Crime and the Minimum Wage*

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Abstract

How does the minimum wage affect crime rates? Empirical research suggests that increasing a worker’s wage can deter him from committing crimes. On the other hand, if that worker becomes displaced as a result of the minimum wage, he may be more likely to commit a crime. In this paper, I describe a frictional world in which a worker’s criminal actions are linked to his labor market outcomes. I calibrate the model to match the aggregate crime rate and the labor market faced by 16-24 year olds in 1998. Using the calibrated model, I show that the relationship between the aggregate crime rate and the minimum wage is non-monotonic. I test for this non-monotonicity using county level crime data and state level minimum wage changes from 1980 to 2012. The results from the empirical analysis as well as the model suggest that any increase in the federal minimum wage may increase the crime rate as the current wage floor is close the the crime minimizing value.

JEL: J08, J38, J64

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1 Introduction

The minimum wage has once again made it to the front lines of political discussion in the United States. Both the Democratic and Republican parties have come out in favor of substantial increases. An unprecedented number of cities have proposed legislation for higher local minimum wages and for the first time ever, a majority of states have minimum wages higher than the federal level. California and New York have passed laws projecting the minimum wage to be $15 within a few years, bringing about some of the largest real increases since 1949. Economists have long debated the labor market effects of a minimum wage, dating back to Stigler (1946) who first drew attention to possible employment effects after a 21% erosion of the real wage floor induced a public outcry for a higher minimum. While nearly all of the arguments hinge on employment, in this paper I ask how changes in the minimum wage affect criminal activity? Given that the policy is primarily aimed at improving labor market conditions for young and unskilled workers, who are also most at risk in terms of criminal activity, see Figure 1, potential changes in crime should be part of the policy debate.

Figure 1: Characteristics of Minimum Wage Workers and Criminals

Many economists have tested how the decision to commit crimes changes with respect to the probability or severity of punishment.1 However, it was not until Schmidt and Witte (1984) and Grogger (1998) that they began to test the effects of labor market changes on people’s criminal actions. The conclusions from these studies are as economic theory suggests: people choose to commit more crimes when unemployment increases and less when they receive higher wages.2

Notes: Plotted in blue is the percent distribution of hourly workers working at or below the minimum wage by age in 2012. The data come from the Bureau of Labor Statistics Characteristics of Minimum Wage Workers Report. Plotted in green is the percent distribution of arrests for Type 1 Property Crimes as defined by the Federal Bureau of Investigation (FBI) by age in 2012. The data come from the FBI’s Uniform Crime Reports.

1See for example: Becker (1968); Ehrlich (1973); Myers (1983); Grogger (1991); Owens (2009); Hansen (2014)
2For a more recent literature reaffirming these results see Gould et al. (2002), Mocan and Unel (2011), Machin and
Therefore, economic theory alone can not determine whether an increase in the minimum wage will increase or decrease the crime rate. Increasing the minimum wage can raise wages for workers, thus deterring them from crime. However, there exists empirical evidence that the minimum wage will displace some workers from jobs, thus enticing them to commit more crimes.

To find the direction of the effect, I use a search-theoretic framework to describe a world in which people make crime and labor market decisions jointly. I calibrate the model to match aggregate statistics of crime and the labor market to analyze the quantitative implications of changing the minimum wage. The existing literature trying to identify and quantify the effect of the minimum wage on crime rates is sparse. Hashimoto (1987) finds evidence of a positive relationship using national time-series data of the minimum wage and teenage arrest rates relative to adults. In a recent micro-level study, Beauchamp and Chan (2014) find a positive effect of minimum wage increases on crime for people employed at a binding wage. I focus on a general equilibrium analysis in which the minimum wage can change all workers’ crime decisions and examine the effect on the aggregate crime rate.

In the labor market workers receive job offers at an exogenous rate and wages are determined by strategic bargaining between workers and firms. The structure of the labor market is closest to Flinn (2006), where matches are ex-post heterogeneous through the existence of match specific productivity. I differ from Flinn (2006) by introducing heterogeneity among workers, an important addition when analyzing the effects of a minimum wage policy on labor market outcomes, since such a policy may not affect all workers identically.

The crime market is as in Burdett et al. (2003), workers receive random crime opportunities while employed and unemployed. I add two levels of heterogeneity to capture two important interactions between changes in the labor market and the crime market. First, in contrast to other models of crime and the labor market, workers are ex-ante heterogeneous in unemployment utility, making the stock of criminals endogenous and allowing changes in the labor market to have an extensive effect on crime. In Huang et al. (2004), for example, only some workers specialize in criminal activities, however among those that commit crimes, their propensity for criminal behavior is identical. In Burdett et al. (2003) all workers are criminals and have the same propensity for criminal behavior and in Engelhardt et al. (2008) all workers commit crimes with propensities differing across employment states. Second, matches are ex-post heterogeneous with respect to productivity, allowing the “quality” of a job to enter into the worker’s crime decision, and creating

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3See Neumark and Wascher (2007) for a review of how changes in the minimum wage affect labor market conditions.

4Hashimoto (1987) is limited by the use of national data which may lose much of its identifying variation through aggregation and is subject to spurious correlations.

5Meyer and Wise (1983a) and Meyer and Wise (1983b) provide evidence of heterogeneities by showing that the effect of a minimum wage on employment and earnings differ across the group of workers for which it is binding.
a range of wages for which he commits crimes, in contrast to a single criminal wage as in Burdett et al. (2003) and Burdett et al. (2004).

Using the benchmark model, I introduce a minimum wage similar to Flinn (2006), imposing a constraint to the bargaining problem faced by firms and workers, and calibrate to match the aggregate crime rate and labor market of 16-24 year olds in 1998. Increasing the minimum wage in the calibrated model shows that the relationship between the aggregate crime rate and the minimum wage is non-monotonic. I vet the model by simulating data and estimating the elasticity of crime with respect to wages and the elasticity of employment with respect to the minimum wage - finding that the model generated elasticities, although not targeted in the calibration, are similar to those found in the empirical literature. Furthermore, I test the model’s prediction of a non-monotonic relationship using county level crime rates and state level minimum wage changes from 1980 to 2012. The results from the empirical analysis, as well as the calibrated model suggest that an increase in the federal minimum wage may increase the crime rate as the current wage floor is close to the crime minimizing level.

2 Model

I describe a world in which people in the labor market receive both exogenous job and crime opportunities and show how they jointly decide whether or not to take a job or act on a crime opportunity in the absence of a binding minimum wage. The question of interest is, how does a binding minimum wage change the behavior of a worker? How does it change his decision to accept jobs and act on crime opportunities, and in turn how do these changes translate into the aggregate crime rate? To answer these questions, I describe the minimum wage as a constraint that workers and firms must consider when bargaining over the wage, and see how the existence of such a constraint changes employment decisions and subsequently wages if an agreement can be reached.

The model is in continuous time and composed of a unit measure of workers, who are: are risk neutral, discount at rate \( r \), and ex-ante heterogeneous in their value of flow unemployment utility. An unemployed worker looking for a job receives utility flow \( b \) given by the cdf \( F(b) \). Workers receive crime opportunities at exogenous rate \( \mu_u \) while unemployed and \( \mu_e \) while employed. A crime is a transfer of utility; the offender receives a gain, \( g \), and is caught with probability \( \pi \). If he is caught, he goes to jail, receives flow utility \( z \) and gets released at exogenous rate \( \gamma \). When released from prison, he enters the labor market as unemployed. All non-incarcerated workers are victims of crime, and lose \( L \) at rate \( \chi \). While unemployed, a worker also receives a job offer with productivity \( \lambda \), given by the cdf \( G(\lambda) \), at exogenous rate \( \mu_j \). Wages are determined by strategic bargaining á la Rubinstein’s alternating offers and jobs separate at exogenous rate \( \delta \).
2.1 Workers

Both unemployed and employed workers receive crime opportunities, and must decide whether or not to act on these opportunities. Their decisions are based on the expected cost and expected utility from committing a crime. For an unemployed worker, the expected utility from committing a crime, \( K_u \), is equal to the instantaneous gain from committing the crime, \( g \), and the weighted average of his continued state: his prison utility if he is caught or his unemployment utility if he is not. The expected utility from committing a crime while employed, \( K_e(w) \), is calculated analogously. Therefore,

\[
K_u = g + \pi V_p + (1 - \pi)V_u \\
K_e(w) = g + \pi V_p + (1 - \pi)V_e(w)
\]

where, \( V_p \) is the value of prison, \( V_u \) is the value of unemployment, and \( V_e(w) \) is the value of being employed at wage \( w \), all defined below. Workers commit crimes rationally; if the expected gain \( (K_u - V_u) \) of committing the crime is greater than zero he will choose to do act on the opportunity. Let \( \phi \) be an indicator that takes on the value 1 if the worker commits a crime. The crime decisions for an unemployed and an employed worker are:

\[
\phi_u = \begin{cases} 
1 & \text{if } g + \pi(V_p - V_u) > 0 \\
0 & \text{if } g + \pi(V_p - V_u) \leq 0 
\end{cases}
\]

\[
\phi_e(w) = \begin{cases} 
1 & \text{if } g + \pi(V_p - V_e(w)) > 0 \\
0 & \text{if } g + \pi(V_p - V_e(w)) \leq 0 
\end{cases}
\]

The crime decision for an employed worker depends on the current wage since it determines his cost of committing a crime.

