Fragile New Economy: The Rise of Intangible Capital and Financial Instability*

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Abstract

This paper studies financial instability in an economy where growth is driven by intangible investment. Firms’ intangible investment creates new productive capital. Once created, capital can be sold to financial intermediaries. Since intangible investment is not pledgeable, firms carry cash, which is inside money issued by intermediaries (short-term safe debt). In good times, well capitalized intermediaries push up the price of capital. This motivates firms to create more capital, but to do so, they must build up cash holdings. As firms’ money demand expands, the yield on inside money (i.e., intermediaries’ debt cost) declines, so intermediaries increase leverage and push up capital price even further. This inside money channel generates several features shared with the U.S. economy before the Great Recession: rising corporate cash holdings, financial intermediaries growing through leverage, increasing asset prices, and declining interest rate. The model also generates endogenous risk accumulation: a longer period of boom and expansion of the financial sector predict a more severe crisis. In crises, the spiral flips, leading to sudden deleveraging of intermediaries, asset price collapse, and investment contraction.

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1 Introduction

Five trends in the United States have attracted enormous attention in the two decades leading up to the Great Recession:

(1) The economy was transforming to an intangible-intensive economy. Production of goods and services increasingly relies on intangible capital, such as brand name, technologies, and organizational capital. Intangible investment overtook physical investment as the largest source of growth in the period of 1995-2007 (Corrado and Hulten (2010)).

(2) An increasing share of non-financial corporations’ assets were cash holdings (Bates, Kahle, and Stulz (2009)). “Cash” is mainly financial intermediaries’ debs, such as bank deposits and repurchase agreements (held through money market funds). The rise of corporate cash holdings is largely due to a growing R&D-intensive sector (Falato and Sim (2014); Bege nau and Palazzo (2015); Pinkowitz, Stulz, and Williamson (2015); Graham and Leary (2015)).

(3) Assets held by the financial sector increased dramatically (Adrian and Shin (2010a); Gorton, Lewellen, and Metrick (2012); Greenwood and Scharfstein (2013)). Schularick and Taylor (2012) find that in advanced economies, bank loan-to-GDP ratio doubled in the last two decades. The growth of financial sector was largely financed by issuing money-like debt securities (Adrian and Shin (2010b); Gorton (2010); Gorton and Metrick (2012); Pozsar (2014)).

(4) Interest rate declined steadily.

(5) The prices of risky assets increased across asset classes.

Despite extensive debates on the causes and implications of these phenomena, there are few theories that analyze them jointly. Motivated by Fact (1), I build a continuous-time model of macroeconomy that highlights the illiquidity of intangible capital and offers a coherent account of Fact (2) to (5). Moreover, the model reveals a new mechanism of endogenous risk accumulation that helps understand recent findings in a surging empirical literature: a longer period of bank expansion precedes a more severe banking and economic crisis (Jordà, Schularick, and Taylor (2013); Baron and Xiong (2016)). At the center of the mechanism are firms’ needs to hoard money for future investments and banks’ role as inside money creators.¹

¹The term, inside money, is borrowed from Gurley and Shaw (1960). From the private sector’s perspective, gold, fiat money, or government securities are in positive supply (“outside money”), while as bank liabilities, deposits are in zero net supply (“inside money”). See Lagos (2008) for a brief review of the related literature.
The model is built upon a simple narrative. The increasing reliance on intangible capital in the production sector implies a shrinkage of pledgeable assets (physical capital, such as properties, plants, and equipments). As a result, producers hoard more cash in anticipation of liquidity needs (e.g., R&D). Cash takes the form of intermediaries’ short-term debts, such as deposits, which are directly used as means of payment, and close substitutes (e.g., repurchase agreements held through money market funds). Corporate money demand feeds leverage to the financial sector, so intermediaries acquire more assets and push up asset prices. The rising corporate money demand also pushes down interest rates, i.e., yields or returns on holdings of money (e.g., deposit rate) or substitutes (e.g., asset-backed commercial paper rate).

The model has two types of agents, bankers and entrepreneurs. They consume generic goods produced by capital. Both can trade capital in a competitive market. They can also trade risk-free debt, i.e., borrow from and lend to each other at the market-clearing interest rate.

Bankers and entrepreneurs differ in two aspects. First, bankers are better at dealing with macro shocks. The only source of aggregate uncertainty is a Poisson shock that destroys a fraction of agents’ capital holdings. Bankers experience a smaller loss than entrepreneurs, which captures bankers’ expertise in rescuing distressed assets (Bolton and Freixas (2000)). This advantage makes bankers the natural buyers of capital (Shleifer and Vishny (2010)). In equilibrium, they borrow from entrepreneurs, holding a levered position in capital. A standard balance-sheet channel arises (as in Brunnermeier and Sannikov (2014) among others): when a macro shock hits, bankers lose wealth because part of their capital holdings are destroyed, so they scale down their demand for capital; as natural buyers retreat, capital price declines and bankers’ wealth is reduced even further. This feedback loop is illustrated in Figure 1.

The second difference between bankers and entrepreneurs, which is the focus of this paper, is that entrepreneurs create new capital. When hit by an idiosyncratic Poisson shock, an entrepreneur is endowed with an investment project. New capital can be created from goods that are bought from other producers (capital owners). The project also allows the entrepreneur to create fake capital out of thin air. Counterfeit is worthless, but is indistinguishable from real capital. This reflects the difficulty to measure intangible capital.\footnote{Corrado, Hulten, and Sichel (2005) and McGrattan and Prescott (2010) discuss the measurement of technology capital. Atkeson and Kehoe (2005) and Eissfeldt and Papanikolaou (2014) measure organization capital.} Capital represents revenue-
**Good times: no macro shocks**

Bank equity grows  \(\rightarrow\)  Capital price increases  \(\rightarrow\)  Investment profitability increases

**Balance-Sheet Channel**

Bankers issue more debt at lower cost  \(\leftarrow\)  Entrepreneurs’ money demand increases

**Inside Money Channel**

**Bad times: triggered by macro shocks**

Bank equity falls  \(\rightarrow\)  Capital price decreases  \(\rightarrow\)  Investment profitability decreases

**Balance-Sheet Channel**

Bankers issue less debt at higher cost  \(\leftarrow\)  Entrepreneurs’ money demand decreases

**Inside Money Channel**

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Figure 1: Model Mechanism.

generating units, which can be a particular design of very common products or simply a catchy brand name. In contrast to manufacturing facilities or physical capital in general, it is harder to check the quality of intangibles.

The purchase of goods has to be partly financed by the entrepreneur’s own wealth, because investment requires her inalienable human capital (Hart and Moore (1994)).\(^3\) The counterfeit problem means that not all components of the entrepreneur’s wealth are equally liquid. Her Holdings of risk-free debt, issued by bankers, are perfectly liquid. Every dollar of it can be sold for goods as investment inputs. Her holdings of existing capital is illiquid. Buyers are only able to examine up to \(\phi\) fraction of her inventory to avoid counterfeits (\(\phi < 1\)).\(^4\) Therefore, only \(\phi\) fraction of wealth invested in capital can be sold for investment inputs.

The illiquidity of capital induces a bias towards bank debt in entrepreneurs’ wealth portfolio. *This bias becomes larger*, when the market price of capital is higher, making investments more profitable. To meet stronger investment needs, entrepreneurs want to hold wealth in the most liquid form, i.e., bank debt that can be readily sold for investment inputs. The rising demand for bank debt pushes down the equilibrium interest rate (bankers’ marginal cost of financing), feeding leverage to bankers and pushing up capital price even further. Held by

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\(^3\)In other words, the new capital to be created cannot be pledged for external funds.

\(^4\)Note that the counterfeit problem only comes with the investment project, so non-investing entrepreneurs do not face the illiquidity of capital holdings (i.e., they can trade capital freely).
entrepreneurs as means of payment, bank debt is the inside money that facilitates resources reallocation. The feedback loop, “inside money channel”, is summarized in Figure 1.

The inside money channel helps explain the fact (2) to (5) in the introduction. Consider a period without the macro shock (i.e., a boom). Bankers accumulate wealth by earning the spread between return on capital holdings and the risk-free rate (debt cost). Their increasing demand for capital drives up capital price, and through the balance-sheet channel, rising capital price further increases bankers’ wealth. As capital is reallocated to bankers, their balance sheet expands, and they issue more debt (inside money) in the process. A rising capital price also motivates entrepreneurs to hoard more money in preparation for opportunities to create new capital, so their money demand drives down the equilibrium interest rate, making cheaper leverage available to bankers, and thus, allowing them to push up capital price even higher. Note that the balance-sheet channel is enough to generate the rising asset price and the growth of banking sector, but it is the inside money channel that brings in the dynamics of interest rate and corporate cash holdings and amplifies asset price increase and bank growth.

