The Hiring Frictions and Price Frictions Nexus in Business Cycle Models

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Abstract

We study the interactions between hiring frictions and price frictions in business cycle models. We find that this interaction matters in a significant way for business cycle fluctuations and for labor market outcomes.

Using a simple DSGE business-cycle model with Diamond-Mortensen-Pissarides (DMP) elements, we derive two main results.

First, introducing hiring frictions into a New Keynesian model offsets the effects of price frictions. As a result, some business cycle outcomes are actually close to the frictionless New Classical-type outcomes; namely, with moderate hiring frictions the response of employment to technology shocks is positive, and the effects of monetary policy shocks are small, if not neutral. Moreover, it generates endogenous wage rigidity.

Second, introducing price frictions into a DMP setting generates amplification of employment and unemployment responses to technology shocks, as well as hump-shaped dynamics.

Both results arise through the confluence of frictions. We offer an explanation of the mechanisms underlying them.

Keywords: hiring frictions; price frictions; business cycles; New Classical model, New-Keynesian model; Diamond, Mortensen and Pissarides (DMP) model; technology shocks; monetary policy shocks; endogenous wage rigidity.

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1 Introduction

Is there a role for hiring frictions in business cycles? Are they consistent with volatile labor market outcomes? While empirical evidence suggests that the answers are positive, key models in the literature seem to be giving negative answers. These discrepancies can be seen as follows.

First, the benchmark New Keynesian business cycle model (as developed and reviewed by Woodford (2003) and Galí (2015, ch. 3)), abstracts from the modelling of unemployment and makes no use of labor market frictions. The benchmark model reflects the view that frictions in the price setting process are key to the propagation of both monetary and non-monetary shocks to real variables. Thus, while the behavior of unemployment along the business cycle is of major importance for policy makers, this workhorse model, which is commonly used to understand business cycle fluctuations and inform policy analysis, does not cater for unemployment or labor market frictions. This has been noted by several authors, for example by Galí and Gertler (2007).¹

Second, models of unemployment dynamics and labor market frictions in the tradition of Diamond, Mortensen and Pissarides (DMP; as surveyed, for example, by Rogerson and Shimer (2011)), have been found to play a negligible role in explaining business cycle fluctuations. Moreover, they typically abstract from price frictions, emphasized by the afore-cited New Keynesian approach. The empirical consistency of frictions with volatile outcomes, such as unemployment and vacancies, was questioned.

Third, when labor market frictions, as modelled in DMP, are explicitly incorporated within New Keynesian models, they still do not contribute directly to the explanation of business cycles.²

In contrast to these points, there is overwhelming empirical evidence, documented in the literature over two decades, that gross labor market flows are large, highly cyclical and volatile, and important for employment and unemployment determination. Given that gross labor market flows have strong relations with labor market frictions, and that employment is key in understanding business cycles, one would expect labor market frictions to play a role in business cycles. Moreover this evidence has been found for many economies, with the seminal work being undertaken by Davis and Haltiwanger and co-authors (starting from their early work, Davis, Haltiwanger and Schuh (1996) and Davis and Haltiwanger (1999), and going up to the recent contribution in Davis and Haltiwanger (2014)). Thus, this kind of empirical findings seems to be in conflict – in this context – with the afore-cited key models.

This paper offers a reconciliation and shows that hiring frictions are indeed important for business cycles and are consistent with volatile labor market variables. Specifically, we examine the effects of technology and monetary policy shocks under alternative specifications of the two

¹Galí (2011, 2015 ch. 7) and related work address this lacuna by setting up the labor market with staggered wage setting, labor force participation decisions, and explicitly-modelled involuntary unemployment, within the New Keynesian model. Unemployment in the model results from market power in labor markets, reflected in positive wage markups. This model does not include hiring frictions, which are at the focus of the analysis here.

²There is an indirect contribution, as these models justify certain wage setting mechanisms, which do have a role in business cycles. We review this literature below and note our innovations relative to it.
sets of frictions. To this end, we adopt a baseline New Classical-type economy, and add price adjustment costs à la Rotemberg (1982) and gross hiring costs in the spirit of the investment model of Lucas and Prescott (1971). The modelling of hiring frictions, which follows empirical work by Merz and Yashiv (2007) and Yashiv (2016 a,b), generates equilibrium unemployment and positive values of jobs, as in the DMP search model.

Our analysis of the interactions between hiring frictions and price frictions produces two key results.

The first is that introducing hiring frictions into a New Keynesian model offsets the effects of price frictions. Specifically, the standard New Keynesian results, whereby employment responds negatively to positive technology shocks, and output responds positively to an expansionary monetary policy shock, only arise if frictions in the labor market are quantitatively small. Introducing empirically-relevant, but moderate, values of labor frictions implies that the precise degree of price frictions becomes less relevant, if not irrelevant, in the propagation of shocks to real variables. As a result, the response of employment to technology shocks is positive and the effects of a monetary expansion are small, if not neutral. These results – whereby a New Keynesian model with hiring frictions can generate outcomes close to the frictionless, New Classical model – do not stem from full flexibility but from a confluence of frictions. Introducing larger – but still empirically reasonable – values of labor frictions, further magnifies the positive response of employment to technology shocks and implies that expansionary monetary policy shocks may even have contractionary effects on the real economy. Introducing hiring frictions also generates endogenous wage rigidity.

The second result is that introducing price frictions into a DMP model with moderate values of hiring frictions, generates amplification in the responses of employment and unemployment to technology shocks as well as hump-shaped dynamics. This result arises even in the presence of a pro-cyclical opportunity cost of work. The latter has been documented by Chodorow-Reich and Karabarbounis (2016), who note that under such conditions, leading models of the labor market, based on either real wage rigidities or a small surplus calibration, fail to generate amplification.

We offer an explanation as to why hiring frictions do matter in a significant way for business cycle fluctuations and are consistent with volatile labor market outcomes. The explanation rests on the idea that hiring involves disruption to production. Hence marginal hiring costs depend, inter-alia, on the relative price of the intermediate good relative to the final good, which is the numeraire. Consider the rule for optimal hiring, which equates these marginal costs of hiring with the expected present value of the hire. Price frictions and shocks affect both sides of this equation. A positive technology shock, for example, generates a fall in the relative price noted above. This makes for a decrease in the value of the intermediate output that is produced by the marginal hire. Ceteris paribus, there is a decrease in both the expected present value of the hire and in marginal hiring costs. In the standard New Keynesian model the latter effect is absent and so hiring falls. In the current model the outcome depends on the relative fall of each side of the equation. In particular, as marginal hiring costs fall, they lead to an increase in hiring,

3Nor does it stem from the selection of a Taylor rule that replicates the optimal policy.
offsetting the former, standard, New-Keynesian effect. The propagation of shocks through relative prices will therefore depend on their net effect on marginal hiring profits vs. costs. The higher the hiring friction costs, the stronger the offset to the standard New-Keynesian mechanism. If hiring costs are sufficiently large – but within plausible empirical estimates – the decrease in marginal hiring costs following a positive technology shock is big enough so as to (a) generate an increase in hiring and employment, akin to the direction of change in a New Classical model; (b) generate significant employment and unemployment fluctuations.

It should be noted that our results do not follow from the obvious argument that if frictions are large, quantities cannot respond. Moreover, we focus on a reasonable spectrum of hiring friction costs. Importantly, the interaction between price and hiring frictions produces non-trivial dynamics, whereby hiring actually increases with labor frictions. Hence hiring frictions amplify some responses and so these cannot be explained by a smoothing effect of frictions. This amplification squares well with volatile labor market outcomes. We explain below how these results fit in the existing literature.

We emphasize at the outset that this paper does not aim to settle the dispute on whether employment falls or increases following a technology shock, or whether monetary policy has material real effects or not, and of what type. Indeed, while the empirical literature on price frictions and in particular on the frequency of price adjustment has reached a relatively mature stage of development, empirical work that tries to measure hiring frictions is still relatively scant, and more work is needed to confidently rely on a specific calibration. We will therefore inspect how the transmission of shocks changes for a grid of plausible parameterizations. This analysis illustrates how labor market frictions, of the kind that give rise to equilibrium unemployment, matter for the transmission of shocks in New Keynesian models. Specifically, if hiring frictions are non-negligible, the precise degree of price rigidity is no longer a sufficient statistic to measure the strength of the New Keynesian propagation mechanism. At the same time, DMP modelling needs to recognize the important role played by the interaction of labor and price frictions. This interaction, or confluence of frictions, is key.

The paper is organized as follows. Section 2 reviews the related literature, and Section 3 presents the baseline model with a minimal set of assumptions. Section 4 discusses the calibration and presents impulse responses. Section 5 investigates the interaction of price and hiring frictions for a grid of parameter values and explains the mechanism. Section 6 explores the robustness of the mechanism to the introduction in the model of a richer set of assumptions and to different parameterizations of the Taylor rule. Section 7 concludes.

2 Relation to the Existing Literature

This paper examines the standard New Classical (NC) and New Keynesian (NK) business cycle models, each with and without labor market frictions. Our specific modelling of price adjustment costs, a key ingredient in NK models, follows Rotemberg (1982). Labor market frictions are modelled here as gross hiring costs.

The use of labor market frictions in general equilibrium, business cycle settings yielded
mixed results. At first, Merz (1995), Andolfatto (1996), and Den Haan, Ramey and Watson (2000) found the DMP model to enhance the performance of the NC model. But Shimer (2005) offered a strong critique of its usefulness, arguing that for realistic productivity shocks, the standard DMP model fails to generate unemployment and vacancy volatility as found in the data. The paper spawned a large body of work on this “Shimer puzzle.” Rogerson and Shimer (2011) argued that in the business cycle context, the main substantive contribution of search models is the presence of match specific rents and hence the opportunity for a richer set of wage setting processes. Yet, relative to a frictionless counterpart, search frictions do not help generate volatility or persistence. Rather, by acting like a labor adjustment cost, search frictions dampen the volatility of employment. If anything then, they exacerbate the difficulties of the frictionless neoclassical paradigm to account for the cyclical behavior of the labor market.

A strand in the literature has embedded labor market frictions in NK models. Prominent contributions include Walsh (2005), Krause, Lopez-Salido and Lubik (2008), Gertler, Sala, and Trigari (2008), Gali (2011), and Christiano, Eichenbaum and Trabandt (2016). Most of these papers, too, found little, if any, direct effect of labor frictions. For example, Krause, Lopez-Salido and Lubik (2008) state that the contribution of labor market frictions to inflation dynamics is small. Gali (2011) showed that labor market frictions per se matter little for the outcomes of macroeconomic variables, and in particular aggregate labor market variables. The role of these frictions, he finds, is to reconcile the presence of wage rigidities with privately efficient employment relations. Christiano, Eichenbaum and Trabandt (2016) too get meaningful effects via the wage setting mechanism, whereby the bargaining set up generates endogenous wage rigidity.

The innovation of the current paper with respect to the afore-cited literature is that we show how hiring frictions matter in a significant way for business cycles, not only through wage setting mechanisms, and that hiring frictions may serve to actually amplify hiring and employment in a DMP context when interacted with price frictions.

Several points need to be noted:

(i) The afore-cited papers postulate different formulations for the labor frictions cost function, including the modelling of its shape (convexity) and its arguments. In modelling these frictions here, we follow the convex function approach of Lucas and Prescott (1971) to investment problems. We rely on the more recent theoretical justifications of King and Thomas (2006) and Khan and Thomas (2008) for convexity, and on the ideas in Alexopoulos (2011) and Alexopoulos and Tombe (2012) for the arguments of the function. Merz and Yashiv (2007) and Yashiv (2016 a,b) have shown, using GMM structural estimation, that the formulation we use fits U.S. data well. Christiano, Trabandt, and Walentin (2011), using Bayesian estimation of a DSGE model, find the same specification works well with Swedish data.

(ii) Labor frictions have been introduced in New-Keynesian models as vacancy costs, so hiring generates rents. However, vacancy costs are typically assumed to be external to the firm, and are therefore expressed in units of the final (numeraire, composite) good; hence they

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4The latter three papers are also related to a strand of the Macro-Finance literature which ties firms investment and/or hiring decisions to financial markets. See Cochrane (2005, Chapter 20, and 2008) for overviews and discussions.
are not affected by a change in relative prices and the mechanism discussed above does not apply (cf. Gertler Sala and Trigari (2008), Gali (2011), Christiano Eichenbaum and Trabandt (2016)). However, empirical studies, cited below, indicate that most of recruitment costs are indeed internal, meaning that they involve disruption in production. Even vacancy costs might involve some form of forgone output, to the extent that workers are engaged in vacancy posting and screening, and their work-time is diverted from production.

(iii) For the mechanism delineated above to operate qualitatively, it is sufficient that frictions generate rents and that friction costs are internal to the firm. That is, the precise degree of convexity in costs does not matter, nor does it matter, when modelling hiring frictions, whether rents derive from the postulation of a matching function as in Diamond, Mortensen and Pissarides\(^5\) or from convex gross hiring costs in the spirit of the Lucas and Prescott (1971) investment model. Quantitatively, moving away from the vacancy cost formulation allows us to inspect the effects of hiring costs under a broader spectrum of parameterizations.