Given the probability is zero that an unemployed worker gets a crime opportunity and job match simultaneously, the flow value of unemployment is:

\[
rV_u = b - \chi L + \mu_u \phi_u[K_u - V_u] + \mu_f \int_{\lambda} \max\{V_e(w) - V_u, 0\} \, dG(\lambda)
\]

As in Burdett et al. (2003), the flow return to being unemployed, \( rV_u \), is equal to the flow utility of unemployment net of becoming a victim of crime plus the expected value of receiving either a crime or job opportunity. Similarly,

\[
rV_e(w) = w - \chi L + \mu_e \phi_e(w)[K_e(w) - V_e(w)] + \delta[V_u - V_e(w)]
\]

\[
rV_p = z + \gamma(V_u - V_p)
\]
are the flow return to employment and prison.

Notice from equation (3) that the crime decision of an unemployed worker is only a function of his unemployment value. Therefore, there exists a unique value of unemployment that makes workers indifferent to committing crimes while unemployed:

\[ V_u^* = \frac{g(r + \gamma)}{r\pi} + \frac{z}{r} \]  

(8)

If \( V_u < V_u^* \) the worker will commit crimes while unemployed and if \( V_u \geq V_u^* \) he will not. Since \( V_u \) is strictly increasing in \( b \), there exists a unique flow utility of unemployment, \( b^* \), such that \( V_u(b^*) = V_u^* \), and workers with \( b < b^* \) commit crimes while unemployed while workers with \( b \geq b^* \) do not. Intuitively, workers with a higher unemployment value are less likely to commit crimes while unemployed since their cost of crime is higher. Proposition 2.1 proves that workers who do not commit crimes while unemployed also never commit crimes while employed. Since workers with a flow utility of unemployment greater than \( b^* \) will never commit crimes, \( F(b^*) \) can be thought of as the stock of criminals in the economy.

**Proposition 2.1.** If with \( b \geq b^* \) then \( \phi_e(w) = 0 \) for all \( w \geq w^R \). Where \( w^R \) is the workers reservation wage defined as \( V(w^R) = V_u \).

**Proof.** If \( b \geq b^* \) then \( V_u > V_u^* \), thus \( \phi_u = 0 \). From (3) this implies \( g + \pi V_p \leq \pi V_u \). The definition of \( w^R \) implies that \( g + \pi V_p \leq \pi V_e(w^R) \). Since (6) is strictly increasing in \( w \) it must be the case that \( g + \pi V_p \leq \pi V_e(w) \) for all \( w \geq w^R \). Thus from (4), \( \phi_e(w) = 0 \) for all \( w \geq w^R \).

\[ \Box \]

### 2.2 Firms

Firms consist of a single job, either vacant or filled. The return to a filled job at productivity \( \lambda \) at wage \( w \) is

\[ rV_f(w) = \lambda - w + \delta[V_v - V_f(w)] + \mu e \phi_e(w) \pi[V_v - V_f(w)] \]  

(9)

where \( V_v \) is the value of posting a vacancy. With free entry \( V_v = 0 \) and the value of a filled job becomes

\[ V_f(w) = \frac{\lambda - w}{r + \delta + \mu e \phi_e(w) \pi} \]  

(10)

Notice that the expected duration of the job depends on the worker’s decision to commit crimes while employed. If he chooses to commit crimes while employed the job can end with him getting caught and going to prison.
2.3 Wages

Along with crime opportunities, an unemployed worker also receives job offers and must decide whether or not to accept the job or continue searching; his decisions is based on the wage. As pointed out by Engelhardt et al. (2008), when the worker can choose to commit crimes while employed, the feasible set of allocations that split the surplus of the match is non-convex, therefore the axiomatic approach to bargaining cannot be implemented. I choose to split the surplus through strategic bargaining: the worker and the firm determine the wage in a two stage game à la Rubinstein’s alternating offers.

In the first stage the firm offers the worker a wage. If he accepts the offer, bargaining ends and the job begins at the offered wage. If he rejects the wage the game moves to the second stage where he gets to set the final wage with probability \( \alpha \) and the firm gets to set the final wage with probability \( 1 - \alpha \). The probability that the match breaks up during negotiations is zero and neither the firm nor the worker discount the future during the bargaining process.

There are two wages that are of particular interest. First the reservation wage, \( w_R \), defined as \( V_u = V_e(w_R) \) such that if \( w \geq w_R \) the worker chooses to stop searching and accept the job. By the value of unemployment, (6), and the fact that a worker who chooses not to commit crimes while unemployed, will never commit a crime while employed, Proposition 2.1, the reservation wage is

\[
    w_R(V_u) = \begin{cases} 
      \chi L + rV_u - \mu e \left[ g + \pi \left( \frac{z-rV_u}{r+\gamma} \right) \right] & \text{if } V_u < V_u^* \\
      \chi L + rV_u & \text{if } V_u \geq V_u^*.
    \end{cases}
\]

(11)

Second, the crime reservation wage, \( w_C \) defined as \( g + \pi[V_p - V_e(w_C)] = 0 \) such that if \( w \geq w_C \) the worker chooses to stop searching, accept the job and does not commit crimes while employed. Again using the value of unemployment, (6), one can solve to the crime reservation wage to find:

\[
    w_C(V_u) = \chi L + \frac{r(r + \delta)}{r + \gamma} V_u^* + \frac{r(y - \delta)}{r + \gamma} V_u
\]

(12)

for \( V_u < V_u^* \). Workers that do not commit crimes while unemployed do not have a crime reservation wage since they forgo crime opportunities for all wages.

To solve for the equilibrium wages I will solve the two stage game through backwards induction, first solving the optimal wages offers in the second stage for the worker and the firm, then solving for the firm’s optimal offer in the first stage given the second stage outcomes. In the first stage the firm offers the profit maximizing wage subject to the worker accepting the offer. Therefore, in equilibrium wages will be determined without delay.

In the second stage, if the worker gets to set the final wage he will choose to set the wage

\( ^6 \)The problem is similar to that of on the job search, see Shimer (2006) for details.

\( ^7 \)I will assumed that workers are moral, such that a worker that is indifferent to committing crimes will chose not to commit crimes.
equal to the productivity of the job, $w = \lambda$, and takes the entire surplus of the job. If the worker is a criminal, then he continues to commit crimes while employed if $\lambda < w_C$ and forges crime if $\lambda \geq w_C$. If the firm matches with a criminal and gets to set the final wage in the second stage, it is optimal for the firm to choose between setting the wage at the reservation wage or setting the wage at the crime reservation wage. So for $V_u < V_u^*$ the firm faces the following problem in the second stage:

$$\arg\max_{w_R, w_C} \left\{ \frac{\lambda - w_R(V_u)}{r + \delta + \mu e \pi}, \frac{\lambda - w_C(V_u)}{r + \delta} \right\}$$

(13)

It is easy to show that $w_R(V_u) < w_C(V_u)$, therefore the firm faces a trade-off between receiving a higher flow value for the job for a shorter expected duration or a lower flow value for the job for a longer expected. Problem (13) has a unique solution for the job productivity that equates the two choices, call it $\lambda^{D2}$, which is:

$$\lambda^{D2}(V_u) = \frac{(r + \delta + \mu e \pi)w_C(V_u) - (r + \delta)w_R(V_u)}{\mu e \pi}.$$  

(14)

If $\lambda < \lambda^{D2}(V_u)$ the firm sets the wage $w_R(V_u)$ and if $\lambda \geq \lambda^{D2}(V_u)$ then the firm sets the wage $w_C(V_u)$. If the firm matches with a non-criminal, $V_u \geq V_u^*$, then it has no choice to make and sets the wage to the reservation wage.

In the first stage the firm offers a wage and chooses to offer the wage that maximizes profits subject to the worker accepting the offer. The worker will accept the offer if it is at least as large as his expected value from the second stage. For non-criminals the expected value of the second stage is $\alpha \lambda + (1 - \alpha)w_R(V_u)$, so the firm faces the following problem in the first stage:

$$\max_w \frac{\lambda - w}{r + \delta} \quad \text{s.t.} \quad w \geq \alpha \lambda + (1 - \alpha)w_R(V_u).$$

(15)

Therefore, the firm offers a wage $w(\lambda, V_u) = \alpha \lambda + (1 - \alpha)w_R(V_u)$ whenever $V_u \geq V_u^*$ and the worker accepts. Since matches are heterogeneous in their productivity, not all matches lead to a filled job. When a worker matches with a firm the productivity must be high enough for him to give up his value of continued search and enter employment. The worker will choose employment whenever $w(\lambda, V_u) \geq w_R(V_u)$, so his reservation match value is $\lambda^R(V_u) = w_R(V_u)$.

If the firm matches with a criminal, the problem it faces in the first stage depends on the productivity of the job. For matches with $\lambda > \lambda^{D2}(V_u)$ the expected value of the second stage is $\alpha \lambda + (1 - \alpha)w_C(V_u)$. If $\alpha > 0$ then the expected value of the second stage is greater than or equal to $w_C(V_u)$, implying that if the firm deters a worker in the second stage the first stage wage will also deter him. Therefore the firm’s first stage problem is:

$$\max_w \frac{\lambda - w}{r + \delta} \quad \text{s.t.} \quad w \geq \alpha \lambda + (1 - \alpha)w_C(V_u).$$

(16)
Again the firm offers the worker the expected value of the second stage \( w(\lambda, V_u) = \alpha \lambda + (1 - \alpha)w_C(V_u) \), and the worker accepts the job and does not commit crimes while employed.