Through the inside money channel, endogenous risk accumulates in booms, suggesting that prolonged booms are followed by severe crises. Entrepreneurs’ liquidity preference is stronger when the level of capital price is higher and investments more profitable. This widens the difference between bankers, as the preferred buyers of capital, and entrepreneurs as the relatively inferior ones. Therefore, when the macro shock hits and triggers a reallocation capital from bankers to entrepreneurs, the decrease in capital price is more significant. Exogenous risk, wealth loss due to capital destruction, is distinguished from endogenous risk, that is wealth loss due to capital price change. The latter increases as a boom prolongs, and materializes when the macro shock flips the feedback loop downward, as shown in Figure 1.

In sum, the model has two key ingredients. First, firms hold cash in the form of bank debt (inside money) in anticipation of illiquid investment projects. Second, capital price increases in bankers’ wealth because they are better at dealing with macro shocks than entrepreneurs. These two ingredients lead to two reinforcing mechanisms, the inside money channel and the typical balance-sheet channel, that create financial fragility. The first element is motivated by the rise of intangible capital and the fact that corporate treasuries are among the largest cash
pools that feed financial intermediaries leverage (Pozsar (2011)).

**Literature.** The structural change towards a new economy that is intangible-intensive has attracted enormous attention (Corrado and Hulten (2010)). Studies explore the implications of this structural change in different areas from both theoretical and empirical perspectives, such as Atkeson and Kehoe (2005) (productivity accounting), McGrattan and Prescott (2010) (current account), Eisfeldt and Papanikolaou (2014) (asset pricing), Peters and Taylor (2016) (corporate investment), and Perez-Orive and Caggese (2017) (secular stagnation). This paper looks into the financial stability implications of intangible capital. There is a large literature on how financial development affects industrial structure (e.g., Levine (1997); Rajan and Zingales (1998)). However, the reverse question, how industrial structure affects the financial system, has not yet been well explored. This paper aims to sheds light on this question.

The model fits into the literature on heterogeneous-agent models. Many frictions manifest themselves into limits on aggregate risk-sharing between sectors. In this paper, the key friction is that between bankers and entrepreneurs, the only form of financial contracting is risk-free debt. Di Tella (2014) notes that perfect risk-sharing shuts down the balance-sheet channel (e.g., Kiyotaki and Moore (1997); Bernanke and Gertler (1989)). Many study the macroeconomic and asset pricing implications of dynamic wealth distribution among heterogeneous agents (e.g., Basak and Cuoco (1998); Krusell and Smith (1998); Longstaff and Wang (2012); He and Krishnamurthy (2013); Brunnermeier and Sannikov (2014); Moll (2014)). This paper contributes to this literature by introducing a new dimension of heterogeneity, that is bankers as money suppliers and entrepreneurs as money demanders. It gives rise to the inside money channel when investment and capital are illiquid.

A recent literature revives the money view of banking that emphasizes the liabilities of banks as inside money that lubricates the economy by facilitating trades (Kiyotaki and Moore (2000); Hart and Zingales (2014); Quadrini (2014); Piazzesi and Schneider (2016)). This paper takes a step further by modeling bankers as private money suppliers in an entrepreneurial economy that suffers from capital and investment illiquidity, and shows how this leads to financial instability.\(^5\) The theoretical framework in this paper is inspired by Kiyotaki and

\(^5\)Safe debt serves as money, which echoes the literature that links the information insensitivity of assets'
Moore (2012) who study the emergence of bubbly outside money (or fiat money) from capital illiquidity (see also Farhi and Tirole (2012)).

This paper is motivated by the large literature on corporate cash holdings (e.g., Opler et al. (1999); Bates, Kahle, and Stulz (2009)). The secular increase of nonfinancial firms’ cash holdings in the last few decades was driven by the R&D-intensive sectors (e.g., Falato and Sim (2014); Begenau and Palazzo (2015); Pinkowitz, Stulz, and Williamson (2015); Graham and Leary (2015)). Physical capital relaxes financial constraints by serving as collateral (Almeida and Campello (2007)). The structural transformation towards an intangible economy shrinks the collateral pool, leading to a stronger incentive to hoard liquidity. Corporate treasuries have become a prominent component of “institutional cash pools” that lend to the financial sector, particularly through the money markets (Pozsar (2011)).

Theoretical models of corporate cash holdings often assume a storage technology (e.g., Froot, Scharfstein, and Stein (1993); Bolton, Chen, and Wang (2011); He and Kondor (2016)). These models characterize a rich environment of corporate decision making, but severs the link between corporate money demand and the equilibrium interest rate (i.e., the yield on money-like assets) by assuming a perfectly elastic supply of storage assets. An exception is Holmström and Tirole (1998) who take a general equilibrium approach to study whether firms are able to supply the assets they themselves need to hold as liquidity buffer. In this paper, firms’ liquidity demand translates into the demand for bank debt, so that interest rate, asset price, bank leverage, and corporate cash holdings can be studied in a unified framework.

The model predicts that a longer period of boom predicts more severe crisis (asset price collapse). Schularick and Taylor (2012) find the expansion of bank asset (loans) precedes financial crises in advanced economies. In line with Bordo et al. (2001) and Reinhart and Rogoff (2009), Schularick and Taylor (2012) define financial crises as events during which a country’s banking sector experiences bank runs, sharp increases in default rates accompanied by large losses of capital that result in public intervention, bankruptcy, or forced merger of financial institutions. Using the information from credit spreads, Krishnamurthy and Muir (2016) are able to achieve a sharper differentiation between financial and non-financial crises. They also find that bank credit expansion precedes financial crises. Using both the information from credit spreads and the composition change

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6 Several papers have argued that the growth of the shadow banking sector is fueled by the demand of money-like securities (e.g., Gorton (2010); Gorton and Metrick (2012); Stein (2012); Pozsar (2014)).

7 Holmström and Tirole (2001) explore the impact of corporate liquidity demand on the prices of liquid assets.

8 In line with Bordo et al. (2001) and Reinhart and Rogoff (2009), Schularick and Taylor (2012) define financial crises as events during which a country’s banking sector experiences bank runs, sharp increases in default rates accompanied by large losses of capital that result in public intervention, bankruptcy, or forced merger of financial institutions. Using the information from credit spreads, Krishnamurthy and Muir (2016) are able to achieve a sharper differentiation between financial and non-financial crises. They also find that bank credit expansion precedes financial crises. Using both the information from credit spreads and the composition change
relationship between bank credit growth and the severity of subsequent recessions. Related, Baron and Xiong (2016) find that bank credit expansion predicts increased bank equity crash risk. The existing theories on endogenous accumulation of risk largely focus on belief distortions (e.g., Gorton and Ordoñez (2014); Moreira and Savov (2014)). This paper emphasizes financial frictions and relates endogenous risk to the balance-sheet dynamics of banking and production sectors.

The rest of the paper is organized as follows. Section 2 introduces the model and discusses the mechanisms. Section 3 solves the model and shows its properties. Section 4 concludes. The solution method is provided in the appendix.

2 Model

2.1 Setup

Consider a continuous-time, infinite-horizon economy with two types of agents, bankers and entrepreneurs. Each type has a continuum of representative agents with measure equal to one.

Preferences. Both types of agents maximize expected logarithm utility with discount rate $\rho$:

$$E\left[\int_{t=0}^{\infty} e^{-\rho t} \ln (c_i^t) \, dt\right], \ i \in \{f, nf\}.$$  \hfill (1)

Throughout this paper, subscripts denote time, and superscripts denote types or sectors with “$f$” for the financial sector (bankers) and “$nf$” for the non-financial sector (entrepreneurs). For example, $c_i^{nf}$ is the representative entrepreneur’s consumption at time $t$.

Capital and aggregate shock. At time $t$, the economy has $K_t$ units of capital that produces non-durable generic goods for consumption and investment. One unit of capital produces $a$ units of goods per unit of time. $a$ is a positive constant. Both entrepreneurs and bankers can own capital, and capital is traded in a competitive market at price $q_t$ (denominated in goods).
At Poisson times with intensity equal to $\lambda$, a certain fraction of capital is destroyed. The random arrival of capital destruction is the only source of aggregate uncertainty in this economy (i.e., the “macro shock”). Bankers and entrepreneurs differ in their ability to deal with the capital destruction shock. When capital is owned by bankers, a fraction $\delta$ of capital is destroyed when the macro shock hits. When capital is owned by entrepreneurs, a fraction $\bar{\delta}$ is destroyed. I assume $\delta < \bar{\delta}$ so that bankers are better at dealing with capital destruction.

The assumption that $\delta < \bar{\delta}$ captures bankers’ expertise in restructuring distressed assets (e.g., Bolton and Freixas (2000)). Consequently, bankers assign a higher value to capital than entrepreneurs. In other words, they are the “natural buyers” (Shleifer and Vishny (1992)). If bankers become undercapitalized, and thus, less willing to hold risky capital, the equilibrium capital price declines.

