Market Power and Informational Efficiency*

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Abstract

Levels and concentration of institutional equity ownership have been growing steadily over the last few decades raising concerns of potential market instability. We study theoretically and empirically the consequences of changes in ownership structure for informational content of prices, on average and across assets with different characteristics. Our theoretical framework is a general equilibrium portfolio-choice model with endogenous information acquisition and market power. We show that, in the cross section, an increase in institutional (informed) ownership increases price informativeness, and an increase in concentration of ownership leads to lower informativeness. We confirm similar effects in the data of U.S. stocks over the period 1980-2015. The policy experiments of changing ownership structure indicate a non-monotonic relation between the levels and concentration of ownership and price informativeness. We conclude that any policy targeting ownership structure should factor in its effects on welfare through price informativeness.

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1 Introduction

In the last few decades, equity ownership of large asset management companies has drastically increased, as has the concentration of those holdings. From 1980 to 2015, the average institutional ownership of U.S. stocks increased from 25% to 60%, and ownership levels among the top 10 largest asset management companies almost doubled, from 18% to 35%.

These striking trends have spurred an active debate among practitioners and policy makers over whether the largest asset managers should be regulated as systemically important financial institutions (SIFIs). Proponents of regulating large asset managers argue that a high concentration in asset ownership may increase systemic risk and cause financial instability, similar to what was observed in the banking sector during the financial crisis of 2007-2008.\footnote{For example, in its 2014 Global Financial Stability Report, the International Monetary Fund stated that the concentration of assets among top managers and concentrated decision making within asset managers would mean that large scale redemptions in one fund could lead to losses across unrelated asset classes, and magnify selling pressure across markets. Similarly, the negative impact of crowded trades on market efficiency has been also studied in Stein (2009).} Further, empirical evidence suggests that large asset managers’ performance may suffer from diseconomies of scale (e.g., Chen, Hong, Huang, and Kubik (2004)). At the same time, many point out that the growth in ownership by large institutions could be healthy as it allows households to access financial products with lower transaction costs and greater operational efficiency.

In this paper, we extend the debate along two dimensions. First, we examine the role of large asset managers from a new angle—the impact that large owners exert on price informativeness, an important component of any welfare analysis. Second, we provide a micro-founded general equilibrium framework that permits analysis of policies that target the ownership structure of financial markets.

Our model features an economy with many assets and many large investors (herein referred to as oligopolists), who each have the ability to internalize the effect of their trades on equilibrium prices. The model also features a continuum of smaller, fringe
investors, each with a smaller capacity to process information. All agents can learn about the future paths of prices, and then oligopolists engage in a Cournot game in the asset market. Inputs into the model are the size of the agents, based on assets under management, and agents’ learning capacities. The presence of large investors is a novel component of portfolio choice models with endogenous information acquisition.

A standard result in the literature is that agents specialize in their learning because returns to learning increase as agents become more informed. We show that while the competitive fringe in our model specializes, large investors need not because they may internalize their market power in their behavior. As market power increases, oligopolists know that the price effect of their trades will be higher, and that the information they collected will be revealed. As a result, they mitigate their price impact by spreading their learning over many assets. Thus, the model endogenously generates diversification in learning, which has important implications for both overall ownership structure and the average price informativeness of each asset.

The model exploits two key cross-sectional mechanisms. The first one is a positive cross-sectional relation between ownership levels and price informativeness. Oligopolists are assumed to be better equipped to learn about assets. The assets that are learned about more by oligopolists will, as a result, also see higher levels of demand. Therefore, assets that have high levels of equilibrium holdings will also have high levels of price informativeness. Moreover, informed investors exhibit a preference for volatility (as the marginal benefit of learning is higher), so assets with higher volatility have greater price informativeness.

The second one is a negative cross-sectional relation between ownership concentration and price informativeness. Controlling for the overall size of oligopolists’ holdings, increasing one oligopolist’s size makes her lessen her learning in that asset to mitigate the price impact. As a result, assets with high levels of concentration in equilibrium also feature lower levels of price informativeness.

To better understand the underlying economic mechanism of the results, we de-
rive closed-form analytic solutions relying on some special cases of the model. Our benchmark is the fully competitive information model with no oligopolists, as in Kacperczyk, Nosal, and Stevens (2017). In this framework, *high-volatility assets are learned about first*. We then introduce a large (monopoly) trader to the competitive market fringe. We show that in the absence of learning, market power increases price volatility and decreases price informativeness. But if the monopolist can learn, she will choose to learn about the most volatile assets first (just as the fringe did). More relevant, we show that *greater market power* causes the monopolist to diversify her attention, which means that the *prices of less volatile assets become relatively more informative, while the prices of more volatile assets become relatively less informative*. Since there are increasing returns to learning, diversifying attention actually *reduces average price informativeness*.

To establish further generality of our results, we simulate the oligopoly model by using institutional ownership as a proxy for informed, and large ownership, consistent with our motivating facts. The numerical exercise derives richer cross-sectional and aggregate implications. First, in the cross section, for a baseline economy, we establish a positive relation between the level of institutional ownership and price efficiency across assets—showing the agents' preference for learning about the most volatile assets. Additionally, we obtain a negative cross-sectional relation between concentration and price informativeness.

Subsequently, we turn to policy experiments for the aggregate market. We show that aggregate institutional ownership has a non-monotonic, hump-shaped relation with average price informativeness: The relation is positive for lower levels of ownership and negative for higher levels. A similar nonlinearity is present when we relate concentration to price informativeness. The intuition for these effects is as follows: When the size of the institutional sector is small, oligopolists' considerations with respect to the impact of their trades on asset prices do not have a great impact on their information allocation decisions. Thus, each oligopolist specializes in learning
and receives precise signals, but because of their small size, not a lot of information is revealed in prices. As the size of the institutional sector increases a little, each oligopolist remains relatively small in size so still mainly specializes, but because they are each a little bigger, more information is revealed in prices. As the sector increases in size a lot, each oligopolist is big enough that the degree of the impact of her trades on asset prices causes her to diversify her learning. The diversification is inefficient from the aggregate perspective, due to increasing returns to scale of learning.

In additional experiments, we further show that the conclusions still hold if we allow for the presence of investors with market power but no ability to learn about their price impact. Finally, we contrast the results of our benchmark model with a model in which the attention allocation is exogenous, as in Kyle (1985). We show that the benchmark model predicts a very different relation of ownership and concentration with price informativeness. In particular, we show that the policy prescriptions about the optimal level of institutional ownership coming from a model with exogenous information choice can be biased upwards or downwards relative to the fully endogenous model, depending on the exogenous information structure one assumes. In terms of the policy prescriptions for optimal concentration, the benchmark model always prescribes low concentration as optimal for price informativeness, while the model with exogenous information predicts optimal levels of concentration that have intermediate, sometimes much higher levels. We conclude that modeling endogenous information choices is crucial when making normative statements about the size and structure of the institutional asset management sector.

Next, we turn to empirical work to illustrate some of the theoretical tradeoffs in the data. Our testing sample covers companies listed on U.S. stock exchanges from 1980 to 2015.² When we sort stocks into deciles by their ownership levels, we find that price informativeness, defined as the predicted variation in cash flows, is highest for high-ownership stocks. This result is consistent with that reported in Bai,

²We are limited by the availability of the institutional ownership data.
Philippon, and Savov (2016), as well as our model. In the cross-section, assets that are learned about more see higher levels of demand, and therefore high equilibrium levels of holdings and informativeness. However in the time series, we cannot rule out other drivers of variation in price informativeness. Depending on the choice of parameter values, the relation between average ownership and price informativeness could be non-monotonic. The results presented in Bai et al. (2016) are thus a special case rather than a general rule on the time-series tradeoffs between the two quantities.

We further decompose price informativeness into two parts: (i) a part related to correlation between prices and fundamentals and (ii) a part related to volatility of cash flows, and show that the monotonicity of our results becomes even stronger for the former component, which supports our information-based story. We also sort stocks according to their concentration levels of institutional ownership, proxied by the Herfindahl-Hirshman Index (HHI). We find that assets with the highest concentration display, on average, lower values of price informativeness. Controlling for levels of institutional ownership preserves the results. We conclude that higher levels of ownership concentration can diminish price informativeness in the cross section, which is the same as in the model. Again, the time-series results are not as clearly tied to the mechanisms of the model. These results jointly leave some space for meaningful policy interventions in asset management.

In Section 2, we present a set of motivating facts from the U.S. data on institutional ownership and its concentration. Section 3 presents the theoretical framework, the equilibrium concept, and derives basic theoretical tradeoffs between the ownership structure and price informativeness. In Section 4, we derive numerical solutions for the more general settings and discuss various policy experiments. Section 5 provides empirical results corroborating some of the model’s predictions. Section 6 concludes. Any omitted proofs and derivations are in the Appendix.
1.1 Related Literature

Our paper spans several research themes. First, our general equilibrium model is anchored in the literature on the endogenous information choice, in the spirit of Sims (1998, 2003). More closely related to our application are the models of costly information of Van Nieuwerburgh and Veldkamp (2009, 2010), Mondria (2010), Kacperczyk, Van Nieuwerburgh, and Veldkamp (2016), and Kacperczyk, Nosal, and Stevens (2017). Ours is the first theoretical study to introduce market power into a model with endogenous information acquisition. This novel aspect allows us to study strategic responses of oligopolistic traders in terms of their demand and information choices.

The literature on informed trading with market power dates back to Kyle (1985) whose setup is one strategic trader, and Holden and Subrahmanyam (1992), which extends the model of Kyle into an oligopolistic framework. Lambert, Ostrovsky, and Panov (2016) extend the Kyle’s model to study the relation between the number of strategic traders and information content of prices.3 In all these studies, information is an exogenous process, which is a key dimension along which our model works. Also, they do not examine the role of concentration of ownership among strategic traders, which is the main focus of our study. Kyle, Ou-Yang, and Wei (2011) allows for endogenous information acquisition but their mechanism depends on differences in risk aversion. Also, they focus on the contracting features of delegation and allow for only one risky asset. In turn, our framework operates through heterogeneity in information capacity and multi-asset economy.

We also contribute to the literature on information production and asset prices. Bond, Edmans, and Goldstein (2012) survey the literature on information production in financial markets, emphasizing the differences between new information produced in markets (revelatory price efficiency: RPE) and what is already known and merely reflected in prices (forecasting price efficiency: FPE). Our focus is solely on RPE and

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3Models in which traders condition on others’ decisions also include Foster and Viswanathan (1996) and Back, Cao, and Willard (2000).
is largely dictated by the modeling framework we use. Stein (2009) develops a model of market efficiency and sophisticated (arbitrage) capital in the presence of capital constraints. Garleanu and Pedersen (2015) examine the role of search frictions in asset management for price efficiency. On an empirical front, Chen, Goldstein, and Jiang (2007) and Bakke and Whited (2010) find that the relation between stock prices and investment is stronger for firms with more informative stock prices, whereas Baker, Stein, and Wurgler (2003) find that it is stronger for firms that issue equity more often. None of the above studies investigates the role of market power and endogenous information acquisition. The exception is Bai, Philippon, and Savov (2016) who show empirically that price informativeness is greater for stocks with greater institutional ownership. We confirm their findings for the range of the ownership values. However, we show that beyond certain levels (not observed in their data) ownership may in fact reduce price informativeness. Separately, we also investigate the role of ownership concentration and provide a micro-founded general equilibrium model that allows us to study the underlying economic mechanism in more depth.

Finally, we add to a growing empirical literature that studies the impact of market structure in asset management on various economic outcomes. Following the diseconomies of scale argument of Chen et al. (2004), Pástor, Stambaugh, and Taylor (2015) show significant diseconomies of scale at the industry level. Using a merger between BlackRock and BGI as a shock to market power, Massa, Schumacher, and Yan (2016) study the asset allocation responses of their competitors. They find that competitors scale down positions which overlap with those held by the merged entity. More broadly, He and Huang (2014) and Azar, Schmalz, and Tecu (2016) study consequences of common asset ownership by large blockholders for product market competition and prices. Our work complements these studies by studying, theoretically and empirically, the effect of ownership structure on price informativeness.

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4Theoretical work on asset prices and real efficiency also includes Dow and Gorton (1997), Subrahmanyam and Titman (1999), Kurlat and Veldkamp (2015), and Edmans, Goldstein, and Jiang (2015).
2 Motivating Facts

In this section, we present the two empirical facts that motivate our study. First, we show that institutional stock ownership has increased widely over the last thirty-five years. Second, we show that the ownership structure is skewed towards the largest owners. The growth in institutional ownership has been previously documented in several studies, including Gompers and Metrick (2001). The evidence on concentration is much more sparse. However, except for the recent paper by Bai, Philippon, and Savov (2016) which emphasizes the first fact, no other study has exploited the implications of these facts for longer-horizon price informativeness.\footnote{A parallel microstructure literature (Boehmer and Kelley (2009)) examines empirically the relation between institutional ownership and price efficiency due to trading intensity. Efficiency there is measured using variance ratios and pricing errors. The conclusions from this literature are akin to those reported in our paper.}

Our data on institutional stock ownership come from Thomson Reuters and span the period 1980–2015. Even though the formal requirements to report holdings allow smaller companies not to report, the representation of institutions in the data is more than 98% in value-weighted terms. We calculate the stock-level institutional ownership by taking the ratio of the number of stocks held by financial institutions at the end of a given year to the number of shares outstanding at the same time. Next, we aggregate the measures across stocks by taking a simple average across all stocks in our sample. Using equal weighting, rather than value weighting, gives a conservative metric of the trends in the data. Subsequently, we calculate a similar measure, but only taking into account the holdings of the top-10 largest holders for a given stock. We present the time-series dynamics of the two quantities in Figure 1.

Both series indicate a clear pattern underlying the recent policy discussions: Institutional ownership has grown and the increase is mostly fueled by the growing concentration of ownership. The magnitudes of the growth are economically large: Over the period of over 35 years, each ownership statistic has more than doubled. While we focus here on the average trends in the data, even stronger effects can be
Figure 1: Institutional Ownership: Unconditional and Top-10 Holders

observed in the cross section of stocks with different characteristics.