(iv) Some recent papers deserve special mention. Christiano, Eichenbaum and Trabandt (2016) have presented a model whereby endogenous wage rigidity, arising in an alternating-offer bargaining set-up, can generate explanations of all key macroeconomic variables in a NK model with labor market frictions. Our paper provides for a very different mechanism, with the key one ensuing from the set up of internal gross hiring costs.\(^6\) Melosi (2016) shows that if economic agents are imperfectly informed about the state of the economy, monetary policy acts as a signalling device, which dampens the transmission of the shocks to real variables. While the dampening mechanism produces effects that are remindful of those that arise in or model, the signaling mechanism cannot lead to a reversal of the New Keynesian outcomes as we get. Our results also remind those in Head, Liu, Menzio and Wright (2012), who develop a new-monetarist model where prices are sticky, and yet money is neutral. In contrast to their paper, the result that nominal rigidities do not logically imply that policy can exploit these rigidities, only arises in our setting in a particular region of the parameter space. More importantly, our results are derived within the standard New Keynesian framework and not within a New Monetarist one.

(v) In canonical NK models hours fall and output rises following a positive neutral technology shock, due to the prominence of the wealth effect on labor supply. Because hours and output are positively correlated, technology shocks cannot be a major source of the business cycle in this class of models. A popular way to give these shocks some chance to matter for business cycle fluctuations is to assume Jaimovich and Rebelo (2009) preferences, which allow wealth effects to be lower in the short than the long run while maintaining balanced growth. In our model, the interaction between price frictions and hiring frictions provides for a mechanism

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\(^5\)The main difference with respect to DMP is that employment rents in our framework originate from the existence of hiring costs that are incurred after a match is formed, and not vacancy costs. Hence, values of jobs arise even without explicitly modelling a matching function. The rationale for focusing on this type of hiring costs is two-fold. First, as indicated in the literature to which we refer below, vacancy posting costs are only a fraction of overall hiring costs, while our formulation of friction costs is more encompassing. Second, these costs are internal to the firm, meaning that they imply disruption in production. This particular feature of hiring costs, as we explain below, is key for the mechanism.

\(^6\)We also use a different bargaining set up.
whereby the positive comovement between employment and output conditional on technology shocks arises endogenously for sufficiently large, but not implausible, parameterizations of hiring friction costs.

(vi) Indeed, our theoretical investigation offers a grid of values in the joint space of price frictions and hiring frictions, which supports the full variety of empirical results obtained in the broad, empirical, VAR literature. For instance, the seminal work by Galí (1999), which sparked a debate in the literature, was the first to identify a negative response of employment to technology shocks. Subsequent VAR analysis in Uhlig (2004), Christiano, Eichenbaum and Vigfusson (2004), Alexopoulos (2011), and Sims (2011) found opposite results, pointing to a positive response of employment. Likewise, the standard, expansionary effects of a decrease in interest rates are consistent with a multitude of results in the VAR literature, which rely on a variety of identification schemes (see Ramey (2016) for a survey). Yet, the findings in Faust, Swanson and Wright (2004), Uhlig (2005), and Amir and Uhlig (2016) are consistent with the view that monetary policy produces small real effects or even no real effects. On a similar note, Ramey (2016) argues that relaxing the conventional assumption in VARs that prices and output cannot respond to interest rate contemporaneously, leads to “puzzling” results, whereby an expansionary monetary policy shock seems to have significant contractionary effects. Furthermore, Nekarda and Ramey (2013) find that markups are procyclical, and argue that the inability of the textbook New Keynesian model to produce pro-cyclical markups is a major shortcoming of this class of models. Our model provides for a possible rationalization of all these findings. In the light of our model, taking a stance on the conflicting VAR evidence can be rationalized as implicitly taking a stance on the joint relevance of price and hiring frictions.

3 The Model

3.1 The Set-Up

The model features two sources of frictions: price adjustment costs and costs of hiring workers. Absent both frictions, the model boils down to the benchmark NC model with labor and capital. Following the Real Business Cycle tradition, capital is included because it plays a key role in producing a positive response of employment to productivity shocks. With standard logarithmic preferences over consumption, income and substitution effects cancel out and a NC model with or without hiring frictions would not produce any change in employment or unemployment to productivity shocks (see Blanchard and Gali (2010)).

Introducing price frictions into the otherwise frictionless model yields the NK benchmark, and introducing hiring frictions into the NK benchmark allows us to analyze how the interplay between these frictions affects the propagation of technology and monetary policy shocks. In this section, and in order to focus on the above interplay, our modeling strategy deliberately abstracts from all other frictions and features that are prevalent in DSGE models and which are typically introduced to enhance propagation and improve statistical fit, namely, habits in consumption, investment adjustment costs, exogenous wage rigidities, etc. In Section 6 below
we examine the robustness of our results with respect to such modifications.

In what follows we look in detail at households, two types of firms, the monetary and fiscal authorities and the aggregate economy.

3.2 Households

The representative household comprises a unit measure of workers searching for jobs in a frictional labor market. At the end of each time period workers can be either employed or unemployed; we therefore abstract from participation decisions and from variation of hours worked on the intensive margin. The household enjoys utility from the aggregate consumption index $C_t$ and disutility from employment, $N_t$. Employed workers earn the nominal wage $W_t$ and hold nominal bonds denoted by $B_t$. Both variables are expressed in units of consumption, which is the numeraire. The budget constraint is:

$$P_tC_t + \frac{B_{t+1}}{R_t} = W_tN_t + B_t + Y_t - T_t,$$

where $R_t = (1 + i_t)$ is the gross nominal interest rate, $P_t$ is the price of the composite consumption good, $Y_t$ denotes dividends from ownership of firms and $T_t$ lump sum taxes.

The labor market is frictional and workers who are unemployed at the beginning of each period $t$ are denoted by $U^0_t$. It is assumed that these unemployed workers can start working in the same period if they find a job with probability $x_t = \frac{H_t}{U^0_t}$, where $H_t$ denotes the total number of matches. It follows that the workers who remain unemployed for the rest of the period, denoted by $U_t$, is $U_t = (1 - x_t)U^0_t$. Consequently, the evolution of aggregate employment $N_t$ is:

$$N_t = (1 - \delta_N)N_{t-1} + x_tU^0_t,$$  

where $\delta_N$ is the separation rate.

The intertemporal problem of the households is to maximize the discounted present value of current and future utility:

$$\max E_t \sum_{j=0}^{\infty} \beta^j \left( \ln C_{t+j} - \frac{\chi}{1 + \varphi} N_{t+j}^{1+\varphi} \right),$$

subject to the budget constraint (1) and the law of motion of employment (2). The parameter $\beta \in (0, 1)$ denotes the discount factor, $\varphi$ is the inverse Frisch elasticity of labor supply, and $\chi$ is a scale parameter governing the disutility of work.

The optimality conditions are:

$$\frac{1}{R_t} = \beta E_t \frac{P_tC_t}{P_{t+1}C_{t+1}},$$

As shown in Rogerson and Shimer (2011), most of the fluctuations in US total hours worked at business cycle frequencies are driven by the extensive margin, so our model deliberately abstracts from other margins of variation.
\[ V_i^N = \frac{W_i}{P_t} - \chi N_i^\theta C_t - \frac{x_t}{1-x_t} V_i^N + \beta (1 - \delta_N) E_t \frac{C_t}{C_{t+1}} V_{i+1}^N, \]  

(5)

where equation (4) is the standard Euler equation and equation (5) is the marginal value of a job to the household net of the value of search. The latter comes in at the bargaining stage, examined below.

The value of a job is equal to the real wage, net of the opportunity cost of work, \( \chi N_i^\theta C_t \), and the flow value of search for unemployed workers, plus a continuation value. It is worth noting that relative to the DMP model, where the opportunity cost of work is assumed to be constant, deriving the net value of employment from a standard problem of the household implies that this opportunity cost equals the marginal rate of substitution between consumption and leisure. As we show later in the text, this feature of the model is key in generating endogenous real wage rigidity in the presence of hiring frictions.

3.3 Firms

We assume two types of firms: intermediate producers and final good producers. Intermediate producers hire labor and invest in capital to produce a homogeneous product, which is then sold to final producers in perfect competition. Final producers transform each unit of the homogeneous product into a unit of a differentiated product facing price frictions à la Rotemberg (1982). This separation between intermediate and final goods firms is a convention in the literature. It is assumed that the same bundle of final goods is used for consumption and investment purposes. So output, consumption and investment have the same price, which is denoted by \( P_t \). Under the common Dixit-Stigliz aggregator of differentiated goods, the expenditure minimizing price index associated with the output index is

\[ P_t = \left( \int_0^1 P_t (i)^{1-\epsilon} \, di \right)^{1/(1-\epsilon)}, \]  

(6)

where \( P_t (i) \) is the price of a variety \( i \) and \( \epsilon \) denotes the elasticity of substitution.

3.3.1 Intermediate Goods Producers

A unit measure of intermediate goods producers sell homogeneous goods to final producers in perfect competition. Intermediate firms combine physical capital, \( K \), and labor, \( N \), in order to produce intermediate output goods, \( Z \). The constant returns to scale production function is \( f(A_t, N_t, K_{t-1}) = A_t N_t^\alpha K_{t-1}^{1-\alpha} \), where \( A_t \) is a standard TFP shock that follows the stochastic process: \( \ln A_{t+1} = \rho_a \ln A_t + \epsilon_t^a \), with \( \epsilon_t^a \sim N(0, \sigma_a) \). Following a convention in DSGE modelling, we have assumed that newly installed capital becomes effective only with a one (quarter) period lag. Capital is firm-specific.\(^8\)

It is assumed that hiring is a costly activity. Specifically, we think of hiring costs as the disruption in economic activity associated with worker recruitment. These costs reflect training

\(^8\)See Woodford (2005) for a discussion of this kind of formulation in the DSGE New Keynesian context.
costs, and other costs that are incurred after a worker is hired, and are not sunk by the time
the match is formed. Typically, labor frictions are modelled as vacancy posting costs, which
are sunk at the time of bargaining. However, training costs are considerably larger than the
costs of advertising a vacancy, and include the time spent by managers and team-workers to
instruct new hires, which is a drag on their ability to produce. Hiring costs might also reflect
the time-costs associated with learning how to operate capital, as well as the implementation of
new organizational structures within the firm and new production techniques; see Alexopoulos
(2011) and Alexopoulos and Tombe (2012).

Micro estimates using Swiss data reported in Blatter et al (2016, Table 1) and structural
macro estimates using U.S. data in Furlanetto and Groshenny (2016, Table 3), Swedish data
in Christiano, Trabandt, and Walentin (2011, p.2039), and Israeli data in Yashiv (2000, Table
2), show that vacancy posting costs are very small compared to other components of hiring
costs, particularly to training costs. Indeed, Christiano, Trabandt, and Walentin (2011), using
Bayesian estimation of a DSGE model of Sweden, conclude, in this same context, that “employ-
ment adjustment costs are a function of hiring rates, not vacancy posting rates.” Acemoglu
and Pischke (1999 a,b) provide empirical evidence as to the importance of worker training in firm
costs. They do so for general training, providing theoretical justification as to why competitive
firms undertake such training and incur these costs.

These costs are thought of as forgone output. In the spirit of the investment model of Lucas
and Prescott (1971), and following Merz and Yashiv (2007), we assume that the friction cost
function is constant returns to scale and quadratic in the hiring rate, in line with estimates by
Yashiv (2016 a,b):

\[ g(A_t, H_t, N_t, K_{t-1}) = \frac{e}{2} \left( \frac{H_t}{N_t} \right)^2 A_t N_t^\alpha K_{t-1}^{1-\alpha}. \]  

(7)

where \( H \) denotes gross hires.

Note that this convex function modelling is consistent with the theoretical analysis of King
and Thomas (2006) and Khan and Thomas (2008). More specifically, the function presented
above may be justified as follows (drawing on Garibaldi and Moen (2009) and Garibaldi, Moen,
and Sommervoll (2016)): suppose each worker \( i \) makes a recruiting and training effort \( h_i \); as
this is to be modelled as a convex function, it is optimal to spread out the efforts equally across
workers so \( h_i = \frac{h}{n} \); formulating the costs as a function of these efforts and putting them in terms
of output per worker one gets \( c \left( \frac{h}{n} \right) \frac{f}{n} \), as \( n \) workers do it then the aggregate cost function is
given by \( c \left( \frac{h}{n} \right) f \). This specification captures the idea that frictions or costs increase with the
extent of hiring activity and needs to be modelled relative to the size of the firm. The intuition
is that hiring 10 workers, for example, means different levels of hiring activity for firms with
100 workers or for firms with 10,000 workers. Hence firm size, as measured by its level of
employment, is taken into account and the costs function is increasing in the hiring rate, \( \frac{h}{n} \). This
formulation accords well with most of the empirical studies surveyed in Blatter at al (2016).

The net output of a representative firm at time \( t \) is:

\[ Z_t = f(A_t, N_t, K_{t-1}) - g(A_t, H_t, N_t, K_{t-1}). \]  

(8)
In every period $t$, the existing capital stock depreciates at the rate $\delta_K$ and is augmented by new investment:

$$K_t = (1 - \delta_K)K_{t-1} + I_t, \quad 0 < \delta_K < 1.$$  \hfill (9)

Similarly, the number of a firm’s employees decreases at the rate $\delta_N$ and it is augmented by new hires $H_t$. The law of motion for employment reads:

$$N_t = (1 - \delta_N)N_{t-1} + H_t, \quad 0 < \delta_N < 1,$$  \hfill (10)

which implies that new hires are immediately productive.

At the beginning of each period, firms hire new workers and invest in capital. Next, wages are negotiated following Nash bargaining. When maximizing its market value, defined as the present discounted value of future cash flows, the representative producer anticipates the impact of its hiring and investment policy on the bargained wage. This is so because with frictions in the labor market, wages are not set competitively and there is bilateral monopoly power in bargaining. Hence the effect of production inputs on the marginal product of labor must be factored in the bargaining (see Cahuc, Marque and Wasmer (2008)).