If the firm does not choose to deter in the second stage, i.e. for productivities \( \lambda < \lambda^{D2}(V_u) \), it still offers a wage that maximizes profits subject to the worker accepting the offer. However, since the firm does not deter in the second stage, the expected value of the second stage might not be high enough to deter the worker from crime. Therefore the firm must choose whether or not to deter the worker in the first stage and faces the following problem:

\[
\max_w \left\{ \begin{array}{ll}
\max_w & \frac{\lambda - w}{r + \delta} \\
\text{s.t.} & w \geq w_C(V_u) & \& & w \geq \alpha \lambda + (1 - \alpha)w_R(V_u)
\end{array} \right\},
\]

Again the firm faces the trade off between a higher flow value for a shorter duration or a lower flow value for a longer duration.

First, one can show that if \((r + \delta)/\mu_e \pi \leq (1 - \alpha)/\alpha\) then \(w_C(V_u) \geq \alpha \lambda + (1 - \alpha)w_R(V_u)\) for all \(\lambda \geq \lambda^{D2}(V_u)\). That is, the expected value of the second stage is always less than the crime reservation wage. If this is the case, there exists a productivity, \(\lambda^{D1}(V_u)\), such that if \(\lambda < \lambda^{D1}(V_u)\) the firm will offer the expected value of the second stage and the worker will accept the offer, at which he continues to commit crimes. If \(\lambda \geq \lambda^{D1}(V_u)\) the firm will offer the crime reservation wage and the worker will accept the offer, since it is above the expected value of the second stage, and will not commit crimes while employed. The productivity above which firms deter workers from crime in the first stage is

\[
\lambda^{D1}(V_u) = \frac{(r + \delta + \mu_e \pi)w_C(V_u) - (1 - \alpha)(r + \delta)w_R(V_u)}{\mu_e \pi + \alpha(r + \delta)}.
\]

The full wage profile for criminals in this case is

\[
w(\lambda, V_u) = \begin{cases} 
\alpha \lambda + (1 - \alpha)w_R(V_u) & \text{if } \lambda^R(V_u) \leq \lambda < \lambda^{D1}(V_u) \\
w_C(V_u) & \text{if } \lambda^{D1}(V_u) \leq \lambda < \lambda^{D2}(V_u) \\
\alpha \lambda + (1 - \alpha)w_C(V_u) & \text{if } \lambda \geq \lambda^{D2}(V_u)
\end{cases}
\]

Figure 2 shows the wage profile for a worker of type \(V_u < V_u^*\). The worker gets the expected value of the second stage for all matches with productivity \(\lambda^R(V_u) < \lambda < \lambda^{D1}(V_u)\), the crime reservation wage for matches with productivity \(\lambda^{D1}(V_u) \leq \lambda < \lambda^{D2}(V_u)\) and the expected value of the second stage for matches with productivity \(\lambda \geq \lambda^{D2}(V_u)\). Proposition 2.2 gives a summary of the worker’s employment decisions and crime decisions for all match values.

**Proposition 2.2.** If \((r + \delta)/\mu_e \pi \leq (1 - \alpha)/\alpha\) then,

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*When \((r + \delta)/\mu_e \pi > (1 - \alpha)/\alpha\), the expected value of the second stage is greater than the crime reservation wage for some productivities. The intuition and mechanism is the same as when \((r + \delta)/\mu_e \pi \leq (1 - \alpha)/\alpha\). Therefore for the sake of brevity the wage profile for this case can be found in the appendix.*
Figure 2: Wage Profile for Workers with $V_u < V_u^*$

$\lambda^R(V_u)$ $\lambda^{D1}(V_u)$ $\lambda^{D2}(V_u)$

$w_R$ $w_C$ $w(\lambda, V_u)$

Figure 3: Reservation match values and decision rules

$\lambda$ $V_u$

$\lambda^R(V_u)$ $\lambda^{D1}(V_u)$ $\lambda^{D2}(V_u)$

$w = w_c$

No Crime and Employed

Crime and Employed

Crime and Unemployed

No Crime and Unemployed

a. If $\phi_u = 0$ then for all $\lambda \geq \lambda^R(V_u)$ the worker accepts the job and $\phi_e(w(\lambda, V_u)) = 0$.

b. If $\phi_u = 1$ then for all $\lambda^R(V_u) \leq \lambda \leq \lambda^{D1}(V_u)$ the worker accepts the job and $\phi_e(w(\lambda, V_u)) = 1$.

c. If $\phi_u = 1$ then for all $\lambda \geq \lambda^{D1}(V_u)$ the worker accepts the job and $\phi_e(w(\lambda, V_u)) = 0$. 
Figure 3 gives a graphical representation of Proposition 2.2. The slopes of the deterrence match values, $\lambda^{D1}(V_u)$ and $\lambda^{D2}(V_u)$, depend on parameter values and can be either negative or positive, however since the worker’s reservation wage is always less than his crime reservation wage one can show that $\lambda^{D1}(V_u) < \lambda^{D2}(V_u)$. The figure shows the case where both $\lambda^{D1}(V_u)$ and $\lambda^{D2}(V_u)$ are decreasing.

3 Steady State

To solve for the steady-state distribution of workers across states. First define for workers with flow utility $b$ the measure $u(b)$, unemployed; $e_c(b)$, employed and committing crimes; $e_{nc}(b)$, employed and not committing crimes; and $p(b)$, in prison. A worker with $b < b^*$ is a potential criminal and can flow between all for states, and a worker with $b \geq b^*$ will never commit a crime and can only flow between $u(b)$ and $e_{nc}(b)$.

For a potential criminal the flow from unemployment to employment and crime is equal to the probability that he receives a job offer times the probability that the productivity of the job is above his reservation match value and below the productivity at which a firm will deter him from crime:

$$\mu_j \left[ G\left(\lambda^C(V_u)\right) - G\left(\lambda^{D1}(V_u)\right) \right] \equiv \mu_j D(b). \quad (20)$$

The flow from unemployment to employment and not committing crimes is equal to the probability that he receives a job offer, times the probability that the productivity of the job is above the value at which a firm will deter him from crime:

$$\mu_j \left[ 1 - G(\lambda^{D1}(V_u)) \right] \equiv \mu_j A(b). \quad (21)$$

For a non-criminal, the flow from unemployment to employment is equal to the probability that he receives a job offer, times the probability that the productivity of the job is above his reservation match value:

$$\mu_j \left[ 1 - G(\lambda^R(V_u)) \right] \equiv \mu_j B(b). \quad (22)$$

Figure 4 shows the labor market flows for both types of workers.

A steady state is a set of measures $\{u(b), e_c(b), e_{nc}(b), p(b)\}$ for all $b$ such that the flows between states are equal. The solution to the steady state measures can be found in the Appendix A.2. The aggregate measure of unemployed criminals and aggregate measure of unemployed non-criminals are:

$$u_c = \int_{b^*}^{b^*} u(b) \, dF(b) \quad (23)$$
The aggregate measure of workers employed and committing crimes and the aggregate measure of workers employed and not committing crimes are:

\[
e_c = \int e_c(b) \, dF(b) \tag{25}
\]

\[
e_{nc} = \int e_{nc}(b) \, dF(b) \tag{26}
\]

The aggregate measure of workers in prison is:

\[
p = \int p(b) \, dF(b) \tag{27}
\]

The steady state unemployment rate is:

\[
U = \frac{u_c + u_{nc}}{1 - p} \tag{28}
\]

and the crime rate is:

\[
C = \frac{\mu_u u_c + \mu_c e_c}{1 - p}. \tag{29}
\]

Here I have use the non-institutionalized population as the denominator for the aggregate unemployment rate and the aggregate crime rate.

### 4 A Binding Minimum Wage

The minimum wage will change the interactions between the firm and the worker by acting as a constraint that each must consider when making a wage offer. I will assume the minimum wage,
$m$, is set exogenously by the government and that all matches are subject to this constraint. Since wages are the only transfer from the firm to the worker, the firm cannot alter any other forms of compensation to undo the effect of the minimum wage. A minimum wage is binding if it alters the outcome of the bargaining problem for at least one type of worker and at least one job productivity. The question of interest is then: how does the minimum wage change wages and in turn a worker’s decision to commit crimes?

4.1 Wages

As the minimum wage enters the bargaining problem as a constraint, firms and workers can never offer a wage below $m$ in the first stage or the second stage of the bargaining process. Under the constrained game, there exists a new value of unemployment for the worker which will depend on the minimum wage, I denote this value as $V_u(m)$. First, the lowest wage payed is $\lambda_R(V_u(m))$, thus any minimum wage for which there exists a $V_u(m)$ such that $m > \lambda_R(V_u(m))$ is binding. An immediate implication of a binding minimum wage is that matches with productivity less than $m$ are no longer feasible.

Starting with the simplest case, if the minimum wage is binding for a non-criminal the firm must offer at least $m$ in the second stage. The expected value of the second stage for the worker is $\alpha \lambda + (1 - \alpha)m$. In the first stage the firm offers a wage that maximizes profits subject to the worker accepting the offer. As before, it offers the value of the second stage and since $m > \lambda_R(V_u(m))$, wages increase for all productivities.