The assumption that $\delta < \bar{\delta}$ is related to the credit view on banking that emphasizes the asset-side of bank balance sheet – banks are important because only they can hold certain assets or finance certain projects (e.g., Bernanke and Blinder (1992)). This view has received tremendous attention recently. There is also the money view on banking, exemplified by the classic account of the Great Depression by Friedman and Schwartz (1963), that emphasizes banks as the suppliers of transaction medium. In this paper, these two views meet. The two functions of banks interact with each other, creating financial fragility. Next, the demand for inside money is introduced through entrepreneurs’ investment needs.

**Investment illiquidity.** While bankers have advantage in restructuring distressed projects or capital, it is entrepreneurs who can create new capital. Investment opportunities arrive at idiosyncratic Poisson times with constant intensity $\lambda_I$, where the subscript “I” stands for both idiosyncratic and investment. Every instant, a constant fraction $\lambda_I dt$ of entrepreneurs invest.

Investing entrepreneurs can convert one unit of goods into one unit of capital instantaneously. Investment is scalable, so when $q_t > 1$ (as will be in equilibrium), entrepreneurs

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9. In Brunnermeier and Sannikov (2014), it is embodied by the enhanced productivity when capital is held by banks (“experts”). Other types of intermediation expertise may serve the same purpose of making banks the natural buyers of capital and capture the credit view on banking, such as lower risk aversion (Longstaff and Wang (2012)), collateralization expertise (Rampini and Viswanathan (2015)), unique ability to hold risky assets (He and Krishnamurthy (2013)), unique ability to monitor projects (Diamond (1984) and Holmström and Tirole (1997)), and advantage in diversification (Brunnermeier and Sannikov (2016)).

10. Idiosyncratic investment shocks are not insurable; otherwise, entrepreneurs will not have liquidity needs.
want to invest as much as possible. The scale of investment is limited by financial frictions.

We can think of creating one unit of capital as a project. The final output of this project is worth \( q_t \). An entrepreneur can pledge up to \( \theta \) fraction of output to competitive investors, including non-investing entrepreneurs and bankers, and raise \( \theta q_t \) because competitive investors break even. Since the entrepreneur enjoys the full surplus, she always exhausts external financing capacity, and invests \((1 - \theta q_t)\) out of her own pocket. The project is not fully pledgeable because it needs the entrepreneur’s inalienable human capital (Hart and Moore (1994)).

Once the project is done and capital created, the entrepreneur delivers \( \theta \) units of capital to outside investors and retain \((1 - \theta)\) units that are worth \((1 - \theta) q_t\). Therefore, the investment return on internal liquidity (“ROI”) is

\[
ROI_t = \frac{(1 - \theta) q_t}{1 - \theta q_t}.
\]  

It increases in the capital price \( q_t \) through the unlevered return \((1 - \theta) q_t\) and the leverage on internal liquidity \(\frac{1}{1-\theta q_t}\).

**Capital illiquidity and intangibility.** An investing entrepreneur’s internal liquidity comes from two sources, holdings of existing capital and inside money. Capital is illiquid, as only a fraction \( \phi \) of investing entrepreneurs’ capital holdings can be readily sold in exchange for goods as investment inputs (\( \phi < 1 \)). Illiquidity arises from an informational friction: in addition to the capital creation technology, the investing entrepreneur is also endowed with a technology to create counterfeits that are indistinguishable from productive capital. Investors can verify the quality of capital up to \( \phi \) fraction of the inventory.\(^{11}\)

The introduction of parameter \( \phi \) is motivated by the increasing reliance of production on intangible capital (Corrado and Hulten (2010)). In this paper, a more intangible economy has a lower \( \phi \). The assumption that \( \phi < 1 \) reflects the difficulty to measure intangible capital.\(^{12}\) Capital represents revenue-generating units, which could be a particular design of very com-

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\(^{11}\)Note that non-investing entrepreneurs do not have access to the counterfeit technology, so they do not suffer from the illiquidity of capital and are able to trade capital freely at market price \( q_t \). Whether an entrepreneur is investing or not is public information.

mon products or simply a brand name. In contrast to manufacturing plants and other types of physical capital, it is often difficult to examine the quality of such intangible capital, and thus, easy for entrepreneurs to claim that they have just discovered the next big thing.

Related, the assumption that investments have to be partially financed with internal liquidity (i.e., $\theta < 1$) is also more relevant in the creation of intangible capital than the creation of tangible or physical capital. In particular, R&D heavily relies on internal financing, which has been well documented since Hall (1992) and Himmelberg and Petersen (1994). Hall and Lerner (2009) review the literature on innovation financing.

**Inside money.** In contrast to capital, the investing entrepreneur can sell all of their holdings of risk-free debts for goods. In equilibrium, these risk-free debts are issued by bankers. It matures at $t + dt$ and pays an interest rate $r_t$. Entrepreneurs carry bank debts as means of payment to purchase investments inputs when opportunities present themselves. Thus, bank debt is inside money that facilitates goods reallocation towards entrepreneurs with investment opportunities.\(^\text{13}\) The superior liquidity of bank debt at investing times makes entrepreneurs prefer holding bank debt in their wealth portfolio than capital. This investment-driven “liquidity preference” is in line with Holmström and Tirole (2001), Eisfeldt and Rampini (2009), and Farhi and Tirole (2012).\(^\text{14}\) This paper emphasizes the resulting demand for inside money.

### 2.2 Markov equilibrium

**State variable evolution.** Let $n^i_t$ denote the wealth of a representative agent in sector $i$ ($i \in \{f, nf\}$). The model has a Markov equilibrium where $\eta_t$, the fraction of aggregate wealth in the banking sector, is the only aggregate state variable:

$$\eta_t = \frac{n^f_t}{n^f_t + n^{nf}_t}, \quad (3)$$

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\(^{13}\)The term is borrowed from Gurley and Shaw (1960). From the private sector’s perspective, fiat money and government securities are in positive supply (“outside money”), while deposits, as bank liabilities, are in zero net supply (“inside money”). See Lagos (2008) for a brief review of the related literature. The moneyness of safe debt echoes the theories that links assets’ information insensitivity to their monetary services (e.g., Gorton and Pennacchi (1990); Holmström (2012); Dang et al. (2014)).

\(^{14}\)Firms’ liquidity demand is likely to be important. Eisfeldt (2007) show that the liquidity premium of Treasury bills cannot be explained by the liquidity demand from consumption smoothing under standard preferences.
Because both production and investment have constant return to scale, the economy is scale-free, so the aggregate capital stock $K_t$ does not affect prices and agents’ choices. Wealth or consumption share is a common state variable in the literature of heterogeneous-agent models (see Basak and Cuoco (1998), Longstaff and Wang (2012), He and Krishnamurthy (2013), and Brunnermeier and Sannikov (2014) among others). In the current setup, it measures the wealth of liquidity suppliers (banks) relative to that of liquidity demanders (entrepreneurs).

Before the macro shock hits, $\eta_t$ has a deterministic rate of change $\mu^\eta_t := \frac{d\eta_t}{d\eta_t}$. In a Markov equilibrium, all endogenous variables are functions of $\eta_t$, so their dynamics are linked to the dynamics of $\eta_t$. For example, before the macro shock hits, the rate of change of capital price $q_t$ is

$$\mu^q_t := \frac{dq_t}{dt} = q' (\eta_t) \frac{\mu^\eta_t}{\eta_t},$$

As will be shown later, the equilibrium capital price increases in $\eta_t$ (i.e., $q' (\eta_t) > 0$). Moreover, $q_t > 1$, because investing entrepreneurs cannot invest unlimited resources to close the wedge between $q_t$, the value of capital, and 1, the cost of creating it.

Bankers and entrepreneurs allocate wealth between safe debt and risky capital. A short position in safe debt means borrowing.$^{15}$ Let $x^i_t$ denote the fraction of wealth invested in capital ($i \in \{ f, nf \}$). Before the macro shock hits, the flow of funds constraint (i.e. the evolution of wealth) is represented by a deterministic growth rate of wealth

$$\mu^{n^i}_t := \frac{dn^i_t}{dt} = (1 - x^i_t) r_t + x^i_t \left( \frac{a}{q_t} + \mu^q_t \right) - \frac{c^i_t}{n^i_t}, i \in \{ f, nf \}. \quad (4)$$

The pre-shock return of capital holdings includes a dividend gain $\frac{a}{q_t}$ and a capital gain $\mu^q_t$. In equilibrium, entrepreneurs hold inside money (bank debt), so we have $x^f_t > 1$ and $x^{nf}_t < 1$.

A constant fraction $\lambda_t dt$ of entrepreneurs can make investments and experience a jump in their wealth. Because when each dollar of liquidity will be worth $ROI_t$ ($> 1$) after investment, investing entrepreneurs want to sell all of these liquid resources for goods to maximize

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$^{15}$Under log-utility, optimal consumption is proportional to wealth, and the Inada condition implies infinite marginal utility when wealth equals to zero. As a result, the agents always try to avoid bankruptcy and keep a positive net worth, and agents never default on debt.
the scale of investment. Given their internal funds

$$\left[ \phi x_i^{nf} + \left( 1 - x_i^{nf} \right) \right] n_t^{nf},$$

i.e., the sum of inside money holdings and their resalable capital holdings, investment is given by the following Proposition.