The intertemporal maximization problem of the firm reads as follows:

$$\max_{I_t, H_t} E_t \sum_{j=0}^{\infty} \Lambda_{t+j} \left\{ mc_{t+j} \left[ f(A_{t+j}, N_{t+j}, K_{t+j}) - g(A_{t+j}, H_{t+j}, N_{t+j}, K_{t+j}) \right] - \frac{W_{t+j}}{P_{t+j}} (K_{t+j}, H_{t+j}, N_{t+j}) - I_{t+j} \right\}$$  \hfill (11)

subject to the laws of motion for capital (9) and labor (10), where $\Lambda_{t+j} = \frac{\beta^{j+1}}{\beta^t}$ is the real discount factor of the households who own the firms.

The relative price of the intermediate firm’s good is $mc_k = P_i^Z / P_t$. This relative price equals the real marginal cost for a final goods producer since, as discussed later, producers transform one unit of intermediate good into one unit of final good. It will play an important role in the analysis below.

The first-order conditions for dynamic optimality are:

$$Q_{t}^N = mc_t (f_{N,t} - g_{N,t}) - \frac{W_t}{P_t} - \frac{W_{N,t}}{P_t} N_t + (1 - \delta_N) E_t \Lambda_{t+1} Q_{t+1}^N,$$  \hfill (12)

$$Q_{t}^N = mc_t g_{H,t} + \frac{W_{H,t}}{P_t} N_t,$$  \hfill (13)

$$1 = E_t \Lambda_{t+1} \left[ mc_{t+1} (f_{K,t+1} - g_{K,t+1}) - \frac{W_{K,t+1}}{P_{t+1}} N_{t+1} + (1 - \delta_K) \right],$$  \hfill (14)

where $Q_{t}^N$ is the Lagrange multiplier associated with the employment law of motion. One can

---

9Our formulation of frictions/costs is consistent with intra-firm bargaining. For theoretical modelling see Stole and Zweibel (1996); this has been implemented to the current context by Cahuc, Marque and Wasmer (2008). Because these costs are not sunk when a worker is hired, the firm correctly anticipates the effects of its hiring policy on the net marginal product, which in turn also depends on marginal hiring costs.
label $Q_t^N$ as Tobin’s Q for labor or the value of labor. For an extensive discussion of its economic significance, see Yashiv (2016 a,b).

Here we notice that the value of a marginal job in equation (12) can be expressed as the sum of current-period profits (the first three terms on the RHS) and a continuation value. The profits equal the marginal revenue product of labor $mc_t (f_{N,t} - g_{N,t})$ less the real wage and the intrafirm bargaining term $\frac{W_{N,t}}{P} N_t$. The latter term appears because the marginal product of labor decreases with the size of the firm, hence the marginal worker decreases the marginal product of labor and the wage bargained by all the infra-marginal workers. In turn, this leads to over-hiring in steady-state. In equation (13), the value of jobs is equated to the real marginal cost of hiring. The real marginal cost of hiring in turn is given by the sum of a frictional component $mc_t g_{H,t}$ and the intra-firm bargaining component $\frac{W_{H,t}}{P} N_t$.

The cost of one unit of capital on the LHS of equation (14) equals the discounted value of the next period marginal revenue product of capital $mc_{t+1} (f_{K,t+1} - g_{K,t+1})$ plus an intrafirm bargaining term and the future value of undepreciated capital. The reason for the appearance of the intrafirm bargaining term is the following: a higher capital stock makes workers more productive, thereby increasing the expected marginal product of labor and the wage bargained by all infra-marginal workers. This term reflects a typical hold-up problem: because workers appropriate part of the rents generated by employment, the capital effect on wages decreases the value of capital, leading to under-investment.

In order to understand the forces driving the relative price, it is worth solving the F.O.C. for employment in equation (12) for $mc_t$, after replacing for $Q_t^N$ using (13):

$$mc_t = \frac{W_t}{P} f_{N,t} - g_{N,t} + \frac{W_{N,t}}{P} N_t + (mc_t g_{H,t} + \frac{W_{H,t}}{P} N_t) - (1 - \delta_N) E_t \Lambda_{t,t+1} \left( mc_{t+1} g_{H,t+1} + \frac{W_{H,t+1}}{P} N_{t+1} \right)$$

(15)

The first term on the RHS of the above equation is the wage component of marginal costs, expressed as the ratio of real wages to the net marginal product of labor. Because the production function is Cobb-Douglas, in the case of a NK model where $g_{N,t} = 0$, the wage component is proportional to the familiar labor share of income $W_t N_t / P_t Y_t$. The second term relates to intrafirm bargaining: the cost of expanding output by raising employment at the margin, decreases with the negative effect of firm size on the negotiated wage bill. The third term shows that with frictions in the labor market, marginal costs also depend on expected changes in real marginal hiring costs, a point already made by Krause, Lopez-Salido and Lubik (2008). So, for instance, an expected increase in marginal hiring costs $E_t \Lambda_{t,t+1} mc_{t+1} g_{H,t+1}$ translates into lower current marginal costs ($mc_t$), reflecting the savings of future recruitment costs that can be achieved by recruiting in the current period.
3.3.2 Final good producers

There is a unit measure of monopolistically competitive final good firms indexed by $i \in [0, 1]$. Each firm $i$ transforms $Z(i)$ units of the intermediate good into $Y(i)$ units of a differentiated good, where $Z(i)$ denotes the amount of intermediate input used in the production of good $i$. Monopolistic competition implies that each final firm $i$ faces the following demand for its own product:

$$Y_t(i) = \left( \frac{P_t(i)}{P_t} \right)^{-\varepsilon} Y_t,$$

where $Y_t$ denotes aggregate demand from households and firms.

We assume price stickiness à la Rotemberg (1982), meaning firms maximize current and expected discounted profits subject to quadratic price adjustment costs and subject to the demand function (16):

$$\max_{P_t, \pi_t} E_t \sum_{s=0}^{\infty} \Lambda_{t+s}^{1/2} \left[ (P_{t+s}(i) - P_{t+s}mc_{t+s}) Y_{t+s}(i) - \frac{\varepsilon}{2} \left( \frac{P_{t+s}(i)}{P_{t+s-1}(i)} - 1 \right)^2 P_{t+s} Y_{t+s} \right].$$

The first order condition with respect to $P_t(i)$ reads as follows:

$$Y_t(i) - \xi \left( \frac{P_t(i)}{P_{t-1}(i)} - 1 \right) \frac{1}{P_{t-1}(i)} P_t Y_t = \frac{1}{P_t(i)} \left\{ - \xi \left( \frac{P_t(i)}{P_{t-1}(i)} - 1 \right) \right\} P_t Y_t.$$

Since all firms set the same price and therefore produce the same output in equilibrium, the above equation can be rewritten to express a law of motion for inflation:

$$\pi_t (1 + \pi_t) = \frac{1 - \varepsilon}{\xi} + \frac{\varepsilon}{\xi} mc_t E_t \frac{1}{1 + \pi_t} (1 + \pi_{t+1}) \pi_{t+1} Y_{t+1},$$

where we have used $E_t \Lambda_{t+1} = \frac{1}{(1+i_t)(1+\pi_{t+1})} = \frac{1}{1+r_t}$, with $i_t$ and $r_t$ denoting the nominal and real net interest rates, respectively. Equation (19) specifies that inflation depends on marginal costs as well as expected future inflation. Solving forward equation (19), it is possible to show that inflation depends on current and expected future real marginal costs.

3.4 Wage Bargaining

Wages are assumed to maximize a geometric average of the household’s and the firm’s surplus weighted by the parameter $\gamma$, which denotes the bargaining power of the households:

$$W_t = \arg \max \left\{ \left( Y_t^{H_t} \right)^{\gamma} \left( Q_t^{K_t} \right)^{1-\gamma} \right\}.$$

The first order condition to this problem leads to the Nash sharing rule:
\[(1 - \gamma)V_t^N = \gamma Q_t^N.\] (21)

Substituting (5) and (12) into the above equation and using the sharing rule (21) to eliminate the terms in \(Q_t^N\) and \(V_t^N\), one gets the following expression for the real wage:

\[
\frac{W_t}{P_t} = \gamma mc_t (f_t - \xi_t, 1 - \alpha) - \gamma \frac{W_t}{P_t} - N_t + (1 - \gamma) \left[ \chi C_t N_t^\theta + \frac{x_t}{1 - x_t} \frac{\gamma}{1 - \gamma} Q_t^N \right]. \tag{22}
\]

Assuming a Cobb-Douglas production function and the frictions cost function in (7), the solution to the differential equation in (22) reads as follows:

\[
\frac{W_t}{P_t} = \gamma mc_t A_t K_t^{1 - \alpha} \left\{ \frac{N_t^{\alpha - 1}}{1 - \gamma(1 - \alpha)} + e^{1 - \frac{\alpha}{2}} H_t^{2 - 3} \right\} + (1 - \gamma) \left[ \chi C_t N_t^\theta + \frac{x_t}{1 - x_t} \frac{\gamma}{1 - \gamma} Q_t^N \right]. \tag{23}
\]

See Appendix A for the details of the derivation.

### 3.5 The Monetary and Fiscal Authorities and Market Clearing

We assume that the government runs a balanced budget:

\[T_t = B_t - \frac{B_{t+1}}{R_t},\] (24)

and the monetary authority sets the nominal interest rate following the Taylor rule:

\[
\frac{R_t}{R^*} = \left( \frac{R_{t-1}}{R^*} \right)^{\rho_r} \left( \frac{1 + \pi_t}{1 + \pi^*} \right)^r \left( \frac{Y_t}{Y^*} \right)^{r_y} \left[ 1 - \gamma \right]^1, \tag{25}
\]

where \(\pi_t\) measures the rate of inflation of the numeraire good, and an asterisk superscript denotes the steady-state values of the associated variables. When linearizing the model around the stationary equilibrium we will assume that \(\pi^* = 0\). The parameter \(\rho_r\) represents interest rate smoothing, and \(r_y\) and \(r_\pi\) govern the response of the monetary authority to deviations of output and inflation from their steady-state values. The term \(\xi_t\) captures a monetary policy shock, which is assumed to follow the process \(\ln \xi_t = \rho_r \ln \xi_{t-1} + e^r_t\), with \(e^r_t \sim N(0, \sigma^2)\).

Consolidating the households and the government budget constraints, and substituting for the profits of intermediate and final good producers yields the market clearing condition:

\[Y_t \left[ 1 - \frac{\xi_t}{2 \pi_t^2} \right] = C_t + I_t = f_t - g_t.\] (26)

### 4 Impulse Response Analysis

We start this section by calibrating the model with price frictions and labor frictions, which will provide a benchmark for the analysis to follow. We then compare how the impulse responses of real variables such as the hiring rate, the investment rate, real wages and net output change...
when we shut down price frictions and/or labor market frictions. In what follows we will look at both technology and monetary policy shocks.

We linearize the model around the non-stochastic steady state and solve for the policy functions, which express the control variables as a function of the states and the shocks. We then shock the stationary equilibrium of the model with a technological or a monetary innovation, and iterate on the policy functions and on the laws of motion for the state variables to trace the expected behavior of the endogenous variables, i.e., we produce impulse responses.

4.1 Calibration

Parameter values are set so that the steady-state equilibrium of our model matches key averages of the 1976Q1-2014Q4 U.S. economy, assuming that one period of time equals one quarter. We start by discussing the parameter values that affect the stationary equilibrium.

Table 1

The discount factor $\beta$ equals 0.99 implying a quarterly interest rate of 1%. The quarterly job separation rate $\delta_{\text{N}}$, measuring separations from employment into either unemployment or inactivity, is set at 0.126, and the capital depreciation rate $\delta_{\text{K}}$ is set at 0.024. These parameters are selected to match the hiring to employment ratio, and the investment to capital ratio measured in the US economy over the period 1976Q1-2014Q4 (see Appendix B in Yashiv (2016a) for details on the computations of these series).

The inverse Frisch elasticity $\phi$ is set equal to 4, in line with the synthesis of micro evidence reported by Chetty et al. (2013), pointing to Frisch elasticities around 0.25 on the extensive margin. The elasticity of substitution in demand $\epsilon$ is set to the conventional value of 11, implying a steady-state markup of 10%, consistent with estimates presented in Burnside (1996) and Basu and Fernald (1997). Finally, the scale parameter $\chi$ in the utility function is normalized to equal 1 and the elasticity of output to the labor input $\alpha$ is set to 0.66 to match a labor share of income of about two thirds.

This leaves us with two parameters to calibrate: the bargaining power $\gamma$ and the scale parameters in the friction costs function $e$. These two parameters are calibrated to match: i) a ratio of marginal hiring friction costs to the average product of labor, $g_{\text{H}} / \left( \frac{(f - g)}{N} \right)$, equal to 0.20 reflecting estimates by Yashiv (2016a); iii) An unemployment rate of 10.6%. This value is the average of the time series for expanded unemployment rates produced by the BLS designed to account also for workers who are marginally attached to the labor force (U-6), consistently with our measure of the separation rate.\footnote{BLS series can be downloaded at: http://data.bls.gov/pdq/SurveyOutputServlet} We also note that the calibration implies a ratio of the opportunity cost of work to the marginal revenue product of labor of 0.77, which is close to the value of 0.745 advocated by Costain and Reiter (2008).