In our model, a more natural way to measure concentration is the Herfindahl-Hirshman Index (HHI), defined as the sum of squared shares of all institutional owners of a given stock. However, the problem with using a raw index value is its mechanical correlation with the number of investors in the data. To the extent that the number of institutions has been growing steadily over the same period the unadjusted index would reflect two effects going in opposite direction. To filter out this mechanical sorting, we take out the predicted component in the HHI accounted by the number of investors. We plot the filtered series in Figure 2.

The results indicate that the concentration levels have been generally going up over time. This pattern has been particularly visible since the early 1990s. The magnitude of the growth is economically large and the large values of concentration, especially in the last few years, reflect the concerns policy makers have voiced with regard to this phenomenon. To conclude, it is important to note that while the motivating facts we present relate to institutional investors, the model we present next is a general theory of asset allocation and information acquisition by investors with market power. We believe institutional investors are natural candidates for this type of investors.
This section presents a noisy rational expectations portfolio choice model in which investors are constrained in their capacity to process information about assets payoffs. The setup departs from the information choice model of Kacperczyk et al. (2017) by introducing market power for some investors. Also, we solve for price informativeness of the aggregate economy and individual assets differentiated by their volatility.

3.1 Setup

The model features a finite continuum of traders, divided into $l+1$ many segments, represented by $\lambda$. The traders in the first segment, $\lambda_0$, are atomistic – these traders act as a competitive fringe, in that they are able to pay attention to innovations in asset prices, but do not have any market power. They are indexed by $h$. Measures $\{\lambda_1, \lambda_2, ..., \lambda_l\}$ of investors act as oligopolists, indexed by $j$. Each measure collects information and trades, as the fringe does, but the oligopolists collect and trade as a unit, and therefore they have market power in information, and market power as traders.
Every member of the fringe, and every oligopolist observe signals about innovations in asset prices. The vector of signals for the oligopolist for asset $j$ is $s_j = (s_{j1}...s_{jl})$. The vector of signals for the fringe for asset $j$ is indexed by $h$. Investors of both types maximize mean-variance utility function, with common risk aversion $\rho$.

The market has one risk-free asset, with a price normalized to one, and a net payout of $r$, and $n > 1$ risky assets, indexed by $i$, with prices $p_i$ and independent payoffs $z_i = \bar{z} + \varepsilon_i$, where $\varepsilon_i \sim N(0, \sigma_i^2)$. The risk-free asset has unlimited supply, and each risky asset has a stochastic supply with mean $\bar{x}$ and variance $\{\sigma_{xi}\}$. We can think of these as noisy supply shocks.

Agents make portfolio decisions and can choose to obtain information about the price innovations for some or all of the risky assets. The capacity to process information for the oligopolists is denoted $\{K_j\}$, while the capacity of each member of the fringe is constant at $K_h$. We place no restrictions on the values of $K_j$ and $K_h$ other than they must be finite. Investors do not learn from prices. Oligopolists and members of the fringe can use their capacities to receive informative signals about the payoff of the asset and reduce that variance accordingly. We model signal choice using entropy reduction as in Sims (2003).

We denote an agent’s posterior variance as $\hat{\sigma}^2$. For simplicity, we also define $\alpha_{ji} \equiv \frac{\sigma^2}{\sigma_{ji}^2} \geq 1$. We conjecture and later verify the following price structure:

$$p_i = a_i + b_i \varepsilon_i - c_i \nu_i - \sum_{j=1}^{n} d_{ji} \zeta_{ji}$$  \hspace{1cm} (1)$$

where $\varepsilon_i$ and $\nu_i$ are the innovations in the payoff and noisy supply shocks, respectively. The first term corresponds to the base price, and the second one to the innovation. The innovation is not typically revealed completely in prices, because agents cannot perfectly observe it. The third term corresponds to noise or liquidity shocks, while the fourth one is defined as follows: First, define $\delta_{ji}$ as the data loss of oligopolist
\( \delta_{ji} \equiv z_i - s_{ji} \), then define \( \zeta_{ji} \equiv \delta_{ji} - \frac{1}{\alpha_{ji}} z_i \) to be the portion of the dataloss that is uncorrelated with the price innovation. Then \( p_i \sim \mathcal{N}(a_i, \sigma^2_{pi}) \) where \( \sigma_{pi} \) can be expressed as:

\[
\sigma^2_{pi} = b_i^2 \sigma^2_i + c_i^2 \sigma^2_{xi} + \sum_{j=1}^{n} d_{ji}^2 \left( 1 - \frac{1}{\alpha_{ji}} \right) \sigma^2_{ji} + \sum_{j=1}^{n} \sum_{k \neq j} 2d_{ji}d_{ki} \left( \frac{1 + \alpha_{ji}\alpha_{ki}}{\alpha_{ji}\alpha_{ki}} \sigma^2_i - \frac{1 + \alpha_{ki}}{\alpha_{ki}} \sigma^2_{ki} - \frac{1 + \alpha_{ji}}{\alpha_{ji}} \sigma^2_{ji} \right) \tag{2}
\]

See the Appendix 7.1.1 for the derivations. Before solving the oligopolists’s problem, we first turn to the problem faced by the competitive fringe.

### 3.1.1 Competitive fringe

**Portfolio problem** The portfolio problem of the fringe is as follows. Given posterior beliefs and equilibrium prices, each competitive investor \( h \) solves the following problem:

\[
U_h = \max_{\{q_{hi}\}_{i=1}^{n}} E_h(W_h) - \frac{\rho}{2} V_h(W_h) \quad \text{s.t.} \quad W_j = r \left( W_{0h} - \sum_{i=1}^{n} q_{hi} p_i \right) + \sum_{i=1}^{n} q_{hi} z_i \tag{3}
\]

where \( E_h \) and \( V_h \) are the perceived mean and variance of investor \( h \) conditional on her information set, and \( W_{0h} \) is initial wealth. Then, optimal portfolio holdings are:

\[
q_{hi} = \frac{\mu_{hi} - rp_i}{\rho \sigma^2_{hi}} \tag{4}
\]

where \( \mu_{hi} \) and \( \sigma^2_{hi} \) are the mean and variance of investor \( h \)’s posterior beliefs about payoff \( z_i \).

Given this ‘second-stage’ problem, the fringe agents have a ‘first-stage’ information choice problem. Each member of the fringe can choose to receive signals \( s_{hi} \) on each asset payoff \( \epsilon_i \). The vector of signals is subject to an information capacity
constraint, based on Shannon (1948)’s mutual information measure: $I(z; s_h) \leq K_h$. Since $K_h$ is finite, this expression constrains the ability of fringe members to reduce the uncertainty of signals.

**Information problem** Each member of the fringe faces the following information problem:

$$
\max_{\{\tilde{\sigma}^2_{hi}\}} U_{0h} \equiv \frac{1}{2\rho} \sum_{i=1}^{n} \frac{E_{0h}(\hat{\mu}_{hi} - r p_i)^2}{\tilde{\sigma}^2_{hi}}
$$

subject to the relative entropy constraint

$$
\prod_{i=1}^{n} \frac{\sigma^2_i}{\tilde{\sigma}^2_{hi}} \leq e^{2K_h}.
$$

The information problem can also be written as:

$$
U_{0h} = \sum_{i=1}^{n} G_i \frac{\sigma^2_i}{\tilde{\sigma}^2_{hi}},
$$

We obtain a corner solution: each investor $h$ learns about one asset $l_h \in \arg \max \{G_i\}$.

The gain to the competitive investors from learning about asset $i$ is:

$$
G_i \equiv \frac{(\bar{z} - ra_i)^2}{\sigma^2_i} + (1 - rb_i)^2 + r^2 \frac{\sigma^2_{x_i}}{\sigma^2_i} +
$$

$$
\frac{r^2}{\tilde{\sigma}^2_{hi}} \left( \sum_{j=1}^{n} d_{ji} \left(1 - \frac{1}{\alpha_{ji}}\right) \hat{\sigma}^2_{ji} + \sum_{j=1}^{n} \sum_{k \neq j}^n 2d_{ji}d_{ki} \left( \frac{1 + \alpha_{ji}\alpha_{ki}}{\alpha_{ji}\alpha_{ki}} \sigma^2_i - \frac{1 + \alpha_{ki} \hat{\sigma}^2_{ki} - 1 + \alpha_{ji}\hat{\sigma}^2_{ji}}{\alpha_{ji}} \right) \right) - \frac{\tilde{\sigma}^2_{hi}}{\sigma^2_i} (1 - 2rb_i)
$$

Derivation in Appendix 7.1.2. The gain from learning about a particular asset is the same across all competitive investors. However, this gain is a function of the learning that the monopolist does in that asset (namely, it is a function of the oligopolist’s posterior variance, $\hat{\sigma}^2_{ji}$). The gains to learning about an asset’s payoff are that the fringe traders can take advantage of deviations in the price from their perception of
the asset’s value. The first term of $G_i$ corresponds to the gains of trade from the fundamental; the second one to the gains of trade from deviations in the innovation; the third one to the gains of trade from noise traders; and the fourth one to alterations in price due to data-loss by the oligopolists.

### 3.1.2 Oligopolist

**Portfolio problem** Oligopolists have a similar trading problem to the fringe and the quantity demanded by each oligopolist is:

$$q_{ji} = \frac{\hat{\mu}_{ji} - r p_i(q_{ji})}{\rho \hat{\sigma}^2_{ji} + \rho \sigma^2_{ji}},$$

where

$$\Phi_{hi} = m_{hi} (e^{2\Phi_{hi}} - 1),$$

and $m_{hi}$ is the mass of competitive investors learning about asset $i$. Hence, how sensitive the price is to an oligopolist’s demand depends (inversely) on how much the competitive fringe is learning about that asset, and how large it is.

The oligopolist’s demand becomes

$$q_{ji} = \frac{\hat{\mu}_{ji} - r p_i}{\rho \left( \hat{\sigma}^2_{ji} + \lambda_{ji} \sigma^2_i \right)}$$

where $\lambda_{ji} = \frac{\hat{\lambda}_{ji}}{\lambda_{0}(1 + \Phi_{hi})}$ - essentially a ratio of the effective shares of the oligopolists to the fringe. Given the expression for quantity, demanded we can then calculate
indirect utility:
\[
U_j = \frac{1}{2\rho} \sum_{i=1}^{n} (\tilde{\mu}_{ji} - r_{pi})^2 \left[ \frac{\tilde{\sigma}_{ji}^2 + 2\tilde{\lambda}_{i}\sigma_i^2}{(\tilde{\sigma}_{ji}^2 + \tilde{\lambda}_i\sigma_i^2)^2} \right],
\]
Derivation in Appendix 7.1.3. As with the fringe, oligopolists’ expected utilities depend positively on the deviations of their personal estimates from the equilibrium price (larger deviations mean larger quantities demanded). The smaller the oligopolists’ posterior variance the larger their utility. The larger the oligopolist’s market power (or conversely the smaller the fringe, or the less informed the fringe), the larger is the oligopolist’s price impact, and therefore the smaller her utility.

**Information problem** The monopolist’s information problem is to solve
\[
\max \{ \sigma_j \}_{j=1}^{n} U_{0j} \quad s.t. \quad \prod_{i=1}^{n} \frac{\sigma_i^2}{\sigma_{ji}^2} \leq e^{2K_j},
\]
We can also write the constraint as
\[
\prod_{i=1}^{n} \alpha_{ji} \leq e^{2K_j} \iff \sum_{i=1}^{n} \ln \alpha_{ji} \leq 2K_j,
\]
with
\[
\ln \alpha_{ji} \geq 0.
\]
The Lagrangean is [dropping $1/2\rho$]
\[
\mathcal{L} = \sum_{i=1}^{n} [u_i(\alpha_{ji}) - \mu \ln \alpha_{ji} + \eta \ln \alpha_{ji}] + n\gamma 2K_j,
\]
The optimality conditions are
\[
u'_i(\alpha_{ji}) - \frac{\mu}{\alpha_{ji}} + \frac{\eta}{\alpha_{ji}} = 0, \quad \forall i = 1, ..., n.
\]
The capacity constraint is always binding, so \( \sum_{i=1}^{n} \ln \alpha_{ji} = 2K_j \) and \( \mu > 0 \). Let \( L \) denote the set of assets that are learned about by the monopolist. We have that

\[
\alpha_{jl} > 1 \quad \text{and} \quad \eta_l = 0 \quad \text{and} \quad \mu = \alpha_{jl}u'_l(\alpha_{jl}) \quad \forall l \in L
\]  

(19)

and

\[
\sum_{l=L} \ln \alpha_{jl} = 2K_j.
\]

(20)

For assets \( i \notin L \),

\[
\alpha_{jl} = 1 \quad \text{and} \quad \eta_l = \mu - u'_l(1) \geq 0 \quad \Leftrightarrow \quad \alpha_{jl}u'_l(\alpha_{jl}) \geq u'_l(1) \quad \forall l \in L.
\]

(21)

These conditions yield the monopolist’s allocation of attention across assets, \( \{\alpha_{ji}\} \), as a function of the equilibrium price coefficients, \( a_i, b_i, c_i, d_i \), and the share of competitive investors learning about each asset, \( m_{hi} \). Given the choice of the monopolist of the set \( \{\alpha_{ji}\} \), variance of the posterior belief of the monopolist is \( \sigma_i^2/\alpha_{ji} \) and the mean is just the signal \( s_{ji} \). The signal is distributed, conditional on the realizations \( z_i = \bar{z} + \varepsilon_i \), as

\[
E(s_{ji}|z_i) = \bar{z} + \left(1 - \frac{\hat{\sigma}_{ji}^2}{\sigma_i^2}\right) \varepsilon_i = \bar{z} + \left(1 - \frac{1}{\alpha_{ji}}\right) \varepsilon_i,
\]

\[
Var(s_{ji}|z_i) = \sigma_i^2 \left(1 - \frac{\hat{\sigma}_{ji}^2}{\sigma_i^2}\right) \frac{\hat{\sigma}_{ji}^2}{\sigma_i^2} = \left(1 - \frac{1}{\alpha_{ji}}\right) \frac{1}{\alpha_{ji}} \sigma_i^2.
\]
3.2 Equilibrium

We can now solve for the coefficients of the equation (1) posited earlier. Doing so yields expressions for $a_i, b_i, c_i, d_{ki},$ and $d_{ji}$ (derivation in Appendix 7.1.4):

$$a_i = \frac{\bar{z}}{r} - \frac{\bar{x}}{r} \lambda_0 \Phi_{hi}$$

(22)

$$b_i = N_i \left( \sum_{j=1}^{n} M_{ji} (\alpha_{ji} - 1) + \frac{\Phi_{hi}}{r(1 + \Phi_{hi})} \right)$$

(23)

$$c_i = \frac{N_i \rho \sigma_i^2}{r \lambda_0 (1 + \Phi_{hi})}$$

(24)

$$d_{ji} = \frac{N_i M_{ji}}{r}$$

(25)

where $M_{ji} \equiv \frac{\lambda_j \sigma_i^2}{(\sigma_j^2 + \lambda_j \sigma_i^2)}$ and $N_i \equiv \frac{1}{1 + \sum_{j=1}^{n} M_{ji}}$. The fundamental component of the price, $a_i$, unsurprisingly depends positively on $\bar{z}$. An increase in supply will also decrease $a_i$, as will increased risk aversion and fundamental volatility. As the fringe’s size or attentional capacity increase, their demand increases, and thus prices increase. As the oligopolists’ size increases, or as their attention to asset $i$ increases, demand goes up, $M_{ji}$ increases, and $N_i$ decreases, again driving up the price.