For a potential criminal, the solution to the constrained bargaining problem depends on whether or not the minimum wage is larger than the crime reservation wage. If $m < w_C(V_u(m))$ then only jobs with productivities at which the firm does not deter the worker in the second stage are constrained. Figure 5a shows the constrained second stage. Since the minimum wage is less than the worker’s crime reservation wage the firm must choose whether or not to deter the worker from crime in the second stage. The firm faces the follow problem in the second stage:

$$\arg\max_{(m, w_C)} \left\{ \frac{\lambda - m}{r + \delta + \mu e \pi}, \frac{\lambda - w_C(V_u)}{r + \delta} \right\}$$  \hspace{1cm} (30)

As with the unconstrained problem, for low productivity jobs the firm will choose to pay the minimum wage and have a shorter job duration. The match value that makes the firm indifferent between deterring and not deterring the worker in the second stage is now,

$$\lambda^{D2}(V_u; m) = \frac{(r + \delta + \mu e \pi)w_C(V_u) - (r + \delta)m}{\mu e \pi}$$  \hspace{1cm} (31)

above which the firm will choose to offer the crime reservation wage and receive a lower flow
Figure 5: Constrained Second Stage

Worker: \( w = \lambda \)

\( \lambda < w_C \) → crime

\( \lambda \geq w_C \) → no crime

\( \alpha \)

\( 1 - \alpha \)

\( \lambda \geq \lambda^{D_2}(m) \) \( w = w_C \) → no crime

Firm:

\( \lambda < \lambda^{D_2}(m) \) \( w = \max\{m,w_R\} \) → crime

(a) If \( m < w_C(V_u) \)

Worker: \( w = \lambda \)

\( \lambda < w_C \) → crime

\( \alpha \)

\( 1 - \alpha \)

Firm: \( w = \max\{m,w_C\} \) → no crime

(b) If \( m \geq w_C(V_u) \)

value for a longer duration. In the case that a firm and a worker match at a productivity less than \( \lambda^{D_2}(V_u;m) \), a binding minimum wage implies that the expected value of the second stage is now \( \alpha \lambda + (1 - \alpha) m \), and the firm faces the following first stage problem:

\[
\max \begin{cases} 
\max_{w} \frac{\lambda - w}{r + \delta}, & \text{s.t.} \ w \geq w_C(V_u(m)) \ \& \ w \geq \alpha \lambda + (1 - \alpha) m \\
\max_{w} \frac{\lambda - w}{r + \delta + \mu \pi}, & \text{s.t.} \ w \geq w_C(V_u(m)) \ \& \ w \geq \alpha \lambda + (1 - \alpha) m 
\end{cases}
\] (32)

The solution is similar to the unconstrained problem: the firm pays the expected value of the second stage for low productivities and there exists some productivity, \( \lambda^{D_1}(V_u;m) \), above which the firm deters the worker from crime by offering the crime reservation wage.

\[
\lambda^{D_1}(V_u;m) = \frac{(r + \delta + \mu \pi)w_C(V_u(m)) - (1 - \alpha)(r + \delta)m}{\mu \pi + \alpha(r + \delta)}.
\] (33)

Figure 6 shows the wage profile with the minimum wage imposed. The new wage offered by the
firm is

\[ \tilde{w}(\lambda, V_u; m) = \begin{cases} 
\alpha \lambda + (1 - \alpha)m & \text{if } m \leq \lambda < \lambda^{D1}(V_u; m) \\
w_C(V_u(m)) & \text{if } \lambda^{D1}(V_u; m) \leq \lambda < \lambda^{D2}(V_u; m) \\
\alpha \lambda + (1 - \alpha)w_C(V_u(m)) & \text{if } \lambda \geq \lambda^{D2}(V_u; m)
\end{cases} \]  

(34)

Figure 6 shows that a binding minimum wage compresses the wage distribution for a worker up to \( \lambda^{D2}(V_u) \). Proposition 4.1 summarizes the effects on the wage distribution. Part a.i. implies that a firm will deter the worker from crime for a larger range of productivities. With the minimum wage, the flow value of a filled job decreases since the expected value of the second stage increases. A reduction in the flow value of the job reduces the benefit to the firm from offering a wage lower than the worker’s crime reservation wage, and therefore the firm will choose to deter the worker from crime for more job productivities.

**Proposition 4.1.**

a. If \((r + \delta)/\mu_c \pi \leq (1 - \alpha)/\alpha\) and \(m < w_C(V_u(m))\) then

i. \( \frac{\partial \lambda^{D1}(V_u; m)}{\partial m} < 0 \)

ii. \( \frac{\partial \lambda^{D2}(V_u; m)}{\partial m} < 0 \)

iii. \( \left| \frac{\partial \lambda^{D1}(V_u; m)}{\partial m} \right| > \left| \frac{\partial \lambda^{D2}(V_u; m)}{\partial m} \right| \)

iv. \( \tilde{w}(\lambda, V_u; m) \geq w(\lambda, V_u) \) for all \( m \leq \lambda < \lambda^{D2}(V_u(m)) \)

b. If \( m \geq w_C(V_u) \) then \( \tilde{w}(\lambda, V_u; m) > w(\lambda, V_u) \) for all matches values that lead to a filled job.

If the minimum wage is above the crime reservation wage the firm has no decision to make
in the second stage since all wages it can offer will deter the worker from crime while employed. Figure 5b shows the constrained second stage for which the expected value is now $\alpha \lambda + (1 - \alpha)m$ for all feasible matches. If there is some positive probability that the worker gets to set the wage in the second stage, then the expected value of the second stage is strictly greater than the crime reservation wage. Therefore, the firm does not need to decide whether or not to deter the worker from crime in the first stage and faces the following problem in the first stage:

$$\max_w \frac{\lambda - w}{r + \delta} \text{ s.t. } w \geq \alpha \lambda + (1 - \alpha)m. \quad (35)$$

The firm maximizes profits by offering the expected value of the second stage which the worker will accept and forgo crimes while employed. The wage is simply $\tilde{w}(\lambda, V_u; m) = \alpha \lambda + (1 - \alpha)m$ for all $\lambda \geq m$. Part b. of Proposition 4.1 summarizes the effect of a minimum wage and Figure 7 shows the effect on the worker’s wage profile, which increases for all feasible matches.

**Figure 7:** Constrained Wage Profile for workers with $V_u < V_u^*$ when $m \geq w_C(V_u(m))$

### 4.2 Workers

Since match rates are exogenous the minimum wage will have no affect on the rate at which a worker matches with a firm. However, the minimum wage will change the range of productivities at which a worker will choose to commit crimes and therefore the rate at which he flows into and out of a criminal state. A potential criminal will commit crimes for all productivity less than $\lambda^{D1}(V_u(m))$; if the productivity is less than $\lambda^R(V_u(m))$ he will commit crimes at rate $\mu_u$ because he is unemployed and if the productivity is greater than $\lambda^R(V_u(m))$ but less than $\lambda^{D1}(V_u(m))$ he will commit crimes at rate $\mu_e$ because the wage offered by such a job is not high enough to deter
him from crime.

A binding minimum wage will have three effects on a worker’s propensity to commit crimes: a wage effect, an unemployment effect, and an indirect effect. The wage effect occurs when workers are deterred from committing crimes due to receiving a higher wage. The unemployment effect occurs when either: (1) a worker is displaced from jobs at which he would not committed crimes or (2) the rate at which he receives crime opportunities differs across states and he is displaced from any job. The indirect effect is driven by changes in the unemployment value, $V_u(m)$. A change in the minimum wage will affect a worker’s value of unemployment and therefore indirectly affect the flows between criminal and non-criminal states.

4.2.1 Wage Effect

Since all workers affected by the minimum wage experience an increase in wages for a range of productivities, the wage effect exists for all workers with a reservation wage less than the minimum wage. In Figure 8a this is all workers with a value of unemployment less than $V^1_u(m)$. For a worker with unemployment value less than $V^2_u(m)$ in Figure 8a, the minimum wage is higher than his crime reservation wage, and he will never commit crimes while employed. Therefore, he flows out of a criminal state if he receive a job offer with productivity greater than or equal to the minimum wage.

For a worker with unemployment value greater than $V^2_u(m)$ but less than $V^1_u(m)$ in Figure 8a, the crime reservation wage is above the minimum wage and he will continue to commit crimes while employed at some jobs. However, the range of productivities for which the commits crimes has decreased (part a.i. of Proposition 4.1.) as shown by the fact that $\lambda^{D1}(V_u)$ is greater than $\lambda^{D1}(V_u; m)$ in Figure 8a. All together, the shaded blue region of Figure 8a shows the matches that no longer lead to crime while employed due to an increase in wages. In Figure 8b the minimum wage is above all workers’ crime reservation wage and therefore no workers commit crimes while employed. Again, the wage effect corresponds to the blue shaded region, as these are matches that which a worker would have committed crimes before the minimum wage.

4.2.2 Unemployment Effect

There are two channels through which a worker will change the amount of crimes he commits due to unemployment. First, if the rate at which he receive crime opportunities differs across states. Specifically, if he receives more crime opportunities while unemployed, $\mu_e < \mu_u$, then when he is displaced from a job, he will commit more crimes. In Figure 8a this corresponds to the red shaded region, these are productivities at which workers would have accepted a job and committed less crime in the absence of the minimum wage.

Second, if a worker is displaced from a job at which he would not have committed a crime, then the minimum wage will increases the amount of crimes he commits. This occurs when the
Figure 8: Minimum Wage Effects on Matches

(a) Low Minimum Wage

(b) High Minimum Wage
minimum wage is above the productivity at which the firm would have chosen to deterred the worker from crime. In Figure 8b these are matches with productivity greater than $\mu D^1(V_u)$ and less than $m$. Only workers with unemployment values greater than $V_u^3(m)$ and less than $V_u^*$ are displaced from jobs at which they would not have committed crimes. The red shaded region of Figures 8a and 8b shows the matches that lead to an increase in crime through both channels.

4.2.3 Indirect Effect

The indirect effect of the minimum wage on a worker’s crime decisions is driven by changes in his value of unemployment. Take, for example, a worker with a value of unemployment such that after the minimum wage becomes binding is greater than $V_u^2(m)$ and less than $V_u^1(m)$. From Figure 8a it is clear that his value of unemployment has changed for two reasons: (1) some matches are no longer feasible and (2) some matches experience a wage increase. The fact that some matches no longer lead to filled jobs decreases his value of unemployment. On the other hand, the wage increase for some matches increase his value of unemployment. Therefore, the overall effect of a minimum wage on the worker’s value of unemployment is ambiguous, depends on the size of the minimum wage, and varies across workers.