This calibration of hiring costs is conservative in the sense that total and marginal frictions costs lie at the low part of the spectrum of estimates reported in the literature. The papers of Krause, Lopez-Salido and Lubik (2008) and Gali (2011) assume that average hiring costs...
equal nearly 5% of quarterly wages, following empirical evidence by Silva and Toledo (2009) on vacancy advertisement costs. Our functional form for frictions costs – discussed extensively in Yashiv (2016 a,b) – allows for hiring costs to be interpreted in terms of training costs as well as all other sources of forgone output associated with hiring and discussed in Section 3.3 above. As reported by Silva and Toledo (2009), average training costs are about 55% of quarterly wages, a figure that is nearly ten times as large as that of vacancy posting costs. Blatter et al (2016) survey the literature and report estimates of hiring costs ranging between 25% and 131% of quarterly wages. In our calibration we follow the structural estimates in Yashiv (2016a) for U.S. data and prefer to err on the conservative side: the calibration target for marginal hiring friction costs in point (ii) above implies a ratio of $mcg_H / (W_t / P_t)$ around 27%, i.e., less than one month of wages. Note that the hiring rate $H_t / N_t$ in the data is in the interval $[0.110, 0.152]$ in the period 1976Q1-2014Q4. Hence the implied ratio of marginal hiring costs over steady state wages $mcA_t / W_t$, using our calibration values, ranges between 24% and 33% of quarterly wages. This represents relatively little variation and an upper bound that is well below the training costs found in the literature cited above. This exercise also shows that the convexity assumed in the hiring cost function (7) is mild. Notice that in the calibration we focus on marginal hiring friction costs, although Silva and Toledo report numbers for average hiring costs. Average hiring friction costs, computed as $\frac{mcA_t}{N_t}$, are close to two weeks of wages in our calibration. Note, too, that in our model the cost of hiring a marginal worker also includes, on top of the training costs discussed above $(g_H)$, the intrafirm bargaining costs $N_t W_{Hi} / P_t$ (see equation 13). These intrafirm marginal costs of hiring are equal to one month of wage payments.

Turning to the remaining parameters that have no impact on the stationary equilibrium, we set the Taylor rule coefficients governing the response to inflation and output to 1.5 and 0.125, respectively, as in Galí (2011), while the degree of interest rate smoothing captured by the parameter $\rho_r$ is set to the conventional value of 0.75 as in Smets and Wouters (2007). The Rotemberg parameter governing price stickiness is set to 120, to match a slope of the Phillips curve of 0.1, as implied by Gali’s (2011) calibration. As for the technology shocks, we assume an autocorrelation coefficient $\rho_a = 0.95$, while monetary policy shocks are assumed to be i.i.d.

### 4.2 Impulse Responses

#### Technology Shocks

Figure 1 plots the impulse responses to a positive technology shock in the following four versions of the model: (i) the model with both frictions – the NK model embodying price frictions together with hiring frictions (discussed in the calibration above, traced out by green

\[^{11}\text{Our value for } \xi \text{ is obtained by matching the same slope of the linearized Phillips Curve as in Gali: } \xi \text{ is uniquely determined by } \frac{\beta - \theta_p}{\theta_p} \text{, where } \theta_p \text{ is the Calvo parameter. Notice that for given values of } \epsilon \text{ and } \beta \text{, this equation implies a unique mapping between } \theta_p \text{ and } \xi. \text{ While Gali (2011) assumes Calvo pricing frictions, with } \theta_p = 0.75 \text{, we adopt Rotemberg pricing frictions, which implies that in our specification prices are effectively reset every quarter.} \]
We emphasize that this simple model is geared to explain a mechanism, and not to get empirical magnitudes or fit that are comparable to VAR outcomes; in Section (6) we look at an extended model and explore robustness of our mechanism in a richer framework.

In the NC model with no frictions (model iv, the black line) the impulse responses are the usual NC-type responses with employment, capital and output increasing following a positive technological innovation. Adding hiring frictions to the NC model (model ii, blue line) generates a mild smoothing in the response of real variables. Independently of hiring frictions, in the NC models, the real marginal cost does not respond. When price frictions are included, the real marginal cost falls following a positive technology shock. Quantitatively, this fall is very similar in the NK model without (model iii, red line) and with (model i, green line) hiring frictions. Yet, on the impact of the technology shock, the response of hiring, employment, and output in the standard NK benchmark is markedly different with respect to the NK model with frictions and the NC models. In Figure 1 there is a clear difference between the red lines and all other lines. Indeed, employment contracts substantially in this model, which attenuates considerably the initial response of output. Because marginal costs are not persistent in this simple NK model, and because, as will become apparent later, the different response of real variables across models is largely driven by their different sensitivity to marginal costs/relative prices, the difference in the responses beyond the first quarter are less pronounced. So over time, the pattern of responses is similar across the four model specifications, pointing to a rise in employment, capital and output, as well as in real wages.

Most importantly, the figure shows that the impulse responses of hiring, employment, investment, capital and output are virtually identical in the two models with hiring frictions (models i and ii above; the green and blue lines in the figure); this implies that in the presence of the hiring frictions assumed in the calibration, the response of these real variables is independent of the level of price frictions. Notably, employment rises following a positive technology shock. When exploring the mechanism below, we show variations on the calibrated frictions in each of the four models.

---

\[ e \] close to zero and not exactly equal to zero for ease of exposition. Notice that in the limit of \( e \to 0 \), the solution does not converge to the frictionless equilibrium because the wage in eq. (22) does not converge exactly to the marginal product of labor due to the intrafirm bargaining term \( W_N^N \). Moreover, for \( e = 0 \) there is no unemployment, and in the frictionless labor market equilibrium the restriction \( n_t + u_t = 1 \) must be lifted to analyze business cycle dynamics. So the model has a discontinuity at \( e = 0 \). Yet, solving the model with \( e = 0 \) for different values of \( \zeta \), would show the same qualitative pattern reported in Figures 3 and 4 below. Hence we abstract from this complication for illustrative purposes.
The magnitude of the real rate increase that follows a positive technology shock is very similar across models, which implies that the response of consumption is also similar. Hence, differences in the response of output across models are mostly explained by the different response of investment. Finally, we emphasize that adding hiring frictions onto the NK benchmark generates a smoother reaction in real wages, meaning that these frictions generate endogenous wage rigidity.

In the next section we explain the mechanisms underlying these results and we show how, for higher friction costs than assumed in the benchmark calibration, the interaction between price frictions and hiring frictions can also generate amplification to technology shocks.

**Monetary Policy Shocks**

Figure 2a plots impulse responses to an i.i.d. expansionary monetary policy shock in the same four versions of the model discussed above.

**Figure 2a**

The results show that in the absence of price frictions, money is neutral, independently of labor market frictions. In the NK benchmark (model iii) instead, the monetary policy shock has real effects, which lead to an increase in employment, investment, output and real wages.\(^{13}\) Most importantly, real variables respond very differently in the NK model without (model iii, red line) and with (model i, green line) hiring frictions: introducing these frictions virtually eliminates all real effects of monetary policy shocks, so that for all real variables except real marginal costs and wages, the response of the NK model with hiring frictions is indistinguishable from the response of the NC benchmark.

The irrelevance of price frictions in the transmission of shocks does not arise because marginal hiring cost are large, and hence quantities cannot move. Indeed, as noted in the case of technology shocks, in the absence of price frictions the real variables respond strongly, even in the presence of hiring frictions. As we explain more in detail in the next section, employment, and therefore output, do not respond because when hiring costs are internal to the firm, a change in marginal costs affects both the marginal revenue product and the marginal cost of hiring, in a way that leaves the marginal incentives to hire unchanged (at the calibrated value of \(c\)). Indeed, both the output that is produced by the marginal worker and the output that is forgone by incurring marginal recruitment costs are expressed in the same units of intermediate good output. Hence a change in the relative price of the intermediate good will affect not just the marginal revenue product, but also the real marginal cost of hiring.

Note that monetary policy shocks have very little persistence, even with the interest rate smoothing in the Taylor rule which we have assumed. So it is natural to investigate the robustness of these results in the case whereby monetary policy shocks display effects beyond the first quarter. We achieve this by assuming autocorrelated monetary policy shocks without interest rate smoothing (using \(\rho_c = 0.5\) and \(\rho_r = 0\)), as in Galí (2011). This is shown in panel

\(^{13}\)These effects do not last for long, as the model lacks propagation. More on this below, when discussing Figure 2b.
b of Figure 2. This alternative parameterization reproduces about the same autocorrelation of marginal costs as in Galí (2011, Figure 4a).

**Figure 2b**

In this case the model generates some real effects of monetary policy shocks, particularly through the response of investment, as the response of hiring is muted. However, the responses of output, employment and capital in the NK model with hiring frictions are small and substantially close to those of the NC benchmark.

We also notice that the response of real wages in both Figures 2a and 2b is smoothed when hiring frictions are introduced into the baseline NK model. Hence, in analogy with the case of technology shocks, hiring frictions generate endogenous real wage rigidity. To sum up, the qualitative effect of hiring frictions in Figure 2b is again to bring the outcomes of the model with both frictions (green line) closer to the frictionless NC case (black line).

We conclude that hiring frictions matter substantially in the transmission of both technology and monetary policy shocks. Specifically, these frictions offset the impact of price frictions on the propagation of shocks. In what follows we elucidate the mechanism that generates these results and explain what brings about the differences. In doing so, we explore variations on the calibration of each of the four cases.

5 Exploring the Mechanism

We aim to study the mechanisms producing the afore-going results: namely, (i) “NC-type outcomes” even with price frictions; (ii) amplification of real responses, in particular of labor market outcomes, as hiring frictions rise; (iii) endogenous wage rigidity; (iv) much smaller real effects for monetary policy. Because these are not the result of full-flexibility, but of a confluence of price and hiring frictions, we find it instructive to use 3D graphs to show how the response of real variables changes for various combinations of price and hiring frictions.

Specifically, we plot the response on impact to technology and monetary policy shocks for four variables: hiring rates, investment rates, real wages and output. For each variable, we look at how the response on impact changes as we change the parameters governing price frictions, $\zeta$, and hiring frictions, $e$. All other parameter values remain fixed at the calibrated values reported in Table 1.

Impulse responses to technology shocks and to monetary policy shocks upon impact are reported in Figures 3 and 4, respectively. In analogy with Figures 2a and 2b in Section (4.2), Figure 4a reports the responses to a monetary policy shock obtained under the benchmark parameterization, while Figure 4b shows the responses obtained in the case where we set interest rate smoothing equal to zero, and we introduce autocorrelation in the monetary policy shock, i.e. we set $\rho_r = 0$ and $\rho_{\xi} = 0.5$ as in Galí (2011), so as to generate persistence in marginal costs/relative prices.

**Figures 3, 4a and 4b**
The area colored in blue (red) denotes the pairs of \((\zeta, e)\) for which the impact response is positive (negative). Colored points mark in the figure four reference points, which correspond to the same four model variants considered in Section (4.2), and associated with different parameterizations: (i) the NC benchmark (black point); (ii) the NK benchmark (red point); (iii) the NC benchmark with hiring frictions (blue point); (iv) the NK benchmark with hiring frictions (green point). One aspect of the analysis to note is that the graphs above feature reasonable ranges of parameter values. For instance, the price stickiness parameter \(\zeta \in (0, 150]\] covers a range of values governing price rigidity that range from full flexibility to considerable stickiness, whereby the upper bound value for \(\zeta\), in Calvo space would correspond to an average frequency of price negotiations of four-and-a-half quarters. The hiring frictions parameter \(e \in (0, 5]\] ranges from the frictionless benchmark to a value of average friction hiring costs equal to approximately two month of wages, which is the average training cost reported in Silva and Toledo (2008). The reader can choose a region in the 3D space conforming to his/her own priors to gauge the results. Thus, while we indicate four points in this space, corresponding to the models under review, these serve as reference points and the graphs offer a “bigger picture”.

There are four notable points, in terms of the results read off the vertical axes, denoting the outcomes of the four real variables:

(i) The NK benchmark (red points) and the NC frictionless benchmark (black points) have very different outcomes in all cases. This is, of course, well known, but serves to place the results in context.

(ii) Adding hiring frictions to the frictionless benchmark, i.e., moving from the black to the blue points, results in relatively small changes, which reflect the moderate size of hiring frictions.

(iii) Starting from the green reference points, i.e., the model with all the frictions included, there are very substantial differences relative to the red points, i.e., the NK model with only price frictions. But there are only moderate to small differences relative to the black points, i.e., the frictionless NC model. The idea, then, is that the model with all the frictions together can yield outcomes that are close to the frictionless benchmark, even with the small values of hiring frictions imposed in the calibration.

(iv) Looking at the first panel of Figure 3, we note that for values of both price and hiring frictions that are relatively high – but within plausible empirical estimates – the hiring rate responds positively to technology shocks, and the response increases with hiring frictions. In this region of the parameter space the model generates amplification of employment responses relative to the NC benchmark.

The counter-intuitive results in points (iii) and (iv) are key and the subject of our analysis and explanations below.

To understand the transmission of both shocks in the presence of hiring frictions, it is important to understand what drives the hiring decision. For this purpose it is useful to write the optimality condition for gross hiring by merging the F.O.C. (12) and (13), expressed in units of consumption goods:
The LHS is an expected, discounted present value expression. This is the expected present value of a marginal job, \( Q^N_t \). This value is made up of current profits from the marginal hire \( (\text{mc}_t(f_{N,t} - g_{N,t}) - \frac{W_t}{P_t} N_t) \) and the expected future discounted profits \( (E_t \Lambda_{t+1} Q^N_{t+1}) \), conditional on no-separation \( (1 - \delta_N) \). The RHS is the real marginal cost of hiring, consisting of the marginal gross hiring costs \( (\text{mc}_t g_{H,t}) \) and the cost effects of the marginal hire on the wage bill \( (\frac{W_{H,t}}{P_t} N_t) \). We shall use this representation in what follows.