$b_i$ depends almost exclusively on the information choices of the fringe and oligopolists. If the fringe cannot pay attention, then $\Phi$ drops to zero, as does the second term of the expression. If the oligopolists cannot pay attention, each $\alpha_{ji}$ goes to zero. $b_i$ is increasing in $\Phi_{hi}$ and $\alpha_{ji}$, because increased attention increases investors’ predictive power of the innovation, and therefore their information will be better reflected in prices.

The same reasons that demand fluctuates in $a_i$ apply to $c_i$, as $c_i$ corresponds to the random component, while $a_i$ corresponds to the mean component.
3.3 Comparative statics

The full model is very large—large enough that a full analytic solution remains elusive. But, if we proceed in stages, different parts of the model can generate some informative comparative statics. We will proceed, specifically, in four stages, each time varying the ability of the agents of the stage to collect information: In stage one, there are no oligopolists, only a fringe. In stage two, there is a fringe and a single monopolist. In stage three, there is a duopoly. In stage four, we move to the full oligopoly model. Throughout we focus on price volatility (defined earlier as $\sigma_{pi}$) and price informativeness (defined as $\frac{\text{Cov}(p_i,z_i)}{\sigma_{pi}}$):

3.3.1 Pure fringe

In a pure fringe setting, we set $\lambda_0 = 1$. If $K_h = 0$, there is no learning, and the model is very simple to solve. We can express price volatility as:

$$\sigma_{pi}^2 = \left( \frac{1}{n} \rho \sigma_i^2 \right)^2 \sigma_{xi}^2$$  \hspace{1cm} (26)

Derivations are presented in Section 7.1.5. Price informativeness is clearly zero, and price volatility varies positively with fundamental volatility, liquidity volatility, and risk aversion, and negatively with the risk-free rate.

If on the other hand, the fringe can learn ($K_h > 0$), then a positive set of assets (call the number $y$) will be learned about. Then we can express the $\Phi_{hi}$ as follows:

$$\Phi_{hi} = \frac{y + (e^{2K_h} - 1) \sqrt{1 + \rho^2 \bar{x}^2 \sigma_i^2 + \rho^2 \sigma_{xi}^2 \sigma_i^2}}{\sum_{k=1}^{y} \sqrt{1 + \rho^2 \bar{x}^2 \sigma_k^2 + \rho^2 \sigma_{xk}^2 \sigma_k^2}}$$  \hspace{1cm} (27)
Then we can express price volatility and price informativeness as:

\[
\sigma_{pi}^2 = \frac{1}{\sigma^2(1 + \Phi_{hi})^2(\Phi_{hi}\sigma_i^2 + \rho^2\sigma_i^4\sigma_{xi}^2)}
\]

\[
\frac{\text{Cov}(p_i, z_i)}{\sigma_{pi}} = \frac{\sigma_i}{\sqrt{1 + \frac{(\sum_{k=1}^{y} \sqrt{1 + \rho^2\sigma_{xk}^2 + \rho^2\sigma_{xi}^2\sigma_{xk}^2})^2}{(y + (e^{2Kh} - 1))^2(1 + \rho^2\sigma_{xk}^2 + \rho^2\sigma_{xi}^2\sigma_{xk}^2)}\rho^2\sigma_i^2\sigma_{xi}^2}}
\]

**Proposition 1.** The fringe wants to learn about high-volatility assets. That is, \( \Phi'_{hi}(\sigma_i) > 0 \).

Proof in the Appendix. The more volatile the asset, the greater the benefit of attending to its payout. Since fringe investors specialize, they cannot adjust on the intensive margin, but the extensive margin, so more agents learn about high-volatility assets, and fewer learn about low volatility assets, until the gains are equalized.

**Proposition 2.** Price informativeness is increasing in underlying volatility.

Proof in the Appendix. Our measure of price informativeness depends (almost) linearly on the standard deviation of the asset’s payoff. If we normalize it by \( \sigma_i \), we would end up with the correlation, which is, in fact, slightly negative in \( \sigma_i \). If \( \sigma_{xi} \) is constant across assets, then the fraction of the fringe that pays attention to an asset depends almost linearly on the standard deviation of the asset. Unsurprisingly price informativeness varies positively with the fringe’s capacity, and negatively with risk aversion, asset volatility, and liquidity volatility.

Now, let us look at the results if we introduce an investor with market power. We will call this investor a monopolist.

**3.3.2 Fringe and monopolist**

Now if we introduce a monopolist, there are four cases: when neither the monopolist nor the fringe can learn, when either can learn, or when both can learn. If **neither the monopolist nor the fringe can learn** - clearly price informativeness
will be zero. If the monopolist’s market power is represented by \( \lambda_1 \), price volatility is:

\[
\sigma_{pi}^2 = \left( \frac{(\lambda_0 + \lambda_1)\rho\sigma_i^2}{r\lambda_0(\lambda_0 + 2\lambda_1)} \right)^2 \sigma_{xi}^2
\]  

(28)

Derivations are in Appendix 7.1.6. Price volatility still increases with liquidity volatility, risk aversion, and fundamental volatility, while it decreases with the risk-free rate.

**Proposition 3.** In the absence of learning, market power increases price volatility in a fixed-size market.

Proof in the Appendix. If \( \lambda_0 = 1 \) as in the previous section, then \( \sigma_{pi}^2 = \left( \frac{(1+\lambda_1)\rho\sigma_i^2}{r(1+2\lambda_1)} \right)^2 \sigma_{xi}^2 \) which is smaller than the volatility of the pure fringe. If \( \lambda_0 + \lambda_1 = 1 \), then \( \sigma_{pi}^2 = \left( \frac{\rho\sigma_i^2}{r\lambda_0(1+\lambda_1)} \right)^2 \sigma_{xi}^2 \) which is larger than the volatility of the pure fringe. These changes are down to demand reasons, because there is no informational content in prices yet.

If we now allow the fringe to learn, but not the monopolist, we get the following expression for price informativeness:

\[
\frac{Cov(p_i, z_i)}{\sigma_{pi}} = \frac{\Phi_{hi} \sigma_i}{\sqrt{\Phi_{hi} + \left( \frac{\rho}{\lambda_0} \right)^2 \sigma_{xi}^2 \sigma_i^2}}
\]  

(29)

**Proposition 4.** When only the fringe can learn, price informativeness increases in fringe size.

Proof in the Appendix. Price informativeness varies negatively with the volatility of demand shocks, and risk aversion. Similarly, if the size of the fringe increases, price informativeness increases, because the fringe can be informed. Price informativeness also increases with \( \Phi_{hi} \).

If the monopolist can learn, but the fringe cannot we have to solve the
monopolist’s problem. We can express the first order conditions as:

\[
\mu = \frac{\alpha_i}{(1 + 2\lambda\alpha_i)^2} X_i \\
\prod_{i=1}^{n} \alpha_i = e^{2K_j}
\]

where \(X_i = \frac{1}{2\rho} \left( \left( \frac{\rho}{\rho_0} \right)^2 (\bar{x}^2 + \sigma^2_{ix})\sigma_i^2 + 1 + 2\lambda \right)\). This cannot be solved in closed form, but some comparative statics can be inferred. For any two assets \(i\) and \(k\) that the monopolist learns about:

\[
\frac{\alpha_k}{(1 + 2\lambda\alpha_k)^2} X_k = \frac{\alpha_i}{(1 + 2\lambda\alpha_i)^2} X_i
\]

(30)

**Proposition 5.** Higher levels of market power for the monopolist increases the price informativeness of low-volatility assets, and reduces the price informativeness of high-volatility assets.

Proof in the Appendix. \(\alpha\) therefore only moves monotonically with volatility, and faces distributional changes in response to movements in all other parameters. The expression for price informativeness is:

\[
\frac{\text{Cov}(p_i, z_i)}{\sigma_{p_i}} = \frac{\lambda_1(\alpha_{ji} - 1)\sigma_i}{\sqrt{\lambda_1^2(\alpha_{ji} - 1)^2 + (1 + \lambda_{ji}\alpha_{ji})^2\rho^2\sigma_i^2\sigma_{z_i}^2 + \lambda_1^2}}
\]

(31)

Higher levels of volatility in the asset correspond to higher learning about the asset, which yields higher levels of price informativeness. Changes in \(\lambda_1\) will affect the distribution of \(\alpha\) across assets, but will not have unconditional effects on any single asset. If however, we shut off the learning adjustment, and look just at the change on size, we can conclude the following:

**Proposition 6.** Holding \(\alpha\) constant, an increase in the monopolist’s market power will increase price informativeness for all assets.
Proof in the Appendix.

If the monopolist and the fringe can both learn, the first order conditions become:

\[ \mu = \frac{\alpha_{ji} Y_i + \tilde{\lambda}_{ji} \Phi_{hi}^2}{(1 + 2\lambda_{ji} \alpha_{ji})(1 + \Phi_{hi})^2} \]  

\( \prod_{i=1}^{n} \alpha_i = e^{2K_j} \)  \hspace{1cm} (33)

where \( Y_i = \frac{1}{2\rho} \left[ 1 + 2\tilde{\lambda}_{ji} \left( \frac{\rho \sigma_i^2}{\lambda_0} \right)^2 (\bar{x}^2 + \sigma_{ix}^2) \right] \). This cannot be solved in closed form, but some comparative statics can be inferred. \( \alpha_{ji} \) is decreasing in \( \Phi_{hi} \)—that is the fringe’s information and the monopolist’s are strategic substitutes.

### 3.3.3 Full model without information

When no one is informed, there is no price informativeness, but we can still look at the volatility of prices:

\[ \sigma_{pi}^2 = \frac{N_i^2 \sigma_i^2}{r^2} \left( \frac{\rho^2 \sigma_i^2 \sigma_{xi}^2}{\lambda_0} - 4 \sum_{j=1}^{n} \sum_{k \neq j} M_{ji} M_{ki} \right) \]  \hspace{1cm} (34)

Derivation in Appendix 7.1.7. Here \( M_{ji} = \frac{\lambda_i}{\lambda_0 + \lambda_j} \). Here for a given size of \( \lambda_0 \), an increase in diversification (perfect diversification is when all the \( \lambda_i \)'s are equal) would decrease volatility. That is, ownership concentration increases price volatility.

### 3.3.4 Summary

We have considered several versions of the model to develop some intuition for how the variables interact with one another. Here, we present a table contrasting the results. Unsurprisingly, an increase in either the fringe’s capacity for information collection, or the oligopolists’ capacity, increases the informativeness of prices. Further, an increase in the size of the fringe, holding fringe member capacity fixed also
increases price informativeness, because increasing the size of the fringe effectively adds fringe members, and therefore adds signals to the price. Increasing the size of the agents with market power can diversify price informativeness (decreasing informativeness for the most volatile assets, while increasing it for less volatile assets), but this result depends on the market structure.

Table 1: Theoretical results on price informativeness: Summary

<table>
<thead>
<tr>
<th>Model Type</th>
<th>$K_h$</th>
<th>$K_j$</th>
<th>$\lambda_0$</th>
<th>$\sum_{j=1}^{n} \lambda_j$</th>
<th>Concentration</th>
</tr>
</thead>
<tbody>
<tr>
<td>Informed Fringe</td>
<td>+</td>
<td>N/A</td>
<td>+</td>
<td>N/A</td>
<td>N/A</td>
</tr>
<tr>
<td>Informed Fringe w/ Uninformed</td>
<td>+</td>
<td>N/A</td>
<td>+</td>
<td>-</td>
<td>-</td>
</tr>
<tr>
<td>Uninformed Fringe w/ Informed</td>
<td>N/A</td>
<td>+</td>
<td>-</td>
<td>diversified diversified</td>
<td>diversified</td>
</tr>
<tr>
<td>Uninformed Fringe w/ Partially</td>
<td>N/A</td>
<td>+</td>
<td>-</td>
<td>diversified variable (depends on which oligopolist is gaining power)</td>
<td>variable</td>
</tr>
</tbody>
</table>

4 Numerical Analysis

In this section, we provide a set of quantitative results from the solution to the equilibrium of the model.\(^6\) We select parameter values for the return distribution $\bar{z}$ and $\{\sigma_i\}_{i=1}^{n}$, the liquidity distribution $\bar{x}$ and $\{\sigma_xi\}_{i=1}^{n}$, the risk-free return $r$, risk aversion $\rho$, learning capacities $K_h$ and $\{K_j\}_{j=1}^{l}$, and fringe and oligopolist sizes $\lambda_0$

\(^6\)This involves solving a fixed point of the best responses of the oligopolists to each other’s learning and trading policies.
and \( \{\lambda_j\}_{j=1}^l \). The simulation generates equilibrium levels of price informativeness, oligopoly holdings, and oligopoly concentration for each asset.