4.3 Equilibrium Crime Rate

The equilibrium crime rate given in equation (29) depends on three aggregate states, $u_c, e_c$, and $p$ given in equations (23), (25), and (27), and the rates at which workers receive crime opportunities while employed, $\mu_e$, and unemployed, $\mu_u$. When the minimum wage changes, the aggregate crime rate is affected by changes in workers’ decision to commit crimes and accept jobs. From Figure 8a it is clear that workers are affected differently by the minimum wage; some workers are deterred from crime for more match productivities and are displaced from more jobs. Therefore, analytical results for a change in the crime rate depend on the distribution of the unemployment utilities, the distribution of match productivities and the size of the minimum wage.

5 Calibration

The unit of time is one month and the rate of time preference is $r = 0.0101$. The probability a worker gets to set the wage in the second round, $\alpha$, acts as the worker’s bargaining power which I set equal to $\alpha = 0.4$ as estimated by Flinn (2006). I calibrate the labor market to 16 to 24 year olds in 1998. The federal minimum wage in 1998 was $5.15 and the average weekly hours worked by 16 to 24 year old workers in the Current Population Survey-Outgoing Rotation Groups for 1998 was 29.59 hours, so the average weekly wage of workers employed at the minimum wage
is $m = 609.55$. I use the monthly separation rate for 16 to 24 year olds estimated by Gorry (2013), $\delta = 0.012$.

The crimes considered are Type 1 property crimes defined by the Federal Bureau of Investigation (FBI) as larceny, burglary and motor vehicle theft. The probability of being caught is derived from the clearance rate and the incarceration rate of these crimes as reported by the FBI’s Uniform Crime Reports (UCR). The UCR defines the clearance rate as the ratio of arrests to crimes reported and the incarceration rate as the ratio of convictions to arrests. In 1998 the clearance rate for property crimes was 17.5% and the incarceration rate for property crimes was 65%, implying the probability a worker goes to prison is $\pi = 0.175 \times 0.65 = 0.114$. The prison release rate is calibrated to target the average time in prison for property crimes as reported by the National Corrections Reporting Program. In 1998, the average time in prison for property crimes was 20 months implying $\gamma = 1/20 = 0.05$. I set prison utility, $z$, equal to zero. The UCR reports that the average loss per property crime in 1998 was $1,407 and the property crime rate per 100,000 individuals was 4.1%, implying the gain from crime is $g = 1,407$ and the expected loss of being victimized is $\chi L = 0.041 \times 1407 = 56.97$.

<table>
<thead>
<tr>
<th>Table 1: Summary of Empirical Moments</th>
</tr>
</thead>
<tbody>
<tr>
<td>Moment</td>
</tr>
<tr>
<td>Uniform Crime reports</td>
</tr>
<tr>
<td>Crime rate</td>
</tr>
<tr>
<td>Gorry 2013</td>
</tr>
<tr>
<td>Job finding rate</td>
</tr>
<tr>
<td>CPS- Monthly</td>
</tr>
<tr>
<td>Unemployment rate</td>
</tr>
<tr>
<td>obs.</td>
</tr>
<tr>
<td>CPS - ORG</td>
</tr>
<tr>
<td>Average Weekly Hours</td>
</tr>
<tr>
<td>$P(5.15 &lt; w^h &lt; 5.25)$</td>
</tr>
<tr>
<td>10/50 percentile</td>
</tr>
<tr>
<td>50/90 percentile</td>
</tr>
<tr>
<td>Min. to Median</td>
</tr>
<tr>
<td>obs.</td>
</tr>
</tbody>
</table>

The remaining set of parameters ($\mu_e, \mu_a, \mu_j, \alpha, \beta, \mu_b, \sigma_b$) are calibrated jointly to match a set of empirical moments. Since the flow utility of prison is set to zero, I choose the log normal distribution with positive real support for the flow utility of unemployment $F(b) \sim \ln N(\mu_b, \sigma_b)$, to ensure that the instantaneous utility of unemployment is no worse than the instantaneous utility of prison. I choose a Weibull distribution for match productivities $G(\lambda) \sim \ln N(\mu_\lambda, \sigma_\lambda)$. The empirical moments I aim to match are: the crime rate, the unemployment rate, the job finding rate, the probability of having an hourly wage within 10 cents of the minimum wage, the 10/50 ratio
of wage percentiles, the 50/90 ratio of wage percentiles and the minimum wage to median wage ratio. The moments are generated from the 1998 CPS basic monthly files and Outgoing Rotation Group for 16 to 24 year old workers and the 1998 UCR. The job finding rate for 16 to 24 year olds is estimated by Gorry (2013). Table 1 summarizes the empirical moments. Table 2 summarizes all parameter values and Table 3 gives the empirical moments and model generated moments.

<table>
<thead>
<tr>
<th>Parameter</th>
<th>Value</th>
<th>Description</th>
</tr>
</thead>
<tbody>
<tr>
<td>r</td>
<td>0.0101</td>
<td>real interest rate</td>
</tr>
<tr>
<td>α</td>
<td>0.4</td>
<td>bargaining power of workers</td>
</tr>
<tr>
<td>δ</td>
<td>0.012</td>
<td>separation rate</td>
</tr>
<tr>
<td>χL</td>
<td>$56.97</td>
<td>expected loss from crime</td>
</tr>
<tr>
<td>z</td>
<td>0</td>
<td>prison utility</td>
</tr>
<tr>
<td>γ</td>
<td>0.05</td>
<td>prison release rate</td>
</tr>
<tr>
<td>π</td>
<td>0.114</td>
<td>probability of getting caught</td>
</tr>
<tr>
<td>m</td>
<td>$609.55</td>
<td>minimum wage job</td>
</tr>
<tr>
<td>g</td>
<td>$1,407</td>
<td>gain from crime</td>
</tr>
<tr>
<td>µu</td>
<td>0.0596</td>
<td>arrival rate of crime opportunities while unemployed</td>
</tr>
<tr>
<td>µe</td>
<td>0.0473</td>
<td>arrival rate of crime opportunities while employed</td>
</tr>
<tr>
<td>µj</td>
<td>1.3632</td>
<td>arrival rate of jobs opportunities</td>
</tr>
<tr>
<td>µλ</td>
<td>6.2029</td>
<td>mean of productivity distribution</td>
</tr>
<tr>
<td>σλ</td>
<td>0.2278</td>
<td>s.d. of productivity distribution</td>
</tr>
<tr>
<td>µb</td>
<td>5.8536</td>
<td>mean of flow unemployment utility</td>
</tr>
<tr>
<td>σb</td>
<td>0.4828</td>
<td>s.d. of flow unemployment utility</td>
</tr>
</tbody>
</table>

Table 3: Moments Matched

<table>
<thead>
<tr>
<th>Moment</th>
<th>Empirical</th>
<th>Model</th>
<th>Data</th>
</tr>
</thead>
<tbody>
<tr>
<td>Crime rate</td>
<td>4.1</td>
<td>3.79</td>
<td>Uniform Crime Reports</td>
</tr>
<tr>
<td>Unemployment rate</td>
<td>10.25%</td>
<td>13.84%</td>
<td>CPS-Monthly, 16-24 yrs</td>
</tr>
<tr>
<td>Job finding rate</td>
<td>0.149</td>
<td>0.1529</td>
<td>Gorry 2013</td>
</tr>
<tr>
<td>P( $5.15 &lt; w^h &lt; $5.25)</td>
<td>0.169</td>
<td>0.1527</td>
<td>CPS-ORG, 16-24 yrs</td>
</tr>
<tr>
<td>10/50 percentiles</td>
<td>0.7556</td>
<td>0.9912</td>
<td>CPS-ORG, 16-24 yrs</td>
</tr>
<tr>
<td>50/90 percentiles</td>
<td>0.5192</td>
<td>0.8620</td>
<td>CPS-ORG, 16-24 yrs</td>
</tr>
<tr>
<td>Min to Median</td>
<td>0.7630</td>
<td>0.9442</td>
<td>CPS-ORG, 16-24 yrs</td>
</tr>
</tbody>
</table>

Note: $w^h$ is the hourly wage and $w$ is the monthly wage.

The model’s crime rate is lower than its empirical counterpart and the unemployment rate is slightly higher. The model matches the job finding rate well. The wage distribution in the model fits well at the lower part of the distribution, with the probability of working within 10 cents of the minimum wage close to its empirical counterpart. However, the remaining three wage ratios are higher than the empirical counterparts. The calibrated crime arrival rates are 0.0596 while unemployed and 0.0473 while employed, implying a monthly probability of finding a crime opportunity of 0.058 while unemployed and 0.046 while employed. The job offer rate is 1.3632,
implying a monthly probability of receiving a job offer of 0.74. The calibrated mean and variance of the productivity distribution imply a mean monthly job productivity of $507.17 and a standard deviation of job productivities of $117.05. The calibrated mean and standard deviation of the unemployment utility distribution imply a mean monthly unemployment utility of $391.56 and standard deviation of $200.62.

5.1 Model Generated Elasticities

Since the effect of the minimum wage on the crime rate is driven through changes in the labor market, I test the model in two dimensions: the response of workers’ crime decisions with respect to changes in the labor market and changes in the labor market with respect to changes in the minimum wage. Specifically, I generate two data sets through simulation of the model, similar to those used by empirical researchers, and estimate the elasticity of crime with respect to unemployment and wages and the elasticity of employment and earnings with respect to the minimum wage. I compare the estimated elasticities that the calibrated model delivers to those found in the empirical literature to validate the relationship between the labor market and criminal propensity and the minimum wage and the labor market. I generate both data sets based on variation in the real minimum wage observed across states from 1990 to 2011. Table 4 summaries the variation in the minimum wage across the sample, where the real binding minimum wage is the maximum of the state and federal minimum wage in 1998 dollars.