5.1 The Effects of Technology Shocks

Before we turn to the analysis of the four specific cases discussed above, it is worth noting that a positive technology shock decreases marginal costs, which serve as a relative price here \( \text{mc}_t \). This fall is induced by the increase in the net marginal product of labor (see equation (15)). Only in the special case where prices are fully flexible, relative prices do not move. The higher are price frictions, the stronger is the fall in marginal costs. The role of price frictions is thus expressed strongly through changes in the marginal costs.

In what follows we depict the mechanism, showing how the different frictions affect the propagation of a technology shock.

The NC Benchmark (black point in Figure 3)

With a frictionless labor market, the labor demand equation is found imposing \( g_{H,t} = g_{N,t} = W_{N,t} = W_{H,t} = Q^N_t = 0 \) for all \( t \) in the F.O.Cs for hiring (27). This yields the following negative relation between employment and the real wage:

\[
mc_t f_{N,t} = \alpha mc_t A_{t} N_t^{\alpha - 1} K_{t-1}^{1-\alpha} = \frac{W_t}{P_t}. \tag{28}
\]

Imposing the above conditions for a frictionless labor market into the wage equation (22) and using eq.(28) we can retrieve the labor supply equation:

\[
\frac{W_t}{P_t} = \chi C_t N_t^\varphi, \tag{29}
\]

which yields a positive relation between employment and the real wage. Equations (28) and (29) characterize the frictionless equilibrium of the labor market, which implies no unemployment. It also shows that in the absence of frictions, the hiring decision solves a static problem, that is, it only depends on the current state of technology. Following a positive technological innovation, i.e. higher \( A_{t} \), the labor demand schedule (28) shifts in such a way that for a given level of employment, the real wage must be higher. Because there is capital in the model, consumption rises proportionately less than real wages, and therefore labor supply in equation

\[\text{mc}_t (f_{N,t} - g_{N,t}) - \frac{W_t}{P_t} N_t + (1 - \delta_N) E_t \Lambda_{t+1} Q^N_{t+1} = \text{mc}_t g_{H,t} + \frac{W_{H,t}}{P_t} N_t. \tag{27}\]
Investment increases too, as $f_{K,t+1}$ rises. Finally, output rises as productivity and employment rise.

The NC Model with Labor Frictions (blue point in Figure 3)

This is a very similar case to the preceding one, but higher labor frictions now generate a slightly weaker response of hiring. Employment increases by less, dampening the response of real wages via the marginal rate of substitution $\chi_t C_t N_t^\theta$, and the response of investment via the marginal product of capital, due to complementarities in the production function. A slightly smaller increase in hiring and employment implies that also output increases a little less. Overall, labor frictions operate as a dampening mechanism, in line with the conclusions of Rogerson and Shimer (2011).

The NK Benchmark (red point in Figure 3)

The relevant F.O.C for the hiring decision is the same as in the NC benchmark, in equation (28). But now when $A_t$ rises $mc_t$ falls. So a positive technology shock, $e^a > 0$, produces two contradictory effects on employment: the direct positive effect on $A_t$, which, everything else equal, implies that employment increases; and an indirect effect through $mc_t$, which implies that, everything else equal, employment should fall. For conventional parameterizations of the monetary policy rule, general equilibrium effects imply that the fall in $mc_t$ is large enough for employment $n_t$ to fall (as the red point in Figure 3 indicates). Hiring rates need to fall to achieve that. Real wages go down as $mc_t$ and $\chi_t C_t N_t^\theta$ fall; investment rises, but by much less as employment falls, so by the complementarities in the production function the marginal product of capital increases by less; output rises but not by as much as in the NC benchmark because hiring (and therefore employment) falls.

The NK model with Labor Frictions and Price Frictions (green point in Figure 3)

The mechanism is somewhat involved so we proceed in steps.

(i) Two contradictory forces on hiring profitability. With frictions in the labor market, the hiring decision becomes a dynamic problem and will depend on the expectation of the entire sequence of the future states of technology and marginal costs. To understand the propagation of technology shocks in this framework it is useful to re-arrange the LHS of equation (27) substituting for the wage function (22) and its derivative $\frac{W_{H,t}}{P_t}$, and iterating forward on $Q_t^N$:

$$\sum_{s=0}^{\infty} (1-\delta_N)^s E_t A_{t+s} (1-\gamma) \left[ mc_{t+s} \left( f_{N,t+s} - g_{N,t+s} + D_{t+s}^1 \right) - D_{t+s}^2 \right] =$$

$$\frac{W_{H,t} (mc_t)}{P_t} N_t + mc_t g_{H,t}. \quad (30)$$

---

15The effect of a higher real wage on labor supply in equation (29) depends on the prevalence of income vs. substitution effects. With the standard assumption of a logarithmic utility function, and if there were no capital in the model, these two effects would cancel out. As a result $N_t$ would remain constant. But with investment, part of the increase in output is saved, and therefore consumption increases less than in the model without capital. Then, by equation (29) it is optimal to increase labor supply.
The term $D_1^1$, which is positive in the calibration, reflects the fall in the wage bill engendered by the marginal hire.\(^{16}\) $D_2^2 = \chi C_t N_t^0 + \frac{\gamma}{1 - \gamma} \frac{Q_t^N}{N_t}$ denotes the outside option of the worker.

The effect of an expansionary technological shock on hiring profitability, the LHS of equation (30), is again the result of two contradictory forces: (a) the direct, positive effect of the sequence of technology states $\{A_{t+s}\}_{s=0}^\infty$ on the present value of the job $Q_t^N$, manifested through the highlighted terms $f_{N,t+s} - g_{N,t+s} + D_{t+s}$, and (b) the indirect, negative effect that goes through the response of the expected sequence of marginal costs $\{mc_{t+s}\}_{s=0}^\infty$. The former is the standard NC mechanism as in Shimer (2005), while the latter reflects a NK channel that is active only in the presence of price frictions. Regarding the latter effect (b), note that by equation (15), an increase in the entire sequence of productivities implies a decrease in the sequence of marginal costs. In turn, the fall in marginal costs will decrease the real value of future expected profits on the LHS of equation (30) in a way that is akin to a negative technology shock, thereby offsetting the direct effect of an increase in productivity.

However, the fall in current period relative prices $mc_t$ will also decrease the real marginal hiring costs on the RHS of equation (30). So overall, the effect of technology shocks on hiring that operates through the sequence of marginal costs will be ambiguous, depending on whether relative prices have a stronger impact on marginal hiring profits (the LHS of (30)) or on costs (the RHS of (30)).

(ii) The relation between marginal costs/relative prices and marginal hiring costs. Recall that there are two distinct concepts here: (a) $mc_{t}$, marginal costs, which equal the relative price of the intermediate goods in terms of final goods $P_t^Z / P_t$; (b) marginal hiring costs, which are the loss of output on the marginal hire. To see how a change in marginal costs or relative prices $mc_t$ affects the marginal hiring cost, it useful to spell out the two terms on the RHS of (30), by replacing for the friction cost $g_{H,t}$ using the functional form in (7):

$$mc_t g_{H,t} = mc_t {A_t (K_{t-1} / N_t)}^{1-a} \frac{H_t}{N_t},$$  \hspace{1cm} (31)

and differentiating the wage function (23) with respect to hiring:

$$\frac{W_{H,t} N_t}{P_t} = mc_t \gamma A_t (K_{t-1} / N_t)^{1-a} \frac{2 - \alpha}{1 - \gamma + \gamma (\alpha - 2)} \frac{H_t}{N_t}.$$  \hspace{1cm} (32)

We note that $mc_t g_{H,t}$ is always positive, and $\frac{W_{H,t} N_t}{P_t}$ is positive in the calibration.

The impact of a change in relative prices $mc_t$ on marginal hiring costs is therefore:

$$\frac{\partial \left( \frac{W_{H,t} N_t}{P_t} + mc_t g_{H,t} \right) }{\partial mc_t} = e A_t \gamma A_t \left( \frac{\gamma (K_{t-1} / N_t)^{1-a} (2 - \alpha)}{1 - \gamma + \gamma (\alpha - 2)} + \frac{(K_{t-1} / N_t)^{1-a}}{1 - \gamma + \gamma (\alpha - 2)} \right) = \frac{Q_t^N}{mc_t}.$$  \hspace{1cm} (33)

\(^{16}\)The term $D_1^1$ is obtained by deriving the wage function in equation (23), and equals:

$$D_1^1 = -\frac{W_{H,t} N_t}{mc_t} = \gamma A_t \left( \frac{K_{t-1}}{N_t} \right)^{1-a} \left[ a \left( 1 - \alpha \right) \left( 1 - \frac{\alpha}{2} \right) + \left( 1 - \frac{\alpha}{2} \right) \frac{e (3 - \alpha)}{1 - \gamma (3 - \alpha)} \right] \frac{H_t}{N_t}.$$
The first equality in the derivative above reveals that a fall in $mc_t$ will decrease both components of the marginal cost of hiring for any $e > 0$, and the extent of this fall increases with the value of $e$. The second equality shows that this effect is proportional to the value of a job to the firm.

(iii) **Comparison to the NK case without hiring frictions.** Consider the response of hiring in the case where hiring frictions are negligible, i.e. $Q^N_t \simeq 0$. In terms of the space in Figure 3, we are looking at the red point, which marks the NK model with no labor frictions discussed above. In this case, the fall in the marginal costs series will decrease profits on the LHS of (30), but, by equations (33), will not decrease costs on the RHS of the same equation. Hence, the response of marginal costs to a positive technology shock will induce a fall in hiring. For conventional parameterizations of the Taylor rule, this effect (effect (b) of point (i)) dominates the direct positive effect of productivity (effect (a) of point (i)), so employment falls.

Now note the changes that take place when moving away from the red point in Figure 3, the NK model with price frictions only, towards the green point, marking the NK model with moderate hiring frictions, as well as price frictions. As $e$ increases, the marginal cost of hiring becomes more sensitive to a change in relative prices. For values of $e$ that are sufficiently large, the fall in marginal hiring costs will be large enough to turn the response of hiring from negative to positive. This effect derives from interacting price frictions (which exist at both red and green points) and hiring frictions (which rise going from the red point to the green point).

(iv) **Key result.** At the calibrated equilibrium, for parameterizations of labor market frictions that reflect conservative estimates of training costs, the response of hiring on the impact of technology shocks is positive. We notice that by offsetting the mechanism at work in the NK model (effect (b) of point (i)), the decline of marginal hiring costs on the RHS of (30) produces the counter-intuitive result whereby hiring actually increases with hiring frictions. It is worth noting that in the region of the parameter space where $e$ takes high, but still reasonable values, the response of hiring is stronger than in the flexible price (NC) economy. Hence, and in contrast to the conclusions reached by Rogerson and Shimer (2011), hiring frictions of the type that give rise to equilibrium unemployment matter for the amplification of employment to technology shocks. This novel result arises because of an interaction between price and hiring frictions, which is absent in DMP models.

The derivative in equation (33) indicates that the offset to the standard NK mechanism is produced by the existence of rents, i.e. $Q^N_t > 0$. Hence, changes in relative prices will affect marginal hiring costs independently of the precise specification that produces rents (for example, the explicit modelling of a matching function as in the standard DMP model, or the precise degree of convexity in our specification). What matters is that frictions involve some form of forgone output. In this case the term $mc_t$ appears on the RHS of equation (30), and the effect of a change in relative prices affects both marginal profits and costs.

(v) **Context in the literature.** In most NK models with hiring frictions, marginal hiring costs have been modelled as external pecuniary costs rather than forgone output, and therefore they are not affected by changes in the relative price $mc_t$ (cf. Gertler Sala and Trigari (2008), Gali, (2011), Christiano Eichenbaum and Trabandt (2016)). In these cases, the NK transmission channel qualitatively works in the same way as in the version with no labor frictions.
While there is no consensus on the effect of technology shocks on employment and hours, the positive response of employment produced by the model with moderate hiring costs is consistent with VAR evidence in Uhlig (2004), Christiano, Eichenbaum and Vigfusson (2004), and Sims (2011). In particular, the latter paper argues that the inability to match the positive response of total hours to a transitory technology shock is a major failure of current DSGE models.

(vi) **Endogenous wage rigidity.** In the NK model with hiring frictions (the green point), the increase in employment implies that wages fall by less than in the NK model with a frictionless labor market; the effect of employment on the marginal rate of substitution endogenously dampens the reaction of real wages. It is also worth noting that for values of $\zeta$ around 60, which map into a Calvo price stickiness of $2 - 2\frac{1}{2}$ quarters, increasing $\varepsilon$ can turn the response of real wages to technology shocks from negative to positive, that is, it reproduces the qualitative response that we observe in a NC benchmark. We note that in Figure 3 the real wage response is noticeably further away from the NC+L friction point, unlike the other cases. This happens because the real wage depends on the marginal revenue product, which in turn depends on the marginal cost. Because the marginal cost falls only with price frictions, the real wage response will be lower than in the NC case with labor frictions.

(vii) **Effects on capital and output.** Investment rises substantially; as hiring rates increase with higher labor market frictions $\varepsilon$, the marginal productivity of capital rises. Finally, output rises substantially as productivity and employment rise.

(viii) **In summary,** as Figure 3 shows, adding conservative estimates of labor (hiring) frictions to price frictions brings the NK model closer to the results of the NC model with labor frictions, i.e., offsets to a significant extent the effects of price frictions. Raising frictions even further generates amplification of hiring and employment responses, relative to the flexible price counterpart.

### 5.2 The Effects of Monetary Policy Shocks

With a monetary expansion, the demand stimulus induced by a fall in the nominal interest rate increases the demand for labor and hence the real wage. In turn, $mc_t \equiv \frac{p_t}{P_t}$ in equation (15) rises, and this response increases with price frictions $\zeta$. Only in the special case where prices are fully flexible, relative prices do not respond.