**Proposition 1** At time $t$, if an investment opportunity arrives, the entrepreneur obtains

$$i_t = \left( \frac{1}{1 - \theta q_t} \right) \left[ 1 - (1 - \phi) x_t^{nf} \right] n_t^{nf},$$

units of goods as investment inputs, and create an equal amount of new capital. The investment depends on internal liquidity, and through $\frac{1}{1 - \theta q_t}$, the leverage on internal liquidity, depends on capital price $q_t$. After investment, the investing entrepreneur’s wealth jumps up to

$$\hat{n}_t^{nf} = \left\{ \left[ \frac{(1 - \theta) q_t}{1 - \theta q_t} \right] \left[ 1 - (1 - \phi) x_t^{nf} \right] + (1 - \phi) x_t^{nf} \right\} n_t^{nf}. \quad (6)$$

The liquid part of wealth (fraction $\left[ 1 - (1 - \phi) x_t^{nf} \right]$) is multiplied by the investment return on internal liquidity (i.e., $ROI_t = \frac{(1 - \theta) q_t}{1 - \theta q_t}$). The value of illiquid wealth, $(1 - \phi) x_t^{nf} n_t^{nf}$, stays the same. By holding more inside money instead of capital (i.e., decreasing $x_t^{nf}$), an entrepreneur enjoys a larger jump in wealth when investment opportunities arrive. This benefit of holding inside money induces entrepreneurs’ liquidity preference.

From bankers’ and entrepreneurs’ pre-shock wealth evolution, we can derive the dynamics of $\eta_t$. Before the macro shock hits, $\eta_t$ follows a deterministic path with the following rate of change:

$$\mu_t^\eta = \frac{d\eta_t}{dt} = \frac{n_t^{nf}}{\eta_t} \left( \frac{dn_t^f / dt}{n_t^f} - \frac{dn_t^{nf} / dt}{n_t^{nf}} \right) \quad (7)$$

$$= (1 - \eta_t) \left\{ \left( \frac{a}{q_t} + \mu_t^q - r_t \right) \left( x^f_t - x_t^{nf} \right) - \lambda_t \left\{ ROI_t \left[ 1 - (1 - \phi) x_t^{nf} \right] + (1 - \phi) x_t^{nf} - 1 \right\} \right\}.$$
where the last line represents the wealth jump of the $\lambda dt$ measure of investing entrepreneurs.

As will be confirmed by the model solution, the Markov equilibrium has a steady state $\eta^*$ at which $\mu^\eta_{n t} = 0$. In equilibrium, bankers hold more capital ($x^f_t > x^nf_t$), and thus, earn larger returns from capital holdings, but entrepreneurs’ wealth grows by creating new capital. When the two forces balance each other at $\mu^\eta_{t} = 0$, the economy reaches its steady state and stays there until the macro shock hits. The model solution will characterize the full dynamics instead of perturbations around the steady state.

When the shock hits, both sectors lose wealth. Denote the post-shock net worth as $\tilde{n}_i^t$ ($i \in \{f, nf\}$). Let “$\tilde{\text{–}}$” denote the post-shock value. Bankers’ wealth changes from $n^f_t$ to $\tilde{n}^f_t$:

$$\tilde{n}^f_t = \left[ \frac{\tilde{q}_t}{q_t} \left( 1 - \delta \right) x^f_t + \left( 1 - x^f_t \right) \right] n^f_t.$$

A fraction $1 - \delta$ of risky capital holdings remain and get re-evaluated at the post-shock price $\tilde{q}_t$. Entrepreneurs’ wealth changes from $n^{nf}_t$ to $\tilde{n}^{nf}_t$:

$$\tilde{n}^{nf}_t = \left[ \frac{\tilde{q}_t}{q_t} \left( 1 - \delta \right) x^{nf}_{t,k} + \left( 1 - x^{nf}_t \right) \right] n^{nf}_t.$$

Therefore, when the macro shock hits, $\eta_t$ jumps to $\tilde{\eta}_t$:

$$\tilde{\eta}_t = \frac{\tilde{n}^f_t}{\tilde{n}^f_t + \tilde{n}^{nf}_t} = \frac{\left[ \frac{\tilde{q}_t}{q_t} \left( 1 - \delta \right) x^f_t + \left( 1 - x^f_t \right) \right] \eta_t}{\left[ \frac{\tilde{q}_t}{q_t} \left( 1 - \delta \right) x^f_t + \left( 1 - x^f_t \right) \right] \eta_t + \left[ \frac{\tilde{q}_t}{q_t} \left( 1 - \delta \right) x^{nf}_t + \left( 1 - x^{nf}_t \right) \right] (1 - \eta_t)} \ (8)$$

Since all endogenous variables are functions of $\eta_t$, when $\eta_t$ jumps to the new value $\tilde{\eta}_t$, all the other variables also jump. For example, the capital price jumps from $q(\eta_t)$ to $q(\tilde{\eta}_t)$.

**Proposition 2** The Markov equilibrium has a single state variable $\eta_t$, whose dynamics are summarized in Equations (7), the pre-shock growth rate, and (8), the jump.

**Banker optimization.** Next, I will solve agents’ optimal decisions. Let $V^f(n^f_t, \eta_t)$ denote a banker’s value function, which depends on the individual’s wealth and the aggregate state.
The Hamilton–Jacobi–Bellman equation (HJB) is:

$$\rho V^f (n_t^f, \eta_t) = \max_{c_t^f \geq 0, x_t^f \geq 0} \ln (c_t^f) + \frac{\partial V^f}{\partial n_t^f} \mu_t^f n_t^f + \frac{\partial V^f}{\partial \eta_t} \mu_t^\eta \eta_t + \lambda \left[V^f (\tilde{n}_t^f, \tilde{\eta}_t) - V^f (n_t^f, \eta_t)\right].$$

The last term reflects the jump in value function when wealth jumps from $n_t^f$ to $\tilde{n}_t^f$ and $\eta_t$ jumps to $\tilde{\eta}_t$ after the macro shock. Log utility implies the functional form of $V^f (n_t^f, \eta_t)$ is

$$V^f (n_t^f, \eta_t) = \frac{1}{\rho} \ln (n_t^f) + U^f (\eta_t).$$

After substituting this function into the HJB equation, we have the following proposition.

**Proposition 3** Bankers’ optimal consumption ($c_t^f$) and leverage (i.e., capital-to-wealth ratio $x_t^f$) satisfy the following first-order conditions (F.O.C.) respectively:

$$c_t^f = \rho n_t^f,$$

and

$$\frac{a}{q_t} + \mu_t^q - r_t \leq \lambda \left(\frac{1 - \frac{\tilde{q}_t}{q_t} (1 - \delta)}{1 - \left[1 - \frac{\tilde{q}_t}{q_t} (1 - \delta)\right] x_t^f}\right),$$

which hold in equality if $x_t^f > 0$.

In equilibrium, the F.O.C. condition for $x_t^f$ holds in equality, because bankers always hold some risky capital (i.e., $x_t^f > 0$). Risk is measured by

$$1 - \frac{\tilde{q}_t}{q_t} (1 - \delta),$$

the total loss per dollar of risky capital holdings (i.e., the difference before pre-shock value, 1, and the post-shock value, $\frac{\tilde{q}_t}{q_t} (1 - \delta)$). Risk has two components, the exogenous risk $\delta$ and the endogenous risk $\tilde{q}_t/q_t$ from capital reevaluation. Increasing one dollar investment in capital adds a potential loss of wealth equal to $1 - \frac{\tilde{q}_t}{q_t} (1 - \delta)$. Due to log utility, the marginal value of wealth is the reciprocal of wealth level, so the right-hand side of Equation (9) is the loss...
multiplied by the marginal value of wealth (wealth level in the numerator and denominator cancel each other out). The left-hand side is the marginal gain (pre-shock return), i.e., the difference between return on capital and the cost of marginal financing $r_t$. $x_t^f$ is bankers’ leverage. Holding the left-hand side of Equation (9)), $x_t^f$ decreases in risk.

**Entrepreneur optimization.** Let $V^{nf}(n_t^{nf}, \eta_t)$ denote an entrepreneur’s value function. The HJB equation is:

$$
\rho V^{nf}(n_t^{nf}, \eta_t) = \max_{c_t^{nf} \geq 0, x_t^{nf} \geq 0} \ln(c_t^{nf}) + \frac{\partial V^{nf}}{\partial n_t^{nf}} \mu_t^{nf} n_t^{nf} + \frac{\partial V^{nf}}{\partial \eta_t} \mu_t^\eta \eta_t \\
+ \lambda \left[ V^{nf}(\hat{n}_t^{nf}, \eta_t) - V^{nf}(n_t^{nf}, \eta_t) \right] \\
+ \lambda_t \left[ V^{nf}(\hat{n}_t^{nf}, \eta_t) - V^{nf}(\hat{n}_t^{nf}, \eta_t) \right].
$$

The second line reflects the wealth jump due to macro shock. The third line reflects the jump in value function when net worth jumps from $n_t^{nf}$ to $\hat{n}_t^{nf}$ due to investment.