<table>
<thead>
<tr>
<th></th>
<th>Mean</th>
<th>St. dev.</th>
<th>Min</th>
<th>Max</th>
</tr>
</thead>
<tbody>
<tr>
<td>Federal Min. Wage</td>
<td>5.24</td>
<td>0.99</td>
<td>3.80</td>
<td>7.25</td>
</tr>
<tr>
<td>State Min. Wage</td>
<td>5.30</td>
<td>1.31</td>
<td>1.6</td>
<td>8.67</td>
</tr>
<tr>
<td>Binding Min. Wage</td>
<td>5.46</td>
<td>1.15</td>
<td>3.80</td>
<td>8.67</td>
</tr>
<tr>
<td>Real Binding Min. W</td>
<td>5.09</td>
<td>0.55</td>
<td>4.34</td>
<td>6.83</td>
</tr>
</tbody>
</table>

The first data set I generate is a panel of 1,000 individuals for every unique realization of the real binding minimum wage; this gives a total sample size of 204,000. For each individual I choose \( b \sim F(\mu_b, \sigma_b) \) and simulate the probability of employment, probability of committing a crime, probability of being in prison, probability of working within 10 cents of the minimum wage, and expected wage using the calibrated parameters. For every value of the minimum wage the aggregate unemployment rate is the average across individuals simulated unemployment probabilities. Panel A of Table 5 gives the summary statistics for the generated sample.

I use the generated sample to estimate the elasticity of workers’ crime decisions with respect to unemployment and wages and compare the model generated elasticities to those found in the
empirical literature. Specifically, I run the following regression:

\[
\ln\text{crime}_{i,m} = \beta_0 + \beta_1 U_m + \beta_2 \ln \mathbb{E}[w]_{i,m} + \beta_3 \ln \text{Min}_m + \varepsilon_{i,m}
\]

where \(\ln\text{crime}_{i,m}\) is the natural log of the simulated probability of committing a crime for worker \(i\) for minimum wage \(m\), \(U_m\) is the aggregate unemployment rate for minimum wage \(m\), \(\ln \mathbb{E}[w]_{i,m}\) is the expected wage for worker \(i\) at minimum wage \(m\), \(\ln \text{Min}_m\) is the log of the hourly minimum wage and \(\varepsilon_{i,m}\) is statistical noise generated in the simulation through the random draw of a crime opportunity and match productivity. Panel B of Table 5 gives the regression results.

### Table 5: Simulated Individual Analysis

<table>
<thead>
<tr>
<th>Panel A: Simulated Data Summary Statistics</th>
</tr>
</thead>
<tbody>
<tr>
<td>Mean</td>
</tr>
<tr>
<td>------</td>
</tr>
<tr>
<td>Crime</td>
</tr>
<tr>
<td>Aggregate Unemp.</td>
</tr>
<tr>
<td>Prison</td>
</tr>
<tr>
<td>(\mathbb{E}[w])</td>
</tr>
</tbody>
</table>

<table>
<thead>
<tr>
<th>Panel B: Regression Results</th>
</tr>
</thead>
<tbody>
<tr>
<td>ln Crime (1)</td>
</tr>
<tr>
<td>Agg Unemployment</td>
</tr>
<tr>
<td>(0.0401)</td>
</tr>
<tr>
<td>ln Expected Wage</td>
</tr>
<tr>
<td>(0.015)</td>
</tr>
<tr>
<td>N</td>
</tr>
</tbody>
</table>

*Note: The regression includes only observations of individuals that choose to commit crimes. Robust standard errors are given in parenthesis.*

Several empirical studies have estimated the elasticity of crime with respect to unemployment and wages and find a semi-elasticity of crime with respect to unemployment, \(\beta_1\), of 1.2 to 2, and an elasticity of crime with respect to wages, \(\beta_2\), of -0.5 to -2 (Gould et al., 2002; Mocan and Unel, 2011; Schnepel, 2014). The model generated elasticity of crime with respect to wages, \(-2.12\), is on the high side of the empirically estimated range. The model captures the elasticity of crime with respect to unemployment well since the generated elasticity, 1.85, is within the range of empirically estimated elasticities.

To estimate the response of the labor market to changes in the minimum wage within the model, I generate is a cross section of aggregate employment probabilities, and expected wages for every realization of the real binding minimum wage within the sample, thus this generated data has a
sample size of 1,122. Using the aggregate sample, I run the following two regressions:

\[
\begin{align*}
(1) \quad \ln \text{Emp}_m &= \xi_0 + \xi_1 \ln \text{MinWage}_m + \epsilon_m \\
(2) \quad \ln \text{Wage}_m &= \psi_0 + \psi_1 \ln \text{MinWage}_m + \epsilon_m
\end{align*}
\]

where \(\ln \text{Emp}_m\) is the natural log of the average employment probability for minimum wage \(m\), \(\ln \text{Wage}_m\) is the natural log of average expected wage for minimum wage \(m\), and \(\epsilon_m\) is statistical noise generated from the random draws from the productivity distribution. Panel A of Table 6 gives summary statistics for the aggregate data and Panel B of Table 6 gives the regression results.

Table 6: Simulated Aggregates Analysis

<table>
<thead>
<tr>
<th>Panel A: Simulated Data Summary Statistics</th>
</tr>
</thead>
<tbody>
<tr>
<td></td>
</tr>
<tr>
<td><strong>Mean</strong></td>
</tr>
<tr>
<td>----------------</td>
</tr>
<tr>
<td>Average Unemp.</td>
</tr>
<tr>
<td>Average Emp.</td>
</tr>
<tr>
<td>Average (\mathbb{E}[w])</td>
</tr>
</tbody>
</table>

<table>
<thead>
<tr>
<th>Panel B: Regression Results</th>
</tr>
</thead>
<tbody>
<tr>
<td></td>
</tr>
<tr>
<td><strong>Dependent Variable</strong></td>
</tr>
<tr>
<td><strong>ln Emp</strong></td>
</tr>
<tr>
<td><strong>ln Wage</strong></td>
</tr>
<tr>
<td>(\ln \text{MinWage})</td>
</tr>
<tr>
<td>(-0.183^{***})</td>
</tr>
<tr>
<td>(0.015)</td>
</tr>
<tr>
<td>(0.406^{***})</td>
</tr>
<tr>
<td>(0.008)</td>
</tr>
<tr>
<td>(N)</td>
</tr>
<tr>
<td>1104</td>
</tr>
<tr>
<td>1104</td>
</tr>
</tbody>
</table>

*Note: Robust standard errors are given in parenthesis.*

The literature on employment effects of the minimum wage is lengthy and mixed, see Neumark and Wascher (2007) for a review. Dube et al. (2010) study employment effects on restaurant workers and find no significant effect. The employment effects from the minimum wage on teen employment is mixed with Allegretto et al. (2010) finding no significant employment effects and Neumark et al. (2014) finding significant employment effects on teens with estimated elasticities around \(-0.3\). The estimated elasticity of employment with respect to the minimum wage within the calibrated model \(-0.183\), lower than the upper bound the empirical literature has estimated but still significant. The empirically estimated elasticity of wages with respect to the minimum wage is between 0.15 and 0.22 (Dube et al., 2010; Allegretto et al., 2010). The model delivers an estimated elasticity of 0.41, higher than the empirical literature.

6 Increasing the Minimum Wage

Although the model is calibrated to match labor market condition and the crime rate for a single minimum wage the previous section shows that it delivers similar elasticities to those estimated in
the empirical literature. The estimates from the empirical literature measure average elasticities and could be used in a back-of-the-envelope calculation to estimate the effect a change in the minimum wage will have on the average worker. However, in this section I show that the relationship between the minimum wage and the aggregate crime rate is non-monotonic and that a change in the minimum wage differentially affects workers across the income distribution.

Using the calibrated parameters, I solve the model for minimum wages between $5 and $9. Figure 9 shows the change in the aggregate crime rate, equation (29), over the range of minimum wages. The figure shows that aggregate crime rate decreases with minimum wages between $5 and $6.20, implying that the wage effect outweighs the unemployment effect over this range. With minimum wages above $6.20, the crime rate begins to increase as the unemployment effect begins to dominate. When the minimum wage increases above $7.50, the crime rate in the model is larger than the crime rate used to calibrate the model, 3.8%. This implies that if the minimum wage were to increase by more than 45%, the aggregate crime rate would increase.

Figure 9: Crime Rate

The fact that the aggregate crime rate responds more to changes in wages than to changes in unemployment for relatively small increases in the minimum wage stems from the fact that employment decreases only marginally. This finding is similar to Imrohoroglu et al. (2004) who find that rising average incomes from 1980 to 1996 alone could account for 20% of the decrease in crime observed over the period, whereas the small increases in youth unemployment over the same period had no affect on the aggregate crime rate. The non-monotonicity of the crime rate is interesting and driven by a similar mechanism as in Engelhardt et al. (2008), who show that the crime rate is non-monotonic in the worker’s bargaining power. For low minimum wages, as for low bargaining powers, a worker has a larger incentive to commit crimes because his labor market outcomes are low in terms of wages. As the minimum wage increases, or bargaining power increases, the worker’s incentive to commit crimes decreases because his labor market outcome in terms of wages increase. However, once the minimum wage increase above a certain point, the probability he finds a feasible match is too low and his labor market outcomes decrease because of high unemployment.
which increases his incentive to commit crime. Similarly in Engelhardt et al. (2008) a high bargaining power for the worker decreases the firms incentive to open vacancies, decrease the workers labor market outcomes through high unemployment and increases his incentive to commit crimes. As Flinn (2006) points out, one can think of the minimum wage as a policy tool that increases the worker’s bargaining power.