**The NC Benchmark (black points in Figures 4 a,b)**

Nothing changes in the F.O.C of the firm, so real variables do not respond. This is an expression of money neutrality.

**The NC Model with Labor Frictions (blue points in Figures 4 a,b)**

Because marginal costs do not respond to monetary policy shocks, money is still neutral.

**The NK Benchmark (red points in Figures 4 a,b)**

\[
\text{See footnote 11.}
\]
The relevant equation is (28). Marginal costs respond to monetary policy shocks; as \( mc_t \) rises, employment \( n_t \) rises and so does hiring. The response of the other real variables follows: real wages rise as \( mc_t \) and \( \chi_t C_t N_t^\phi \) rise; the response of the investment rate is positive as the marginal cost rises and the marginal product of capital increases with employment; output rises when hiring rates and employment rise.

The NK Model with Labor Frictions (green points in Figures 4 a, b)

Again we proceed in steps.

(i) Two contradictory forces. The key equation to be used here is (30), which we reproduce below for convenience:

\[
\sum_{s=0}^{\infty} (1 - \delta)^s E_t \Lambda_{l,t+s} (1 - \gamma) \left[ mc_{t+s} \left( f_{N,t+s} - g_{N,t+s} + D^1_{t+s} \right) - D^2_{t+s} \right] = \frac{W_{H,t} (mc_t)}{P_t} N_t + mc_t g_{H,t}.
\]

An expansionary monetary policy shock produces an increase in the sequence of relative prices \( \{ mc_{t+s} \}_{s=0}^{\infty} \). In analogy with the previous discussion of technology shocks, this will increase marginal profits on the LHS of equation (34), which, everything else equal, implies that employment increases; concurrently, it will also increase marginal hiring costs on the RHS of the same equation, which, everything else equal, implies that hiring decreases. Importantly, as with equation (33), the effect of an increase in \( mc_t \) on the two components of marginal hiring costs increases with \( e \). The main difference, relative to the case of technology shocks, is that monetary policy shocks affect hiring only through their impact on marginal costs, with no direct effect on productivity.

(ii) Comparison to the NK case without hiring frictions. Consider the case where \( e \approx 0 \) (the red points in Figures 4 a, b). Hiring will increase, since the rise in marginal costs increases profits, leaving marginal hiring costs unaffected. Moving from the red point to the green point, as \( e \) rises and the value of a job \( Q^N \) increases, marginal hiring costs become more sensitive to the relative price \( mc_t \) (eq.(33)). In a region of the parameter space where \( e \) is associated with moderate friction costs, the change in marginal hiring costs will be approximately equal to the change in marginal profits, so real variables hardly move. Increasing \( e \) even further towards the region where friction costs are relatively high, but still reasonable and in line with the evidence reported in Silva and Toledo (2009), implies that the response of hiring eventually turns negative.

(iii) Key results. Under the assumption of hiring frictions equal to roughly one month of wages, a monetary stimulus is effectively neutral (see green points in Figures 4 a, b).

For values of hiring frictions that are higher than assumed in our calibration, marginal costs, conditional on monetary policy shocks, are contractionary and thus countercyclical, in line with empirical evidence by Nekarda and Ramey (2013), and in contrast to the predictions of the textbook NK model.

(iv) Effects on real wages, capital, and output. Because employment is virtually unaffected by
a monetary policy shock, the marginal rate of substitution, and therefore the real wage, do not respond as much as in the NK model. Thus labor frictions generate endogenous real wage rigidity by containing movements in the marginal rate of substitution. Because employment does not respond, the productivity of capital remains unchanged, hence investment does not respond. Output also remains unchanged and money is virtually neutral.

(v) Serial correlation in monetary policy shocks. As shown in Figure 2a, the response of the marginal cost is not persistent when the monetary policy shock is i.i.d. Hence, a natural question to explore is whether higher persistence in marginal costs would restore real effects of monetary policy that are quantitatively similar to those produced in the NK benchmark. To tackle this issue, we repeat the analysis in Figure 4b and assume a serial correlation in the monetary policy shocks. This alternative parameterization reproduces about the same autocorrelation of marginal costs as in Galí (2011, his Figure 4a). The result is that monetary policy shocks generate some real effects, mostly through investment, but overall the impact of monetary policy shocks is substantially lower than in the baseline NK model. The main insight from Figures 4a and 4b, then, is that the NK real effects of monetary shocks taper-off with mild deviations from the frictionless NK benchmark, even with reasonable persistence in marginal costs/relative prices. The main effect of increased persistence is to raise the threshold value of $e$ at which expansionary monetary policy shocks become contractionary.

(vi) The role of hiring frictions and VAR evidence. The simple model presented here highlights the importance of labor frictions in the transmission of monetary policy shocks. The precise threshold of $e$ that delivers no response of real variables on the impact of the shock will depend both on the parameterizations and on the modelling assumptions. Quite clearly, the simple model we use (presented in Section (3) above) abstracts from many assumptions that are prevalent in DSGE modelling, and which we include in the robustness Section (6) below. But Figures 4a and 4b reveal a main theme that remains valid even in larger scale versions of the model presented here: on the one hand, these figures show that there exists a range of reasonable joint parameterizations for $e$ and $\zeta$, such that the model produces real effects of money. These results are consistent with a multitude of VAR studies based on a number of identification hypotheses. Many of these studies rely on the recursiveness assumption, meaning that output and inflation cannot react contemporaneously to changes in the interest rate (see Ramey, 2016). On the other hand, Figures 4a and 4b show that there also exists a wide range of alternative reasonable parameterizations under which monetary policy has smaller real effects, or even contractionary ones, which is consistent with the results based on an agnostic VAR identification scheme, as proposed in Uhlig (2005), and also reported in Faust, Swanson and Wright (2004) and Amir and Uhlig (2016). As noted by Ramey (2016), the conventional effects of monetary policy in VAR analysis are significantly overturned when restricting the sample, so as to include only the great moderation, or lifting the recursiveness assumption, which is violated by standard assumptions on the Taylor rule, as in eq.(25). For instance, using the Romer and Romer’s (2004) monetary policy shock as an instrument, or the proxy SVAR method, leads to a significant increase in industrial production following a contractionary policy shock in its first year (Ramey (2016)).
Here we do not take a stance on whether monetary policy has real effects or not, and in what way. What we take away from this analysis is that hiring frictions matter for the transmission of monetary policy shocks, and a structural assessment of the transmission of such shocks cannot abstract from a careful quantitative evaluation of labor market frictions.

6 The Extended Model

6.1 A Larger Scale Model

The model laid-out in Section 3 is relatively simple and abstracts from various features that are prevalent in medium-scale DSGE models. In this sub-section we augment the simple model with investment adjustment costs, habits in consumption, exogenous wage rigidity, trend inflation and indexation to past inflation. We do not aim to produce a fully-fledged DSGE model that should be considered as our best characterization of the actual US economy; rather, we want to show that the effects generated by internal hiring frictions remain important even in a richer model. We relegate the full description of the model to Appendix B.

In what follows we summarize the main additions to the model of Section (3). We now assume that the law of motion for physical capital evolves following the process:

$$K_t = (1 - \delta_K)K_{t-1} + 
1 - S \left( \frac{I_t}{I_{t-1}} \right) I_t,$$

where $S$ is an investment adjustment cost function, and it is assumed that $S(1) = S'(1) = 0$, and $S''(1) \equiv \phi > 0$.}

We assume that the Rotemberg price adjustment costs faced by final good firms depend on the ratio between the new reset price and the one set in the previous period, adjusted by a geometric average of steady state inflation and past inflation. Specifically, final good firms maximize the following expression:

$$\max E_t \sum_{s=0}^{\infty} \Lambda_{t, t+s} \frac{P_t}{P_{t+s}} \left[ (P_{t+s}(i) - P_{t+s}mc_{t+s}) Y_{t+s}(i) - \frac{\zeta}{2} \left( \frac{P_{t+s}(i)}{(\pi_{t+s-1})^{\psi}} \left( \frac{P_{t+s}(i)}{P_{t+s-1}(i)} \right)^{1-\psi} - 1 \right) \right] \left( \frac{P_{t+s}}{P_{t+s}(i)} \right)^2 P_{t+s} Y_{t+s},$$

where $\pi$ denotes steady-state inflation and $\psi$ denotes the degree of indexation to past inflation. This specification gives rise to a backward looking term in the NK Phillips curve.

As for the household, we now assume external habits in consumption, meaning that the preferences of a household indexed by $j$ are described by the following utility function:

$$U_{j,t} = \ln \left( C_{j,t} - \theta C_{j,t-1} \right) - \frac{N_{j,t}}{1 + \phi},$$

where $\theta \in [0, 1)$ is the habit parameter.

We remove the assumption that wages are bilaterally renegotiated in every period, thereby
abandoning the intra-firm bargaining protocol and the underlying assumption that firms correctly anticipate the impact of their hiring and investment policy on the negotiated wage. We instead assume wage rigidity in the form of a Hall (2005) type wage norm:

$$\frac{W_t}{P_t} = \omega \frac{W_{t-1}}{P_{t-1}} + (1 - \omega) \frac{W_t^{NASH}}{P_t}, \quad (38)$$

where $\omega$ is a parameter governing real wage stickiness, and $W_t^{NASH}$ denotes the Nash reference wage

$$W_t^{NASH} = \arg \max \left\{ \left( V_t^N \right)^\gamma \left( Q_t^N \right)^{1-\gamma} \right\}. \quad (39)$$

This simple wage-setting rule allows for targeting the persistence of the real wage data series in the calibration of the model.

The model is calibrated following the same steps as in Section (4.1). The parameter values for the friction cost scale parameter $e$ and the bargaining power $\gamma$ are set so as to hit the same targets as in the calibration of the simple model. The scale parameter in the utility function $\chi$ is no longer normalized to equal one, but is set so as to target the same replacement ratio of the opportunity cost of work over the marginal revenue product (77%), as implied by the benchmark calibration in Section (4.1). All other parameter values that are common to the simple model are set to the same value reported in Table 1. As for the new parameters, the investment adjustment cost parameter $\phi$ is set to 2.5, and the habit parameter to $\vartheta = 0.8$, reflecting the estimate by Christiano, Eichenbaum and Trabandt (2016). The parameter governing trend inflation is set to $\bar{\pi} = 0.783\%$, which corresponds to the average of the US GDP deflator over the calibration period. Given that, the value of the discount factor $\beta$, is set so as to match a 1% nominal rate of interest. Finally, we set the degree of indexation to a moderate value of $\psi = 0.5$, and the parameter governing wage rigidity to $\omega = 0.8$, as in Chistoffel and Linzert (2010), in order to match the persistence of the US real wage data. Parameter values and calibration targets for the extended model are reported in Table 2.

Table 2

Figure 5 reports impulse responses for a technology shock obtained under the benchmark parameterization with moderate friction costs, $e = 1.5$ (the green solid line), and an alternative parameterization with a higher, but still reasonable friction cost (the purple broken line). For the “high” friction case we assume a value of $e = 5$, which implies average friction costs equal to two months of wages as in Silva and Toledo (2009), and marginal hiring cost equal to about three months of wages, which is still below the upper bound for hiring costs reported by Blatter et al. (2016).

Figure 5

Figure 5 shows that the lower calibration of friction costs is not sufficient to turn the response of employment from negative to positive on the impact of the technology shock. A
larger friction parameterization instead can. Yet, most strikingly, increasing hiring costs implies
a much stronger expansionary response of employment, investment, output and consumption,
which increase over the impulse response horizon showing persistent, hump-shaped dynam-
ics. This counterintuitive result, whereby higher frictions magnify the response of real variables
in a NK model, is in accordance with the discussion of the mechanism presented in Section (5.1),
whereby labor market frictions increase the sensitivity of marginal hiring costs to changes in
relative prices. As a result, higher labor frictions generate a stronger response of employment
and production on the supply side, and a stronger reaction of consumption and investment on
the demand side. Summing up, Figure 5 shows that moderate changes of hiring frictions in the
extended model with price frictions produce dramatic effects on the transmission of technology
shocks.

A complementary and insightful approach to identify and visualize the effect of the interac-
tion between price frictions and hiring frictions is to show how price frictions affect the trans-
mission of technology shocks in a model with hiring frictions. The natural focus, in this context,
is on the behavior of unemployment, which has sparked a large literature since Shimer (2005),
as discussed in Section (2). We do so in Figure 6, where we compare the impulse responses ob-
tained under the same “high” labor friction case reported in Figure 5 (traced out by the purple
broken lines), with the otherwise identical model where we shut down price frictions, i.e. we
set \( \zeta \approx 0 \) (this is traced out by the yellow solid lines). In Figure 6 we label the rigid price model
as NK+L Frictions, and the flexible price model as NC + L Frictions.

Because the latter is effectively a rich specification of the DMP model with capital, Figure
6 allows us to pin down the effects of introducing price frictions into this DMP benchmark.
The figure reveals that the mechanism produces strong amplification of unemployment to the
underlying TFP shock, with an impact elasticity around 6 and a peak elasticity around 8 in the
presence of both hiring frictions and price frictions. This compares with an impact – and peak –
elasticity around 1.8 under flexible prices. Indeed, the hump-shaped impulse response of
unemployment to technology shocks disappears when prices are made fully flexible. Hence,
introducing price frictions into a model with hiring frictions generates both volatility and end-
dogenous persistence in the response of unemployment to technology shocks. The mechanism,
once again, is the one discussed in Section (5.1): price rigidities increase the sensitivity of rela-
tive prices to the technology shock. In turn, when hiring costs are internal, relative prices affect
the incentives for job creation by inducing changes in the marginal cost of hiring. If the friction
costs are large enough, the fall in relative prices induced by an expansionary technology shock,
decreases the marginal cost of hiring by more than the marginal profit, amplifying the increase
in hiring.