In our simulations, we choose the parameters with two goals in mind: they have to be in an empirically relevant region of the parameter space and the solution needs to involve some degree of learning. Specifically, we consider parameters such that the benchmark model exhibits: (i) learning about all assets, (ii) aggregate institutional holding share of between 60 and 70% (which corresponds to the information in Figure 1), (iii) market excess real return of around 7% (which corresponds to the average over 1980-2015). For the results reported below, we set the number of assets to \( n = 10 \) and the number of oligopolists to \( l = 6 \). We report parameter values in Table 2.

<table>
<thead>
<tr>
<th>Parameter</th>
<th>Symbol</th>
<th>Value</th>
</tr>
</thead>
<tbody>
<tr>
<td>Mean payoff, supply</td>
<td>( \tilde{z}_i, \overline{x}_i )</td>
<td>10, 5 for all ( i )</td>
</tr>
<tr>
<td>Number of assets</td>
<td>( n, l )</td>
<td>10, 6</td>
</tr>
<tr>
<td>Risk-free rate</td>
<td>( r )</td>
<td>2.5%</td>
</tr>
<tr>
<td>Vol. of noise shocks</td>
<td>( \sigma_{x_i} )</td>
<td>0.41 for all ( i )</td>
</tr>
<tr>
<td>Vol. of asset payoffs</td>
<td>( \sigma_i )</td>
<td>( \in [1, 1.5] ), linear distribution</td>
</tr>
<tr>
<td>Risk aversion</td>
<td>( \rho )</td>
<td>1.3</td>
</tr>
<tr>
<td>Information capacities</td>
<td>( K_h, {K_j} )</td>
<td>0, 4.5, constant</td>
</tr>
<tr>
<td>Investor masses</td>
<td>( \lambda_0, \lambda_l/\lambda_1 )</td>
<td>0.45, 4 ( \lambda_j )s linearly distributed</td>
</tr>
</tbody>
</table>

### 4.1 Cross-sectional patterns

We begin by analyzing the cross section of equilibrium output variables across assets for the benchmark parameter values in Table 2. Figure 3 presents the relation between equilibrium price informativeness per asset (on the \( y \)-axis) and equilibrium oligopoly holdings per asset (on the \( x \) axis). The intuition for this result is based on our analysis in Section 3.3—agents want to learn about high-volatility assets first, because those are the most rewarding. Therefore, price informativeness is increasing in underlying volatility, and so are total oligopoly holdings.
Figure 3: Price informativeness and institutional ownership

Figure 4 presents the relation between equilibrium price informativeness and equilibrium oligopoly concentration. The larger an oligopolist’s presence in a particular asset’s market, the more likely she is to internalize the price effect of her trade. As such, she would like to be less informed than she would be if she had a small presence. As a result, concentration in a particular asset is associated with lower levels of price informativeness.

Figure 4: Price informativeness and concentration
In Figure 5, we present the above cross-sectional relations for the part of price informativeness due to only the correlation of prices and shocks. That is the part of the information measure that is endogenous to the information choices of agents, and does not come from pure cross-sectional dispersion of the exogenous shocks (see, equation (31)). As the figure indicates, the positive relation with institutional holdings and the negative relation with concentration hold for the correlation part of price informativeness, consistently with the empirical patterns documented before.

![Figure 5: Price informativeness: correlation only](image)

4.2 Policy experiments

The different signs of the relation suggest an interesting interaction between high ownership and high concentration for the overall effect on price informativeness. We now move on to analyze the effect of policy on the aggregate price informativeness. While in Figures 3 and 4, each point corresponded to one asset, in the following exercise, each point corresponds to one iteration of a full financial market (with several assets). The experiments are useful as a way to isolate the relative effects of institutional concentration and holdings on price informativeness.
The size of the oligopoly In our first experiment, we look at how average price informativeness across assets changes in response to different levels of $\lambda_0$. Holding the relative distribution of $\lambda_j$ fixed, we look at simulations of the model for varying $\lambda_0$ from 0.05 to 0.95. The type of policy being tested here could be thought of as a limits on entry, or limits on a per-agent size in a given market.

Figure 6 shows the relation between the size of the institutional sector (parameterized by $1 - \lambda_0$) and endogenous variables of interest. The price informativeness in Panel (a) shows a hump-shaped relation, on average and also for each asset (as indicated by interquantile 10-90 range) with the parameterized size, and hence also with the actual realized ownership which is monotonically increasing (Panel (b)). The model results point to an interior solution to optimal institutional sector size. This result can be explained by a tradeoff between the deviation from efficiency of signals due to market power and the information reflected in prices via traded quantities of informed traders. When initially the size of the institutional sector is small, the oligopolists are small and hence their price impact considerations are not very important in their information allocation decisions, which leaves capacity allocation undistorted. In this case, each oligopolist learns in a relatively efficient way and receives precise signals, but because of their small size, not a lot of information gets revealed in prices. As the size of the institutional sector increases, additional oligopolists start learning (Panel (d)) and trading any given asset, which results in a large drop in concentration (Panel (c)), and increased ownership (Panel (b)). The increased diversification in learning means less efficient signals per oligopolist, but that negative effect is offset by more equally sized active traders, and hence we have more of the less efficient signals which are then reflected more in prices. As the size of the institutional sector increases further, the information choice gets increasingly distorted by price impact considerations, and hence the lower efficiency of signals outweighs the larger number of signals to average out; thus, price informativeness starts falling.
These effects give rise to a hump-shaped relation in the model between price informativeness and both institutional ownership and the concentration measure of that ownership. We present these results in Figure 7.

![Graphs showing the response of model to changing size of institutional sector](image)

(a) Price Informativeness, average and 10, 90 percentiles  
(b) Institutional Ownership  
(c) Concentration  
(d) Average Num. of Active Traders per Asset

Figure 6: Response of model to changing size of institutional sector $(1 - \lambda_0)$.

**The concentration of the oligopoly**  
Now, we consider the effects of a policy that affects the concentration of the actively trading oligopolists. Holding $\lambda_0$ constant, we vary the size distribution of $\{\lambda_j\}$ in order to measure an impact on the concentration measure. Specifically, we vary $\lambda_t/\lambda_1$ from 1.05 to 10, with intermediate $\lambda_j$s growing linearly from $\lambda_1$ to $\lambda_t$. In doing so, the sum of all $\lambda_j$s is kept equal to $1 - \lambda_0$ to isolate the effect of concentration on endogenous variables. Figure 8 presents the results for
price informativeness and concentration.\textsuperscript{7} The results with respect to concentration are roughly monotonic: Holding ownership relatively constant, a decrease in concentration increases the price informativeness in the aggregate. This is in line with the intuition from the previous exercise. If ownership is relatively stable, then there is no change in average market power across these markets. However, changing the size distribution of the oligopolists towards a more unequal one increases the concentration of ownership and hence increases market power of some of the oligopolists, distorting their information choices more. That leads to a negative relation between concentration and price informativeness, while keeping the ownership stable.

**Passive investors** We further explore the predictions of the model by considering the role played by passive investors—those who have market power, but no capacity for informational investment. The growing importance of such investors (e.g., BlackRock, Vanguard) has been an important element of the asset management landscape in the last few decades. To this end, we consider two versions of the model. In one, the smallest half of the institutional investors has zero information capacity (small passive); in the other, the largest half of the institutional investors has zero capacity (big passive). In Figure 9, we repeat the first experiment, and plot average price

\textsuperscript{7}Institutional ownership in this case varies only by 1.4\% of the mean—by design—and hence we do not show its graph explicitly.
informativeness across different institutional ownership levels (via $\lambda_0$ variation). Not surprisingly, informativeness is always higher in the small passive case. In the model, size is an impediment to learning efficiently, as it makes more information get revealed via actual trades. Hence, having small informed agents implies their allocation of learning capacity is more efficient, which is reflected in aggregate informativeness.

Figure 8: Average price informativeness and concentration relative to dispersion $\lambda_i/\lambda_1$

Figure 9: Aggregate price informativeness and institutional ownership: The case of big or small passive investors
4.3 The role of endogenous learning choice

In this section, we present a comparison of the model with endogenous learning choice (our benchmark) to a model in which the information structure is given and set the same as in the benchmark model. The model with a fixed information structure is similar in spirit to that presented in Kyle (1985), in that the effect of market power in the absence of endogenous reoptimization of information choices depends entirely on how the quantities adjust.

Figure 10 presents the interaction of institutional ownership and price informativeness, where the different points are generated by varying $\lambda_0$. The black dots represent the benchmark case in which we allow both quantities and information choices to adjust in response to changing $\lambda_0$. The red crosses correspond to a case with a fixed learning structure. For the fixed learning cases, the information choice is either fixed at the benchmark value (i.e., $\lambda_0 = 0.45$, Panel (a)), or at values such that information structures are optimal at $\lambda_0 = 0.999$ (small oligopolists, Panel (b)) and $\lambda_0 = 0.05$ (large oligopolists, Panel (c)). In all the fixed-information cases, the level of price informativeness is below that of the benchmark model for which capacity choice adjusts optimally. The gains in price informativeness from optimal learning can be quite large. For example, for the benchmark specification of fixed alphas, price informativeness is reduced by up to 40%. More important, fixing the learning choices leads to very different conclusions about the optimal size of the institutional sector. Depending on what values of learning one exactly fixes, the optimal size lies either below or above the actual optimum derived when all the choices are endogenous. This finding underscores the importance of modeling the information choice margin when making normative statements about the size of the institutional sector.

Next, we evaluate the ‘concentration of oligopoly’ exercise of Section 4.2, in which we hold $\lambda_0$ fixed but vary $\lambda_l/\lambda_1$. Figure 11 presents the relation between concentration of ownership and price informativeness for the benchmark model with endogenous information choice (black dots), as well as three cases of fixing the information choice.
Figure 10: Aggregate price informativeness and institutional ownership with varying $\lambda_0$

at the benchmark values (i.e., for $\lambda_l/\lambda_1 = 4$, Panel (a)), as well as at values that are optimal at two extremes of the size distribution of the oligopolists, $\lambda_l/\lambda_1 = 9$ (Panel (b)) and $\lambda_l/\lambda_1 = 2$ (Panel (c)). For all the three cases, the exogenous and endogenous information models give remarkably different predictions in terms of the relation of concentration and price informativeness. In particular, for the benchmark model, lower concentration always increases price informativeness. In contrast, models with fixed information structure exhibit a hump-shaped relation between concentration and price informativeness. Similar to the previous exercise, the two models give very different recommendations regarding the level of concentration that maximizes price informativeness. The exogenous information models optimally imply an intermediate level of concentration, while at the same time the fully endogenous model prescribes
a concentration level that is at the lower bound of the potential values.

![Graph](image1)

(a) Fixed alpha at benchmark value

![Graph](image2)

(b) Fixed alpha for $\lambda_l/\lambda_1 = 9$

![Graph](image3)

(c) Fixed alpha for $\lambda_l/\lambda_1 = 2$

Figure 11: Aggregate price informativeness and concentration of institutional ownership with varying $\lambda_l/\lambda_1$

Overall, we conclude that the predictions resulting from a model with endogenous learning choices are not a simple extension of the model where information choices are fixed. The differences are not only quantitatively important but also qualitatively relevant from the perspective of policy making.

5 Empirical Results

In this section, we provide an empirical verification of the main predictions coming from our model. In particular, we focus on the relations between ownership levels and its concentration, and price informativeness. Our goal is a more modest qualitative
assessment of comparative statics rather than an attempt to match the quantities from the model. The specifics of our empirical methodology follow closely those in Bai, Philippon, and Savov (2016).

We begin by constructing an empirical measure of price informativeness. The measure captures the covariance between the price and fundamental information and is formally defined as:

\[ PI_{t,h} = b_{t,h} \ast \sigma_t(\log(M/A)), \]  

(35)

where \( M \) denotes the market value of equity, \( A \) denotes the book value of assets, \( h \) is the investment horizon, and \( b_{t,h} \) is a coefficient obtained from the following regression:

\[ \frac{E_{i,t+h}}{A_{i,t}} = a_{t,h} + b_{t,h} \log(\frac{M_{i,t}}{A_{i,t}}) + c_{t,h} \log(\frac{E_{i,t}}{A_{i,t}}) + d_{t,h} SIC_{i,t} + \epsilon_{i,t,h}, \]  

(36)

\( E \) is the value of earnings (EBIT) and \( SIC \) is an indicator variable for each one-digit SIC code. In our empirical tests, we set the horizon period \( h \) to be equal to one year.

To estimate the measure, we consider all companies in Compustat with valid financial information. Our data are recorded at an annual frequency and cover the period of 1980-2015. Following the observation in Bai et al. (2016) who argue that large firms have the most stable characteristics we also considered a subset of only large firms and also a set that excludes financial companies. The results remain qualitatively similar in all those cases and are available upon request. The \( PI \) measure is defined for each time period using a given cross-section of firms. In our tests, we sort companies into portfolio based on various characteristics of interest and thus the respective \( PI \) measures correspond to a particular portfolio.

Specifically, each year, we sort companies into deciles according to their ownership levels. For each decile portfolio, we calculate the equal-weighted \( PI \) measure. Next,
we aggregate information for each individual portfolio using the time-series dimension. We present the results in column (2) of Table 3.