The log utility implies the functional form of $V^{nf}(n_t^{nf}, \eta_t)$ is

$$
V^{nf}(n_t^{nf}, \eta_t) = \frac{1}{\rho} \ln(n_t^{nf}) + U^{nf}(\eta_t).
$$

Substituting this conjecture into the HJB equation, we have the following proposition.

**Proposition 4** Entrepreneurs’ optimal consumption ($c_t^{nf}$) and capital-to-wealth ratio ($x_t^{nf}$) satisfy the following first-order conditions (F.O.C.) respectively:

$$
c_t^{nf} = \rho n_t^{nf},
$$

and

$$
\frac{a}{q_t} + \mu_t^q - r_t \leq \lambda \left\{ \frac{1 - \frac{q_t}{q_t} (1 - \delta)}{1 - \left[ 1 - \frac{q_t}{q_t} (1 - \delta) \right] x_t^{nf}} \right\} + \lambda_t \left\{ \frac{(ROI_t - 1) (1 - \phi)}{ROI_t \left[ 1 - (1 - \phi) x_t^{nf} \right] + (1 - \phi) x_t^{nf}} \right\},
$$

(10)
which holds in equality if $x_t^{nf} > 0$.

In comparison with bankers’ F.O.C. condition (Equation (9)), the right-hand side of Equation (10) has a similar risk compensation term and a new illiquidity compensation term. The second line of Equation (10) reflects the illiquidity of capital. By switching one dollar from money to capital, an entrepreneur loses $(1 - \phi)$ dollars of liquidity for investments, which has a net impact of $(ROI_t - 1)(1 - \phi)$ on the size of wealth jump (the numerator in the second line of Equation (10)). Due to log utility, the marginal value of wealth is the reciprocal of wealth level, so the last line of Equation (10) is the reduction in wealth jump multiplied by the marginal value of wealth. For entrepreneurs to hold capital, i.e., $x_t^{nf} > 0$ and the F.O.C. condition in equality, the excess return of capital holdings must be sufficient to compensate for both risk and illiquidity.

**Aggregation.** In aggregate, investing entrepreneurs converts $i_t \lambda_I dt$ units of goods into new capital, where individual entrepreneur’s investment, $i_t$, is given by Equation (5). Thus, the aggregate amount of goods invested is

$$
\left( \frac{1}{1 - \theta q_t} \right) \left[ 1 - (1 - \phi) x_t^{nf} \right] N_t^{nf} \lambda_I dt,
$$

where $N_t^{nf}$ is the aggregate wealth of entrepreneurs. Similarly, let $N_t^f$ denote the aggregate wealth of bankers. The aggregate consumption is given by $\rho N_t^f dt + \rho N_t^{nf} dt$. In equilibrium, the aggregate production, $aK_t dt$, is equal to the sum of consumption and investment:

$$
aK_t dt = \rho N_t^f dt + \rho N_t^{nf} dt + \left[ \frac{1 - (1 - \phi) x_t^{nf}}{1 - \theta q_t} \right] N_t^{nf} \lambda_I dt.
$$

Note that because debt is in zero net supply, the aggregate wealth of the economy is $q_t K_t$ (i.e., the total value of productive capital). Therefore, the aggregate consumption is $\rho q_t K_t dt$. Dividing both sides by $K_t dt$, we have the goods market clearing condition:

$$
a = \rho q_t + \left[ \frac{1 - (1 - \phi) x_t^{nf}}{1 - \theta q_t} \right] (1 - \eta_t) q_t \lambda_I.
$$

(11)
The capital market clearing conditions are:

\[ x_t^f N_t^f + x_t^{nf} N_t^{nf} = q_t K_t, \]

which we can simplify by dividing both sides by the aggregate wealth:

\[ x_t^f \eta_t + x_t^{nf} (1 - \eta_t) = 1. \]  

(12)

The debt market clears by Walras’ law. Given any initial capital stock \( K_0 \) and wealth distribution, the Markov equilibrium is formally defined as follows.

**Proposition 5** Given initial capital endowments of bankers and entrepreneurs, there is a rational expectation Markov equilibrium that is described by the stochastic processes of bankers’ consumption \((c_t^f)\) and portfolio choice \((x_t^f)\), entrepreneurs’ consumption \((c_t^{nf})\), portfolio choice \((x_t^{nf})\), and investment \((i_t)\), and price variables (capital price \(q_t\) and interest rate \(r_t\)) on the filtered probability space generated by the Poisson process of macro shock, such that

1. agents know and take as given the processes of price variables;
2. agents make optimal choices as stated in Proposition 1, 3, and 4;
3. price variables adjust to clear the markets;
4. all the choice and price variables are functions of \(\eta_t\), so Proposition 2 gives an autonomous law of motion that maps any path of macro shocks to the current state \(\eta_t\).

### 2.3 The mechanism and its empirical implications

The model has two mechanisms that amplify each other and lead to financial instability. Bankers add value to the economy in two ways. First, the macro shock destroys a smaller fraction of capital, when capital is held by bankers instead of entrepreneurs \((\delta < \delta)\). This emphasizes the asset side of bankers’ balance-sheet. Second, bankers are money creators. By issuing debt that serves as inside money, they provide the most liquid securities that entrepreneurs can hold against liquidity shocks. Inside money allows investing entrepreneurs to
purchase goods that are used to create new capital, and thus, directly affects economic growth. To what extend bankers can provide these two services depends on their wealth.

**Boom.** Consider a calm period without any macro shocks. As shown in the wealth evolution equation (Equation (4)), bankers accumulate wealth by earning the spread between return on capital and debt cost. As bankers get richer, they hold more capital, and as capital gets reallocated towards its natural buyers, the market price of capital increases. An upward reevaluation of capital increases bankers’ wealth even further, and the impact is amplified by leverage ($x_t^f > 1$). This is a typical feedback loop that arises in models of balance-sheet channel (e.g., Brunnermeier and Sannikov (2014)): in good times, the increasing asset price and natural buyers’ risk-taking capacity reinforce each other. However, the story does not end here.

The model features a unique inside money channel. When asset price increases, the return on internal liquidity increases ($dROI_t/dq_t > 0$), so entrepreneurs’ liquidity preference is stronger. Intuitively, when capital becomes more valuable, entrepreneurs want to create more capital, but to do so, they need to build up savings, ideally in the most liquid form – bank debt. Therefore, their portfolio is more biased towards bank debt. The increasing money demand pushes down interest rate $r_t$, which is banks’ debt cost, and pushes up the equilibrium quantity of bank debt, allowing bankers to expand their balance-sheet even faster than what is implied by the balance-sheet channel. As a result, bankers have ammunition to push up capital price even further. The inside money channel feeds back into the balance-sheet channel by further increasing bankers’ wealth.

The mutually reinforcing mechanisms are illustrated by Figure 1 in the introduction. During a calm period without macro shocks, the economy exhibits the following features:

1. *Entrepreneurs hold more cash in the form of bank debt;*

2. *Banks expand their asset holdings;*

4. *Interest rate declines.*

5. *Capital price increases;*

The model’s calm period reproduces the four features of U.S. economy (summarized in the introduction) in the two decades before the 2007-09 financial crisis. During that time,
asset prices rose across asset classes. The financial sector grows dramatically (Schularick and Taylor (2012); Greenwood and Scharfstein (2013)), funded by the issuance of short-term debt (Adrian and Shin (2010b); Gorton and Metrick (2012); Pozsar (2014); Carlson et al. (2016)).

There is a large empirical literature that documents the increases of cash holdings in the non-financial corporate sector (e.g., Opler et al. (1999); Bates, Kahle, and Stulz (2009)), and many have attributed the increase to firms in the new-economy industries that are R&D intensive (Falato and Sim (2014); Begenau and Palazzo (2015); Pinkowitz, Stulz, and Williamson (2015); Graham and Leary (2015)).

Another salient feature during this period is the secular decline of interest rate. A prominent explanation is the global imbalance (Caballero, Farhi, and Gourinchas (2008)). This paper turns the attention to domestic money demand, and in particular, the corporate liquidity hoarding for investment needs.\footnote{Investment need is a key determinant of the cross-sectional variation in corporate cash holdings (e.g., Denis and Sibilkov (2010); Duchin (2010)), especially for firms with less collateral (e.g., Almeida and Campello (2007); Li, Whited, and Wu (2016)) and more intensive R&D activities (Falato and Sim (2014))).}

The repeal of Regulation Q essentially introduces interest-paying money (Lucas and Nicolini (2015)). Therefore, when the money demand is strong, the yield on money (i.e., the interest rate) tends to decline. The low interest rate allows banks to borrow at a low cost, and thus, to take on more risks.

