that can deliver it at the lowest price, the price in country $i$ is $p_{mi}(v) = \min_{j=1,\ldots,I} [p_{mjj}(v)d_{mij}]$. The price of the composite intermediate good in country $i$ at time $t$ is then

$$P_{mit} = \gamma \left[ \sum_{j=1}^I (u_{jt}d_{ij})^{-\theta} T_{mj} \right]^{-\frac{1}{\theta}}, \quad (A.3)$$

where $u_{jt} = \left( \frac{r_{jt}}{\alpha \nu_m} \right)^{\alpha \nu_m} \left( \frac{w_{jt}}{(1-\alpha) \nu_m} \right)^{(1-\alpha) \nu_m} \left( \frac{P_{jt}}{1-\nu_m} \right)^{1-\nu_m}$ is the unit cost for a bundle of inputs for intermediate goods producers in country $n$ at time $t$.

Next we define total factor usage in the intermediates sector by aggregating across the individual varieties.

$$K_{mit} = \int_0^1 K_{mit}(v)dv, \quad L_{mit} = \int_0^1 L_{mit}(v)dv,$$

$$M_{mit} = \int_0^1 M_{mit}(v)dv, \quad Y_{mit} = \int_0^1 Y_{mit}(v)dv.$$
The term $L_{mit}(v)$ denotes the quantity of labor used in the production of variety $v$ at time $t$. If country $i$ imports variety $v$ at time $t$, then $L_{mit}(v) = 0$. Hence, $L_{mit}$ is the total quantity of labor used in sector $m$ in country $i$ at time $t$. Similarly, $K_{mit}$ is the total quantity of capital used, $M_{mit}$ is the total quantity of intermediates used as an input, and $Y_{mit}$ is the total quantity of output of intermediate goods.

Cost minimization by firms implies that, within each sector $b \in \{c, m, x\}$, factor expenses exhaust the value of output:

\[
\begin{align*}
  r_{it}K_{bit} &= \alpha \nu_b P_{bit} Y_{bit}, \\
  w_{it}L_{bit} &= (1 - \alpha) \nu_b P_{bit} Y_{bit}, \\
  P_{mit}M_{bit} &= (1 - \nu_b) P_{bit} Y_{bit}.
\end{align*}
\]

That is, the fraction $\alpha \nu_b$ of the value of each sector’s production compensates capital services, the fraction $(1 - \alpha) \nu_b$ compensates labor services, and the fraction $1 - \nu_b$ covers the cost of intermediate inputs; there are zero profits.

**Trade flows** The fraction of country $i$’s expenditures allocated to intermediate varieties produced by country $j$ is given by

\[
\pi_{ijt} = \frac{(u_{mjt}d_{ijt})^{-\theta} T_{mj}}{\sum_{j=1}^{J} (u_{mjt}d_{ij})^{-\theta} T_{mj}},
\]

where $u_{mjt}$ is the unit cost of a bundle of factors faced by producers of intermediate varieties in country $j$.

**Market clearing conditions** We begin by describing the domestic factor market clearing conditions.

\[
\begin{align*}
  \sum_{b \in \{c, m, x\}} K_{bit} &= K_{it}, \\
  \sum_{b \in \{c, m, x\}} L_{bit} &= L_{i}, \\
  \sum_{b \in \{c, m, x\}} M_{bit} &= M_{it}.
\end{align*}
\]

The first two conditions impose that the capital and labor markets clear in country $i$ at each time $t$. The third condition requires that the use of the composite intermediate good equals its supply. Its use consists of intermediate demand by firms in each sector. Its supply is the quantity of the composite good, which consists of both domestically and foreign-produced varieties.
The next conditions require that goods markets clear.

\[ C_{it} = Y_{cit}, \quad X_{it} = Y_{xit}, \quad \sum_{j=1}^{I} P_{mjt} (M_{cjt} + M_{mjt} + M_{xjt}) \pi_{jit} = P_{mit} Y_{mit}. \]

The first condition states that the quantity of consumption demanded by the representative household in country \( i \) must equal the quantity produced by country \( i \). The second condition says the same for the investment good. The third condition imposes that the value of intermediates produced by country \( i \) has to be absorbed globally. Recall that \( P_{mjt} M_{bjt} \) is the value of intermediate inputs that country \( i \) uses in production in sector \( b \). The term \( \pi_{jit} \) is the fraction of country \( j \)'s intermediate good expenditures sourced from country \( i \). Therefore, \( P_{mjt} M_{bjt} \pi_{jit} \) denotes the total value of trade flows from country \( i \) to country \( j \).