It is worth noting that in the case where there are no price frictions (the yellow line), the
model lacks amplification, despite the high level of real wage rigidities imposed in the cal-
ibration \( \omega = 0.8 \). This is because the opportunity cost of work, \( \chi C_t N_t^\phi \), is procyclical in
our model. Using detailed microdata, Chodorow-Reich and Karabarbounis (2016) provide evidence that the opportunity cost of work is indeed procyclical; they show that under this assumption many leading models of the labor market, including models with rigid wages, fail to generate amplification, irrespective of the level of the opportunity cost. The amplification of labor market outcomes generated in our model by the interaction of hiring and price frictions is instead robust to the procyclicality of the opportunity cost of work.

In analogy with Figure 5, Figure 7 reports impulse responses for a monetary policy shock obtained under the same “low” and “high” parameterizations of friction costs.

**Figure 7**

The impulse response analysis reveals that at the lower level of friction costs (green line), an expansionary monetary policy shock produces real effects, reducing the real rate of interest and increasing output, consumption, employment, investment, and real wages. At the higher level of friction costs instead (purple line), monetary policy shocks still produce real effects, but in the opposite direction. These results are consistent with those that were obtained with the simple model of Section (3). As explained in Section (5.2), increasing labor friction costs implies that the marginal cost of hiring becomes more sensitive to the increase in the relative price. Increasing friction costs reduces the effectiveness of monetary policy until a threshold where most real variables do not respond on the impact of the shock. Beyond that threshold, the NK propagation of monetary policy shocks is reversed, with a negative shock to the nominal interest rate leading to a contraction in real economic activity.

In the “high” frictions case, the incentives for job creation fall on the impact of an expansionary monetary shock, so the production of intermediate and final output must also fall. Given this fall in supply, in equilibrium, the relative price of intermediate goods $m_{c_t} = P^Z_t/P_t$ must increase strongly so that inflation curbs the aggregate demand stimulus generated by the fall in the interest rate. In the “low” frictions case instead, the aggregate demand stimulus is absorbed by an increase in output supply, and as a result marginal costs, and therefore prices, do not need to increase as much.

We emphasize that the parameterization of friction costs underlying the purple line, which corresponds to the survey evidence of hiring costs reported in Silva and Toledo (2011), is a

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18We note that the response of the real marginal cost is not persistent in the “high” frictions case, even in the presence of wage rigidities. This is because the marginal cost is mostly explained by the frictional component, i.e. the third term on the R.H.S. of eq.(15), which is not persistent. This result is in contrast to the dynamics generated by the canonical, frictionless NK model, where marginal costs are only driven by the first term in eq.(15), the “labor share.” In this frictionless model, real wage rigidities directly induce persistence in the real wage and therefore in the marginal cost. In the “low” parameterization, the contribution of the frictional component to the variation of marginal costs is relatively less important. This model is thus relatively closer to the NK frictionless benchmark, and therefore the real marginal cost will reflect more closely the response of the “labor share.” In this “low” frictions case, wage rigidities imply that the response of marginal costs will also be persistent.
perfectly reasonable parameterization, and is labeled in Figures 5 and 7 as “high” friction cost purely for comparative reasons. So the bottom line of the analysis presented in this Section, is that changing hiring costs within a reasonable, moderate range of parameterizations, has dramatic implications for the propagation of shocks even in a relatively rich specification of the model.

6.2 Variations on the Taylor Rule

It is well known that in NK models the dynamics of the endogenous variables are sensitive to the precise parameterization of the Taylor rule coefficients. In particular, the negative response of employment to technology shocks is a fragile result, which relies on specific assumptions on the parameters of the Taylor rule. Specifically, the simple model of Section (3) requires positive and high enough values of interest rate smoothing to achieve this result. So, in order to show that the offsetting effect of frictions on the standard NK dynamics does not depend on the parameters of the Taylor rule, we have carried out the following robustness exercise.

We take as a benchmark the version of the extended model parameterized with comparatively high frictions, i.e. \( e = 5 \). As discussed in the previous sub-section, under this parameterization an expansionary technology shock produces an increase in employment and an expansionary monetary policy shock produces a contraction in output. To show that these results are a genuine manifestation of the offsetting effect of friction costs, and not an artifact of a specific Taylor rule, we inspect impulse responses obtained by randomizing the Taylor rule coefficients over a broad parameter space, leaving all other parameters fixed at the values reported in Table 2.

Specifically, we have generated 10,000 parameterization vectors, which differ only in the coefficients governing the Taylor rule. These parameter values are assigned by drawing randomly from uniform distributions defined over the support of \( r_y \sim U(0, 0.5) \), \( r_\pi \sim U(1.1, 3) \) and \( \rho_r \sim U(0, 0.8) \). Our results indicate that output responded negatively on the impact of a monetary stimulus in every single parameterization, and the sign of the response was never overturned one year or two years after the impact. Similarly, on the impact of the technology shock instead, employment responded positively in every single parameterization. The sign of the response was not overturned after one year in any of the parameterizations and remained in positive territory, after two years, in 99.8% of the parameterizations.

The intuition for the role of smoothing in the textbook NK model without hiring frictions is the following: a positive technology shock implies that the same level of demand can be achieved with less labor, so the demand for labor falls. At the same time, the marginal product of labor increases, and therefore marginal costs and inflation fall. Because inflation falls, the Central Bank responds by lowering the nominal interest rate, which stimulates aggregate demand and counteracts the afore-mentioned effects. With a simple Taylor rule (no smoothing), the net effect is a negligible change in employment. With a simple Taylor rule (no smoothing), the net effect is a negligible change in employment. With a simple Taylor rule (no smoothing), the net effect is a negligible change in employment. The sign of the response was not overturned after one year in any of the parameterizations and remained in positive territory, after two years, in 99.8% of the parameterizations.

\(^{19}\)The intuition for the role of smoothing in the textbook NK model without hiring frictions is the following: a positive technology shock implies that the same level of demand can be achieved with less labor, so the demand for labor falls. At the same time, the marginal product of labor increases, and therefore marginal costs and inflation fall. Because inflation falls, the Central Bank responds by lowering the nominal interest rate, which stimulates aggregate demand and counteracts the afore-mentioned effects. With a simple Taylor rule (no smoothing), the net effect is a negligible change in employment. The sign of the response was not overturned after one year in any of the parameterizations and remained in positive territory, after two years, in 99.8% of the parameterizations.
7 Conclusions

This paper has shown that hiring and price frictions interactions matter. Moderate deviations from the standard assumption of frictionless labor markets have dramatic implications for the outcomes of real variables in NK models. Hence, we need to have relatively precise estimates of both price frictions and hiring frictions in order to gauge the true real effects of demand and supply shocks in a DSGE framework. Most of the empirical research in this field has focused on measuring price rigidities, under the prevalent belief that this is a necessary statistic to gauge the strength of the New-Keynesian mechanism. Our results indicate that if hiring frictions are more than tiny, but still moderate, the precise degree of price rigidity is less relevant, if not irrelevant, in the propagation of shocks to real variables. For higher, yet not implausible values of frictions costs, the conventional New Keynesian propagation mechanism is turned upside-down. Therefore, a correct assessment of hiring costs is key in the calibration of business cycle models. At the same time, we showed that price frictions, interacted with these hiring frictions, are important for DMP-type analysis.

Our model features internal costs of hiring that are incurred after a match is formed. This differs from a prevalent formulation, whereby the costs of hiring are external. The latter was typically used for simplicity. By changing the notion of the hiring costs, we reverse the conclusions obtained by different papers, reviewed above, which found a negligible role for hiring frictions in business cycle models. We argue that it is the gross hiring rate, and the costs associated with it, which should feature in the estimation of models that assume labor market frictions either explicitly, i.e. structural DSGE models, or implicitly, i.e., reduced-form VAR models. The quantification of hiring costs is among the key objectives in the estimation of the model, which is in progress.

This paper has explored the theoretical mechanisms by which frictions in labor markets affect the propagation of shocks to real variables in DSGE models with price rigidities. There may be implications also for inflation and monetary policy. Sbordone (2005) has empirically confirmed the importance of forward-looking terms in accounting for inflation dynamics. Given the New Keynesian Phillips Curve in equation (19), the current paper has demonstrated the potential role of hiring frictions in this context. These frictions enter through current and expected future marginal costs. Hence, one would need to re-examine the role of labor frictions in DSGE models used in central banks. Possibly, the formulation of monetary policy itself could be affected. Thus, the analysis may inform policymakers of variables, such as those related to hiring frictions, that need to be taken into account when setting monetary policy strategies under inflation-targeting policy.20

References


20For the modelling of this policy see the discussion in Giannoni and Woodford (2005).


Appendix A
Solving for the Wage with Intrafirm Bargaining

We rewrite below for convenience the wage sharing rule consistent with Nash bargaining as derived in equation (21):

\[(1 - \gamma) V_t^N = \gamma Q_t^N, \tag{40}\]

where we make use of subscripts \(i\) and \(j\) to denote a particular household \(i\) and firm \(j\) bargaining over the wage \(W_{jt}\). Substituting (5) and (12) into the above equation one gets:

\[
\begin{align*}
\gamma \left\{ mc_{jt} \left( f_{N,jt} - g_{N,jt} \right) - \frac{W_{jt}}{P_t} - \frac{W_{N,jt}}{P_t} N_{jt} \right\} + (1 - \delta_N) E_t \Lambda_{t, t+1} Q_{jt+1}^N &= \\
(1 - \gamma) \left[ \frac{W_{jt}}{P_t} - \chi N_{it}^q C_t - \frac{x_t}{1 - x_t} V_{jt}^N + (1 - \delta_N) E_t \Lambda_{t, t+1} V_{jt+1}^N \right].
\end{align*}
\]

Using the sharing rule in (40) to cancel out the terms in \(Q_{jt+1}^N\) and \(V_{jt+1}^N\) we obtain the following expression for the real wage:

\[
W_{jt} = \gamma mc_{jt} \left( f_{N,jt} - g_{N,jt} \right) - \frac{W_{N,jt}^N}{P_t} N_{jt} + (1 - \gamma) \left[ \chi C_{it} N_{it}^q + \frac{x_t}{1 - x_t} \frac{\gamma}{1 - \gamma} Q_{it+1}^N \right]. \tag{41}\]

Ignoring the terms in square brackets, which are independent of \(N_{jt}\) and can therefore be treated as a constant, and dropping all subscripts from now onward with no risk of ambiguity, we can rewrite the above equation as follows:

\[
W_N + \frac{1}{\gamma N} W - Pmc \left( \frac{f_N}{N} - \frac{g_N}{N} \right) = 0 \tag{42}\]

The solution of the homogeneous equation, \(W_N + \frac{1}{\gamma N} W = 0\), is

\[
W(N) = CN^{-\frac{1}{\gamma}}, \tag{43}\]

where \(C\) is a constant of integration of the homogeneous equation. Assuming that \(C\) is a function of \(N\) and deriving (43) w.r.t. \(N\), yields:

\[
W_N = C_N N^{-\frac{1}{\gamma}} - \frac{1}{\gamma} CN^{-1-\frac{1}{\gamma}}. \tag{44}\]

Substituting (43) and (44) into (42) one gets:

\[
C_N = N^{1 - \frac{1}{\gamma}} Pmc (f_N - g_N). \tag{45}\]

Integrating (45) yields:

\[
C = Pmc \int_0^N z^{1-\gamma} (f_z - g_z) dz + D, \tag{46}\]

38
where $D$ is a constant of integration. Let's solve for the two integrals in $f_z$ and $g_z$, one at a time. Assuming that $f(Az, K) = Az^\alpha K^{1-\alpha}$, we can write:

$$Pmc \int_0^N \frac{1}{\gamma} f_z dz = Pmc \alpha \gamma AN^{\frac{1-\gamma(1-\alpha)}{\gamma}} K^{1-\alpha}.$$ (47)

Given our assumptions on the functional form of $g$ as in (7), the function $g_N$ can be rearranged as follows:

$$g_N = -AK^{1-\alpha} e^{H_2 N^\alpha-3} + \alpha AK^{1-\alpha} e^{H_2 N^\alpha-3}. (48)$$

Integrating the first term in the RHS of the above equation yields:

$$Pmc \int_0^N \frac{1}{\gamma} AK^{1-\alpha} e^{H_2 z^{\alpha-3}} dz = Pmc e^{H_2 N} \frac{\gamma}{1-\gamma(\alpha-2)} N^{\frac{1-\gamma(\alpha-2)}{\gamma}}. (49)$$

Integrating the second term in squared brackets on the RHS of equation (48) yields:

$$-Pmc \int_0^N \frac{1}{\gamma} \alpha AK^{1-\alpha} e^{H_2 z^{\alpha-3}} dz = -Pmc e^{H_2 N} \frac{\gamma}{1-\gamma(\alpha-2)} N^{\frac{1-\gamma(\alpha-2)}{\gamma}}. (50)$$

Denoting $A_1 \equiv \frac{\gamma}{1-\gamma(1-\alpha)}$ and $A_2 \equiv \frac{\gamma}{1-\gamma+\gamma(\alpha-2)}$, we can now rewrite (46) as follows:

$$C = D + Pmc AK^{1-\alpha} \left[ \alpha A_1 N^{1/A_1} + \left( 1 - \frac{\alpha}{2} \right) H_2^2 A_2 N^{1/A_2} \right]. (51)$$

Plugging (51) into (43) one gets:

$$W(N) = D N^{-\frac{1}{\gamma}} + Pmc AK^{1-\alpha} \left( \alpha A_1 N^{a_{-1}} + \left( 1 - \frac{\alpha}{2} \right) e^{H_2^2 A_2 N^{a_{-3}}} \right). (52)$$

In order to eliminate the constant of integration $D$ we assume that $\lim_{N \to 0} NW(N) = 0$. The solution to (41) therefore is:

$$\frac{W_t}{P_t} = \gamma mc A_1 K^{1-\alpha} \left\{ \frac{\alpha N_t^{a_{-1}}}{1-\gamma(1-\alpha)} + \left( 1 - \frac{\alpha}{2} \right) e^{H_2^2 N_t^{a_{-3}}} \right\}$$

$$+ (1-\gamma) \left[ \chi C_i N_t^\gamma + \frac{x_t}{1-\gamma} \frac{\gamma}{1-\gamma} Q_i^N \right]. (53)$$
Appendix B
The Extended Model

This Appendix characterizes the extended model used to derive the results reported in Figures 5 and 6. The model augments the simple set-up of Section (3) to include habits in consumption to the problem of the households, trend inflation and inflation indexation in the problem of final good firms, investment adjustment costs in the problem of the intermediate good firms, and exogenous wage rigidity in the wage rule.