Many have argued that the development of shadow banking sector is driven by the strong demand for money-like securities.\footnote{As pointed out by Gorton (2010), Gorton and Metrick (2012), and Stein (2012), circumventing regulations on leverage to capture the money premium is one of driving forces behind the growth of shadow banking.}

The financial intermediaries’ privilege to create money is a double-edged sword: on the one hand, inside money facilitates transaction and resources reallocation; on the other hand, it fuels the financial sector’s risk-taking. When the macro shock hits, the feedback loops in Figure 1 turn into vicious cycles.

\textbf{Crisis.} Consider the response of the model economy to the macro shock. A fraction of capital holdings are destroyed, and bankers lose their wealth. As the natural buyers retreat from the market, capital price declines, which erodes bankers’ wealth even further. This balance-sheet channel triggers a downward feedback loop.

Moreover, as capital price decreases, entrepreneurs find investments less profitable (i.e., the investment return on internal liquidity, $ROI_t$, is lower), which decreases entrepreneurs’
incentive to hoard money. Their money demand contracts, leading to higher debt cost for the bankers and a smaller equilibrium quantity of bank debt, which precipitates the balance-sheet contraction in the banking sector and results in even lower capital price.

A key assumption is that bankers cannot raise equity from entrepreneurs. Otherwise, the effect of macro shock will be dissipated across the whole economy. In fact, if bankers can raise equity from entrepreneurs, they always hold all the capital.18 The model imposes restriction that the only financial contract between bankers and entrepreneurs is risk-free debt. Allowing banks to issue equity can improve the model’s quantitative performance and may introduce new mechanisms as in He and Krishnamurthy (2013), but as long as there are certain frictions on bankers’ equity issuance, the model’s qualitative implications carry through.

**Endogenous risk.** When the macro shock hits, the state variable $\eta_t$ will jump downward to $\tilde{\eta}_t$, because in equilibrium, bankers have a larger exposure to macro shocks (i.e., $x^f_t > x^{nf}_t$). And, the capital price decreases, jumping from $q_t$ to $\tilde{q}_t$. In contrast to the exogenous capital destruction, capital price change is an endogenous risk. The strength of the model’s shock amplification mechanism is measured by the following ratio,

$$\text{Shock amplification strength: } \frac{1 - \frac{\tilde{q}_t}{q_t} (1 - \delta)}{\delta}. \quad (13)$$

From bankers’ perspective, it is the ratio of total loss to exogenous loss per dollar of risky capital holdings. A large value indicates the shock amplification mechanisms are strong.

### 3 Solution

Based on the numeric solution (details in the appendix), this section shows the model’s properties graphically, and in particular, how the model generates the stylized facts in booms and how the feedback mechanisms lead to endogenous risk that materializes in crises. The model solution characterizes the full dynamics instead of perturbations around the steady state.

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18 A limit on equity issuance is a limit on risk-sharing. Di Tella (2014) show that perfect risk-sharing shuts down the balance-sheet channel.
3.1 Parameters

One unit of time corresponds to a month. I set $\lambda_t$ equal to 0.0125, which implies 5% arrival rate quarterly, following the calibration of Kiyotaki and Moore (2012). $\lambda$ is $\frac{1}{40}$, so the macro shock arrives every five years, which is roughly at the business cycle frequency. I set $\delta = 0.025$ and $\delta = 0.05$, which is broadly consistent with the estimates of default loss in Andrade and Kaplan (1998) and Chen (2010). $\rho$ is set to 0.0025 (annualized to 3%). $\alpha$ set to 0.0167, which implies a capital price-to-production ratio around 8 over the cycle. $\theta$, the external financing capacity of investment projects, is set to 10%.

$\phi$ is set to 0.9 in the baseline calibration. It is the key parameter in this model, as it measures the fraction of existing capital that can be pledged or readily sold when an investment opportunity arrives. Fixing other parameters, we can think of the economy as indexed by $\phi$. Decreasing $\phi$ corresponds to an unexpected and permanent increase in the intangibility of productive capital in this economy.

If $\phi = 1$, the inside money channel is shut down. An increase in the capital price and the profitability of investment does not increase entrepreneurs’ preference for bank debt, because bank debt and existing capital are equally liquid when investment opportunities arrive.

3.2 Equilibrium properties

Asset price. Panel A of Figure 2 shows the equilibrium capital price as a function of bankers’ wealth share $\eta_t$. Capital price increases in bankers’ wealth. Bankers assign a higher value to capital than entrepreneurs because they experience a smaller loss when the macro shock hits. As a result, bankers are willing to pay a higher price for capital. As their wealth grows relative to entrepreneurs (i.e., $\eta_t$ increases), bankers bid up capital price.

Because bankers have a levered position in capital (i.e., $x^f_t > 1$), the rising capital price has a larger impact on their wealth than on the entrepreneurs’ wealth, which feeds into a further increase of $\eta_t$, bankers’ wealth share. The rising capital price also stimulates entrepreneurs’ investment-driven money demand by making investment opportunities more profitable (i.e., a higher return on internal liquidity $ROI_t$). As entrepreneurs bias their wealth portfolio more towards bank debt, the equilibrium interest rate $r_t$ is pushed downward, feeding banks cheap
leverage, and thereby, enhancing their purchasing power in the market of capital. This money demand channel further increases capital price.

**State variable dynamics.** The pre-shock growth rate of state variable $\eta_t$ is shown in Panel B of Figure 2. A steady state exists around $\eta_t = 0.34$, which is marked by the dotted vertical lines in the four panels of Figure 2 and the upcoming figures. When the financial sector’s wealth share reaches 34%, the drift of state variable is equal to zero, so the economy will stay there until hit by a macro shock. This steady state is stable, because to the left, the growth rate of $\eta_t$ is positive (i.e., $\mu_\eta > 0$), and to the right, $\mu_\eta < 0$. The growth rate is large when $\eta_t$ is small because bankers earn a higher return on capital holdings, and the return on capital is
high because the capital price is low when $\eta_t$ is low.

According to simulation results, if the economy starts at $\eta_t = 5\%$, it takes an average of 125 years to reach the steady state $\eta = 34\%$. During the process towards the steady state (if not interrupted by the macro shock), we observe the expansion of the financial sector (i.e., $\eta_t$ increases), increasing asset price $q_t$, and an increasing share of capital being held by the financial sector ($x^f_k\eta_t$). These predictions are consistent with the dynamics of the U.S. and other advanced economies before the financial crisis.

**Capital allocation.** Panel C of Figure 2 shows the fraction of capital held by bankers. When the banking sector is small, entrepreneurs have to hold capital, which happens when $\eta_t < 0.2$. Entrepreneurs’ first-order condition for $x^{nf}_t$ holds in equality (Equation (10)). The expected return on capital must compensate for both exposure to macro shocks and illiquidity at investment times. Because of $\hat{\delta} < \tilde{\delta}$, it is inefficient for the entrepreneurs to hold existing capital, but they have to do so to clear market when the capacity of banks is limited.

Close to the steady state, all the existing capital is held by bankers, so entrepreneurs focus on creating new capital and invest all of its wealth in the inside money issued by bankers. In other words, when $\eta_t$ is high, the economy achieves an efficient *specialization*: bankers hold existing capital, and entrepreneurs hold money and create new capital (Panel D of Figure 2). The subsequent figures mark the inefficiency point of $\eta_t$, below which entrepreneurs have to hold capital to clear the market.

**Endogenous risk and risk pricing.** Panel A of Figure 3 shows the pre-shock excess return of capital

$$\frac{\alpha}{q_t} + \mu^q_t - r_t.$$  

Panel B shows bankers’ total loss faced per dollar of risky capital holdings when the macro shock hits

$$1 - \frac{\tilde{q}}{q} (1 - \delta).$$

We use bankers’ total loss as a measure of risk, because bankers are always the marginal investor, while entrepreneurs exit the market when $\eta_t$ is large enough. As the banking sector grows and the economy moves towards the steady state, endogenous risk accumulates. The
bankers’ total loss per dollar increases from around 2.5%, that is close to the exogenous risk $\delta$, to around 9%. As a consequence, bankers require a higher pre-shock excess return as compensation, so we see in Panel A of Figure 3, the pre-shock excess return increases.

When $\eta_t$ is small and entrepreneurs hold risky asset, Equation (10) holds in equality, which decomposes the pre-shock excess return into two components: the macro risk compensation

$$\lambda \left( \frac{1 - \tilde{q}_t}{q_t} (1 - \delta) \right) \left( 1 - \left[ 1 - \frac{\tilde{q}_t}{q_t} (1 - \delta) \right] x_t \eta_t \right),$$

Figure 3: Endogenous Risk and Risk Pricing.
and the illiquidity compensation

$$\lambda_I \left( \frac{(ROI_t - 1)(1 - \phi)}{ROI_t \left( 1 - (1 - \phi) x_t^{nf} \right) + (1 - \phi) x_t^{nf}} \right). \tag{14}$$

Because entrepreneurs can only invest in two assets, risky and illiquid capital and safe debt, the illiquidity compensation also reflects the liquidity premium that assigned to the safe debt issued by bankers. The red dotted line in Panel A of Figure 3 shows the macro risk compensation required by entrepreneurs, so the gap between this dotted line and the pre-shock excess return is the illiquidity compensation. It grows as $\eta_t$ increases and capital price increases, because higher capital price means a higher return on internal liquidity, and thus, it is more costly to hold illiquid capital that cannot be fully sold for investment inputs when needed.