Finally, we impose an aggregate resource constraint in each country: Net exports equal zero. Equivalently, gross output equals gross absorption:

\[ P_{mit} Y_{mit} = P_{mit} M_{it}. \]

The left-hand side denotes the gross output of intermediates in country \( i \) and the right-hand side denotes total expenditures on intermediates.

**B Data**

This Appendix describes the sources of data and any adjustments we make to the data to map it to the model.

**B.1 Production and trade data**

Mapping the trade dimension of our model to the data requires data on both production and international trade flows. Our focus is on manufactured intermediate goods. We interpret manufacturing broadly as defined by the International Standard Industrial Classification (ISIC).

We obtain production data from multiple sources. First, we use value added and gross output data from the INDSTAT database, which are reported at the two-digit level using ISIC. The data for countries extend no further than 2010 and even less for many countries. We turn to data on value added output in UN National Accounts Main Aggregates Database.
(UNNAMAD, [http://unstats.un.org/unsd/snaama/Introduction.asp](http://unstats.un.org/unsd/snaama/Introduction.asp)), which reports value added output for 2011. For countries that report both value added and gross output in INDSTAT, we use the ratio from the year closest to 2011 and apply that ratio to the value added from UNNAMAD to recover gross output. For countries with no data on gross output in INDSTAT for any years, we apply the average ratio of value added to gross output across all countries and apply that ratio to the value added figure in UNNAMAD for 2011. In our dataset, the ratio of value added to gross output does not vary significantly over time and is also not correlated with level of development or country size.

Our source of trade data is the UN Comtrade Database ([http://comtrade.un.org](http://comtrade.un.org)). Trade is reported for goods using revision 2 Standard International Trade Classification (SITC2) at the four-digit level. We use the correspondence tables created by Affendy, Sim Yee, and Satoru (2010) to map SITC2 to ISIC. We also omit any petroleum-related products from the trade data.

Using the trade and production data, we construct bilateral trade shares for each country pair by following Bernard, Eaton, Jensen, and Kortum (2003) as follows:

$$\pi_{ij} = \frac{X_{ij}}{\text{ABS}_i},$$

where $i$ denotes the importer and $j$ denotes the exporter. $X_{ij}$ denotes manufacturing trade flows from $j$ to $i$, and $\text{ABS}_i$ is country $i$’s absorption defined as gross output less net exports of manufactures.

### B.2 National accounts and price data

**GDP and population** For our calibration, we collect data on output-side real GDP at current Purchasing Power Parity (2005 U.S. dollars) from version 8.1 of the Penn World Tables (PWT hereafter; see Feenstra, Inklaar, and Timmer, 2015) using the variable $\text{cgdpo}$.

We use the variable $\text{pop}$ from PWT to measure the population in each country. The ratio $\frac{\text{cgdpo}}{\text{pop}}$ corresponds to GDP per capita, $y$, in our model.

In our counterfactuals, we compare changes over time with past trade liberalization episodes using national accounts data from the PWT: $\text{rgdpna}$, $\text{rkna}$, and $\text{rtfpna}$.