Table 3: Decile Sorts and Price Informativeness

| Decile | Ownership Concentration Residualized Concentration |
|--------|--------------------------------------------------|--------------------------------------------------|
| 1      | 0.0214                                          | 0.0127                                          | 0.0075                                          |
| 2      | 0.0147                                          | 0.0105                                          | 0.0039                                          |
| 3      | 0.0083                                          | 0.0065                                          | 0.0020                                          |
| 4      | 0.0061                                          | 0.0018                                          | 0.0006                                          |
| 5      | -0.0020                                         | -0.0011                                         | -0.0012                                         |
| 6      | 0.0010                                          | -0.0076                                         | -0.0023                                         |
| 7      | 0.0068                                          | -0.0089                                         | -0.0050                                         |
| 8      | 0.0073                                          | -0.0117                                         | -0.0063                                         |
| 9      | 0.0093                                          | -0.0161                                         | -0.0085                                         |
| 10     | 0.0115                                          | -0.0185                                         | -0.0126                                         |

We observe that price informativeness is monotonically increasing with ownership levels. Companies with lowest ownership have the least informative prices and companies with highest ownership have the most informative prices. This result is consistent with predictions of our model and also confirms the empirical results in Bai et al. (2016).

Next, we perform a similar sort, this time based on measure of concentration. Our measure of concentration is the Herfindahl-Hirshman Index (HHI), defined in Section 2. We present the results from the sort in column (3). Consistent with our predictions, we observe that price informativeness is decreasing in the level of concentration. Companies with the highest levels of ownership concentration have the least informative prices. The opposite is true for companies which have the most dispersed ownership.

Since the HHI mechanically depends on the number of institutional investors, the issue arises whether the results we identify are a function of pure concentration or
result from the relation between number of institutions and price informativeness. Given that ownership is positively related to $PI$, this would suggest the pure concentration effect should be in fact even larger than the one we identify. To evaluate this claim, we obtain residuals from the regression of the index on the number of participants and use it as a sorting variable in our exercise. We present the results in column (4).

The $PI$ measure we use in our analysis nests two economic effects: the effect on the correlation structure between prices and fundamentals and the effect of volatility of volatility of prices. While the correlation is directly related to information story, volatility in the data could change for reasons other than information. To assess whether our analysis is not driven by any non-information component, we consider the correlation between prices and fundamentals as an alternative information measure. We perform similar three sorts and before and present our results in Table 4.

<table>
<thead>
<tr>
<th>Decile</th>
<th>Ownership</th>
<th>Concentration</th>
<th>Residualized Concentration</th>
</tr>
</thead>
<tbody>
<tr>
<td>1</td>
<td>-0.0769</td>
<td>0.1626</td>
<td>0.0679</td>
</tr>
<tr>
<td>2</td>
<td>-0.0533</td>
<td>0.1147</td>
<td>0.0386</td>
</tr>
<tr>
<td>3</td>
<td>-0.0312</td>
<td>0.0629</td>
<td>0.0253</td>
</tr>
<tr>
<td>4</td>
<td>-0.0257</td>
<td>0.0246</td>
<td>0.0068</td>
</tr>
<tr>
<td>5</td>
<td>-0.0026</td>
<td>0.0004</td>
<td>-0.0004</td>
</tr>
<tr>
<td>6</td>
<td>0.0146</td>
<td>-0.0327</td>
<td>-0.0108</td>
</tr>
<tr>
<td>7</td>
<td>0.0525</td>
<td>-0.0360</td>
<td>-0.0203</td>
</tr>
<tr>
<td>8</td>
<td>0.0682</td>
<td>-0.0445</td>
<td>-0.0232</td>
</tr>
<tr>
<td>9</td>
<td>0.1004</td>
<td>-0.0585</td>
<td>-0.0314</td>
</tr>
<tr>
<td>10</td>
<td>0.1270</td>
<td>-0.0692</td>
<td>-0.0433</td>
</tr>
</tbody>
</table>

In column (2), we present the results for ownership-based sorts. Again, we find a very strong monotonic relation between the levels of ownership and correlation between prices and fundamentals. The magnitude of the differences between the top and
the bottom decile is economically very large and equals approximately 0.2. Similarly, in column (3), we report the results for the sorts based on levels of ownership concentration and document a similar negative relation between concentration and the correlation measure. The most concentrated portfolio has a correlation value which is lower by approximately 0.23 than the correlation of the least concentrated portfolio. Finally, in column (4), we report the results for the sorts based on residualized concentration measure and find very similar results. Overall, our results suggest that the variation in $PI$ that we document in the paper is unlikely to be driven only by non-information forces. Moreover, our results strongly confirm the theoretical predictions coming from our model. Since we do not attempt the model and the data on quantities, we cannot determine whether the information effect is the sole driver of the variation in price informativeness.

6 Concluding Remarks

The last few decades have observed important changes in institutional equity ownership structure, with significant consequences for financial stability and social welfare. These trends have triggered an active response from financial regulators and finance industry. While several participants in the debate have raised important reasons for or against regulatory changes, the ultimate verdict is difficult to reach in the absence of a well-specified economic model. This paper aims to take one step in this direction by offering a general equilibrium model in which asymmetric information, market power, and heterogeneity of assets play an important role. We think this setting is a good way to characterize the world of equity ownership. Our goal is to rank various equilibria by comparing their average price informativeness.

We show that for the level of ownership equal to the currently observed levels in the U.S. (roughly 60%), to average price efficiency is positively related to the levels of institutional ownership and negatively related to its concentration. This
cross-sectional result is strongly supported by the data. Further, we show that the average price informativeness across assets is maximized for the values of ownership and concentration that are strictly within the range of admissible outcomes. This result suggests an interesting role for policy makers to enforce optimal structure of equity ownership.

Our model can be flexibly applied to settings with rich cross-section of assets, differences in information asymmetry across agents, and differences in market power. Hence, it can generate interesting policy implications at the aggregate and cross-sectional dimensions. It can also be a good tool to evaluate asset pricing implications in the presence of market power and information asymmetry. We leave these questions for future research. At the same time, while our research can inform the debate for the role of institutional owners for price informativeness and learning in the economy, it naturally abstracts from other important dimensions relevant for policy makers, such as investment costs or flows of funds in and out of the sector.

References


Garleanu, Nicolai, and Lasse H. Pedersen, 2015, Efficiently inefficient markets for assets and asset management, Working Paper, CBS.


Massa, Massimo, David Schumacher, and Wang Yan, 2016, Who is afraid of Black-Rock?, Working Paper INSEAD.


7 Appendix: Proofs

7.1 Model

7.1.1 Derivation of Equation 2

\[
\sigma_{pi}^2 = b_i^2 \sigma_i^2 + c_i^2 \sigma_{zi}^2 + \sum_{j=1}^{n} \left( \tilde{\sigma}_{ji}^2 + \frac{2}{\alpha_{ji}} \text{Cov}(\varepsilon_i, \delta_{ji}) - 2b_j d_{ji} \text{Cov}(\varepsilon_i, \zeta_{ji}) - 2c_j d_{ji} \text{Cov}(\nu_i, \delta_{ji}) \right) \\
+ \sum_{j=1}^{n} \sum_{k \neq j} 2d_{ji} d_{ki} \text{Cov}(\zeta_{ji}, \zeta_{ki}) \\
= b_i^2 \sigma_i^2 + c_i^2 \sigma_{zi}^2 + \sum_{j=1}^{n} \left( \sigma_{ji}^2 + \frac{2}{\alpha_{ji}} \tilde{\sigma}_{ji}^2 \right) - 2b_j d_{ji} \left( \frac{\sigma_{ji}^2}{\tilde{\sigma}_{ji}^2} - \frac{\sigma_i^2}{\tilde{\sigma}_i^2} \right) - 2c_j d_{ji} \left( \frac{\sigma_{ji}^2}{\tilde{\sigma}_{ji}^2} - \frac{\sigma_i^2}{\tilde{\sigma}_i^2} \right) \\
+ \sum_{j=1}^{n} \sum_{k \neq j} 2d_{ji} d_{ki} \text{Cov}(\delta_{ji} - \frac{1}{\alpha_{ji}} \varepsilon_i, \delta_{ki} - \frac{1}{\alpha_{ki}} \varepsilon_i) \\
= b_i^2 \sigma_i^2 + c_i^2 \sigma_{zi}^2 + \sum_{j=1}^{n} d_{ji}^2 \left( 1 - \frac{1}{\alpha_{ji}} \right) \tilde{\sigma}_{ji}^2 + \\
\sum_{j=1}^{n} \sum_{k \neq j} 2d_{ji} d_{ki} \left( \frac{1}{\alpha_{ji} \alpha_{ki}} \sigma_i^2 - \frac{\tilde{\sigma}_{ji}^2}{\tilde{\sigma}_i^2} - \frac{\tilde{\sigma}_{ki}^2}{\tilde{\sigma}_i^2} - \frac{1}{\alpha_{ji}} \tilde{\sigma}_{ji}^2 \right) \\
= b_i^2 \sigma_i^2 + c_i^2 \sigma_{zi}^2 + \sum_{j=1}^{n} d_{ji}^2 \left( 1 - \frac{1}{\alpha_{ji}} \right) \tilde{\sigma}_{ji}^2 + \\
\sum_{j=1}^{n} \sum_{k \neq j} 2d_{ji} d_{ki} \left( \frac{1 + \alpha_{ji} \alpha_{ki}}{\alpha_{ji} \alpha_{ki}} \sigma_i^2 - \frac{1 + \alpha_{ki} \tilde{\sigma}_{ji}^2}{\alpha_{ki} \tilde{\sigma}_i^2} - \frac{1 + \alpha_{ji} \tilde{\sigma}_{ji}^2}{\alpha_{ji} \tilde{\sigma}_i^2} \right)
\]

7.1.2 Derivation of Equation 8

The objective is \( U_{0h} = \frac{1}{2p} \sum_{i=1}^{n} \left( \frac{E_{0h} (\tilde{\mu}_{hi} - r_{pi})}{\sigma_{hi}^2} \right)^2 = \frac{1}{2p} \sum_{i=1}^{n} \frac{\tilde{R}_{i}^2 + \tilde{V}_{hi}}{\tilde{\sigma}_{hi}^2} \), where

\( \tilde{R}_{i} \equiv E_{0h} (\tilde{\mu}_{hi} - r_{pi}) = \bar{z} - r \bar{p}_i = \bar{z} - r \alpha_i \), \( \tilde{V}_{hi} \equiv V_{0h} (\tilde{\mu}_{hi} - r_{pi}) = \text{Var} (\tilde{\mu}_{hi}) + r^2 \sigma_{pi}^2 - 2r \text{Cov} (\tilde{\mu}_{hi}, p_i) \).

\[
\text{Var} (\tilde{\mu}_{hi}) = \sigma_{hi}^2 - \tilde{\mu}_{hi}^2, \quad \sigma_{pi}^2 = b_i^2 \sigma_i^2 + c_i^2 \sigma_{zi}^2 + \sum_{j=1}^{n} d_{ji}^2 \left( 1 - \frac{1}{\alpha_{ji}} \right) \tilde{\sigma}_{ji}^2 + \\
\sum_{j=1}^{n} \sum_{k \neq j} 2d_{ji} d_{ki} \left( \frac{1 + \alpha_{ji} \alpha_{ki}}{\alpha_{ji} \alpha_{ki}} \sigma_i^2 - \frac{1 + \alpha_{ki} \tilde{\sigma}_{ji}^2}{\alpha_{ki} \tilde{\sigma}_i^2} - \frac{1 + \alpha_{ji} \tilde{\sigma}_{ji}^2}{\alpha_{ji} \tilde{\sigma}_i^2} \right).
\]
Posterior beliefs and prices are conditionally independent given payoffs.

\[
\text{Cov} \left( \mu_{hi}, p_i \right) = \frac{1}{\sigma_i^2} \text{Cov} \left( \mu_{hi}, z_i \right) \text{Cov} \left( z_i, p_i \right)
\]

\[
= \frac{1}{\sigma_i^2} \left( \sigma_i^2 - \tilde{\sigma}_{hi}^2 \right) \left( \text{Cov}(\varepsilon_i, b_i \varepsilon_i) - \sum_{j=1}^{n} \text{Cov}(\varepsilon_i, d_{ji} \varepsilon_{ji}) \right)
\]

\[
= \frac{1}{\sigma_i^2} \left( \sigma_i^2 - \tilde{\sigma}_{hi}^2 \right) \left( b_i \sigma_i^2 - \sum_{j=1}^{n} d_{ji} \text{Cov} \left( \varepsilon_i, \delta_{ji} - \frac{1}{\alpha_{ji}} \varepsilon_i \right) \right)
\]

\[
= \frac{1}{\sigma_i^2} \left( \sigma_i^2 - \tilde{\sigma}_{hi}^2 \right) \left( b_i + \sum_{j=1}^{n} \frac{d_{ji}}{\alpha_{ji}} \right) \sigma_i^2 - \sum_{j=1}^{n} d_{ji} \tilde{\sigma}_{ji}^2
\]

\[
= \left( \sigma_i^2 - \tilde{\sigma}_{hi}^2 \right) b_i
\]

Hence

\[
\tilde{V}_{hi} = \sigma_i^2 - \tilde{\sigma}_{hi}^2 + r^2 \left( b_i^2 \sigma_i^2 + \alpha_i^2 \sigma_{xi}^2 + \sum_{j=1}^{n} d_{ji}^2 \left( 1 - \frac{1}{\alpha_{ji}} \right) \tilde{\sigma}_{ji}^2 + \sum_{j=1}^{n} \sum_{k \neq j} 2d_{ji}d_{ki} \left( \frac{1 + \alpha_{ji} \alpha_{ki}}{\alpha_{ji} \alpha_{ki}} \sigma_i^2 - \frac{1 + \alpha_{ki} \tilde{\sigma}_{ki}^2}{\alpha_{ki}} - \frac{1 + \alpha_{ji} \tilde{\sigma}_{ji}^2}{\alpha_{ji}} \right) \right) - 2r \left( \sigma_i^2 - \tilde{\sigma}_{hi}^2 \right) b_i
\]

Expected utility becomes

\[
U_{0h} = \frac{1}{2p} \sum_{i=1}^{n} G_i \frac{\sigma_i^2}{\tilde{\sigma}_{hi}^2} - \frac{1}{2p} \sum_{i=1}^{n} (1 - 2rb_i),
\]

where

\[
G_i = G^{KNS}_{i} + r^2 \left( \sum_{j=1}^{n} d_{ji}^2 \left( 1 - \frac{1}{\alpha_{ji}} \right) \tilde{\sigma}_{ji}^2 + \sum_{j=1}^{n} \sum_{k \neq j} 2d_{ji}d_{ki} \left( \frac{1 + \alpha_{ji} \alpha_{ki}}{\alpha_{ji} \alpha_{ki}} \sigma_i^2 - \frac{1 + \alpha_{ki} \tilde{\sigma}_{ki}^2}{\alpha_{ki}} - \frac{1 + \alpha_{ji} \tilde{\sigma}_{ji}^2}{\alpha_{ji}} \right) \right).
\]

Note: \( G^{KNS}_{i} = \frac{\tilde{R}_i^2}{\sigma_i^2} + (1 - rb_i)^2 + r^2 \frac{\sigma_{xi}^2}{\sigma_i^2} \).
7.1.3 Derivation of Equation 13

Market clearing for each asset $i$ is

$$x_i = \sum_{j=1}^{n} \lambda_j q_{ji} + \int_{H_i} q_{hi} dh$$

$$= \sum_{j=1}^{n} \lambda_j q_{ji} + \int_{H_i} \frac{\hat{\mu}_{hi} - r_{pi}}{\rho \sigma^2_{hi}} dh$$

$$= \sum_{j=1}^{n} \lambda_j q_{ji} \left[ e^{2K_h} \int_{H_i} \hat{\mu}_{hi} dh - m_{hi} e^{2K_h} r_{pi} + (1 - m_{hi}) (z - r_{pi}) \right]$$

where $H_i$ is the mass of competitive investors learning about asset $i$, of measure $m_{hi}$.