Households

Let $\vartheta \in [0, 1)$ be the parameter governing external habit formation. The intertemporal problem of a household indexed by subscript $j$ is to maximize the discounted present value of current and future utility:

$$\max_{\{C_{t+s}\}_{s=0}^{\infty}} E_t \sum_{s=0}^{\infty} \beta^s \left[ \ln \left( C_{j,t+s} - \vartheta C_{t+s-1} \right) - \frac{X}{1 + \varphi} N_{j,t+s}^{1+\varphi} \right],$$

subject to the budget constraint (1) and the law of motion for employment (2).

Denoting by $\lambda$ the Lagrange multiplier associated with the budget constraint, and assuming that all households are identical in equilibrium, the conditions for dynamic optimality are:

$$\lambda_t = \frac{1}{P_t (C_t - \vartheta C_{t-1})},$$

$$\frac{1}{R_t} = \beta E_t \lambda_{t+1} \lambda_t,$$

$$V_{t} = \frac{W_t}{P_t} - \frac{X N_t^\varphi}{\lambda_t P_t} - \frac{\lambda_t}{1 - \lambda_t} V_{t} + \frac{E_t \Lambda_{t,j} (1 - \delta_N)}{1 - \delta_N} V_{t+1}^{N},$$

where the Euler equation (54) and the value of a marginal job to the household (55) correspond to equations (4) and (5) in the simple model of Section (3), respectively.

Intermediate good firms

The problem of the intermediate good firm is to maximize the present discounted value of cashflows:

$$\max E_t \sum_{j=0}^{\infty} \Lambda_{t,j} \left\{ mc_{t,j} \left[ f(A_{t+j}, N_{t+j}, K_{t+j-1}) - g(A_{t+j}, H_{t+j}, N_{t+j}, K_{t+j-1}) \right] \right\}$$

$$- \frac{W_{t+j}}{P_{t+j}} N_{t+j} - I_{t+j},$$

subject to the friction cost function in eq. (7), the law of motion for capital

$$K_t = (1 - \delta_K) K_{t-1} + \left[ 1 - S \left( \frac{I_t}{I_{t-1}} \right) \right] I_t, \quad 0 \leq \delta_K \leq 1,$$
and the law of motion for employment in (10), where \( \Lambda_{t,t+1} = \beta j \frac{\bar{F}_{t+1}}{F_t} \) is the real discount factor of the households who own the firms, and \( S \) is the investment adjustment cost function. It is assumed that \( S(1) = S'(1) = 0 \), and \( S''(1) \equiv \phi > 0 \).

We obtain the following conditions for dynamic optimality:

\[
Q_t^N = mc_t (f_{N,t} - g_{N,t}) - \frac{W_t}{P_t} + (1 - \delta_N)E_t \Lambda_{t,t+1} Q_{t+1}^N
\]

(57)

\[
Q_t^K = mc_t g_{H,t} \]

(58)

\[
Q_t^K = E_t \Lambda_{t,t+1} \left[ mc_{t+1} (f_{K,t+1} - g_{K,t+1}) + (1 - \delta_K) Q_{t+1}^K \right],
\]

(59)

and

\[
Q_t^K \left[ 1 - S \left( \frac{l_t}{l_{t-1}} \right) - S' \left( \frac{l_t}{l_{t-1}} \right) \frac{l_t}{l_{t-1}} \right] + E_t \Lambda_{t,t+1} Q_{t+1}^K S' \left( \frac{l_{t+1}}{l_t} \right) \left( \frac{l_{t+1}}{l_t} \right)^2 = 1.
\]

(60)

**Final good firms**

We assume price stickiness à la Rotemberg (1982), meaning firms maximize current and expected discounted profits subject to quadratic price adjustment costs. We assume that adjustment costs depend on the ratio between the new reset price and the one set in the previous period, adjusted by a geometric average of steady state inflation, \( \bar{\pi} \), and past inflation. We denote by \( \psi \) the parameter that captures the degree of indexation to past inflation.

Final good firms maximize the following expression:

\[
\max E_t \sum_{s=0}^{\infty} \Lambda_{t,t+s} \frac{P_t}{P_{t+s}} \left[ (P_{t+s}(i) - P_{t+s}mc_{t+s}) Y_{t+s}(i) - \frac{\zeta}{2} \left( \frac{P_{t+s}(i)}{(\pi_{t+s-1})^\psi (\bar{\pi})^{1-\psi} P_{t+s-1}(i)} - 1 \right)^2 P_{t+s} Y_{t+s} \right],
\]

subject to the demand function (16). The optimality condition is:

\[
\left( \frac{P_t(i)}{P_t} \right)^{-\epsilon} Y_t - \epsilon \left( \frac{P_t(i)}{P_t} \right)^{-\epsilon-1} \frac{P_{t+s}(i) - P_{t+s}mc_{t+s}}{P_t} Y_t
\]

\[
- \zeta \left( \frac{P_t(i)}{(\pi_{t-1})^\psi (\bar{\pi})^{1-\psi} P_{t-1}(i)} - 1 \right) \frac{P_t Y_t}{(\pi_{t-1})^\psi (\bar{\pi})^{1-\psi} P_{t-1}(i)}
\]

\[
+ E_t \Lambda_{t,t+1} \frac{P_t}{P_{t+1}} \left[ -\zeta \left( \frac{P_{t+1}(i)}{(\pi_{t})^\psi (\bar{\pi})^{1-\psi} P_{t}(i)} - 1 \right) P_{t+1} Y_{t+1} - \frac{(\pi_t)^\psi (\bar{\pi})^{1-\psi} P_{t+1}(i)}{(\pi_{t})^\psi (\bar{\pi})^{1-\psi} P_{t}(i)} \right]^2 = 0.
\]

Since all firms set the same price and therefore produce the same output in equilibrium, the above equation can be rearranged as follows:

\[
\left( \frac{\pi_t}{(\pi_{t-1})^\psi (\bar{\pi})^{1-\psi} - 1} \right) \frac{\pi_t}{(\pi_{t-1})^\psi (\bar{\pi})^{1-\psi}} = \frac{1 - \epsilon}{\zeta} + \frac{\epsilon}{\zeta} mc_t
\]
$+E_t \frac{1}{R_t/\pi_{t+1}} \left[ \left( \frac{\pi_{t+1}}{(\pi_t)^{1-\psi}} - 1 \right) \frac{\pi_{t+1}}{(\pi_t)^{1-\psi}} \frac{Y_{t+1}}{Y_t} \right]. \quad (61)$

Wage norm

We assume wage rigidity in the form of a Hall (2005) type wage norm:

$$\frac{W_t}{P_t} = \omega \frac{W_{t-1}}{P_{t-1}} + (1 - \omega) \frac{W_{t}^{\text{NASH}}}{P_t},$$

where $\omega$ is a parameter governing real wage stickiness, and $W_{t}^{\text{NASH}}$ denotes the Nash reference wage

$$W_{t}^{\text{NASH}} = \arg \max \left\{ \left( V_t^N \right)^{\gamma} \left( Q_t^N \right)^{1-\gamma} \right\}.$$

The Monetary and Fiscal Authorities and Market Clearing

The model is closed by assuming that the government runs a balanced budget, as per eq. (24), the monetary authority follows the Taylor rule in eq.(25), and the market clears as per eq.(26).
Table 1: Calibrated Parameters and Steady State Values, Baseline Model

<table>
<thead>
<tr>
<th>Description</th>
<th>Parameter</th>
<th>Value</th>
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<tbody>
<tr>
<td>Discount factor</td>
<td>$\beta$</td>
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<td>Separation rate</td>
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<td>Capital depreciation rate</td>
<td>$\delta_K$</td>
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<td>Elasticity of output to labor input</td>
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<tr>
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<td>Price frictions (Rotemberg)</td>
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<tr>
<td>Autocorrelation monetary shock</td>
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Panel B: Steady State Values

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<tr>
<th>Definition</th>
<th>Expression</th>
<th>Value</th>
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</thead>
<tbody>
<tr>
<td>Total adjustment cost/ output</td>
<td>$g / (f - g)$</td>
<td>0.012</td>
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<tr>
<td>Marginal hiring cost</td>
<td>$g_H / [(f - g) / N]$</td>
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</tr>
<tr>
<td>Opportunity cost of work/ marginal revenue prod.</td>
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<tr>
<td>Unemployment rate</td>
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## Table 2: Calibrated Parameters and Steady State Values, Extended Model

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<td>Trend inflation</td>
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### Panel B: Steady State Values

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<tr>
<td>Unemployment rate</td>
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</table>
Figure 1: Impulse Responses to the Technology Shock

Notes: impulse responses to a 1% positive technology shock obtained in four benchmark parameterizations of the model: 1) the New Classical model (NC, black line); 2) the New Classical model with labor frictions (NC & L frictions, blue line); 3) the New-Keynesian model (NK, red line); 4) the New-Keynesian model with labor frictions (NK & L frictions, green line). All variables are expressed in % deviations, except hiring and investment rates, which are expressed in percentage points deviations.
Figure 2a: Impulse Responses to the Monetary Policy Shock: Benchmark Calibration

Notes: impulse responses to a 25bp expansionary monetary shock obtained in four benchmark parameterizations of the model: 1) the New Classical model (NC, black line); 2) the New Classical model with labor frictions (NC & L frictions, blue line); 3) the New-Keynesian model (NK, red line); 4) the New-Keynesian model with labor frictions (NK & L frictions, green line). All variables are expressed in % deviations, except hiring and investment rates, which are expressed in percentage points deviations.
Figure 2b: Impulse Responses to the Monetary Policy Shock:

- No Interest Rate Smoothing ($\rho_r = 0$), Auto-Correlated MP Shock ($\rho_x = 0.5$).

Notes: See Figure 2a.
Figure 3: Impulse Responses on Impact of a Technology Shock

Notes: The figure shows impulse responses on the impact of a positive technology shock for various parameterizations of the model where we allow price rigidities, $\zeta$, and hiring frictions, $e$, to vary. The real wage and output are expressed in percent deviations from steady state, hiring and investment rates in percentage points deviations.
Figure 4a: Impulse Responses on Impact of a Monetary Policy Shock
Benchmark Calibration

Notes: The figure shows impulse responses on the impact of an expansionary monetary shock for various parameterizations of the model where we allow price rigidities, $\zeta$, and hiring frictions, $\epsilon$, to vary. The real wage and output are expressed in percent deviations from steady state, hiring and investment rates in percentage point deviations.
Figure 4b: Impulse Responses on Impact of a Monetary Policy Shock
No interest rate smoothing ($\rho_r = 0$), auto-correlated MP shock ($\rho_\xi = 0.5$)

Notes: The figure shows impulse responses on the impact of an expansionary monetary shock for various parameterizations of the model where we allow price rigidities, $\zeta$, and hiring frictions, $\epsilon$, to vary. The real wage and output are expressed in percent deviations from steady state, hiring and investment rates in percentage points deviations.
Figure 5: Impulse Responses to a Technology Shock: Extended Model with “Low” vs. “High” Labor Frictions

Notes: impulse responses to a 1% positive technology shock obtained for two different parameterizations: “high” labor frictions (purple broken line; $e = 5$) and “low” frictions (solid green line; $e = 1.5$). All variables are expressed in % deviations, except hiring and investment rates, which are expressed in percentage points deviations.
Figure 6: Impulse Responses to a Technology Shock: Extended Model with Rigid vs. Flexible Prices

Notes: impulse responses to a 1% positive technology shock obtained for two different parameterizations: The rigid price model with labor frictions (NK + L Frictions, purple broken line; $\zeta = 120$ and $\epsilon = 5$) and the flexible price model with labor frictions (NC + L Frictions, solid yellow line; $\zeta \simeq 0$ and $\epsilon = 5$). All variables are expressed in % deviations, except hiring and investment rates, which are expressed in percentage points deviations.
Figure 7: Impulse Responses to a Monetary Policy Shock: Extended Model with “Low” vs. “High” Labor Frictions

Notes: impulse responses to a 25 basis point expansionary monetary policy shock obtained for two different parameterizations: “high” labor frictions (purple broken line; $e = 5$) and “low” frictions (solid green line; $e = 1.5$). All variables are expressed in % deviations, except hiring and investment rates, which are expressed in percentage points deviations.