We take the price level of household consumption and the price level of capital formation (both relative to the price of output-side GDP in the Unites States in constant prices) from PWT using variables $\text{pl.c}$ and $\text{pl.i}$, respectively. These correspond to $P_c$ and $P_z$ in our model.
We construct the price of intermediate goods (manufactures) by combining disaggregate price data from the World Bank’s 2011 International Comparison Program (ICP; http://siteresources.worldbank.org/ICPEXT/Resources/ICP_2011.html). The data have several categories that fall under what we classify as manufactures: “Food and nonalcoholic beverages,” “Alcoholic beverages, tobacco, and narcotics,” “Clothing and foot wear,” and “Machinery and equipment.” The ICP reports expenditure data for these categories in both nominal U.S. dollars and real U.S. dollars. The conversion from nominal to real dollars uses the PPP price; that is, the PPP price equals the ratio of nominal expenditures to real expenditures. As such, we compute the PPP for manufactures as a whole of manufactures for each country as the sum of nominal expenditures across categories divided by the sum of real expenditures across categories.

There is one more step before we take these prices to the model. The data correspond to expenditures and thus include additional margins such as distribution. To adjust for this, we first construct a price for distribution services. We assume that the price of distribution services is proportional to the overall price of services in each country and use the same method as above to compute the price across the following categories: “Housing, water, electricity, gas, and other fuels,” “Health,” “Transport,” “Communication,” “Recreation and culture,” “Education,” “Restaurants and hotels,” and “Construction.”

Now that we have the price of services in hand, we strip it away from the price of goods computed above to arrive at a measure of the price of manufactures that better corresponds to our model. In particular, let $P_d$ denote the price of distribution services and let $P_g$ denote the price of goods that includes the distribution margin. We assume that $P_g = P_d \psi P_m^{1-\psi}$, where $P_m$ is the price of manufactures. We set $\psi = 0.45$, which is a value commonly used in the literature.

### B.3 Computing the steady-state equilibrium in the balanced trade model

The steady-state equilibrium consists of 22 objects: $\vec{w}^*, \vec{r}^*, \vec{P}_c^*, \vec{P}_m^*, \vec{P}_x^*, \vec{C}^*, \vec{X}^*, \vec{K}^*, \vec{M}^*$, $\vec{Y}_c^*, \vec{Y}_m^*, \vec{Y}_x^*, \vec{K}_c^*, \vec{K}_m^*, \vec{K}_x^*, \vec{L}_c^*, \vec{L}_m^*, \vec{L}_x^*, \vec{M}_c^*, \vec{M}_m^*, \vec{M}_x^*, \vec{\pi}_c^*, \vec{\pi}_m^*, \vec{\pi}_x^*$. Table 1 provides a list of 23 equilibrium conditions that these objects must satisfy. One market clearing equation is redundant (condition 12 in our algorithm).
Table B.1: Steady-state equilibrium conditions in balanced trade model