Note that bankers’ pre-shock excess return on capital is not the expected excess return, which is the pre-shock excess return adjusted by the expected loss:

$$\frac{\alpha}{q_t} + \mu_t - r_t + \lambda (1 - \delta) \left( \frac{\tilde{q}_t}{q_t} - 1 \right) + \lambda \delta (-1).$$

The expected excess return in Panel C of Figure 3 follows roughly the same pattern as the pre-shock excess return.

Panel D of Figure 3 shows the “price of risk”, which is the ratio of expected excess return divided by total loss per dollar of investment. It decreases as the bankers’ wealth share increases. Thus, the model economy generates counter-cyclical price of risk, which is consistent with the empirical asset pricing literature (e.g., Lettau and Ludvigson (2010)). The dynamics of risk price is dominated by the dynamics of quantity of risk (Panel B). As a result, the dynamics of expected excess return (Panel C) follows the quantity of risk more closely.

**Endogenous risk accumulation in booms.** Panel A of Figure 4 shows $\frac{\tilde{\eta}_t}{\eta_t}$, the ratio of post-shock state to pre-shock state. It shows the impact of macro shock on $\eta_t$. This ratio increases as $\eta_t$ increases. Therefore, as the banking sector grows, the economy becomes more stable in the sense that bankers’ wealth share is less sensitive to shocks. However, this does not necessarily imply to a smaller endogenous risk.
Panel B of Figure 4 shows $\frac{\tilde{q}_t}{q_t}$, the ratio of post-shock capital price to pre-shock asset price. When $\eta_t$ is relatively small, this ratio decreases as $\eta_t$ increases, meaning that endogenous risk accumulates as the banking sector grows. When the macro shock hits, the change of capital price depends on (1) the state variable jump from $\eta_t$ to $\tilde{\eta}_t$ and (2) the sensitivity of capital price to state variable, i.e., $q'(\eta_t)$. The impact of state variable jump is certainly mitigated when bankers are richer (i.e., higher $\eta_t$) as shown in Panel A. But the sensitivity of capital price to bankers’ wealth increases through the inside money channel. And when this effect dominates, the macro shock hits capital price harder when bankers are richer.

Bankers and entrepreneurs differ in two aspects, and both aspects make bankers the better capital holders when entrepreneurs. The first difference is exogenous and time-invariant: when macro shocks hit, bankers lose a smaller fraction of capital holdings.

The second difference widens as capital price increases: when $q_t$ is high, entrepreneurs’ investment return on internal liquidity is high, so it is very costly to hold capital that cannot be readily sold for investment inputs when needed. Therefore, as $\eta_t$ rises, stronger liquidity preference makes entrepreneurs the increasingly inferior capital owners, and relatively speaking, bankers the increasingly superior ones. As a result, capital price becomes more sensitive to $\eta_t$, the wealth distribution between two types of agents. This explains why when $\eta_t$ increases,
the negative impact of macro shock on capital price actually becomes larger.

Panel C of Figure 3 shows the strength of shock amplification in the model. The ratio of total loss (i.e., \(1 - \frac{\tilde{w}}{\tilde{q}}(1 - \tilde{\delta})\)) to exogenous loss (i.e., \(\tilde{\delta}\)) increases to above 3.5, and then decreases because eventually, the smaller shock impact on state variable \(\eta_t\) dominates the increasing sensitivity of capital price to \(\eta_t\). This amplification effect is quantitatively large in comparison with the state-of-art models in the macro finance literature (e.g., Brunnermeier and Sannikov (2014); He and Krishnamurthy (2014)).

The accumulation of endogenous risk in the model sheds light on the recent empirical findings that a long period of boom and banking expansion precedes severe crises (e.g., Jordà, Schularick, and Taylor (2013); Baron and Xiong (2016)). The existing theories attribute to the accumulation of endogenous risk to learning (Moreira and Savov (2014)) or the lack of information production (Gorton and Ordoñez (2014)). This paper emphasizes financial friction instead of informational distortions, and moreover, it offers a model that accounts for several phenomena in the pre-crisis period and links them to the structural transformation towards intangible-intensive economy. The inside money channel is built upon the illiquidity of intangible capital and the resulting demand for bank debt as money in the production sector.

**Inside money channel.** Panel A of Figure 5 shows an entrepreneur’s return on internal liquidity. As \(\eta_t\) grows, capital price grows, which leads to higher \(ROI_t\), and thus, a higher illiquidity compensation required by entrepreneurs to hold capital (Panel B). Note that after the inefficiency point, entrepreneurs no longer hold capital, so the plot in Panel B stops. This confirms the previous explanation of endogenous risk accumulation through the widening difference between the first-best (bankers) and second-best holders (entrepreneurs) of capital that is due to entrepreneurs’ increasing aversion to capital illiquidity.

As shown in Panel C of Figure 5, a stronger money demand of entrepreneurs pushes down the interest rate (i.e., the risk-free rate), which reduces the leverage cost for bankers. Panel D plots \((1 - \eta_t)(1 - x_t^{nf})\), i.e., the inside money to total wealth ratio. \((1 - \eta_t)\) is entrepreneurs’ share of total wealth. \((1 - x_t^{nf})\) is the fraction of their wealth invested in bank debt. When the banking sector is small, its money creation capacity is limited, because it does not have enough wealth to buffer the macro shock. The money supply increases as
the banking sector grows and its leverage cost decreases due to a rising money demand from entrepreneurs’ investment needs.

Before the inefficiency point, the asset side of their balance-sheet can grow through both the upward reevaluation of capital and reallocation of capital from entrepreneurs to bankers. After the inefficiency point, bankers already hold all the capital, so the asset side of their balance-sheet can only grow through the upward reevaluation of capital, a further increase of bank equity crowds out bank debt. In other words, bank equity grows faster than asset.

Panel C and D of Figure 5 show that through the rising demand for money in the production sector, the interest rate declines, and at the same time, the indebtedness of the banking sector increases. As already shown in Figure 4, endogenous risk accumulates along with the
increasing capital price, setting the stage of severe crises when the macro shock hits.

**Simulation.** Figure 6 shows a simulated path of the economy for 30 years (360 months). The economy starts from $\eta_t = 5\%$. After being hit by the macro shock at the 40th month, the economy experiences a prolonged period of boom. Panel A shows the trajectory of the state variable $\eta_t$. The banking sector’s wealth share grows to more than 20%. Bankers increase risk-taking, holding a larger share of capital (Panel C). By issuing more inside money, bankers provide liquidity to entrepreneurs (Panel D). Along the process, the economy accumulates endogenous risk (Panel B). The boom is followed by three recessions. Each recession lasts for three to four years, during which the banking sector cannot hold all the capital, so
entrepreneurs have to hold capital and the economy deviates from the efficient specialization.

**Intangibility and instability.** The strength of inside money channel depends on $\phi$, capital intangibility. It is shut down when $\phi = 1$, because bank debt and capital are equally liquid. When $\phi < 1$, bank debt is more liquid than capital, and thus, entrepreneurs bias their wealth allocation towards bank debt, which triggers the inside money channel. A lower value of $\phi$ means capital is more illiquid when investments are needed. A decrease in $\phi$ captures a structural transformation towards a more intangible economy.\(^\text{19}\)

Capital intangibility heightens endogenous risk. Figure 7 compares the strength of shock amplification for different values of $\phi$ (0.7, 0.9, and 1). Moving from $\phi = 1$ to $\phi = 0.9$ and 0.7, the inside money channel is strengthened (Panel A). In the case of $\phi = 1$, the only shock amplification mechanism is the typical balance-sheet channel. Therefore, comparing it with the cases of $\phi < 1$ reveals the contribution of inside money channel to shock amplification that is beyond the typical balance-sheet channel.

When the economy becomes more intangible (lower $\phi$), bankers cater to entrepreneurs’ money demand by issuing more safe debt. Panel B of Figure 7 shows the bank debt to total

\(^{19}\)Investing entrepreneurs can either sell existing capital for investment inputs, or equivalently, pledging existing capital to borrow investments inputs. The latter form of capital liquidity emphasizes collateralizability, which is more closely related to the capital tangibility.
wealth ratio. As the economy becomes more intangible-intensive, the capital illiquidity problem becomes more severe. This drives up the production sector’s demand for inside money in anticipation of investment needs, and thereby, pushes down the leverage cost for banks and leads to a more indebted banking sector. Thus, a transition towards intangible-intensive economy is accompanied by rising bank money-to-wealth (and GDP) ratio, which has been documented in advanced economies (e.g., Schularick and Taylor (2012)).