Using $E[s_{hi} | z_i] = \begin{cases} \bar{z} + (1 - e^{-2K_h}) \varepsilon_i & \text{if } i = l_h \\ \bar{z} & \text{if } i \neq l_h, \end{cases}$

$$\int_{H_i} \hat{\mu}_{hi} dh = m_{hi} [\bar{z} + (1 - e^{-2K_h}) \varepsilon_i].$$

Then market clearing becomes

$$x_i = \sum_{j=1}^{n} \lambda_j q_{ji} + \frac{\lambda_0}{\rho \sigma^2_i} \left[ (1 - m_{hi} + e^{2K_h} m_{hi}) \bar{z} + (e^{2K_h} - 1) \varepsilon_i m_{hi} - (1 - m_{hi} + e^{2K_h} m_{hi}) r_{pi} \right]$$

Defining $\Phi_{hi} \equiv m_{hi} (e^{2K_h} - 1)$,

$$x_i = \sum_{j=1}^{n} \lambda_j q_{ji} + \frac{\lambda_0}{\rho \sigma^2_i} \left[ \bar{z} (1 + \Phi_{hi}) + \Phi_{hi} \varepsilon_i - r_{pi} (1 + \Phi_{i}) \right]$$

which becomes

$$\frac{\rho \sigma^2_i}{\lambda_0} \bar{z}_i = \frac{\rho \sigma^2_i}{\lambda_0} \sum_{j=1}^{n} \lambda_j q_{ji} + \bar{z} (1 + \Phi_{hi}) + \Phi_{hi} \varepsilon_i - r_{pi} (1 + \Phi_{i})$$

and then

$$r_{pi} = \frac{\rho \sigma^2_i}{\lambda_0 (1 + \Phi_{hi})} \sum_{j=1}^{n} \lambda_j q_{ji} + \bar{z} + \frac{\Phi_{hi}}{1 + \Phi_{hi}} \varepsilon_i - \frac{\rho \sigma^2_i}{\lambda_0 (1 + \Phi_{hi})} x_i$$

Hence,

$$\frac{d\pi(q_{ji})}{dq_{ji}} = \frac{\lambda_j \rho \sigma^2_i}{\lambda_0 (1 + \Phi_{hi})} > 0$$

Let $\lambda_{ji} \equiv \frac{\lambda_j}{\lambda_0 (1 + \Phi_{hi})}.$

Then $q_{ji} = \frac{\hat{\mu}_{ji} - r_{pi}}{\rho (\sigma^2_{ji} + \lambda_{ji} \sigma^2_i)}$, and similarly for $k$.

Plugging in the expression for $q_{ji}$:

$$r_{pi} = \sum_{j=1}^{n} \lambda_{ji} \rho \sigma^2_i \frac{\hat{\mu}_{ji} - r_{pi}}{\rho (\sigma^2_{ji} + \lambda_{ji} \sigma^2_i)} + \bar{z} + \frac{\Phi_{hi}}{1 + \Phi_{hi}} \varepsilon_i - \frac{\rho \sigma^2_i}{\lambda_0 (1 + \Phi_{hi})} x_i$$

which becomes

$$r_{pi} \left(1 + \sum_{j=1}^{n} \frac{\lambda_{ji} \rho \sigma^2_i}{\rho (\sigma^2_{ji} + \lambda_{ji} \sigma^2_i)}\right) = \sum_{j=1}^{n} \frac{\lambda_{ji} \rho \sigma^2_i}{\rho (\sigma^2_{ji} + \lambda_{ji} \sigma^2_i)} \mu_{ji} + \bar{z} + \frac{\Phi_{hi}}{1 + \Phi_{hi}} \varepsilon_i - \frac{\rho \sigma^2_i}{\lambda_0 (1 + \Phi_{hi})} x_i$$

dividing through gives
The indirect utility function $U_j = \sum_{i=1}^{n} q_{ji} (\tilde{\mu}_{ji} - r_{pi}) - \frac{\rho}{2} \sum_{i=1}^{n} q_{ji}^2 \tilde{\sigma}_{ji}^2$ becomes

$U_j = \sum_{i=1}^{n} \left[ q_{ji}^2 \rho \left( \tilde{\sigma}_{ji}^2 + \tilde{\lambda}_{ji} \sigma_i^2 \right) - \frac{\rho}{2} q_{ji}^2 \tilde{\sigma}_{ji}^2 \right]$

$U_j = \sum_{i=1}^{n} \left[ \rho q_{ji}^2 \left( \tilde{\sigma}_{ji}^2 + \tilde{\lambda}_{ji} \sigma_i^2 - \frac{1}{2} \sigma_i^2 \right) \right]$

$U_j = \sum_{i=1}^{n} \left\{ \left( \tilde{\mu}_{ji} - r_{pi} \right)^2 \left( \frac{\tilde{\sigma}_{ji}^2 + 2 \tilde{\lambda}_{ji} \sigma_i^2}{\tilde{\sigma}_{ji}^2 + \lambda_i \sigma_i^2} \right) \right\}$

More detailed expression for $U$: We can rewrite $E_{0j}(\tilde{\mu}_{ji} - r_{pi})^2$ as $\tilde{R}_i + \hat{V}_{ji}$, where $\tilde{R}_i$ and $\hat{V}_{ji}$ denote the ex-ante mean and variance of expected excess returns, which means:

$\tilde{R}_i \equiv E_{0j}(\tilde{\mu}_{ji} - r_{pi}) = \frac{\lambda_{ji} \sigma_i^2}{\rho(\tilde{\sigma}_{ji}^2 + \lambda_i \sigma_i^2)} \bar{x}$ Define $M_{ji} \equiv \frac{\lambda_{ji} \sigma_i^2}{\rho(\tilde{\sigma}_{ji}^2 + \lambda_i \sigma_i^2)} \ N_i \equiv \frac{1}{1 + \sum_{j=1}^{n} M_{ji}}$

$\hat{V}_{ji} \equiv V_{0j}(\tilde{\mu}_{ji} - r_{pi})$ simply:

$= V_{0j} \left( \tilde{\mu}_{ji} - N_i \sum_{k=1}^{n} M_{ki} \tilde{\mu}_{ki} - N_i \tilde{z} - N_i \frac{\Phi_{hi}}{1 + \Phi_{hi}} e_i + N_i \frac{\rho \sigma_i^2}{\sigma(\tilde{\sigma}_{ji}^2 + \lambda_i \sigma_i^2)} x_i \right)$

$= V_{0j} \left( N_i \tilde{\mu}_{ji} + \sum_{k=1}^{n} M_{ki} (\tilde{\mu}_{ji} - \tilde{\mu}_{ki}) - N_i \frac{\Phi_{hi}}{1 + \Phi_{hi}} e_i + N_i \frac{\rho \sigma_i^2}{\sigma(\tilde{\sigma}_{ji}^2 + \lambda_i \sigma_i^2)} x_i \right)$

$= N_i^2 V_{0j} \left( \tilde{\mu}_{ji} + \sum_{k \neq j}^{n} M_{ki} \tilde{\mu}_{ji} - \frac{\Phi_{hi}}{1 + \Phi_{hi}} e_i + \frac{\rho \sigma_i^2}{\sigma(\tilde{\sigma}_{ji}^2 + \lambda_i \sigma_i^2)} x_i \right)$

$= N_i^2 \left( \frac{1}{1 + \sum_{k=1}^{n} M_{ki}} \right)^2 \left( \sigma_i^2 - \tilde{\sigma}_{ji}^2 \right) \left( \frac{2 \tilde{\sigma}_{ji}^2}{\sigma(\tilde{\sigma}_{ji}^2 + \lambda_i \sigma_i^2)} \right) \sigma_i^2 + \left( \frac{N_i \Phi_{hi}}{1 + \Phi_{hi}} \right)^2 \sigma_i^2 - \frac{2 N_i^2 \frac{\Phi_{hi}}{1 + \Phi_{hi}}}{1 + \Phi_{hi}} \left( \frac{\rho \sigma_i^2}{\sigma(\tilde{\sigma}_{ji}^2 + \lambda_i \sigma_i^2)} \right) \sigma_i^2$

$U_{0j} = \frac{1}{2 \rho} \sum_{i} N_i^2 \left( \sigma_i^2 - \tilde{\sigma}_{ji}^2 \right) \left( \frac{\rho \sigma_i^2}{\lambda_0(1 + \Phi_{hi})} \right)^2 \left( \sigma_i^2 + \sigma_{ix}^2 \right) + \left( \frac{\Phi_{hi}}{1 + \Phi_{hi}} \right)^2 \sigma_i^2$

$= \frac{1}{2 \rho} \sum_{i} N_i^2 \left( \frac{2 \tilde{\lambda}_{ji} \alpha_{ji} \sigma_i^2}{1 + \lambda \alpha_{ji}^2} \right) \left( \frac{\Phi_{hi}}{1 + \Phi_{hi}} \right)^2 \alpha_{ji} + \left( \frac{\rho \sigma_i^2}{\lambda_0(1 + \Phi_{hi})} \right)^2 \left( \sigma_i^2 + \sigma_{ix}^2 \right) \sigma_i^2$

$= \frac{1}{2 \rho} \sum_{i} \left( \frac{N_i^2 (1 + 2 \lambda \alpha_{ji})}{1 + \lambda \alpha_{ji}^2} \right) \left( \frac{\Phi_{hi}}{1 + \Phi_{hi}} \right)^2 \alpha_{ji} + \left( \frac{\rho \sigma_i^2}{\lambda_0(1 + \Phi_{hi})} \right)^2 \left( \sigma_i^2 + \sigma_{ix}^2 \right) \sigma_i^2$
7.1.4 Derivation of Equations 22

The market clearing condition is

\[ rp_i = \frac{\sum_{j=1}^{n} \frac{\hat{\lambda}_{ji} \rho \sigma_i^2}{\rho (\sigma_j^2 + \lambda_{ji} \sigma_i^2)} \hat{\mu}_{ji} + \bar{z} + \frac{\Phi_{hi} \epsilon_i - \rho \sigma_i^2}{\lambda_0 (1 + \Phi_{hi})} x_i}{1 + \sum_{j=1}^{n} \frac{\hat{\lambda}_{ji} \rho \sigma_i^2}{\rho (\sigma_j^2 + \lambda_{ji} \sigma_i^2)}} \]  

(37)

From here we identify the price coefficients as a function of the monopolist learning and the competitive fringe learning. Now, conditionally on \( z_i \), we have

\[ \hat{\mu}_{ji} = s_{ji} \]

and \( s_{ji} \) is normally distributed with mean \( \bar{z} + (1 - \frac{1}{\alpha_{ji}}) \epsilon_i \) and variance \( (1 - \frac{1}{\alpha_{ji}}) \frac{1}{\alpha_{ji}} \sigma_i^2 \). What we want is to express the posterior mean in terms of delta as in \( z_i = s_i + \delta_i \). Given that,

\[ \delta_{ji} = z_i - s_{ji} = -\frac{1}{\alpha_{ji}} \epsilon_i + \text{noise} \]

\[ rp_i = N_i \sum_{j=1}^{n} M_{ji} \left( \bar{z} + \left(1 - \frac{1}{\alpha_{ji}}\right) \epsilon_i - \zeta_{ji} \right) + N_i \left[ \bar{z} + \frac{\Phi_{hi} \epsilon_i - \rho \sigma_i^2}{\lambda_0 (1 + \Phi_{hi})} x_i \right] \]  

(38)

\[ rp_i = \bar{z} - \bar{x} \frac{N_i \rho \sigma_i^2}{\lambda_0 (1 + \Phi_{hi})} + \epsilon_i N_i \left( \sum_{j=1}^{n} M_{ji} (\alpha_{ji} - 1) \frac{\alpha_{ji}}{\alpha_{ji} + 1 + \Phi_{hi}} \right) \]

\[ - \frac{N_i \rho \sigma_i^2}{\lambda_0 (1 + \Phi_{hi})} x_i - N_i \sum_{j=1}^{n} M_{ji} \zeta_{ji} \]

7.1.5 Derivations for Section 3.3.1

In a pure fringe, the \( \lambda_1, ..., \lambda_l = 0 \), and we can normalize \( \lambda_0 \) to 1 for simplicity. Then we quickly get that \( M_{ji} = 0 \) and \( N_i = 1 \) for all \( i, j \). Then we can calculate the price coefficients:

\[ a_i = \frac{1}{\rho} \left( \bar{z} - \bar{x} \sigma_i^2 \right) \]

\[ b_i = 0 \]

\[ c_i = \frac{1}{\rho} \sigma_i^2 \]

\[ d_{ji} = 0 \]