<p>| | | |</p>
<table>
<thead>
<tr>
<th></th>
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</tr>
</thead>
<tbody>
<tr>
<td>1</td>
<td>$r_i^* K_{ci}^* = \alpha \nu_c P_{ci}^* Y_{ci}^*$</td>
<td>( \forall(i) )</td>
</tr>
<tr>
<td>2</td>
<td>$r_i^* K_{mi}^* = \alpha \nu_m P_{mi}^* Y_{mi}^*$</td>
<td>( \forall(i) )</td>
</tr>
<tr>
<td>3</td>
<td>$r_i^* K_{xi}^* = \alpha \nu_x P_{xi}^* Y_{xi}^*$</td>
<td>( \forall(i) )</td>
</tr>
<tr>
<td>4</td>
<td>$w_i^* L_{ci}^* = (1 - \alpha) \nu_c P_{ci}^* Y_{ci}^*$</td>
<td>( \forall(i) )</td>
</tr>
<tr>
<td>5</td>
<td>$w_i^* L_{mi}^* = (1 - \alpha) \nu_m P_{mi}^* Y_{mi}^*$</td>
<td>( \forall(i) )</td>
</tr>
<tr>
<td>6</td>
<td>$w_i^* L_{xi}^* = (1 - \alpha) \nu_x P_{xi}^* Y_{xi}^*$</td>
<td>( \forall(i) )</td>
</tr>
<tr>
<td>7</td>
<td>$P_{mi}^* M_{ci}^* = (1 - \nu_c) P_{ci}^* Y_{ci}^*$</td>
<td>( \forall(i) )</td>
</tr>
<tr>
<td>8</td>
<td>$P_{mi}^* M_{mi}^* = (1 - \nu_m) P_{mi}^* Y_{mi}^*$</td>
<td>( \forall(i) )</td>
</tr>
<tr>
<td>9</td>
<td>$P_{mi}^* M_{xi}^* = (1 - \nu_x) P_{xi}^* Y_{xi}^*$</td>
<td>( \forall(i) )</td>
</tr>
<tr>
<td>10</td>
<td>$K_{ci}^* + K_{mi}^* + K_{xi}^* = K_i^*$</td>
<td>( \forall(i) )</td>
</tr>
<tr>
<td>11</td>
<td>$L_{ci}^* + L_{mi}^* + L_{xi}^* = L_i$</td>
<td>( \forall(i) )</td>
</tr>
<tr>
<td>12</td>
<td>$M_{ci}^* + M_{mi}^* + M_{xi}^* = M_i^*$</td>
<td>( \forall(i) )</td>
</tr>
<tr>
<td>13</td>
<td>$C_i^* = Y_i^*$</td>
<td>( \forall(i) )</td>
</tr>
<tr>
<td>14</td>
<td>$\sum_{j=1}^I P_{mj}^* (M_{cj}^* + M_{mj}^* + M_{xj}^<em>) \pi_{ji} = P_{mi}^</em> Y_{mi}^*$</td>
<td>( \forall(i) )</td>
</tr>
<tr>
<td>15</td>
<td>$X_i^* = Y_{xi}^*$</td>
<td>( \forall(i) )</td>
</tr>
<tr>
<td>16</td>
<td>$P_{ci}^* = \left( \frac{1}{A_{ci}} \right) \left( \frac{r_i^<em>}{\alpha r_c} \right) ^{\alpha r_c} \left( \frac{w_{ij}^</em>}{(1-\alpha) \nu_c} \right) ^{(1-\alpha) \nu_c} \left( \frac{P_{mi}^*}{1-\nu_c} \right) ^{1-\nu_c}$</td>
<td>( \forall(i) )</td>
</tr>
<tr>
<td>17</td>
<td>$P_{mi}^* = \gamma \left[ \sum_{j=1}^I (u_{mj}^* d_{ij})^{-\theta T_{mj}} \right]^{-\frac{1}{\theta}}$</td>
<td>( \forall(i) )</td>
</tr>
<tr>
<td>18</td>
<td>$P_{xi}^* = \left( \frac{1}{A_{xi}} \right) \left( \frac{r_i^<em>}{\alpha r_x} \right) ^{\alpha r_x} \left( \frac{w_{ij}^</em>}{(1-\alpha) \nu_x} \right) ^{(1-\alpha) \nu_x} \left( \frac{P_{mi}^*}{1-\nu_x} \right) ^{1-\nu_x}$</td>
<td>( \forall(i) )</td>
</tr>
<tr>
<td>19</td>
<td>$\pi_{ij}^* = \frac{\sum_{j=1}^I (u_{mj}^* d_{ij})^{-\theta T_{mj}}}{\sum_{j=1}^I (u_{mj}^* d_{ij})^{-\theta T_{mj}}}$</td>
<td>( \forall(i, j) )</td>
</tr>
<tr>
<td>20</td>
<td>$P_{mi}^* Y_{mi}^* = P_{mi}^* M_i^*$</td>
<td>( \forall(i) )</td>
</tr>
<tr>
<td>21</td>
<td>$P_{ci}^* C_i^* + P_{xi}^* X_i^* = r_i^* K_i^* + w_i^* L_i^*$</td>
<td>( \forall(i) )</td>
</tr>
<tr>
<td>22</td>
<td>$X_i^* = \delta K_i^*$</td>
<td>( \forall(i) )</td>
</tr>
<tr>
<td>23</td>
<td>$r_i^* = \left( \frac{1}{\gamma} - (1 - \delta) \right) P_{xi}^*$</td>
<td>( \forall(i) )</td>
</tr>
</tbody>
</table>

Note: $u_{mj}^* = \left( \frac{r_i^*}{\alpha r_m} \right) ^{\alpha r_m} \left( \frac{w_{ij}^*}{(1-\alpha) \nu_m} \right) ^{(1-\alpha) \nu_m} \left( \frac{P_{mi}^*}{1-\nu_m} \right) ^{1-\nu_m}$. 