Although the crime rate is decreasing for small increases in the minimum wage, workers across the wage distribution are affected differentially. Figure 10 shows the changes in the crime rate for minimum wages at which the wage effect dominates the unemployment effect, for a worker at the 10th, 75th and 90th percentile of the unemployment utility distribution. A worker at the 10th percentile of the unemployment utility distribution has an expected wage of $641 when the minimum wage is $5.15, a worker at the 75th percentile has an expected wage of $684, and a worker at the 90th percentile of the unemployment utility distribution has an expected wage of $751. Figure 10 shows that the minimum wage will have differential affects across the wage distribution. An increase in the minimum wage will have no effect on high wage workers propensity to commit crimes while from some lower wage workers it will decrease in their propensity to commit crimes while for others it will increase their propensity.

Figure 10: Crime Rate by Percentile of Unemployment Utility

![Figure 10: Crime Rate by Percentile of Unemployment Utility](image)

The fact that the minimum wage decreases a low wage worker’s probability of committing crimes by more than a high wage worker and that the wage effect dominates for small changes in the minimum wage is consistent with findings in Imrohoroglu et al. (2000) who show that there is a positive correlation between income inequality and crime and in Autor et al. (2010) who show that the minimum wage reduces inequality in the lower tail (50/10) of the wage distribution. Although both low and high wage workers experience increases in wages as the minimum wage increase, the increase is larger for low wages workers, compressing the aggregate wage distribution from the
bottom and decreasing inequality across workers.

6.1 Empirical Evidence

Figure 9 shows that the model predicts the minimum wage to have a non-monotonic effect on the crime rate. In this section I use county level crime data from 1980 to 2012 to test this prediction. The county level crime data come from the FBI’s UCR and the data include the number of Type 1 Property Crimes (burglary, larceny, and motor vehicle theft) and the number of Type 1 Violent Crimes (murder, robbery, forcible rape and aggravated assault) reported to the police. I use county level demographic data from the Survey of Epidemiology and End Results (SEER) that provide estimates of the total population, and estimates of the population by 19 age groups, sex and 3 race groups - white, black and other. I use the estimates of the total population by county to calculate the property and violent crime rates per 100,000 individuals for each county in each year. Using the estimates for population by age\textsuperscript{9}, sex and race, I create demographic shares for each group at the county-year level. I find the real median wage for 16 to 24 year olds in each state using the CPS Outgoing Rotation Group and for each county I find the real binding minimum (RBM) wage as the maximum of the federal and state minimum wage in 1998 dollars.

I estimate the correlation between the RBM and the property crime rate using three different empirical specifications: (1) linear, (2) quadratic and (3) non-parametric. For the linear relationship between the RBM and the property crime rate I estimate,

\[ \text{pc-rate}_{ct} = \beta_0 + \beta_1 \text{RBM}_{st} + \beta_2 \text{RBM} \times \text{MW}_{st} + \beta_3 \text{MW}_{st} + \beta_4 X_{ct} + t_s + \gamma_c + \epsilon_{ct} \]  \hspace{1cm} (36)

where \( \text{pc-rate}_{ct} \) is property crime rate per 100,000 individuals in county \( c \) in year \( t \), \( \text{MW} \) is the median wage in state \( s \) at time \( t \), \( t_s \) are state specific linear time trends, \( \gamma_c \) are county fixed effects and \( X_{ct} \) are the county level demographic variables and the county level violent crime rate in county \( c \) in year \( t \). I include the county level violent crime rate to proxy for the counties overall propensity for crime, which may be time varying and therefore not controlled for by the county fixed effects. I include the interaction term between the RBM and the median wage within each state to allow the minimum wage to have a different effect on crime depending on how many workers it binds for. For example, in some states a $5 minimum wage may bind for more people and thus have a larger effect on the crime rate.

For the quadratic relationship I estimate

\[ \text{pc-rate}_{ct} = \beta_0 + \beta_1 \text{RBM}_{st} + \beta_2 \text{RBM} \times \text{MW}_{st} + \beta_3 \text{RBM}^2 + \beta_4 \text{RBM}^2 \times \text{MW}_{st} + \beta_5 X_{ct} + t_s + \gamma_c + \epsilon_{ct} \]  \hspace{1cm} (37)

\textsuperscript{9}I aggregate the age estimates to 6 groups : 0 to 14, 15 to 24, 25 to 39, 40 to 59, 60 to 79 and 80 plus
Table 7: Mean Real Binding Minimum Wage by Quantile

<table>
<thead>
<tr>
<th>Quantile</th>
<th>Mean RBM</th>
<th>Quantile</th>
<th>Mean RBM</th>
</tr>
</thead>
<tbody>
<tr>
<td>1</td>
<td>4.34</td>
<td>6</td>
<td>5.97</td>
</tr>
<tr>
<td>2</td>
<td>4.58</td>
<td>7</td>
<td>5.20</td>
</tr>
<tr>
<td>3</td>
<td>4.71</td>
<td>8</td>
<td>5.35</td>
</tr>
<tr>
<td>4</td>
<td>4.87</td>
<td>9</td>
<td>5.55</td>
</tr>
<tr>
<td>5</td>
<td>4.96</td>
<td>10</td>
<td>6.04</td>
</tr>
</tbody>
</table>

where the covariates are the same as above.

For the non-parametric relationship I include dummy variables which take on a value of 1 if the RBM falls into one of 10 bins and 0 otherwise. The 10 bins are the quantiles of the observed RBM wage distribution across counties from 1980 to 2012. Table 7 shows mean RBM within each quantile. Using these dummy variables I estimate

\[ \text{pc-rate}_{ct} = \beta_0 + \alpha_1 1\{\text{RBM}_{st} \in (q(j-1),q(j)]\} + \alpha_2 1\{\text{RBM}_{st} \in (q(j-1),q(j)]\} \times \text{MW}_{st} \]

\[ + \beta_1 \text{MW}_{st} + \beta_2 X_{ct} + t_s + \gamma_c + \varepsilon_{ct} \]  

(38)

where \( q(j) \) is the \( j^{th} \) quantile of the RBM, \( j \in \{1,2,\ldots,10\} \), \( 1 \) is the indicator function and the other covariates are the same as above. Again the dummies are interacted with the median wage within each state.

Table 8 shows the estimated coefficient on RBM from the linear and quadratic regression. The coefficients from the linear model in column (1) imply that an increase in the minimum wage will decrease the crime rate and less so in areas that have higher median wages; however the coefficients are not significant. Adding a quadratic term to the regression as in column (2) makes all coefficients significant at the 5% level. Figure 11 plots the change in the crime rate from increasing the RBM from $2.50 to $9 at the average median wage within the sample, $6.65. The grey shaded region shows the 95% confidence interval. Figure 11 shows a non-monotonic relationship between the real minimum wage and the property crime rate similar to that delivered by the model.

Table 9 gives the estimated coefficients on the dummy variables of the non-parametric specification, Equation 38, and Figure 12 plots the predicted effect on the crime rate for the average median wage at the mean RBM of each quartile. The grey shaded area gives the 95% confidence interval of the estimated coefficients. Again the figure shows that increasing the minimum wage from the first quantile to the second has a negative effect on crime, while increasing more, to higher quantiles may have a positive effect on crime. I argue that the results from the quadratic relationship and the non-parametric approach provide empirical evidence for the existence of the non-monotonic relationship between the minimum wage and the crime rate - as predicted by the model.

In 2015 the US real median wage for 16 to 24 year olds in 1998 dollars was $6.88 and the fed-
Table 8: Regression Results: Linear & Quadratic

<table>
<thead>
<tr>
<th></th>
<th>(1)</th>
<th>(2)</th>
</tr>
</thead>
<tbody>
<tr>
<td></td>
<td>Linear</td>
<td>Non-Linear</td>
</tr>
<tr>
<td>RBM</td>
<td>-44.44</td>
<td>-12474.0**</td>
</tr>
<tr>
<td></td>
<td>(442.2)</td>
<td>(4669.8)</td>
</tr>
<tr>
<td>RBM×MW</td>
<td>13.82</td>
<td>1436.3**</td>
</tr>
<tr>
<td></td>
<td>(65.82)</td>
<td>(651.8)</td>
</tr>
<tr>
<td>RBM²</td>
<td></td>
<td>1301.3**</td>
</tr>
<tr>
<td></td>
<td></td>
<td>(1981.7)</td>
</tr>
<tr>
<td>RBM²×MW</td>
<td></td>
<td>-150.8**</td>
</tr>
<tr>
<td></td>
<td></td>
<td>(67.85)</td>
</tr>
<tr>
<td>Time trends</td>
<td>x</td>
<td>x</td>
</tr>
<tr>
<td>County FE</td>
<td>x</td>
<td>x</td>
</tr>
<tr>
<td>Demographics</td>
<td>x</td>
<td>x</td>
</tr>
<tr>
<td>Mean pc-rate</td>
<td>2348.4</td>
<td>2348.4</td>
</tr>
<tr>
<td>N</td>
<td>94,919</td>
<td>94,919</td>
</tr>
</tbody>
</table>