And therefore, \( \sigma_{pi}^2 = \left( \frac{1}{\rho} \sigma_i^2 \right)^2 \sigma_{x_i}^2 \)
If we introduce learning, then the coefficients become:

\[ a_i = \frac{1}{r} \left( \frac{\tilde{z} - \bar{x}}{\rho \sigma_i^2} \right) \quad \Phi_{hi} \]

\[ b_i = \frac{\Phi_{hi}}{r(1 + \Phi_{hi})} \]

\[ c_i = \frac{\rho \sigma_i^2}{r(1 + \Phi_{hi})} \]

\[ d_{ji} = 0 \]

Then we can calculate \( G_i, \Phi_{hi}, \) and price volatility:

\[ G_i = \frac{G_i^{KNS}}{1 + \Phi_{hi}} \]

\[ \frac{1 + \Phi_{hi}}{1 + \Phi_{hk}} = \frac{\rho^2 \bar{x}_i^2 \sigma_i^2 + 1 + \rho^2 \sigma_i^{2} \sigma_i^{2}}{(1 + \Phi_{hi})^2} \]

\[ \sum_{i=1}^{y} \frac{1 + \Phi_{hi}}{1 + \Phi_{hk}} = \sum_{i=1}^{y} \frac{\rho^2 \bar{x}_i^2 \sigma_i^2 + 1 + \rho^2 \sigma_i^{2} \sigma_i^{2}}{\rho^2 \bar{x}_k \sigma_k^2 + 1 + \rho^2 \sigma_k^{2} \sigma_k^{2}} \]

\[ \frac{1}{1 + \Phi_{hk}} \left( \frac{y + (1 - e^{2K_h})}{1 + \Phi_{hk}} \right) = \sum_{i=1}^{y} \sqrt{\frac{\rho^2 \bar{x}_i^2 \sigma_i^2 + 1 + \rho^2 \sigma_i^{2} \sigma_i^{2}}{\rho^2 \bar{x}_k \sigma_k^2 + 1 + \rho^2 \sigma_k^{2} \sigma_k^{2}}} \]

\[ \Phi_{hk} = \frac{(y + (e^{2K_h} - 1)) \sqrt{1 + \rho^2 \bar{x}_i^2 \sigma_i^2 + \rho^2 \bar{x}_k \sigma_k^2}}{\sum_{i=1}^{y} \sqrt{1 + \rho^2 \bar{x}_i^2 \sigma_i^2 + \rho^2 \bar{x}_k \sigma_k^2}} \]

\[ \sigma_{\hat{\nu}_i}^2 = b_i^2 \sigma_i^2 + c_i^2 \sigma_{xi}^2 - 1 \]

\[ = \frac{\Phi_{hi}^2}{r^2(1 + \Phi_{hi})^2} \sigma_i^2 + \frac{\rho^2 \sigma_i^4 \sigma_{xi}^2}{r^2 (1 + \Phi_{hi})^2} \]

\[ = \frac{1}{r^2(1 + \Phi_{hi})^2} \left( \Phi_{hi}^2 \sigma_i^2 + \rho^2 \sigma_i^4 \sigma_{xi}^2 \right) \]
\[
\frac{\text{Cov}(p_i, z_i)}{\sigma_{pi}} = \frac{\text{Cov}(a_i + b_i \epsilon_i - c_i \nu_i, z_i)}{\sigma_{pi}}
\]
\[
= \frac{b_i \sigma_i^2 r (1 + \Phi_{hi})}{\sqrt{\Phi_{hi}^2 \sigma_i^2 + \rho^2 \sigma_i^4 \sigma_{xi}^2}}
\]
\[
= \frac{\Phi_{hi} \sigma_i}{\sqrt{\Phi_{hi}^2 + \rho^2 \sigma_i^2 \sigma_{xi}^2}}
\]
\[
= \frac{1 + \Phi_{hi}^2 \rho^2 \sigma_i^2 \sigma_{xi}^2}{\sigma_i}
\]
\[
= \frac{\left( \sum_{k=1}^{b} \sqrt{1 + \rho^2 \sigma_i^2 \sigma_k^2 + \rho^2 \sigma_k^2 \sigma_{xi}^2} \right)^2}{\left( (y + (e^{2Ko} - 1)) \right)^2 (1 + \rho^2 \sigma_i^2 \sigma_{zi}^2)} \rho^2 \sigma_i^2 \sigma_{xi}^2
\]

Proof of Proposition 1

Proof.

\[
\Phi_{hi}^2 (\sigma_i^2) = \frac{(y + (e^{2Ko} - 1) (\rho^2 (\bar{x}^2 + \sigma_{zi}^2) \left( \sum_{k \neq i}^{b} \sqrt{1 + \rho^2 \sigma_i^2 \bar{x}^2 + \sigma_{zi}^2} \right))}{2 \sqrt{1 + \rho^2 \sigma_i^2 (\bar{x}^2 + \sigma_{zi}^2) \left( \sqrt{1 + \rho^2 \sigma_i^2 (\bar{x}^2 + \sigma_{zi}^2) + \sum_{k \neq i}^{b} \sqrt{1 + \rho^2 \sigma_k^2 (\bar{x}^2 + \sigma_{zi}^2)} \right)}^2} > 0
\]

Proof of Proposition 2

Proof. Define \(\frac{\left( \sum_{k=1}^{b} \sqrt{1 + \rho^2 \sigma_i^2 \sigma_k^2 + \rho^2 \sigma_k^2 \sigma_{zi}^2} \right)^2}{(y + (e^{2Ko} - 1))^2 (1 + \rho^2 \sigma_i^2 \sigma_{zi}^2)} = Z\).

\[
A(X, Y, Z) = \frac{\partial A}{\partial \sigma_i} \sigma_i \sqrt{1 + X \rho^2 \sigma_i^2 \sigma_{zi}^2}
\]
\[
= \frac{1}{(1 + X \rho^2 \sigma_i^2 \sigma_{zi}^2)^{1.5}} > 0
\]
7.1.6 Derivations for section 3.3.2

Introducing a monopolist means that $\lambda_1 > 0$. When no one can learn, the definitions of $M_{ji}$ and $N_i$ become: $M_{ji} = \frac{\lambda_1}{\lambda_0 + \lambda_1}$ and $N_i = \frac{\lambda_0 + \lambda_1}{\lambda_0 + 2\lambda_1}$ respectively. And the price coefficients and price volatility are:

\[
\begin{align*}
a_i &= \frac{\bar{z}}{r} - \frac{\bar{x}(\lambda_0 + \lambda_1)\rho\sigma_i^2}{r\lambda_0(\lambda_0 + 2\lambda_1)} \\
b_i &= 0 \\
c_i &= \frac{(\lambda_0 + \lambda_1)\rho\sigma_i^2}{r\lambda_0(\lambda_0 + 2\lambda_1)} \\
d_{ji} &= \frac{\lambda_1}{r(\lambda_0 + 2\lambda_1)} \\
\sigma_{pi}^2 &= \left(\frac{(\lambda_0 + \lambda_1)\rho\sigma_i^2}{r\lambda_0(\lambda_0 + 2\lambda_1)}\right)^2 \sigma_{xi}^2
\end{align*}
\]

If the fringe can learn, the new price coefficients are:

\[
\begin{align*}
a_i &= \frac{\bar{z}}{r} - \frac{\bar{x}(\lambda_0(1 + \Phi_{hi}) + \lambda_1)\rho\sigma_i^2}{r\lambda_0(\lambda_0(1 + \Phi_{hi}) + 2\lambda_1)(1 + \Phi_{hi})} \\
b_i &= \frac{(\lambda_0(1 + \Phi_{hi}) + \lambda_1)\Phi_{hi}}{(\lambda_0(1 + \Phi_{hi}) + 2\lambda_1)(1 + \Phi_{hi})r} \\
c_i &= \frac{r\lambda_0(\lambda_0(1 + \Phi_{hi}) + 2\lambda_1)(1 + \Phi_{hi})}{r\lambda_0(\lambda_0(1 + \Phi_{hi}) + 2\lambda_1)(1 + \Phi_{hi})} \\
d_{ji} &= \frac{\lambda_1}{r(\lambda_0(1 + \Phi_{hi}) + 2\lambda_1)}
\end{align*}
\]

And the gains to the fringe can be expressed as:

\[
G_i = G_i^{KNS} \\
= \frac{(\bar{z} - ra_i)^2}{\sigma_i^2} + (1 - rb_i)^2 + r^2 c_i^2 \frac{\sigma_{xi}^2}{\sigma_i^2} \\
= \left(\frac{(\lambda_0(1 + \Phi_{hi}) + \lambda_1)\rho\sigma_i^2}{\lambda_0(\lambda_0(1 + \Phi_{hi}) + 2\lambda_1)(1 + \Phi_{hi})}\right)^2 \left(\frac{\bar{x}^2 + \sigma_{xi}^2}{\sigma_i^2}\right) \\
+ \left(\frac{\lambda_0(1 + \Phi_{hi}) + 2\lambda_1 + \lambda_1\Phi_{hi}}{(\lambda_0(1 + \Phi_{hi}) + 2\lambda_1)(1 + \Phi_{hi})}\right)^2
\]

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And then price informativeness and volatility are:

\[
\begin{align*}
\sigma_{pi}^2 &= \left( \frac{(\lambda_0(1 + \Phi_{hi}) + \lambda_1)\Phi_{hi}}{(\lambda_0(1 + \Phi_{hi}) + 2\lambda_1)(1 + \Phi_{hi})r} \right)^2 \sigma_i^2 + \left( \frac{(\lambda_0(1 + \Phi_{hi}) + \lambda_1)\rho \sigma_i^2}{(r\lambda_0(1 + \Phi_{hi}) + 2\lambda_1)(1 + \Phi_{hi})} \right)^2 \sigma_{zi}^2 \\
\frac{Cov(p_i, z_i)}{\sigma_{pi}} &= \frac{(\lambda_0(1 + \Phi_{hi}) + \lambda_1)\Phi_{hi}}{(\lambda_0(1 + \Phi_{hi}) + 2\lambda_1)(1 + \Phi_{hi})r} \sigma_i^2 \\
&= \sqrt{\left( \frac{(\lambda_0(1 + \Phi_{hi}) + \lambda_1)\Phi_{hi}}{(\lambda_0(1 + \Phi_{hi}) + 2\lambda_1)(1 + \Phi_{hi})r} \right)^2 \sigma_i^2 + \left( \frac{(\lambda_0(1 + \Phi_{hi}) + \lambda_1)\rho \sigma_i^2}{(r\lambda_0(1 + \Phi_{hi}) + 2\lambda_1)(1 + \Phi_{hi})} \right)^2 \sigma_{zi}^2} \\
&= \frac{\Phi_{hi} \sigma_i}{\sqrt{\Phi_{hi}^2 + \left( \frac{\rho}{\lambda_0} \right)^2 \sigma_{zi}^2}} \\
\end{align*}
\]

Proof of Proposition 4:

Proof.

\[ P^I_i(\Phi_{hi}) = \frac{\partial P_{I_i}}{\partial \Phi_{hi}} = \frac{1 + \lambda_{ji} \alpha_{ji}}{1 + 2\lambda_{ji} \alpha_{ji}} \left( \lambda_0 + \frac{\rho \sigma_i^2}{\lambda_0} \right)^2 \left( \frac{\alpha_{ji}}{\sigma_i^2} + \frac{1}{\sigma_{zi}^2} \right) \]

If only the monopolist can learn, then \( \Phi_{hi} = 0 \). First we can write: \( M_{ji} = \frac{\lambda_{ji} \alpha_{ji}}{1 + \lambda_{ji} \alpha_{ji}}, N_i = \frac{1 + \lambda_{ji} \alpha_{ji}}{1 + 2\lambda_{ji} \alpha_{ji}} \). Then we need to solve the monopolist’s information problem:

\[
0 = \frac{\partial}{\partial \alpha_{ji}} \frac{1}{2\rho} \sum_i \frac{1}{1 + 2\lambda_{ji} \alpha_{ji}} \left[ \left( \frac{\rho \sigma_i^2}{\lambda_0} \right)^2 \left( \bar{x}^2 + \sigma_{ix}^2 \right) \frac{\alpha_{ji}}{\sigma_i^2} + \left( \alpha_{ji} - 1 \right) \right] \\
0 = \frac{1}{1 + 2\lambda_{ji} \alpha_{ji}} \left[ \left( \frac{\rho \sigma_i^2}{\lambda_0} \right)^2 \left( \bar{x}^2 + \sigma_{ix}^2 \right) \frac{1}{\sigma_i^2} + 1 \right] - \frac{2\lambda_{ji}}{(1 + 2\lambda_{ji} \alpha_{ji})^2} \left[ \left( \frac{\rho \sigma_i^2}{\lambda_0} \right)^2 \left( \bar{x}^2 + \sigma_{ix}^2 \right) \frac{\alpha_{ji}}{\sigma_i^2} + \left( \alpha_{ji} - 1 \right) \right] + 2\rho \frac{\eta_i - \mu}{\alpha_{ji}}
\]
$$\frac{2\rho\mu(1 + 2\hat{\lambda}_j\alpha_{ji})}{\alpha_{ji}} = \left[ \left( \frac{\rho\sigma^2_i}{\lambda_0} \right)^2 (\bar{x}^2 + \sigma^2_{iz}) + 1 \right] - \frac{2\hat{\lambda}_j}{1 + 2\hat{\lambda}_j\alpha_{ji}} \left[ \left( \frac{\rho\sigma^2_i}{\lambda_0} \right)^2 (\bar{x}^2 + \sigma^2_{iz}\alpha_{ji}) + (\alpha_{ji} - 1) \right]$$

$$\frac{2\rho(1 + 2\hat{\lambda}_j\alpha_{ji})^2}{\alpha_{ji}} = \left[ \left( \frac{\rho}{\lambda_0} \right)^2 (\bar{x}^2 + \sigma^2_{ii}) + 1 \right] + 2\hat{\lambda}_j$$