27
C Derivations

This Appendix shows the derivations of key structural relationships in the balanced trade model. We omit time subscripts to ease notation. We begin by deriving an expression for $\frac{w_i}{P_{mi}}$ that will be used repeatedly.

Combining conditions 17 and 19, we obtain

$$\pi_{ii} = \gamma^{-\theta} \left( \frac{u_{mi} - \theta T_{mi}}{P_{mi}^\theta} \right).$$

Use the fact that $u_{mi} = B_m r^\alpha w_i (1 - \alpha) \nu_{mi} P_{mi}^{1 - \nu_{mi}}$, where $B_m$ is a collection of constants; then rearrange to obtain

$$P_{mi} = \left( \frac{T_{mi}}{\pi_{ii}} \right)^{\frac{1}{\theta}} \left( \frac{r_i}{w_i} \right)^{\alpha \nu_{mi}} \left( \frac{w_i}{P_{mi}} \right)^{\nu_{mi}} P_{mi},$$

$$\Rightarrow \frac{w_i}{P_{mi}} = \left( \frac{T_{mi}}{\pi_{ii}} \right)^{\frac{1}{\theta}} \left( \frac{w_i}{P_{mi}} \right)^{\alpha \nu_{mi}} \left( \frac{r_i}{w_i} \right)^{\nu_{mi}} \gamma B_m.$$

Note that this relationship holds in both the steady state and along the transition.

Relative prices We show how to derive the price of consumption relative to intermediates; the relative price of investment is analogous. Begin with condition 16 to obtain

$$P_{ci} = \left( \frac{B_c}{A_{ci}} \right) \left( \frac{r_i}{w_i} \right)^{\alpha \nu_c} \left( \frac{w_i}{P_{mi}} \right)^{\nu_c} P_{mi},$$

where $B_c$ is a collection of constants. Substitute equation (C.1) into the previous expression and rearrange to obtain

$$\frac{P_{ci}}{P_{mi}} = \left( \frac{B_c}{A_{ci}} \right) \left( \frac{T_{mi}}{\pi_{ii}} \right)^{\frac{\nu_c}{\nu_{mi}}} \gamma B_m.$$

Analogously,

$$\frac{P_{xi}}{P_{mi}} = \left( \frac{B_x}{A_{xi}} \right) \left( \frac{T_{mi}}{\pi_{ii}} \right)^{\frac{\nu_x}{\nu_{mi}}} \gamma B_m.$$
Note that these relationships hold in both the steady state and along the transition.

**Capital-labor ratio** We derive a structural relationship for the capital-labor ratio in the steady state only and make reference to conditions in Table B.1. Conditions 1-6 together with conditions 10 and 11 imply that

$$\frac{K_i}{L_i} = \left( \frac{\alpha}{1 - \alpha} \right) \left( \frac{w_i}{r_i} \right).$$

Using condition 23, we know that

$$r_i = \left( \frac{1}{\beta} - (1 - \delta) \right) P_{xi},$$

which, by substituting into the prior expression, implies that

$$\frac{K_i}{L_i} = \left( \frac{\alpha}{(1 - \alpha) \left( \frac{1}{\beta} - (1 - \delta) \right)} \right) \left( \frac{w_i}{P_{xi}} \right),$$

which leaves the problem of solving for $\frac{w_i}{P_{xi}}$. Equations (C.1) and (C.3) imply

$$\frac{w_i}{P_{xi}} = \left( \frac{w_i}{P_{mi}} \right) \left( \frac{P_{mi}}{P_{xi}} \right) = \left( \frac{A_{xi}}{B_x} \right) \left( \frac{\left( \frac{T_{mi}}{\pi_{ii}} \right)^{\frac{1}{\gamma}}}{\gamma B_m} \right) \left( \frac{w_i}{r_i} \right) \alpha.$$