Standard errors clustered at the state level in parentheses

* p < 0.10, ** p < 0.05, *** p < 0.01

Figure 11: Non-Linear Model

The federal minimum wage in 1998 dollars was $4.99. Both the estimated coefficients from the quadratic regression and the non-parametric regression imply that given a median wage of $6.88 an approximately $5 per hour minimum wage minimizes the crime rate. That is, the model and empirical results imply that any increase in the federal minimum wage may increase the property crime rate.
Table 9: Regression Results: Non-Parametric

<table>
<thead>
<tr>
<th>Quantile (j)</th>
<th>$\hat{a}_j^1$</th>
<th>$\hat{a}_j^2$</th>
</tr>
</thead>
<tbody>
<tr>
<td>2</td>
<td>-1042.9****</td>
<td>144.0**</td>
</tr>
<tr>
<td></td>
<td>(379.1)</td>
<td>(56.37)</td>
</tr>
<tr>
<td>3</td>
<td>-1412.6****</td>
<td>187.1****</td>
</tr>
<tr>
<td></td>
<td>(341.8)</td>
<td>(48.51)</td>
</tr>
<tr>
<td>4</td>
<td>-667.3</td>
<td>83.20</td>
</tr>
<tr>
<td></td>
<td>(476.2)</td>
<td>(70.80)</td>
</tr>
<tr>
<td>5</td>
<td>-924.1**</td>
<td>117.9*</td>
</tr>
<tr>
<td></td>
<td>(425.6)</td>
<td>(62.53)</td>
</tr>
<tr>
<td>6</td>
<td>-823.2**</td>
<td>97.69*</td>
</tr>
<tr>
<td></td>
<td>(340.5)</td>
<td>(49.62)</td>
</tr>
<tr>
<td>7</td>
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<td>5.993</td>
</tr>
<tr>
<td></td>
<td>(369.3)</td>
<td>(56.07)</td>
</tr>
<tr>
<td>8</td>
<td>184.9</td>
<td>-27.12</td>
</tr>
<tr>
<td></td>
<td>(592.1)</td>
<td>(84.97)</td>
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<tr>
<td>9</td>
<td>178.7</td>
<td>-25.21</td>
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<td></td>
<td>(505.4)</td>
<td>(77.23)</td>
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<tr>
<td>10</td>
<td>618.7</td>
<td>-65.85</td>
</tr>
<tr>
<td></td>
<td>(1035.6)</td>
<td>(140.9)</td>
</tr>
</tbody>
</table>

| N            | 94919        | 94919        |

Standard errors clustered at the state level in parentheses
* p < 0.10, ** p < 0.05, *** p < 0.01

7 Conclusion

The minimum wage has been discussed extensively around the country, leading many states and cities to increase their minimum wages by amounts that we have not seen in the past. The increases are targeted to improve labor market condition primarily for young and unskilled workers; however, increasing the minimum wage may have unforeseen effects on these workers’ decisions to commit crimes. I have shown that the relationship between the aggregate crime rate and the minimum wage is non-monotonic due to two opposing effects: the wage effect and the unemployment effect. Which effect dominates, and ultimately how the aggregate crime rate will change depends on how much the minimum wage increases. In the calibrated model small increases in the minimum wage, up to 20%, the aggregate crime rate will decrease. However, with real increases above 45% the minimum wage may have the unintended effect of increasing the crime rate.

The goal of this paper is to establish the relationship between the minimum wage and the crime rate, and quantify the effects. The model identifies a non-monotonic effect of the minimum wage on crime rate and the empirical analysis corroborates this finding. The empirical findings also
suggest that any increase in the federal minimum wage may have lead to increase in the crime rate, as the current federal level may be the crime minimizing value. Both the Democratic and Republican party have recently endorsed a higher federal minimum wage that is well above a 45% real increase. It is my hope that policy makers use the ideas presented in this paper and consider the consequences on workers’ decisions to commit crimes when choosing to implement policies that change labor market conditions – specifically the minimum wage.

References


A Appendix

A.1 Wage profile if \((r + \delta)/\mu_e \pi > (1 - \alpha)/\alpha\)

If \((r + \delta)/\mu_e \pi > (1 - \alpha)/\alpha\) then there exist some productivities for which the expected value of the second stage is greater than the crime reservation wage, and for these productivities the worker will not commit crimes when offered the expected value of the second stage. Let \(\lambda^E(V_u)\) be the productivity for which the expected value of the second stage is exactly equal to the crime reservation wage,

\[
\lambda^E(V_u) = \frac{w_C(V_u) - (1 - \alpha) w_R(V_u)}{\alpha}. \tag{39}
\]

With some algebra one can show that \(\lambda^E(V_u) < \lambda^{D1}(V_u)\). The firm still faces the same problem as before, should it offer the value of the second stage or the crime reservation wage? The answer is similar to the first case. If the productivity is below \(\lambda^{D1}(V_u)\), it chooses not to deter and offers the value of the second stage. If the productivity is \(\lambda^{D1}(V_u) \leq \lambda < \lambda^E(V_u)\), it maximizes profits by offering the worker his crime reservation wage, which in turn he will accept and forgo crimes while employed. If the productivity of the job is greater than \(\lambda^E(V_u)\) the expected value of the second stage is greater than the crime reservation wage and the firm must offer the expected value, which again the worker will accept and forgo crimes while employed. Putting the pieces together, the full wage profile for the worker is:

\[
w(\lambda, V_u) = \begin{cases} 
\alpha \lambda + (1 - \alpha) w_R(V_u) & \text{if } \lambda^R(V_u) \leq \lambda < \lambda^{D1}(V_u) \\
w_C(V_u) & \text{if } \lambda^{D1}(V_u) \leq \lambda < \lambda^E(V_u) \\
\alpha \lambda + (1 - \alpha) w_C(V_u) & \text{if } \lambda^E(V_u) \leq \lambda < \lambda^{D2}(V_u) \\
\alpha \lambda + (1 - \alpha) w_C(V_u) & \text{if } \lambda \geq \lambda^{D2}(V_u)
\end{cases} \tag{40}
\]

Figure 13 shows the wage profile of a worker with \(V_u < V_u^*\). In both cases the expected value of the second stage increases at \(\lambda^{D2}(V_u)\).
To solution to the constrained bargaining problem in this case is:

\[
\begin{align*}
    w(\lambda, V_u; m) &= \begin{cases} 
            \alpha \lambda + (1 - \alpha)m & \text{if } m \leq \lambda < \lambda^{D1}(V_u; m) \\
            w_C(V_u) & \text{if } \lambda^{D1}(V_u; m) \leq \lambda < \lambda^{E}(V_u; m) \\
            \alpha \lambda + (1 - \alpha)m & \text{if } \lambda^{E}(V_u; m) \leq \lambda < \lambda^{D2}(V_u; m) \\
            \alpha \lambda + (1 - \alpha)w_C(V_u(m)) & \text{if } \lambda \geq \lambda^{D2}(V_u; m)
        \end{cases}
\end{align*}
\]  

(41)

where \( \lambda^{E}(V_u; m) \) is the productivity at which the expected value of the second stage is equal to the crime reservation wage.

\[
\lambda^{E}(V_u; m) = \frac{w_C(V_u(m)) - (1 - \alpha)m}{\alpha}
\]  

(42)

Figure 14 shows the wage profile. Again \( \frac{\partial \lambda^{D1}(V_u; m)}{\partial m} < 0 \) and therefore the minimum wage compresses the wage distribution from the bottom. The effects of a binding minimum wage in this case are:

**Proposition A.1.** If \((r + \delta)/\mu_\pi (1 - \alpha)/\alpha \text{ and } m < w_C(V_u(m))\) then

i. \( \frac{\partial \lambda^{D1}(V_u; m)}{\partial m} < 0 \)

ii. \( \frac{\partial \lambda^{E}(V_u; m)}{\partial m} < 0 \)

iii. \( \left| \frac{\partial \lambda^{D1}(V_u; m)}{\partial m} \right| > \left| \frac{\partial \lambda^{E}(V_u; m)}{\partial m} \right| \)

iv. \( \tilde{w}(\lambda, V_u; m) \geq w(\lambda, V_u) \text{ for all } m \leq \lambda < \lambda^{D2}(V_u) \)
A.2 Steady State Distributions

Equating the flows from Figure 4 gives the following steady state distributions:

\[
\begin{align*}
  u(b) &= \begin{cases} 
    \frac{\delta \gamma(\mu_e \pi + \delta) f(b)}{\Omega(b)} & \text{if } b < b^* \\
    \frac{\delta f(b)}{\mu_j B(b) + \delta} & \text{if } b \geq b^* 
  \end{cases} \\
  e_{nc}(b) &= \begin{cases} 
    \frac{\mu_j A(b) \gamma(\mu_e \pi + \delta) f(b)}{\Omega(b)} & \text{if } b < b^* \\
    \frac{\mu_j B(b) f(b)}{\mu_j B(b) + \delta} & \text{if } b \geq b^* 
  \end{cases} \\
  e_c(b) &= \begin{cases} 
    \frac{\delta \gamma \mu_j D(b) f(b)}{\Omega(b)} & \text{if } b < b^* \\
    0 & \text{if } b \geq b^* 
  \end{cases} \\
  p(b) &= \begin{cases} 
    \frac{\delta \pi [\mu_u (\mu_e \pi + \delta) + \mu_e \mu_j D(b)] f(b)}{\Omega(b)} & \text{if } b < b^* \\
    0 & \text{if } b \geq b^* 
  \end{cases}
\]

where \( \Omega(b) = (\mu_e \pi + \delta)[\delta(\mu_u \pi + \gamma) + \gamma \mu_j A(b)] + \delta \mu_j D(b)(\mu_u \pi + \gamma) \).