4 Conclusion

This paper aims to provide a coherent account of several trends in the two decades leading up to the Great Recession, such as the rising corporate cash holdings, the expansion of financial sector, the declining interest rate, and the rising prices of risky assets. At the center is the dynamic interaction between inside money suppliers (banks) and demanders (entrepreneurs).

The money demand of firms arises from investment and capital illiquidity, which are in turn motivated by the increasing reliance on intangible capital in advanced economies. The financial stability implications of this structural change has not yet been explored in the existing literature. This paper proposes an inside money channel that links rising asset price with the accumulation of endogenous risk in booms. It sheds light on the recent empirical findings that a long period of banking expansion precedes severe crises. This new channel of instability and the typical balance-sheet channel reinforce each other, creating a quantitatively significant amplification mechanism for macro shocks.

The model leaves out the provision of outside money (liquid government securities or central bank liabilities) for future research. By expanding money supply in booms, the government can crowd out bank leverage (Krishnamurthy and Vissing-Jorgensen (2015)), and thereby, weaken the upward spiral in asset price that leads to endogenous risk accumulation. In contrast to the current practice of monetary expansion in recessions, the optimal supply of outside money can be procyclical in this setting. However, since entrepreneurs’ incentive to invest is tied to asset price, a government faces the trade-off between growth and stability.

The model also leaves out banks’ default. The empirical literature on financial crises...
commonly use banks’ default or a high possibility of default as a crisis indicator. A theoretical model of crisis should ideally accommodate default, and by doing so, it opens up the question of optimal government intervention, for instance, through equity injection into the banking sector, in order to prevent a sudden evaporation of inside money. To finance the intervention in bad times, the government may increase the issuance of outside money (e.g., short-term government securities), suggesting countercyclical supply of outside money.
Appendix - Solving the Equilibrium

The fully solved Markov equilibrium is a set of functions that map $\eta_t$ to the values of endogenous variables, such as capital price, interest rate, and bank leverage. In the description of solution method, time subscripts are suppressed to save notations.

First, we need to solve the capital price $q(\eta)$ as the fixed point of a contraction mapping in $C^1$ space. Once we know $q(\eta)$, the other endogenous variables can be solved easily. To solve the capital price, we need to construct a contraction functional from individual optimality and market clearing conditions. We may pick a starting candidate capital price function $q^1(\eta) \in C^1$, and give it an index equal to “1” (in the superscript). The contraction functional maps $q^1(\eta)$ to $q^2(\eta)$ in $C^1$. Then, we apply the functional to $q^2(\eta)$ to get $q^3(\eta)$, and repeat until convergence. The fixed point $q^\infty(\eta)$ is the equilibrium capital price function $q(\eta)$ that satisfies all the equilibrium conditions.

The contraction mapping can be constructed in five steps. First, from the goods market clearing condition,

$$a - \rho q^1 = \lambda_f \left( \frac{1 - (1 - \phi) x^{nf,1}}{1 - \theta q^1} \right) (1 - \eta) q^1,$$

we can solve the implied $x^{nf,1} = x^{nf,1}(\eta)$, i.e. the entrepreneurs’ capital-to-wealth ratio as a function of $\eta$.\(^{20}\) Second, using the capital market clearing condition,

$$x^{f,1} \eta + x^{nf,1} (1 - \eta) = 1,$$

we can solve the implied $x^{f,1} = x^{f,1}(\eta)$, i.e. the bankers’ capital-to-wealth ratio as a function of $\eta$. Third, we can solve $\eta^1 = \eta^1(\eta)$, which is the state to which the economy jumps from $\eta$ when the macro shock hits, using the equation

$$\eta^1 = \frac{\eta^f}{\eta^f + \eta^{nf}} = \frac{\left[ \frac{\eta^f}{q^1} (1 - \delta) x^{f,1} + (1 - x^{f,1}) \right] \eta}{\left[ \frac{\eta^f}{q^1} (1 - \delta) x^{f,1} + (1 - x^{f,1}) \right] \eta + \left[ \frac{\eta^{nf}}{q^1} (1 - \delta) x^{nf,1} + (1 - x^{nf,1}) \right] (1 - \eta)},$$

\(^{20}\)The starting candidate $q^1(\eta)$ should be chosen such that the implied $x^{nf,1}(\eta) \geq 0.$

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where \( \tilde{q}^1 = q^1(\tilde{\eta}^1) \). Since the function \( q^1(\eta) \) is known, the equation above solves \( \tilde{\eta}^1 \) as a function of \( \eta \). Fourth, we can solve the pre-shock excess return of capital using bankers’ F.O.C. condition for \( x^f \) (which always holds in equality),

\[
\lambda \left( \frac{1 - \frac{\tilde{q}^1}{q^1} (1 - \delta)}{1 - \left[1 - \frac{\tilde{q}^1}{q^1} (1 - \delta)\right] x^f,1} \right).
\]

The final step is to check entrepreneurs’ F.O.C. condition for \( x^{nf} \) at every value of \( \eta \) (i.e. the inequality (10)). We have already solved the pre-shock excess return of capital, which is the left-hand side of the inequality (10). The right-hand side can be solved by substituting in \( q^1, \tilde{q}^1 \) and \( x^{nf,1} \),

\[
\lambda \left( \frac{1 - \frac{\tilde{q}^2}{q^2} (1 - \delta)}{1 - \left[1 - \frac{\tilde{q}^2}{q^2} (1 - \delta)\right] x^{nf,2}} \right) + \lambda_I \left( \frac{(ROI^1 - 1) (1 - \phi)}{ROI^1 [1 - (1 - \phi) x^{nf,1}] + (1 - \phi) x^{nf,1}} \right),
\]

where \( ROI^1 = \frac{(1 - \theta)q^1}{1 - \theta q^1} \), the return on internal liquidity given \( q^1 \). If the inequality (10) is not violated, the value of \( q^2 \) at \( \eta \) is set to equal \( q^1 \); if it is violated, we set \( q^2 \) at \( \eta \) according to the following procedure.

To construct \( q^2(\eta) \), consider the following scenarios for every value of \( \eta \). First, the left-hand side of the inequality (10) is larger than the right-hand side. In this case, entrepreneurs prefer to hold more capital at the current price \( q^1(\eta) \). By setting entrepreneurs’ F.O.C. to equality, we solve a new (and higher) \( x^{nf,2} \), and then by substituting \( x^{nf,2} \) into the goods market clearing condition, we solve the implied \( q^2(\eta) \). Second, the left-hand side of the inequality (10) is smaller than the right-hand side. If \( x^{nf,1} = 0 \), we do not need to update \( q^1 \) because the F.O.C. is not violated. If \( x^{nf,1} > 0 \), as before, we calculated an updated \( x^{nf,2} \) by setting entrepreneurs’ F.O.C. to equality, and then calculate \( q^2(\eta) \) by substituting \( x^{nf,2} \) into the goods market clearing condition. Finally, if the left-hand side of the inequality (10) is equal

\[21\text{Effectively, this means to adjust } q^1 \text{ upward (i.e., making capital less attractive for entrepreneurs to hold). The root in } (1, \frac{1}{\theta}) \text{ is selected. When } q < 1, \text{ aggregate investment is zero, so goods market clearing implies } q = \frac{\rho}{\rho}, \text{ which is larger than one under the calibrated parameter values, a contradiction. Therefore, } q \geq 1. \text{ } q \text{ has to be small than } \frac{1}{\theta}. \text{ Otherwise, the investment project becomes self-financing, in which case the scale of investment is positive infinite.}\]
to the right-hand side, we do not need to update \( q^1 \) because the F.O.C. is not violated.

We have constructed a functional that maps \( q^1 (\eta) \) to \( q^2 (\eta) \). I The proof of this mapping being a contraction mapping in \( \mathbb{C}^1 \) is beyond the scope of the paper, but under the calibrated parameter values, the recursive algorithm converges quickly, starting from \( q^1 (\eta) \) that is given by the goods market clearing condition with \( x^{nf} = 0 \) for all \( \eta \) in \((0, 1)\).

After \( q (\eta) \) is solved, we can solve \( x^{nf}, x^f, \tilde{q}, \hat{q} \), and the pre-shock excess return of capital as previously discussed. Since \( q (\eta) \) is known, we can solve \( \mu^q (\eta) \) using Itô’s lemma as follows,

\[
\mu^q (\eta) = \frac{dq (\eta) / q (\eta)}{d\eta / \eta} \mu^\eta (\eta),
\]

where \( \mu^\eta (\eta) \) is given by Equation (7). The interest rate \( r (\eta) \) is solved as the difference between the pre-shock return on capital (i.e. \( \frac{\alpha}{q (\eta)} + \mu^q (\eta) \)) and pre-shock excess return.
References