$$\mu = \frac{\alpha_{ji}}{(1 + 2\hat{\lambda}_j\alpha_{ji})^2}X_i$$

$$X_i = \frac{1}{2\rho} \left( \left[ \left( \frac{\rho}{\lambda_0} \right)^2 (\bar{x}^2 + \sigma^2_{iz}\sigma^2_i + 1) + 2\hat{\lambda}_j \right] \right)$$

$$\frac{(1 + 2\hat{\lambda}_j\alpha_{ji})^2}{(1 + 2\hat{\lambda}_k\alpha_{jk})^2} = \frac{\alpha_{ji}X_i}{\alpha_{jk}X_k}$$

$$\prod_{k=1}^{n} \frac{(1 + 2\hat{\lambda}_j\alpha_{ji})^2}{(1 + 2\hat{\lambda}_k\alpha_{jk})^2} = \prod_{k=1}^{n} \frac{\alpha_{ji}X_i}{\alpha_{jk}X_k}$$

$$\frac{(1 + 2\hat{\lambda}_j\alpha_{ji})^{2n}}{\prod_{k=1}^{n}(1 + 2\hat{\lambda}_k\alpha_{jk})^2} = \frac{(\alpha_{ji}X_i)^n}{e^{2K}n!} \prod_{k=1}^{n} X_k$$

$$M_{ji} = \frac{\lambda_1\alpha_{ji}}{\lambda_0 + \lambda_1\alpha_{ji}}$$

$$N_t = \frac{\lambda_0 + \lambda_1\alpha_{ji}}{\lambda_0 + 2\lambda_1\alpha_{ji}}$$

$$a_i = \frac{\bar{z}}{r} - \frac{x}{r} \frac{\lambda_0 + \lambda_1\alpha_{ji}}{\lambda_0 + 2\lambda_1\alpha_{ji}}$$

$$b_i = \frac{\lambda_1(\alpha_{ji} - 1)}{r(\lambda_0 + 2\lambda_1\alpha_{ji})}$$

$$c_i = \frac{(\lambda_0 + \lambda_1\alpha_{ji})\rho\sigma^2_i}{r\lambda_0(\lambda_0 + 2\lambda_1\alpha_{ji})}$$

$$d_{ji} = \frac{\lambda_1\alpha_{ji}}{r(\lambda_0 + 2\lambda_1\alpha_{ji})}$$

$$\alpha_{ji} = \frac{\lambda_0 + \lambda_1\alpha_{ji}}{\lambda_0 + 2\lambda_1\alpha_{ji}}$$

$$\sigma_{ji}^2 = b_i^2\sigma^2_i + c_i^2\sigma^2_{zi} + d_{ji}^2 \left( 1 - \frac{1}{\alpha_{ji}} \right) \sigma^2_{ji}$$

$$= \frac{1}{r^2(\lambda_0 + 2\lambda_1\alpha_{ji})^2} \left[ \lambda_1^2(\alpha_{ji} - 1)^2\sigma^2_i + (\lambda_0 + \lambda_1\alpha_{ji})^2\rho^2\sigma^4_i\sigma^2_{zi}\lambda_0^2 + \lambda_1^2\sigma^2_i \right]$$
\[
\frac{\text{Cov}(p_i, z_i)}{\sigma_{pi}} = \frac{\text{Cov}(a_i + b_i \varepsilon_i - c_i \omega_i - h_i \zeta_{ji}, z_i)}{\sigma_{pi}}
\]
\[
= \frac{\lambda_1 (\alpha_{ji} - 1) \sigma_i^2}{\sqrt{\left[ \lambda_1^2 (\alpha_{ji} - 1)^2 \sigma_i^2 + (\lambda_0 + \lambda_1 \alpha_{ji})^2 \rho^2 \sigma_i^4 \sigma_{xi}^2 \lambda_0^{-2} + \lambda_1^2 \sigma_i^2 \right]}}
\]
\[
= \frac{\lambda_1 (\alpha_{ji} - 1) \sigma_i}{\sqrt{\left[ \lambda_1^2 (\alpha_{ji} - 1)^2 + (1 + \hat{\lambda}_{ji} \alpha_{ji})^2 \rho^2 \sigma_i^2 \sigma_{xi}^2 + \lambda_1^2 \right]}}
\]

Proof of Proposition 5

Proof. Define \( f(\lambda, \alpha_j) = \frac{\alpha_j}{(1 + 2 \lambda \alpha_j)^2} X_j \). Then \( f_\lambda = \frac{-2 \alpha_j (2 \alpha_j \hat{\lambda} + 2 \alpha (X_j - \hat{\lambda}) - 1)}{(2 \alpha \alpha_j + 1)^2} \) which is decreasing in \( \alpha_j \). Since \( \alpha_j' (\sigma_j) > 0 \) therefore assuming wlog that \( \sigma_j > \sigma_i \), we get that \( \alpha_j > \alpha_i \), and therefore \( f_\lambda(i) > f_\lambda(j) \). Similarly \( f_\alpha = \frac{-1 + 2 \hat{\alpha} \hat{\lambda}}{(1 + 2 \alpha \lambda)^2} X \) is decreasing in \( \alpha_j \). Therefore in order to satisfy the first order conditions \( \alpha_j(\lambda) < 0 \) and \( \alpha_i(\lambda) > 0 \).

Proof of Proposition 6

Proof.

\[
PI_i'(\lambda_1) = \frac{\partial PI_i}{\partial \lambda_1}
\]
\[
= \frac{(\alpha - 1) \sigma_i^3 \rho^2 \sigma_{xi}^2 (\hat{\lambda} + 1)}{\left[ \lambda_1^2 (\alpha_{ji} - 1)^2 + (1 + \hat{\lambda}_{ji} \alpha_{ji})^2 \rho^2 \sigma_i^2 \sigma_{xi}^2 + \lambda_1^2 \right]^{1.5}}
\]
\[
> 0
\]

If both the monopolist and the fringe can learn, we need to solve the monopolist’s information problem when \( \Phi_{hi} > 0 \). First we can write: \( M_{ji} = \frac{\hat{\lambda}_{ji} \alpha_{ji}}{1 + \hat{\lambda}_{ji} \alpha_{ji}}, N_i = \frac{1 + \hat{\lambda}_{ji} \alpha_{ji}}{1 + 2 \lambda \alpha_{ji}} \)

\[
0 = \frac{\partial}{\partial \alpha_{ji}} \left[ \frac{1}{2 \rho} \sum_i \frac{1}{1 + 2 \lambda \alpha_{ji}} \left[ \left( \frac{\Phi_{hi}}{1 + \Phi_{hi}} \right)^2 \alpha_{ji} + \left( \frac{\rho \sigma_i^2}{\lambda_0 (1 + \Phi_{hi})} \right)^2 \left( \hat{x}^2 + \sigma_{\hat{x}}^2 \right) \alpha_{ji} \sigma_i^2 \right] \right]
\]
\[
+ \left( 1 + \Phi_{hi} \right) \left( 1 + \sum_{k \neq j} M_{ki} \right) \left( 2 \Phi_{hi} \right) \left( 1 + \sum_{k \neq j} M_{ki} \right) \left( \alpha_{ji} - 1 \right)
\]

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\[
\begin{align*}
2\mu &= \frac{-2\hat{\lambda}_{ji}}{(1 + 2\hat{\lambda}_{ji}\alpha_{ji})^2} \left[ \left( \frac{\Phi_{hi}}{1 + \Phi_{hi}} \right)^2 \alpha_{ji} + \left( \frac{\rho \sigma_i^2}{\lambda_0(1 + \Phi_{hi})} \right)^2 (\bar{x} \Phi_{hi}^2 + \sigma_{\bar{x}}^2 \sigma_i^2 \alpha_{ji} + \frac{1 - \Phi_{hi}}{1 + \Phi_{hi}} (\alpha_{ji} - 1) \right) \\
&+ \frac{1}{1 + 2\hat{\lambda}_{ji}\alpha_{ji}} \left[ \left( \frac{\Phi_{hi}}{1 + \Phi_{hi}} \right)^2 + \left( \frac{\rho \sigma_i^2}{\lambda_0(1 + \Phi_{hi})} \right)^2 (\bar{x}^2 + \sigma_{\bar{x}}^2 \sigma_i^2) + \frac{1 - \Phi_{hi}}{1 + \Phi_{hi}} \right] \\
2\rho \mu &= \frac{1}{(1 + 2\hat{\lambda}_{ji}\alpha_{ji})^2} \left[ \left( \frac{\Phi_{hi}}{1 + \Phi_{hi}} \right)^2 + \left( \frac{\rho \sigma_i^2}{\lambda_0(1 + \Phi_{hi})} \right)^2 (\bar{x}^2 + \sigma_{\bar{x}}^2 \sigma_i^2) + \frac{1 - \Phi_{hi}}{1 + \Phi_{hi}} \right] \\
Y_i &= \frac{1}{2\rho} \left[ 1 + 2\hat{\lambda}_{ji} + \left( \frac{\rho \sigma_i^2}{\lambda_0} \right)^2 (\bar{x}^2 + \sigma_{\bar{x}}^2 \sigma_i^2) \right] \\
\mu &= \frac{\alpha_{ji} Y_i + \hat{\lambda}_{ji} \Phi_{hi}^2}{(1 + 2\hat{\lambda}_{ji}\alpha_{ji})^2(1 + \Phi_{hi})^2} \\
\lambda_i &= 0 \\
M_{ji} &= \frac{\lambda_1 \alpha_{ji}}{\lambda_0(1 + \Phi_{hi}) + \lambda_1 \alpha_{ji}} \\
N_i &= \frac{\lambda_0(1 + \Phi_{hi}) + \lambda_1 \alpha_{ji}}{\lambda_0(1 + \Phi_{hi}) + 2\lambda_1 \alpha_{ji}} \\
a_i &= \frac{\hat{z} - \bar{x}}{r} \frac{\lambda_0(1 + \Phi_{hi}) + \lambda_1 \alpha_{ji}}{\lambda_0(1 + \Phi_{hi})(\lambda_0(1 + \Phi_{hi}) + 2\lambda_1 \alpha_{ji})} \\
b_i &= \frac{\lambda_1(\alpha_{ji} - 1)}{r(\lambda_0(1 + \Phi_{hi}) + 2\lambda_1 \alpha_{ji})} + \frac{\lambda_0(1 + \Phi_{hi}) + \lambda_1 \alpha_{ji} \Phi_{hi}}{\lambda_0(1 + \Phi_{hi}) + 2\lambda_1 \alpha_{ji}} \frac{1}{r(1 + \Phi_{hi})} \\
c_i &= \frac{(\lambda_0(1 + \Phi_{hi}) + \lambda_1 \alpha_{ji}) \rho \sigma_i^2}{\lambda_0(1 + \Phi_{hi})(\lambda_0(1 + \Phi_{hi}) + 2\lambda_1 \alpha_{ji}) \frac{1}{r}} \\
d_{ji} &= \frac{\lambda_1 \alpha_{ji}}{r(\lambda_0(1 + \Phi_{hi}) + 2\lambda_1 \alpha_{ji})} \\
\bar{\sigma}_{pi}^2 &= b_i^2 \sigma_i^2 + c_i^2 \sigma_{\bar{x}}^2 + d_{ji}^2 \left( 1 - \frac{1}{\alpha_{ji}} \right) \bar{\sigma}_{ji}^2 \\
\text{Cov}(p_i, z_i) &= \text{Cov}(a_i + b_i \epsilon_i - c_i \nu_i - d_i \zeta_{ji}, z_i) \\
\sigma_{pi} &= \sqrt{\left( \lambda_1(\alpha_{ji} - 1) + \lambda_0(1 + \Phi_{hi}) + \lambda_1 \alpha_{ji} \frac{\Phi_{hi}}{1 + \Phi_{hi}} \right) \sigma_i} \\
&= \sqrt{\left( \lambda_1(\alpha_{ji} - 1)^2 + \frac{(\lambda_0 + \Phi_{hi}) + \lambda_1 \alpha_{ji})^2}{(1 + \Phi_{hi})^2} + (\lambda_0(1 + \Phi_{hi}) + \lambda_1 \alpha_{ji})^2 \rho^2 \bar{\sigma}_{\bar{x}}^2 \lambda_0^{-2} + \lambda_1^2 \right)} \\
\end{align*}
\]
7.1.7 Derivation for 3.3.3

\[\lambda_1, ..., \lambda_n > 0\]

\[M_{ji} = \frac{\lambda_j}{\lambda_0 + \lambda_j}\]

\[N_i = \frac{1}{1 + \sum_{j=1}^{n} M_{ji}}\]

\[a_i = \bar{z} - \frac{\bar{x} N_i \sigma_i^2}{r \lambda_0}\]

\[b_i = 0\]

\[c_i = \frac{N_i \rho \sigma_i^2}{r \lambda_0}\]

\[d_{ji} = \frac{N_i M_{ji}}{r}\]

\[\sigma_{pi}^2 = c_i^2 \sigma_{xi}^2 + \sum_{j=1}^{n} \sum_{k \neq j} 2d_{ji}d_{ki} \left( \frac{1 + \alpha_{ji} \alpha_{ki}}{\alpha_{ji} \alpha_{ki}} \sigma_{i}^2 - \frac{1 + \alpha_{ki} \sigma_{i}^2}{\alpha_{ki}} - \frac{1 + \alpha_{ji} \sigma_{i}^2}{\alpha_{ji}} \right)\]

\[= \left( \frac{N_i \rho \sigma_i^2}{r \lambda_0} \right)^2 \sigma_{xi}^2 - \sum_{j=1}^{n} \sum_{k \neq j} 4d_{ji}d_{ki} \sigma_i^2\]

\[= \frac{N_i^2 \sigma_i^2}{r^2} \left( \frac{\rho^2 \sigma_i^2 \sigma_{xi}^2}{\lambda_0^2} - 4 \sum_{j=1}^{n} \sum_{k \neq j} M_{ji} M_{ki} \right)\]