Substituting once more for $\frac{w_i}{r_i}$ in the previous expression yields

$$\left( \frac{w_i}{P_{xi}} \right)^{1 - \alpha} = \left( \frac{1}{\beta} - (1 - \delta) \right)^{-\alpha} \left( \frac{A_{xi}}{B_x} \right) \left( \frac{\left( \frac{T_{mi}}{\pi_{ii}} \right)^{\frac{1}{\gamma}}}{\gamma B_m} \right)^{1 - \frac{\nu_x}{\nu_m}}.$$

Solve for the aggregate capital-labor ratio

$$\frac{K_i}{L_i} = \left( \frac{\alpha}{1 - \alpha} \right) \left( \frac{A_{xi}}{B_x} \right)^{\frac{1}{\alpha}} \left( \frac{\left( \frac{T_{mi}}{\pi_{ii}} \right)^{\frac{1}{\gamma}}}{\gamma B_m} \right)^{1 - \frac{\nu_x}{(1 - \alpha)\nu_m}}.$$

(C.4)
Note that we invoked steady-state conditions, so this expression does not necessarily hold along the transition path.

**Income per capita** We define (real) income per capita in our model as

\[ y_i = \frac{r_i K_i + w_i L_i}{L_i P_{ci}}. \]

We invoke conditions from Table ?? for the remainder of this derivation. Conditions 1-6, 10, and 11 imply that

\[ r_i K_i + w_i L_i = \frac{w_i L_i}{1 - \alpha} \Rightarrow y_i = \left( \frac{1}{1 - \alpha} \right) \left( \frac{w_i}{P_{ci}} \right). \]

To solve for \( \frac{w_i}{P_{ci}} \), we use condition 16:

\[ P_{ci} = \frac{B_c}{A_{ci}} \left( \frac{r_i}{w_i} \right)^{\alpha \nu_c} \left( \frac{w_i}{P_{mi}} \right)^{\nu_c} P_{mi} \Rightarrow \frac{P_{ci}}{w_i} = \frac{B_c}{A_{ci}} \left( \frac{r_i}{w_i} \right)^{\alpha \nu_c} \left( \frac{w_i}{P_{mi}} \right)^{\nu_c - 1}. \]

Substituting equation (C.1) into the previous expression and exploiting the fact that \( \frac{w_i}{r_i} = \left( \frac{1 - \alpha}{\alpha} \right) \left( \frac{K_i}{L_i} \right) \) yields

\[ y_i = \left( \frac{1}{1 - \alpha} \right) \left( \frac{w_i}{P_{ci}} \right) = \alpha^{-\alpha} (1 - \alpha)^{\alpha - 1} \left( \frac{A_{ci}}{B_c} \right) \left( \frac{\frac{\tau_{mi}}{\gamma B_m}}{\frac{1}{\gamma B_m}} \right)^{\frac{1 - \nu_c}{\gamma B_m}} \left( \frac{K_i}{L_i} \right)^{\alpha}. \]

(C.5)

Note that this expression holds both in the steady state and along the transition path.

The steady-state income per capita can be expressed more fundamentally by invoking
equation (C.4) as

\[
y_i = \left( \frac{\frac{1}{\beta} - (1 - \delta)}{1 - \alpha} \right)^{-\frac{\alpha}{1 - \alpha}} \left( \frac{A_{c_i}}{B_c} \right)^\frac{\alpha}{1 - \alpha} \left( \frac{T_{m_i}}{\pi_{i1}} \right)^\frac{1}{\gamma} \left( \frac{\gamma B_m}{\nu_m} \right)^{1 - (1 - \nu_a)} \left( \frac{\alpha}{1 - \alpha} \right)^{(1 - \nu_a)} . \tag{C.6}
\]